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*Reference Points Based on Dynamic  
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Fishery Management with Bio-economic Age-  
structured Models*

# Reference points based on dynamic optimisation: a versatile algorithm for mixed fishery management with bio-economic age-structured models

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## Abstract

Single-species management objectives may not be consistent within mixed fisheries. They may lead species to unsafe situations, promote discarding of over-quota and/or misreporting of catches. We provide an algorithm for characterising bio-economic reference points for a mixed fishery as the steady-state solution of a dynamic optimal management problem. The optimisation problem takes into account: i) that species are fishing simultaneously in unselective fishing operations and ii) intertemporal discounting and fleet costs to relate reference points to discounted economic profits along optimal trajectories. We illustrate how the algorithm can be implemented by applying it to the European Northern Stock of Hake (*Merluccius merluccius*), where fleets also capture Northern megrim (*Lepidorhombus whiffiagonis*) and Northern anglerfish (*Lophius piscatorius* and *Lophius budegassa*). We find that optimal mixed management leads to a target reference point that is quite similar to the  $2/3$  of the  $F_{msy}$  single-species (hake) target. Mixed management is superior to single-species management because it leads the fishery to higher discounted profits with higher long-term SSB for all species. We calculate that the losses due to the use of the  $F_{msy}$  single-species (hake) target in this mixed fishery account for 11.4% of total discounted profits.

**Keywords:** optimisation in age-structured models, bio-economic reference points, mixed fisheries.

**Running title:** Mixed fisheries reference points based on dynamic optimisation.

## 1.- Introduction

The aim of this article is to characterise fishing reference points as the steady-state solution of a dynamic optimal management problem defined over an infinite-time horizon considering a mixed fishery and an age-structured population model based on Baranov's catch equation (1918).

Since Beverton and Holt (1957), target reference points have been one of the main tools used by fishery managers to make decisions about future catch options. Prominent among the classical "target reference points" are those associated with Maximum Sustainable Yield (MSY), such as  $B_{msy}$  and  $F_{msy}$ . The use of MSY reference points was criticised by Larkin (1977) in his famous paper *An epitaph for the concept of maximum sustainable yield*, where he identifies some situations where management based on MSY fails, e.g. mixed fisheries, multispecies interactions and economic performance. The introduction of the "Precautionary Approach" to fisheries in the 90's (Rio Summit, 1992), emphasised the use of "limit reference points" to constrain harvesting within safe biological limits. In other words, there are limits to exploitation (e.g. spawning stock biomass) below which it is considered that stock sustainability cannot be ensured or below which the likelihood of a negative outcome (e.g. stock collapse) is unacceptable. MSY and the precautionary approach are unified in UN (1995) where Annex II, Point 7 says: "The fishing mortality rate which generates maximum sustainable yield should be regarded as a minimum standard for limit reference point". The World Summit on Sustainable Development (WSSD) aims to maintain or restore stocks to levels that can produce MSY by 2015 (COFI, 2003), and indeed this is now the one of the main goals of fishery management. However, the deficiencies identified by Larkin (1977) continue to pose a challenge for fishery management.

There are two important shortcomings in this classical reference point. The first one is that many fisheries are mixed fisheries, i.e. fleets target different species together. Therefore what may be "safe" for one stock may be "dangerous" for another stock caught together with it. In this regard, Mace (2001) points out that in a mixed fishery the reference target should be lower than the fishing mortality associated with the MSY,  $F_{msy}$ . Moreover, single-species objectives may not be consistent with one another in mixed fisheries where species are caught simultaneously in relatively unselective fishing operations, and can lead to over-quota catches and misreporting of catches (Vinther *et al.*, 2004; Rijnsdorp *et al.*, 2005; Mackinson *et al.*, 2009; Baudron *et al.*, 2010; Reiss *et al.*, 2010; Ulrich *et al.*, 2011). Other studies as Agar and Sutinen (2004) also suggest that the failure to recognize the mixed nature of the fisheries can have profound impacts on the success of the recovering strategies for fisheries in rebuilding process.

The second shortcoming of the MSY reference point is that it is a time-independent, long-term strategy which is determined without taking into account the intertemporal aspects of economic variables. However, any economic assessment of future variables requires discounting to be introduced into the analysis (Koopmas 1960, Sumaila *et al.*, 2010, Duncan *et al.*, 2011). In this sense, for years it was thought that targeting fishing mortality in order to maximise net present value (NPV) of profits (also called discounted profits) would cause stock depletion. However, Grafton *et al.* (2007) find that in practice the biomass associated with maximisation of discounted profits is higher than the biomass associated with MSY. This suggests that the use of reference points lower than  $F_{msy}$  may be a win-win strategy: higher profits and safer biomass.

Age-structured population models have been extensively used to assess fisheries, but they have not been used so often for computing optimal fishing mortality trajectories. Some exceptions are Hanneson (1975), Gurtin and Murphy (1981), Hoorwood and Whittle (1986) and Hoorwood (1990), and more recently Grafton *et al.*, (2006; 2007; 2010), Gröger *et al.* (2007) Kulmala *et al.* (2008), Da Rocha *et al.* (2010), Da Rocha and Gutiérrez (2011), Dichmont *et al.* (2010), Kompas *et al.* (2010) and Tahvonen (2009).

Economic assessment for mixed fishery management frequently focuses on single-stock biological reference points (F or SSB targets). Once these targets have been set,

management rules based on them are proposed and the biological and economical impacts of those rules on other stocks (F or SSB) are evaluated (Kraak *et al.*, 2008, Ulrich *et al.*, 2011). This approach has two main features: i) the selection of the optimal management rule is based on a balance of performance of single stocks with no objective measure that quantifies their joint value; and ii) from the economic point of view biological targets are the basis for setting management rules and obviously this does not guarantee optimal economic performance for single stocks or indeed for the whole fishery.

In this paper we address these problems through an algorithm that analytically calculates mixed fishery economic reference points. We illustrate how the algorithm can be implemented by applying it to the European Northern Stock of Hake fishery, where fleets also capture megrim (*Lepidorhombus whiffiagonis*) and anglerfish (*Lophius piscatorius* and *Lophius budegassa*).

The algorithm highlights the intertemporal dimension of the issue by computing reference points that maximise the NPV of profits. The assumption that the manager's objective function is the NPV of profits could be considered a simplification. However, it allows us to illustrate how the algorithm can be implemented to reach conservation and NPV objectives.

In practice, fishery managers look at many objectives: NPV of profits, employment, trade-offs with other fisheries, etc. In this sense, the algorithm must be understood as a method by which broader optimisation problems can be addressed. The advantage of the method proposed is that it quantifies the trade-off between conservation and managers' objectives within a mixed fishery. To show how the algorithm can be implemented to quantify these trade-offs, we redefine the objective function as MSY and compute the "shadow prices" (the Lagrangian multipliers). In fact, as suggested by a referee, in real-world applications the quantification of these trade-offs could be more useful to managers than the optimal values themselves.

## 2.- Material and methods

We use a standard age-structured approach that is used in many stock assessment models. Assume that there are  $n$  species in the fishery. The stock of species  $j = 1, 2, \dots, n$  is broken into  $A^j$  cohorts. That is, for species  $j$  in each period  $t$ , there are  $A^j - 1$  initial old cohorts and one new cohort is born.

Let  $z_t^{a,j}$  be the mortality rate that affects the population of species  $j$  at age  $a$  during period  $t$ . This mortality rate can be decomposed into fishing mortality,  $F_t^{a,j}$  and natural mortality,  $m^{a,j}$ ,  $z_t^{a,j} = F_t^{a,j} + m^{a,j}$ . The population of species  $j$  decreases at an exponential rate in accordance with the mortality rate  $z_t^{a,j}$ . Formally,  $N_{t+1}^{a+1,j} = \exp(-z_t^{a,j})N_t^{a,j}$ , where  $N_t^{a,j}$  represents the abundance of species  $j$  for age  $a$  at the beginning of the time period  $t$ .

We assume that species are fished simultaneously in relatively unselective fishing operations,  $q^j$ , and that the fishing selection pattern,  $\bar{p}^{-a,j}$ , of each species is constant. Therefore, for each unit of effort,  $E_t$ , the fishing mortality over each age and species is given  $F_t^{a,j} = \bar{p}^{-a,j} q^j E_t$ . While the fishing mortality rate may vary from one period and one age to another, natural mortality is constant over periods.

Notice that in this mixed fishery context, we are modelling a multiproduct technology such that for a given level of effort each species is captured in fixed proportions (Leontief, 1941) at each age. Therefore, with no loss of generality, we can define the fishing mortality multiplier as effort,  $F_t = E_t$ , and rescale the original fishing selection patterns,  $p^{a,j} = \bar{p}^{-a,j} q^j$ , to rewrite the fishing mortality over each age and species as  $F_t^{a,j} = \bar{p}^{-a,j} q^j E_t = p^{a,j} F_t$ .

Notice that by backward substitution  $N_t^{a,j}$  can be expressed as a function of recruitment

$$N_t^{a,j} = \phi_t^{a,j} N_{t-(a-1)}^{1,j}, \quad \forall a = 1, \dots, A(j), \quad (1)$$

where  $\forall j = 1, \dots, n$ ,

$$\phi_t^{a,j} = \begin{cases} \prod_{i=1}^{a-1} \exp(-z_{t-i}^{a-1}) & \text{if } a > 1, \\ 1 & \text{if } a = 1. \end{cases} \quad (2)$$

$\phi_t^{a,j}$  can be understood as the survival function that shows the probability of a recruit of species  $j$  born in period  $t - (a - 1)$  reaching age  $a$  for a given fishing mortality (effort) path  $F_{t-1}, F_{t-2}, F_{t-3}, \dots, F_{t-(a-1)}$ . Notice that the survival function at any period depends upon the  $a - 2$  previous mortality rates.

The size of a new cohort (recruitment) of species  $j$  is given by the Stock Recruitment relationship (S-R),

$$N_{t+1}^{1,j} = \Psi(SSB_t^j), \quad (3)$$

where,  $SSB_t^j = \sum_{a=1}^{A(j)} \omega^{a,j} \mu^{a,j} N_t^{a,j}$  is the spawning stock biomass, which is a function of the spawning stock weight at age  $\omega^{a,j}$  and the maturity fraction  $\mu^{a,j}$ . Finally, the fishing yield for each species and age is given by Baranov's equation (1918).

### 3.- Discounted reference points in mixed fisheries

The objective of this section is to characterise reference points as steady-state solutions of dynamic management problems. Formally, let the NPV of a fishery economic indicator (discounted economic indicator) be expressed as

$$\sum_{t=0}^{\infty} \beta^t \left( \sum_{j=1}^n \sum_{a=1}^{A(j)} pr^{a,j} y_t^{a,j} \phi_t^{a,j} N_{t-(a-1)}^{1,j} - C(F_t) \right), \quad (4)$$

where  $y_t^{a,j}$ ,  $pr^{a,j}$ ,  $C(F_t)$  and  $\beta$  are the yield per fish of species  $j$  for age  $a$ , the price of species  $j$  for age  $a$ , the total cost function (which depends positively on fishing mortality at time  $t$ ,  $F_t$ ) and the discount factor,  $0 \leq \beta \leq 1$ , respectively. The discount factor in this model represents the willingness of the manager (or society) to trade-off the value of fishing today against the benefits of increased profits in the future, measured by higher biomass and recruitment.

Some relevant aspects of the fishery economic indicator (4) must be highlighted. First, it takes into account the mixed aspect of the fishery. In fact the economic indicator represents the discounted profits of the whole fishery considering the  $n$  species that are caught together. Second, the economic indicator can be interpreted in several ways from the economic point of view. For instance, if the cost is zero the economic indicator represents the discounted revenues of the fishery. Alternatively, if  $pr^{a,j}$  is one, for all  $a$  and for all  $j$ , and the cost is zero, the economic indicator represents the discounted yield of the fishery. And thirdly, if the discount factor is one,  $\beta = 1$ , then the future is not discounted and all future profits are equally weighted in calculating NPV of profits.

The objective is to find the fishing rate trajectory that maximises the NPV of the mixed fishery's profits (equation (4)) taking into account the S-R relationship (equation (3)) and biological precautionary limits. Formally, the maximisation problem consists of solving

$$\max_{\{F_t, N_{t+2}^{1,j}, \dots, \infty\}} \sum_{t=0}^{\infty} \beta^t \left( \sum_{j=1}^n \sum_{a=1}^{A(j)} pr^{a,j} y_t^{a,j} \phi_t^{a,j} N_{t-(a-1)}^{1,j} - C(F_t) \right), \quad (5)$$

$$s.t. \begin{cases} N_{t+1}^{1,j} = \Psi(SSB_t^j), & \forall t, j, \\ SSB_t^j \geq B_{pa}^j, & \forall t, j. \end{cases}$$

An optimal steady-state solution of problem (5) is defined by a vector  $\{F_{SS}, N_{SS}^{1,1}, N_{SS}^{1,2}, \dots, N_{SS}^{1,n}\}$  such that for any future period,  $F_{SS} = F_t = F_{t+1}$  and  $N_{SS}^{1,j} = N_t^{1,j} = N_{t+1}^{1,j}$ . Notice that this optimal solution determines a unique value for the long-term fishing rate that, if applied, will generate stationary recruitment for all species leading to the maximum long-term profits of the mixed fishery.

We show in the Supplementary Materials that the steady state solution must satisfy the following  $(n+1)$  equation system when the precautionary constraints are not binding for any species,

$$\sum_{j=1}^n \sum_{a=1}^{A(j)} pr^{a,j} \frac{dy^{a,j}(F_{SS})}{dF} \phi_{SS}^{a,j} N_{SS}^{1,j} - \frac{dC(F_{SS})}{dF} = \left[ \sum_{j=1}^n \left( \sum_{a=1}^{A(j)} pr^{a+j} y_{SS}^{a,j} \phi_{SS}^{a,j} N_{SS}^{1,j} \left( \sum_{i=1}^{a-1} \beta^{a-i} p^{i,j} \right) - \frac{dN_{SS}^{1,j}}{dF} \sum_{a=1}^{A(j)} \beta^{1+a} pr^{a+j} y_{SS}^{a,j} \phi_{SS}^{a,j} \right) \right], \quad (6)$$

$$N_{SS}^{1,j} = \Psi \left( \sum_{a=1}^{A(j)} \omega^{a,j} \mu^{a,j} N_{SS}^{a,j} \right), \quad \forall j = 1, \dots, n, \quad (7)$$

where  $\phi_{SS}^{a,j}$  is given by expression (2) valued in the steady-state.

Once  $\{F_{SS}, N_{SS}^{1,1}, N_{SS}^{1,2}, \dots, N_{SS}^{1,n}\}$  is known, the steady-state cohort size of any age and species can be calculated as  $N_{SS}^{a,j} = \phi_{SS}^{a,j} N_{SS}^{1,j}$ .

An important issue to be analysed is the relationship between the solution of the discounted maximisation problem (5) and the standard single-species reference points. In the Supplementary Materials we prove that  $F_{SS}$ , the optimal steady-state mortality rate of problem (5), is just a generalisation of  $F_{msy}$ . In particular,  $F_{msy}$  coincides with  $F_{SS}$  for the case of one species in which price is one, marginal cost is zero ( $dC(F)/dF = 0$ ) and  $\beta=1$ .

#### 4.- A numerical algorithm

The steps below outline the method for searching for the discounted reference points,  $F_{SS}$ .

Step 1: Collect all the exogenous parameters describing biological and economic characteristics of all species. This includes:

- The biological parameters of the species:  $p^{a,j}$ ,  $\mu^{a,j}$ ,  $\omega^{a,j}$ ,  $m^{a,j}$ .
- The initial population distribution of all species:  $N_0^{a,j}$ .
- The precautionary limit reference point,  $SSB_{pa}^j$ .
- The multi-sepecies technology,  $q^j$ .
- The economic parameters,  $pr^{a,j}$  and  $C(F)$ . An example of cost function could be a linear cost function,  $C(F) = cF$ . In this case, the marginal cost,  $c$ , has to be reported. This step can be skipped if one is interested in the solution from revenue maximisation. In that case  $C(F) = 0$ .
- The discount factor used to calculate variables in present terms,  $\beta$ .

Step 2: Using outside information, select the S–R relationship to be used. Some examples:

- If the S–R relationship of species  $j$  is defined as in Shepherd (1982),

$$N^{1,j} = \frac{\alpha^j SSB^j}{I + \left( \frac{SSB^j}{K^j} \right)^{b^j}}, \quad (8)$$

recruitment is determined by

$$N^{1,j} = K^j \frac{\left( \alpha^j \sum_{a=1}^{A(j)} \omega^{a,j} \mu^{a,j} \phi^{a,j} - I \right)^{1/b^j}}{\sum_{a=1}^{A(j)} \omega^{a,j} \mu^{a,j} \phi^{a,j}}. \quad (9)$$

So,  $\alpha^j$ ,  $b^j$  and  $K^j$  have to be reported.

- If the S–R relationship is not well defined then recruitment may be considered as a fixed variable that does not depend on the fishing rate, that is  $N^{1,j} = \bar{N}^{1,j}$ .

Step 3: Guess a value for the steady-state fishing rate,  $F$ .

Step 4: Calculate recruitment for all species,  $N_{ss}^{1,j}$  for the steady-state fishing rate guessed,  $F$ , taking into account the S–R relationship selected in step 2 and the optimal condition (7).

Step 5: Calculate the survival functions,  $\phi_{ss}^{a,j}$ , and yield per fish functions,  $y_{ss}^{a,j}$  for the steady-state fishing rate guessed,  $F$ , using expression (2) and

$$y_{ss}^{a,j} = pr^{a,j} \omega^{a,j} \frac{p^{a,j} F}{z^{a,j}} [1 - \exp(-z^{a,j})]. \quad (10)$$

Step 6: Calculate the derivatives of yield per fish for any species and age, and the cost function for the steady-state fishing rate guessed,  $F$ . In particular,

$$\frac{dy^{a,j}(F_{ss})}{dF} = \left( \frac{m^{a,j}}{z^{a,j}} + p^{a,j} F \frac{\exp(-z^{a,j})}{1 - \exp(-z^{a,j})} \right) \frac{y_{ss}^{a,j}}{F}. \quad (11)$$

Step 7: Calculate the  $\frac{dN_{ss}^{1,j}}{dF}$  using the following expression

$$\frac{dN_{ss}^{1,j}}{dF} = \frac{N^{1,j} (\Psi^j)' \sum_{a=1}^{A(j)} \mu^{a,j} \omega^{a,j} \frac{d\phi_{ss}^{a,j}}{dF}}{I - (\Psi^j)' \sum_{a=1}^{A(j)} \mu^{a,j} \omega^{a,j}}. \quad (12)$$

To that end it is advisable first to calculate:

- The derivative of the survival functions:

$$\frac{d\phi_{ss}^{a,j}}{dF} = \begin{cases} \sum_{i=1}^{a-1} (p^{i,j}) \phi_{ss}^{i,j} < 0 & \text{if } a > 1, \\ 0 & \text{if } a = 1. \end{cases} \quad (13)$$

- The derivative of the cost function:

$$\frac{dC(F)}{dF} = \begin{cases} c & \text{if } C(F) = \text{Fixcost} + cF, \\ 0 & \text{if } C(F) = \text{Fixcost}. \end{cases} \quad (14)$$

- The derivative of the S–R relationship: If the S–R relationship is not well defined, then this derivative is zero. Otherwise, the derivative may be calculated analytically. For instance, for the Shepherd relationship (equation (8)),

$$(\Psi^j)' = K^j \frac{\sum_{a=1}^{A(j)} \omega^{a,j} \mu^{a,j} \frac{d\phi_{ss}^{a,j}}{dF} \left( \alpha^j \sum_{a=1}^{A(j)} \omega^{a,j} \mu^{a,j} \phi^{a,j} - I \right)^{(1-b^j)/b^j}}{\sum_{a=1}^{A(j)} \omega^{a,j} \mu^{a,j} \phi^{a,j}} - \left[ \alpha^j \frac{(1-b^j)}{b^j} \sum_{a=1}^{A(j)} \omega^{a,j} \mu^{a,j} \phi^{a,j} + I \right] \quad (15)$$

Step 8: Solve equation (6) using a one-dimensional equation-solving routine.

Step 9: Once  $F_{ss}$  has been solved, the steady-state recruitment,  $N_{ss}^{1,j}$ , is obtained with the S–R relationship as in Step 4, and steady-state cohort sizes,  $N_{ss}^{a,j}$ , are calculated using the survival functions (2) and the steady-state recruitment,  $N_{ss}^{a,j} = \phi_{ss}^{a,j} N_{ss}^{1,j}$

Step 10: Check whether the precautionary limit reference point is satisfied for all species. If it is not  $F_{ss}$  must be selected as the corner solution of problem (5). Formally, if  $F_{pa}^j$  is referred to as the fishing mortality rate that guarantees that  $SSB^j$  is equal to  $SSB_{pa}^j$  then the corner solution is given by

$$F_{pa} = \min_j \{ F_{pa}^j \}, \quad (18)$$

$$N_{pa}^{1,j} = \Psi \left( \sum_{a=1}^{A(j)} \omega^{a,j} \mu^{a,j} \phi^{a,j} N_{ss}^{1,j} F_{pa} \right), \quad (19)$$

$$N_{pa}^{a,j} = \phi^{a,j} N_{pa}^{1,j}. \quad (20)$$

## 5.- An example: the Northern Stock of Hake fishery

We use the European Northern Stock of Hake as an example of how to find long-term reference points in mixed fisheries. This is a fishery where commercial fleets capture mainly hake (*Merluccius merluccius*) and other species such as megrim (*Lepidorhombus whiffiagonis*) and anglerfish (*Lophius piscatorius* and *Lophius budegassa*). Furthermore, a long-term management plan is currently being designed for this fishery after its recovery from strong depletion in the late 90s.

### 5.1 Calibrating the model

Tables S1 and S2 in the Supplementary Materials provide the multi-species age structure for the four main species caught in the fishery. This information comes from the ICES Report of the Working Group on Hake, Monkfish and Megrim (ICES, 2007). Since we have been unable to find price information by ages for the secondary species, we use the following prices for all ages: €4.35 per kg for megrim and €6.29 per kg for anglerfish, both *L. budegassa* and *L. piscatorius*. Taking into account these prices and a fishing mortality rate of 0.25 for hake, 0.38 for megrim, 0.26 for *L. budegassa* and 0.21 for *L. piscatorius*, we calculate a value with a yield of €523.38 million. Finally, the mixed fishery technology is obtained by normalizing  $q^{hake} = 1$ , and matching the initial fishing mortality ratios. That is,  $q^{megrim} = 1.52$ ,  $q^{budegassa} = 1.04$  and  $q^{piscatorius} = 0.84$ .

For computing long-term marginal cost we use a standard practice in economics which consists of considering that capital income share is roughly constant at around 70%. This fact has been used in other applied fisheries studies such as Hannesson (2007). Therefore we assume a total cost of €366.37 million (70% of the value of total landings from this fishery). For the current fishing rate,  $F = 0.25$ , this implies a marginal cost,  $c$ , of €1.46 million.

Finally, the precautionary reference points for this simulation were set according to ICES (2007). 140 000 t for Northern Hake; that is the result of applying a precautionary buffer to  $B_{lim}$ , the lowest observed biomass in the 2003 assessment ( $B_{pa} = 100\ 000 * 1.4$ ). 55 000 t for Northern megrim, that is the lowest observed SSB in 2003 assessment, since there



is not evidences of reduced recruitment at the lowest observed SSB. The same rationale was applied to both anglerfishes, resulting in  $B_{pa}$  of 31 000 t for *Lophius piscatorius* and 22 000 for *Lophius budegassa*.

Continuing with Step 2, the S–R relationship for Northern hake is fitted using the Shepherd relationship (1982) described by (8) using recruitment and SSB data for 1978 to 2006. This fit gives  $\alpha = 2.4879$ ,  $K = 168\,270$  and  $b = 1.7602$ . For the rest of the species the expected recruitment is considered constant over time. In particular, recruitment for Northern megrim is 279 630 th, for *Lophius budegassa* it is 14 330 th and for *Lophius piscatorius* it is 21 630 th.

Following the working group STECF/SGBRE-07-03, which analysed the impact of the hake management plan on the mixed fishery, we assume that the mortality rate of any of these species is proportional to the hake mortality rate. Note that in that case the yield per fish of any species can be drawn as a function of the hake fishing mortality.

## 5.2 Findings

Reference targets for the multi-species European Northern Stock of hake are calculated assuming two alternative management scenarios. The first is the classical single-species scenario where each species is regulated with an independent reference point associated with its own MSY.  $F_{msy}^j$  denotes the target of this scenario for species  $j = \{\text{hake, megrim, budegassa, piscatorius}\}$ . The second scenario focuses on maximising the NPV of the fishery's profits taking into account its mixed nature and considering a discount factor of  $\beta = 0.95$ . The reference point associated with this mixed scenario is denoted by,  $(F_{\beta=0.95}^{\text{profits}})$ .

Formally all the targets of both scenarios are solutions of equations (6)-(7) under different assumptions, which are calculated using the algorithm shown in Section 4. For the single-species scenario it is assumed that the target is only species  $j$  with  $pr^{a,j} = 1$ ,  $pr^{a,i} = 0$  for  $i \neq j$ ,  $c = 0$  and  $\beta = 1$ . Notice that although the management problem is solved considering only one of the species the other species are also affected, giving the multiproduct fishing technology considered. For the mixed scenario  $c \neq 0$ ,  $\beta = 0.95$  and  $pr^{a,j} \neq 0$  are assumed for all species  $j$ .

Figure 2 summarises the various elements involved in the solution of the mixed scenario where the NPV of profits is maximised. We show that variations in the steady-state fishing rate  $F$  affect the revenues of the fishery in three ways. First, changes in  $F$  affect current catches in weight of all species. This effect is represented by the first sum on the left hand side of equation (6) and is drawn in the top left plot of Figure 2 for all species. Second, changes in  $F$  affect future weighted catches due to future changes in the size of the cohorts currently alive. This effect is represented by the first sum on the right hand side of equation (6) and is drawn in the top right plot of Figure 2 for all species, considering a discount factor of  $\beta = 0.95$ . Third, changes in  $F$  affect the number of future recruits. This effect is represented by the second sum on the right hand side of equation (6) and is drawn in the bottom left plot of Figure 2, considering a discount factor of  $\beta = 0.95$ . Note that this plot represents only hake because the recruitment for megrim and anglerfish is assumed to be constant. These three effects of variations in  $F$  on the revenues are illustrated in the bottom-right plot, where the dashed line represents the sum of the three elements for all species. Formally, the dashed line represents

$$\begin{aligned}
f_{zero} = & \underbrace{\sum_{j=1}^n \sum_{a=1}^{A(j)} pr^{a,j} \frac{dy^{a,j}(F_{ss})}{dF} \phi_{ss}^a N_{ss}^{1,j}}_{\text{Above-left plot}} - \underbrace{\sum_{j=1}^n \sum_{a=1}^{A(j)} pr^{a+j} y_{ss}^{a,j} \phi_{ss}^{a,j} N_{ss}^{1,j} \left( \sum_{i=1}^{a-1} \beta^{a-i} p^{i,j} \right)}_{\text{Above-right plot}} \\
& + \underbrace{\sum_{j=1}^n \frac{dN_{ss}^{1,j}}{dF} \sum_{a=1}^{A(j)} \beta^{1+a} pr^{a+j} y_{ss}^{a,j} \phi_{ss}^{a,j}}_{\text{Below-left plot}}. \tag{21}
\end{aligned}$$

Comparing equations (6) and (21) it can be seen that the optimal fishing rate  $F$  is determined by  $f_{zero} = dC(F_{ss})/dF$ . This comparison is illustrated in the bottom right plot.

$f_{zero}$  is compared with marginal cost,  $c=1.46$ , to obtain  $F_{\beta=0.95}^{\text{profits}}$ . This reference point can be seen to be 0.12.

To compare the scenarios, we also calculate for each of them the discounted profits and the SSB value for all four species. Formally, discounted profits are calculated as

$$\text{Profits} = \sum_{t=0}^{\infty} \beta^t \left( \sum_{j=1}^n \sum_{a=1}^{A(j)} pr^{a,j} y_t^{a,j} \phi_t^{a,j} N_{t-(a-1)}^{1,j} - C(F_t) \right). \tag{22}$$

Notice that to obtain these values we need to simulate the whole optimal path of fishing mortality that drives the fishery from the initial conditions to the steady-state.

Table 1 shows the reference targets, the discounted profits, the yields and the biomass for all species under the single-species and mixed management scenarios. The main findings observed are the following. First, the lowest target reference point appears in the mixed management scenario, in which the NPV of the whole fishery's profits is maximised. Second, the mixed management scenario is superior to single-species management because it leads the fishery to higher discounted profits with higher long-term SSB for all species. In fact mixed management implies 11.4% more in discounted profits than single-species (hake) management. This win-win result is along the lines of Grafton *et al.* (2007). Third, as in Ulrich *et al.* (2011) our results also indicate that single-species management may lead the other species into unsafe situations. For instance, if the fishery were regulated with the anglerfish (*L. budegassa*) as the single species to be considered, then hake SSB would drop below the precautionary level of 140,000 t.

Finally, the mixed management scenario determines a target reference point that is quite similar to the 2/3 of the single-species reference point when hake is the species regulated. That is,  $F_{\beta=0.95}^{\text{profits}}$  converges towards 2/3  $F_{\text{msy}}^{\text{hake}}$ . Table 2 compares the performance of the fishery in the two situations. The discounted profits of the fishery associated with the 2/3  $F_{\text{msy}}^{\text{hake}}$  have been calculated simulating the evolution of the fishery, considering a fishing rate path of 2/3 of the optimal path for  $F_{\text{msy}}^{\text{hake}}$ .

### 5.3 Versatile use of the algorithm

In this section we highlight the intertemporal dimension of the algorithm. In particular, we illustrate how the algorithm can be used to quantify the "cost" of trade-offs between conservation and managers' objectives within a mixed fishery. In fact, in real-world applications this quantification of trade-offs could be more useful to managers than the optimal values themselves.

To illustrate how to quantify the trade-offs, assume that MSY is the appropriate target. Figure 1 shows the dilemma faced by managers when single-species reference points ( $F_{\text{msy}}$  or  $F_{\text{max}}$ ) are used in this mixed fishery. With Shepherd's S-R function, the steady-state reference point for hake is  $F_{\text{msy}} = 0.1715$ . However, for this value only *Lophius piscatorius* is close to its maximum yield per fish. By contrast, the  $F_{\text{max}}$  of *Lophius budegass* is close to the  $F_{\text{pa}}$  (0.25) for hake while  $F_{\text{max}}$  for megrim is outside the safety limits for hake.

If MSY is the target, the objective function can be expressed as

$$\sum_{t=0}^{\infty} \sum_{j=1}^n \sum_{a=1}^{A(j)} pr^{a,j} y_t^{a,j} \phi_t^{a,j} N_{t-(a-1)}^{1,j}. \quad (23)$$

Note that equation (23) is obtained from equation (4) when the discount factor is one,  $\beta = 1$ , and  $C(F_t) = 0$ . If  $pr^{a,j}$  are market prices, the objective function of the mixed fishery is the total yield in value of the four species. However,  $pr^{a,j}$  can be interpreted in a more versatile way: it represents the willingness of the manager (or society) to trade-off the value of fishing one species against the others.

In order to measure these trade-offs, the maximisation problem (5) can be rewritten as

$$\begin{aligned} \max_{\{\bar{F}_t^{hake}, \bar{N}_t^{1,hake}, \bar{F}_t^k, N_t^{1,k}\}} J = & J_t^{hake}(\bar{F}_t^{hake}, \bar{N}_t^{1,hake}) + \sum_{k=1}^3 J_t^k(\bar{F}_t^k, \bar{N}_t^{1,k}) \\ - \sum_{t=0}^{\infty} \left\{ & \lambda_t^{hake} [N_{t+1}^{1,hake} - \Psi(SSB_t^{hake})] + \sum_{k=1}^3 \lambda_t^k [\bar{N}_t^k - N_t^{1,k}] + \sum_{k=1}^3 \delta_t^k [F_t^k - q^k F_t^{hake}] \right\} \quad (24) \end{aligned}$$

where  $J_t^k(\bar{F}_t^k, \bar{N}_t^{1,k}) = \sum_{t=0}^{\infty} \sum_{a=1}^{A(k)} pr^{a,k} y_t^{a,k} \phi_t^{a,j} N_{t-(a-1)}^{1,k}$  is the contribution of species  $k$  to total yield and  $\lambda_t^{hake}$ ,  $\lambda_t^k$  and  $\delta_t^k$  are the Lagrangian multipliers. Notice that these multipliers are the “shadow prices” of the trade-offs faced by managers. In particular  $\delta_t^k = \frac{\partial J_t^k(\bar{F}_t^k, \bar{N}_t^{1,k})}{\partial F_t^k}$ , represents the losses associated with an increase in fishing

mortality of species  $k$ . For example, to quantify the losses associated with use, in period  $t$ , of the single species reference point,  $F_{max}^k$ ,  $\Delta J = -\delta_t^k q^k (F_t - F_{max}^k)$  is computed. Finally, note that the fishing mortality that solves problem (24) satisfies

$$\frac{\partial J}{\partial F_t^{hake}} = - \sum_{k=bude, pisca, megrim} q^k \delta_t^k.$$

Therefore, to quantify the losses associated with the use, in period  $t$ , of the hake reference point,  $F_{msy}^{hake}$ , one must compute  $\Delta J = - \sum_{k=bude, pisca, megrim} q^k \delta_t^k (F_t - F_{msy}^{hake})$ .

The “shadow price” of a hake recruit is  $\lambda_t^{hake}$ . The Supplementary Materials show that in the steady state solution this shadow price is

$$\lambda_{ss}^{hake} = \frac{\sum_{a=1}^{A(hake)} \beta^{1+a} pr^{a,hake} y_{ss}^{a,hake} \phi_{ss}^{a,hake}}{\left[ I - (\Psi_{ss}^{hake})' \sum_{a=1}^{A(hake)} \omega^{a,hake} \mu^{a,hake} \phi_{ss}^{a,hake} \right]}.$$

It measures the cost of distorting the SSB of hake. Formally, the losses associated with an increase of  $\alpha$  percent the SSB are  $\Delta J = \alpha \lambda_{ss} \Psi(SSB_{ss})$ .

Finally,  $\lambda_t^k = \frac{\partial J_t^k(\bar{F}_t^k, \bar{N}_t^{1,k})}{\partial N_t^{1,k}}$  is the “shadow price” of one recruit for the other

species. Remember that for the other species expected recruitment is considered constant over time. Therefore,  $\Delta J = \alpha \lambda_t^k \bar{N}^k$  is the profit from modifying the expected recruitment of species  $k$  by  $\alpha$  percent.

Table 3 quantifies these trade-offs. The steady-state reference point that maximizes the total yield in value of the four species is 0.1649. The optimum global fishing mortality is lower than the fishing mortality that maximises the single specie reference point for *Lophius budegassa* and megrim and higher for the case of *Lophius piscatorius*. Therefore,  $\delta_t^k$  multipliers quantify why the fishing mortality of each species is set below or above its  $F_{max}$  level. Setting the fishing mortality rate of *Lophius budegassa* equal to its reference point reduces total yield by 3.9 million Euros.

Moreover,  $\lambda_t^{hake}$  is the “shadow price of a hake recruit”. Therefore, if the total SSB of hake is modified by 1% the impact on the number of recruits will reduce total yield by 104.35 thousand of Euros. Furthermore, the “shadow prices” of the recruits of the other species show how sensitive the numerical results of the model are to the expected number of recruits used in the simulations.

## 6.- Discussion

Reference points are one of the main tools used by fishing managers to make decisions about future catch options. The Johannesburg Summit requires that all fishery stocks be at levels capable of producing MSY by 2015. As a first step the European Union, through the Common Fisheries Policy (CFP), wants all stocks to be fished at  $F_{msy}$  by 2015. At the same time, the CFP also promotes a gradual change in management to consider the mixed nature of many fisheries. It is not possible to fish all the stocks in a mixed fishery at  $F_{msy}$  (Mace, 2001). The mixed nature of many fisheries requires that ad hoc reference points be adopted for their management. These reference points become an additional reference to help managers to make decisions. In this regard, we have developed an algorithm that allows reference targets for a mixed fishery to be calculated using basic biological information about the age structure of the stocks involved. The algorithm is easily translated to similar situations where many stocks managed with different F targets are caught together.

In this paper target reference points for mixed age-structured fisheries are characterised as the steady-state solution of a maximisation problem where the objective function may represent an economic indicator that accounts for the future in discounted terms. The algorithm deals with two shortcomings of current mixed fisheries management: TACS based on single-species objectives are not usually consistent with one another and the inter-temporal aspects of economic variables are not taken into account.

This algorithm is applied to the European Northern Stock of Hake, where fleets also capture megrim and anglerfish. Data from these stocks are used as an example to illustrate the applicability of the algorithm: the results cannot be considered for management purposes since models for the stocks considered have been reviewed (or are under review) in the wake of problems with data and changes in the perception of their biological properties, but they do illustrate the advantages of applying the algorithm. We find that under reasonable prices and costs the fishing mortality reference point that maximises discounted profits taking into account the mixed character of the fishery is close to  $2/3 F_{msy}^{hake}$  using a discount rate of 5%. This implies a larger spawning biomass and higher discounted profits than those associated with a single-species reference point,  $F_{msy}^{hake}$ . In fact, losses due to the use of the single-reference target  $F_{msy}^{hake}$  in this mixed fishery are quantified at 11.4% of total discounted profits. Therefore, as Grafton *et al.* (2007) suggest, the use of reference points lower than  $F_{msy}$  may be a win-win strategy: higher profits and safer biomass.

The example is developed assuming a deterministic linear change in F for all the stocks. This is tantamount to assuming effort management instead of the current TAC management. Effort management has been seen as a more effective measure for mixed fisheries by many authors (Kraak *et al.*, 2008; Ulrich *et al.*, 2011).

The algorithm is applied to maximise the NPV of fishery profits deterministically with no constraints. However constraints can be easily implemented. For example, if a limit

reference point is identified for any of the stocks concerned, it can be incorporated as a constraint that guarantees that the risk of reaching the limit is low. This may also be done by extending the deterministic algorithm to a stochastic one considering the main sources of uncertainty.

Our findings are consistent with those of other authors who recognise that  $F_{msy}$  may be too high to be an adequate management reference (Larkin, 1997; Mace, 2001). These findings have been followed by many fishery regulations. For instance, according to the UN Fish Stock Agreement (1995),  $F_{msy}$  should be a limit rather than a target reference point. In this sense the Fish Stock Sustainability Index, used as a performance measure for the sustainability of 230 U.S. stocks, considers that a stock is subject to overfishing if it has a fishing mortality rate above the level that provides for the level of MSY. Other analyses consider using fractions of  $F_{msy}$  as targets to be safer for stocks. For instance NAFO (Northwest Atlantic Fisheries Organisation) uses  $2/3 F_{msy}$  for the yellowtail flounder (see Maddock Parsons *et al.*, 2008); Mehanna (2004, 2007) also considers  $2/3 F_{msy}$  as a target reference point for some species in the Gulf of Suez fishery; Jensen (2002) finds that in surplus production models the use of  $3/4 F_{msy}$  maximises yield while minimising the impact of fishing on population biomass. Moreover Christiansen (2010) shows that the win-win strategy appears to a lesser extent when profits are calculated considering the overall fishing sector including processing, distribution and marketing of fish products. All these results can be interpreted as an explicit recognition that  $F_{msy}$  is too high to be considered a target reference point (Quinn and Collie, 2005).

Finally, the algorithm proposed is a versatile tool: it can be used by summarising the management target in a single objective function. We highlight the temporal dimension by using the NPV of the fishery's profits as our objective function. However the algorithm can easily be applied taking targets based for instance on MSY as its objective function. It is well known that quantifying economic values is not a simple task in real-world applications and that it may even be considered an inappropriate objective. For many fishery stocks MSY is considered the appropriate target. Moreover, targets based on MSY have at least one advantage over NPV: there are specific ICES Working Groups that collect data in order to quantify it.

Independently of the objective function used to summarise the management target, the algorithm can be used to quantify the "cost" of trade-offs. As a referee suggests, this could be more important in a real-world application than the optimal values themselves because these trade-offs could be more useful to managers than the optimal values.

Future research may focus on the extension of this dynamic approach to other contexts such as predator-prey relationships, other meta-population models or more state dependent fishing technologies, as in van Oostenbrugge *et al.* (2008).

### **Supplementary material**

Supplementary material is available at ICESJMS in the online version of this manuscript. It shows in detail how the calibration of the model has been prepared using the data set reported by ICES (2007).

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Table 1: Reference points, discounted profits, yields and SSB in the long term under single and mixed management scenarios

	Single-species Management Scenario				Mixed Management Scenario
Reference point ( $F$ )	$F_{msy}^{hake}$	$F_{msy}^{megrim}$	$F_{msy}^{budegassa}$	$F_{msy}^{piscatorius}$	$F_{\beta=0.95}^{profits}$
	0.1715	0.3800	0.2400	0.1400	0.1205
Discounted profits ('000 Euros)	5 936	-2 052	3 803	6 468	6 613
SSB (tonnes)					
Hake	198 169	40 656	134 219	232 440	255 898
Megrim	96 529	54 448	77 991	107 421	115 098
Anglerfish ( <i>Lophius budegassa</i> )	33 716	14 184	24 599	39 383	43 496
Anglerfish ( <i>Lophius piscatorius</i> )	66 707	20 150	43 362	8 2597	94 788
Yield (tonnes)					
Hake	60 842	25 723	55 673	59 540	57 293
Megrim	13 621	15 863	15 105	12 465	11 544
Anglerfish ( <i>Lophius budegassa</i> )	6 826	6 674	7 062	6 479	6 146
Anglerfish ( <i>Lophius piscatorius</i> )	19 514	14 860	18 026	19 802	19 683

Source: Own calculations using the algorithm proposed

Table 2: Mixed management against 2/3 of single-species (hake) reference point

	2/3 of single-species (hake) Management	Mixed Management
Reference point ( $F$ )	$2/3 F_{msy}^{hake}$	$F_{\beta=0.95}^{profits}$
	0.1143	0.1205
Discounted profits ('000 Euros)	6 599	6 613
SSB (tonnes)		
Hake	263 667	255 898
Megrim	117 678	115 098
Anglerfish ( <i>Lophius budegassa</i> )	44 907	43 496
Anglerfish ( <i>Lophius piscatorius</i> )	99 087	94 788

Source: Own calculations using the algorithm proposed



Table 3: Lagrange multipliers and shadow costs in the long term under mixed management scenario

Objective	Total MSY in value ('000 Euros)	
Reference point $F_{\beta=1}$	0.1649	
<b>Changes in fishing mortality of:</b>	$\delta_{ss}^k$	Cost $F_{max}^k$
Anglerfish ( <i>Lophius budegassa</i> )	49 962	3 901
Anglerfish ( <i>Lophius piscatorius</i> )	- 488	10
Megrim	52 674	17 344
	$\sum_{k=bude, pisca, megrim} -q^k \delta_{ss}^k$	Cost $F_{msy}^{hake}$
<b>Changes in fishing mortality of hake</b>	131 614	868
<b>Changes in recruits of:</b>	$\lambda_{ss}^k$	<i>Profit</i> $\Delta 1\%$ <i>Expected</i> <i>Recruitment</i>
Anglerfish ( <i>Lophius budegassa</i> )	19.12	2 740
Anglerfish ( <i>Lophius piscatorius</i> )	44.99	9 732
Megrim	1.27	3 552
	$\lambda_{ss}^{hake}$	Cost $\Delta 1\%$ <i>SSB</i>
<b>Changes in recruits of hake</b>	1.56	-104.35

Cost  $F_{max}^k$  : Losses associated with implementing the single species  $k$  reference point, i.e.  $\Delta J = -\delta_t^k q^k (F_t - F_{max}^k)$ ; Cost  $\Delta 1\%$  *SSB*: Losses associated with incrementing the SSB of hake by 1%, i.e.  $\Delta J = .01 \lambda_{ss} \Psi' SSB_{ss}$ ; *Profit*  $\Delta 1\%$  *Expected Recruitment*: Profits associated with incrementing by 1% the expected recruitment  $\Delta J = .01 \lambda_t^k \bar{N}^k$

Source: Own calculations using the algorithm proposed

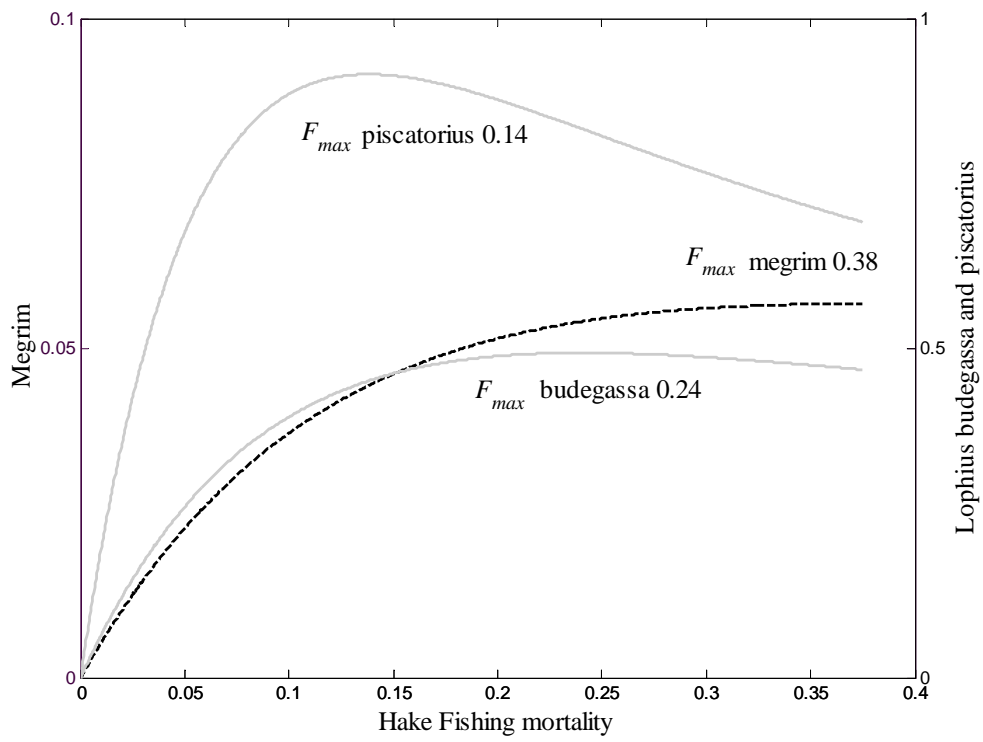


Figure 1:  $F_{max}$  for megrim (dotted black line), and Lophius budegassa and piscatorius (grey lines).

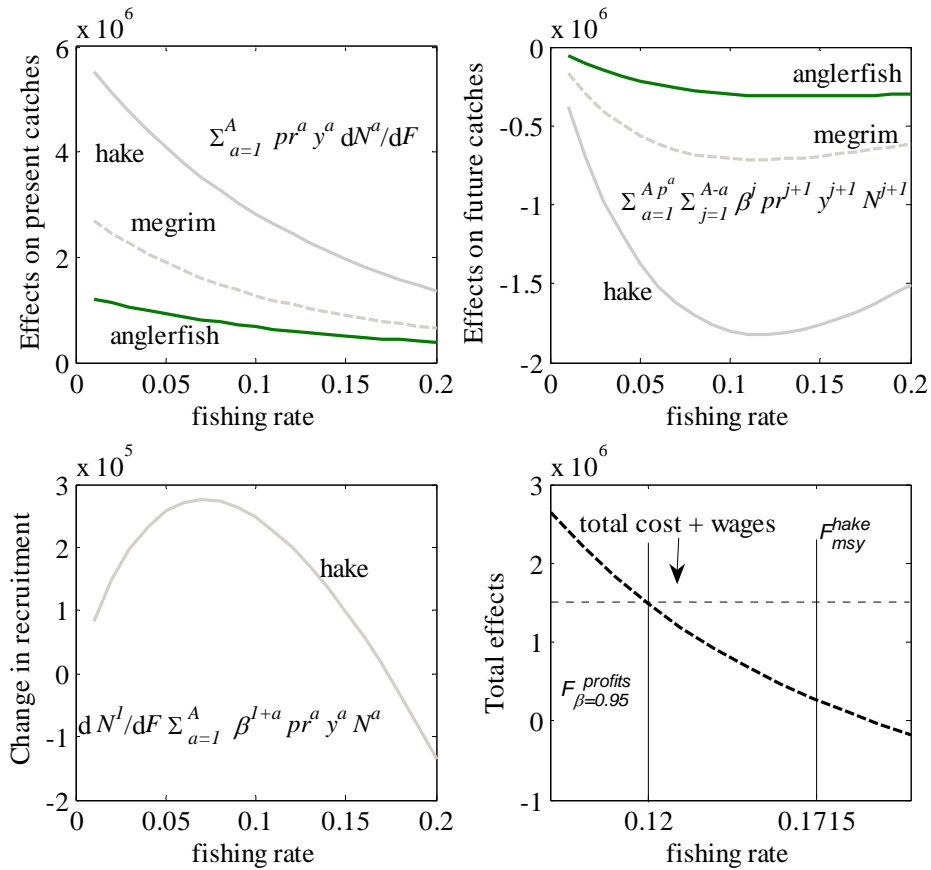


Figure 2. Target reference point that maximises net present value of profits in the mixed Northern Stock considering a discount factor of  $\beta = 0.95$ . Changes in  $F$  affect the net present value of the fishery's profits through variations in: i) current yields (top left plot), ii) future yields derived from variations in future size of the cohorts currently alive (top right plot), and iii) future yield derived from changes in future recruits of hake (bottom left plot). The optimal reference point that maximises the net present value of profits is given by the  $F$  that offsets all these effects (i, ii plus iii) against the marginal cost (bottom right plot).

## Reference points as the steady-state solutions of dynamic management problems in mixed age-structured fisheries

José-María Da Rocha, María-José Gutiérrez and Santiago Cerviño

In this supplementary material we show:

- i) How to solve the maximisation problem (5) in order to obtain the equation system (6)-(7) that characterises the steady state solution. We also prove that  $F_{msy}$  coincides with  $F_{ss}$  for the case of one species in which price is one, marginal cost is zero ( $dC(F)/dF = 0$ ) and  $\beta=1$ .
- ii) Tables S1, S2, S3 and S4 show the parameters of the biological model for the four species harvested in the Northern Stock: northern hake, northern megrim, *lophius budegassa*, *lophius piscatorius*.

Assume without loss of generality that  $A = 3$  and  $n=2$ . In this context, the function to be maximised can be expressed as

$$L = \sum_{t=0}^{\infty} \beta^t \left( \begin{array}{l} pr^{1,1} y_t^{1,1} (F_t) \phi_t^{1,1} N_t^{1,1} + pr^{2,1} y_t^{2,1} (F_t) \phi_t^{2,1} N_{t-1}^{1,1} + pr^{3,1} y_t^{3,1} (F_t) \phi_t^{3,1} N_{t-2}^{1,1} \\ + pr^{1,2} y_t^{1,2} (F_t) \phi_t^{1,2} N_t^{1,2} + pr^{2,2} y_t^{2,2} (F_t) \phi_t^{2,2} N_{t-1}^{1,2} + pr^{3,2} y_t^{3,2} (F_t) \phi_t^{3,2} N_{t-2}^{1,2} - C(F_t) \\ + \lambda_t^1 \left[ \Psi_t^1 (\omega^{1,1} \mu^{1,1} \phi_t^{1,1} N_t^{1,1} + \omega^{2,1} \mu^{2,1} \phi_t^{2,1} N_{t-1}^{1,1} + \omega^{3,1} \mu^{3,1} \phi_t^{3,1} N_{t-2}^{1,1}) - N_{t+1}^{1,1} \right] \\ + \lambda_t^2 \left[ \Psi_t^2 (\omega^{1,2} \mu^{1,2} \phi_t^{1,2} N_t^{1,2} + \omega^{2,2} \mu^{2,2} \phi_t^{2,2} N_{t-1}^{1,2} + \omega^{3,2} \mu^{3,2} \phi_t^{3,2} N_{t-2}^{1,2}) - N_{t+1}^{1,2} \right] \\ + \theta_t^1 \left[ \omega^{1,1} \mu^{1,1} \phi_t^{1,1} N_t^{1,1} + \omega^{2,1} \mu^{2,1} \phi_t^{2,1} N_{t-1}^{1,1} + \omega^{3,1} \mu^{3,1} \phi_t^{3,1} N_{t-2}^{1,1} - SSB_p^1 \right] \\ + \theta_t^2 \left[ \omega^{1,2} \mu^{1,2} \phi_t^{1,2} N_t^{1,2} + \omega^{2,2} \mu^{2,2} \phi_t^{2,2} N_{t-1}^{1,2} + \omega^{3,2} \mu^{3,2} \phi_t^{3,2} N_{t-2}^{1,2} - SSB_p^2 \right] \end{array} \right), \quad (IS)$$

where  $\lambda$  and  $\theta$  are the Lagrange multipliers associated with the first and second restrictions of the maximisation problem (5), respectively. Notice that for the general case of  $n$  species, there would be  $2n$  constraints.

Moreover, taking into account the survival functions, (2), it is known that

$$\begin{aligned}
\phi_t^{1,1} &= 1 \\
\phi_t^{2,1} &= \phi(F_{t-1}) = e^{-p^{1,1}F_{t-1}-m^{1,1}} \\
\phi_t^{3,1} &= \phi(F_{t-1}, F_{t-2}) = e^{-p^{2,1}F_{t-1}-m^{2,1}} e^{-p^{1,1}F_{t-2}-m^{1,1}} \\
\phi_t^{1,2} &= 1 \\
\phi_t^{2,2} &= \phi(F_{t-1}) = e^{-p^{1,2}F_{t-1}-m^{1,2}} \\
\phi_t^{3,2} &= \phi(F_{t-1}, F_{t-2}) = e^{-p^{2,2}F_{t-1}-m^{2,2}} e^{-p^{1,2}F_{t-2}-m^{1,2}}
\end{aligned}$$

Note that  $F_t$  appears only in the sums multiplied by  $\beta^t$ ,  $\beta^{t+1}$  and  $\beta^{t+2}$  in equation (1S). That is

$$\begin{aligned}
L = & \dots + \beta^t \left( \begin{aligned} & pr^{1,1} y_t^{1,1}(F_t) N_t^{1,1} + pr^{2,1} y_t^{2,1}(F_t) e^{-p^{1,1}F_{t-1}-m^{1,1}} N_{t-1}^{1,1} + pr^{3,1} y_t^{3,1}(F_t) e^{-p^{2,1}F_{t-1}-m^{2,1}} e^{-p^{1,1}F_{t-2}-m^{1,1}} N_{t-2}^{1,1} \\ & + pr^{1,2} y_t^{1,2}(F_t) N_t^{1,2} + pr^{2,2} y_t^{2,2}(F_t) e^{-p^{1,2}F_{t-1}-m^{1,2}} N_{t-1}^{1,2} + pr^{3,2} y_t^{3,2}(F_t) e^{-p^{2,2}F_{t-1}-m^{2,2}} e^{-p^{1,2}F_{t-2}-m^{1,2}} N_{t-2}^{1,2} \\ & - C(F_t) \\ & + \lambda_t^1 [\Psi_t^1 (\omega^{1,1} \mu^{1,1} \phi_t^{1,1} N_t^{1,1} + \omega^{2,1} \mu^{2,1} \phi_t^{2,1} N_{t-1}^{1,1} + \omega^{3,1} \mu^{3,1} \phi_t^{3,1} N_{t-2}^{1,1}) - N_{t+1}^{1,1}] \\ & + \lambda_t^2 [\Psi_t^2 (\omega^{1,2} \mu^{1,2} \phi_t^{1,2} N_t^{1,2} + \omega^{2,2} \mu^{2,2} \phi_t^{2,2} N_{t-1}^{1,2} + \omega^{3,2} \mu^{3,2} \phi_t^{3,2} N_{t-2}^{1,2}) - N_{t+1}^{1,2}] \\ & + \theta_t^1 [\omega^{1,1} \mu^{1,1} \phi_t^{1,1} N_t^{1,1} + \omega^{2,1} \mu^{2,1} \phi_t^{2,1} N_{t-1}^{1,1} + \omega^{3,1} \mu^{3,1} \phi_t^{3,1} N_{t-2}^{1,1} - SSB_p^1] \\ & + \theta_t^2 [\omega^{1,2} \mu^{1,2} \phi_t^{1,2} N_t^{1,2} + \omega^{2,2} \mu^{2,2} \phi_t^{2,2} N_{t-1}^{1,2} + \omega^{3,2} \mu^{3,2} \phi_t^{3,2} N_{t-2}^{1,2} - SSB_p^2] \end{aligned} \right) \\
& + \beta^{t+1} \left( \begin{aligned} & pr^{1,1} y_{t+1}^{1,1}(F_{t+1}) N_{t+1}^{1,1} + pr^{2,1} y_{t+1}^{2,1}(F_{t+1}) e^{-p^{1,1}F_t-m^{1,1}} N_t^{1,1} + pr^{3,1} y_{t+1}^{3,1}(F_{t+1}) e^{-p^{2,1}F_t-m^{2,1}} e^{-p^{1,1}F_{t-1}-m^{1,1}} N_{t-1}^{1,1} \\ & + pr^{1,2} y_{t+1}^{1,2}(F_{t+1}) N_{t+1}^{1,2} + pr^{2,2} y_{t+1}^{2,2}(F_{t+1}) e^{-p^{1,2}F_t-m^{1,2}} N_t^{1,2} + pr^{3,2} y_{t+1}^{3,2}(F_{t+1}) e^{-p^{2,2}F_t-m^{2,2}} e^{-p^{1,2}F_{t-1}-m^{1,2}} N_{t-1}^{1,2} \\ & - C(F_{t+1}) \\ & + \lambda_{t+1}^1 [\Psi_{t+1}^1 (\omega^{1,1} \mu^{1,1} \phi_{t+1}^{1,1} N_{t+1}^{1,1} + \omega^{2,1} \mu^{2,1} \phi_{t+1}^{2,1} N_t^{1,1} + \omega^{3,1} \mu^{3,1} \phi_{t+1}^{3,1} N_{t-1}^{1,1}) - N_{t+2}^{1,1}] \\ & + \lambda_{t+1}^2 [\Psi_{t+1}^2 (\omega^{1,2} \mu^{1,2} \phi_{t+1}^{1,2} N_{t+1}^{1,2} + \omega^{2,2} \mu^{2,2} \phi_{t+1}^{2,2} N_t^{1,2} + \omega^{3,2} \mu^{3,2} \phi_{t+1}^{3,2} N_{t-1}^{1,2}) - N_{t+2}^{1,2}] \\ & + \theta_{t+1}^1 [\omega^{1,1} \mu^{1,1} \phi_{t+1}^{1,1} N_{t+1}^{1,1} + \omega^{2,1} \mu^{2,1} \phi_{t+1}^{2,1} N_t^{1,1} + \omega^{3,1} \mu^{3,1} \phi_{t+1}^{3,1} N_{t-1}^{1,1} - SSB_p^1] \\ & + \theta_{t+1}^2 [\omega^{1,2} \mu^{1,2} \phi_{t+1}^{1,2} N_{t+1}^{1,2} + \omega^{2,2} \mu^{2,2} \phi_{t+1}^{2,2} N_t^{1,2} + \omega^{3,2} \mu^{3,2} \phi_{t+1}^{3,2} N_{t-1}^{1,2} - SSB_p^2] \end{aligned} \right) \\
& + \beta^{t+2} \left( \begin{aligned} & pr^{1,1} y_{t+2}^{1,1}(F_{t+2}) N_{t+2}^{1,1} + pr^{2,1} y_{t+2}^{2,1}(F_{t+2}) e^{-p^{1,1}F_{t+1}-m^{1,1}} N_{t+1}^{1,1} + pr^{3,1} y_{t+2}^{3,1}(F_{t+2}) e^{-p^{2,1}F_{t+1}-m^{2,1}} e^{-p^{1,1}F_t-m^{1,1}} N_t^{1,1} \\ & + pr^{1,2} y_{t+2}^{1,2}(F_{t+2}) N_{t+2}^{1,2} + pr^{2,2} y_{t+2}^{2,2}(F_{t+2}) e^{-p^{1,2}F_{t+1}-m^{1,2}} N_{t+1}^{1,2} + pr^{3,2} y_{t+2}^{3,2}(F_{t+2}) e^{-p^{2,2}F_{t+1}-m^{2,2}} e^{-p^{1,2}F_t-m^{1,2}} N_t^{1,2} \\ & - C(F_{t+2}) \\ & + \lambda_{t+2}^1 [\Psi_{t+2}^1 (\omega^{1,1} \mu^{1,1} \phi_{t+2}^{1,1} N_{t+2}^{1,1} + \omega^{2,1} \mu^{2,1} \phi_{t+2}^{2,1} N_{t+1}^{1,1} + \omega^{3,1} \mu^{3,1} \phi_{t+2}^{3,1} N_t^{1,1}) - N_{t+3}^{1,1}] \\ & + \lambda_{t+2}^2 [\Psi_{t+2}^2 (\omega^{1,2} \mu^{1,2} \phi_{t+2}^{1,2} N_{t+2}^{1,2} + \omega^{2,2} \mu^{2,2} \phi_{t+2}^{2,2} N_{t+1}^{1,2} + \omega^{3,2} \mu^{3,2} \phi_{t+2}^{3,2} N_t^{1,2}) - N_{t+3}^{1,2}] \\ & + \theta_{t+2}^1 [\omega^{1,1} \mu^{1,1} \phi_{t+2}^{1,1} N_{t+2}^{1,1} + \omega^{2,1} \mu^{2,1} \phi_{t+2}^{2,1} N_{t+1}^{1,1} + \omega^{3,1} \mu^{3,1} \phi_{t+2}^{3,1} N_t^{1,1} - SSB_p^1] \\ & + \theta_{t+2}^2 [\omega^{1,2} \mu^{1,2} \phi_{t+2}^{1,2} N_{t+2}^{1,2} + \omega^{2,2} \mu^{2,2} \phi_{t+2}^{2,2} N_{t+1}^{1,2} + \omega^{3,2} \mu^{3,2} \phi_{t+2}^{3,2} N_t^{1,2} - SSB_p^2] \end{aligned} \right) \\
& + \dots
\end{aligned}$$

Therefore, the first order conditions from  $dL/dF_t = 0$  are given by

$$\begin{aligned}
\frac{dL}{dF_t} = & \beta^t \left\{ pr^{1,1} \frac{dy_t^{1,1}(F_t)}{dF_t} N_t^{1,1} + pr^{2,1} \frac{dy_t^{2,1}(F_t)}{dF_t} \phi_t^{2,1} N_{t-1}^{1,1} + pr^{3,1} \frac{dy_t^{3,1}(F_t)}{dF_t} \phi_t^{3,1} N_{t-2}^{1,1} \right. \\
& \left. + pr^{1,2} \frac{dy_t^{1,2}(F_t)}{dF_t} N_t^{1,2} + pr^{2,2} \frac{dy_t^{2,2}(F_t)}{dF_t} \phi_t^{2,2} N_{t-1}^{1,2} + pr^{3,2} \frac{dy_t^{3,2}(F_t)}{dF_t} \phi_t^{3,2} N_{t-2}^{1,2} - \frac{dC(F_t)}{dF_t} \right\} \\
& + \beta^{t+1} \left\{ pr^{2,1} y_{t+1}^{2,1}(F_{t+1})(-p^{1,1}) \phi_{t+1}^{2,1} N_t^{1,1} + pr^{3,1} y_{t+1}^{3,1}(F_{t+1})(-p^{2,1}) \phi_{t+1}^{3,1} N_{t-1}^{1,1} \right. \\
& + pr^{2,2} y_{t+1}^{2,2}(F_{t+1})(-p^{1,2}) \phi_{t+1}^{2,2} N_t^{1,2} + pr^{3,2} y_{t+1}^{3,2}(F_{t+1})(-p^{2,2}) \phi_{t+1}^{3,2} N_{t-1}^{1,2} \\
& + \lambda_{t+1}^1 (\Psi^1)' \left[ \omega^{2,1} \mu^{2,1} (-p^{1,1}) \phi_{t+1}^{2,1} N_t^{1,1} + \omega^{3,1} \mu^{3,1} (-p^{2,1}) \phi_{t+1}^{3,1} N_{t-1}^{1,1} \right] \\
& + \lambda_{t+1}^2 (\Psi^2)' \left[ \omega^{2,2} \mu^{2,2} (-p^{1,2}) \phi_{t+1}^{2,2} N_t^{1,2} + \omega^{3,2} \mu^{3,2} (-p^{2,2}) \phi_{t+1}^{3,2} N_{t-1}^{1,2} \right] \\
& + \theta_{t+1}^1 \left[ \omega^{2,1} \mu^{2,1} (-p^{1,1}) \phi_{t+1}^{2,1} N_t^{1,1} + \omega^{3,1} \mu^{3,1} (-p^{2,1}) \phi_{t+1}^{3,1} N_{t-1}^{1,1} \right] \\
& \left. + \theta_{t+1}^2 \left[ \omega^{2,2} \mu^{2,2} (-p^{1,2}) \phi_{t+1}^{2,2} N_t^{1,2} + \omega^{3,2} \mu^{3,2} (-p^{2,2}) \phi_{t+1}^{3,2} N_{t-1}^{1,2} \right] \right\} \\
& + \beta^{t+3} \left\{ pr^{3,1} y_{t+2}^{3,1}(F_{t+2})(-p^{1,1}) \phi_{t+2}^{3,1} N_t^{1,1} + pr^{3,2} y_{t+2}^{3,2}(F_{t+2})(-p^{1,2}) \phi_{t+2}^{3,2} N_t^{1,2} \right. \\
& \left. + \lambda_{t+2}^1 (\Psi^1)' \left[ \omega^{3,1} \mu^{3,1} (-p^{1,1}) \phi_{t+2}^{3,1} N_t^{1,1} \right] + \lambda_{t+2}^2 (\Psi^2)' \left[ \omega^{3,2} \mu^{3,2} (-p^{1,2}) \phi_{t+2}^{3,2} N_t^{1,2} \right] \right\} = 0.
\end{aligned}$$

A generalisation of this example for any  $j = 1, \dots, n$  and  $a = 1, \dots, A(j)$  can be expressed as

$$\begin{aligned}
\frac{dL}{dF_t} = 0 & \Rightarrow \sum_{j=1}^n \left[ \sum_{a=1}^{A(j)} pr^{a,j} \frac{dy_t^{a,j}(F_t)}{dF_t} N_t^{a,j} - \frac{dC(F_t)}{dF_t} \right] \\
& = \sum_{j=1}^n \sum_{a=1}^{A(j)-1} p^{a,j} \left\{ \sum_{i=1}^{A(j)-a} \beta^i \left[ pr^{a+i,j} y_{t+i}^{a+i,j}(F_{t+i}) + \left( (\Psi_{t+i}^j)' \lambda_{t+i}^j + \theta_{t+i}^j \right) \omega^{a+i,j} \mu^{a+i,j} \right] N_{t+i}^{a+i,j} \right\}. \quad (2S)
\end{aligned}$$

The other first order condition comes from  $dL/dN_{t+2}^{1,j} = 0 \quad \forall j = 1, \dots, n$  is given by

$$\begin{aligned}
\frac{dL}{dN_{t+2}^{1,j}} = & -\beta^{t+2} \lambda_{t+1}^j + \beta^{t+2} \left\{ pr^{1,j} y_{t+2}^{1,j}(F_{t+2}) + \left[ \lambda_{t+2}^j (\Psi_{t+2}^j)' + \theta_{t+2}^j \right] \omega^{1,j} \mu^{1,j} \right\} \\
& + \beta^{t+3} \left\{ pr^{2,1} y_{t+3}^{2,1}(F_{t+3}) \phi_{t+3}^{2,j} + \left[ \lambda_{t+3}^j (\Psi_{t+3}^j)' + \theta_{t+3}^j \right] \omega^{2,j} \mu^{2,j} \phi_{t+3}^{2,j} \right\} \\
& + \beta^{t+4} \left\{ pr^{3,1} y_{t+4}^{3,1}(F_{t+4}) \phi_{t+4}^{3,j} + \left[ \lambda_{t+4}^j (\Psi_{t+4}^j)' + \theta_{t+4}^j \right] \omega^{3,j} \mu^{3,j} \phi_{t+4}^{3,j} \right\} = 0.
\end{aligned}$$

Notice that given the S-R relationship (equation (3) in the main text), recruitment at period 1 is determined by the fish abundance  $\phi$  at time 0. Therefore  $N_1^{1,j}$  cannot be chosen at period 0. The relevant state variable is  $N_{t+2}^{1,j} \quad \forall t = 0, 1, \dots$

A generalisation of this example for any  $j = 1, \dots, n$  and  $a = 1, \dots, A(j)$  can be expressed as

$$\begin{aligned}
\frac{dL}{dN_{t+2}^{1,j}} = 0 & \Rightarrow \sum_{a=1}^{A(j)} \beta^{1+a} pr^{a,j} y_{t+1+a}^{a,j}(F_{t+1+a}) \phi_{t+1+a}^{a,j} \\
& = \beta^{t+2} \lambda_{t+1}^j - \sum_{a=1}^{A(j)} \beta^{1+a} \left[ (\Psi_{t+1+a}^j)' \lambda_{t+1+a}^j + \theta_{t+1+a}^j \right] \omega^{a,j} \mu^{a,j} \phi_{t+1+a}^{a,j}. \quad (3S)
\end{aligned}$$

If the precautionary level restriction is not binding then  $\theta_{t+i}^j = 0$ , and the first order conditions (2S)-(3S) valued in the steady state can be written as the following  $j+1$  equation system

$$\sum_{j=1}^n \sum_{a=1}^{A(j)} pr^{a,j} \frac{dy_{ss}^{a,j}}{dF_{ss}} N_{ss}^{a,j} - \frac{dC(F_{ss})}{dF_{ss}} = \sum_{j=1}^n \sum_{a=1}^{A(j)-1} p^{a,j} \left\{ \sum_{i=1}^{A(j)-a} [\beta^i pr^{a+i,j} y_{ss}^{a+i,j} + (\Psi_{ss}^j)' \lambda_{ss}^j \omega^{a+i,j} \mu^{a+i,j}] N_{ss}^{a+i,j} \right\}, \quad (4S)$$

$$\sum_{a=1}^{A(j)} \beta^{1+a} pr^{a,j} y_{ss}^{a,j} \phi_{ss}^{a,j} = \lambda_{ss}^j \left[ 1 - (\Psi_{ss}^j)' \sum_{a=1}^{A(j)} \omega^{a,j} \mu^{a,j} \phi_{ss}^{a,j} \right], \quad (5S)$$

where  $\lambda_{ss}^j = \lambda_t^j = \beta^i \lambda_{t+i}^j$ ,  $\forall i = 1, \dots, A(j) + 1$ .

Taking into account (1) and substituting (5S) into (4S) the following expression is obtained

$$\begin{aligned} \sum_{j=1}^n \sum_{a=1}^{A(j)} pr^{a,j} \frac{dy_{ss}^{a,j}}{dF_{ss}} N_{ss}^{a,j} - \frac{dC(F_{ss})}{dF_{ss}} &= \sum_{j=1}^n \sum_{a=1}^{A(j)-1} p^{a,j} \left\{ \sum_{i=1}^{A(j)-a} \beta^i pr^{a+i,j} y_{ss}^{a+i,j} \phi_{ss}^{a+i,j} N_{ss}^{1,j} \right\} + \\ &+ \sum_{j=1}^n \left[ \frac{(\Psi_{ss}^j)' \sum_{a=1}^{A(j)} \beta^{1+a} pr^{a,j} y_{ss}^{a,j} \phi_{ss}^{a,j}}{1 - (\Psi_{ss}^j)' \sum_{a=1}^{A(j)} \omega^{a,j} \mu^{a,j} \phi_{ss}^{a,j}} \sum_{a=1}^{A(j)-1} p^{a,j} \left\{ \sum_{i=1}^{A(j)-a} \omega^{a+i,j} \mu^{a+i,j} \phi_{ss}^{a+i,j} N_{ss}^{1,j} \right\} \right]. \end{aligned} \quad (6S)$$

On the one hand, it can be seen that

$$\begin{aligned} &\sum_{a=1}^{A(j)-1} p^{a,j} \left\{ \sum_{i=1}^{A(j)-a} \beta^i pr^{a+i,j} y_{ss}^{a+i,j} \phi_{ss}^{a+i,j} \right\} = \\ &p^{1,j} [\beta pr^{2,j} y_{ss}^{2,j} \phi_{ss}^{2,j} + \beta^2 pr^{3,j} y_{ss}^{3,j} \phi_{ss}^{3,j} + \dots + \beta^{A(j)-1} pr^{A(j),j} y_{ss}^{A(j),j} \phi_{ss}^{A(j),j}] + \\ &p^{2,j} [\beta pr^{3,j} y_{ss}^{3,j} \phi_{ss}^{3,j} + \beta^2 pr^{4,j} y_{ss}^{4,j} \phi_{ss}^{4,j} + \dots + \beta^{A(j)-2} pr^{A(j),j} y_{ss}^{A(j),j} \phi_{ss}^{A(j),j}] + \dots + \\ &p^{A(j)-2,j} [\beta pr^{A(j)-1,j} y_{ss}^{A(j)-1,j} \phi_{ss}^{A(j)-1,j} + \beta^2 pr^{A(j),j} y_{ss}^{A(j),j} \phi_{ss}^{A(j),j}] + p^{A(j)-1,j} [\beta pr^{A(j),j} y_{ss}^{A(j),j} \phi_{ss}^{A(j),j}] = \\ &pr^{A(j),j} y_{ss}^{A(j),j} \phi_{ss}^{A(j),j} [p^{1,j} \beta^{A(j)-1} + p^{2,j} \beta^{A(j)-2} + \dots + p^{A(j)-1,j} \beta] + \\ &pr^{A(j)-1,j} y_{ss}^{A(j)-1,j} \phi_{ss}^{A(j)-1,j} [p^{1,j} \beta^{A(j)-2} + p^{2,j} \beta^{A(j)-3} + \dots + p^{A(j)-2,j} \beta] + \dots + \\ &pr^{3,j} y_{ss}^{3,j} \phi_{ss}^{3,j} [p^{1,j} \beta^2 + p^{2,j} \beta] + pr^{2,j} y_{ss}^{2,j} \phi_{ss}^{2,j} [p^{1,j} \beta] = \\ &\sum_{a=1}^{A(j)} pr^{a,j} y_{ss}^{a,j} \phi_{ss}^{a,j} \left\{ \sum_{i=1}^{a-1} \beta^{a-i} p^{i,j} \right\}. \end{aligned} \quad (7S)$$

On the other hand, taking into account the survival functions, (2), valued in the steady state, it can be seen that

$$\begin{aligned}
& \sum_{a=1}^{A(j)-1} p^{a,j} \left\{ \sum_{i=1}^{A(j)-a} \omega^{a+i,j} \mu^{a+i,j} \phi_{ss}^{a+i,j} \right\} = \\
& p^{1,j} \left[ \omega^{2,j} \mu^{2,j} \phi_{ss}^{2,j} + \omega^{3,j} \mu^{3,j} \phi_{ss}^{3,j} + \dots + \omega^{A(j),j} \mu^{A(j),j} \phi_{ss}^{A(j),j} \right] + \\
& p^{2,j} \left[ \omega^{3,j} \mu^{3,j} \phi_{ss}^{3,j} + \omega^{4,j} \mu^{4,j} \phi_{ss}^{4,j} + \dots + \omega^{A(j),j} \mu^{A(j),j} \phi_{ss}^{A(j),j} \right] + \dots + p^{A(j)-1,j} \omega^{A(j),j} \mu^{A(j),j} \phi_{ss}^{A(j),j} = \\
& p^{1,j} \omega^{2,j} \mu^{2,j} \phi_{ss}^{2,j} + [p^{1,j} + p^{2,j}] \omega^{3,j} \mu^{3,j} \phi_{ss}^{3,j} + \dots + [p^{1,j} + p^{2,j} + \dots + p^{A(j)-1,j}] \omega^{A(j),j} \mu^{A(j),j} \phi_{ss}^{A(j),j} = \\
& - \sum_{a=1}^{A(j)} \omega^{a,j} \mu^{a,j} \frac{d\phi_{ss}^{a,j}}{dF_{ss}}. \quad (8S)
\end{aligned}$$

Moreover, taking derivatives in the S-R relationship, (3), valued in the steady state and considering (8S), the following is obtained:

$$\frac{dN_{ss}^{1,j}}{dF_{ss}} = \frac{(\Psi_{ss}^j)' \sum_{a=1}^{A(j)} \omega^{a,j} \mu^{a,j} \frac{d\phi_{ss}^{a,j}}{dF_{ss}}}{1 - (\Psi_{ss}^j)' \sum_{a=1}^{A(j)} \omega^{a,j} \mu^{a,j} \phi_{ss}^{a,j}} N_{ss}^{1,j} = \frac{(\Psi_{ss}^j)' \sum_{a=1}^{A(j)-1} p^{a,j} \left\{ \sum_{i=1}^{A(j)-a} \omega^{a+i,j} \mu^{a+i,j} \phi_{ss}^{a+i,j} \right\}}{1 - (\Psi_{ss}^j)' \sum_{a=1}^{A(j)} \omega^{a,j} \mu^{a,j} \phi_{ss}^{a,j}} N_{ss}^{1,j}. \quad (9S)$$

Substituting (7S) and (9S) into (6S), the following is obtained:

$$\sum_{j=1}^n \sum_{a=1}^{A(j)} pr^{a,j} \frac{dy_{ss}^{a,j}}{dF_{ss}} N_{ss}^{a,j} - \frac{dC(F_{ss})}{dF_{ss}} = \sum_{j=1}^n \left[ \sum_{a=1}^{A(j)} pr^{a,j} y_{ss}^{a,j} \phi_{ss}^{a,j} N_{ss}^{1,j} \left\{ \sum_{i=1}^{a-1} \beta^{a-i} p^{i,j} \right\} - \frac{dN_{ss}^{1,j}}{dF_{ss}} \sum_{a=1}^{A(j)} \beta^{1+a} pr^{a,j} y_{ss}^{a,j} \phi_{ss}^{a,j} \right],$$

which is equation (6) in the main text.

Equation (7) in the main text corresponds to the S-R relationship, (3), valued in the steady state.

Finally, notice that for the case of one species in which price is one, marginal cost is zero ( $dC(F)/dF = 0$ ) and  $\beta=1$ , the first order condition (7) can be expressed as

$$\sum_{a=1}^A pr^a \frac{dy_{ss}^a}{dF_{ss}} N_{ss}^a = \sum_{a=1}^A y_{ss}^a \phi_{ss}^a N_{ss}^1 \left\{ \sum_{i=1}^{a-1} p^i \right\} - \frac{dN_{ss}^1}{dF_{ss}} \sum_{a=1}^A y_{ss}^a \phi_{ss}^a.$$

where superscript  $j$  has disappeared.

Moreover, taking into account the definition of the survival function, (2), it is known that

$d\phi_{ss}^a / dF_{ss} = \phi_{ss}^a \left\{ \sum_{i=1}^{a-1} p^i \right\}$ . Therefore, under these conditions the steady state solution satisfies

$$\sum_{a=1}^A pr^a \frac{dy_{ss}^a}{dF_{ss}} N_{ss}^a + \sum_{a=1}^A y_{ss}^a \frac{d\phi_{ss}^a}{dF_{ss}} N_{ss}^1 + \frac{dN_{ss}^1}{dF_{ss}} \sum_{a=1}^A y_{ss}^a \phi_{ss}^a = 0.$$



It is immediately apparent that the  $F_{ss}$  that solves this equation corresponds to  $F_{msy}$  because this expression is the first order condition of the maximisation problem

$$\begin{aligned} \max_{\{F_t, N_{t+2}^I\}_{t=0, \dots, \infty}} & \sum_{t=0}^{\infty} \left( \sum_{a=1}^A pr^a y_t^a \phi_t^a N_{t-(a-1)}^I \right), \\ \text{s.t.} & \begin{cases} N_{t+1}^I = \Psi(SSB_t), & \forall t, \\ SSB_t \geq B_{pa}, & \forall t. \end{cases} \end{aligned}$$

**Table S1.** Parameters of the age-structured model for Northern Hake.

Parameter	Age 0	Age 1	Age 2	Age 3	Age 4	Age 5	Age 6	Age 7	Age 8	Age 9	Age 10
Initial conditions											
N (thousands)	186 213	152 458	123 457	100 213	67 409	35 551	19 674	10 206	9 147	4 078	1 819
Population dynamics											
Natural mortality	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
Selectivity parameter	0.00	0.06	0.05	1.15	1.03	1.52	2.09	2.43	2.43	2.43	2.43
Weights at age (Kg)	0.06	0.13	0.22	0.34	0.60	0.98	1.44	1.83	2.68	2.68	2.68
Maturity	0.00	0.00	0.00	0.23	0.60	0.60	0.90	1.00	1.00	1.00	1.00
Initial population											
s.d.	0.200	0.200	0.166	0.086	0.061	0.063	0.076	0.084	0.084	0.084	0.084
Prices											
€/per kg	2.36	2.93	3.42	3.85	4.55	5.22	5.81	6.22	6.92	6.92	6.92

Source: Meeting on long-term management plans for northern hake (STECF/SGBRE-07-03 and SGBRE-07-05). Prices: Own calculations from 2007 daily sales for the Spanish "300 fleet".

**Table S2.** Parameters of the age-structured model for Northern Megrim.

Parameter	Age 1	Age 2	Age 3	Age 4	Age 5	Age 6	Age 7	Age 8	Age 9	Age 10
Initial conditions										
N (thousands)	279 630	225 980	182 965	176 583	192 375	54 215	15 537	8 254	3 312	9 480
Population dynamics										
Natural mortality	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
Selectivity parameter	0.03	0.15	0.42	0.82	1.21	1.54	1.04	0.98	0.67	0.67
Weights at age (Kg)	0.02	0.05	0.08	0.11	0.15	0.20	0.31	0.38	0.59	0.78
Maturity	0.04	0.21	0.60	0.90	0.98	1.00	1.00	1.00	1.00	1.00
Initial population										
s.d.	0.37	0.37	0.37	0.43	0.32	0.25	0.23	0.23	0.25	0.18

Source: Meeting on long-term management plans for northern hake (STECF/SGBRE-07-03 and SGBRE-07-05).

**Table S3.** Parameters of the age-structured model for *lophius budegassa*.

Parameter	Age 1	Age 2	Age 3	Age 4	Age 5	Age 6	Age 7	Age 8	Age 9	Age 10	Age 11	Age 12	Age 13
Initial conditions													
N (thousands)	14 329	19 309	9 983	13 735	5 431	3 090	2 275	1 015	1 159	997	676	406	475
Population dynamics													
Natural mortality	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
Selectivity parameter	0.01	0.05	0.24	0.58	0.64	1.13	1.03	1.15	1.05	0.93	0.97	1.22	1.22
Weights at age (Kg)	0.20	0.17	0.33	0.50	0.76	1.06	1.49	2.11	2.64	3.48	3.99	4.49	6.49
Maturity	0.03	0.07	0.12	0.21	0.34	0.50	0.66	0.79	0.88	1.00	1.00	1.00	1.00
Initial population													
s.d.	0.20	0.20	0.20	0.20	0.20	0.18	0.17	0.13	0.12	0.08	0.08	0.08	0.08

Source: Meeting on long-term management plans for northern hake (STECF/SGBRE-07-03 and SGBRE-07-05).

**Table S4.** Parameters of the age-structured model for *lophius piscatorius*.

Parameter	Age 1	Age 2	Age 3	Age 4	Age 5	Age 6	Age 7	Age 8	Age 9	Age 10	Age 11	Age 12	Age 13
Initial conditions													
N (thousands)	21 633	26 019	5 055	5 882	10 246	9 942	8 300	5 259	3 052	1 689	678	429	400
Population dynamics													
Natural mortality	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
Selectivity parameter	0.10	0.40	0.50	0.72	0.91	1.13	1.32	1.41	1.57	1.56	2.09	2.53	2.53
Weights at age (Kg)	0.25	0.34	0.50	0.81	1.30	1.95	2.91	3.88	4.89	5.96	6.94	8.75	12.58
Maturity	0.00	0.00	0.00	0.00	0.00	0.54	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Initial population													
s.d.	0.20	0.20	0.20	0.20	0.20	0.17	0.13	0.12	0.12	0.08	0.08	0.08	0.08

Source: Meeting on long-term management plans for northern hake (STECF/SGBRE-07-03 and SGBRE-07-05).