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Transient thermal parameters correlation of spacecraft thermal models against test results

Iñaki Garmendia [a,*], Eva Anglada [b]

[a] Mechanical Engineering Department, University of the Basque Country UPV/EHU, Engineering School of Gipuzkoa. Plaza de Europa, 1. E-20018 Donostia, San Sebastián, Spain

[b] TECNALIA, Basque Research and Technology Alliance (BRTA), Mikeletegi Pasealekua, 2, E-20009, Donostia, San Sebastián, Spain

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ABSTRACT

There are always differences between the computer predicted temperatures of a spacecraft thermal model and the temperatures measured during the mandatory laboratory thermal tests. As a consequence, the model must be correlated before the spacecraft is launched to space, in order to identify the correct parameters that match the experimentally measured temperatures.

A new technique is presented to identify the parameters, based on the minimization of the error of the transient equations which governs the heat transfer in the spacecraft. The steady state minimization was presented in a previous work, but the transient techniques presented hereafter enable a better and more extended identification of parameters despite the higher complexity of the computational problem.

The use of a set of available subroutines (TOLMIN), which permits the constrained optimization of a general function, makes possible to ensure that the obtained parameters are non-negative, a requirement to have physical sense. The gradient function must be calculated for each problem, but this can be done automatically.

Results show that for small and medium size transient Thermal Mathematical Models (TMM), a good correlation of thermal parameters can be achieved even if some of the nodes temperatures are not measured in the thermal tests.

1. Introduction

All space missions are very demanding from a technological point of view. The external conditions that are to be found in space make any space mission very challenging. Spacecraft Thermal Control is not an exception to this general rule and, as a consequence, all the design, fabrication and test phases before launching must be dealt with very carefully [1–3].

The usual way to design the thermal control subsystem of a space mission comprises several activities. One of these tasks is the elaboration of the Thermal Mathematical Model (TMM), which represents, from a computational point of view, the thermal behaviour of the spacecraft or payload under design. Several load cases can be studied with this model (hot case, cold case, transient cases …) in such a way that the temperatures of different parts of the spacecraft can be predicted with reasonable accuracy. Then, these predicted temperatures that are foreseen for different on orbit scenarios, can be compared with the maximum or minimum allowable temperatures of the spacecraft components. This way the thermal engineers can assess if the thermal design is appropriate for the different situations that are to be found [4,5].

Nevertheless, the computational models (TMMs), even if they are carefully constructed, can have errors, or can be poor when predicting temperatures in some cases. A thermal test campaign is, for this reason, always needed in the thermal control design.

The different thermal tests try to represent the most extreme thermal cases that the spacecraft will find in its mission. Temperatures and other parameters are then measured in tests and compared with the results predicted by the TMMs. As it could be expected, there are always differences between both sets of temperatures (measured and computationally calculated ones) and the origin of these mismatches is to be investigated and corrected [6–10].

It is generally accepted that the origin of the differences can be attributed, on the one hand, to the inherent errors of experimental measurements and, on the other hand, to some miscalculations on the parameters that define the TMMs. Different techniques can be used for

* Corresponding author. Engineering School of Gipuzkoa, Plaza de Europa, 1, E-20018, Donostia, San Sebastián, Spain.
E-mail addresses: inaki.garmendia@ehu.es (I. Garmendia), eva.anglada@tecnalia.com (E. Anglada).

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parameters identification (Genetic Algorithms, Gradient Based Algorithms …) [11–13].

In previous works by the authors, gradient based techniques were used in steady state parameter identification, with some encouraging results for small and medium sized thermal models [14]. However, difficulties appeared also when identifying the real thermal parameters of the TMMs, as there is not always a unique solution for the minimization problem that gives the real thermal parameters.

In this paper, we will study the improvements that can be achieved in the thermal parameters identification problem when using transient thermal tests results and their corresponding transient thermal temperatures coming from the TMM's.

2. Thermal Mathematical Models (TMM)

The set of nonlinear equations that describe the temperatures of a transient thermal mathematical model for a node i is given by Eq. (1).

$$\sum_{j=1}^{j=n} GL(i,j)\left(T_i - T_j\right) + \sum_{j=1}^{j=n} \sigma GR(i,j)\left(T_i^4 - T_j^4\right) + M_i C_i \frac{dT_i}{dt} = q_i \tag{1}$$

where n is the number of nodes of the TMM, $GL(i,j)$ is the conductive conductance (W/m) between nodes i and j, σ is the Stefan-Boltzmann constant ($5.67 \cdot 10^{-8}$ W/(m$^2\cdot$K^4)), $GR(i,j)$ is the radiative conductance (m^2) between nodes i and j, T_i and T_j are the temperatures (K) of nodes i and j, M_iC_i is the product of the i node mass (kg) times the heat capacity (J/(kg·K)) and q_i is the power (W) that enters into node i. The subscripts i and j go from 1 to n. It is usual to call thermal inertia to the product M_iC_i as it describes the "opposition" to change the temperature of i node when subjected to a power input.

The time derivative of the temperature of node i can be approximated by Eq. (2).

$$\frac{dT_i}{dt} = \frac{T_i^{t+\Delta t} - T_i^t}{\Delta t} \tag{2}$$

For a node i and for a general time step $(t + \Delta t)$ it is possible to write Eq. (3).

$$\sum_{j=1}^{j=n} GL(i,j)\left(T_i^{t+\Delta t} - T_j^{t+\Delta t}\right) + \sum_{j=1}^{j=n} \sigma GR(i,j)\left({T_i^4}^{t+\Delta t} - {T_j^4}^{t+\Delta t}\right) + M_i C_i \frac{\left(T_i^{t+\Delta t} - T_i^t\right)}{\Delta t} - q_i = 0 \tag{3}$$

In order to simplify the notation, Eq. (4) will be used, obtaining Eq. (5).

$$T_i^{t+\Delta t} = T_i \; ; \quad T_i^t = T_i^t \tag{4}$$

$$\sum_{j=1}^{j=n} GL(i,j)\left(T_i - T_j\right) + \sum_{j=1}^{j=n} \sigma GR(i,j)\left(T_i^4 - T_j^4\right) + M_i C_i \frac{\left(T_i - T_i^t\right)}{\Delta t} - q_i = 0 \tag{5}$$

For a general time step $(t + \Delta t)$ and for one load case, the set of n equations that must be solved is shown in (6):

$$\left.\begin{array}{l} GL(1,2)(T_1 - T_2) + \ldots + GL(1,n)(T_1 - T_n) + \sigma \cdot GR(1,2)\left(T_1^4 - T_2^4\right) + \ldots + \sigma \cdot GR(1,n)\left(T_1^4 - T_n^4\right) + \dfrac{M_1 C_1\left(T_1 - T_1^t\right)}{\Delta t} - q_1 = 0 \\ GL(i,1)(T_i - T_1) + \ldots + GL(i,n)(T_i - T_n) + \sigma \cdot GR(i,1)\left(T_i^4 - T_1^4\right) + \ldots + \sigma \cdot GR(i,n)\left(T_i^4 - T_n^4\right) + \dfrac{M_i C_i\left(T_i - T_i^t\right)}{\Delta t} - q_i = 0 \\ GL(n,1)(T_n - T_1) + \ldots + GL(n,n-1)(T_n - T_{n-1}) + \sigma \cdot GR(n,1)\left(T_n^4 - T_1^4\right) + \ldots + \sigma \cdot GR(n,n-1)\left(T_n^4 - T_{n-1}^4\right) + \dfrac{M_n C_n\left(T_n - T_n^t\right)}{\Delta t} - q_n = 0 \end{array}\right\} \tag{6}$$

This set of equations is solved for each time step $(t + \Delta t)$, taking into account the temperature distribution in the previous time step. For the first time step, initial temperatures at time 0 are used.

It is interesting to note that the number of *GLs* and *GRs* different from 0 are less than their maximum number value $(n \cdot (n-1)/2)$, so the number of terms of Eq. (6) that are different from 0 is in general small. It has also to be noted that if boundary conditions (temperatures) are imposed in *NSINK* nodes, the corresponding equations disappear from the set of non-linear Eq. (6), making a total of $(n - NSINK)$ equations for each time step. Finally, in the general approach that has been adopted to solve this transient problem, it has been assumed that Δt is constant during the whole calculation. This has been done by simplicity, but it is not strictly necessary.

3. TMM correlation methodology

As explained in Ref. [14], the TMM correlation tries to minimize the differences existing between the temperatures predicted by the TMM and the temperatures measured in the thermal tests. This is done by modifying the thermal parameters, that is, the *GLs*, *GRs* and *MCs*.

If in the set of Eq. (6) we use the values of the measured temperatures $T_1, T_2, \ldots T_n$ as a data, the values of the *GLs*, *GRs* and *MCs* become the unknowns. For a particular time step, and for a particular load case, equilibrium equation of node i becomes

$$f(x_i) = GL(i,1)(T_i - T_1) + \ldots + GL(i,n)(T_i - T_n) + \sigma GR(i,1)\left(T_i^4 - T_1^4\right) + \ldots + \sigma GR(i,n)\left(T_i^4 - T_n^4\right) + M_iC_i\frac{(T_i - T_i^t)}{\Delta t} - q_i = 0 \quad (7)$$

Eq. (7) must be fulfilled by each node, in each load case and in each time step, so the function to be minimized is given by Eq. (8).

$$F = \left(\sum_{icase=1}^{ncase}\left(\sum_{istep=1}^{nstep}\left(\sum_{i=1}^{i=n-NSINK}\left(f(x_i)^2\right)\right)\right)\right) \quad (8)$$

subjected to next conditions, needed to have physical sense:

$$GL(i,j) \geq 0 \; GR(i,j) \geq 0 \; T_i^{icase} > 0 \; M_iC_i > 0$$

It is important to underline that, to have an overdetermined system of equations that make the minimization of Eq. (8) possible, the number of linear independent equations must be greater than the number of unknowns.

We can calculate the number of equations following Eq. (9).

$$nequations = nstep \cdot ncase \cdot (n - NSINK) \quad (9)$$

where *nstep* is the number of time steps used in the calculations, *ncase* the number of load cases (hot case, cold case …), *n* is the number of nodes and *NSINK* the number of nodes whose temperature has been imposed.

We can also calculate the number of unknowns following Eq. (10).

$$nunknowns = NGL + NGR + NMCP + nstep \cdot ncase \cdot (n - NSINK - NTC) \quad (10)$$

where *NGL* is the number of *GLs* present in the model, *NGR* the number of *GRs*, *NMCP* the number of *MCs* present in the equations ($NMCP = n - NSINK$) and *NTC* the number of nodes where we have measurements of temperatures (*NTC* stands for number of thermocouples). The last term of Eq. (10) ($nstep \cdot ncase \cdot (n - NSINK - NTC)$) quantifies the number of unknown temperatures, as sometimes it will be not possible to measure all the temperatures. In an ideal case, if all the temperatures of the nodes are measured, $n = NSINK + NTC$, so the last term becomes 0.

The key point is to determine *ncase*, the number of load cases (hot case, cold case, stay alive case …) that have to be tested in the laboratories.

According to the previous paragraphs it is necessary that:

$$nstep \cdot ncase \cdot (n - NSINK) \geq NGL + NGR + NMCP + nstep \cdot ncase \cdot (n - NSINK - NTC) \quad (11)$$

Doing some algebra and taking into account that $NMCP = n - NSINK$ we can write

$$nstep \cdot ncase \cdot NTC \geq NGL + NGR + n - NSINK \quad (12)$$

And the value of *ncase* is given by Eq. (13).

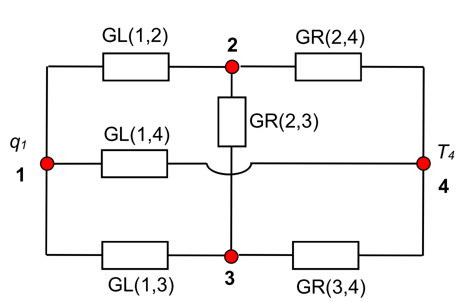

Fig. 1. 4 nodes model.

$$ncase \geq \frac{NGL + NGR + n - NSINK}{nstep \cdot NTC} \quad (13)$$

In a first approach, Eq. (13) should be fulfilled in an easy way, as *nstep* tends to be big and, as a consequence, the number of needed load cases *ncase* should be small. However, it must be remembered that the equations must be linearly independent, and this is not fully ensured in the present context.

The set of subroutines TOLMIN developed by M.J.D. Powell [15] and freely available in the internet [16] were used to minimize function *F* defined in Eq. (8). TOLMIN demands from the user the writing of a subroutine FGCALC which evaluates the function to be minimized *F* and evaluates also the gradient vector of function *F* with respect to each unknown (*GLs*, *GRs*, *MCs* and unknown temperatures). The authors wrote a program to automatically write the FGCALC subroutine, as this can be extremely time consuming and prone at errors, even if the thermal models are not that big. The most challenging part of this writing was to correctly take into account the derivatives of *F* with respect to the unknown temperatures, as values of temperatures in different time steps appear in the mathematical expressions.

The vector of unknowns is composed by the *GLs*, *GRs*, *MCs* and the node temperatures unknown in the different time steps. The values of the *GLs*, *GRs* and *MCs* are considered constant through the calculations (do not vary with time or temperature). If the number of *nstep* is big, the number of temperature unknowns becomes very large. This makes the minimization work of TOLMIN more challenging, due to the big number of unknowns. At the same time, it must be stressed that the real objective of the minimization is the thermal parameter identification (*GLs*, *GRs*, *MCs*) and no the unknown temperatures, which could be calculated once the thermal parameters are correctly correlated.

4. Inverse heat transfer problems

Before showing the results obtained with the previously explained methodology, it is important to mention some mathematical issues that appear when, as it is our case, we are trying to solve an inverse heat transfer problem. Specifically, these points are the existence of the solution, uniqueness of the solution and stability of the solution. As it is clearly and in depth explained in Refs. [17,18], inverse heat transfer problems tend to be ill-conditioned, that is, one or more of these three points are not fulfilled. Specifically, when talking about ill posed systems we are underlining the fact that small changes in the data produce big changes in the solution. If the problem is linear (which is not our case), the conditioning of the problem is measured by the ratio between the maximum and minimum eigenvalues of the parameter matrix. If the problem is non linear, more complex considerations are to be taken into account.

One of the solutions suggested by Refs. [17,18] is to use regularization techniques, which will help the gradient algorithm to find the real solution of the inverse problem (that is, the reals GLs, GRs and MCs). This solution has given very good results reducing the impact of the noise measurements in the temperatures employed for the algorithms. In short, regularization helps to reduce the conditioning number and build quasi-solutions less sensitive to measurements errors.

After careful consideration, we decided not to use these regularization techniques, mainly because, at this stage of our investigation, we are using “exact-reference” temperatures instead of measured temperatures, so in theory, there are no measurements errors. Also, it would be extremely difficult to modify successfully the algorithm TOLMIN,

Table 1
Parameter values. 4 nodes model. 1 load case.

Label	Initial value (base model)	Reference value (correct value)	Correlated value	Initial error (%)	Final error (%)
GL(1,2)	2.00	8.00	8.00	75.00	4.6000E-03
GL(1,3)	1.00	6.00	6.00	83.33	5.2000E-03
GL(1,4)	4.00	5.00	5.00	20.00	2.0000E-04
GR(2,3)	0.03	0.04	0.04	25.00	4.8890E-01
GR(2,4)	0.05	0.08	0.08	37.50	8.3000E-03
GR(3,4)	0.08	0.03	0.03	166.67	3.2800E-02
MC(1)	3570.00	3000.00	3000.00	19.00	2.0000E-04
MC(2)	850.00	2500.00	2500.00	66.00	4.2000E-03
MC(3)	1600.00	2000.00	2000.00	20.00	3.3000E-03
Mean Error				56.94	6.0856E-02

Table 2
Parameter values. 4 nodes model. 2 load cases.

Label	Initial value (base model)	Reference value (correct value)	Correlated value	Initial error (%)	Final error (%)
GL(1,2)	2.00	8.00	8.00	75.00	3.7300E-02
GL(1,3)	1.00	6.00	6.00	83.33	4.3400E-02
GL(1,4)	4.00	5.00	5.00	20.00	1.7000E-03
GR(2,3)	0.03	0.04	0.04	25.00	9.0000E-04
GR(2,4)	0.05	0.08	0.08	37.50	3.8000E-02
GR(3,4)	0.08	0.03	0.03	16.67	4.0900E-02
MC(1)	3570.00	3000.00	3000.00	19.00	1.0000E-03
MC(2)	850.00	2500.00	2499.00	66.00	3.7000E-02
MC(3)	1600.00	2000.00	2001.00	20.00	4.3800E-02
Mean Error				56.94	2.7111E-02

Temperatures obtained using the correlated thermal parameters values and the reference temperatures match perfectly well.

Table 3
Parameter values. 4 nodes model. 1 load case. 1 temperature unknown.

Label	Initial value (base model)	Reference value (correct value)	Correlated value	Initial error (%)	Final error (%)
GL(1,2)	2.00	8.00	11.25	75.00	40.67
GL(1,3)	1.00	6.00	2.70	83.33	54.99
GL(1,4)	4.00	5.00	5.02	20.00	0.41
GR(2,3)	0.03	0.04	0.03	25.00	34.85
GR(2,4)	0.05	0.08	0.09	37.50	15.42
GR(3,4)	0.08	0.03	0.01	16.67	54.88
MC(1)	3570.00	3000.00	3002.00	19.00	0.06
MC(2)	850.00	2500.00	3588.00	66.00	43.52
MC(3)	1600.00	2000.00	900.10	20.00	54.99
Mean Error				56.94	33.31

devised in Ref. [16], and used in our work. Finally, as it will be shown in next section, in general we obtained good results both in the parameters space and even better in the temperatures space. In any case, this question remains open for further investigation in the future.

5. Results obtained for different case studies

It is necessary to evaluate the presented methodology with some case studies to ascertain its validity. The ideal case would be to have a complete set of experimental tests results for different TMMs and load cases and apply the correlation methodology explained in the previous sections of the paper. However, as a complete set of tests results is not available, an equivalent procedure will be used.

First, we will build a 'reference' TMM. This TMM will have the thermal parameters considered as correct values. With this reference model, different sets of reference temperatures will be obtained for the different transient load cases. These reference temperatures would represent the ones measured in a perfectly done thermal test.

Second, we build a 'base' TMM. This TMM will be obtained varying a certain percentage the thermal parameters of the reference model (*GLs*, *GRs* and *MCs*). This base model represents the one that thermal engineers would construct with the CAD files, thermal material properties, view factors … With this base model, different sets of base temperatures will be obtained for the different transient load cases. As it can be expected, the set of base temperatures is somehow different from the set of reference temperatures. The differences between both sets are to be minimized changing the values of the thermal parameters of the base TMM.

Several transient case studies, formed by three different TMMs (4, 7 and 16 nodes models), combined with several subcases, have been used to evaluate the methodology. The models are based on the steady state cases presented in Ref. [14].

5.1. 4 nodes model

A simple 4 nodes model (nodes 1 to 4) has been used for this first case. The thermal model, that can be seen in Fig. 1, has 3 linear conductances (*GLs*), 3 radiative conductances (*GRs*) and 3 thermal inertias (*MCs*). Power is applied in node number 1 and a constant temperature of 20 °C is maintained in sink node 4, for all the load cases. The initial temperature for all the nodes is 20 °C and the transient case extends 7200 s (2 h). Time step used is 600 s (10 min).

5.1.1. 4 nodes model. Correlation with no unknown temperatures and 1 or 2 load cases

It is assumed that the reference temperatures of nodes 1, 2 and 3 are

Table 4
Node 2 temperature values. 4 nodes model. 1 load case. 1 temperature unknown.

Label	Initial value (base model)	Reference value (correct value)	Correlated value	Initial error (°C)	Final error (°C)
T(600, 2, 1)	22.26	22.09	22.09	0.17	0.00
T(1200, 2, 1)	24.38	23.88	23.90	0.50	0.01
T(1800, 2, 1)	25.94	25.20	25.23	0.75	0.03
T(2400, 2, 1)	27.02	26.13	26.18	0.89	0.04
T(3000, 2, 1)	27.75	26.80	26.85	0.95	0.05
T(3600, 2, 1)	28.24	27.26	27.32	0.98	0.06
T(4200, 2, 1)	28.58	27.59	27.66	0.98	0.07
T(4800, 2, 1)	28.81	27.83	27.90	0.98	0.07
T(5400, 2, 1)	28.96	28.00	28.07	0.97	0.07
T(6000, 2, 1)	29.07	28.11	28.19	0.96	0.08
T(6600, 2, 1)	29.15	28.20	28.27	0.96	0.08
T(7200, 2, 1)	29.21	28.25	28.33	0.95	0.08
Mean Error				0.84	0.05

known ($NTC = 3$). Therefore, the minimum number of load cases needed ideally to make the correlation is 1. The cold case ($q_1 = 50\ W$) will be used.

$$ncase \geq \frac{NGL + NGR + N - NSINK}{nstep \cdot NTC} = \frac{3 + 3 + 4 - 1}{12 \cdot 3} = 0.25$$

The values obtained for the correlated conductances and thermal inertias, as well as the relative errors, the base, and the reference values, are collected in Table 1. As can be seen, the values obtained for the correlated conductances and thermal inertias fit almost perfectly the reference values, going from a mean initial error of 56.94% to almost 0%. The error values have been calculated following Eq. (14) for parameter values, and Eq. (15) for temperature values.

$$error = \frac{reference\ value - predicted\ value}{reference\ value}\ x\ 100 \tag{14}$$

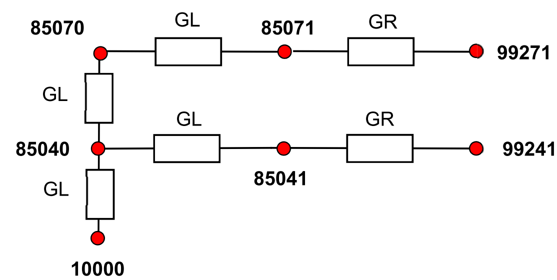

Fig. 2. 7 nodes model.

Table 7
Power values. 7 nodes model.

Node	Q $_{Cold\ case}$ (W)	Q $_{Hot\ case}$ (W)	Q $_{Stay\ alive}$ (W)	Q $_{TEM}$ (W)
85040	5.59	33.39	20.0	25.0
85041	1.01	7.61	2.0	6.0
85070	11.18	1.20	4.0	8.0
85071	1.83	12.55	10.0	5.0

Table 8
Sink temperatures. 7 nodes model.

Node	T $_{Cold\ case}$ (°C)	T $_{Hot\ case}$ (°C)	T $_{Stay\ alive}$ (°C)	T $_{TEM}$ (°C)
10000	−26.95	47.60	25.0	0.0
99241	−129.62	−61.28	−100.0	−80.0
99271	−156.48	−110.31	−120.0	−110.0

Table 5
Parameter values. 4 nodes model. 2 load cases. 1 temperature unknown.

Label	Initial value (base model)	Reference value (correct value)	Correlated value	Initial error (%)	Final error (%)
GL(1,2)	2.00	8.00	8.35	75.00	4.34
GL(1,3)	1.00	6.00	5.65	83.33	5.90
GL(1,4)	4.00	5.00	5.00	20.00	0.00
GR(2,3)	0.03	0.04	0.04	25.00	1.46
GR(2,4)	0.05	0.08	0.08	37.50	2.11
GR(3,4)	0.08	0.03	0.03	166.67	5.91
MC(1)	3570.00	3000.00	3000.00	19.00	0.02
MC(2)	850.00	2500.00	2618.00	66.00	4.72
MC(3)	1600.00	2000.00	1882.00	20.00	5.90
Mean Error				56.94	3.37

Table 6
Parameter values. 4 nodes model. 3 load cases. 1 temperature unknown.

Label	Initial value (base model)	Reference value (correct value)	Correlated value	Initial error (%)	Final error (%)
GL(1,2)	2.00	8.00	8.47	75.00	5.87
GL(1,3)	1.00	6.00	5.52	83.33	7.96
GL(1,4)	4.00	5.00	5.00	20.00	0.01
GR(2,3)	0.03	0.04	0.04	25.00	2.23
GR(2,4)	0.05	0.08	0.08	37.50	2.84
GR(3,4)	0.08	0.03	0.03	166.67	7.97
MC(1)	3570.00	3000.00	3001.00	19.00	0.02
MC(2)	850.00	2500.00	2659.00	66.00	6.37
MC(3)	1600.00	2000.00	1841.00	20.00	7.96
Mean Error				56.94	4.58

For this case, the mean error in the temperatures of node 2 goes from 1.34 °C to 0.01 °C.

Table 9
Parameter values. 7 nodes model. Subcase: No unknown temperatures and 2 load cases.

Label	Initial value (base model)	Reference value (correct value)	Correlated value	Initial error (%)	Final error (%)
GL(10000,85040)	0.0416	0.0333	0.0333	25.00	0.0134
GL(85040,85041)	0.8047	3.2190	3.2190	75.00	0.0004
GL(85040,85070)	0.8008	0.4883	0.4883	64.00	0.0001
GL(85070,85071)	7.0910	4.4310	4.4310	60.04	0.0001
GR(85041,99241)	0.0230	0.0383	0.0383	39.97	0.0002
GR(85071,99271)	0.0967	0.0612	0.0612	57.92	0.0002
MC(85040)	5907.0000	4964.0000	4964.0000	19.00	0.0007
MC(85041)	100.0000	182.3000	182.3000	45.16	0.0021
MC(85070)	2569.0000	4847.0000	4847.0000	47.00	0.0001
MC(85071)	125.0000	365.7000	365.7000	65.82	0.0023
Mean Error				49.89	0.0020

$$error = reference\ value - predicted\ value \tag{15}$$

If two load cases are used in the correlation, such the hot case ($q_1 = 120W$) and the cold case ($q_1 = 50W$), results improve a little bit as can be seen in Table 2.

5.1.2. 4 nodes model. Correlation with one unknown temperature

The results showed in the previous section suggest that the methodology and the minimization algorithm could be used in the 4 nodes model even if one of the temperatures is unknown, for instance, that of node 2 ($NTC = 3 - 1 = 2$). The rest of parameters have been maintained.

$$ncase \geq \frac{NGL + NGR + N - NSINK}{nstep \cdot NTC} = \frac{3 + 3 + 4 - 1}{12 \cdot 2} = 0.375$$

In this case, theoretically, with only one load case it might be possible to obtain good results. The chosen case was the cold case ($q_1 = 50W$). However, as can be seen in Table 3, although results improve, the mean error of the parameters is big (33.31%).

Next, the temperatures of node 2 for the different time steps have been calculated using the correlated parameters shown in Table 3. The comparison between the reference temperatures and the correlated temperatures can be seen in Table 4. Results improve significantly. The temperatures have been labelled as T (time, node number, load case).

A possible solution to improve the results is to make the correlation with 2 or 3 load cases. For 2 load cases (cold case $q_1 = 50\ W$ and hot case $q_1 = 120\ W$) results improve significantly, as can be seen in Table 5.

The mean error of the parameters (9 unknowns) goes from 56.94% to 3.37%. The temperatures obtained for node 2 in the 12 time steps and for the 2 load cases (this makes a total of 24 unknowns) also improve, from a mean error of 1.36 °C with the base model to a 0.01 °C with the correlated model.

It is interesting to note that to find the 'correct' values of the 9 thermal parameters, one must also calculate 24 unknown temperatures, making a total of 33 unknowns, which increases the elapsed time in the computer. These additional temperatures can also be seen as a useful information, because they are temperatures of nodes that could not be measured in the tests.

For 3 load cases (cold case $q_1 = 50\ W$, hot case $q_1 = 120\ W$ and stay alive case $q_1 = 80\ W$) results do not improve, as can be seen in Table 6. The authors have not a clear explanation about this fact, although some sort of 'optimal' load case number seems to exist.

5.2. 7 nodes model

This is a reduced 7 nodes model of the Tribolab instrument, a space tribometer that was flown on board the International Space Station [19]. Three of the nodes are sink nodes: two radiation sink nodes (nodes 99241 and 99271) and one conduction sink node (node 10000). The model consists of 4 linear conductances, 2 radiation conductances, and 4 thermal inertias which can be seen in Fig. 2. The nodes with temperature imposed (sink nodes) do have MC parameters, but their values do not appear in the equations, as these equations are deleted from the system when the temperatures are imposed. The calculation runs for 86.400 s (that is, one day) and time step $\Delta t = 600\ s$. This makes a total of 144 time steps. The initial temperature considered for $t = 0$ is $T = 20^{\circ}C$.

Powers applied in no sink nodes (85040, 85041, 85070, and 85071) are collected in Table 7 for 4 different load cases (cold, hot, stay alive and TEM cases), where values are expressed in watts. Sink temperatures for these load cases are collected in Table 8, with values expressed in °C.

Table 10
Number of unknowns and maximum initial error.

ncase	NGL + NGR + N–NSINK + nstep·NNOTC·ncase	Initial base average error	Maximum initial average error in temperatures
1	4 + 2+7–3+144·2·1 = 298	9.29 °C	0.929 °C
2	4 + 2+7–3+144·2·2 = 586	8.01 °C	3.2 °C
3	4 + 2+7–3+144·2·3 = 874	7.58 °C	6.14 °C
4	4 + 2+7–3+144·2·4 = 1162	7.62 °C	6.10 °C

Once more, it seems that a maximum number of load cases can be used without increasing too much the number of unknowns and having at the same time convergence of results. Once this maximum is reached, further adding of load cases do not improve the solution.

5.2.1. 7 nodes model. Correlation with no unknown temperatures

In this case study, it is assumed that reference temperatures in no sink nodes are known ($NTC = 4$). Therefore, the minimum number of load cases needed to make the correlation is 1.

$$ncase \geq \frac{NGL + NGR + N - NSINK}{nstep \cdot NTC} = \frac{4 + 2 + 7 - 3}{144 \cdot 4} = 0.0174$$

However, as data were available, 2 load cases were used, to improve the chances of convergence to the true thermal parameters. The two load cases used in the correlation are the cold and hot cases, whose heat fluxes and sink temperatures are included in Tables 7 and 8. The values of the base, reference and correlated thermal parameters, as well as the relative errors, are collected in Table 9.

As it can be seen, the correlation is excellent (final mean error 0.0020%). As a consequence, the correlation with only one load case (cold case) was also done. The results are also very good (final mean error 0.0065%).

5.2.2. 7 nodes model. Correlation with 1 unknown temperature

This case study is equal to the previous case study (section 4.2.1), but now one of the inner temperatures (that of node 85040) is considered unknown ($NTC = 3$). Therefore, the minimum number of load cases needed to make the correlation is again 1.

$$ncase \geq \frac{NGL + NGR + N - NSINK}{nstep \cdot NTC} = \frac{4 + 2 + 7 - 3}{144 \cdot 3} = 0.0231$$

Using 1 load case, a somehow unexpected result was obtained. It has

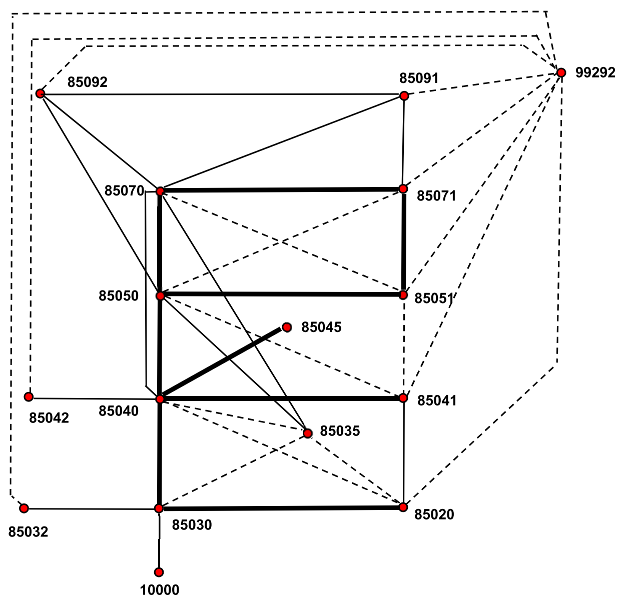

Fig. 3. 16 nodes model.

been previously stated that an important difficulty in the optimization process is to make sure that the employed equations are linearly independent. In fact, if only the cold case is used to the correlation, no convergence of the thermal parameters is obtained. However, if only the hot case is used, convergence is obtained, with no special difficulties and a mean error of 0.02% is reached for the parameters, while the mean error of the temperatures of node 85040 goes from 6.13 °C to 0.0005 °C. A possible explanation of this fact is that the variations of temperatures in the different nodes and timesteps in the cold case is small and the independency of equations is not guaranteed, while bigger temperature variations are obtained in the hot case, which permits the correct functioning of the optimization algorithm.

Using 2 load cases (the cold and the hot cases) correct thermal parameters are obtained, with a mean error of 0.005%. The temperatures obtained for node 85040 in both cold and hot cases go from a mean error of 6 °C (using the base model) to a mean error of 0.0001 °C. It is interesting to note now that in this case we have 298 unknowns just to correlate the 10 thermal parameters whose values we are looking for. 10 unknowns (4 GLs, 2 GR y 4 RMCPs) plus 288 unknowns (1 node · 2 load cases · 144 time steps). Elapsed computer times increased accordingly.

5.2.3. 7 nodes model. Correlation with 2 unknown temperatures

We are now supposing that temperatures of inner nodes 85040 and 85070 are unknown. As a consequence, $NTC = 4 - 2 = 2$. Once more, the theoretical expression for the needed number of load cases gives us a value of 1.

$$ncase \geq \frac{NGL + NGR + N - NSINK}{nstep \cdot NTC} = \frac{4 + 2 + 7 - 3}{144 \cdot 2} = 0.0347$$

Four different optimization problems have been defined, according to the number of load cases employed in each problem (1, 2, 3 or 4). These cases are collected in Tables 7 and 8. If *NNOTC* is the number of nodes that have no thermocouples (in our case, 2), the total number of unknowns that the optimization algorithm has to calculate is $NGL + NGR + N - NSINK + nstep \cdot NNOTC \cdot ncase$. Table 10 collects the number of unknowns as a function of the *ncase* load cases used. As it can be seen, the number of unknowns has increased dramatically.

It is convenient to state from the beginning that not a single one of these four optimization cases has converged in the usual conditions employed as far as now, that is, initial solution for the optimization

Table 11
Results for the 16 nodes model. Correlation with no unknown temperatures.

Number of unknown temperatures (NNOTC)	Number of load cases considered (ncase)	Initial average error (%)	Final average error (%)	Number of unknown parameters correlated	Number of improved thermal parameters
0	1	25.87	14.56	58	45/58
0	2	25.87	3.29	58	54/58
0	3	25.87	2.35	58	56/58
0	4	25.87	2.06	58	57/58

Table 12
Results for the 16 nodes model. Correlation with unknown temperatures.

Number of unknown temperatures (NNOTC)	Number of load cases considered (ncase)	Initial average error (%)	Final average error (%)	Number of unknown parameters correlated	Number of improved thermal parameters	T initial error (°C)	T final error (°C)
1 (node 85040)	1	25.87	158.8	202	30/58	5.22	0.10
	2	25.87	2.73	346	56/58	3.38	5.62E-05
	3	25.87	2.49	490	57/58	3.58	1.36E-05
2 (nodes 85040, 85070)	1	25.87	269.17	346	27/58	4.01	0.065
	2	25.87	5.89	634	51/58	2.55	3.56E-05
	3	25.87	3.88	922	56/58	3.68	1.54E-04
	4	25.87	8.70	1210	54/58	3.62	1.11E-04
3 (nodes 85030, 85040, 85070)	2	25.87	No Conv	922	–	–	–
	3	25.87	4.55	1354	56/58	3.71	4.0E-04
	4	25.87	13.84	1786	54/58	3.71	3.0E-04
4 (nodes 85030, 85040, 85050, 85070)	3	25.87	5.59	1786	52/58	3.65	2.8E-04
	4	25.87	12.18	2362	54/58	3.66	1.8E-04
	5	25.87	25.91	2938	54/58	3.92	2.8E-04
5 (nodes 85030, 85035, 85040, 85050, 85070)	3	25.87	No Conv	2218	–	–	–
	4	25.87	4.05	2938	54/58	3.64	1.35E-02
	5	25.87	7.46	3658	54/58	3.88	1.10E-03
	6	25.87	7.05	4378	54/58	3.62	1.10E-03
6 (nodes 85030, 85035, 85040, 85045, 85050, 85070)	3	25.87	No Conv	2650	–	–	–
	4	25.87	7.15	3514	49/58	3.56	9.8E-03
	5	25.87	No Conv	4378	–	–	–
	6	25.87	No Conv	5242	–	–	–

process is that of base values, both thermal parameters and unknown temperatures. It has been possible to devise alternative initial solutions for these four cases. These alternative initial solutions have led to correct convergence of the four optimization cases. It has been done linearly combining the base and the reference temperatures. The last column of Table 10 shows the maximum initial average error that permits to obtain convergence. For the 3 and 4 load cases, almost the base value initial solution reaches a convergence.

5.3. 16 nodes model

A 16 nodes model was selected to validate the proposed transient correlation methodology. It is called Tribolab compact model, because the model represents in a more accurate way the real geometry and thermal behaviour of Tribolab. The number of nodes is $N = 16$ and, between them, there are 22 linear conductances ($NGL = 22$) and 25 radiative conductances ($NGR = 25$). There is also one conductive sink node (node 10000, the ISS) and one radiative sink node (node 99292, the Space). Taking into account the thermal inertias ($N - NSINK = 14$), the total number of thermal parameters to be correlated are 61 ($NGL + NGR + N - NSINK$). However, as 3 of the *MCs* are 0 (correspond to nodes composed of MLI), their value will be maintained constant and the total number of thermal parameters will be 58.

The model is shown in Fig. 3, where the red dots represent the nodes, the dash lines the radiative conductances (GRs), the solid thin lines the conductive conductances (GLs) and the solid thick lines represent the presence of conductive and radiative conductances (GLs and GRs) between the nodes.

5.3.1. 16 nodes model. Correlation with no unknown temperatures

In this case study, it will be assumed that all the reference temperatures of Tribolab are known, that is, $NTC = 16 - 2 = 14$. As a consequence, a minimum of 1 load case is needed. However, considering the results obtained in the case studies of the 7 nodes model, the correlation has been also performed using 2, 3 and 4 load cases.

$$ncase \geq \frac{NGL + NGR + N - NSINK}{nstep \cdot NTC} = \frac{22 + 25 + 16 - 2}{144 \cdot 14} = 0.0303$$

The obtained results are, in general terms, quite good for the thermal parameters, if the number of load cases considered at the same time is 2 or bigger. Table 11 summarizes the initial and final mean error in thermal parameters values, the number of unknowns correlated and the number of thermal parameters whose value improves with the correlation (out of 58).

Once the thermal parameters were obtained with the correlations techniques already described, these values were used in the thermal models instead of the base values. The obtained temperatures match extremely well with the reference temperatures, as it could be expected for the different load cases.

Table 11 shows also that the best correlations are obtained when a higher number of different load cases are used for the correlation.

5.3.2. 16 nodes model. Correlation with unknown temperatures

This final case study is equal to the previous case study (section 4.3.1), but in this occasion some of the inner temperatures of Tribolab will be unknown. Specifically, correlations with 1, 2, 3, 4, 5 or even 6 inner unknown temperatures were attempted, with some interesting and encouraging results. It is good to remember that the corresponding number of measured temperatures would be 13, 12, 11, 10, 9 or 8 respectively ($N - NSINK = 16 - 2 = 14$). Different number of load cases were used in each situation and results are summarized in Table 12. Two columns (the last two) have been added in this Table 12, if it is compared with Table 11. These columns reflect the improvement in the unknown values of the temperatures.

In general terms, convergence has been reached in all the cases, sometimes with any number of load cases considered. Convergence has been more difficult in those cases where the number of nodes whose temperatures were unknown is high.

It is interesting to note the capacity of the method to improve dramatically the expected values of the nodes whose temperatures are not measured, when compared with the foreseen results predicted by the base model.

Finally, it is also interesting to note the high number of variables that must be correlated (sometimes, more than 4.000) in order to have good values for the 58 thermal parameters, real objective of our work.

6. Conclusions

A methodology to optimize the values of spacecraft thermal models parameters has been presented. This methodology is based on the thermal transient equations that govern the heat transfer process and on the transient measurements of temperatures done in thermal tests.

The methodology has been implemented using a gradient based set of minimization subroutines called TOLMIN, freely available in internet. The methodology is based on a previously developed steady state technique. In the transient version, the thermal inertias of the models can also be optimized. As a bonus, some temperatures in nodes with no test measurements can be predicted with accuracy.

The number of theoretically minimum number of needed load cases for the optimization has been derived. However, this minimum number seems to be insufficient for real calculations, as the equations present seem to be no linearly independent. It can be stated also that if additional load cases are used, in general, the values found for the thermal parameters will be more *correct*.

The vector of unknowns in the optimization process can have not only linear and radiative conductances and thermal inertias but also unknown temperatures for the different load cases used. The TOLMIN algorithm deals well with these different types of parameters to be identified: absolute values are quite different, but it seems that the algorithm is robust enough to cope with this difficulty.

Three case studies of increasing complexity have been presented and solved to test the accuracy and power of the designed methodology. In general terms, the results are really good, with some points which will need further studies to make the methodology fully operational.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Nomenclature

GL: Linear conductances
GR: Radiative conductances
MC: Thermal Inertias
MLI: Multi Layer Insulation
ISS: International Space Station
TMM: Thermal mathematical model
TLP: Thermal Lumped Parameter Method