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COUNTERVAILING INCENTIVES IN ADVERSE SELECTION MODELS: A SYNTHESIS

by

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Countervailing Incentives in Adverse Selection Models: A Synthesis

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ABSTRACT

In this paper we propose a simple method of characterizing countervailing incentives in adverse selection problems. The key element in our characterization consists of analyzing properties of the full information problem. This allows solving the principal problem without using optimal control theory. Our methodology can also be applied to different economic settings: health economics, monopoly regulation, labour contracts, limited liabilities and environmental regulation.

Key words: adverse selection, countervailing incentives.
JEL: D82, L50.

I. **Introduction**

In this paper, we propose a simple method of characterizing countervailing incentives in adverse selection problems. The key element in our proposal is to analyze properties of the full information problem.

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Most of the existing principal-agent models under adverse selection deal with settings where the agent (he) has a systematic incentive to always overstate or to always understate his private information. The results are well known in the literature: the principal (she) deviates from the efficient contract (either below or above the efficient levels for all types of agent) in order to reduce informational rents. This incentive to exaggerate private information may, in certain circumstances, be tempered by a countervailing incentive to understate private information. That is, the agent might be tempted either to overstate or to understate his private information, depending upon his specific realization. When countervailing incentives arise, performance is distorted both above and below efficient levels and the agent's informational rents generally increase with the realization of his private information over some ranges, and decrease over others.

The way countervailing incentives affect some specific agency problems has been analyzed by several authors including Lewis and Sappington (1989), Maggi and Rodriguez-Clare (1995a) and Jullien (2000). However, we are not aware of general results in the literature characterizing the presence of countervailing incentives in a general framework. The main aim of this paper is to complete the analysis of sufficient and necessary conditions for the existence of countervailing incentives under adverse selection. We illustrate this new characterization of the principal-agent problem with countervailing incentives with some examples coming from the literature on monopoly regulation, partially altruistic agents in health economics, labour contracts, limited liability and environmental regulation.

The paper is organized as follows. Section II presents the general model. In Section III, we characterize the full information case. In Section IV we analyze the general contract under asymmetric information and state the main result of the paper. In Theorem 1 we identify the exact conditions under which general incentive problems are characterized by the existence of countervailing incentives. We also state a general and very simple method to obtain the optimal contract under asymmetric information. Then we illustrate how many economic problems analyzed in literature may be seen as particular cases of our general benchmark. Finally, Section V presents some concluding remarks.

II. **The model**

We consider that the relationship between the principal and the agent involves an action variable, denoted as *l*, which is observable to both, and a monetary transfer, denoted as *t*, from the principal to the agent. Moreover, there is a one-dimensional parameter, denoted as θ, which is known to the agent but unobservable to the principal. The principal's uncertainty about the parameter θ is represented by a probability distribution $F(\theta)$ with associated density function $f(\theta)$ strictly positive on the support $[\underline{\theta}, \bar{\theta}]$. This function is assumed to be common knowledge.

The agent's welfare is represented by a utility function *U* depending upon the action variable *l*, the transfer *t*, and the unknown parameter θ; that is $U(l, t, \theta)$. In particular, we assume that the agent's utility depends linearly on transfers:

$$U(l, t, \theta) = u(l, \theta) + t. \tag{1}$$

We consider a principal's welfare function that incorporates a linear cost of transfers:[1]

$$W(l, t, \theta) = w(l, \theta) - \mu t. \tag{2}$$

For example, if the principal is a regulatory agency which takes into account distributive considerations and public funds are costly then the principal's function is:

$$W(l, t, \theta) = CS(l) + \alpha U(l, t, \theta) - (1 + \lambda)t = CS(l) + \alpha u(l, \theta) + \alpha t - (1 + \lambda)t.$$

So in that case $\mu = 1 + \lambda - \alpha$ (Laffont and Tirole, 1990a, 1990b, consider $\alpha = 1$ and $\mu = \lambda$) and $w(l, \theta) = CS(l, \theta) + \alpha u(l, \theta)$. If the principal does not take into account the agent's utility and public funds are not costly then $\mu = 1$. Finally, we assume that the principal is endowed with the power to set both *l* and *t*.

[1] The parameter μ is the shadow cost of public funds. Transfers between a firm and either consumers or the state may involve administrative costs, tax distortions or inefficiencies that must be taken into account in the design of the regulatory mechanism. See, for example, Laffont and Tirole (1986), (1993) and Caillaud et al. (1988).

III. **The full information case: a benchmark**

Consider the benchmark case in which the regulator knows the parameter θ. The problem of the principal under full information is then given by:

$$\max_{l,t} W(l,t,\theta)$$
$$\textit{Subject to}\ \ U(l,t,\theta) \geq 0.$$

Solving condition (1) for *t* and substituting *t* in condition (2), the problem is equivalent to:

$$\max_{l,U} W(l,U,\theta)$$
$$\textit{Subject to}\ \ U \geq 0.$$

That is,

$$\max_{l,U} w(l,\theta) + \mu u(l,\theta) - \mu U$$
$$\textit{Subject to}\ \ U \geq 0. \tag{3}$$

First order conditions are given by:

$$W_l(l^*,U^*) = w_l(l^*,\theta) + \mu u_l(l^*,\theta) = 0, \tag{4}$$
$$W_U(l^*,U^*) = -\mu \tag{5}$$

The full information policy consists of $l^*(\theta)$ defined by (4) and payment transfers such that firms obtain zero profits, $t^*(\theta) = u(l^*(\theta)\,,\theta)$. Note that,

$$\frac{dl^*(\theta)}{d\theta} = -\frac{W_{l\theta}}{W_{ll}},$$

where $W_{l\theta}(l^*,U^*) = w_{l\theta}(l^*,\theta) + \mu u_{l\theta}(l^*,\theta)$. As a consequence, the sign of $\frac{dl^*(\theta)}{d\theta}$ is the same as the sign of $W_{l\theta}$.

IV. **Characterization of optimal contracts under private information**

We now analyze the optimal policy when the agent has private information concerning the parameter θ. The parameter θ is continuously distributed on the support $\Theta = [\underline{\theta}, \overline{\theta}]$ according to the cumulative distribution function $F(\theta)$ and strictly positive density $f(\theta)$. We assume that $F(\theta)$ satisfies the monotone hazard rate condition; that is, the ratios $\frac{f(\theta)}{1-F(\theta)}$ and $\frac{F(\theta)}{f(\theta)}$ are non-decreasing functions of θ.[2]

The single-crossing property, which states that the greater the parameter θ, the more systematically willing an agent is to forego transfer payments to obtain a higher value for *l*, holds if the firm's marginal rate of substitution (MRS) of the action variable for transfer payment grows with θ.[3] Given the agent's utility defined by (3), the marginal rate of substitution is $MRS_{lt} = -\frac{U_l}{U_t} = -u_l$. Without loss of generality we assume $\frac{\partial |MRS_{lt}|}{\partial \theta} = u_{\theta l} > 0$.

To characterize the optimal regulatory policy under private information we first determine the class of feasible policies and then select the optimal policy from that class.[4] At the first stage, we restrict the analysis to direct revelation mechanisms by the revelation principle.[5] A direct revelation mechanism is composed of transfer functions and associated price levels given by $\{l(\theta), t(\theta)\}_{\theta \in \Theta}$. Therefore, we may be restricted to regulatory policies which require the firm to report its private information parameter truthfully, that is, incentive compatible policies, to determine the class of feasible policies. The principal maximizes the expected social welfare subject to the following incentive compatibility and individual rationality constraints:

Incentive compatibility constraints (*IC*): the agent reports θ truthfully if the utility it expects to obtain by announcing his type is at least as great as the expected utility from any other report. That is,

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[2] These properties require the density function not to increase too rapidly. They are satisfied by frequently used distribution functions (for example, Uniform, Normal and Exponential).

[3] Araujo and Moreira (2010) study a class of adverse selection problems where the agent's utility function does not satisfy the Spence-Mirrlees Condition or, also named, the single-crossing property.

[4] We adopt the approach of Baron and Myerson (1982) and Guesnerie and Laffont (1984).

[5] The revelation principle was established by Myerson (1979) and Dasgupta, Hammond and Maskin (1979).

(*IC*) $U(\theta) \geq U(\hat{\theta},\theta)\ \forall(\hat{\theta},\theta) \in \Theta^2$, where $U(\hat{\theta},\theta) = u(l(\hat{\theta}),\theta) + t(\hat{\theta})$ and $U(\theta) = U(\theta,\theta)$.

Individual rationality constraints (*IR*): the principal cannot force the agent to participate if it expects negative profits. That is,

(*IR*) $U(\theta) \geq 0\ \ \forall\theta \in \Theta$.

The regulator's problem can be written as:

$$\max_{l(\theta),t(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} W(l(\theta),t(\theta),\theta)d\theta \tag{6}$$

Subject to (*IR*) and (*IC*).

The following lemma characterizes the class of policies that satisfies (*IC*).

Lemma 1. Necessary and sufficient conditions for (*IC*) are: [6]

(i) $\frac{dU(\theta)}{d\theta} = u_\theta(l(\theta),\theta)$.

(ii) $u_{\theta l}(l(\theta),\theta)\frac{dl(\theta)}{d\theta} \geq 0$.

We assume that the agent is responsive (see Cailleau, Guesnerie et all, 1988). This implies that $l^*(\theta)$ is a non-decreasing function of θ and therefore can be implemented under private information through a transfer $t(l^*(\theta))$ such that $\frac{dU(\theta)}{d\theta} = u_\theta(l^*(\theta),\theta)$.

When $u_\theta(l^*(\theta),\theta) = 0\ \forall\theta \in [\underline{\theta},\overline{\theta}]$, the optimal allocation under complete information is implementable through the transfer $t(l^*(\theta)) = t^*(\theta)$ and therefore it would be optimal under private information (see Lewis & Sappington, 1988a.). When $u_\theta(l^*(\theta),\theta) \neq 0$ for some

[6] The proof of Lemma 1 is standard. See, for example, Baron and Myerson (1982) and Guesnerie and Laffont (1984).

$\theta \in [\underline{\theta}, \overline{\theta}]$ and transfers are costly, the optimal allocation under private information is, however, different from $l^*(\theta)$ because the expected value of the associated transfer $t(l^*(\theta))$ is too high. The principal faces a trade off between the cost of informational rents and the welfare loss generated by the departure from the complete information allocation. In order to solve this trade off, the principal would distort $l^*(\theta)$. The sign of this distortion is related to the sign of $u_\theta(l^*(\theta), \theta)$. Let $l^{PI}(\theta)$ be the optimal allocation under private information and let $\tilde{l}(\theta)$ the allocation such that $u_\theta\big(\tilde{l}(\theta)\,, \theta\big) = 0\ \forall\theta \in [\underline{\theta}, \overline{\theta}]$.

IV.1. *Necessary and sufficient conditions for countervailing incentives*

The next lemma states the direction of the informational rents.

Lemma 2. *The direction of the informational rents.*

(i) When $u_\theta(l^*(\theta), \theta) \geq 0\ \forall\theta \in [\underline{\theta}, \overline{\theta}]$, the optimal allocation under private information must be such that $u_\theta(l^*(\theta), \theta) \geq u_\theta(l^{PI}(\theta), \theta) \geq 0\ \forall\theta \in [\underline{\theta}, \overline{\theta}]$, and therefore $\tilde{l}(\theta) \leq l^{PI}(\theta) \leq l^*(\theta)$. Therefore, for any $\theta \in [\underline{\theta}, \overline{\theta}]$, there is an incentive to understate the true value of the private information parameter, θ. As a consequence, agents with higher types receive higher informational rents.

(ii) When $u_\theta(l^*(\theta), \theta) \leq 0\ \forall\theta \in [\underline{\theta}, \overline{\theta}]$, the optimal allocation under private information must be such that $u_\theta(l^*(\theta), \theta) \leq u_\theta(l^{PI}(\theta), \theta) \leq 0\ \forall\theta \in [\underline{\theta}, \overline{\theta}]$, and therefore $l^*(\theta) \leq l^{PI}(\theta) \leq \tilde{l}(\theta)$. Therefore, for any $\theta \in [\underline{\theta}, \overline{\theta}]$, there is an incentive to overstate the true value of the private information parameter, θ. As a consequence, agents with lower types receive higher informational rents.

We consider the following definition of countervailing incentives.

Definition 1. *Countervailing incentives.*

There are countervailing incentives when the incentive of the agent to understate or overstate his type depends upon his realization. [7]

[7] See, for instance, Lewis and Sappington (1989) and Maggi and Rodríguez-Clare (1995)..

The next theorem states almost tautologically the conditions under which the optimal policy under private information exhibits countervailing incentives. We only require the function u_θ to be a monotone (non-increasing or non-decreasing) function.

Theorem 1. The principal's optimal policy under private information presents countervailing incentives if and only if under full information there exists a type $\tilde{\theta} \in (\underline{\theta}, \overline{\theta})$ such that $u_\theta\left(l^*(\tilde{\theta}), \tilde{\theta}\right) = 0$, that is $\tilde{l}(\tilde{\theta}) = l^*(\tilde{\theta})$.

Proof. Assume that u_θ is a nondecreasing monotone function. (The proof when a non-increasing function is similar).

(i) Necessity is almost direct. Assume that there does not exist a type $\tilde{\theta} \in (\underline{\theta}, \overline{\theta})$ such that $u_\theta\left(l^*(\tilde{\theta}), \tilde{\theta}\right) = 0$. Then from monotony or $u_\theta(l^*(\theta), \theta) < 0\ \forall\theta \in (\underline{\theta}, \overline{\theta})$, and therefore from Lemma 2, any type would have an incentive to overstate or $u_\theta(l^*(\theta), \theta) > 0\ \forall\theta \in (\underline{\theta}, \overline{\theta})$ and in consequence, from Lemma 2, any type would have an incentive to understate. As a consequence, there are not countervailing incentives.

(ii) It is also straightforward to show sufficiency. Assume that there exists a type $\tilde{\theta} \in (\underline{\theta}, \overline{\theta})$ such that $u_\theta\left(l^*(\tilde{\theta}), \tilde{\theta}\right) = 0$. If u_θ is a strictly monotone increasing function, then $u_\theta(l^*(\theta), \theta) < 0\ \forall\theta \in (\underline{\theta}, \tilde{\theta})$ and, from Lemma 2, these types would have incentives to overstate and $u_\theta(l^*(\theta), \theta) > 0\ \forall\theta \in (\tilde{\theta}, \overline{\theta})$ and the incentive would be, therefore, to understate. As a consequence, the incentive of the agent to understate or overstate his type depends on its realization. So there are countervailing incentives. □

IV.2. *Characterization of the optimal policy under private information*

The optimal policy under private information depends crucially on the curvature of *U* for any implementable policy. This curvature is given by:

$$\frac{d^2U(\theta)}{d\theta^2} = u_{\theta l}\frac{dl}{d\theta} + u_{\theta\theta}.$$

Given that $u_{\theta l}\frac{dl}{d\theta} \geq 0$ for any implementable policy, then the sign of $\frac{d^2U(\theta)}{d\theta^2}$ depends on the sign and relative magnitude of $u_{\theta\theta}$.[8] In order to solve the problem under countervailing incentives, we distinguish three cases: [9]

a) $u_{\theta\theta} \leq 0$ (small enough in absolute value)

In this case, U is convex for any implementable allocation and U reaches its minimum at $\tilde{\theta}$ when there exist countervailing incentives. Furthermore, $\tilde{l}(\theta)$ is a non-decreasing function of θ and, therefore, is implementable under private information. Note that in this case we have that under private information $u_\theta(l(\theta),\theta) \leq 0$ and $l^*(\theta) \leq l^{PI}(\theta) \leq \tilde{l}(\theta)$ $\forall\theta \in [\underline{\theta},\tilde{\theta})$ and $u_\theta(l(\theta),\theta) \geq 0$ and $\tilde{l}(\theta) \leq l^{PI}(\theta) \leq l^*(\theta)$ $\forall\theta \in (\tilde{\theta},\overline{\theta}]$. In that case, we can write:

$$U(\theta) = U(\tilde{\theta}) - \int_{\theta}^{\tilde{\theta}} u_\theta(l(v),v)dv \quad \forall\theta \in [\underline{\theta},\tilde{\theta}),$$

And

$$U(\theta) = U(\tilde{\theta}) + \int_{\tilde{\theta}}^{\theta} u_\theta(l(v),v)dv \quad \forall\theta \in (\tilde{\theta},\overline{\theta}].$$

The principal's problem can be written as:

$$\max_{l(\theta),U(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} W(l(\theta),t(\theta),\theta)f(\theta)d\theta \tag{7}$$

$$Subject\ to \quad \text{(a)}\ U(\theta) = U(\tilde{\theta}) - \int_{\theta}^{\tilde{\theta}} u_\theta(l(v),v)dv \quad \forall\theta \in [\underline{\theta},\tilde{\theta}),$$

[8] Note that the curvature of $U(\theta)$determines who obtains the highest informational rent at equilibrium.

[9] Note that our classification of cases is similar of that used by Maggi and Rodríguez-Clare (1995) even though our approach considers a general utility function for the agent.

(b) $U(\theta) = U(\tilde{\theta}) + \int_{\tilde{\theta}}^{\theta} u_\theta(l(v), v)dv \quad \forall\theta \in (\tilde{\theta}, \overline{\theta}]$,

(c) $U(\tilde{\theta}) \geq 0$,

(d) $\frac{dl}{d\theta} \geq 0$,

(e) $l^*(\theta) \leq l(\theta) \leq \tilde{l}(\theta) \quad \forall\theta \in [\underline{\theta}, \tilde{\theta})$,

(f) $\tilde{l}(\theta) \leq l(\theta) \leq l^*(\theta) \quad \forall\theta \in (\tilde{\theta}, \overline{\theta}]$.

Plugging (a) and (b) into the objective function and taking into account that (c) is binding at the optimum, we can rewrite the social welfare in state θ as:

$$\widetilde{W}(l(\theta), \theta) = \begin{cases} w(l(\theta), \theta) + \mu u(l(\theta), \theta) + \mu \frac{F(\theta)}{f(\theta)} u_\theta(l(\theta), \theta) \quad \forall\theta \in [\underline{\theta}, \tilde{\theta}) \\ w(l(\theta), \theta) + \mu u(l(\theta), \theta) - \mu \frac{1 - F(\theta)}{f(\theta)} u_\theta(l(\theta), \theta) \quad \forall\theta \in (\tilde{\theta}, \overline{\theta}]. \end{cases}$$

The principal's problem becomes:

$$\max_{l(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \widetilde{W}(l(\theta), \theta) f(\theta) d\theta \tag{8}$$

Subject to (d), (e) and (f).

The next proposition characterizes the principal's optimal policy under private information.

Proposition 1. The optimal policy under private information $\{l^{PI}(\theta), t^{PI}(\theta)\}_{\theta \in [\underline{\theta}, \overline{\theta}]}$ is given by:

$$l^{PI}(\theta) = \begin{cases} \hat{l}_1(\theta) & \underline{\theta} \leq \theta \leq \theta_1 \\ \tilde{l}(\theta) & \theta_1 \leq \theta \leq \theta_2, \\ \hat{l}_2(\theta) & \theta_2 \leq \theta \leq \overline{\theta} \end{cases}$$

$$t(l^{PI}(\theta)) = \begin{cases} -\int_{\theta}^{\tilde{\theta}} u_\theta(l^{PI}(v), v)dv - u(l^{PI}(\theta), \theta) & \underline{\theta} \leq \theta \leq \tilde{\theta} \\ \int_{\tilde{\theta}}^{\theta} u_\theta(l^{PI}(v), v)dv - u(l^{PI}(\theta), \theta) & \tilde{\theta} \leq \theta \leq \overline{\theta} \end{cases},$$

where $\hat{l}_1(\theta)$ solves $w_l(\hat{l}_1(\theta), \theta) + \mu u_l(\hat{l}_1(\theta), \theta) + \mu \frac{F(\theta)}{f(\theta)} u_{\theta l}(\hat{l}_1(\theta), \theta) = 0$, and $\hat{l}_2(\theta)$ solves $w_l(\hat{l}_2(\theta), \theta) + \mu u_l(\hat{l}_2(\theta), \theta) - \mu \frac{1-F(\theta)}{f(\theta)} u_{\theta l}(\hat{l}_2(\theta), \theta) = 0$. The types θ_1 and θ_2 are such that $\hat{l}_1(\theta_1) = \tilde{l}(\theta_1)$ and $\hat{l}_2(\theta_2) = \tilde{l}(\theta_2)$, respectively.

The interval $[\theta_1, \theta_2]$ contains the types that do not receive informational rents. Note that when $u_{\theta\theta} = 0$ then $\tilde{l}(\theta) = l^*(\tilde{\theta})$ $\forall \theta \in [\underline{\theta}, \overline{\theta}]$. As a consequence, in this case there would be a pooling equilibrium in the interval $[\theta_1, \theta_2]$, as illustrated in Figure 1.

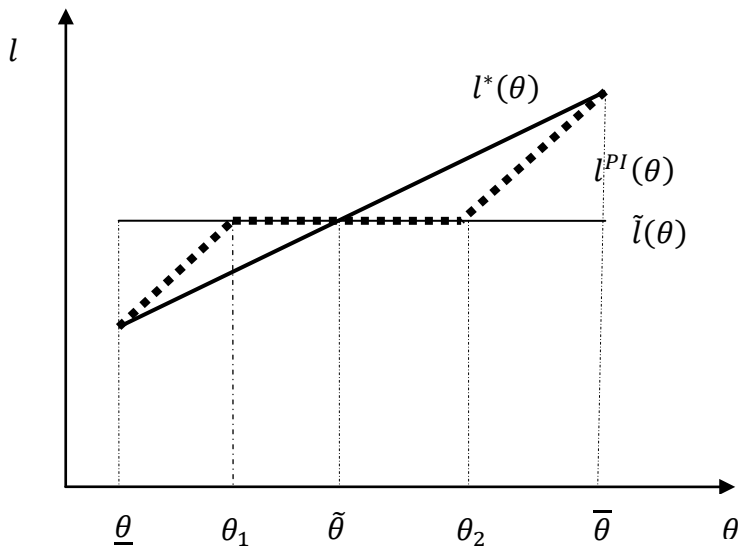

Figure 1. Optimal Policies under full information and under private information when $u_{\theta\theta} = 0$. Pooling equilibrium appears under private information.

Nevertheless, when $u_{\theta\theta} < 0$, $\tilde{l}(\theta)$ is a strictly increasing function and therefore the optimal solution under private information depends on θ even though some types do not receive informational rents, as illustrated in Figures 2 and 3.

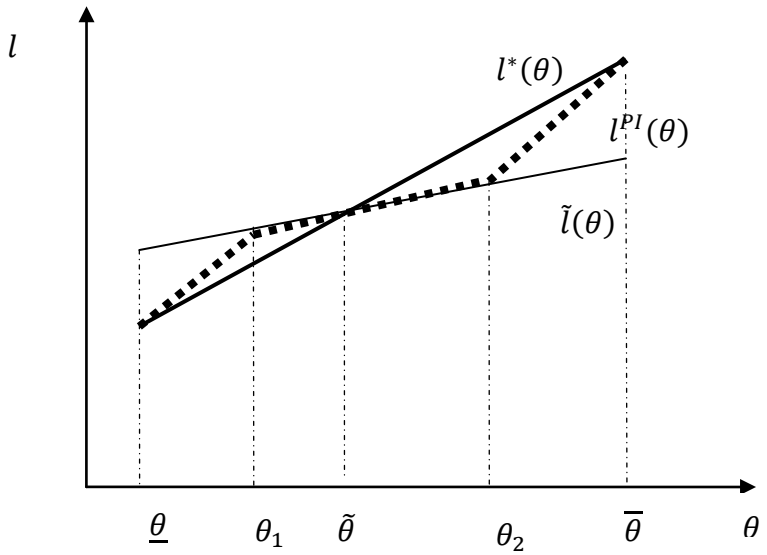

Figure 2. Optimal Policies under full information and under private information when *u* is not too concave.

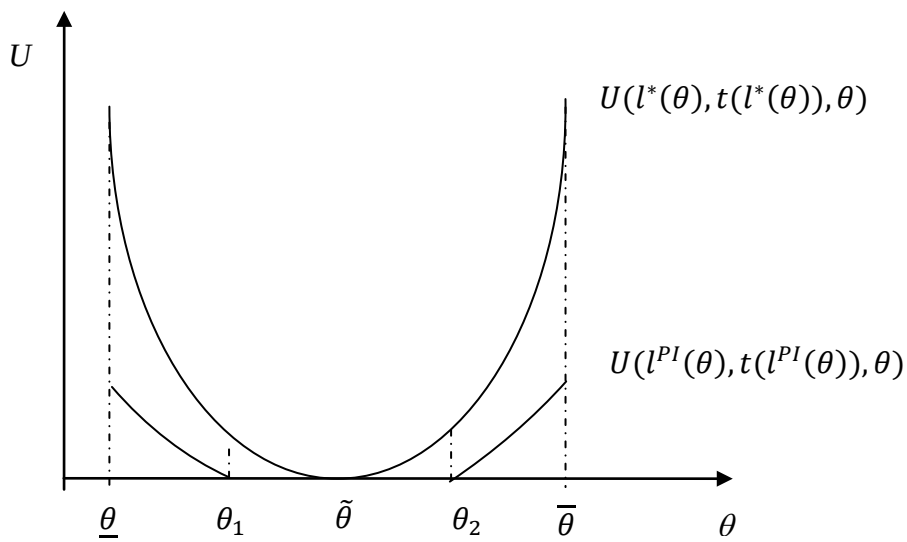

Figure 3. Agent's utility $l^*(\theta)$ and $l^{PI}(\theta)$ are implemented under private information and $u_{\theta\,\theta} \leq 0$.

Figure 3 represents the agent's utility under private information when the optimal level of the action variable under full information, $l^*(\theta)$, is implemented through the transfer needed to guaranty incentive compatibility $t\big(l^*(\theta)\big)$. Given that the implementation $l^*(\theta)$ under private information generates high informational rents, the principal deviates from this level and in

order to reduce these rents, they will be zero in the interval $[\theta_1, \theta_2]$.

This kind of optimal contract has characterized many incentive problems in the literature. Aguirre and Beitia (2008) consider the regulation of a multiproduct monopolist with unknown demand and show that when the firm sells demand complements then countervailing incentives characterize the optimal contract in contexts where the firm would want to practice cross subsidization under full information.

In a labor economics context, Kübler (2002) analyzes the optimal contract between an employer and a worker when the productiviy of the worker is not observable by the principal (the employer). In a context in which the more productive a worker is, the higher his reservation utility, she shows that optimal contracts may be characterized by countervailing incentives (compare for example her Figure 1 with our Figure 1). In a context of health economics, Choné and Ma (2011) also comment in their appendix that countervailing incentives of this type may characterize optimal contract.

b) $u_{\theta\theta} > 0$.

The principal faces the same maximization problem as in the previous case (U is strictly convex for any implementable allocation) but now $\tilde{l}$ is a decreasing function of θ and, as a consequence, it is not implementable under private information.

The next proposition summarizes the optimal contract under private information.

Proposition 2. The optimal policy under private information $\{l^{PI}(\theta), t^{PI}(\theta)\}_{\theta\in[\underline{\theta},\overline{\theta}]}$ is:

$$l^{PI}(\theta) = \begin{cases} \hat{l}_1(\theta) & \underline{\theta} \leq \theta \leq \theta_1 \\ l^*(\tilde{\theta}) & \theta_1 \leq \theta \leq \theta_2, \\ \hat{l}_2(\theta) & \theta_2 \leq \theta \leq \overline{\theta} \end{cases}$$

where $\hat{l}_1(\theta)$ solves $w_l(\hat{l}_1(\theta), \theta) + \mu u_l(\hat{l}_1(\theta), \theta) + \mu \frac{F(\theta)}{f(\theta)} u_{\theta l}(\hat{l}_1(\theta), \theta) = 0$, and $\hat{l}_2(\theta)$ solves $w_l(\hat{l}_2(\theta), \theta) + \mu u_l(\hat{l}_2(\theta), \theta) - \mu \frac{1-F(\theta)}{f(\theta)} u_{\theta l}(\hat{l}_2(\theta), \theta) = 0$. The types θ_1 and θ_2 are such that $\hat{l}_1(\theta) = \hat{l}_2(\theta) = l^*(\tilde{\theta})$.

Figure 4 represents the optimal policy under full information and under private information and Figure 5 illustrates the agent's utility when $l^*(\theta)$ and $l^{PI}(\theta)$ are implemented under private information. Proposition 2 states that as in the case where $u_{\theta\theta} = 0$, there is pooling equilibrium in the interval $[\theta_1, \theta_2]$ (see Figure 4) but now the only type that does not receive informational rents is $\tilde{\theta}$ (see Figure 5).

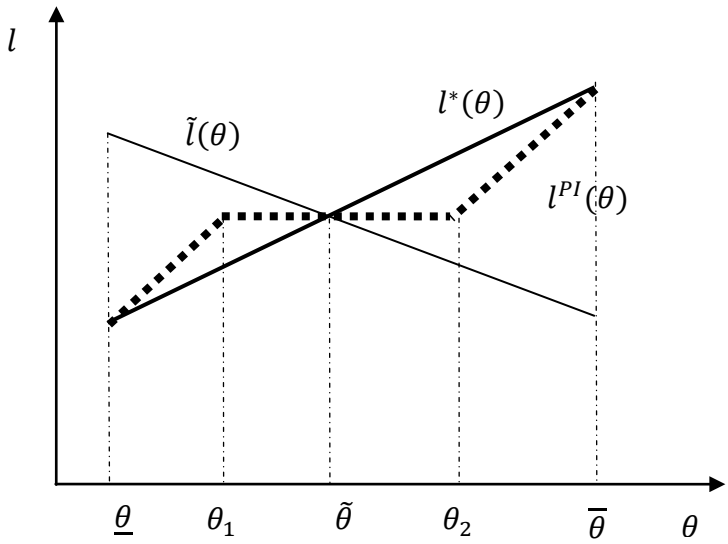

Figure 4. Optimal policies when *u* is strictly convex.

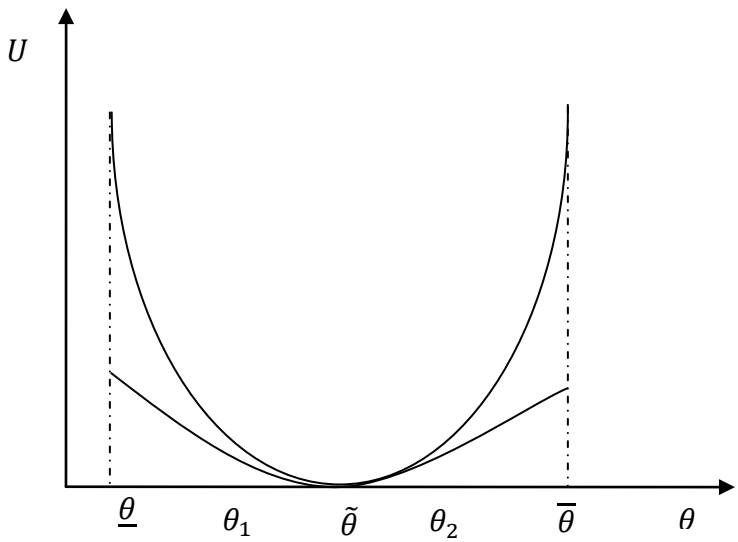

Figure 5. Agent's utility when $l^*(\theta)$ and $l^{PI}(\theta)$ are implemented under private information when *u* is strictly convex.

This kind of contract is obtained in Lewis and Sappington (1989). They consider the regulation of a single product firm with unknown total cost. The firm has private information on constant marginal cost and on fixed cost, and they assume that both variables move in opposite direction: the higher the marginal cost of a firm is, the lower is its fixed cost. The optimal regulatory policy exhibits countervailing incentives: for low realizations of θ the firm's incentive to overstate θ will dominate its incentive to understate θ, while for higher realizations the dominant incentive will be to understate θ. Under private information, the optimal contract is such that an interior type obtains zero profits and there is pooling equilibrium for an interior interval of types.

c) $u_{\theta\theta} < 0$ (high enough in absolute value).

In this case U is concave for any implementable allocation and reaches its maximum at $\tilde{\theta}$ when there exist countervailing incentives and $U(\underline{\theta}) = U(\overline{\theta}) = 0$. Moreover, $\tilde{l}$ is an increasing function of θ and, therefore, is implementable under private information.

By Lemma 2, $u_\theta(l(\theta), \theta) \geq 0$ and $\tilde{l}(\theta) \leq l^{PI}(\theta) \leq l^*(\theta)$ $\forall\theta \in [\underline{\theta}, \tilde{\theta})$ and $u_\theta(l(\theta), \theta) \leq 0$ and $l^*(\theta) \leq l^{PI}(\theta) \leq \tilde{l}(\theta)$ $\forall\theta \in (\tilde{\theta}, \overline{\theta}]$. Therefore we have:

$$U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} u_\theta(l(v), v)dv \quad \forall\theta \in [\underline{\theta}, \tilde{\theta}),$$

and

$$U(\theta) = U(\overline{\theta}) - \int_{\theta}^{\overline{\theta}} u_\theta(l(v), v)dv \quad \forall\theta \in (\tilde{\theta}, \overline{\theta}].$$

The principal's problem can be written as:

$$\max_{l(\theta),U(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} W(l(\theta),t(\theta),\theta)f(\theta)d\theta \tag{9}$$

Subject to (a′) $U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} u_\theta(l(v),v)dv \quad \forall\theta \in [\underline{\theta},\tilde{\theta})$,

(b′) $U(\theta) = U(\overline{\theta}) - \int_{\theta}^{\overline{\theta}} u_\theta(l(v),v)dv \quad \forall\theta \in (\tilde{\theta},\overline{\theta}]$,

(c′) $U(\underline{\theta}) \geq 0$,

(d′) $U(\overline{\theta}) \geq 0$,

(e′) $\frac{dl}{d\theta} \geq 0$,

(f') $\tilde{l}(\theta) \leq l(\theta) \leq l^*(\theta) \quad \forall\theta \in [\underline{\theta},\tilde{\theta})$,

(g') $l^*(\theta) \leq l(\theta) \leq \tilde{l}(\theta) \quad \forall\theta \in (\tilde{\theta},\overline{\theta}]$.

Plugging (a') and (b') into the objective function and taking into account that (c') and (d') are binding at the optimum, we can rewrite the social welfare in state θ as:

$\widetilde{W}(l(\theta),\theta) = w(l(\theta),\theta) + \mu u(l(\theta),\theta) - \mu\frac{F(\tilde{\theta})-F(\theta)}{f(\theta)}u_\theta(l(\theta),\theta) \quad \forall\theta \in [\underline{\theta},\overline{\theta}]$.

The principal's problem becomes:

$$\max_{l(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \widetilde{W}(l(\theta),\theta)f(\theta)d\theta \tag{10}$$

Subject to (e'), (f') and (g').

The next proposition characterizes the principal's optimal policy under private information.

Proposition 3. The optimal policy under private information is $\{l^{PI}(\theta),t^{PI}(\theta)\}_{\theta\in[\underline{\theta},\overline{\theta}]}$ where $l^{PI}(\theta)$ solves $w_l(l^{PI}(\theta),\theta) + \mu u_l(l^{PI}(\theta),\theta) - \mu\frac{F(\tilde{\theta})-F(\theta)}{f(\theta)}u_{\theta l}(l^{PI}(\theta),\theta) = 0$,

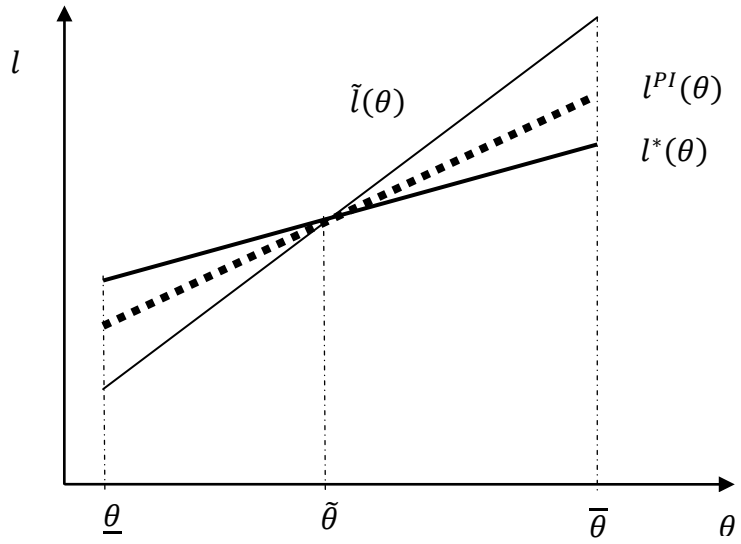

Figure 6. Optimal policies when *U* is strictly concave.

Figure 6 illustrates the optimal policies both under full information and under private information, and Figure 7 shows the agent's utility when $l^{*}(\theta)$ and $l^{PI}(\theta)$ are implemented under private information.

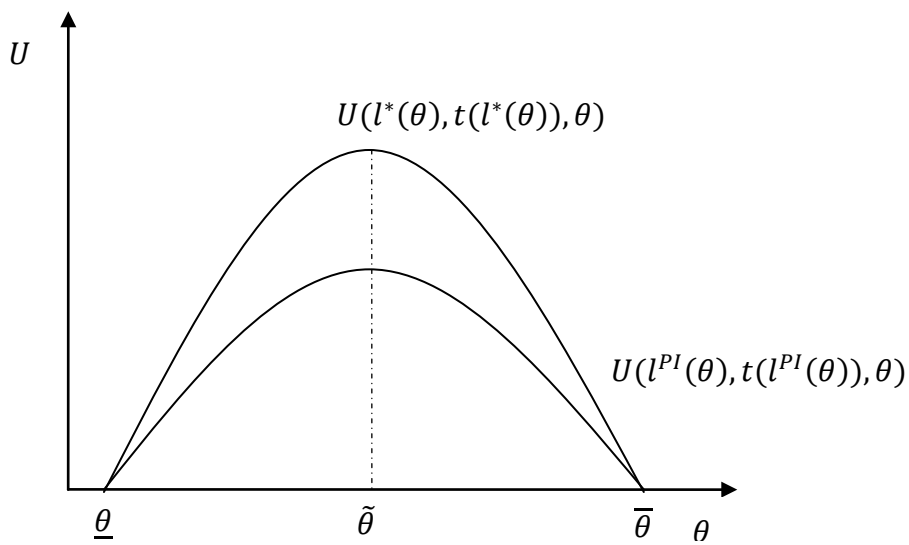

Figure 7. Agent's utility when $l^{*}(\theta)$ and $l^{PI}(\theta)$ are implemented under private information when u is strictly concave.

Maggi and Rodríguez-Clare (1995) consider a situation where a principal contracts with an agent to produce a certain amount of output and compensates him with a monetary transfer. Constant marginal cost (increasing with the type) is privately observed by the agent and it is

assumed that the agent also has an outside opportunity that provides him a reservation utility decreasing with marginal costs. When the reservation utility is highly convex then $u_{\theta\theta} < 0$ (high enough in absolute value) and the analysis of Maggi and Rodríguez-Clare (1995) would be well described by Figure 7 where informational rents are bell-shaped with both extreme types earning no rents. In an environmental economics context, Sheriff (2008) considers the socially optimal policy for reducing emissions in politically influential sectors. He shows that countervailing incentives can exist if high productivity is correlated with high foregone profit from abatement, and the incentive of over-state or under-state productivity depends crucially of the realization of private information.

V. Concluding remarks

In this paper, we develop a simple method of characterizing countervailing incentives in adverse selection problems. The key element in our characterization consists of analyzing properties of the full information problem. This allows solving the principal problem without using optimal control theory. We illustrate our methodology with several examples arising from health economics, monopoly regulation, labour contracts, limited liabilities and environmental regulation.

There is also a literature on multidimensional screening that considers incentive mechanisms when private information concerns more than one variable (see, for instance, Rochet and Stole, 2003) where countervailing incentives have been also studied. Boone and Schottmüller (2013) analyze optimal procurement mechanisms when firms are specialized. They assume that the procurement agency has incomplete information concerning the firms' cost functions and values high quality as well as low price. They analyze a two-dimensional screening model with countervailing incentives. Szalay (2013) considers the regulation of a two-product monopolist when private information concerns two variables and characterizes the optimal policy that again exhibits countervailing incentives. As an extension we leave for future research, it would be very valuable to obtain a general theorem and a complete characterization of such multidimensional problems along the lines of our Theorem 1 and Propositions 1, 2 and 3.

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