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by

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Optimal Taxation and Indeterminacy in the Uzawa-Lucas Model with Sector-Specific Externalities

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Abstract

In an extended Uzawa-Lucas model that includes labor-leisure decisions, sector-specific externalities in the production of goods generate a market failure relative to the socially optimal decisions. We show that, regardless of whether agents value pure or effective units of leisure, the first best solution can be attained either by using a time-varying subsidy to the human capital employed to produce goods or by combining consumption and labor income taxes with this type of subsidy. Moreover, when leisure is defined as raw time, we find that even when there is global determinacy, local indeterminacy may arise for several combinations of the parameters that are consistent with empirical evidence and previous literature. Importantly, under local indeterminacy the optimal policy does not ensure that identical economies will converge to the same per capita levels. Thus, not only the size and type of human capital externalities are important for optimal policy but also the indeterminacy aspects are relevant.

JEL classification: E62, H21, 041.

Keywords: Endogenous Growth; Externalities; Optimal Policy; Indeterminacy.

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1 Introduction

The contribution of endogenous growth models in explaining economic growth in the long-run has been studied in a large literature, including Lucas (1988), Rebelo (1991) and King and Rebelo (1990) and many others. The focus of the literature is trying to explain a number of observed growth patterns that the standard exogenous growth model fails to account for.[1] Since the pioneering work of Uzawa (1965), much of this literature suggests human capital as the engine of growth. Lucas (1988) extends Uzawa's model by considering that the average human capital produces a positive external effect on the final good producing sector. In the presence of this type of externality, if there is no government intervention the competitive equilibrium is sub-optimal. García-Castrillo and Sanso (2000), Gómez (2003), and Gorostiaga et al. (2013), for instance, design the optimal fiscal policies in the Uzawa-Lucas model when there is an externality *à la Lucas*.

Gómez (2008) considers, among other external effects, alternative human capital externalities in the final good producing sector that are associated with the average human capital employed in the production of goods: *sector-specific externalities*. He shows that when agents value both consumption and leisure, the competitive equilibrium is not optimal. In this paper, beyond this non-optimality result, we are interested in studying the properties of local and global stability of equilibria. Benhabib and Perli (1994), Chamley (1993), and others, find that indeterminacy of equilibria can arise in the Uzawa-Lucas model. Indeterminacy may explain differences in growth rates for similar economies (global indeterminacy), or differences in per capita income, consumption levels and time allocations between activities even when they grow at the same rate (local indeterminacy). The issues associated with indeterminacy are important and can be applied not only to the study of the dynamics of growth, but also to the impact of the optimal fiscal policy.

In this paper we study the Uzawa-Lucas model with sector-specific externalities in the production of goods when pure leisure and qualified leisure are considered. The specification of leisure in the utility function may drastically change the stability properties of the equilibrium. The dynamics of the equilibrium when no externalities are present has been studied in Ladrón-de-Guevara, Ortigueira and Santos (1997, 1999) and Ortigueira (2000). These authors characterize the set of necessary and sufficient

[1] In particular, these authors focus their analysis on the ability of the endogenous growth models to explain the observed heterogeneity in long-run growth rates across countries.

conditions for the existence of a unique BGP (global determinacy) with pure leisure and a logarithmic utility function. However, they do not analyze the stability properties of the equilibrium. When leisure is human capital-adjusted, Ortigueira (2000) shows that there is a unique globally stable BGP; that is, a unique equilibrium path converging to a unique BGP. We extend and complete their analysis by including sector-specific external effects and also by studing the local indeterminacy of the centralized equilibrium.

Using a numerical analysis, we find that when pure leisure is considered the centralized economy has an interior and a unique BGP for several combinations of parameters that are consistent with empirical evidence and with previous literature. Even though there is no global indeterminacy, we show that local indeterminacy may arise. In the presence of local indeterminacy two countries with the same initial conditions, technology and preferences that implement the same optimal fiscal policy may choose different allocations of consumption, labor, leisure and human capital accumulation. In the long-run they will grow at the same rate but they can display different per-capita levels and time allocations between activities. We find that when qualified leisure and constant returns to scale in the aggregate level are considered, the global and local determinacy result in Ortigueira (2000) still holds even when sector-specific externalities associated with the average human capital employed in the production of goods are included in the Uzawa-Lucas model.

Finally, we show that, regardless of whether leisure is specified as raw time or as quality time, the first best solution can be obtained either by using a time-varying subsidy to the human capital employed to produce goods or by combining consumption and labor income taxes with this type of subsidy. In both cases, we find that lump-sum taxes are required to balance the government budget. This result complements previous theoretical findings in Gómez (2008). In addition, the dynamics of the subsidy will depend on the specification of the leisure activity.

The rest of the paper is organized as follows. Section 2 describes the decentralized economy. The centralized economy is described in Section 3. Section 4 studies the stability properties of the equilibria. The optimal fiscal policy is analyzed in Section 5. Section 6 concludes.

2 The competitive equilibrium

We consider the standard Uzawa-Lucas model with two modifications: first, we assume that the average human capital employed in the production of goods has an external effect on the goods sector and, second, that agents derive utility not only from consumption but also from leisure. In particular, our framework considers two leisure specifications (pure leisure and qualified leisure) as in Gómez (2008) who also considers other type of externalities.

The economy consists of two sectors: the final good sector and the human capital accumulation sector. We assume that population remains constant and is normalized to one. The typical household enters period t endowed with a stock of physical capital, k, a stock of human capital, h, and one unit of time. A fraction of time is allocated to the production of the final good, n, a fraction to leisure, l, and the remaining to human capital accumulation, $1 - l - n$.

The final sector combines qualified labor and physical capital to produce a single and homogeneous good that can be allocated either to consumption or investment. The technology in this sector is described by the following Cobb-Douglas production function:

$$y = F[k, nh, \overline{nh}] = Ak^{\alpha}(nh)^{\nu}(\overline{nh})^{1-\alpha-\nu}, \text{ with } 0 < \alpha < 1, 0 < \nu < 1, \text{ and } \alpha + \nu < 1,$$

where nh represents the efficient labor units, and A measures the productivity of this sector. Following Ben-Gad (2003), Benhabib and Farmer (1996), Gómez (2008), Mino (2001), we also consider *sector-specific external effects* derived from the human capital employed in the final good production. This external effect is referred to as $\overline{nh}$ and measures the average human capital in the goods sector. In the specification we consider, the production of goods exhibits constant returns to scale at the aggregate level but decreasing returns to scale at the private level.

Let r and w denote the rental prices of physical capital and efficiency units of labor, respectively. The problem faced by a representative firm is to choose input demands $\{k, nh\}$ that maximize each period profits given the prices $\{r, w\}$:

$$\max_{k,nh}\{F[k, nh, \overline{nh}] - rk - wnh\}.$$

Firms treat $\overline{nh}$ as exogenously determined, since the positive effect that human capital employed in this sector has on productivity can only be observed at the aggregate level. The marginal product of each production factor is equalized to its price:

$F_1 = \alpha A k^{\alpha-1}(nh)^{\nu}(\overline{nh})^{1-\alpha-\nu} = r$, $F_2 = \nu A k^{\alpha}(nh)^{\nu-1}(\overline{nh})^{1-\alpha-\nu} = w$. Note that, in equilibrium, competitive firms earn positive profits: $\pi = (1-\alpha-\nu)y$. We assume that firms distribute these profits to households as dividends.

In the human capital accumulation sector new human capital is produced using time and human capital as inputs according to:

$$\dot{h} = \phi(1-l-n)h, \tag{1}$$

where ϕ measures the productivity of this sector.

Households derive their utility from consumption and from a measure of the efficiency units of leisure. Future utility is discounted at a rate β and preferences are described by: $U(c, lh^{\lambda})$ with $\lambda = \{0, 1\}$, where c is consumption. When $\lambda = 0$ agents derive utility from pure leisure, while when $\lambda = 1$ the level of human capital affects the productivity of leisure and enters into the utility function symmetrically with leisure.[2] The utility function $U(c, lh^{\lambda})$ is increasing in both arguments and strictly concave.

Let τ_w denote the tax rate on labor income, τ_r the tax rate on capital income, τ_c the tax rate on consumption (when $\tau_w, \tau_c, \tau_r < 0$ they can be interpreted as subsidies), s_n the subsidy per efficient labor units (that is, the government subsidizes the human capital employed in the production of goods, nh), and T lump-sum taxes (transfers when $T > 0$). Each household receives a wage income in exchange for labor, an interest income for physical capital and dividends. Taking these payments as given, they decide the fraction of time to be devoted to each activity. Households maximize the value of a discounted stream of utility and choose optimum decisions by taking into account the following budget constraint:

$$\dot{k} + (1+\tau_c)c \leq (1-\tau_r)rk + (1-\tau_w)wnh + s_n nh + \pi - T. \tag{2}$$

The problem faced by a representative household is to maximize the discounted stream of utility through her choice of paths $\{c, n, l, k, h\}_{t=0}^{\infty}$, the prices and dividends $\{r, w, \pi\}$, given the policy $\{\tau_w, \tau_c, \tau_r, s_n, T\}$ and initial holdings ($k(0)$ and $h(0)$):

$$Max_{c,n,l} \int_0^{\infty} e^{-\beta t} U(c, lh^{\lambda}),$$

subject to the restrictions (1) and (2). To solve the representative's household problem we define the current value Hamiltonian as:

[2] This type of preferences were first proposed by Becker (1965) and later have been used by Heckman (1976), Rebelo (1991), Ladrón-de-Guevara et al. (1997, 1999) and Gómez (2008), among others.

$$U(c, lh^{\lambda}) + \theta\left[(1-\tau_r)rk + (1-\tau_w)wnh + s_n nh + \pi - T - (1+\tau_c)c\right] + \eta\left[\phi(1-l-n)h\right],$$

where θ and η are the costate variables. The optimality conditions for an interior solution are:

$$\frac{U_1}{(1+\tau_c)} = \theta, \tag{3}$$

$$\theta\left[(1-\tau_w)wh + s_n h\right] = \eta\phi h, \tag{4}$$

$$U_2 h^{\lambda} = \eta\phi h, \tag{5}$$

$$\dot{\theta} = (\beta - r(1-\tau_r))\theta, \tag{6}$$

$$\dot{\eta} = [\beta - \phi(1-l-n)]\eta - \theta[(1-\tau_w)wn + s_n n] - \lambda U_2 h^{\lambda-1} l, \tag{7}$$

$$\lim_{t\to\infty} e^{-\beta t}\theta k = 0, \tag{8}$$

$$\lim_{t\to\infty} e^{-\beta t}\eta h = 0, \tag{9}$$

where subscripts denote the corresponding derivative.

At the margin, the two uses of goods (consumption and capital accumulation) must be equally valuable (equation (3)), non-leisure time must be equally valuable in its two income-directed activities (equation (4)), and the allocation of time between leisure and any of the two income-directed activities must be equally valuable (equations (4) and (5)).

The government collects revenues from consumption, labor income, capital income and lump-sum taxes, and it subsidizes effective working time maintaining its budget balanced every period: $\tau_c c + \tau_r rk + \tau_w wnh + T = s_n nh$.

A competitive equilibrium consists of paths for quantities $\{c, n, l, k, h\}_{t=0}^{\infty}$, and prices and dividends $\{r, w, \pi\}_{t=0}^{\infty}$ such that: *i*) the problem faced by a representative household given the policy variables and her initial holdings is solved; *ii*) the problem faced by a representative firm is solved; *iii*) the government fulfills its budget constraint; *iv*) all markets clear; *v*) the consistency condition $\overline{nh} = nh$ holds, for all t.

Combining equations (3), (4) and (5), in equilibrium the marginal rate of substitution between leisure and consumption must be equal to the return to working time:

$$\frac{[(1-\tau_w)F_2 + s_n]h}{(1+\tau_c)} = \frac{U_2 h^{\lambda}}{U_1}. \tag{10}$$

From equations (4), (5), (6), and (7), the rates of change of the shadow prices θ and η of the two kinds of capital are given by:

$$\dot{\theta} = (\beta - F_1(1-\tau_r))\theta, \text{ where } \theta = \frac{U_1}{(1+\tau_c)}, \tag{11}$$

$$\begin{aligned} \dot{\eta} &= [\beta - \phi(1-l-n)]\eta - \eta\phi n - \eta\phi\lambda l = \\ &= [\beta - \phi(1-l(1-\lambda)]\eta, \quad \text{where } \eta = \frac{U_2 h^{\lambda-1}}{\phi} = \frac{\theta\left[(1-\tau_w)F_2 + s_n\right]}{\phi}. \end{aligned} \tag{12}$$

In order to ensure the existence of a BGP for the decentralized economy, the following functional forms of the utility function are considered:[3]

$$U(c, lh^{\lambda}) = \begin{cases} \frac{[c^{\omega}(lh^{\lambda})^{1-\omega}]^{1-\sigma}}{1-\sigma}, \; 0 \le \omega \le 1 \;,\; \sigma > 0 \;,\; 0 \le \lambda \le 1 \;, \sigma \neq 1, \\ \omega \log c + (1-\omega)\log(lh^{\lambda}), \; 0 \le \omega \le 1 \;,\; 0 \le \lambda \le 1, \; \sigma = 1. \end{cases}$$

Setting $\tau_c = \tau_w = \tau_r = s_n = 0$ (fiscal policy will be used to attain the first best solution as shown below), the long-run values for the decentralized economy are determined from the following system of equations:

$$\frac{1-\omega}{\omega}\frac{(c/k)^*}{l^*} = \frac{\nu(y/k)^*}{n^*}, \tag{13}$$

$$\omega(1-\sigma)\gamma^* + [(1-\omega)(1-\sigma)-1]\lambda\gamma^* - (1-\lambda)\gamma^* = \beta - \phi(1-l^*(1-\lambda)), \tag{14}$$

$$[\omega(1-\sigma)-1]\gamma^* + (1-\omega)(1-\sigma)\lambda\gamma^* = \beta - \alpha(y/k)^*, \tag{15}$$

$$\gamma^* = \phi(1-l^*-n^*). \tag{16}$$

An *interior* BGP is characterized by a constant positive growth rate of per-capita variables (γ), while hours worked and leisure remain constant. The constant rate of growth is determined by the rate of accumulation of human capital. We assume that $\beta - (1-\sigma)[\omega + \lambda(1-\omega)]\gamma > 0$ which ensures that the attainable utility is bounded and that the transversality conditions hold.

When Beckerian-type preferences are considered ($\lambda = 1$), there is a unique BGP. Note that when $\lambda = 1$ from equation (14) $\gamma^* = \frac{\phi-\beta}{\sigma}$ and hence the time devoted to accumulate human capital can be unambiguously determined from equation (16): $(1-l^*-n^*) = \frac{\phi-\beta}{\phi\sigma}$. Using equations (14) and (15), we have $F_1^* = \alpha(y/k)^* = \phi$, and from the resource constraint $(c/k)^* = (y/k)^* - \gamma^* = \frac{\phi}{\alpha} - \frac{\phi-\beta}{\sigma}$ is obtained. The allocation of time to leisure and to the production of the final good can be derived from equation

[3] See King, Plosser and Rebelo (1988), Rebelo (1991) and Ladrón-de-Guevara, Ortigueira and Santos (1997, 1999) for a more detailed discussion on this issue.

(13) joint with $(1-l^*-n^*)=\frac{\phi-\beta}{\phi\sigma}$. Note that from equations (14) and (16) the long-run productivity of human capital is unambiguously determined by the time devoted to accumulate human capital. In order to ensure an interior BGP with a positive constant growth rate, we must have $\phi-\beta>0$.

However, when pure leisure is considered ($\lambda=0$), these equations give rise to multiple equilibria for some parameter values. In this case, since the stock of human capital affects asymmetrically the time spent on the various activities, the long-run productivity of human capital is not unambiguously determined by the time devoted to accumulate human capital. It also depends on the time devoted to leisure and work. Such long-run productivity would be highest if time is devoted to those activities where human capital is most profitable. From equations (14) and (15) we obtain that $F_1^*=\alpha(y/k)^*=\phi(1-l^*)$ and $[\omega(1-\sigma)-1]\gamma^*=\beta-\phi(1-l^*)$. Taking into account these expressions, since $(c/k)^*=(y/k)^*-\gamma^*$, by combining equations $(13),(14)$ and (16) we obtain the following quadratic equation for $l^*:$

$$a{l^*}^2-bl^*+d=0,$$

where $a=\frac{\nu}{\alpha}\phi^2[\omega(1-\sigma)-1]-\frac{1-\omega}{\omega}\phi^2\omega(1-\sigma)e$, $\quad b=\frac{\nu}{\alpha}\phi^2[\omega(1-\sigma)-1]+(2\phi\omega(1-\sigma)-\beta)\phi\frac{1-\omega}{\omega}e-\frac{\beta\phi\omega(1-\sigma)}{\omega(1-\sigma)-1}\frac{1-\omega}{\omega}$, $d=\frac{1-\omega}{\omega}(\beta-\omega(1-\sigma)\phi)\left[\frac{\beta}{\omega(1-\sigma)-1}-\phi e\right]$ and $e=\left[\frac{1-\alpha}{\alpha}+\frac{\omega(1-\sigma)}{\omega(1-\sigma)-1}\right]$. This quadratic equation may contain up to two positive roots with $0<l_1^*<l_2^*<1$. In order to ensure an interior BGP with a positive constant growth rate, we must have $\frac{\phi(1-l^*)-\beta}{1-\omega(1-\sigma)}>0$. Note that for some parameter values this equation can give rise to multiple interior equilibria. Below, we use a numerical analysis and find that this economy has an interior unique BGP for several combinations of a wide range of parameters that are consistent with empirical evidence and with previous literature.

3 The Centralized Economy

A social planner maximizes the discounted stream of utility subject to the resource constraint of the economy, the equation that describes the evolution of human capital, and the constraint that $\overline{nh}=nh$ for all t. That is, a central planner internalizes the external effect by solving the following problem:

$$Max_{c,n,l}\int_0^\infty e^{-\beta t}U(c,lh^\lambda),$$

subject to (1), $\overline{nh}=nh$ and $\dot{k}+c\leq F[k,nh,\overline{nh}]=Ak^\alpha(nh)^\nu(\overline{nh})^{1-\alpha-\nu}$. The first order conditions for the first best allocation are:

$$U_1 = \widetilde{\theta}, \tag{17}$$

$$\widetilde{\theta}(F_2 + F_3)h = \widetilde{\eta}\phi h, \tag{18}$$

$$U_2 h^{\lambda} = \widetilde{\eta}\phi h, \tag{19}$$

$$\dot{\widetilde{\theta}} = (\beta - F_1)\widetilde{\theta}, \tag{20}$$

$$\dot{\widetilde{\eta}} = [\beta - \phi(1 - l - n)]\widetilde{\eta} - \widetilde{\theta}(F_2 + F_3)n - \lambda U_2 h^{\lambda-1} l. \tag{21}$$

$$\lim_{t\to\infty} e^{-\beta t}\widetilde{\theta}k = 0, \tag{22}$$

$$\lim_{t\to\infty} e^{-\beta t}\widetilde{\eta}h = 0, \tag{23}$$

In equilibrium the marginal rate of substitution between leisure and consumption must be equal to the return to work, and the rates of change of $\widetilde{\theta}$ and $\widetilde{\eta}$ are given by:

$$(F_2 + F_3)h = \frac{U_2}{U_1}h^{\lambda}, \tag{24}$$

$$\dot{\widetilde{\theta}} = (\beta - F_1)\widetilde{\theta}, \quad \text{with } \; \widetilde{\theta} = U_1, \tag{20}$$

$$\begin{aligned} \dot{\widetilde{\eta}} &= [\beta - \phi(1 - l - n)]\widetilde{\eta} - \widetilde{\eta}\phi n - \widetilde{\eta}\phi\lambda l \\ &= [\beta - \phi(1 - l(1 - \lambda)]\widetilde{\eta}, \quad \text{with } \widetilde{\eta} = \frac{U_2 h^{\lambda-1}}{\phi} = \frac{U_1(F_2 + F_3)}{\phi}. \end{aligned} \tag{25}$$

Since the production of goods exhibits constant returns to scale at the aggregate level, it is easy to verify that this system of equations is exactly the same as that in Ladrón-de-Guevara, Ortigueira and Santos (1997).[4] As Ortigueira (2000) argues, when qualified leisure is considered ($\lambda = 1$), there is a unique BGP which is globally stable. However, when pure leisure is considered ($\lambda = 0$), there are multiple equilibria for some parameter values. When $\lambda = 0$ a quadratic equation for l^* is obtained: $a'l^{*2} - b'l^* + d = 0$ with $a' = \frac{1-\alpha}{\alpha}\phi^2[\omega(1-\sigma) - 1] - \frac{1-\omega}{\omega}\phi^2\omega(1-\sigma)e > a$, and $b' = \frac{1-\alpha}{\alpha}\phi^2[\omega(1-\sigma) - 1] + (2\phi\omega(1-\sigma) - \beta)\phi\frac{1-\omega}{\omega}e - \frac{\beta\phi\omega(1-\sigma)}{\omega(1-\sigma)-1}\frac{1-\omega}{\omega} > b$. As shown by Ladrón-de-Guevara, Ortigueira and Santos (1999), necessary and sufficient conditions

[4] These authors consider the Uzawa-Lucas model with pure and qualified leisure but without any type of externality. As their production function exhibits constant returns to scale at the aggregate level, the system of equations in their framework is the same as the one obtained for the centralized economy here.

for the existence of a unique interior BGP can be obtained when a logarithmic utility function is considered ($\sigma = 1$): the parameter condition $\frac{1-\alpha}{\alpha} > \frac{(1-\omega)\beta}{\omega\phi-\beta}$ ensures the uniqueness of the BGP and $\phi(1-l^*) > \beta$ is required to ensure that the BGP is interior. Following the same procedure we find that the parameter condition that guarantees the uniqueness of the BGP for the decentralized economy is $\nu > \frac{\beta}{\phi-\beta}\frac{1-\omega}{\omega}$.

When $\lambda = 1$ in the long-run, the time devoted to accumulate human capital, the growth rate, the consumption/physical-capital ratio and the output/physical-capital ratio in the centralized economy are the same as those corresponding to the decentralized economy. The reason is that the long-run productivity of human capital will be the same in both economies. This means that the sum of the time devoted to leisure and to the production of goods ($l + n$) is the same in both economies. As the only difference between the centrally-planned and the decentrally-planned economies is that the private return to working time is lower than its social return, the time devoted to the production of the final good in the former will be higher than that in the latter. Hence, the time devoted to leisure in the centrally planned economy will be lower than that in the decentralized economy.

However, when $\lambda = 0$ human capital is not equally productive in every activity. In this case, the long-run growth rate is positively related to the time allocated to non-leisure activities. In the long-term the allocation of time devoted to leisure in the centrally-planned economy will be lower than that in the decentralized economy as the central planner will internalize the external effect associated with non-leisure activities. Thus the long-run growth rate in the centrally-planned economy will be higher than that in the decentralized economy, with a higher allocation of time devoted to accumulate human capital in the former than in the latter.[5]

Even when the parameter conditions that ensure the uniqueness of the equilibria hold, there might exist multiple transition paths all of which converge to the same BGP. In the next section we study the global and local indeterminacy of equilibria.

[5] The time devoted to producing goods in both centrally- and decentrally-planned economies will be the same when a logarithmic utility function is considered ($\sigma = 1$). In this particular case, $n^* = \frac{\beta}{\phi}$. Otherwise, for higher (lower) values of σ, the time devoted to the production of goods in the centrally-planned economy will be higher (lower) than that in the decentrally-planned economy. Regardless of σ, the sum of the time allocated to both non-leisure activities in the former will be higher than that in the latter.

4 Indeterminacy of Equilibria

As mentioned before, when Beckerian-type preferences are considered ($\lambda = 1$), there is a unique BGP which is globally stable; that is, there is a unique transition path converging towards the long-run equilibrium. However, when pure leisure is considered ($\lambda = 0$) global stability is lost. In order to study the stability properties of equilibria we calibrate a benchmark economy. Looking at the market sector, the parameter α is set to 0.36 which is the capital's average share of per capita GNP. Labor's average share is set at 0.6 ($\nu = 0.6$). The scale parameter A is normalized to unity. To guarantee that the steady state equilibria is interior and unique we consider $\sigma = 1$. An annual rate of time preference of 5% is standard in the RBC literature ($\beta = 0.05$). Per capita growth rate of output is set at $\gamma^* = 0.02$ which is consistent with the observed average annual growth rate of per capita output in the US for the period 1960-2014.[6] Parameters ϕ and ω are chosen so that the growth rate of output in the decentralized economy matches the observed 2 percent annual growth rate. Consumption's share in the utility function is set at $\omega = 0.45$. When $\lambda = 0$ we set $\phi = 0.162$, while for $\lambda = 1$, ϕ is set at 0.07. Higher values for parameters ϕ and ω would increase the long-term growth rate. The numerical values considered for the benchmark economy satisfy the parameter conditions that ensure the existence of a unique interior BGP for both the centralized and the decentralized economies.

Taking into account that $\sigma = 1$ and $\frac{\dot{h}}{h} = \phi(1-l-n)$, the dynamics of the system for the centrally-planned economy can be described by the following equations:

$$\beta - \phi[1 - l(1-\lambda)] + (1-\lambda)\phi(1-l-n) = -[\frac{\dot{l}}{l} + \lambda\phi(1-l-n)], \tag{26}$$

$$\alpha A\widehat{k}^{\alpha-1}n^{1-\alpha} - \beta = \frac{\dot{\widehat{c}}}{\widehat{c}} + \phi(1-l-n), \tag{27}$$

$$\frac{1-\omega}{\omega}\frac{\widehat{c}}{l} = (1-\alpha)A\widehat{k}^{\alpha}n^{-\alpha}, \tag{28}$$

$$\frac{\dot{\widehat{k}}}{\widehat{k}} + \phi(1-l-n) = A\widehat{k}^{\alpha-1}n^{1-\alpha} - \frac{\widehat{c}}{\widehat{k}}, \tag{29}$$

where stationary time series are obtained by expressing growing variables in relation to the stock of human capital: $\widehat{k} = \frac{k}{h}$ and $\widehat{c} = \frac{c}{h}$. By taking logs in equation (28) and differentiating with respect to time, we obtain $\frac{\dot{\widehat{c}}}{\widehat{c}} = \alpha\frac{\dot{\widehat{k}}}{\widehat{k}} + \frac{\dot{l}}{l} - \alpha\frac{\dot{n}}{n}$, which combined with

[6] Source: World Development Indicators in World Data Bank.

equations $(26), (27)$ and (29), results in an expression for the dynamics of n. Thus, a three-dimensional system in $\widehat{k}, n$, and l is obtained:

$$\frac{\dot{\widehat{k}}}{\widehat{k}} = A\widehat{k}^{\alpha-1}n^{1-\alpha} - \frac{\widehat{c}}{\widehat{k}} - \phi(1-l-n),$$

$$\alpha\frac{\dot{n}}{n} = \phi[1-l(1-\lambda)] - \alpha\frac{\widehat{c}}{\widehat{k}} - \alpha\phi(1-l-n),$$

$$\frac{\dot{l}}{l} = \phi[1-l(1-\lambda)] - \beta - \phi(1-l-n),$$

with $\widehat{c} = \frac{\omega}{1-\omega}(1-\alpha)A\widehat{k}^{\alpha}n^{-\alpha}l$.

By linearizing these equations in the neighborhood of the steady state, the equilibrium solution is locally unique if the Jacobian of the system has one negative eigenvalue. If the number of negative eigenvalues is greater than one, then there are multiple equilibrium paths converging to the same BGP.[7]

When qualified leisure is considered ($\lambda = 1$), the number of negative eigenvalues is equal to the number of pre-determined state variables, and this entails that there is a unique equilibrium path converging to the unique BGP (saddle-path stability). However, when pure leisure is included in the utility function ($\lambda = 0$), the number of negative eigenvalues is higher than the number of pre-determined state variables and, hence, there are multiple optimal trajectories converging to the same BGP (local indeterminacy). As shown in Table 1 below this result is robust to changes in ω and ϕ.

The policy implication behind local indeterminacy of the centralized equilibrium is the following: Even though two economies with the same preferences, technology and initial conditions that implement the optimal taxation may converge to the same grow rate, they could readily have different allocations of time between leisure, working and education as well as different per capita levels of output, consumption, physical and human capital in the long-run.

[7] Details on the Jacobian matrix are shown in the appendix.

Table 1. Eigenvalues for the benchmark case

$\lambda = 1$				$\lambda = 0$			
ω	eigenvalue 1	eigenvalue 2	eigenvalue 3	ω	eigenvalue 1	eigenvalue 2	eigenvalue 3
0.45	0.176783	-0.124444	0.0292337	0.45	0.18765	-0.0988234	-0.0388262
0.6	0.178765	-0.124444	0.0198332	0.6	0.26813	-0.205034	-0.0130967
0.7	0.180478	-0.124444	0.0127318	0.7	0.296909	-0.236662	-0.0102466
ϕ	eigenvalue 1	eigenvalue 2	eigenvalue 3	ϕ	eigenvalue 1	eigenvalue 2	eigenvalue 3
0.08	0.1944	-0.142222	0.0289682	0.18	0.229131	-0.156874	-0.0222564
0.09	0.212034	-0.16	0.0287659	0.2	0.270737	-0.204697	-0.0160394
0.1	0.229684	-0.177778	0.0286078	0.25	0.367651	-0.307833	-0.00981761

5 The Optimal Fiscal Policy

The market failure caused by any type of externalities generates the opportunity for fiscal policy to improve efficiency. Under an optimal fiscal policy the decentralized economy must replicate the equilibrium time path of the centrally-planned economy. Sector-specific externalities in the production of goods decrease the private return to working time relative to the social return. Thus we characterize a set of tax structures that close the wedge between the private and social returns to working time. To this end we consider consumption taxes, labor and capital income taxes, lump-sum taxes as well as a subsidy per unit of effective leisure, and we allow them all to be time-varying.

We first note that the model in Gómez (2008) collapses to our framework when utility externalities, sector-specific externalities in the educational sector, and sector-specific externalities associated with physical capital employed by the sector producing goods are absent from the analysis. He finds that in this setting the first best equilibrium can be attained if labor income and consumption are taxed at a *constant* rate satisfying $(1 - \alpha)(1 + \tau_c) = \nu(1 - \tau_w)$. Here, we show that the first best can also be obtained by combining these taxes with a *time-varying* subsidy to the human capital employed to produce goods and also by using this type of subsidy alone. The dynamics of the subsidy will depend on the specification of the leisure activity. In both cases, lump-sum taxes are required to balance the government budget at least in the transition phase. The following proposition summarizes this result.

Proposition. *When consumption taxes are constant, regardless of whether agents value pure leisure or effective units of leisure, the optimal fiscal policy has the following characteristics: (a) capital income taxes must be equal to zero, (b) labor income taxes must be constant and can be combined with consumption taxes and a direct time-varying subsidy to the human capital employed to produce goods:*

$$s_n = \frac{y}{nh}[(1-\alpha)(1+\tau_c) - \nu(1-\tau_w)], \quad \textit{with} \;\; 0 \leq s_n < 1,$$

(c) the dynamics of the subsidy must be:

$$\frac{\dot{s_n}}{s_n} = \alpha\frac{y}{k} - \phi(1 - l(1-\lambda)),$$

and (d) lump-sum taxes are needed to balance the government budget at least in the transition path.

Proof. First, by comparing equations (10) and (24) we see that for the decentralized dynamic equilibrium time path to replicate the centralized one, the condition $\frac{(1-\tau_w)F_2+s_n}{(1+\tau_c)} = F_2 + F_3$ is required. By rewriting this expression we obtain $s_n = \frac{y}{nh}[(1-\alpha)(1+\tau_c) - \nu(1-\tau_w)]$. Second, as equations (12) and (25) coincide when $\frac{(1-\tau_w)F_2+s_n}{(1+\tau_c)} = F_2 + F_3$, the decentralized economy can replicate the first best equilibrium dynamics if the capital income tax rate is set to zero and consumption taxes are constant (see equations (11) and (20)). By doing so, the valuation of physical and human capital for the market and optimal equilibrium allocations will coincide. Third, the dynamics of the subsidy rate s_n are obtained by the following procedure. Log-differentiating equation (18) and using equations (20) and (25) for the centrally-planned economy we obtain $\alpha\left[\frac{c}{k} + \frac{\dot{n}}{n} + \frac{\dot{h}}{h}\right] = \phi(1 - l(1-\lambda))$. Proceeding in the same manner with equations (4), (11) and (12) for the market economy we obtain $\alpha\left[\frac{c}{k} + \frac{\dot{n}}{n} + \frac{\dot{h}}{h}\right] = \frac{1}{(1-\tau_w)F_2}\left\{-F_1 s_n - \dot{\tau}_w F_2 + \dot{s_n} + \phi(1-l(1-\lambda))\left[(1-\tau_w)F_2 + s_n\right]\right\}$. These two expressions are equivalent when labor income taxes are constant and $\frac{\dot{s_n}}{s_n} = \alpha\frac{y}{k} - \phi(1-l(1-\lambda))$. Finally, a balanced government budget requires that $\tau_c c + \tau_w wnh + T = s_n nh$. Substituting the optimal tax rate s_n and the equilibrium wage $w = F_2$, and dividing both sides of the equation by the final output we have $\tau_c\frac{c}{y} + \tau_w\nu + \frac{T}{y} = [(1-\alpha)(1+\tau_c) - \nu(1-\tau_w)]$. The optimal share of public expenditure or taxes in output are constant and less than one at any time. In the transition phase to the long-run equilibrium lump-sum taxes are needed to balance the government budget.

If the government uses a single policy instrument, the amount that has to be collected depends on the size of the externality and can easily be obtained from the government budget: when effective working time or labor income are subsidized alone by using $\tau_w = \frac{\nu+\alpha-1}{\nu}$ or $s_n = \frac{y}{nh}(1-\nu-\alpha)$, this amounts to $T = (1-\nu-\alpha)y$, and when a single consumption subsidy $\tau_c = \frac{\nu+\alpha-1}{1-\alpha}$ is used, then $T = \frac{(1-\nu-\alpha)}{(1-\alpha)}c$. Note that a tax on profits would act as a lump-sum tax.■

6 Conclusions

Externalities are known to cause market failure that may justify government intervention on efficiency grounds. *Sector-specific* externalities cause a market failure relative to the socially optimal consumption-leisure decisions since the private return to effective working time is lower than its social return. In this paper we consider an extended Uzawa-Lucas model with labor-leisure decisions in the presence of sector-specific externalities derived from the human capital employed in the final good production. We find that regardless of whether agents value pure leisure or effective units of leisure, the first best solution can be obtained either by combining consumption and labor income taxes with a *time-varying* subsidy to the human capital employed to produce goods or simply by using this type of subsidy alone. In both cases, we find that lump-sum taxes are required to balance the government budget at least in the transition phase. Moreover, the dynamics of the subsidy will depend on the specification of leisure activity. Thus, these results complement previous theoretical findings in Gómez (2008).

The existence of market imperfections such that productive externalities as well as the presence of leisure can generate multiple equilibrium paths on two-sector models with endogenous growth. The issues associated with indeterminacy can be applied not only to the study of the volatility of the growth rates over time and their dispersion across countries both in the short- and long-run, but also to the impact of the optimal fiscal policy. Indeterminacy can explain why countries with the same initial conditions differ so much after some years not only in levels, but also in the rate of growth.

We have also shown that when qualified leisure and constant returns to scale in the aggregate level are considered, the global and local determinacy result in Ortigueira (2000) still holds even when sector-specific externalities associated with the average human capital employed in the production of goods are included in the Uzawa-Lucas model. However, when leisure is defined as raw time, global stability is lost. Following Ladrón-de-Guevara et al. (1997) we find the parameter conditions that ensure the uniqueness of the equilibria when a logarithmic utility function is considered. Our analysis shows that the centralized economy has an interior and unique BGP for several combinations of the parameters that are consistent with empirical evidence and previous literature. Even when there is global determinacy we show that local indeterminacy may arise. That is, two economies with the same preferences, technology and initial conditions that implement the optimal taxation may converge to the same growth rate, but readily have different allocations of time between leisure, working and education as well as per capita levels of output, consumption, physical and human capital in the long-run. It should be possible to rank the equilibrium paths in a neighborhood of the BGP using a selection device based on cultural, social or historical considerations. A social planner should target the equilibrium ranked first.

Our results strongly suggest that not only the size and the type of human capital externalities but also the indeterminacy aspects are important in the debate of fiscal policy.

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Appendix: Stability Properties of the Equilibrium

We can write the Jacobian matrix of the reduced system evaluated at the steady state as:

$$J^* = \begin{pmatrix} J_{11}^* & J_{12}^* & J_{13}^* \\ J_{21}^* & J_{22}^* & J_{23}^* \\ J_{31}^* & J_{32}^* & J_{33}^* \end{pmatrix},$$

where,

$$\begin{aligned}
J_{11}^* &= \frac{\partial \dot{\widehat{k}}}{\partial \widehat{k}}|_{ss} = A\alpha \left(\widehat{k}^*\right)^{\alpha-1} (n^*)^{1-\alpha} - \phi(1-l^*-n^*) - \frac{\omega}{1-\omega}(1-\alpha)\alpha A \left(\widehat{k}^*\right)^{\alpha-1} (n^*)^{-\alpha} l^*, \\
J_{12}^* &= \frac{\partial \dot{\widehat{k}}}{\partial n}|_{ss} = A(1-\alpha) \left(\widehat{k}^*\right)^{\alpha} (n^*)^{-\alpha} + \phi\widehat{k}^* + \frac{\omega}{1-\omega}(1-\alpha)\alpha A \left(\widehat{k}^*\right)^{\alpha} (n^*)^{-\alpha-1} l^*, \\
J_{13}^* &= \frac{\partial \dot{\widehat{k}}}{\partial l}|_{ss} = -\frac{\omega}{1-\omega}(1-\alpha) A \left(\widehat{k}^*\right)^{\alpha} (n^*)^{-\alpha} + \phi\widehat{k}^*, \\
J_{21}^* &= \frac{\partial \dot{n}}{\partial \widehat{k}}|_{ss} = -\frac{\omega}{1-\omega}(1-\alpha)(\alpha-1) A \left(\widehat{k}^*\right)^{\alpha-2} (n^*)^{1-\alpha} l^*, \\
J_{22}^* &= \frac{\partial \dot{n}}{\partial n}|_{ss} = \frac{\phi}{\alpha}[1-l(1-\lambda)] - \frac{\omega}{1-\omega}(1-\alpha)^2 A \left(\widehat{k}^*\right)^{\alpha-1} (n^*)^{-\alpha} l^* + 2\phi n^* - \phi(1-l^*), \\
J_{23}^* &= \frac{\partial \dot{n}}{\partial l}|_{ss} = -\frac{\omega}{1-\omega}(1-\alpha) A \left(\widehat{k}^*\right)^{\alpha-1} (n^*)^{1-\alpha} + \phi n^* - \frac{\phi}{\alpha}(1-\lambda) n^*, \\
J_{31}^* &= \frac{\partial \dot{l}}{\partial \widehat{k}}|_{ss} = 0, \\
J_{32}^* &= \frac{\partial \dot{l}}{\partial n}|_{ss} = \phi l^*, \\
J_{33}^* &= \frac{\partial \dot{l}}{\partial l}|_{ss} = 2\phi l^* \lambda - \beta.
\end{aligned}$$