EARNINGS-BASED VS ASSETS-BASED BORROWING CONSTRAINTS AND THEIR IMPACT ON OUTPUT

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ABSTRACT

We present an overlapping generations model in order to assess the quantitative effects of financial frictions on output depending on the type of loan contract. Financial frictions are one of the major obstacles of the business environment because firms cannot borrow all what they need. This fact creates a misallocation of resources what reduces TFP and consequently GDP. We show that the type of loan contract plays an important role in order to quantify the effects of financial frictions on TFP and per capita GDP. Recent literature has begun to distinguish between asset-based and those earnings-based loan contracts. The traditional literature used to consider only asset-based loan contracts. However, over the last years, empirical evidence shows the relevance of earnings-based loan contracts. Asset-based borrowing constraints overstate the effects of misallocation due to financial frictions. But earnings increase with productivity, so when the earnings are pledgeable, the effects on TFP caused by financial frictions are lower than when only assets are pledgeable.

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INDEX

INTRODUCTION

Per capita income is the main indicator of wealth and well-being in countries. Differences in this indicator across countries are closely related to differences in living standards. Most of cross-country differences in per capita income are due to differences in Total Factor Productivity (TFP), rather than differences in production factors (See, among others, Hall & Jones (1999), Caselli (2005)). Furthermore, most of these differences are due to misallocation of resources rather than differences in the level of technology (See Jones (2011)). Misallocation of resources can arise because of several distortions, such as: bad policies, financial frictions and, in general, distortions that affect different firms in a differently way. In this paper we consider that there are financial frictions in those economies in which contracts cannot be completely enforced. Furthermore, we are going to distinguish between asset-based and earnings-based borrowing constraints and their different impact on the misallocation of resources and hence on income per capita.

Our objective is to quantify the effects of the financial frictions on TFP and per capita GDP under different scenarios. We distinguish between three type of loan contract: The first one is the one developed by Amaral & Quintin (2010), where either the assets and the earnings are pledgeable. The second one is the assets-based borrowing constraint. And the last type is the earnings-based borrowing constraint. In order to carry out our work, we develop an overlapping generations model model based on Amaral & Quintin (2010). For each of the three frameworks we introduce financial frictions via contractual enforcement to compare the results across the three scenarios. What we find that assets-based borrowing constraints distort more than earnings-based borrowing constraints.

REVIEW OF THE LITERATURE

This section will review the literature of the effect of financial frictions on aggregate variables, such as: Total Factor Productivity (TFP) and per capita GDP. This work is placed within the literature that focuses on the analysis of those factors that can explain cross-country differences in per capita GDP. On the one hand, Hall and Jones (1999) show that cross-country differences in TFP is the main cause explaining differences in per capita income between developed and developing countries. Caselli (2005) also shows that cross-country differences in per capita income are due to cross-country differences in TFP. On the other hand, there is a consensus that cross-country differences in TFP are due either to the technological gap or to differences in efficiency.

In recent years, a new literature is focusing on misallocation of resources as a factor causing cross-country differences in TFP. For an economy to produce at an optimal level, resources must be efficiently allocated at the microeconomic level, that is, at the firm's level. The allocation will be efficient when the marginal product of each production factor is equalized across firms. However, there are a number of reasons why this does not take place and would generate the aforementioned, misallocation of resources, which would mean a reduction in TFP and therefore in per capita income. Specifically, weak institutions are associated with greater distortions. Among the distortions that have been considered in the literature, which generate misallocation of resources, are: financial frictions, public firms (some of them may be less efficient than private ones and yet use more factors of production), corruption, political instability, informal sector, competition, crime, excess bureaucracy. Hsieh & Klenow (2009) is one of the first papers that quantifies how misallocation of resources across firms and industries can explain the difference in TFP between the US, China and India. They consider distortions, through a tax (subsidy) which favour some firms (with good political connections) over others and financial frictions through a capital tax. Jones (2011) reviews the literature that analyses the misallocation of resources as one of the main causes of cross-country differences in TFP.

Financial frictions are one of the causes of misallocation of resources, so an economy with a developed financial system will have fewer financial frictions, and therefore a higher TFP. Levine et al. (2000) show how a developed financial system corresponds with a high level of enforcement of loan contracts and causes positive economic growth. Amaral & Quintin (2010) carry out an analysis across countries, showing that there is a monotonous relationship between the parameter that captures the level of financial development (that is, the degree at which financial contracts can be enforced), the level of private credit over GDP and the per capita GDP of a country. Those countries with greater financial development increase their output by increasing the capital used in production and using it to its best use. Furthermore, following the model of Lucas (1978), Amaral and Quintin show how these financial frictions can generate a misallocation of talent. Bah & Fang (2015) use the World Bank Entreprise Survey (ESWB) database and show how for a number of Sub-Saharan African countries, financial limitations (along with other distortions) are one of the causes that generate misallocation of resources, causing a large difference in per capita income levels and TFP across countries[2]. Most recently, Ranasinghe & Restuccia (2018), using the same database (ESWB), study how crime and lack of access to finance are the two biggest obstacles to operating at the optimal level. Rajan & Zingales (1998) show that industries that require more external financing grow faster in more financially developed countries. Additionally, small establishments are more severely affected by poor financial development, and industries dominated by small firms for technological reasons grow faster in countries with greater financial development.

[2] Bah and Fang (2015) build a simple model based in Amaral et al. (2010).

These papers show the importance of the quality of financial institutions when contributing to the TFP. When we talk about financial frictions, traditional literature has considered asset-based borrowing restrictions as the cornerstone. However, recent literature has begun to distinguish between two types of restrictions: asset-based (the traditional approach) and earnings-based borrowing constraints. Drechsel (2019), using US data, claims that US firms borrowing capacity not only depends on firms' assets but also on their current earnings. This finding comes from the observed facts, which are compatible with the empirical implications that earnings-based borrowing constraints imply that the borrowing capacity expands in response to investment positive shocks, whereas an asset-based constraint implies the opposite[3]. Moreover Li (2016) shows for a large panel of firms in Japan that the borrowing capacity increases with productivity, that is, borrowing capacity depends not only on assets but also in earnings. Aggregate productivity loss due to financial frictions is also smaller if taking into account that borrowing capacity not only depends on assets but also on earnings. Chen & Ma (2019), for a panel firms of Japan and the US, find that only the 20% of the debt of non-financial US firms is based on assets, and that there are differences between both countries. They also point out that for the least financially developed countries, earnings-based loans are more complicated to take place.

EMPIRICAL EVIDENCE

The availability of comparable data at the microeconomic level across countries, quantifying variables such as the likelihood of vandalism, bribery of licenses, lack of electricity, infrastructure or access to finance has made it possible to analyse how differences across countries, in these factors, can explain differences in TFP or GDP per capita. One of the best databases is the Enterprise Survey of the World Bank (ESWB)[4] which records factors that affect the firm's business environment. The surveys cover a broad range of business environment topics including access to finance, corruption, infrastructure, crime, competition, and performance measures. Enterprise Surveys offers an expansive array of economic data on 164,000 firms in 144 countries. According to ESWB, access to finance is one of the major obstacles of firms, especially for young and small firms.

Access to finance is one of the 12 topics that this database records. This topic includes several indicators assessing credit sources, loan requirements, and access to financial services for 144 countries, and by country regions. These indicators provide us the possibility to analyse and observe the financial development of the countries and regions. Next table is going to show the different indicators of finance considered by ESWB, for each country group or geographic region:

[3] See Drechsel (2019) Appendix B for more information about earnings-based constraint.
[4] Available at *https://www.enterprisesurveys.org/*

TABLE 1

Indicators of Finance by Regions of Countries

Economy	Percent of firms with a checking or savings account	Percent of firms with a bank loan/line of credit	Proportion of loans requiring collateral (%)	Value of collateral needed for a loan (% of the loan amount)	Percent of firms not needing a loan	Percent of firms whose recent loan application was rejected
East Asia & Pacific	81.5	33.7	82.3	229.7	48.5	7.6
Europe & Central Asia	93.4	37.5	72.8	176.2	57.9	9.1
High income: OECD	96.4	47.3	57.8	88.1	74.8	10.9
Latin America & Caribbean	92.9	47.7	70.7	199	45.4	3.3
Middle East & North Africa	79.2	26.5	78.5	193.9	52.8	11.9
South Asia	77.6	27	81.1	236	44.7	14.4
Sub-Saharan Africa	87.4	21.5	84.3	203.7	38.6	15.6

Source: ESWB

As we can observe in Table 1, in high income countries (OECD) 47.3% of the firms have a loan of credit with a bank, meanwhile in South Asia countries or Sub-Saharan Africa countries is less than 30%. Consequently, high income countries have higher Private Debt / GDP ratio than less developed countries, being consistent with the quantitative results of Amaral et al. (2010). Furthermore, the third column give us the proportion of loans requiring collateral, that is, percentage of loan based on assets. OECD countries are the one who require less collateral (57.8%), while Sub-Saharan Africa countries are who require the most collateral (84.3%). Therefore, the relevance of asset-based versus earnings-based borrowing constraints across countries is not the same.

THE MODEL

The model is the same as in Bah & Fang (2015), based on Amaral & Quintin (2010). The framework is a discrete-time overlapping generations model. Each period consists in 20 years and a mass one of heterogeneous individuals, that live two periods, is born. Over the first period, all individuals are workers and they supply their labour force inelastically. Over their second period individuals must decide between remaining as workers or becoming entrepeneurs[5]. In order to carry out this distinction, all individuals are endowed with a particular managerial skill level $z \in Z$ which is constant along the individual's lifetime and follows a distribution g(z) which is constant across generations.

This occupational choice is based on Lucas (1978) where individuals decide to become in entrepreneurs if their expected value function as entrepreneurs is higher than their expected value function as workers. The work value function is constant along workers lifetime and independent of their level of managerial skill. However, entrepreneurs' value function increases strictly with the level of managerial skill. So, individuals become entrepeneurs if their managerial skill level is above an ability threshold (for which the value function of worker and of entrepreneur cross each other). Then more productive individuals will become in managers.

The model is analyzed at the steady state equilibrium for a closed economy with constant wage and interest rates. The interest rate is exogenously determined. The calibration will be done in order to ensure that savings are higher than capital demand, that is, the interest rate is not adjusted to guarantee that savings are equal to capital. The differences between savings and capital will be interpreted as the residential investment. In all our simulations this storage (residential investment) is higher than zero, as in Amaral et al (2010). Furthermore, since the economy is closed, the model does not allow individuals to go to international markets.

THE UTILITY FUNCTION

Individuals live for two periods. Individuals' preferences are represented by the following utility function:

$$U(c_1, c_2) = \log(c_1) + \beta \log(c_2)$$ [6]

Where β denotes the subjective discount factor. In the first period, all individuals are workers and supply labour force inelastically. In the second period they must choose between remaining as workers or becoming a manager.

[5] Amaral et al. (2010) consider an overlapping generations model formed by three generations. The first two period are the same as in Bah & Fang (2015), and in the third periods all the individuals are retired.

[6] In the US log corresponds to the neperian logarithm.

THE PRODUCTION FUNCTION

In the second period, each entrepreneur with an ability z combines inputs of labor l>0 and physical capital k>0 into a single consumption good producing according to a Cobb-Douglas technology with decreasing return to scale:

$$F(k,l,z) = y = z[k^{\alpha} l^{1-\alpha}]^{\gamma}$$

Where $\alpha \in (0,1)$ and $\gamma < 1$ is the span of control parameter as in Lucas (1978), and is lower than one in order to firms have positive profits even when the markets are competitive[7]. We can rewrite the above function as follows:

$$F(k,l,z) = y = zk^{\theta} l^{\mu}$$

Where $\theta = \alpha\gamma$ is the capital share, $\mu = (1-\alpha)\gamma$ is the labor share and $\theta + \mu < 1$.

THE MARKET FOR LOANS

Managers can self-finance part of their capital using savings from the first period of their lifetime $a \geq 0$. They also can finance their capital through external resources, that is, borrowing an amount $d \geq 0$ from an intermediary agent. This intermediary can borrow and lend at an exogenous gross rate 1+r. Intermediaries store resources from workers with exogenous net return r, and can lend to the entrepreneurs at that same rate. The production sector carried out by managers is the production activity, while savings channeled outside the production sector is the storage technology which can be interpreted as residential investment. This assumption is the same done by Amaral et al. (2010), because residential capital is linearly related to GDP[8].

The financial market is imperfect and the managers have the option to default on their debt liabilities with the intermediary agent. If a manager decides to defaults, he loses a fraction η >0 of his earnings and/or assets. This parameter records the quality of the financial institutions. Higher the quality of enforcement, higher the quality of financial institutions and higher the fraction lost. So, we can think in the parameter η as the degree of contractual enforcement.[9] There is no uncertainty in the model and the financial intermediary behaves competitively. In equilibrium, intermediaries will impose a maximum debt limit such that managers find rational not to default.

[7] When $\gamma = 1$ we have constant return to scale, and only the most productive manager obtains positive profits.
[8] See Erosa (2001).
[9] In a developed country there clearly the parameter η would be high, because in case of default, the lender will have a court with high grants against the firm in order to recover his loan, Drechsel (2019).

So, there exist a level equal or higher than η at which the market of loan is perfect and then all firms produce at an optimal scale because the borrowing needs of all firms are lower than their debt limit.

Managers select the solution where their own investment a is the highest, and if they can self-finance all their capital they do so, even though they could choose to borrow some of it[10]. As we have just said above, there exist a value of the parameter η such that all the firms produce at an optimal scale. However, below that value, managers are constrained to operate at a suboptimal scale, unless their savings exceed a threshold a^{*}, then they could self-finance all their capital demand. Nonetheless, under that threshold a^{*} the managers access to financial market, and this outside finance increases with the savings a or with the managerial skill z, depending of course on the type of lending contract.

ENDOGENOUS DEBT LIMITS

In this paper we consider three frameworks regarding the borrowing constraints. The first scenario is the Amaral & Quintin (2010) loan contract, which combines assets-based and earnings-based borrowing constraints. The second scenario is the assets-based borrowing constraint, the most familiar in the literature. Finally, the third scenario is the loan contract based on earnings as in Drechsel (2019). For the three scenarios, if η = 0, the institutional quality is so weak that the cost of default is zero, and firms decide, rationally, do not pay their debt. In equilibrium there will not be lenders willing to lend and entrepreneurs will not be able to borrow. At the other extreme, if η = 1; the institutional quality is very strong, and entrepreneurs lose everything if they default. In this case, there will not be any limit on their debt. So, if $\eta = 0$ the amount of debt that entrepeneurs can borrow is zero, and if $\eta = 1$ entrepreneurs are not restricted.

Amaral & Quintin borrowing constraint

Each entrepreneur with ability $z \in Z$ and savings a ≥0, faces the following incentive-compatibility constraint:

$$\pi(a+d, z; w, r) + a(1+r) \geq (1-\eta)[\pi(a+d, z; w, r) + (a+d)(1+r)]$$

where $k = a + d$, and $\pi(a+d, z; w, r)$ denotes manager's profits.

This inequality states that it must be individually rational for the managers to repay their loan. The left hand side denotes entrepreneur's income in their second period when they pay their

[10] In the "Appendix.2" section is shown why managers decide to invest all their savings in physical capital.

loan, that is, if the managers abide the contract they receive their profits $\pi(a+d,z;w,r)$ and the accrued value of their own investment, a(1+r). On other hand, on the right hand side entrepreneurs' default and do not pay their loan, and they lose a fraction η (which denotes the cost of default) of the resulting resources. That is, if the managers do not abide the contract fraction η of their profits, undepreciated capital and its return is lost.

In order to obtain the entrepreneurs' maximum debt limit, we solve the previous inequality in terms of d such that the managers choose not to default and repay the loan:

$$d^{max} = \frac{\eta}{(1-\eta)(1+r)}\left[\pi(a+d,z;w,r) + a(1+r)\right] \tag{7}$$

This constraint of the maximum debt limit states that managers with more savings and better productivity can borrow more from the financial market. So, managers with more savings can finance more capital with their own savings, and those with more savings can also borrow more too from financial market. Furthermore, debt limit depends positively on the parameter η, the contractual enforcement. If $\eta = 0$, managers cannot borrow any amount. If $\eta = 1$ managers are not restricted to borrow any positive amount.

So, the profit maximization problem of a manager with ability z, savings a, and capital k=a+d is given by:

$$\max_{\{l,k\}} \pi(a+d,z;w,r) = zk^{\theta}l^{\mu} - wl - (r+\delta)k$$

$$\text{s.t} \qquad d \leq \frac{\eta}{(1-\eta)(1+r)}\left[\pi(k,z;w,r) + a(1+r)\right]$$

Assets-based borrowing constraint

In this subsection we focus on the assets-based borrowing constraint framework. The lender has a legal claim on assets, and if the entrepreneur defaults, he loses a fraction η of their assets. The incentive-compatibility constraint of entrepreneurs of ability $z \in Z$ with savings a ≥0 is given by the following expression:

$$\pi(a+d,z;w,r) + a(1+r) \geq \pi(a+d,z;w,r) + (1-\eta)(a+d)(1+r)$$

where $k = a + d$, $\pi(a+d,z;w,r)$ denotes manager's profits, w the wage rate and r the interest rate.

That inequality states that it must be individually rational for the managers to repay their loan. The left hand side denotes entrepreneur's income in their second period when they pay their loan, that is, if the managers abide the contract they receive their profits $\pi(a+d,z;w,r)$ and the accrued value of their own investment, a(1+r), exactly the same as in the previous scenario. On other hand, on the right hand side entrepreneurs' default, do not pay their loan and lose a fraction η (which denotes the cost of default) of their undepreciated capital and its return is lost.

In order to obtain the entrepreneurs debt limit, we solve the previous inequality in terms of d such that the managers choose not to default and repay the loan:

$$d^{max} = \frac{\eta}{(1-\eta)} a \tag{8}$$

This constraint of the maximum debt limit states that managers with more savings can borrow more from the financial market. So, managers with more savings can finance more capital with their own savings, and those with more savings can also borrow more from financial market. Furthermore, debt limit depends positively on the parameter η, the contractual enforcement. In contrast with the previous scenario, now the debt limit only depends on savings and not in productivity too, what is going to have effects on our quantitative exercise.[11]

So, the profits maximization problem of a manager with ability z, savings a, and capital k=a+d is given by:

$$\max_{\{l,k\}} \pi = zk^{\theta} l^{\mu} - wl - (r+\delta)k$$

$$\text{s.t} \qquad d \leq \frac{\eta}{(1-\eta)} a$$

Earnings-based borrowing constraint

In the case of earnings-based constraint, the lender has a legal claim of the entrepreneur's operational profits. In order to assess the operational profits, Drechsel (2019) use a fixed multiple of the term EBDITA[12]. We use the same method. So, each entrepreneur with ability $z \in Z$ and savings a ≥0, faces the following incentive-compatibility constraint (ICC):

$$\pi_n(a+d,z;w) - (r+\delta)k + a(1+r) \geq [(1-\eta)\pi_n(a+d,z;w) - (r+\delta)k] + (a+d)(1+r)$$

where $k = a + d$, and $\pi_n(a+d,z;w,r) = y - wl$ denotes manager's EBITDA

Therefore: $\pi_n(a+d,z;w,r) - (r+\delta)k = \pi(a+d,z;w,r)$

That inequality states that it must be individually rational for the managers to repay their loan. The left hand side denotes entrepreneur's income in their second period when they pay their

[11] In "Results" section we are going to show how when firms when they are constrained by assets-based borrowing constraint, the effect of financial frictions on aggregate TFP is higher than we consider earnings-based constraint, as in Li (2016).

[12] EBITDA = Earnings Before Interest Depreciation and Amortization

loan. That is, if managers abide the contract, they receive their profits $\pi(a+d,z;w,r)$ and the accrued value of their own investment, a(1+r), exactly the same as in the two previous scenarios. On other hand, on the right hand side, if entrepreneurs' default and do not pay their loan, they lose a fraction η (which denotes the cost of default) of their resulting operational profits (y-wl).

In order to obtain the entrepreneurs' maximum debt limit, we solve the previous ICC in terms of d such that the managers choose not to default and repay the loan:

$$d^{max} = \frac{\eta}{(1+r)}\pi_n \tag{9}$$

This constraint of debt limit states that managers with more productivity can borrow more from the financial market. Furthermore, indirectly managers with more savings can borrow more too because the profits depend positively on savings. So, managers with more savings can finance more capital with their own savings, and those with more savings can also borrow more too from financial market. As in the previous cases, the debt limit depends positively on the parameter η, the contractual enforcement.

So, the profit maximization problem of a manager with ability z, savings a, and capital k=a+d is given by:

$$\max_{\{l,k\}} \pi(a+d,z;w,r) = zk^{\theta}l^{\mu} - wl - (r+\delta)k$$

$$\text{s.t} \qquad d \leq \frac{\eta}{(1+r)}\pi_n(a+d,z;w,r)$$

ENTREPNEURS' PROFIT MAXIMIZATION PROBLEM

For the three frameworks, the maximum debt limit of firms d^{max} can be higher or lower than entrepreneurs' borrowing needs. If they can self-finance all their capital they do so, even though they could choose to borrow some of it[13]. If the borrowing needs are lower than the maximum debt limit, the firm will demand capital at an optimal scale, producing at an optimal scale. On the other hand, if the borrowing needs are higher than the debt limit, firms will demand capital below the optimal level, producing at a level below the optimal. Therefore, entrepreneurs can be unconstrained, and produce at their optimal scale, or they can be constrained and produce a lower level of production than the optimal. Consequently, entrepreneurs' profit maximization problem is solved in two steps. In the first step, it is obtained the optimal labor demand, for a given amount of capital. In the second step, the capital demanded is obtained (it can be the optimal one or lower than the optimal).

[13] In the "Appendix.2" section is shown why managers decide to invest all their savings in physical capital.

Then, the problem of profits maximization problem faced by a firm is:

$$\max_{\{l,k\}} \pi = y - wl - (r+\delta)k$$

We solve this maximization problem in two stages, first we get the optimal labor force for a given capital demand and then we obtain the optimal capital:

1) Optimal labour demand in terms of capital[14].

$$\max_{\{l\}} \pi = y - wl - (r+\delta)k$$

$$\text{s.t } y = zk^{\theta} l^{\mu}$$

Maximizing in terms of l we obtain the optimal expression of labour demand as a function of capital:

$$l^{*} = \left[\frac{\mu z k^{\theta}}{w}\right]^{\frac{1}{1-\mu}} \tag{1}$$

As we can observe labor demand depends negatively on wage rate, that is, the price of labor factor, and positively on entrepreneur's managerial ability. Once we have obtained the optimal labor demand expression, we move to the second stage to get the expression of capital demand:

2.a) The optimal capital demand when entrepreneurs are unconstrained

$$\max_{\{k\}} \pi = y - wl - (r+\delta)k$$

Maximizing in terms of k and using (1) we obtain the optimal expression of capital demand:

[14] The level of capital could be the optimal level or lower, depending on each firm borrowing needs and debt limit.

$$k^* = \left[\left(\frac{\mu}{w}\right)^{\mu} z \left(\frac{\theta}{r+\delta}\right)^{1-\mu} \right]^{\frac{1}{1-\theta-\mu}} \qquad (2)$$

Capital demand depends negatively on factor prices w, r and δ. Moreover, it depends positively on entrepreneur managerial ability[15]. Then, more productive firms demand more capital.

2.b) Optimal capital demand for constrained entrepreneurs: k=a+d^{max}

The expression for the output of a firm in terms of capital:

$$y = zk^{\theta} l^{\mu}$$

$$y = \left[\left(\frac{\mu}{w}\right)^{\mu} zk^{\theta} \right]^{\frac{1}{1-\mu}} \qquad (3)$$

For entrepreneurs unconstrained, output is negatively related to factor prices and positive related to managerial skill. Replacing (2) we obtain the optimal output expression:

$$y^* = \left[\left(\frac{\mu}{w}\right)^{\mu} z \left(\frac{\theta}{r+\delta}\right)^{1-\mu} \right]^{\frac{1}{1-\theta-\mu}} \qquad (4)$$

Finally, we obtain the expression for profits in terms of capital:

$$\pi = zk^{\theta} l^{\mu} - wl - (r+\delta)k$$

Replacing (1):

$$\pi = (1-\mu) \left[\left(\frac{\mu}{w}\right)^{\mu} zk^{\theta} \right]^{\frac{1}{1-\mu}} - (r+\delta)k \qquad (5)$$

Expression (3) is the profits of a firm in terms of capital, which can be the optimal level or lower. For unconstrained entrepreneurs, profits are negatively related to factor prices and positive related to managerial skill. We use (2) to get the optimal expression of profits:

[15] Remember that θ + μ < 1 then 1- θ- μ >0

$$\pi^* = (1-\mu-\theta)\left[\left(\frac{\mu}{w}\right)^{\mu} z \left(\frac{\theta}{r+\delta}\right)^{1-\mu}\right]^{\frac{1}{1-\theta-\mu}} \tag{6}$$

HOUSEHOLD'S UTILITY MAXIMIZATION PROBLEM

Individuals live for two periods. In the first one all individuals are workers and they supply their labour force inelastically. In their second period they must choose between remaining as workers or becoming a manager. We begin by solving the model for those individuals who decide to remain as workers in their second period.

WORKERS

Workers' utility maximization problem is given by:

$$\max_{\{c_1\, c_2\}} U = \log c_1 + \beta \log c_2$$

$$\text{s.t} \begin{cases} c_1 = w - a \\ c_2 = (1+r)a + w \\ a \geq 0 \end{cases}$$

We establish non-negativeness of savings as in Amaral et al. (2010). Maximizing in terms of c_1 and c_2 we get:

1) The interior solutions are characterized by the following expressions:

$$a^{workers} = \frac{w(\beta(1+r)-1)}{(1+r)(1+\beta)} \tag{7}$$

$$c_1^{workers} = \frac{w(2+r)}{(1+r)(1+\beta)} \tag{8}$$

$$c_2^{workers} = \frac{w\beta(2+r)}{(1+\beta)} \tag{9}$$

2) The corner solutions are characterized by the following ones:

$$a^{workers} = 0;\ c_1^{workers} = w; c_2^{workers} = w \tag{10}$$

ENTREPRENEURS

Similarly, entrepreneurs solve the following utility maximization problem:

$$\max_{\{c_1\ c_2\}} U = \log c_1 + \beta \log c_2$$

$$\text{s.t} \begin{cases} c_1 = w - a \\ c_2 = (1+r)a + \pi^*(a, z; w, r) \\ a \geq 0 \end{cases}$$

We establish non-negativeness of savings as in the workers' case. Maximizing in terms of c_1 and c_2 we get:

1) The interior solutions are characterized by the following expressions:

$$a^{entrepeneurs} = \frac{w\beta(1+r) - \pi}{(1+r)(1+\beta)} \tag{11}$$

$$c_1^{entrepeneurs} = \frac{w(1+r) + \pi}{(1+r)(1+\beta)} \tag{12}$$

$$c_2^{entrepeneurs} = \frac{\beta(w(1+r) + \pi)}{(1+\beta)} \tag{13}$$

1) The corner solutions are characterized by the following ones:

$$a^{entrepreneurs} = 0, \qquad c_1^{entrepreneurs} = w, \qquad c_2^{entrepreneurs} = \pi \tag{14}$$

OCCUPATIONAL CHOICE

In the second period individuals have an occupation choice. For a given level of contractual enforcement, wage and interest rate, $\pi(a+d, z; w, r)$ increases with z. It can be shown that there exists a unique ability threshold $\bar{z}$ at which individuals are indifferent between remaining as workers or becoming managers. If individuals are above the threshold, they become managers because the value function as entrepreneurs (which increases with z) is higher than the value function as worker[16]. Furthermore, it can also be shown that the level of k(z; η,w,r) is unique for each z, because for each z the managers select the solution where their own investment a is the highest, and then, the results for each z is going to be unique at that specific level z.

PARAMETERIZATION

In this section we are going to present the choice of the parameter values of the model. Some of the parameters are taken exogenously as given from other papers. Some other parameters will be calibrated.

EXOGENOUS PARAMETERS

Table 2 shows the parameters taken outside of the model, their values and the papers from where they have been taken:

TABLE 2

EXOGENOUS PARAMETERS

Parameters	Value	Paper
r	0.04	Bah & Fang (2015)
Zmin	1	Bah & Fang (2015)
Zmax	10	Bah & Fang (2015)
Gridz	100	Bah & Fang (2015)
α	0.33	Amaral & Quintin (2010)
δ	0.05	Bah & Fang (2015)
γ	0.85	Bah & Fang (2015)

Source: Own elaboration. Notice that r and δ are annualized values. See more in the text below.

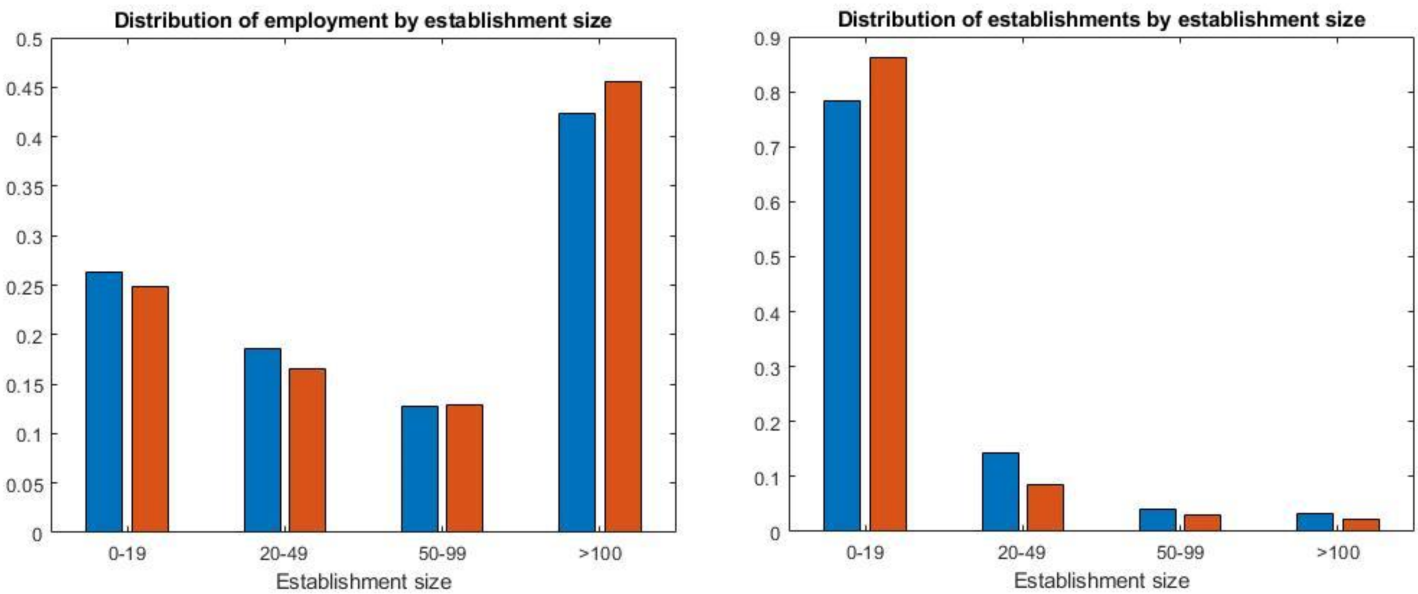
[16] Amaral et al. (2010), in Lemma 1 show in more detail the results of this occupational choice.

A period in the model correspond to 20 years. The values of the parameters δ and r are annualized. Parameter γ is the span-of-control, Lucas (1978), which in Bah & Fang is equal to 0.85. With respect to parameter α, this determines the capital share. In our model, the share of capital is αγ which is equal to θ, and share of labor is equal to (1- α)γ which is the parameter μ. With respect to Zmin and Zmax, those parameters correspond to the minimum and the maximum value of the managerial skill, respectively. Gridz is the grid of the managerial ability. As in Bah & Fang (2015), we consider that there are 100 levels of managerial ability between 1 and 10, both included.

PARAMETERS CALIBRATED

The economy is calibrated according to two main features of the US economy, the employment distribution along the establishment size, and the distribution of the establishments. In order to replicate these features, we calibrate the mean and the standard deviation of the managerial ability. As in Bah & Fang (2015), we assume that managerial skill follows a lognormal distribution with mean m, standard deviation σ. Next figure shows how our model matches the data compared to the US data. Orange color correspond to the US data and blue color to the model. So, we can say that the calibration of m and σ is quite good.

FIGURE 1



Source: Own elaboration. The graph of the left-hand side shows the distribution of establishment by establishment size, that is, the most of the establishments have less than 20 workers. On the right-hand side we have the employment distribution along the establishment size. Big establishments (with more than 100 workers) are the establishments which employ more workers.

We calibrate the parameter β as in Amaral et al. (2010) to get a ratio of Storage/Savings close to 40%, as observed in the US economy in 1994. The annualized Value of β is equal to 1.123[17].

TABLE 3

PARAMETERS CALIBRATED

Parameters	Value
m	-2.12
σ	0.382
β	1.123
η	0.7 (Amaral & Quintin)
	1 (Assets-based)
	0.79 (Earnings-based)

Source: Own elaboration.

With respect to the parameter η, we compute to the minimum value at which all firms produce at an optimal scale. Then, we have three different values of η, each for each of the three scenarios. At these levels or above, all firms are producing at an optimal scale because they are not constrained. So, the results at the optimal level are the same for the three scenarios, independently of the type of borrowing constraint. In all equilibria, the amount of storage is positive, therefore savings are always higher than physical capital, $S > K$.

RESULTS

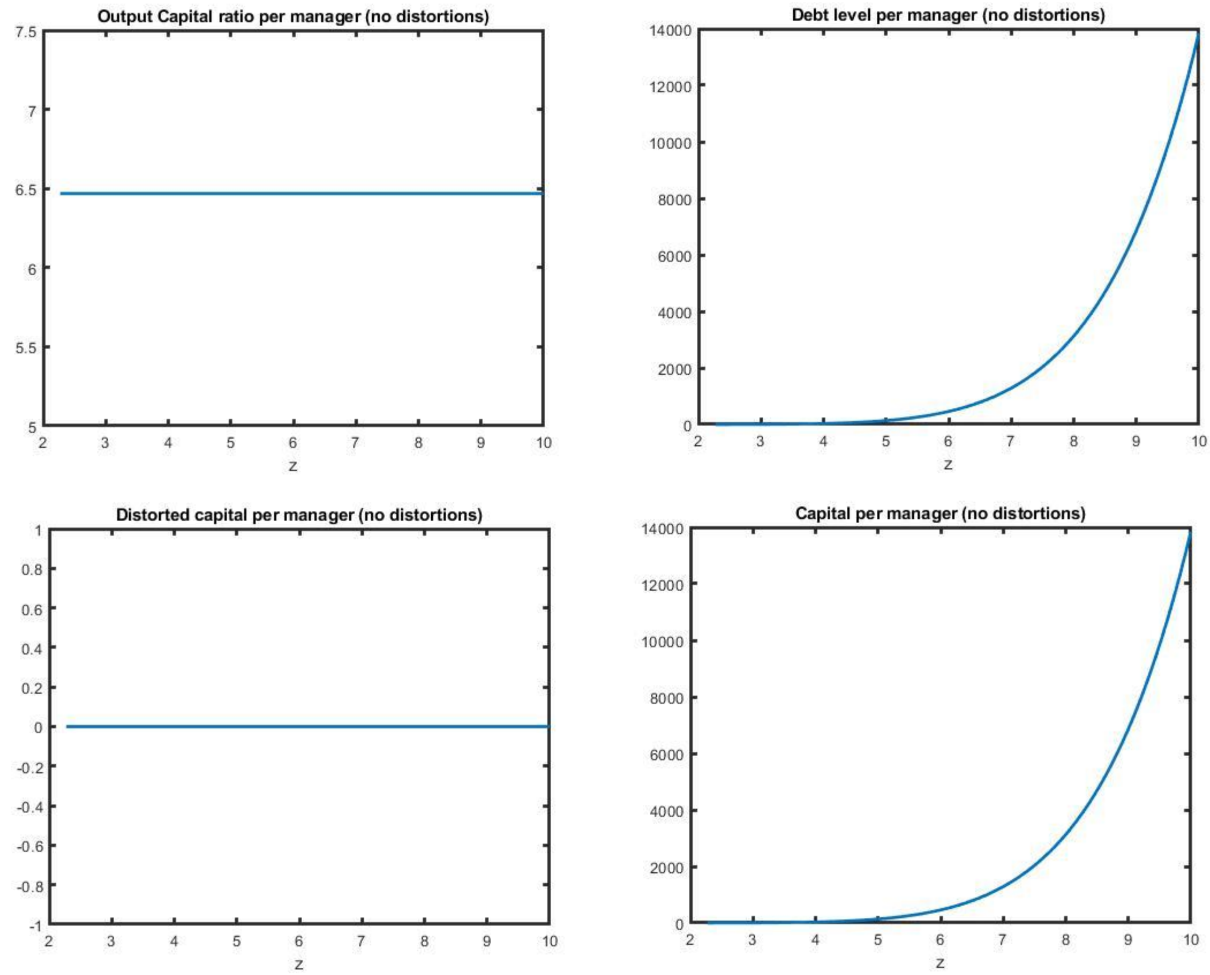
In this section we show the results of our quantitative exercise that have been computed using the Matlab software. We compare steady-state equilibria of different economies that differ on the type of loan contract and the quality of enforcement. For each scenario (type of loan contract), we decrease the quality enforcement (decreasing parameter η), starting from the case where all the firms produce at an optimal level. Then we compare the results along the three different frameworks. Firstly, we present our results at individual level, that is, at firm's level. Secondly, we present the results at aggregate level.

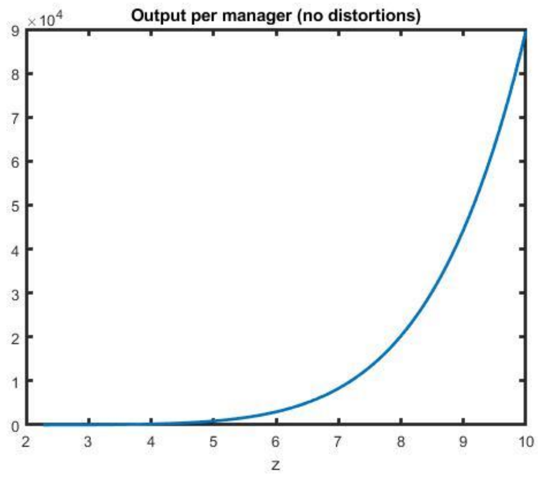
[17] In an OLG model the parameter β can be higher than one. Nonetheless, in a dynastic model β must be lower than one in order to fulfill the transversality condition.

INDIVIDUAL LEVEL

The first distortion of the financial friction is the misallocation of capital. More productive managers (managers with high managerial skill) demand more capital than less productive managers. Consequently, the most productive managers are going to borrow more from the financial market under the absence of financial frictions. However, if the contractual enforcement is low (high financial frictions), the debt limit decreases and these high productive managers cannot get all the capital they need to operate at an optimal scale. In order to measure this first distortion, we use the Output/Capital ratio as a proxy variable of the marginal product of capital. The next figure shows the Output/Capital ratio, debt level, capital, distorted capital (difference between the optimal and the level of capital demanded) and output across managerial levels, when there are no distortions.

FIGURE 2





Source: Own elaboration. X axis goes from 2.27 to 10 due to below 2.27 (ability threshold) individuals are workers and this figure records the results for managers. Then 2.27 is the ability threshold in the optimal case.

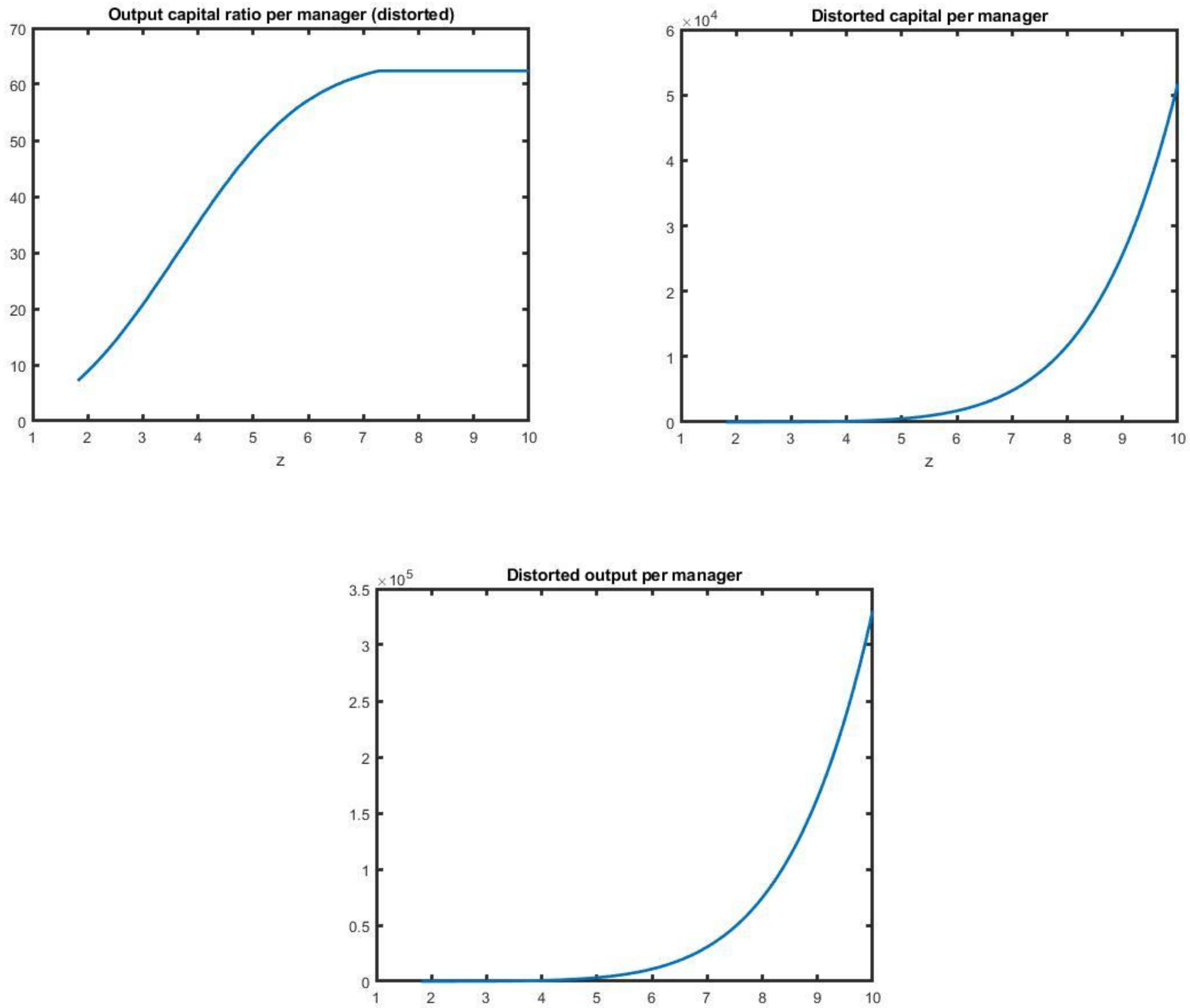
Looking at the first graph of Figure 2, we can see that the Output/Capital ratio is the same across entrepreneurs when they are producing at their optimal level. That is, the marginal product of capital is equalized across firms, because the resources are efficiently allocated. When firms are operating at an optimal scale, the distortion in capital is zero for all managers (third graph), that is, the difference between the optimal capital and the capital demanded across all managerial levels is zero. Furthermore, the second graph of Figure 2 shows that debt increases across managerial skills, that is, as we have just said, outside financing increases with productivity because higher the ability of the entrepreneur, higher his/her capital demand (fourth graph of Figure 2), and then he/she requires more to borrow. Finally, output is higher as manager's productivity increases, because these individuals manage more capital and use them to its best uses.

Once we have analyzed the case when the financial frictions are zero, we are focus on the distortions generated from the financial frictions for each type of loan contract.

AMARAL & QUINTIN BORROWING CONSTRAINTS

In this scenario, all firms produce at an optimal level when $\eta \geq 0.7$. As the value of η decreases, the financial frictions suffered by entrepreneur's increase. Figure 3 shows the quantitative results for a value η=0.08

FIGURE 3



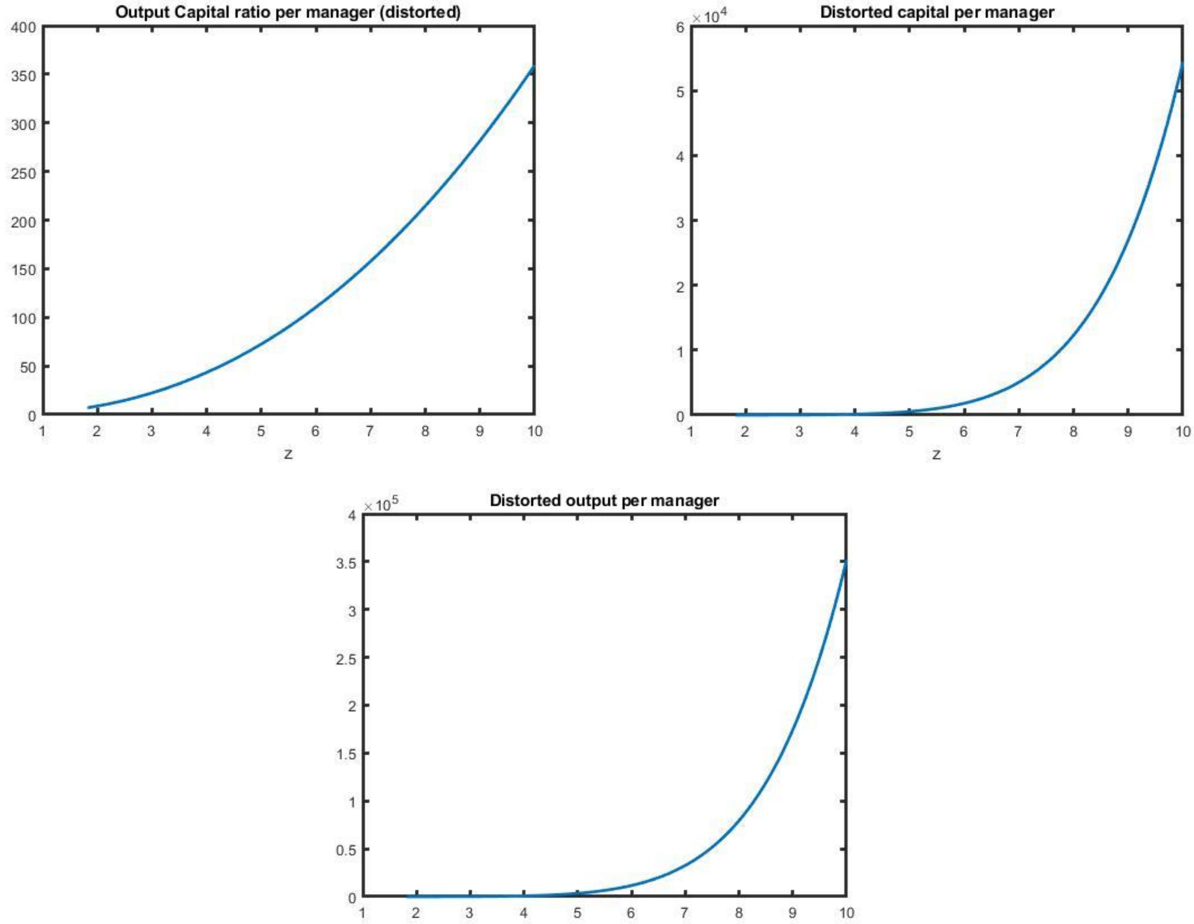
Source: Own elaboration.

In the first graph of Figure 3, we can observe that as the value of η decreases, less contracts become enforceable, and manager's access to outside financing get worse. The marginal product of capital is not equalized across firms. The most skilled managers are more constrained than less skilled managers and then their level of capital is further from the optimal case. For this reason, the Output/Capital ratio for the most skilled entrepreneurs is higher. This misallocation of resources is one of the distortions that arises because of financial frictions. The output/capital ratio would be equal across entrepreneurs if more capital would be used among more productive entrepreneurs. The second graph shows the distortion in the capital demanded, and this distortion increases with the skill level. The distortion of less productive entrepreneurs is very low, because their optimal capital demand is lower than for more productive managers. These less skilled managers can self-finance all the capital they demand and if they go to the financial market, they are going to borrow and small amount (compared with more productive managers) due to the fact that the outside financing increases with managerial ability. So, more productive entrepreneurs operate at a suboptimal scale, meanwhile less productive entrepreneurs produce at an optimal scale, being a misallocation of the capital. Finally, the last graph shows how the distortion in the output is higher for the most skilled managers. Consequently, the lost in the output for these entrepreneurs is very high[18].

ASSETS-BASED BORROWING CONSTRAINT

Now we focus in the assets-based borrowing constraint case. Under this framework, all firms produce at an optimal level when $\eta \geq 1$. As the value of η decreases, the financial frictions suffered by entrepreneur's increase. Figure 4 shows the quantitative results for a value of η=0.23.

FIGURE 4



Source: Own elaboration.

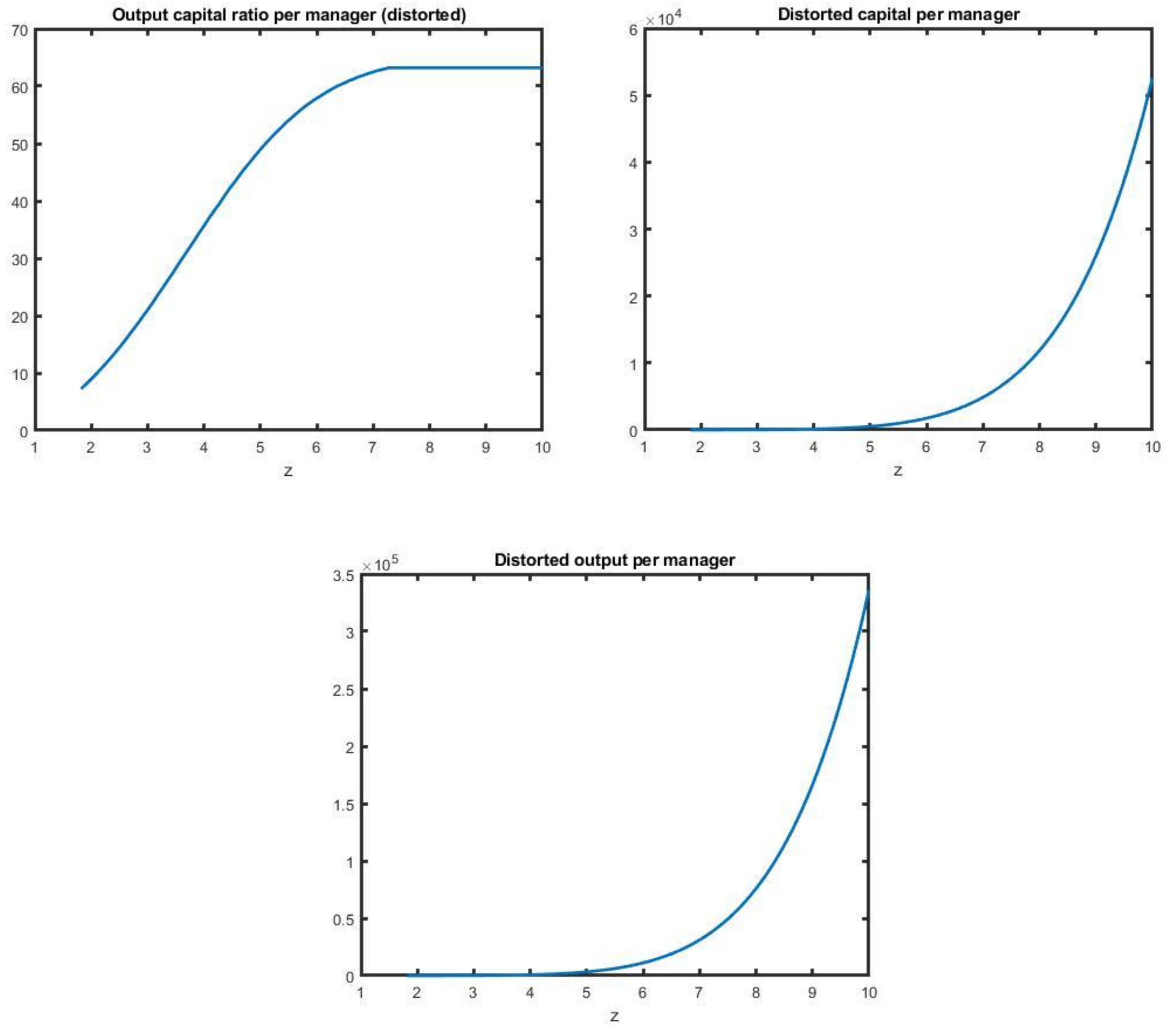
[18] Interesting to remark that when a firm demands the optimal level of capital, the labor demand is also optimal. However, when the level of capital demanded is suboptimal, since labor demand depends on the capital level, labor demand is also below the optimal level.

These results are very similar to the previous framework.

EARNINGS-BASED BORROWING CONSTRAINT

Finally, we see the results for earnings-based borrowing constraint case. Under this framework, all firms produce at an optimal level when η ≥ 0.79. As the value of η decreases, the financial frictions suffered by entrepreneurs increase. Figure 5 shows the quantitative results for a value of η=0.08.

FIGURE 5



Source: Own elaboration.

As we can observe, Figure 5 reports similar results as in the two previous cases.

The misallocation of capital is one of the two distortions at individual level. **The second distortion** is on the occupational choice. When the contractual enforcement decreases (η decreases), the equilibrium wage rate decreases, and the opportunity cost of becoming in manager decreases. So, more individuals become entrepreneurs. As a consequence, the ability threshold decreases and individuals with lower ability become managers. Then, less productive technologies need to be activated for labor market to clear. Now, not only we have more entrepeneurs with low ability operating, in addition, managers with low ability but high savings can borrow more capital and operate at an optimal scale, while managers with high ability but low savings cannot acquire enough capital and they operate at a suboptimal scale. Table 5 shows the fall in ability threshold and the increasing in the percentage of entrepreneurs due to the decrease in the contractual enforcement.

TABLE 5

η	Framework	Ability Threshold	Entrepreneurs %
Optimal case	The Three scenarios	2.27	8.23
0.08	Amaral & Quintin	1.818	23.97
0.23	Assets-based	1.818	23.97
0.08	Earnings-based	1.818	23.97

Source: Own elaboration.

AGGREGATE LEVEL

Once we have presented the effects of the financial frictions at individual level, we start showing the consequences at aggregate level, focusing on the effect of the GDP per capita, TFP and average size of the plants. Then we conclude our quantitative experiments doing a comparison among the different loan type contracts.

TFP, it is obtained from the aggregate production function, $Y = AK^{\theta}L^{\mu}$. A corresponds to the aggregate productivity, that is TFP is the sum of all particular managerial skill levels. Y is the aggregate output, K the aggregate capital and L the aggregate number of workers. Then:

$$TFP = \frac{Y}{K^{\theta}L^{\mu}}$$

Next tables show the results of our aggregates variables of interest for the three frameworks the contractual enforcement increases. We start from a low value of the parameter η and we increase until the value at which there are no financial frictions. Values of η not need to be the same across the three scenarios. For the same contractual enforcement level, depending on the loan contract the results can be different[19]. However, for the three scenarios the results are exactly the same when for each case the financial frictions are zero. This is because in each scenario all the firms are producing at an optimal scale, they are not constrained by the debt limit (the debt limit is not binding).

TABLE 6

Amaral & Quintin borrowing constraint

η	GDP per capita	Debt/GDP ratio	TFP	% of entrepreneurs	Plants' average size
0.27	1.0838	0.0439	1.9636	15.74	11.7050
0.3	1.1064	0.0482	1.9750	15.74	11.7069
0.33	1.1269	0.0521	1.9864	15.74	11.7014
0.4	1.1653	0.0653	2.0052	12.70	14.7428
0.44	1.1872	0.0709	2.0153	12.70	14.7473
0.5	1.2115	0.0834	2.0200	10.23	18.5319
0.55	1.2321	0.0908	2.0273	10.23	18.5342
0.6	1.2447	0.1024	2.0230	8.23	23.2763
0.65	1.2591	0.1099	2.0261	8.23	23.2929
0.7	1.2675	0.1155	2.0270	8.23	23.2796

Source: Own elaboration.

Table 6 shows the aggregate results for Amaral & Quintin borrowing constraint case. When the contractual enforcement increases, GDP per capita, TFP, Debt/GDP ratio and plant average size increase. Lower financial frictions reduce capital distortions because entrepreneur's debt limit increases. Consequently, more productive managers can manage more capital, reducing the distortions they face. This fact increases the aggregate productivity, TFP, which contributes to increase GDP. Furthermore, the correlation between the parameter η and the Debt/GDP ratio is almost one as in Amaral et al. (2010). Moreover, the percentage of entrepreneurs is negatively related to contractual enforcement, which is compatible with the results at individual level. Higher the contractual enforcement, higher the ability threshold and lower the percentage of entrepreneurs. Managers with more talent hire more workers and consequently the average size of establishments increases.

[19] The values of η chosen for each case are not the unique equilibria. These values have been chosen to illustrate the relationship between η and the variables of interest.

TABLE 7

Assets-based borrowing constraint

η	GDP per capita	Debt/GDP ratio	TFP	% of entrepreneurs	Plants' average size
0.55	1.008	0.042	1.845	19.45	9.278
0.6	1.027	0.046	1.856	19.45	9.283
0.63	1.038	0.048	1.864	19.45	9.277
0.71	1.054	0.058	1.870	15.74	11.703
0.75	1.074	0.062	1.884	15.74	11.710
0.8	1.099	0.067	1.903	15.74	11.711
0.85	1.115	0.076	1.910	12.70	14.743
0.9	1.149	0.082	1.938	12.70	14.750
0.95	1.184	0.094	1.962	10.23	18.544
1	1.267	0.115	2.027	8.23	23.280

Source: Own elaboration.

Table 7 shows the results for the Assets-based scenario. As we can see, the results are similar to the previous case. The relationship of contractual enforcement with respect to the aggregate variables is positive, except for the percentage of entrepreneurs. We can appreciate that the results for η=1 are the same as in the previous scenario when η=0.7, because at those values the firms are producing optimally.

TABLE 8

Earnings-based borrowing constraint

η	GDP per capita	Debt/GDP ratio	TFP	% of entrepreneurs	Plants' average size
0.34	1.119	0.050	1.984	15.74	11.709
0.4	1.148	0.061	2.000	12.70	14.743
0.44	1.169	0.066	2.009	12.70	14.743
0.48	1.188	0.071	2.017	12.70	14.739
0.54	1.208	0.082	2.020	10.23	18.535
0.59	1.227	0.089	2.026	10.23	18.541
0.63	1.239	0.094	2.030	10.23	18.538
0.69	1.251	0.105	2.025	8.23	23.289
0.74	1.261	0.111	2.026	8.23	23.299
0.79	1.267	0.115	2.027	8.23	23.280

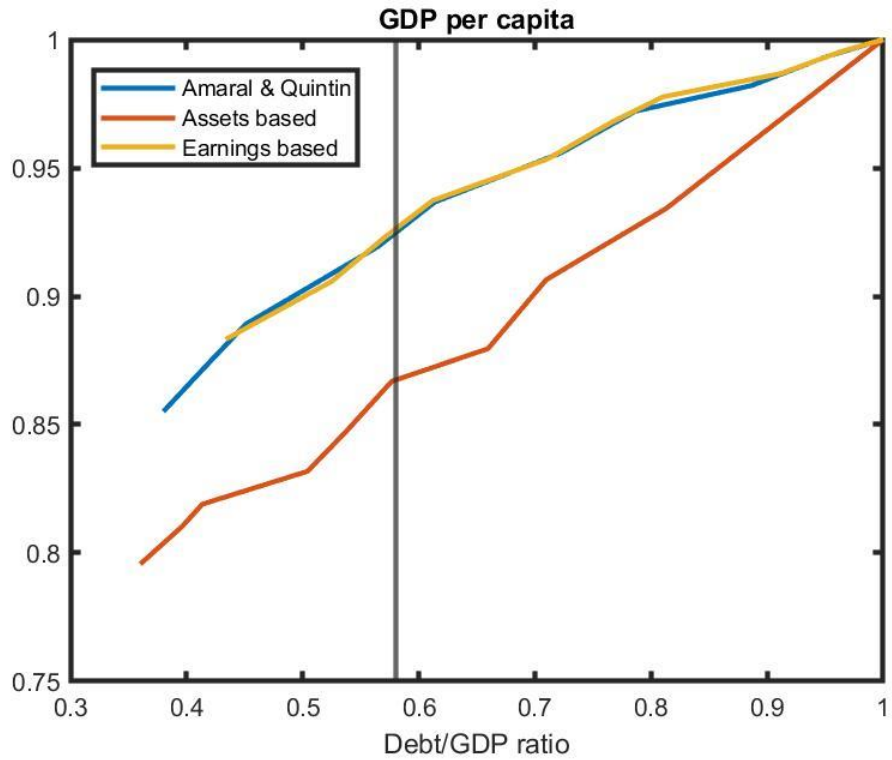
Source: Own elaboration.

Finally, the results for earnings-based case are similar to the two previous scenarios. The optimal situation of this case (η=0.79) also gives the same results as the two previous cases.

Next graphs plot the GDP per capita, TFP and plants average size against Debt//GDP ratio for each framework. We use Debt/GDP ratio because, for each type of loan contract, this empirical measure increases monotonically with contractual enforcement[20]. The four variables are indexed to their respective highest value, that is, for each variable the maximum value is one. Furthermore, we fixed a vertical line at a specific value of Debt/GDP ratio in order to emphasize the differences across the three scenarios.

[20] From now on we use the Debt/GDP ratio as a proxy variable of the contractual enforcement, as in Amaral & Quintin (2010).

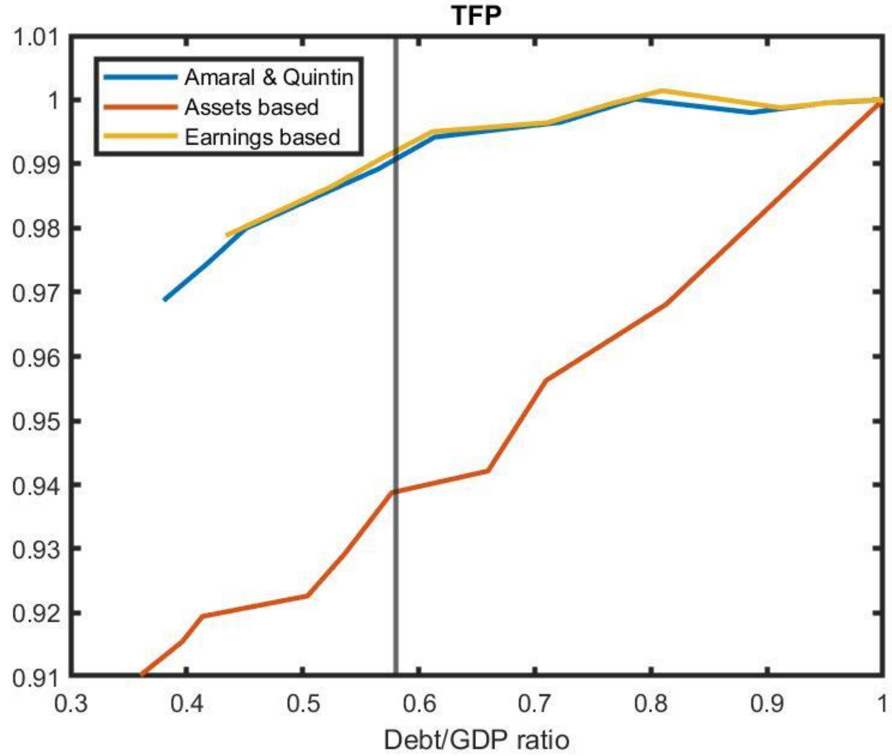
FIGURE 6



Source: Own elaboration.

Figure 6 shows the evolution of the GDP per capita when the Debt/GDP ratio increases for each of the frameworks. For the three cases, the relationship is positive. Higher the Debt/GDP ratio, closer is the GDP per capita to its optimal level. However, we can observe that the distortion in GDP per capita is higher under the assets-based borrowing constraint case. When the Debt/GDP ratio is 0.58 of its optimal value, GDP per capita under assets-based scenario falls until 0.87 of the optimal per capita GDP, while for the other two scenarios fall approximately until 0.93. As in Li (2016), when earnings are not pledgeable, the borrowing capacity of a firm is proportional of its own resources and does not vary with the firm's productivity. On the other hand, when earnings are pledgeable, more productive firms have higher earnings and hence can borrow more than less productive firms, even if they all have the same inside funds. Finally, for earnings-based and Amaral & Quintin cases the relation between GDP per capita and Debt/GDP ratio is concave, meanwhile for assets-based case is linear.

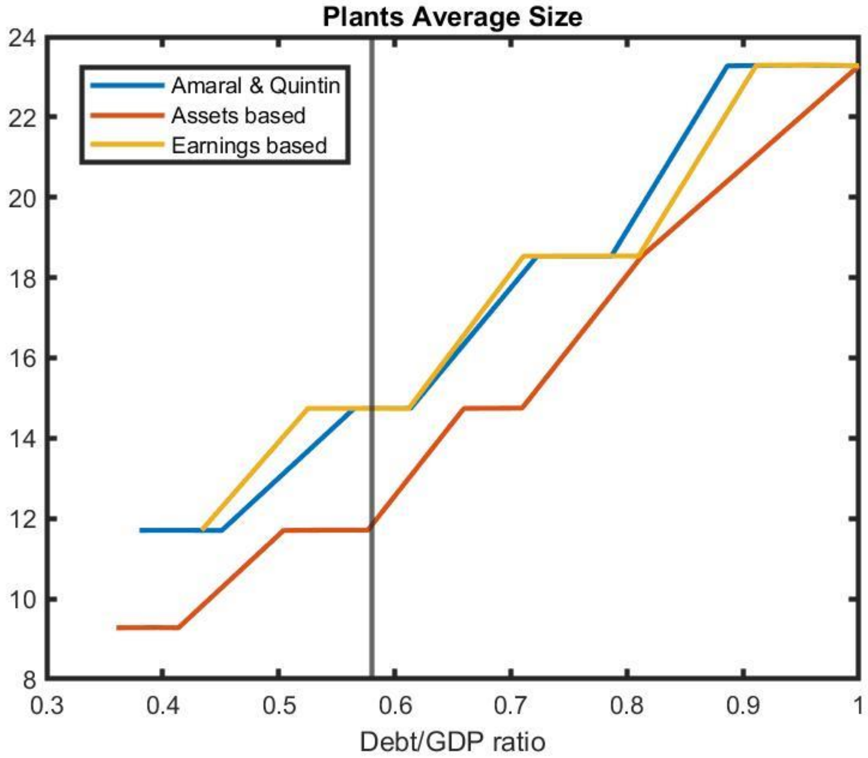
FIGURE 7



Source: Own elaboration.

Figure 7 shows the relationship between TFP and the Debt/GDP ratio. The results are similar to those shown in Figure 6 GDP. Positive relationship in the three scenarios, but in the assets-based case the distortion is higher than for the other two scenarios. Furthermore, the shape of the curves for each case are the same as in the previous Figure. So, financial frictions generate misallocation of the resources that reduce the aggregate productivity, TFP, causing a decrease in the per capita GDP.

FIGURE 8



Source: Own elaboration.

Finally, Figure 8 shows the relationship between the plants' average size and the Debt/GDP ratio. We can observe a clear positive relationship, what is consistent with the results at individual level. When the Debt/GDP ratio rises, more individuals remain as workers because the opportunity cost of become in a manager increases. Consequently, the percentage of entrepreneurs decreases (as we have showed Tables 6-8) and the firm's size increase. Once more time, the distortion under assets-based case is the highest.

CONCLUSIONS

We show a clear positive relationship between GDP per capita, TFP, plant average size and the financial development for a given type of lending contract. Consequently, we can ensure that financial frictions are one of the principal causes of misallocation of resources that affect TFP, as in Bah & Fang (2015) and Amaral et al. (2010). Moreover, borrowing constraint based on assets distort more compared with borrowing constraints where the earnings are pledgeable. So, assets-based borrowing constraint causes an overstatement of the TFP and per capita GDP loss due to the financial frictions. Earnings increase with the productivity, so if earnings are pledgeable then more productive managers can borrow more from the financial market.

For the same level of Debt/GDP ratio, GDP per capita is lower under the assets-based borrowing constraint. Under assets-based borrowing constraint, more productive entrepreneurs are more constrained than under the earnings-based case, because they cannot pledge their earnings. So, if there is a decrease in the financial development, the effect of misallocation of resources is higher under assets-based scenario, what generates a decrease in the TFP and as a consequence, in the GDP per capita. Furthermore, the gain in increasing the financial development level (the degree at which loan contracts can be enforced) is different across the three frameworks. The gain is more concave under Amaral & Quintin and earnings-based borrowing constraints, and more linear under the assets-based borrowing constraint. So, increases in the degree at which loan contracts can be enforced are proportionally higher under assets-based case. According to Chen & Ma (2019), in the US only 20% of the firms have covenants based on assets. This fact suggests that traditional literature, which has considered assets-based as the main borrowing constraint, has overstated the effects of financial frictions in the aggregate productivity and per capita GDP. However, the access of small firms and less developed countries' firms to earnings-based loans is more difficult. Therefore, to quantify the gain in TFP and per capita GDP of increasing the degree at which loan contracts can be enforced should take into account the proportion of the different type of loan contracts in the economy. In conclusion, we could quantify the effects of different loan contracts, and the results have been consistent with the disposable literature.

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APPENDICES

APPENDIX 1

FIRMS MAX. PROBLEM

Optimal labor demand in terms of capital.

$$\max_{\{l\}} \pi = y - wl - (r+\delta)k$$

$$\text{s.t } y = zk^{\theta} l^{\mu}$$

$$\max_{\{l\}} \pi = zk^{\theta} l^{\mu} - wl - (r+\delta)k$$

F.O.C:

$$\frac{\partial \pi}{\partial l} = 0 \rightarrow \mu z k^{\theta} l^{\mu-1} = w$$

$$l^* = \left[\frac{\mu z k^{\theta}}{w}\right]^{\frac{1}{1-\mu}} \qquad (1.1)$$

a) The optimal capital demand when entrepreneurs are unconstrained

$$\max_{\{k\}} \pi = zk^{\theta} l^{\mu} - wl - (r+\delta)k$$

F.O.C:

$$\frac{\partial \pi}{\partial k} = 0 \rightarrow \theta z k^{\theta-1} l^{\mu} = r + \delta$$

Using optimal labor demand expression (1.1):

$$\theta z k^{\theta-1} \left[\frac{\mu z k^{\theta}}{w}\right]^{\frac{\mu}{1-\mu}} = r + \delta$$

$$k^* = \left[\left(\frac{\mu}{w}\right)^{\mu} z \left(\frac{\theta}{r+\delta}\right)^{1-\mu}\right]^{\frac{1}{1-\theta-\mu}} \qquad (2.1)$$

Profits for constrained and unconstrained entrepreneurs:

$$\pi = zk^{\theta}l^{\mu} - wl - (r+\delta)k \;\rightarrow \pi = zk^{\theta}\left[\frac{\mu zk^{\theta}}{w}\right]^{\frac{\mu}{1-\mu}} - w\left[\frac{\mu zk^{\theta}}{w}\right]^{\frac{1}{1-\mu}} - (r+\delta)k$$

$$\pi = (1-\mu)\left[\left(\frac{\mu}{w}\right)^{\mu} zk^{\theta}\right]^{\frac{1}{1-\mu}} - (r+\delta)k$$

Replacing optimal capital demand expression (2.1) we obtain profits for unconstrained firms:

$$\pi^{*} = (1-\mu-\theta)\left[\left(\frac{\mu}{w}\right)^{\mu} z\left(\frac{\theta}{r+\delta}\right)^{1-\mu}\right]^{\frac{1}{1-\theta-\mu}}$$

Output for constrained and unconstrained entrepreneurs is obtained using optimal labor demand function (1.1):

$$y = zk^{\theta}l^{\mu}$$

$$y = \left[\left(\frac{\mu}{w}\right)^{\mu} zk^{\theta}\right]^{\frac{1}{1-\mu}}$$

Output for unconstrained entrepreneurs can be obtained replacing the optimal level of capital (2.1):

$$y^{*} = \left[\left(\frac{\mu}{w}\right)^{\mu} z\left(\frac{\theta}{r+\delta}\right)^{1-\mu}\right]^{\frac{1}{1-\theta-\mu}}$$

ENDOGENOUS DEBT LIMIT

AMARAL & QUINTIN BORROWING CONSTRAINT

The incentive-compatibility constraint is given by the following expression:

$$\pi + a(1+r) \geq (1-\eta)[\pi + (a+d)(1+r)]$$

$$\frac{\pi + a(1+r)}{1-\eta} \geq \pi + (a+d)(1+r)$$

$$\frac{\pi + a(1+r)}{(1-\eta)(1+r)} - \frac{\pi + a}{1+r} \geq d$$

Firm's maximum level of debt would be:

$$d \leq \frac{\eta}{(1-\eta)(1+r)}[\pi + a(1+r)]$$

ASSETS BASED BORROWING CONSTRAINT

The incentive-compatibility constraint is given by the following expression:

$$\pi + a(1+r) \geq \pi + (1-\eta)(a+d)(1+r)$$

Where:

$$k = a + d$$

$$a(1+r) \geq (1-\eta)(a+d)(1+r)$$

$$\frac{a(1+r)}{(1-\eta)(1+r)} - a \geq d$$

Firm's maximum level of debt would be:

$$d \leq \frac{\eta}{(1-\eta)}a$$

EARNINGS BASED BORROWING CONSTRAINT

The incentive-compatibility constraint is given by the following expression:

$$\pi_n - (r+\delta)k + a(1+r) \geq [(1-\eta)\pi_n - (r+\delta)k] + (a+d)(1+r)$$

$$\pi_n - (r+\delta)k + a(1+r) \geq \pi_n - \eta\pi_n - (r+\delta)k + a(1+r) + d(1+r)$$

$$\eta\pi_n \geq d(1+r)$$

Firm's maximum level of debt would be:

$$d \leq \frac{\eta}{(1-r)}\pi_n$$

APPENDIX 2

In order to check that our results correspond to the steady-state equilibrium values, we prove not only that the labour market clears, but also that there is equilibrium in the goods market. That is, aggregate GDP must equal aggregate demand. Aggregate GDP is given by:

$$\mathrm{Ys} = \mathrm{Y} + (\mathrm{r} + \delta)\mathrm{Storage}.$$

On the other hand, the aggregate demand:

$$Y_d = C_1 + C_2 + \delta S$$

Where S is aggregate savings, and C_1 and C_2 are aggregate consumptions in period 1 and period 2, respectively. Finally, we build GDP per capita as GDP/2, because the population in each period is equal two and the grow rate of population is zero.

APPENDIX 3

Managers select the solution where their own investment a is the highest, and if they can self-finance all their capital they do so, even though they could choose to borrow some of it. This is because the marginal product of capital exceeds its opportunity cost (1+r) in establishments operated at a suboptimal scale. When we solve firm's profit maximization problem, we obtain that marginal product of the capital equals to its price.

$$MPK = r + \delta$$

However, when the firm is operated at a suboptimal scale:

$$k^* > k$$

Where k^* is the optimal capital and k the level of capital demanded by the firm when is operating at a suboptimal scale. Then:

$$MPK^* < MPK$$

Taking into account that:

$$MPK^* = r + \delta$$

Then:

$$MPK > r + \delta$$

So, entrepreneurs will invest all their savings in physical capital because the net return of doing this is higher than the net return to invest in the Storage technology, which is 1+r.

APPENDIX 4

<u>WORKERS MAX. PROBLEM</u>

$$\max_{\{c_1\, c_2\}} U = \log c_1 + \beta \log c_2$$

$$\text{s.t} \begin{cases} c_1 = w - a \\ c_2 = (1+r)a + w \\ a \geq 0 \end{cases}$$

$$\max_{\{a \geq 0\}} U = \log(\mathrm{w} - \mathrm{a}) + \beta \log(a(1+r) + w)$$

Lagrange function:

$$£ = \log(\mathrm{w} - \mathrm{a}) + \beta \log(a(1+r) + w) + \lambda(a - 0)$$

Kuhn-Tucker F.O.C:

$$\frac{\partial £}{\partial a} = 0 \rightarrow \ -\frac{1}{c_1} + \frac{\beta(1+r)}{c_2} + \lambda = 0$$

$$\lambda(a - 0) = 0 \text{ and } \lambda \geq 0$$

1. Interior solution:

$\lambda = 0 \ \rightarrow a \geq 0$, and from (5) we have the Euler's equation:

$$c_2 = \beta(1+r)c_1$$

Then the optimal solutions are given by:

$$a^{workers} = \frac{w(\beta(1+r) - 1)}{(1+r)(1+\beta)}$$

$$c_1^{workers} = \frac{w(2+r)}{(1+r)(1+\beta)}$$

$$c_2^{workers} = \frac{w\beta(2+r)}{(1+\beta)}$$

2. Corner solution:

$\lambda \geq 0 \ \rightarrow a = 0$, and from (5) we have:

$$\frac{\beta(1+r)}{c_2}+\lambda=\frac{1}{c_1} \rightarrow \lambda \geq 0 \rightarrow \frac{\beta(1+r)}{c_2} \leq \frac{1}{c_1} \rightarrow c_2 \geq \beta(1+r)c_1$$

$$a^{workers}=0; c_1^{workers}=w; c_2^{workers}=w$$

ENTREPENEURS MAX. PROBLEM

$$\max_{\{c_1\, c_2\}} U=\log c_1+\beta \log c_2$$

$$\text{s.t}\begin{cases} c_1=w-a \\ c_2=(1+r)a+\pi \\ a \geq 0 \end{cases}$$

$$\max_{\{a \geq 0\}} U=\log(\mathrm{w}-\mathrm{a})+\beta \log(a(1+r)+\pi)$$

$$£=\log(\mathrm{w}-\mathrm{a})+\beta \log(a(1+r)+\pi)+\lambda(a-0)$$

F.O.C:

$$\frac{\partial £}{\partial a}=0 \rightarrow -\frac{1}{c_1}+\frac{\beta(1+r)}{c_2}+\lambda=0$$

$$\lambda(a-0)=0 \text{ and } \lambda \geq 0$$

1. Interior solution:

$\lambda=0 \rightarrow a \geq 0$, and from (10) we have the Euler's equation:

$$c_2=\beta(1+r)c_1$$

Then the optimal solutions are given by:

$$a^{entrepeneurs}=\frac{w\beta(1+r)-\pi}{(1+r)(1+\beta)}$$

$$c_1^{entrepeneurs}=\frac{w(1+r)+\pi}{(1+r)(1+\beta)}$$

$$c_2^{entrepeneurs}=\frac{\beta(w(1+r)+\pi)}{(1+\beta)}$$

2. Corner solution:

$\lambda \geq 0 \ \rightarrow a = 0$, and from (5) we have:

$$\frac{\beta(1+r)}{c_2} + \lambda = \frac{1}{c_1} \ \rightarrow \lambda \geq 0 \rightarrow \frac{\beta(1+r)}{c_2} \leq \frac{1}{c_1} \rightarrow c_2 \geq \beta(1+r)c_1$$

$$a^{entrepeneurs} = 0; c_1^{entrepeneurs} = w; c_2^{entrepeneurs} = \pi$$