DOCUMENTOS DE TRABAJO

BILTOKI

D.T. 2007.05

Consumption-Leisure Trade-offs and Persistency in Business Cycles.

Ilaski Barañano y Paz Moral

eman ta zabal zazu

Universidad del País Vasco
Euskal Herriko Unibertsitatea

Facultad de Ciencias Económicas.
Avda. Lehendakari Aguirre, 83
48015 BILBAO.

Documento de Trabajo BILTOKI DT2007.05
Editado por el Departamento de Economía Aplicada III (Econometría y Estadística) de la Universidad del País Vasco.

Depósito Legal No.: BI-3813-07
ISSN: 1134-8984
URN: RePEc:ehu:biltok:200705

Consumption-Leisure Trade-offs and Persistency in Business Cycles

Ilaski Barañano
Dpt. Fundamentos Análisis Económico I
UPV/EHU

M. Paz Moral*
Dpt. Economía Aplicada III
UPV/EHU

Abstract

This paper studies whether nonseparabilities between consumption and leisure may help to explain the observed persistence in GNP growth. We consider an extended version of Lucas' (1988) human capital investment model that includes labor adjustment costs and compare its performance under different utility specifications with different degrees of complementarity and substitutability between consumption and leisure. We find that when consumption and leisure are complements the model succeeds in matching not only the autocorrelation of output growth but also the important trend-reverting component found in US data. These results hold even if low adjustment costs of labor are considered. Hence, we conclude that an arguably simple margin not studied previously can provide useful insights into observed business cycle patterns.

JEL classification: E32; O41; C52.
Keywords: Real Business Cycle Models; Endogenous Growth; Propagation Mechanism; Persistence.

*We thank Ignacio Palacios-Huerta for valuable comments and suggestions. Financial support from Ministerio de Educación y Ciencia (SEJ2005-08269) and the Department of Education of the Basque Government (IT-334-07) is gratefully acknowledged.

Correspondence: Ilaski Barañano, Dept. Fundamentos del Análisis Económico I, UPV-EHU, Lehendakari Agirre Etorbidea 83, 48015 Bilbao, Spain. Email: ilaski.baranano@ehu.es.

1 Introduction

Even though preference specifications that are nonseparable in consumption and leisure have already been formally considered in fiscal policy studies (McGrattan, 1994; Finn, 1998), international RBC models (Baxter, 1995), the asset pricing literature (Mankiw, 1985; Eichenbaum, 1988), life-cycle models (Low, 2005) and monetary policy studies (Matheny, 1998), among other areas, most RBC models typically consider constant relative risk aversion preferences with leisure and consumption entering the utility function separably. Additively separable utility functions are used for the sake of analytic simplicity. As is already known, if utility is separable in consumption and labor, a logarithmic utility consumption is needed if a balanced growth path (BGP hereafter) is to exist, implying a relative risk aversion (measured as the inverse of the intertemporal elasticity of substitution in consumption) equal to one.[1] As a result, most RBC models assume an additively separable utility function in which case the RRA must equal one.

As is also known, in static models labor supply depends on the degree of complementarity or substitutability between consumption and leisure. An increase in consumption shifts downward the short run labor supply curve when the marginal disutility of labor is a decreasing function of consumption (i.e., when consumption and leisure are substitutes). If consumption and leisure are complements the opposite effect occurs, and when consumption and leisure enter the utility function separably, no such labor supply effect arises. As noted by de Hek (1998, p. 255): "In a dynamic context of economic growth this acquires great significance as the nature of consumption leisure trade-off determines the intertemporal accumulation paths for the economy."

The empirical literature documents two stylized facts about U.S. output dynamics: first, GNP growth is positively autocorrelated over short horizons and has a weak and possibly insignificant negative autocorrelation over longer horizons; second, GNP appears to have an important trend-reverting component that has a hump-shaped moving average representation. Several modelling strategies have

[1]In the long run the effect of productivity growth on real wages and consumption must generate offsetting income and substitution effects to ensure the absence of a trend growth in per capita labor supply. King et al. (1988) showed that for this to be achieved, when additively separable utility functions are considered they must be logarithmic in consumption implying a relative risk aversion (RRA hereafter) parameter equal to one, and for non-additively separable utility functions the RRA parameter can be greater or lower than one.

been adopted in order to explain these patterns: capital and labor adjustment costs (Cogley and Nason, 1995), variable factor utilization rates (Burnside and Eichenbaum, 1996), the combination of habit formation in leisure and increasing returns to scale (Wen, 1998), external increasing returns and indeterminacy (Benhabib and Wen, 2004; Schmitt-Grohe, 2000), externalities and multiple equilibria (Perli, 1998), multisector models (Benhabib et al., 2006), 'skilled' and 'unskilled labor' with low elasticity of substitution in the production of human capital (Perli and Sakellaris, 1998) and job search (Andolfatto, 1996), among others.

All these papers can be broadly classified into three groups. Some papers rely on strong increasing returns. However, as Schmitt-Grohe (2000) finds these returns do not seem empirically plausible and as Wen (1998) shows they may even generate an upward sloping aggregate labor demand. Other papers such as Andolfatto (1996), Burnside and Eichenbaum (1996) and Cogley and Nason (1995) rely on 'rigidities' in the labor market. In Andolfatto (1996)) these frictions arise endogenously, whereas in the other two they are an assumption. Among these papers only the job search approach by Andolfatto (1996) succeeds in matching both stylized facts. Finally, other papers such as Benhabib et al. (2006) and Jones et al. (2005) emphasize the role played by intratemporal substitution across sectors, but they both fail to generate the autocorrelation found in the data.

Combinations of these three strategies have also been studied by Perli (1998) and Perli and Sakellaris (1998), among others. Perli (1998) considers a home sector coupled with increasing returns and Perli and Sakellaris (1998) rely on the low elasticity of substitution in the human capital sector relative to the physical sector which generates an adjustment cost effect. They both obtain autocorrelation properties similar to those observed in the data. Our paper pursues this line of research, but we consider an arguably simpler strategy: we study the role that intratemporal nonseparabilities between consumption and leisure may have in explaining the above mentioned patterns.

In particular, we consider an extended version of Lucas' (1988) human capital investment model that includes 'effective' labor adjustment costs.[2] We then gen-

[2] In our model we impose labor adjustment costs following Cogley and Nason (1995) in order to achieve an improvement in the autocorrelation of GNP growth. As shown by Perli and Sakellaris (1998) this improvement could also be obtained endogenously without imposing labor market frictions by considering that labor is an aggregation of 'skilled' and 'unskilled labor' and that these two types of labor are substitutable in different degrees in each sector.

eralize the utility function by including leisure. Further, we next study different utility specifications which satisfy the conditions needed to allow the existence of a BGP following King et al. (1988), as is common in the RBC literature. In particular, we compare a log-separable specification of the utility function with a multiplicatively separable one in characterizing persistence in output growth, and assess the sensitivity of the results of the model to the degree of complementarity or substitutability between consumption and leisure. When the multiplicatively separable specification is considered, the RRA parameter determines the degree of complementarity or substitutability between consumption and leisure. Hence, standard constant RRA specifications allow us to study the role that the degree of substitutability or complementarity may play by varying a single parameter. Interestingly, we find that when consumption and leisure are complements the model is able to reproduce the persistence of output growth found in the data, even with low adjustment costs of labor.

The rest of the paper is as follows: We briefly describe the endogenous growth model considered in Section 2. Section 3 analyzes the persistence in output growth, and Section 4 concludes.

2 The Model

We consider a stochastic discrete time version of Lucas' (1988) model in the absence of externalities with two modifications. First, we allow agents to derive utility not only from consumption but also from leisure.[3] Second, as suggested by Shapiro (1986) and Cogley and Nason (1995), labor adjustment costs are included. Here we study whether persistence results depend on the nature of how consumption and leisure enter the utility function. In particular, we consider two utility function specifications: a log-separable specification and a multiplicatively separable specification.

The economy consists of a large number of productive families which own both the production factors and the technology used in two productive activities: the production of the final good (market sector) and the production of new human

[3] As shown by Ladrón de Guevara et al. (1999), the concavity of the representative agent's problem is not guaranteed. In the simulations reported below, existence and uniqueness of the BGP is verified for each calibration. For all models and parameter choices, the equilibrium was found to be saddlepath stable.

capital (human capital sector).[4] Population size is assumed to be constant. At any point in time, individuals must decide what fraction of their time they devote to each of these activities, and how much time they set aside for leisure. The time endowment is normalized to one, so that l_t denotes the fraction of time given over to leisure and n_t the fraction of time devoted to the production of the consumption good.

The technology of the consumption good is described by a production function with constant returns to scale with respect to physical capital and efficient labor. As already mentioned, we also consider labor adjustment costs. In particular, the production function includes quadratic adjustment costs in labor input measured in efficiency units. Formally, the production technology is made (log) linear in the cost of adjustment,

$$\begin{aligned} \ln\ y_t &= \ln[F^m(k_t, Z_t, n_t h_t)] - \frac{\eta}{2}\left[\frac{\Delta(n_t h_t)}{n_{t-1}h_{t-1}}\right]^2 \\ &= \ln[A_m Z_t k_t^{\alpha}(n_t h_t)^{1-\alpha}] - \frac{\eta}{2}\left[\frac{\Delta(n_t h_t)}{n_{t-1}h_{t-1}}\right]^2, \end{aligned} \tag{1}$$

where $n_t h_t$ represents the qualified labor units, the term in brackets represents the percentage change in efficient labor, A_m is the parameter which measures the productivity of this sector, k_t and h_t are the stocks of physical capital and human capital in per-capita terms, respectively, η is the labor adjustment cost parameter, and finally Z_t is a technology shock characterized by the following autoregressive process:

$$\ln(Z_t) = \rho_1\ \ln(Z_{t-1}) + (1-\rho_1)\ln(\bar{Z}) + \varepsilon_t, \tag{2}$$

where $\ln(\bar{Z})$ is the mean of $\ln(Z_t)$ and ε_t is a serially independent innovation with standard deviation σ_Z.

The law of motion for physical capital is:

$$k_{t+1} = y_t - c_t + (1-\delta_k)k_t, \tag{3}$$

where c_t denotes consumption and δ_k represents the depreciation rate of physical capital, which is assumed to be constant.

[4]The introduction of a non-market sector competing with the market sector has already been used in RBC literature. See for example Benhabib et al. (1991) and Greenwood and Hercowitz (1991).

New human capital is assumed to evolve according to the following process:

$$h_{t+1} = A_h \theta_t (1 - l_t - n_t) h_t + (1 - \delta_h) h_t, \tag{4}$$

where A_h measures the productivity of this sector, δ_h denotes the depreciation rate of human capital and θ_t is a shock which follows a first order autoregressive process given by:

$$\ln(\theta_t) = \rho_2 \; \ln(\theta_{t-1}) + (1 - \rho_2) \ln(\bar{\theta}) + \epsilon_t, \tag{5}$$

where $\ln(\bar{\theta})$ is the mean of $\ln(\theta_t)$ and ϵ_t follows a white noise process with standard deviation σ_θ. It is further assumed that θ_t is uncorrelated with the shock Z_t.

Consumers derive utility from the consumption of the final good and from leisure. Future utility is discounted at a rate β and preferences are described by the following utility function:[5]

$$U(c_t, l_t) = \begin{cases} \frac{(c_t^{\lambda} l_t^{1-\lambda})^{1-\gamma} - 1}{1-\gamma}, & 0 \leq \lambda \leq 1, \quad \gamma > 0 \;\text{ and }\; \gamma \neq 1 \\ \lambda \ln c_t + (1 - \lambda) \ln l_t & \text{for } \;\gamma = 1 \end{cases} \tag{6}$$

This utility function is increasing and concave in both arguments: $U_c > 0$, $U_l > 0$, $U_{cc} < 0$, $U_{ll} < 0$. Note that when $\gamma = 1$ we have that $U_{cl} = 0$ (i.e., the marginal utility of leisure is independent of consumption). However, for $\gamma \neq 1$, the marginal utility of leisure depends on the level of consumption, $U_{cl} \neq 0$. Further, depending on the value of γ, consumption and leisure will be substitutes or complements: for $\gamma > 1$ we have $U_{cl} < 0$, whereas for $\gamma < 1$ we have $U_{cl} > 0$.

As is well known, in the absence of externalities, public goods and distortionary taxation, the solution to the planner's problem is the competitive equilibrium allocation. The problem faced by the central planner is to choose the sequences of consumption, hours worked, leisure, physical capital and human capital that maximize the expected discounted stream of utility given by:

$$\max_{n_t, c_t, l_t, k_{t+1}, h_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t),$$

subject to (1), (2), (3), (4), (5), (6), the usual non-negativity constraints, $0 \leq l_t \leq 1$, $0 \leq 1 - l_t - n_t \leq 1$, and given an initial condition $(Z_0, \theta_0, k_0, h_0, n_0)$. The first-order conditions are shown in Appendix A.

[5]Note that this function satisfies the conditions needed to ensure the existence of a BGP. For further details, see King et al. (1988, pp. 201-202).

2.1 Calibration

We follow the calibration procedure suggested by Kydland and Prescott (1982). The steady state values of the variables are approached by averaging the corresponding US time series for the period 1954 to 1989. The values for structural parameters and some steady state variables are displayed in Table 1.

Table 1: Benchmark parameter and steady state values [a]

Parameter	Value[b]	Interpretation
α	0.36	Share of physical capital in the final good technology
δ_k	0.025	Depreciation rate of physical capital
δ_h	0.005	Depreciation rate of human capital
A_m	1	Scale parameter in the final good technology
A_h	0.0266666	Scale parameter in the human capital production function
λ	0.3769	Consumption weight in utility function
η	0.36	Size of labor adjustment costs
σ_Z	0.007	Standard deviation of ε_t
σ_θ	0.004	Standard deviation of ϵ_t
ρ_2	0.95	Persistence of θ_t
ρ_1	0.95	Persistence of Z_t
v	0.0036	Growth rate
$\bar{n}$	0.24	Hours worked
$\bar{r}$	0.01	Real interest rate
β	$\beta = \frac{(1+v)^\sigma}{1+\bar{r}}$	Subjective discount factor
σ	0.9, 1, 2	Relative risk aversion (RRA) parameter
γ	$(1-\gamma)\lambda = 1-\sigma$	Risk aversion parameter

[a] For parameters with a time dimension, the unit of time is a quarter of a year.

[b] Following RBC tradition, when changing σ we recalibrate other parameters such as β and γ.

Looking at the market sector, the parameter α is set at 0.36, which is the capital's average share of per capita US GNP during the period under study. The rate of depreciation for physical capital, δ_k, is set equal to 0.025 which is equivalent to the 10% annual rate used by Kydland and Prescott (1982). The parameter A_m is normalized to unity.

The parameters of the autoregressive process which characterize the technology shock dynamics (Z_t) are usually chosen on the basis of calibration studies well known in the literature. As Prescott (1986) suggests, the value assigned to ρ_1 is 0.95. The value for σ_Z is chosen in order to replicate the volatility of per capita GNP observed in U.S. data.

The labor adjustment parameter, η, has been calibrated from estimates in Shapiro (1986). Following Cogley and Nason (1995) we take $\eta = 0.36$ as the baseline value. These authors point out that this value probably overstates the size of aggregate labor adjustment costs. Here, labor is measured in efficiency units and, as a consequence, not only the hours worked but also human capital are subject to adjustment costs. Hence, the same baseline value seems to be more suitable when human capital is included.

Given that the human capital sector here is a non-market sector, the calibration of the parameters involved is no trivial. We have chosen those parameter values that reproduce the observed average of US time series. Parameter A_h is chosen to match the 1.4% annual growth rate. Estimates for human capital depreciation rate, δ_h, range from approximately 0.6% to 13.3% per year (Heckman, 1976; Rosen, 1976). We consider $\delta_h = 0.005$ which is equivalent to 2% per year. Coefficient ρ_2 has been assigned the same value as the one assigned to the technology shock (0.95). In order to examine the robustness of the results to changes in σ_θ, we have experimented with different values for this parameter. The results are shown in Appendix B. It is assumed that both shocks are uncorrelated.

Looking at household preferences, and following the suggestion by Greenwood and Hercowitz (1991) the value of parameter λ is established to guarantee that the fraction of time allocated to the market sector is 0.24, which is the fraction of time spent working by the U.S. working-age population.

Since the utility function is multiplicatively separable, it can be written as $U(c, l) = u(c)v(l)$, where $u(c)$ is homogeneous of degree $1-\sigma$. Note that $(1-\gamma)\lambda = 1 - \sigma$. When $\sigma = 1$ (i.e., $\gamma = 1$) the utility function is logarithmic and $U_{cl} = 0$.

But, for $\sigma > 1$ (i.e., $\gamma > 1$) then $U_{cl} < 0$, and for $\sigma < 1$ (i.e., $\gamma < 1$) then $U_{cl} > 0$, respectively. Mehra and Prescott (1985) consider σ in the interval [0, 10]. We consider a smaller interval [0.9, 2] to show how sensitive certain statistics displayed by the model are to changes in the degree of complementarity or substitutability between consumption and leisure.[6] The value for γ can be derived from the expression $(1-\gamma)\lambda = 1-\sigma$.

The discount factor is chosen so that the annual real interest rate is equal to 4%. This value is derived from fullfiling the following first-order condition in the deterministic steady state:

$$\beta(1+v)^{-\sigma}1.01 = 1,$$

given the homogeneity properties of the utility function.

This model has no closed-form solution. Hence, a numerical approximation method is required to solve it. The resolution method we follow is Uhlig's (1999) Log-linear Method.

3 Persistence in output growth

The simulation procedure we employ to study the dynamic properties can be summarized as follows. We generate artificial time series for output by simulating various RBC models. This allows us to evaluate the contribution of each utility specification in explaining the persistence in output growth. In particular, the performance of the model is compared with U.S. quarterly data from 1955:3 to 1984:1. The autocorrelation function and impulse response functions are estimated for each artificial sample which is 115 periods long (each model was simulated 1,000 times) and the corresponding empirical probability distributions are computed.

[6]Most empirical studies on the RRA coefficient suggest a moderate range for σ that includes the [0.5, 3] interval. For example, Hansen and Singleton (1982) estimate that σ ranges between 0.3502 and 0.9903, σ being less than 1. But, Friend and Blume (1975) estimate that σ lies between 1 and 3. As mentioned above, in this paper all parameter choices for σ are consistent with the evidence cited by Mehra and Prescott (1985).

3.1 Autocorrelation Functions

We analyze whether all the different models replicate the sample autocorrelation function (ACF) for output growth. Given a model, we have computed the average c and the covariance matrix V_c of the ACF's of the artificial time series. We apply the following generalized test statistic to analyze the goodness of fit between the actual and the theoretical ACF's:

$$Q_{acf} = (\widehat{c} - c)' V_c^{-1} (\widehat{c} - c),$$

where $\widehat{c}$ stands for the actual ACF and c for the model-generated one. A high value of Q_{acf} indicates a poor fit between the theoretical ACF and the actual ACF. The generalized Q_{acf} statistic is approximately $\chi^2(p)$, where p is the number of lags in c.

Following Cogley and Nason (1995) (CN hereafter), we compute the generalized statistic for the first $p = 8$ autocorrelations. The results are summarized in Table 2 and Figure 1. The first three columns of Table 2 report values of the Q_{acf} statistic for each choice of σ with probability values in parentheses. Lines 1-5 show how sensitive the results are to changes in the labor adjustment costs parameter, η.

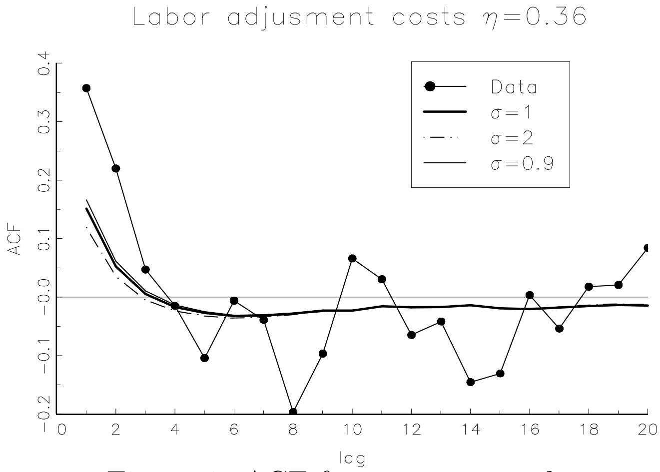

Figure 1: ACF for output growth

As σ becomes higher, the p-values decrease. We find the same result as η decreases. Numerical results show that regardless of the value of σ, the introduction of labor adjustment costs is crucial for obtaining realistic ACF results for output growth. However, when $\sigma = 0.9$ (i.e., consumption and leisure are complements in the sense that $U_{cl} > 0$) the model is not rejected even for $\eta = \frac{0.36}{4}$. Figure 1 shows that the lower the RRA parameter σ, the higher the first autocorrelation coefficients are. The internal propagation mechanism of the model provides some intuition for these results.

We have also analyzed the sensitivity of the ACF results to changes in the standard deviation of the human capital shock, σ_θ. As shown in Appendix B, the ACF results do not depend on human capital accumulation process being stochastic since when $\sigma = 0.9$ and $\eta = \frac{0.36}{3}$ the model is not rejected even for a human capital shock that is half as large.

Table 2: Sensitivity analysis to changes in σ and η

	Q_{acf} (p−value)			$Q_{irf}(Y_P)$ (p−value)			$Q_{irf}(Y_T)$ (p−value)		
Costs\RRA	$\sigma = 0.9$	$\sigma = 1$	$\sigma = 2$	$\sigma = 0.9$	$\sigma = 1$	$\sigma = 2$	$\sigma = 0.9$	$\sigma = 1$	$\sigma = 2$
$\eta = 0.36$	9.45	10.09	11.99	18.88	25.60	47.84	8.61	11.65	25.71
	(0.30)	(0.26)	(0.15)	(0.08)	(0.048)	(0.01)	(0.32)	(0.20)	(0.06)
$\eta = \frac{0.36}{2}$	11.68	12.46	14.63	18.39	25.26	48.82	8.96	12.11	27.38
	(0.16)	(0.13)	(0.07)	(0.09)	(0.05)	(0.01)	(0.30)	(0.20)	(0.05)
$\eta = \frac{0.36}{3}$	13.56	14.40	16.71	18.24	25.17	49.22	9.82	13.11	29.32
	(0.09)	(0.07)	(0.03)	(0.09)	(0.05)	(0.01)	(0.27)	(0.17)	(0.048)
$\eta = \frac{0.36}{4}$	15.06	15.95	18.33	18.19	25.16	49.49	10.55	13.95	30.81
	(0.06)	(0.04)	(0.02)	(0.09)	(0.05)	(0.01)	(0.24)	(0.16)	(0.045)
$\eta = 0$	28.58	28.66	32.45	18.47	25.69	51.36	15.94	19.91	40.13
	(0.00)	(0.00)	(0.00)	(0.09)	(0.05)	(0.01)	(0.12)	(0.09)	(0.03)

3.2 Impulse Response Functions

We also analyze whether these models replicate observed impulse response functions (IRF's). The IRF's are obtained by using the structural VAR model with long-run restrictions developed by Blanchard and Quah (1989). To implement this technique, a third-order VAR for per-capita output growth and ln(hours) is estimated. We analyze whether the theoretical IRF is close to the actual IRF by using the following test statistic:

$$Q_{irf} = (\widehat{r} - r)' V_r^{-1} (\widehat{r} - r),$$

where $\widehat{r}$ is the actual IRF and r is the model-generated one, which is computed by the average of IRF's across the artificial time series. The matrix V_r denotes the covariance matrix of the IRF's of the Monte Carlo draws.

A high value of Q_{irf} indicates that the performance of the theoretical model is not consistent with actual data. We compute this statistic for endogenous growth models with coefficients up to lag 8. Exogenous growth models are driven by a single shock, so their bivariate VAR models have stochastic singularities. Table 2 reports the sensitivity of the Q_{irf} statistics to changes in σ, with Monte Carlo probability values in parentheses. Figure 2 illustrates the transitory IRF's for this generalized endogenous growth model with different values of σ.

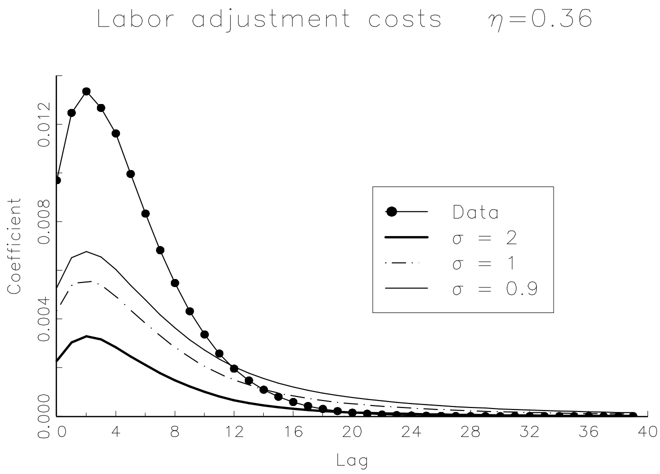

Figure 2: Transitory IRF function for $\ln(GNP)$

Let us consider our benchmark value $\sigma = 1$ in which case the utility function is logarithmic. In this case, the results are partially successful since the model is able to generate the pattern of the transitory IRF found in the data, but fails to reproduce the permanent IRF. In contrast with well-known transitory IRF results on standard RBC models, our setting is not rejected at conventional significance levels even when $\eta = 0$ (see the fifth column of Table 2). Hence, this human capital investment model not only generates the right qualitative response to transitory shocks but it is also able to match the magnitude of the transitory IRF.

The results are very sensitive to changes in the value of the RRA parameter. When $\sigma = 2$ labor adjustment costs are needed in order to generate the transitory IRF found in the data. As σ decreases the performance of the model improves. Note that when $\sigma = 0.9$ the model is not rejected at conventional significance levels even in the absence of labor adjustment costs (i.e., when $\eta = 0$). Hence, when consumption and leisure are complements there is no need to include labor adjustment costs with endogenous growth in order to match not only the large transitory IRF, but also the permanent IRF found in the data. Figure 2 shows that as the RRA decreases (i.e., as the degree of complementarity between consumption and leisure increases) the hump displayed by the transitory IRF increases.

The IRF results are sensitive to changes in σ_θ but, as shown in Appendix B, when $\sigma = 0.9$ (i.e., consumption and leisure are complements) and $\eta = \frac{0.36}{3}$, the model is able to match the observed transitory IRF even for $\sigma_\theta = 0.002$.

3.3 The dynamic response functions

In order to assess the importance of those properties of the model that amplify the effect of the shocks and cause the deviation from the steady state to persist, and also to get some intuition for the results, we analyze how the internal propagation mechanism is affected by changes in σ.

Figures 3 and 4 report the responses of certain interesting variables to both transitory (Z_t) and permanent (θ_t) shocks, respectively. As shown by these figures, the dynamic response functions depend on the value of σ, which determines the degree of complementarity or substitutability between consumption and leisure.

When a favorable technology shock (Z_t) takes place, the greater the degree of risk aversion is, the fewer the resources that are devoted to producing goods. This, in the end, leads to a smaller reduction of the growth rate (see g_t in Figure 3). Due to the existence of labor adjustment costs, output not only rises at the time of the impact but also in subsequent periods. As shown in this figure, not only the impact effect on output of Z_t but also the lagged effects become smaller the greater σ is. This generates not only a smaller serial correlation in output growth but also a smaller hump in the transitory IRF (see Figure 2). In addition to this effect, we must take into account that which results from considering human capital shocks (θ_t). In that case, the greater the degree of risk aversion, the fewer the resources that are devoted to human capital accumulation, which will lead to a smaller increase of the growth rate (see g_t in Figure 4). Output falls during some periods and subsequently rises back towards its initial trend,

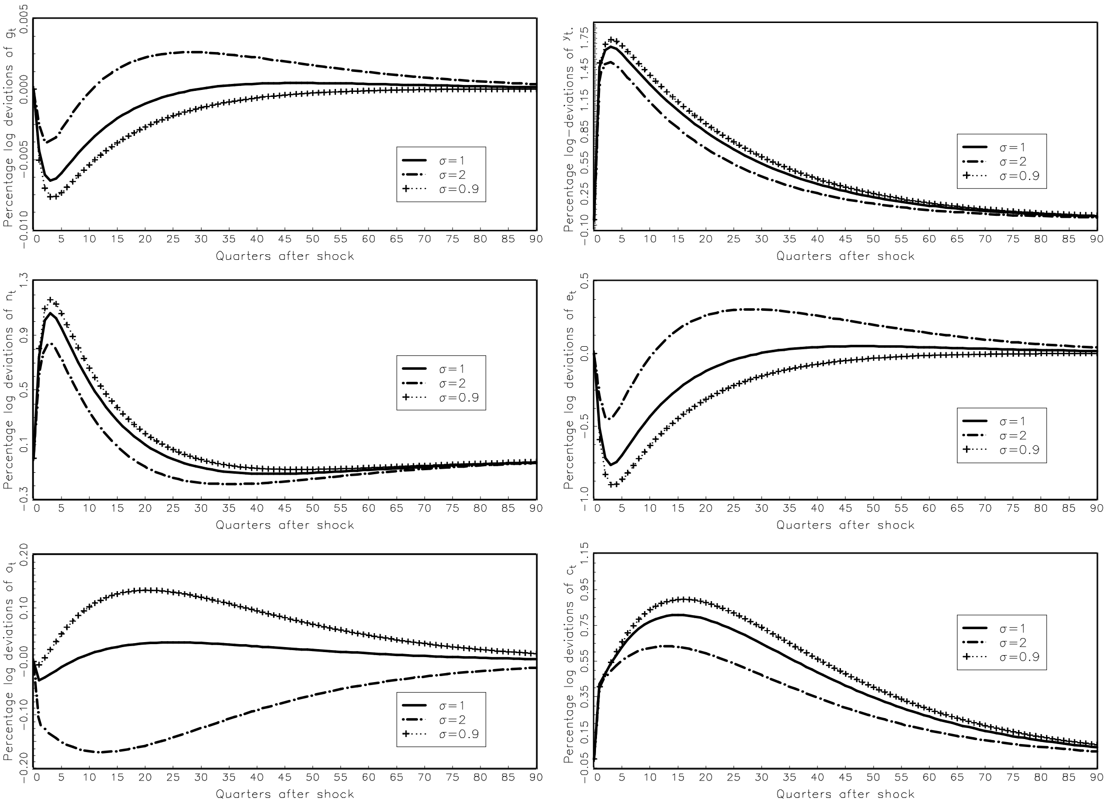

Figure 3: Response functions to a 1% technology shock

but the greater σ is, the smaller is the response. Hence, this second effect also generates a smaller correlation in output growth.[7] As a result, increasing RRA leads to decreasing the serial correlation in output growth. Figure 3 illustrates how the hump displayed by hours increases as σ decreases, since hours worked not only increase at the time of the impact but also in subsequent periods, and the same holds for output.

These sensitivity results are consistent with the results obtained in a recent paper by Jones et al. (2005) (JMS hereafter), in a similar endogenous growth model.[8] They show that in contrast to well-known results for exogenous growth

[7]Note that this second effect is only observed when the human capital production sector is considered. As noted by Barañano and Moral (2003), this effect explains why introducing endogenous growth in non-standard RBC model with labor adjustment costs enhances the model's ability to reproduce the observed persistence in US output.

[8]Their human capital investment model differs from ours in many aspects. First, they not only consider that physical capital enters the human capital production function, but also that

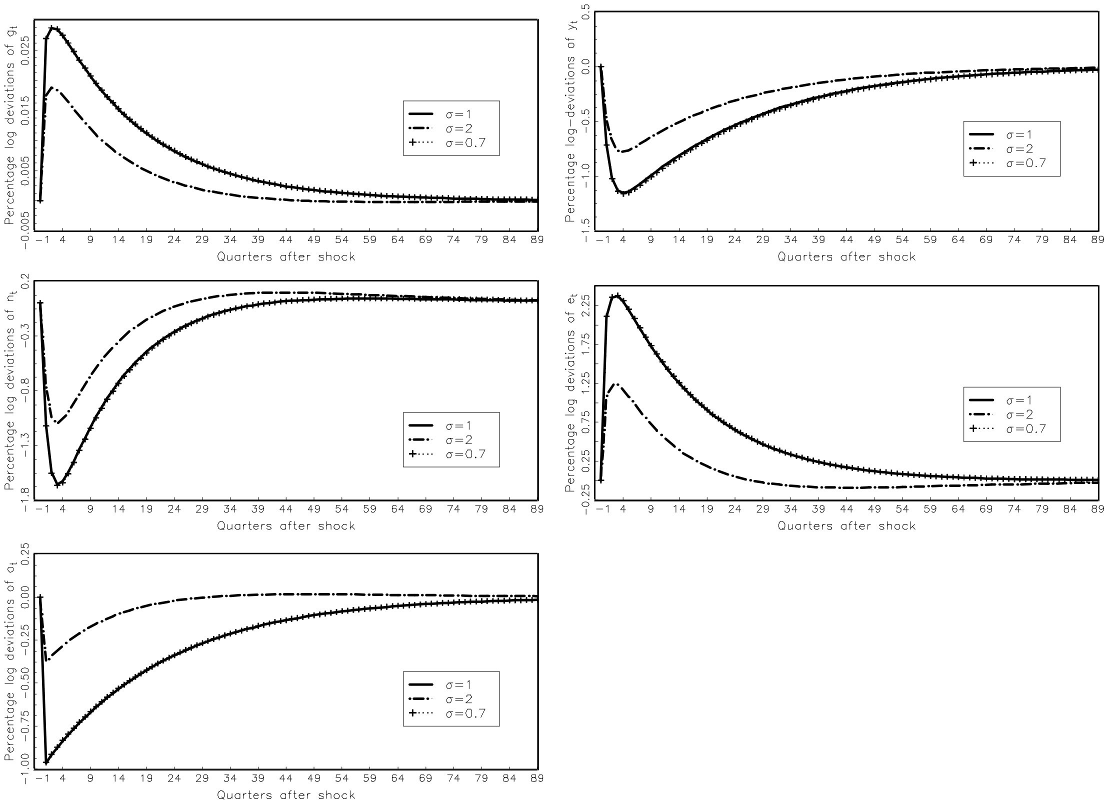

Figure 4: Response functions to a 1% human capital shock

models, the RRA parameter plays a major role in determining the second moment properties of several macroeconomic time series in a human-capital based endogenous growth model.[9] As argued by JMS (2005), these sensitivity results

both capitals are generated by the same technology, implying that they are perfect substitutes. In our setting the production of human capital involves no physical capital. Second, they do not include labor adjustment costs and, as a consequence, their model fails to generate the ACF found in the data.

[9] JMS (2005) show how sensitive several second moments statistics are to changes in the RRA parameter. They study: (i) the standard deviations of the growth rate of output, the growth rate of labor productivity, the investment-output ratio and labor; (ii) the first-order autocorrelations of the growth rate of output, the growth rate of labor productivity and labor; and (iii) several cross-correlations. Note that we study the whole ACF as suggested by CN (1995), whereas JMS consider only the first autocorrelation coefficient of output growth. Maury and Tripier (2003) show the importance of preserving the whole CN's empirical setup. These authors analyze the properties of JMS's model from the viewpoint of CN's analysis and conclude that although this model improves the first positive value of the ACF over the standard RBC model, it is however unable to reproduce the negative values of the ACF for higher orders.

arise in human capital investment models since the total share of all capital is large relative to the share in otherwise usual RBC models, which has important consequences for the response to a shock.[10]

From this discussion it is clear that in human capital based growth models the RRA parameter plays an important role in generating the kind of internal propagation mechanism needed to obtain realistic output dynamics of GNP. Further, our results highlight how the degree of complementarity or substitutability between consumption and leisure is a key factor in determining the properties of the model.[11] The numerical results show that dynamics of the variables of the model may vary substantially. For instance, let us consider the dynamic response of leisure to a 1% technology shock. As shown by Figure 3, when $\sigma = 1$ individuals maintain a smooth path for leisure, implying that they respond to fluctuations by varying the time allocated to each sector. As we move from this bechmark value, the behavior of leisure changes. When $\sigma > 1$ the time devoted to leisure falls at the impact period and remains under the steady state value for longer than fifty periods. As Figure 3 shows, the hump displayed by consumption is smaller, since consumption and leisure are substitutes. However, when $\sigma < 1$ (i.e., $U_{cl} > 0$), although the technology shock causes leisure to decrease at the impact period, it rises in subsequent periods and remains over the steady state value for longer than fifty periods. As also shown in Figure 3, the hump displayed by consumption increases, since an increase in leisure raises the marginal utility of consumption and induces individuals to consume more.

To sum up, this subsection shows how the degree of complementarity or substitutability between consumption and leisure may be a crucial determinant of the performance of this generalized endogenous growth model. We find that both ACF of output growth and IRF results are sensitive to changes in the RRA parameter. Our ACF results depend on the size of the labor adjustment costs, although the smaller the RRA parameter, the smaller the costs required. However, our IRF results do not depend on the size of these costs when leisure and consumption are complements.

[10]As they explain, in a non-stochastic version of this endogenous growth model when a surprise increase in capital stocks takes place, output increases at impact but there is no dampened return for output in levels to the original time path, as occurs in the exogenous version. For a detailed discussion see JMS (2005, p. 816).

[11]In our case, due to the specification considered, this degree of complementarity or substitutability depends on the value considered for σ.

These results naturally raise the question of whether this complementarity between consumption and leisure has a counterpart in the data. There are different types of leisure activities.[12] Following Hurd and Rohwedder (2003), some types of leisure such as watching TV seem to be neutral with consumption, others such as home production seem to be substitutes, and others such as travel would seem to be complements: "Everyday observation and introspection say that we have all types, and it is an empirical question as to which dominates" (p. 8). Hence, complementarities between consumption and leisure seem empirically plausible, although the extent of their magnitude is an open empirical question which, unfortunately, exceeds the scope of this paper.

4 Conclusions

This paper studies the role that nonseparabilities between leisure and consumption may have in explaining the persistence in output growth in postwar US data. It considers an extended version of Lucas' (1988) human capital investment model which includes 'effective' labor adjustment costs.

When preferences are additively separable in consumption and leisure, the RRA must be equal to one if a BGP is to exist, in which case the marginal utility of consumption does not depend on leisure. This is the most common specification used in the RBC literature. In our paper we compare a log-separable specification with a multiplicatively separable specification in explaining the persistence in output growth. As argued by King et al. (1988), in the latter case the RRA can be greater or lower than one, implying that the marginal utility of consumption can be a decreasing or an increasing function of leisure, respectively. In this sense, given the specification considered, the RRA parameter determines the degree of complementarity or substitutability between leisure and consumption. We find that the persistence in GNP growth is sensitive to changes in the RRA parameter. Hence, the link between consumption and leisure built into the utility function may help to explain its persistence. In particular, when consumption and leisure are complements the model succeeds in matching not only the autocorrelation of output growth but also the trend-reverting component found in US data, even

[12]For a detailed description of different definitions of leisure see Aguiar and Hurst (2007). These definitions range from the narrowest one, which include any activity that yield direct utility, to the broadest one, that is the residual of market plus non-market work.

if low adjustment costs of labor are considered. We conclude that a seemingly simple margin not studied previously, but which occupies an important role in other areas, offers the potential to provide useful insights into observed patterns of business cycles.

Appendix A

The first-order conditions for this problem are:

$$\begin{aligned}
U_2(c_t, l_t) &= U_1(c_t, l_t)\left[\tfrac{1-\alpha}{n_t} - \eta\tfrac{\Delta(n_t h_t) h_t}{(n_{t-1}h_{t-1})^2}\right] y_t \\
&+ \beta\, E_t\left\{U_1(c_{t+1}, l_{t+1})\,\eta\tfrac{\Delta(n_{t+1}h_{t+1}) n_{t+1}h_{t+1}h_t}{(n_t h_t)^3}\, y_{t+1}\right\}, \qquad (7)\\
U_1(c_t, l_t) &= \beta\, E_t\left\{U_1(c_{t+1}, l_{t+1})\left[\frac{\alpha}{k_{t+1}}\, y_{t+1} + 1 - \delta_k\right]\right\}, \qquad (8)\\
\frac{U_2(c_t, l_t)}{A_h\theta_t h_t} &= \beta\, E_t\left\{\frac{U_2(c_{t+1}, l_{t+1})}{A_h\theta_{t+1}h_{t+1}}\left[A_h\theta_{t+1}(1-l_{t+1}) + 1 - \delta_h\right]\right\}, \qquad (9)\\
h_{t+1} &= A_h\theta_t(1 - l_t - n_t)h_t + (1-\delta_h)h_t,\\
y_t &= \frac{A_m Z_t k_t^{\alpha}(n_t h_t)^{1-\alpha}}{\exp\left\{\frac{\eta}{2}\left[\frac{\Delta(n_t h_t)}{n_{t-1}h_{t-1}}\right]^2\right\}},\\
k_{t+1} + c_t &= y_t + (1-\delta_k)k_t,\\
\lim_{t\to\infty} E_t\beta^t U_1 k_{t+1} &= 0,\\
\lim_{t\to\infty} E_t\beta^t \frac{U_2}{A_h\theta_t h_t} h_{t+1} &= 0,
\end{aligned}$$

where E_t is an operator whose expectations are conditional on the information available at time t.

Equation (7) shows the optimal way of determining the fraction of time devoted to the production of goods. The marginal utility of an additional labor unit has to be equal to its marginal disutility. Labor adjustment costs affect not only current marginal utility but also expected utility via future output. Hence, due to the presence of labor adjustment costs, firms do not adjust labor input completely in the current quarter. Their optimal response is to defer part to the subsequent quarter.

Equation (8) governs the accumulation of physical capital. It establishes that, at the margin, the expected return to acquiring an additional unit of physical capital must be equal to the cost it causes in utility terms today.

Equation (9) governs the accumulation of human capital. Given that $1 - l_t$ denotes the fraction of time not allocated to leisure, this equation establishes that, at the margin, the expected return in current period utility from an additional unit of human capital must be equal to its cost.

In the steady state, the variables k_t, y_t and c_t grow at a constant rate, which is equal to the human capital growth rate, while n_t and l_t remain constant. Therefore, non-stationary time series are obtained from the first order conditions characterizing the social planner problem. For the sake of simplicity, the first-order conditions can be rewritten as:

$$\begin{aligned}
U_1(\hat{c}_t, l_t) &= \beta \left(\frac{h_{t+1}}{h_t}\right)^{-\sigma} E_t \left\{ U_1(\hat{c}_{t+1}, l_{t+1}) \left[\frac{\alpha \hat{y}_{t+1}}{\hat{k}_{t+1}} + 1 - \delta_k\right]\right\}, \\
U_2(\hat{c}_t, l_t) &= U_1(\hat{c}_t, l_t) \left[\frac{1-\alpha}{n_t} - \eta \frac{n_t - n_{t-1}\frac{h_{t-1}}{h_t}}{n_{t-1}\frac{h_{t-1}}{h_t}} \frac{h_t}{n_{t-1}h_{t-1}}\right] \hat{y}_t + \\
&\quad \beta \left(\frac{h_{t+1}}{h_t}\right)^{1-\sigma} E_t \left\{ U_1(\hat{c}_{t+1}, l_{t+1})\, \eta \frac{n_{t+1} - n_t \frac{h_t}{h_{t+1}}}{n_t \frac{h_t}{h_{t+1}}} \frac{n_{t+1} h_{t+1} \hat{y}_{t+1}}{{n_t}^2 h_t} \right\}, \\
\frac{U_2(\hat{c}_t, l_t)}{A_h \theta_t} &= \beta \left(\frac{h_{t+1}}{h_t}\right)^{-\sigma} E_t \left\{ \frac{U_2(\hat{c}_{t+1}, l_{t+1})}{A_h \theta_{t+1}} \left[A_h \theta_{t+1}(1 - l_{t+1}) + 1 - \delta_h\right]\right\}, \\
\frac{h_{t+1}}{h_t} &= A_h \theta_t (1 - l_t - n_t) + 1 - \delta_h, \\
\hat{c}_t + \hat{k}_{t+1} \frac{h_{t+1}}{h_t} &= \frac{A_m Z_t \hat{k}_t^{\alpha} n_t^{1-\alpha}}{\exp\left\{ \frac{\eta}{2} \left[\frac{n_t - n_{t-1}\frac{h_{t-1}}{h_t}}{n_{t-1}\frac{h_{t-1}}{h_t}}\right]^2 \right\}} + (1 - \delta_k)\hat{k}_t,
\end{aligned}$$

where $\hat{c}_t = \dfrac{c_t}{h_t}$ and $\hat{k}_t = \dfrac{k_t}{h_t}$.

Appendix B

Sensitivity analysis to changes in σ_θ and η

	Q_{acf} [(a)]		$Q_{irf}(Y_P)$ [(b)]		$Q_{irf}(Y_T)$ [(c)]	
Costs \ Shock	$\sigma_\theta = 0.004$	$\sigma_\theta = 0.002$	$\sigma_\theta = 0.004$	$\sigma_\theta = 0.002$	$\sigma_\theta = 0.004$	$\sigma_\theta = 0.002$
$\eta = 0.36$	9.45	10.55	18.88	36.05	8.61	18.88
	(0.305)	(0.228)	(0.084)	(0.019)	(0.318)	(0.094)
$\eta = \frac{0.36}{2}$	11.68	12.90	18.39	36.02	8.96	19.43
	(0.165)	(0.115)	(0.088)	(0.023)	(0.302)	(0.088)
$\eta = \frac{0.36}{3}$	13.56	14.81	18.24	36.31	9.82	20.57
	(0.093)	(0.063)	(0.09)	(0.023)	(0.268)	(0.075)

References

Aguiar, M., Hurst, E., 2007. Measuring Trends in Leisure: The Allocation of Time Over Five Decades. Quarterly Journal of Economics, 122(3), 969-1006.

Andolfatto, D., 1996. Business Cycles and Labor Market Search. American Economic Review, 86(1), 112-132.

Barañano, I., Moral, P., 2003. Output Dynamics in an Endogenous Growth Model. Economics Bulletin, 5, 1-13.

Baxter, M., 1995. International Trade and Business Cycles. NBER Working Papers WP5025, National Bureau of Economic Research.

Benhabib, J., Perli, R., Sakellaris, P., 2006. Persistence of Business Cycles in Multisector Real Business Cycle Models. International Journal of Economic Theory, 2, 181-197.

Benhabib, J., Rogerson, R., Wright, R., 1991. Homework in Macroeconomics: household production and aggregate fluctuations. Journal of Political Economy, 99(6), 1166-1187.

Benhabib, J., Wen, Y., 2004. Indeterminacy, Aggregate Demand and the Real Business Cycle. Journal of Monetary Economics, 51(3), 503-530.

Blanchard, O. J., Quah, D., 1989. The Dynamic Effects of Aggregate Demand and Supply Disturbances. American Economic Review, 79(4), 655-673.

Burnside, C., Eichenbaum, M., 1996. Factor-Hoarding and the Propagation of Business-Cycle Shocks. American Economic Review, 86(5), 1154-1174.

Cogley, T., Nason, J. M., 1995. Output Dynamics in Real-Business-Cycle Models. American Economic Review, 85(3), 492-511.

de Hek, P.A., 1998. An Aggregative Model of Capital Accumulation with Leisure-dependent Utility. Journal of Economic Dynamics and Control, 23(2), 255-276.

Eichenbaum, M.S., Hansen, L.P., Singleton, K.J., 1988. A Time Series Analysis of Representative Agents Models of Consumption and Leisure Choice under Uncertainty. Quarterly Journal of Economics, 103(1), 51-78.

Finn, M., 1998. Cyclical Effects of Government's Employment and Goods Purchases, International Economic Review, 39(3), 635-657.

Friend, I., Blume, M.E., 1975. The Demand for Risky Assets. American Economic Review, 65(5), 900-922.

Greenwood, J., Hercowitz, Z., 1991. The Allocation of Capital and Time over Business Cycle. Journal of Political Economy, 99(6), 1118-1214.

Hansen, L.P., Singleton, K.J., 1982. Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models. Econometrica, 50(5), 1269-1286.

Heckman, J.J., 1976. A Life-Cycle Model of Earnings, Learning and Consumption. Journal of Political Economy, 84(4), 11-44.

Hurd, M., Rohwedder, S., 2003. The Retirement-Consumption Puzzle: Anticipated and Actual Declines in Spending at Retirement. NBER Working Papers WP9586, National Bureau of Economic Research.

Jones, L., Manuelli, R., Siu, H., 2005. Fluctuations in Convex Models of Endogenous Growth, II: Business Cycle Properties. Review of Economic Dynamics, 8(4), 805-828.

King, R., Plosser, C., Rebelo, S., 1988. Production, Growth and Business Cycles: I. The Basic Neoclassical Model. Journal of Monetary Economics, 21, 195-232.

Kydland, F.E., Prescott, E.C., 1982. Time to Build and Aggregate Fluctuations. Econometrica, 50(6), 1345-1370.

Ladrón-de-Guevara, A., Ortigueira, S., Santos, M.S., 1999. A Two-Sector Model of Endogenous Growth with Leisure. The Review of Economic Studies, 66(3), 609-631.

Low, H.W., 2005. Self Insurance in a Life-Cycle Model of Labour Supply and Savings. Review of Economic Dynamics, 8(4), 945-975.

Lucas, R. E., Jr., 1988. On the Mechanics of Economic Development. Journal of Monetary Economics, 22(1), 3-42.

Mankiw, N.G., Rotemberg, J.J., Summers, L.H., 1985. Intertemporal Substitution in Macroeconomics. Quarterly Journal of Economics, 100(1), 225-51.

Matheny, K.J., 1998. Non-neutral Responses to Money Supply Shocks when Consumption and Leisure Are Pareto Substitutes. Economic Theory, 11(2), 379-402.

Maury, T.P., Tripier, F., 2003. Output Persistence in Human Capital-based Growth Models. Economics Bulletin, 5(11), 1-8.

Mehra, R., Prescott, E.C., 1985. The Equity Premium: A Puzzle. Journal of Monetary Economics, 15, 145-161.

McGrattan, E.R., 1994. The Macroeconomic Effects of Distortionary Taxation. Journal of Monetary Economics, 33(3), 573-601.

Perli, R., 1998. Indeterminacy, home production and the business cycle: A Calibrated Analysis. Journal of Monetary Economics, 41(1), 105-125.

Perli, R., Sakellaris, P., 1998. Human Capital Formation and Business Cycles Persistence. Journal of Monetary Economics, 42, 67-92.

Prescott, E. C., 1986. Theory Ahead of Business Cycle Measurement. Federal Reserve Bank of Minneapolis, Quarterly Review, 10(4), 9-22.

Rosen, S., 1976. A Theory of Life Earnings. Journal of Political Economy, 84(4), s45-67.

Shapiro, M.D., 1986. The Dynamic Demand for Capital and Labor. Quarterly Journal of Economics, 101(3), 513-542.

Schmitt-Grohé, S., 2000. Endogenous Business Cycles and the Dynamics of Output, Hours and Consumption. American Economic Review, 90(5), 1136-1159.

Uhlig, H., 1999. A toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily. In Marimon, R., Scott, A. (Eds.). Computational Methods for the Study of Dynamic Economies. Oxford: Oxford University Press, chapter 3.

Wen, Y., 1998. Can a Real Business Cycle Model Pass the Watson Test?. Journal of Monetary Economics, 42(1), 185-203.