Choice of Product Variety for the

Durable Goods Monopolist

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Abstract

This paper analyzes the strategic choice of variety by a monopolist seller of a durable good as a means to mitigate his commitment problem. The monopolist chooses his product variety with a goal of ensuring that a strong reduction in future prices will not be profitable because it allows the firm to attract few additional consumers. The main result that emerges from considering product variety as an endogenous variable is that, contrary to the case in which it is exogenously determined, social welfare is always higher when the monopolist cannot commit that when he can.

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1. Introduction

The power held by a monopolist in the production and sale of a durable good can be substantial but is significantly less than the power held by a monopolist who produces a non-durable good. The monopolist seller of a durable good faces the problem of time inconsistency when deciding on his optimal production path. In the case of the durable goods monopolist, the credibility problem rests on whether or not he can commit to a future schedule of production. In a dynamic theory of the durable goods monopoly, the time path of prices will generally not be the one which, if a commitment to future prices were possible, would generate sales that maximize the present value of the monopolist's profits.

This paper analyzes the strategic choice of product variety by a monopolist seller of a durable good as a means to mitigate his commitment problems. The literature has examined different possibilities which solve or mitigate this commitment problem: the good can be rented rather than sold (Coase (1972)); capacity restrictions (Bulow (1982)); the establishment of exclusive contracts in serving the product, that is, the transfer of monopoly power to services, which are not durable (Bulow (1982)); planned obsolescence, that is, choice of product durability (Coase (1972), Bulow (1986)); and the use of best-price provisions (Butz (1990)). In our analysis, the monopolist who cannot commit to future production chooses to produce a variety such that he credibly commits not to reduce future prices drastically.

It is very common in the durable goods monopolist literature to impose the existence of an exogenously given demand (see Bulow (1982, 1986), Kahn (1986), Malueg and Solow (1989)). However, firms frequently choose the variety of the good they will produce and, as a result, they determine their own demand. For this reason it is interesting to analyze the monopolist's choice of variety (or demand) from a strategic point of view. The more tractable way to analyze this problem is to deal with the linear city model proposed by Hotelling (1929). This frame, which is very useful for analyzing competition between rivals, is reasonable to use here because in the case of a durable goods monopolist, the firm faces its own future competition. Moreover, this model provides locally linear demands, which allows us to compare the results with the existing results in the durable goods monopolist literature.

The literature on durable goods has also analyzed the implications that the ability or inability of commitment to a future schedule of production has for social welfare in different scenarios (Bulow (1982), Kahn (1986), Kahn, Malueg and Solow (1988), Malueg and Solow (1987, 1989), Bond and Samuelson (1987)). This paper analyzes the implications that the choice of product variety has for social welfare by comparing the cases of the monopolist renter and the monopolist seller. The main result that emerges from the analysis is that when product variety is considered an endogenous variable, contrary to the case in which it is exogenously determined, social welfare is always higher when the monopolist cannot commit than when he can. The assumption that the variety is exogenously determined is implicit in the literature on durable goods, e.g. Malueg and Solow (1989).

The paper is organized as follows. Section 2 sets out the model and solves for the optimal choices of the monopolist who can commit (renter) and the monopolist who cannot (seller). Section 3 compares the implications for social welfare under endogenous and exogenous demand. Section 4 concludes with some final remarks.

2. The Model

Consider a monopolist in the production and sale of a durable good. The monopolist must decide the variety of the good and the quantities to be produced. The good does not depreciate over time. There are two discrete periods of time (j=1,2) and production occurs only at the beginning of each period. For the sake of simplicity we assume that the marginal cost of production of the good in each period is zero and the discount factor is one.[1]

[1] The qualitative results of the paper do not change if the discount factor is assumed to be less than one.

Purchasers are assumed to be price takers and to have perfect foresight. Each period each consumer wishes to make use of one unit of the durable good. Each consumer has a different preferred variety of the good which does not change over time and consumers' tastes are distributed uniformly over the varieties interval [0,1]. The number of consumers is normalized to one. There is perfect and complete information about consumer tastes distribution and the monopolist's production costs.

Let x denote the consumer whose favourite variety is at a distance x from the left limit of the interval [0,1]. The reservation price for the rental services provided by the good for consumer x is equal to $1 - t|x - a|$, where t is a positive constant and a is the variety of the durable good produced by the monopolist. Consumers are thus modeled as in Hotelling's standard model (Hotelling (1929)). The problems related to the non-existence of an equilibrium price solution for some locations in Hotelling's model (described by D'Aspremont, Gabszewicz and Thisse (1979)) do not appear when the monopolist faces his own future price competition. Note that the consumer's reservation price must be interpreted not in terms of transportation costs but in terms of the difference between consumer's preferred variety and the variety produced by the monopolist.[2] For simplicity but without loss of generality, we assume that $t \geq 2$.

The analysis is modeled as a game with two stages: First, the monopolist decides the variety of the good to produce. Second, the firm decides the quantity to be produced in each period. The solution concept is that of the subgame perfect Nash equilibrium. To solve the problem we must then proceed by backward induction from the last stage of the game.

We assume that the monopolist's choice of product variety is irreversible, that is, it cannot be changed over time. One may think, for instance, that the firm is adopting a

[2]Contrary to recent applications, Hotelling (1929) does not fix a maximum reservation price for the consumer whose preferred variety is exactly the one produced by the firm. We need to establish such a level because otherwise the monopolist's equilibrium price could not be determined. In our model, and without loss of generality, it is normalized to 1.

technology that will allow it to produce just that variety of the good and not any other. Without loss of generality we assume that $a \in [0,0.5]$.

Let p be the rental price of the good. The consumer whose preferred variety is x, such that $x > a$, and who is indifferent between renting and not renting one unit of the good is determined from the equation: $1 - t(x - a) - p = 0$. Consequently, $x = \frac{1 - p}{t} + a$. Note that:

- If $x \leq 2a$, the total amount rented is given by $q = 2(x - a) = \frac{2}{t}(1-p)$.

- If $1 \geq x > 2a$, the total amount rented is given by $q = x = \frac{1 - p}{t} + a$.

As a result, the inverse demand function for the services yielded by the good is:

$$p = \begin{cases} 1 - \frac{t}{2}q & \text{if } q \leq 2a \\ 1 + ta - tq & \text{if } 2a \leq q \leq 1 \end{cases} \tag{1}$$

Figure 1 shows the rental demand:

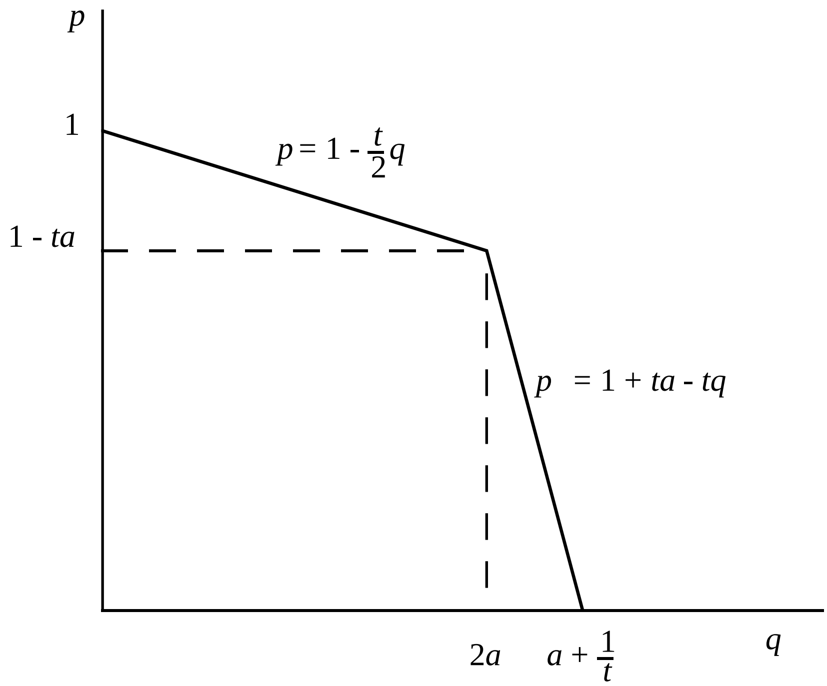

Figure 1: Rental demand of the durable good.

It is important to note that the change in the shape of the demand curve (its kink) is due to the existence of potential buyers only on one side of the market when the quantity produced is higher than $2a$. In this case, an increase of production implies a strong reduction in the price charged.

Let q_j^i denote the quantity produced by the monopolist i (where $i=r$ for the monopolist renter and $i=s$ for the monopolist seller) in period j, j=1,2. Let p_j^i denote the corresponding price charged by monopolist i in period j. Let a^i denote the corresponding product variety produced by monopolist i. Considered, next are the case of the monopolist renter and the case of the monopolist seller.

A. Monopolist renter

This type of monopolist can commit to a future schedule of production. As a result, he will try to guarantee the highest demand for his product and, accordingly, he will try to locate far enough from the left end of the variety distribution. In other words, the monopolist wants to sell to the same number of consumers on both sides of the variety produced (a). This type of monopolist solves the following problem:

$$\underset{q_1^r,\, q_2^r}{Max}\ (1 - \frac{t}{2}q_1^r)q_1^r + [1 - \frac{t}{2}(q_1^r + q_2^r)](q_1^r + q_2^r)$$

The solution is: $q_1^{r*} = \frac{1}{t}$, $q_2^{r*} = 0$, $p_1^r = p_2^r = \frac{1}{2}$, the monopolist produces a variety $a^{r*} \geq \frac{1}{2t}$ and obtains profits $\pi^r = \frac{1}{t}$.

The assumption that $t \geq 2$ guarantees that the market will not be covered by the monopolist. Note that when $t \leq 1$ the monopolist renter covers the market. This case is uninteresting since there is no commitment problem. Moreover, when $1 < t < 2$, the qualitative results of the paper do not change.

B. Monopolist seller

This type of monopolist, who cannot commit to a future schedule of production, will choose the intertemporally consistent plan of production that maximizes the present value of revenues. Note that this type of monopolist has four possibilities:

I.- $q_1^s < 2a^s < q_1^s + q_2^s$

II.- $q_1^s + q_2^s < 2a^s$

III.- $q_1^s < 2a^s = q_1^s + q_2^s$

IV.- $q_1^s \geq 2a^s$

Next, we proceed to solve the monopolist seller problem considering these four possibilities.

Case I: $q_1^s < 2a^s < q_1^s + q_2^s$.

The maximization problem of the monopolist must be solved by backward induction; that is, we first solve for the monopolist's optimal choice in period two and then, given this optimal solution, we solve his problem in period one: find the q_1^s that maximizes the present value of his revenues. Finally, we determine the value of a^s that maximizes his profits.

At time j=2, taking into account (1) and given q_1^s, the monopolist solves:

$$\underset{q_2^s}{Max} \left(1 + ta^s - tq_1^s - tq_2^s\right)q_2^s$$

subject to $q_1^s + q_2^s > 2a^s$.

The solution to this problem is: $q_2^s = \dfrac{1 + ta^s - tq_1^s}{2t}$, $\dfrac{3ta^s - 1}{t} < q_1^s < 2a^s$.

At time j=1 the monopolist will solve for the q_1^s that maximizes the present value of his revenues, taking into account (1):

$$\underset{q_1^s}{Max}\left(\frac{3 + ta^s}{2} - tq_1^s\right)q_1^s + \left(\frac{1 + ta^s}{2} - \frac{tq_1^s}{2}\right)\left(\frac{1 + ta^s}{2t} - \frac{q_1^s}{2}\right)$$

From the first order condition of this problem we get: $q_1^s = \frac{2}{3t}$, $q_2^s = \frac{a^s}{2} + \frac{1}{6t}$, $\pi^s = \frac{7}{12t} + \frac{a^s}{2} + \frac{t(a^s)^2}{4}$, $\frac{1}{3t} < a^s < \frac{5}{9t}$. As a result, in case I the firm chooses the highest value of a^s and its profits have an upper limit in the value $\bar{\pi}^s = \frac{76}{81t}$.

Case II: $q_1^s + q_2^s < 2a^s$.

At $j = 2$, taking into account (1) and given q_1^s, the monopolist solves the following problem:

$$\underset{q_2^s}{Max}\left[1 - \frac{t}{2}(q_1^s + q_2^s)\right]q_2^s$$

From the first order condition of this problem we obtain: $q_2^s = \frac{1}{t} - \frac{q_1^s}{2}$, $q_1^s < \frac{4ta^s - 2}{t}$.

When $j = 1$ the monopolist will solve for the q_1^s that maximizes the present value of his revenues, taking into account (1):

$$\underset{q_1^s}{Max}\left(\frac{3}{2} - \frac{3tq_1^s}{4}\right)q_1^s + \left(\frac{1}{2} - \frac{tq_1^s}{4}\right)\left(\frac{1}{t} - \frac{q_1^s}{2}\right)$$

subject to $q_1^s < \frac{4ta^s - 2}{t}$.

The solution to this problem is: $q_1^s = \frac{4}{5t}$, $q_2^s = \frac{3}{5t}$, $\pi^s = \frac{0.9}{t}$, $\forall\ a^s > \frac{0.7}{t}$.

Case III: $q_1^s < 2a^s = q_1^s + q_2^s$.

The consistent schedule of production such that $q_1^s + q_2^s = 2a^s$ must satisfy the following conditions:

First, given q_1^s, the q_2^s in which the marginal revenue corresponding to the rental demand $p_2^s = 1 + ta^s - tq_1^s - tq_2^s$ is zero must be smaller than or equal to $2a^s - q_1^s$;

Second, given q_1^s, the q_2^s in which the marginal revenue corresponding to the rental demand $p_2^s = 1 - \frac{tq_1^s}{2} - \frac{tq_2^s}{2}$ is zero must be greater than or equal to $2a^s - q_1^s$, $\left(\frac{1}{3t} + \frac{q_1^s}{3} \leq a^s \leq \frac{1}{2t} + \frac{q_1^s}{4}\right)$.[3] Thus, at j=1 the monopolist solves the following problem:[4]

$$\underset{q_1^s,\, a^s}{Max} \left(2 - ta^s - \frac{tq_1^s}{2}\right)q_1^s + (1 - ta^s)(2a^s - q_1^s) + \lambda\left(\frac{3ta^s - 1}{3t} - \frac{q_1^s}{3}\right) + \mu\left(\frac{-2ta^s + 1}{2t} + \frac{q_1^s}{4}\right)$$

From the first order conditions of this problem we get the following solution:

$$a^s = \frac{8}{13t},\ q_1^s = \frac{11}{13t},\ q_2^s = \frac{5}{13t},\ \pi^s = \frac{25}{26t}.$$

Case IV: $q_1^s \geq 2a^s$.

At j=2 the monopolist solves the following problem:

$$\underset{q_2^s}{Max} \left(1 + ta^s - tq_1^s - tq_2^s\right)q_2^s$$

Then, $q_2^s = \frac{1 + ta^s - tq_1^s}{2t}$, $q_1^s \geq 2a^s$.

At time j=1 the monopolist will solve for the q_1^s that maximizes the present value of his revenues, taking into account (1):

$$\underset{q_1^s}{Max} \left(2 + 2ta^s - 2tq_1^s - tq_2^s\right)q_1^s + \left(1 + ta^s - tq_1^s - tq_2^s\right)q_2^s + \lambda\left(q_1^s - 2a^s\right)$$

subject to: $q_2^s = \frac{1 + ta^s - tq_1^s}{2t}$.

From the first order condition of this problem we get the following solution:

$$a^s = \frac{5}{11t},\ q_1^s = \frac{10}{11t},\ q_2^s = \frac{3}{11t},\ \pi^s = \frac{9}{11t}.$$

Comparing the solutions obtained in the different cases, the following proposition can be established:

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[3] $q_1^s < 2a^s$ implies that $\frac{1}{3t} + \frac{q_1^s}{3} < \frac{1}{2t} + \frac{q_1^s}{4}$.

[4] Given that the firm is a monopolist, the solution choosing first a^s and then q_1^s remains the same if a^s and q_1^s are chosen simultaneously.

Proposition 1: *If the monopolist seller cannot commit to a future schedule of production he will produce the variety* $a^{s*} = \frac{8}{13t}$, *and the quantities* $q_1^{s*} = \frac{11}{13t}$, $q_2^{s*} = \frac{5}{13t}$.

Proof: Comparing the profits corresponding to the different cases it is straightforward to verify that the optimal variety and schedule of production is such that: $a^s = \frac{8}{13t}$, $q_1^s = \frac{11}{13t}$, $q_2^s = \frac{5}{13t}$. ■

The monopolist who cannot commit to future production chooses to produce a variety such that he credibly commits not to reduce future prices drastically. This can be guaranteed by moving away from the central varieties and by deciding to produce a variety such that a strong reduction in future prices would allow the firm to attract fewer additional consumers that in the previous period.

In the first period the monopolist renter would produce a higher quantity than the monopolist seller, but finally, the accumulated quantity produced would be lower than the one chosen by the monopolist who can commit. Note also that both types of monopolists may produce the same variety. The choice of product variety for the monopolist has not been considered in the extensive literature on durable goods. However, as this paper shows, from the monopolist seller's point of view, the choice of product variety is a means at his disposal to mitigate his commitment problem. More precisely, the monopolist seller has an incentive to locate himself far enough from the central variety (a = 0,5) to get a lower residual demand for the second period. He chooses a variety that guarantees that the accumulated production is such that all consumers with a preferred variety lower than that chosen by the seller always buy the good. In this case, in order to produce an additional unit, the monopolist will have to drastically reduce the price because the potential buyers are located only on one side of the market. Although it increases the commitment ability, if we analyze the equilibrium variety chosen ($a^{s*} = 8/13t$), this drastic price reduction is never interesting from the monopolist seller's point of view.

3. Social Welfare: Endogenous versus Exogenous Demand

Social welfare may be defined as the sum of the present value of consumer surplus and monopolist's profits. Comparing social welfare under both types of monopolists, the following proposition can be established:

Proposition 2: *Social welfare is higher when the monopolist cannot commit to a future schedule of production than when he can.*

Proof: In the case of the monopolist seller, social welfare (W^s) is: $W^s = \frac{79}{52t}$. Social welfare in the case of the monopolist renter (W^r) is: $W^r = \frac{3}{2t}$. Therefore, $W^s > W^r$. ■

The result obtained in Proposition 2 is standard in the literature on durable goods when both linear demands and the assumption that the monopolist may only choose the quantities are considered simultaneously (e.g. Bulow (1982), Kahn (1986)). However, the literature shows that this result relies crucially on these assumptions (see for instance, Bulow (1982), Bond and Samuelson (1984), Bulow (1986) and Malueg and Solow (1989)). Malueg and Solow (1989), for example, show that the result that social welfare is higher when the monopolist cannot commit to a future schedule of production is not robust to changes from linear to kinked demands. This paper shows that Malueg and Solow's result might change when the monopolist is also allowed to choose the variety to be produced, that is, when the kinked demand is determined endogenously.

Malueg and Solow (1989) find that the existence of a kink in the rental demand may imply that the "social welfare may be raised or lowered by requiring the monopolist to sell, rather than rent, its output." In their analysis the kink and the shape of the demand curve are determined exogenously. The shape of the demand curve to the right of the kink is crucial to get different results: social welfare is higher when the monopolist sells the good than when he rents it if and only if the change in the shape of the rental demand is mild enough.

Malueg and Solow (1989) consider the following rental demand (figure 2):

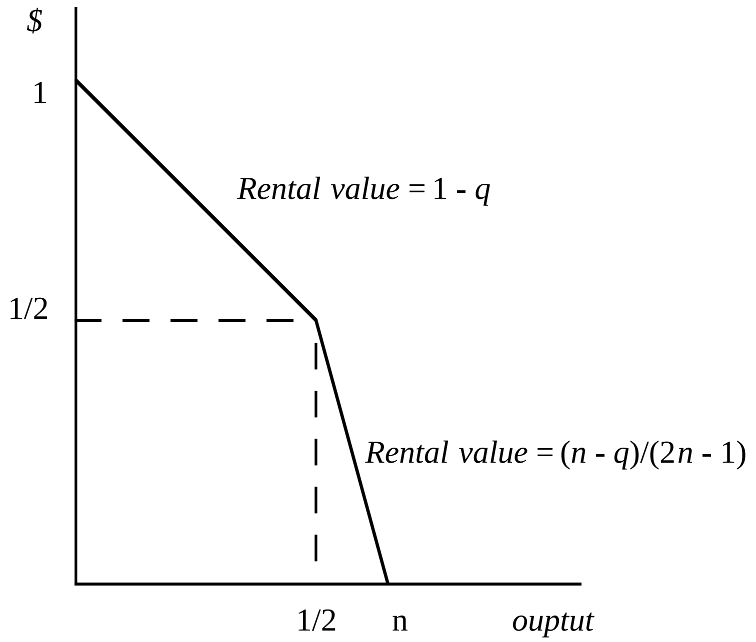

Figure 2: Malueg and Solow's rental demand.

Note, for instance, that when $n = 0.75$ their rental demand coincides with the one in our model when $t = 2$, product variety a is exogenous and $a = 0.25$. With these parameter values, as Malueg and Solow conclude, social welfare is higher when the monopolist can commit to a future schedule of production than when he cannot. However, when the product variety is a choice of the monopolist, social welfare will be greater for the monopolist seller than for the monopolist renter. In our model, for instance, for $t = 2$, the monopolist seller would choose $a^{s*} = 4/13$, and the corresponding social welfare would be $W^s = 79/104$. The monopolist renter would choose $a^{r*} \geq 0.25$, and social welfare would be $W^r = 0.75$. Therefore, social welfare is higher under the monopolist seller than under the monopolist renter. This example shows that the consideration of endogenous rental demands may play an important role in the implication that the ability or inability of commitment to a future schedule of production has for the analysis of social welfare in the durable goods monopolist literature.

4. Concluding Remarks

The choice of product variety by a durable goods monopolist can be very important from a strategic point of view. When the monopolist decides on the variety of the good to be produced, he may choose between situations in which the demand is high and the firm cannot commit not to flood the market with the product in the future and situations in which the demand is low but he can commit not to flood the market in the future.

Contrary to the case in which the monopolist who can commit produces a variety such that he has the highest demand (e.g. the central variety), the monopolist who cannot commit to a future schedule of production finds it more profitable to sell a variety of the durable good which allows him to mitigate his commitment problem. The reason is that with respect to the central variety choice, this solution generates new intertemporally consistent production schedules which increase monopolist seller profits. The monopolist chooses his product variety with a goal of making sure that a strong reduction in future prices will not be profitable because it allows the firm to attract few additional consumers.

This paper also shows that contrary to the case in which product variety is exogenously determined, under endogenous choice of product variety, social welfare is always higher when the monopolist cannot commit to future production levels.

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