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Abstract

This paper analyzes the process of endogenous union formation in the context of a sequential bargaining model between a firm and several unions and tries to explain why workers may be represented by several unions of different sizes. We show that the equilibrium number of unions and their relative size depend on workers' attitudes toward the risk of unemployment and union configuration is independent of labor productivity.

Keywords*: Endogenous union formation; Constant relative risk aversion; Sequential bargaining; Monopoly union.

1. Introduction

The labor market is not a perfectly competitive market. There may be considerable market power on the demand side and also on the supply side since workers group into unions to improve their bargaining position.

Previous work has analyzed different aspects of union configuration (De la Rica and Espinosa, 1997; Horn and Wolinsky, 1988; MacDonald and Solow, 1981; Manning, 1987c; Naylor, 1995; Nickell and Andrews, 1983; Oswald, 1985); however, most of the literature on the labor market assumes one or several unions but the number of unions is exogenously given and never determined in the model. In this paper we formalize the process of union formation and try to explain why workers may be represented by several unions instead of joining forces in a single union with more market power. More precisely, we look at labor productivity and workers' preferences as potential factors for the equilibrium number of unions and their relative size.

The formation of groups or coalitions in the presence of spillovers or externalities among coalitions has been the focus of attention of the literature on endogenous group formation and has been applied to various fields in economics like mergers in Cournot markets, partnership formation or international environmental agreements, among others (Bloch, 1995; Carraro and Siniscalco, 1998; Espinosa, and Macho-Stadler, 2003; Ray and Vohra, 1999; Salant, Switzer and Reynolds, 1983). This non cooperative theory of coalition formation uses the following framework: There is a first stage in which players take actions leading to a given group structure; when that coalition formation process is over, a non cooperative game is played and payoffs for each player depend on the group structure formed in the first stage. The fact that individual payoffs depend on the whole coalition structure is a consequence of the external effects (spillovers) that the formation of coalitions imposes on the other

players.

In this context, the present paper tries to contribute towards the understanding of the process of endogenous union formation. In our model, in the first stage workers participate in an open membership game of union formation (Yi and Shin, 1995; Yi, 1997; Belleflamme, 2000), i.e. each worker decides which union she wants to belong to, and no worker can be excluded from a union. After the number and composition of unions is so determined, the unions and the firm engage in a wage and employment determination game under the rules of the monopoly union model, that is, we consider a bilateral monopoly in the labor market in which unions make a "take-it-or-leave-it" offer concerning the wage and the firm decides employment from each union (Dunlop 1944; Farber, 1986; Oswald, 1985).

In the first part of the paper, we assume identical workers who will form unions that only differ in size. Later on, we analyze the case of workers that group into unions with different behavior in their bargaining with the firm (unions may be more or less "tough" in this bargaining). This game-theoretic perspective proves useful to explain the number and size of unions. We show that union configuration depends on workers' attitudes toward the risk of unemployment and it is independent of labor productivity.

The paper is organized as follows. Section 2 describes the model and the bargaining process. Section 3 solves the model under the assumption of symmetric workers. Section 4 assumes heterogeneous unions. Section 5 concludes with the main results.

2. The model

2.1. The union formation game

Assume n workers ($i \in N$) decide which unions they are going to form before bargaining with a firm. We formalize this process as a simultaneous open membership game. Each worker i announces x_i from the set $(1, 2, \dots n)$. All unions with a non-empty set of announcements are formed provided they reach the minimum size (which will be defined later on as a fraction of the total number of workers),

$$S_j = \{i \in N : x_i = j\}$$

As a result, we have a union structure:
$$\{S_1, S_2, \dots S_r\}, \quad r \leq n$$

These r unions will bargain with the firm.

2.2. The workers

Each worker i has a utility function that depends on wage (w) and employment (l):

$$V_i(w, l) = u_i(w)\tfrac{l}{t} + u_i(0)\left(1 - \tfrac{l}{t}\right)$$

where l/t is the probability of being employed; l is employment measured by the number of employed workers, t is the total number of workers (employed and unemployed) and reservation wage has been normalized to zero. We assume that the number of hours of work per period is not a decision variable but it is fixed; $u_i(w)$ is a concave function to represent risk aversion on workers' preferences. In particular, we will assume that workers have a constant relative risk aversion (CRRA) utility function (Blanchard and Fisher, 1989), so:

$$u_i(w) = \frac{w^{1-\sigma_i}}{1-\sigma_i}$$

where $\sigma_i \in (0,1)$ is the risk aversion coefficient of worker i; a greater value of σ_i indicates more risk

aversion.

With this formulation for $u_i(w)$, and normalizing $u_i(0) = 0$, the utility function for worker i can be expressed as:

$$V_i(w,l) = \left(\frac{w^{1-\sigma_i}}{1-\sigma_i}\right)\left(\frac{l}{t}\right)$$

(1)

2.3. The unions

When a union S_j is formed, the union maximizes the utility function of the median member:

$$U_j(w,l) = \frac{w_j^{\eta_j}}{\eta_j}\frac{l_j}{t_j}$$

(2)

where l_j is the number of members of union S_j who are employed; t_j is the total number of members of union S_j; $\eta_j = 1 - \sigma_j$ and σ_j is the CRRA coefficient of the median member of union S_j. A higher value of η_j corresponds to a higher preference for high wages, then, η_j is a measure of the "toughness" of union S_j in the bargaining with the firm.

2.4. The firm

The firm has a production function:

$$F(L) = \theta L - \frac{L^2}{2}$$

where L is the total number of workers employed by the firm, and θ is the productivity parameter.

The profits can be expressed as:

$$\pi(w,l) = pF(L) - wL$$

We will assume the firm behaves as a price taker in the product market, so that p is fixed, and it will be normalized to $p = 1$. The demand for labor is then:

$$L(w) = \theta - w$$

2.5. The bargaining process

From the union formation game we have a coalition structure $\{S_1, S_2, \dots S_r\}$. Assume for simplicity that unions are ordered so that $t_1 \geq t_2 \geq \dots \geq t_r$.

Wage and employment determination is formalized through a sequential game similar to the monopoly union model (Banerji, 2002; Chica and Espinosa, 2009; Dobson, 1994; Manning, 1987a). When several unions are formed we will assume that the firm deals with them sequentially so that it bargains with the largest union S_1 first, then with S_2, … and it bargains with S_r last.[1] Each union S_j announces a wage w_j and then, the firm decides on the employment for members of that union l_j. We assume that forming a union has a small fixed cost so that unions will have a minimum size. In particular, for a union to form, membership should be at least a fraction a of the total number of workers: $t_i \geq \alpha T,$ where $T = \sum_{i=1}^{r} t_i$ and $\alpha \in (0,1)$

3. Symmetric workers

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[1]It can be checked that the firm could not benefit from a change in this order.

In this section we will solve the model for the case $\sigma_i = \sigma$ for all $i \in N$, so that $\eta_j = \eta$ for all $j \in \{1, 2, ..., r\}$.

3.1. The equilibrium of the bargaining process

We solve the game backwards to calculate the subgame perfect equilibrium.

The profit function for the firm is:

$$\pi(w_1,\ldots,w_r,l_1,\ldots,l_r) = \left[\theta\left(\sum_{j=1}^{r} l_j\right) - \frac{\left(\sum_{j=1}^{r} l_j\right)^2}{2} \right] - \left(\sum_{j=1}^{r} w_j l_j\right)$$

When the firm bargains with union S_k its labor demand is given by the first order condition of the maximization problem:

$$\underset{l_k}{Max} \quad \left[\theta\left(\sum_{j=1}^{r} l_j\right) - \frac{1}{2}\left(\sum_{j=1}^{r} l_j\right)^2 \right] - \left(\sum_{j=1}^{r} w_j l_j\right)$$

$$s.t. \quad (l_1,\ldots,l_{k-1}) = \left(\overline{l}_1,\ldots,\overline{l}_{k-1}\right)$$

The first order condition of this maximization problem is:

$$\theta\left(1 + \sum_{j=k+1}^{r} \frac{\partial l_j}{\partial l_k}\right) - \left(\sum_{j=1}^{k-1} \overline{l}_j + l_k + \sum_{j=k+1}^{r} l_j\right)\left(1 + \sum_{j=k+1}^{r} \frac{\partial l_j}{\partial l_k}\right) - \left[w_k + \sum_{j=k+1}^{r} \frac{\partial(w_j l_j)}{\partial l_k}\right] = 0$$

So, when $k = r$: [2]

$$l_r = \theta - w_r - \sum_{j=1}^{r-1} \overline{l}_j \tag{3}$$

and union S_r should ask for a wage such that:

$$\underset{w_r}{Max} \quad \left(\frac{w_r^{\eta}}{\eta}\right)\left(\frac{l_r}{t_r}\right)$$
$$s.t. \quad (3)$$

The first order condition of this maximization problem is:

$$w_r^{\eta}(-1) + \eta w_r^{\eta-1}\left(\theta - w_r - \sum_{j=1}^{r-1} \overline{l}_j\right) = 0$$

which yields the optimal wage:

$$w_r = \frac{\eta\left(\theta - \sum_{j=1}^{r-1} \overline{l}_j\right)}{1 + \eta} \tag{4}$$

Substituting (4) in (3):

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[2] Note that this formulation is for $r > 1$; when $r = 1$: $w_r = \dfrac{\eta\theta}{(1+\eta)}$ and $l_r = \dfrac{\theta}{(1+\eta)}$

$$l_r = \frac{\theta - \sum_{j=1}^{r-1} \overline{l}_j}{\eta + 1} \tag{5}$$

Solving the game backwards, we obtain that in equilibrium union S_{r-h} asks for a wage: [3]

$$w_{r-h} = \frac{\eta^{h+1}(\eta + 2)^h \left(\theta - \sum_{j=1}^{(r-h)-1} \overline{l}_j \right)}{(1 + \eta)^{2h+1}} \tag{6}$$

with h: 0, 1, 2, ….$(r - 1)$

The employment level for union S_{r-h} is:

$$l_{r-h} = \frac{\theta - \sum_{j=1}^{(r-h)-1} \overline{l}_j}{1 + \eta} \tag{7}$$

Note that the higher the priority of the union (the higher h) the higher the wage and the employment level. This implies that with symmetric workers we may obtain in equilibrium more than one union only if the larger union bargains first: Being a member of a larger union implies a higher wage and a higher employment but, since membership is also larger, possibly a higher probability of being unemployed.

Substituting the equilibrium levels of employment, (6) and (7) can be expressed in terms of the parameters of the model as:

$$l_{r-h} = \theta \frac{\eta^{(r-h)-1}}{(1 + \eta)^{r-h}} \tag{8}$$

$$w_{r-h} = \theta \frac{\eta^r (2 + \eta)^h}{(1 + \eta)^{r+h}} \tag{9}$$

with h: 0, 1, 2, ….$(r - 1)$

Wage for union $(r-h)$ is negatively related to the number of unions, r, but it is higher the higher the toughness of the unions η and the priority of the union in the sequential process, h. Employment level for union $(r-h)$, however, is independent of the total number of unions, but higher the higher the priority of that union; moreover, in general, each union´s employment level it is positively related to the toughness of the unions, η, except for the first union to take part in negotiations (the union that gets the highest wages and the highest employment level). Therefore, the higher the priority of the union in negotiations, the higher both the wage and the employment level. This is due to the effect of sequential negotiation; in fact, it can be checked that the firm could not benefit form a change in this order.

On the other hand, total employment is positively related to the total number of unions, r

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[3] Note that h representes the priority in negotiations; for example, the priority for union r is 0 and for union 1 is $(r-1)$.

(although each union´s employment is not affected by this change), but it is lower the higher the toughness of the unions, η. That is, the more unions there are, the more competitive the labor market becomes, so that for a given level of employment, the lower the wage. Regarding the relationship between total employment and the toughness of the unions, the more aggressive the unions are the lower the total employment.

3.2. The equilibrium of the union formation game

From Subsection 3.1, we have that a worker i belonging to a union S_{r-h} with t_{r-h} members gets utility:

$$\frac{(w_{r-h})^{1-\sigma}}{1-\sigma} \frac{l_{r-h}}{t_{r-h}}$$

Substituting (8) and (9) we obtain utility as a function of the number of unions, r, the order of union in the negotiations, h, and membership t_{r-h}:

$$V_i(t_{r-h}, h, r) = \theta^{\eta+1} \frac{\eta^{r(\eta+1)-(h+2)}(2+\eta)^{h\eta}}{t_{r-h}(1+\eta)^{r(1+\eta)-h(1-\eta)}}$$

(10)

A Nash equilibrium coalition structure $(S_1, S_2, ... S_r)$ is such that no worker wishes to change unions. Since workers are identical, this means that the utility of any worker is the same no matter the union she belongs to. First, we consider that there are at most two unions. If all the workers have the same risk aversion, will they group into a single union? We assume that the firm always bargains first with the largest union. Thus, a worker i in union i with t_i members compares the utility in union 1 to that of union 2. We also suppose that to form the second union a minimum number of workers, l_2, is needed.[4] The firm does not bargain with individual workers.

Therefore, two unions can form in equilibrium if the following conditions hold:

$$V_i(t_1, 1, 2) = V_i(t_2, 0, 2)$$

(11)

$$t_1 \geq t_2 \quad \text{and}$$

$$t_2 \geq l_2 = \theta \frac{\eta}{(1+\eta)^2}$$

Substituting (8) and (9) into (11) we obtain the following result:

$$\frac{t_1}{t_2} = \frac{(\eta+2)^\eta(1+\eta)^{1-\eta}}{\eta}$$

(12)

Note that t_1 / t_2 does not depend on h nor r, and it is greater than one, so that, the first union is always larger than the second one.

Taking into account (12) and $T = t_1 + t_2$ we obtain the total number of members of each union:

$$t_1^* = \frac{(\eta+2)^\eta(1+\eta)^{1-\eta}}{\left[\eta+(\eta+2)^\eta(1+\eta)^{1-\eta}\right]}T$$

(13)

$$t_2^* = \frac{\eta}{\left[\eta+(\eta+2)^\eta(1+\eta)^{1-\eta}\right]}T$$

(14)

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[4]The minimun number of workers is chosen so as to avoid corner solutions.

Note that the size of the unions does not depend on the productivity parameter θ. However, for $t_2^* \geq l_2$ to hold:

$$\frac{\eta}{\left[\eta + (\eta + 2)^\eta (1 + \eta)^{1-\eta}\right]} T \geq \theta \frac{\eta}{(1 + \eta)^2}$$

Thus, for an interior solution with $r = 2$ we have to impose the following condition on the total number of unionized workers:

$$T \geq \frac{\theta\left[\eta + (\eta + 2)^\eta (1 + \eta)^{1-\eta}\right]}{(1 + \eta)^2} \tag{15}$$

From (13) and (14) we obtain the number of unemployed members at each union:

$$t_1^* - l_1 = \frac{(1 + \eta)(\eta + 2)^\eta (1 + \eta)^{1-\eta} T - \theta\left[\eta + (\eta + 2)^\eta (1 + \eta)^{1-\eta}\right]}{\left[\eta + (\eta + 2)^\eta (1 + \eta)^{1-\eta}\right](1 + \eta)} \tag{16}$$

$$t_2^* - l_2 = \frac{\eta(1 + \eta)^2 T - \theta\eta\left[\eta + (\eta + 2)^\eta (1 + \eta)^{1-\eta}\right]}{\left[\eta + (\eta + 2)^\eta (1 + \eta)^{1-\eta}\right](1 + \eta)^2} \tag{17}$$

It can be checked that unemployment is larger at the first union and also that the first union has a higher probability of unemployment.[5] The higher the productivity parameter, the lower the unemployment in each union.

We have obtained a Nash equilibrium of the open membership game with two unions. This is not, however, the only Nash equilibrium. We find that all workers in a single union is also a Nash equilibrium, since there is no profitable individual deviation (an individual deviation would yield a zero utility level). However, this equilibrium is not robust to group deviations: if a group of l_2 workers deviate and form another union their utility may increase, as long as the grand coalition is large enough, that is, if:

$$T > \frac{\theta}{(1 + \eta)^{1-\eta}\eta^\eta} \tag{18}$$

Then:

$$V_i(t_2 = l_2, 0, 2) > V_i(T, 0, 1)$$

This second union would ask for a lower wage so as to increase its employment probability.[6] Also, if a group of l_1 workers deviate to form another union their utility may increase also as long as T is large enough, in this case, if:

$$T > \theta \frac{(1 + \eta)^{2\eta - 1}}{(2 + \eta)^\eta \eta^\eta} \tag{19}$$

Then:

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[5]See Appendix A.I.
[6]See Appendix A.II.

$$V_i(t_1 = l_1, 1, 2) > V_i(T, 0, 1)$$

Condition (18) is more restrictive than condition (19), so that if condition (18) holds there would be at least two groups of members with incentives to deviate. Therefore, all workers in a single union is a strong Nash equilibrium only if T is not very large, in particular, if (19) does not hold.

Similarly, when we consider a maximum of r unions, for all of them to exist in equilibrium the utility of any worker has to be the same no matter the union she belongs to. Therefore, r unions can form in equilibrium if the following conditions hold:

$$V_i(t_{r-(h+1)}, h+1, r) = V_i(t_{r-h}, h, r) \qquad (20)$$

$$t_{r-(h+1)} \geq t_{r-h}$$

and

$$t_{r-h} \geq l_{r-h} = \theta \frac{\eta^{r-(h+1)}}{(1+\eta)^{r-h}}$$

From (10) and (20) we obtain:

$$\frac{t_{r-(h+1)}}{t_{r-h}} = \frac{(\eta+2)^\eta (1+\eta)^{1-\eta}}{\eta} \qquad (21)$$

For the sake of notational simplicity, we denote:

$$K = \frac{(\eta+2)^\eta (1+\eta)^{1-\eta}}{\eta}$$

Note that K is greater than one and negatively related to the toughness parameter, η.

Therefore $\frac{t_{r-(h+1)}}{t_{r-h}}$ decreases with η.

To find the equilibrium, we solve the following system:

$$t_{r-(h+1)} = t_{r-h}K \qquad \forall h : 0, 1, \ldots, (r-1)$$

$$T = t_1 + t_2 + \ldots t_r$$

Thus, in equilibrium, the number of members of each union r-h is given by:

$$t^*_{r-h} = \frac{(K)^h}{\sum\limits_{j=0}^{r-1}(K)^j} T \qquad (22)$$

as long as:

$$\frac{1}{\sum\limits_{j=0}^{r-1}(K)^j} \geq \alpha.$$

Hence, the equilibrium number of unions is:

$$r^* = \max. \; r \text{ such that } \left[\frac{1}{\sum_{j=0}^{r-1}(K)^j} \right] \geq \alpha$$

(23)

We can now state our main result:

Result 1. When unions maximize the utility of the representative worker, the equilibrium number of unions is decreasing in the workers' aversion to the risk of unemployment, σ. The lower the parameter α, which represents the cost of forming a union, the larger the equilibrium number of unions. Union configuration is independent of the productivity parameter θ.

The equilibrium configuration of r^* unions of $t_{r-h}{}^*$ members, is not the unique Nash equilibrium; the grand coalition is also a trivial Nash equilibrium since a worker cannot deviate and form a union by himself. However, the equilibrium configuration of r^* unions of $t_{r-h}{}^*$ members is not a strong Nash equilibrium because the utility of a representative member of union $t_{r-h}{}^*$ will increase if unions form the grand coalition. As shown in the case of $r = 2$, there may be other Nash equilibria with lower r but they are not robust to group deviations.

4. Heterogeneous Unions
In this section we assume that unions have their own preferences $\eta_j \in (0, 1)$, $j \in \{1,..., r\}$, and do not maximize the utility of the median worker. The parameter η_j is a measure of the toughness of union S_j in the bargaining with the firm and unions may be different in this respect.

4.1. The equilibrium of the bargaining process
Similarly to section 3.1. we solve the game backwards to calculate the subgame perfect equilibrium. Thus, we obtain that in equilibrium union S_{r-h} with a toughness η_{r-h} asks for a wage:

$$w_{r-h} = \theta \prod_{j=1}^{r-h} \left[\frac{\eta_j}{(1+\eta_j)} \right] \prod_{j=r-(h-1)}^{r} \left[\frac{\eta_j(2+\eta_j)}{(1+\eta_j)^2} \right] \qquad r\text{-}h \neq r$$

(24)

$$w_{r-h} = \theta \prod_{j=1}^{r-h} \frac{\eta_j}{(1+\eta_j)} \qquad r\text{-}h = r$$

and gets employment:

$$l_{r-h} = \frac{\theta}{\eta_{r-h}} \prod_{j=1}^{r-h} \frac{\eta_j}{(1 + \eta_j)}$$

(25)

The equilibrium wage for union $(r-h)$ is positively related to its priority in negotiations h, and negatively to the number of unions at the firm r. In this case, it is also positively related to both its toughness η_{r-h} and other unions' toughness in negotiations with the firm η_j.
Similarly, employment for union $(r-h)$ does not depend on the total number of unions r and is positively related to its priority in negotiations h. In this case, however, for all unions $(r-h)$ the equilibrium employment level is negatively related to its toughness η_{r-h} but given $l_{r-(h+1)}$, positively related to the other unions' toughness.
Concerning total employment, it increases with the total number of unions, r (reflecting a more competitive labor market), but it is lower the higher the toughness of the unions, η_i (in fact, the effect of the unions' toughness on total employment is the same regardless the priority of the union).

4.2. The equilibrium of the union formation game

When unions have different behavior in their bargaining with the firm, a worker i belonging to a union S_{r-h} with t_{r-h} members gets utility:

$$V_i(t_{r-h}, h, r) = \frac{(w_{r-h})^{1-\sigma_i}}{1 - \sigma_i} \frac{l_{r-h}}{t_{r-h}} \tag{26}$$

Substituting (24) and (25) in (26) we have:

$$V_i(t_{r-h}, h, r) = \frac{\left[\prod_{j=r-(h-1)}^{r} \left[\frac{\eta_j(2+\eta_j)}{(1+\eta_j)^2} \right] \right]^{1-\sigma_i} \left[\theta \prod_{j=1}^{r-h} \frac{\eta_j}{(1+\eta_j)} \right]^{2-\sigma_i}}{(1 - \sigma_i)\eta_{r-h} t_{r-h}} \tag{27}$$

Note that a Nash equilibrium coalition structure $(S_1, S_2, ... S_r)$ is such that no worker wishes to change unions. For the sake of simplicity we assume that all workers have the same risk aversion, that is, $\sigma_i = \sigma$.

We consider, first, that there are two unions. We assume that the firm always bargains first with the largest union. Thus, a worker i in a union S_{r-h} with t_{r-h} members compares the utility of belonging to union 1 to that of belonging to union 2. We also suppose that to form the second union at least l_2 members are needed (defined below). Therefore, to have two unions in equilibrium the following condition should hold:

$$V_i(t_1, 1, 2) = V_i(t_2, 0, 2) \tag{28}$$

$$t_1 \geq t_2$$

and

$$t_2 \geq l_2 = \frac{\theta \eta_1}{(1 + \eta_1)(1 + \eta_2)}$$

Substituting (24) and (25) in (28) we obtain the following result:[7]

$$\frac{t_1}{t_2} = \frac{(2 + \eta_2)^{1-\sigma}(1 + \eta_2)^{\sigma}}{\eta_1} \tag{29}$$

Note that t_1 / t_2 does not depend on r.

Taking into account (29) and $T = t_1 + t_2$ we obtain the number of members of each union:

$$t_1 = \frac{(2 + \eta_2)^{1-\sigma}(1 + \eta_2)^{\sigma}}{\left[\eta_1 + (2 + \eta_2)^{1-\sigma}(1 + \eta_2)^{\sigma} \right]} T \tag{30}$$

$$t_2 = \frac{\eta_1}{\left[\eta_1 + (2 + \eta_2)^{1-\sigma}(1 + \eta_2)^{\sigma} \right]} T \tag{31}$$

To check whether the condition $t_1 \geq t_2$ holds, suppose that η_1 is very large so that

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[7]See Appendix B.I.

$\eta_1 > (1 + \eta_2)^\sigma (2 + \eta_2)^{1-\sigma}$, then $t_2 > t_1$. Therefore, the first union in negotiation (the largest according to our asumption) cannot be much tougher than the second, that is:

$$t_1 \geq t_2 \quad if \quad \eta_1 \leq (1 + \eta_2)^\sigma (2 + \eta_2)^{1-\sigma}$$

That is, since workers are identical, unions cannot be too different for an equilibrium configuration to exist. When the two unions have the same behavior in bargaining, $\eta_1 = \eta_2 = \eta$, we have that $t_2 \geq t_1 \quad if$ $\eta \geq (1 + \eta)^\sigma (2 + \eta)^{1-\sigma}$. It can be checked that for $\sigma \in (0, 1)$ and $\eta \in (0, 1)$ this inequality does not hold. Thus, in equilibrium there will be a big union (union 1) and a small union (union 2).

Moreover, for an interior solution and $r = 2$ we have to impose the following condition on the total number of workers:

$$T \geq \frac{\theta\left[\eta_1 + (2 + \eta_2)^{1-\sigma}(1 + \eta_2)^\sigma\right]}{(1 + \eta_1)(1 + \eta_2)} \tag{32}$$

Taking into account (30) and (31) the number of unemployed members at each union is given by:

$$t_1 - l_1 = \frac{(1 + \eta_2)^\sigma (2 + \eta_2)^{1-\sigma}(1 + \eta_1)T - \theta\left[\eta_1 + (1 + \eta_2)^\sigma (2 + \eta_2)^{1-\sigma}\right]}{\left[\eta_1 + (1 + \eta_2)^\sigma (2 + \eta_2)^{1-\sigma}\right](1 + \eta_1)} \tag{33}$$

$$t_2 - l_2 = \frac{T\eta_1(1 + \eta_1)(1 + \eta_2) - \eta_1\theta\left[\eta_1 + (1 + \eta_2)^\sigma (2 + \eta_2)^{1-\sigma}\right]}{\left[\eta_1 + (1 + \eta_2)^\sigma (2 + \eta_2)^{1-\sigma}\right](1 + \eta_1)(1 + \eta_2)} \tag{34}$$

Unemployment is larger at the first union.[8]

We have obtained a Nash equilibrium of the open membership game with two unions. In equilibrium all the workers enjoy the same expected utility level. The members of the first union have higher wages but also a higher probability of unemployment. This is not, however, the only Nash equilibrium. We find that all workers in the first union is also a Nash equilibrium, since there is not a profitable individual deviation (an individual deviation would yield a zero utility level). However, this equilibrium is not robust to group deviations: if a group of l_2 workers deviate and form another union their utility may increase, as long as the grand coalition is large enough, that is, if:

$$T > \frac{\left(1 + \eta_2\right)^{1-\sigma}}{\left(\eta_2\right)^{1-\sigma}\left(1 + \eta_1\right)}\theta \tag{35}$$

Then:

$$V_i(T, 0, 1) \leq V_i(t_2, 0, 2)$$

Therefore, if the grand coalition is large enough, a group of workers form a second union that asks for a lower wage and increases the probability of employment.[9] We are assuming that the firm

bargains first with the largest union, but when unions differ in their parameter η_i, it is possible that the firm is interested in negotiating first with the smaller union either because it has lower or a higher η. It can be checked that whether it bargains first with the most aggressive or the less aggressive union, the firm supports the same labor costs.

Similarly, when we consider r unions, for all of them to form in equilibrium, the utility of any worker has to be the same no matter the union she belongs to. The necessary conditions for an r-union configuration are:

$$V_i(t_{r-(h+1)}, h+1, r) = V_i(t_{r-h}, h, r) \tag{36}$$

$$t_{r-(h+1)} \geq t_{r-h}$$

and

$$t_{r-h} \geq l_{r-h} = \frac{\theta}{\eta_{r-h}} \prod_{j=1}^{r-h} \frac{\eta_j}{(1+\eta_j)}$$

From (36) we obtain:

$$\frac{t_{r-(h+1)}}{t_{r-h}} = \frac{(2+\eta_{r-h})^{1-\sigma}(1+\eta_{r-h})^{\sigma}}{\eta_{r-(h+1)}} \tag{37}$$

For the sake of notational simplicity, we denote:

$$k_{r-(h+1)} = \frac{(2+\eta_{r-h})^{1-\sigma}(1+\eta_{r-h})^{\sigma}}{\eta_{r-(h+1)}} \tag{38}$$

It can be checked that $k_{r-(h+1)}$ is greater than one and negatively related to the workers' aversion to the risk of unemployment, σ. Regarding the toughness of the unions, $k_{r-(h+1)}$ decreases with $\eta_{r-(h+1)}$ but it is positively related to η_{r-h}. Nervertheless, when the toughness of all unions increases the parameter $k_{r-(h+1)}$ decreases.

To find the equilibrium, we solve the following system:

$$t_{r-(h+1)} = t_{r-h} k_{r-(h+1)} \qquad \forall h : 0, 1, \dots, (r-1)$$

$$T = t_1 + t_2 + \dots t_r$$

We obtain that for any number of unions r, the number of members of union r-h is given by:

$$t_{r-h} = \frac{\prod_{\substack{i=r-h \\ i \neq r}}^{r-1} k_i}{\sum_{i=1}^{r-1}\left[\prod_{j=i}^{r-1} k_j\right]+1} T \qquad r - h \neq r$$

$$t_{r-h} = \frac{1}{\sum_{i=1}^{r-1}\left[\prod_{j=i}^{r-1} k_j\right]+1} T \qquad r - h = r$$

as long as

$$\frac{1}{\sum_{i=1}^{r-1}\left[\prod_{j=i}^{r-1} k_j\right]+1} \geq \alpha.$$

We can now determine the equilibrium number of unions:

$$r^* = \text{max. } r \text{ such that } \left[\frac{1}{\sum_{i=1}^{r-1}\left(\prod_{j=i}^{r-1}k_j\right)+1}\right] \geq \alpha$$

To conclude, we state the main result of this section:

Result 2. When unions do not maximize the utility of the median worker but have their own preferences, the lower the workers' aversion to the risk of unemployment, σ, the lower the equilibrium number of unions. The equilibrium number of unions decreases with the parameter α, which represents the cost of forming a union, and increases with the toughness of the unions.

5. Concluding Remarks

In this paper we analyze endogenous union formation in the context of a sequential bargaining model between a firm and several unions, under the rules of the monopoly union model. We deal with the process of union formation and try to explain why workers may end up being represented by one or several unions. The characteristics of the market and the workers' preferences are potential factors in the determination of the number of unions and their relative size. We show that union configuration depends on workers' attitudes toward the risk of unemployment but it is independent of labor productivity.

In the first part of the paper we assume that workers have the same aversion to the risk of unemployment and that once a union is formed the union maximizes the utility function of the median member. As a result, we obtain that unions will only differ on size. In this case, the lower the workers' aversion to the risk of unemployment, the higher the toughness of the unions and the larger the equilibrium number of unions.

In the second part we analyze the case of workers that group into unions with different preferences in their bargaining with the firm, that is, unions may be more or less "tough" in their bargaining, but they do not maximize the utility of the median worker. This assumption on union' behaviour yields the opposite result: the number of unions is increasing with the workers' aversion to the risk of unemployment.

In both cases the higher the toughness of the unions the higher the equilibrium number of unions and the smaller their sizes. The cost of forming a union, has always a negative effect on the equilibrium number of unions.

We have ignored many interesting aspects of the labor market like the comparison of patterns of bargaining (simultaneous or sequential) and institutional environments (different rules of the negotiation process). A next step is to formalize the question of endogenous union formation under the assumption of the Nash bargaining rules, and to check in which direction simultaneous negotiations could change the results. We leave this analysis for further research.

Appendix A (Homogeneous unions)

A.I. Unemployment is larger in the first union if:

$$(t_1 - l_1) \geq (t_2 - l_2)$$

$$(1 + \eta)(\eta + 2)^\eta (1 + \eta)^{1-\eta} T - \theta\big[\eta + (\eta + 2)^\eta (1 + \eta)^{1-\eta}\big]$$

$$\geq \frac{\eta(1 + \eta)^2 T - \theta\eta\big[\eta + (\eta + 2)^\eta (1 + \eta)^{1-\eta}\big]}{1 + \eta}$$

$$T \geq \frac{\theta\big[(\eta + 2)^\eta (1 + \eta)^{1-\eta} + \eta\big]}{\big[(\eta + 2)^\eta (1 + \eta)^{1-\eta} - \eta\big](1 + \eta)^2} \tag{A.1}$$

For $t_2 \geq l_2$ and $r = 2$ we have to impose the following condition:

$$T \geq \frac{\theta\big[\eta + (\eta + 2)^\eta (1 + \eta)^{1-\eta}\big]}{(1 + \eta)^2} \tag{A.2}$$

Taking into account (A.2.) we have that:

$$\frac{\theta\big[(\eta + 2)^\eta (1 + \eta)^{1-\eta} + \eta\big]}{(1 + \eta)^2} > \frac{\theta\big[(\eta + 2)^\eta (1 + \eta)^{1-\eta} + \eta\big]}{\big[(\eta + 2)^\eta (1 + \eta)^{1-\eta} - \eta\big](1 + \eta)^2}$$

Then (A.1) holds and unemployment in the first union is larger than in the second one.

A.II. If there is a single union, the utility of worker i would be:

$$V_i(T, 0, 1) = \frac{\theta^{\eta+1} \eta^{\eta-1}}{(1 + \eta)^{\eta+1} T}$$

There would be a deviation of t_2 if:

$$V_i(t_2, 0, 2) > V_i(T, 0, 1)$$

That is, if:

$$t_2 < \frac{\eta^{\eta+1}}{(1 + \eta)^{\eta+1}} T \tag{A.3}$$

In the case of the best deviation, if l_2 workers change union to form union 2, the utility of the representative worker i would be:

$$V_i(t_2, 0, 2) = \theta^\eta \frac{\eta^{2\eta-1}}{(1 + \eta)^{2\eta}}$$

The utility in the second union is higher if:

$$T > \frac{\theta}{(1 + \eta)^{1-\eta} \eta^\eta} \tag{A.4}$$

Similarly, there would be a deviation of t_1 workers if:

$$V_i(t_1, 1, 2) > V_i(T, 0, 1)$$

that is, if:

$$t_1 < \frac{\eta^\eta (2 + \eta)^\eta}{(1 + \eta)^{2\eta}} T$$

(A.5)

More precisely, if l_1 workers change union and form union 1, the utility of a worker i would be:

$$V_i(t_1, 1, 2) = \theta^\eta \frac{\eta^{2\eta-1}(2 + \eta)^\eta}{(1 + \eta)^{3\eta}}$$

Utility in union 1 is higher if:

$$T > \theta \frac{(1 + \eta)^{2\eta-1}}{(2 + \eta)^\eta \eta^\eta}$$

(A.6)

Condition (A.4) is more restrictive than condition (A.6) and condition (A.3) and condition (A.5) cannot hold simultaneously.

APPENDIX B (Heterogeneous unions)

B.I. If there are two unions with t_1 and t_2 members, the utility that a worker obtains in union 1 is given by:

$$V_1 = \frac{1}{(1 - \sigma)} \left[\frac{\eta_1 \eta_2 (2 + \eta_2)\theta}{(1 + \eta_1)(1 + \eta_2)^2} \right]^{1-\sigma} \frac{\theta}{(1 + \eta_1)t_1}$$

Denoting

$$A = \frac{\eta_1 \eta_2 (2 + \eta_2)}{(1 + \eta_1)(1 + \eta_2)^2}$$

$$V_1 = \frac{1}{(1 - \sigma)} [\theta A]^{1-\sigma} \frac{\theta}{(1 + \eta_1)t_1}$$

(B.1)

Thus, the utility that the worker obtains in union 2 is:

$$V_2 = \frac{1}{(1 - \sigma)} \left[\frac{\eta_1 \eta_2 \theta}{(1 + \eta_1)(1 + \eta_2)} \right]^{1-\sigma} \frac{\eta_1 \theta}{(1 + \eta_1)(1 + \eta_2)t_2}$$

Denoting

$$B = \frac{\eta_1 \eta_2}{(1 + \eta_1)(1 + \eta_2)}$$

$$V_2 = \frac{[B\theta]^{2-\sigma}}{(1 - \sigma)\eta_2 t_2}$$

(B.2)

We can check that:

$$A = B\frac{(2+\eta_2)}{(1+\eta_2)}.$$

Substituiting A in (B.1):

$$V_1 = \left[\frac{(2+\eta_2)}{(1+\eta_2)}\right]^{1-\sigma} \frac{B^{1-\sigma}\theta^{2-\sigma}}{(1-\sigma)(1+\eta_1)t_1}$$

(B.3)

As workers are identical, to obtain two unions in equilibrium we need:

$$V_i(t_1,1,2) = V_i(t_2,0,2)$$

$$\left[\frac{(2+\eta_2)}{(1+\eta_2)}\right]^{1-\sigma} \frac{1}{(1+\eta_1)t_1} = \frac{B}{1-\eta_2 t_2}$$

Substituiting B we get:

$$\frac{t_1}{t_2} = \frac{(1+\eta_2)^\sigma (2+\eta_2)^{1-\sigma}}{\eta_1}$$

(B.4)

B.II. Unemployment at the first union is higher than at the second if:

$$(t_1 - l_1) \geq (t_2 - l_2)$$

$$\left[(1+\eta_2)^\sigma(2+\eta_2)^{1-\sigma}(1+\eta_1)T - \theta\left[\eta_1 + (1+\eta_2)^\sigma(2+\eta_2)^{1-\sigma}\right]\right](1+\eta_2)$$
$$\geq T\eta_1(1+\eta_1)(1+\eta_2) - \eta_1\theta\left[\eta_1 + (1+\eta_2)^\sigma(2+\eta_2)^{1-\sigma}\right]$$

$$T \geq \theta\frac{\left[(1+\eta_2) - \eta_1\right]\left[(1+\eta_2)^\sigma(2+\eta_2)^{1-\sigma} + \eta_1\right]}{(1+\eta_1)(1+\eta_2)\left[(1+\eta_2)^\sigma(2+\eta_2)^{1-\sigma} - \eta_1\right]}$$

(B.5)

For $t_2 \geq l_2$ and $r = 2$ we have to impose the following condition:

$$T \geq \frac{\theta\left[\eta_1 + (2+\eta_2)^{1-\sigma}(1+\eta_2)^\sigma\right]}{(1+\eta_1)(1+\eta_2)}$$

(B.6)

Taking into account (B.6) we have that if:

$$\frac{\theta\left[\eta_1 + (2+\eta_2)^{1-\sigma}(1+\eta_2)^\sigma\right]}{(1+\eta_1)(1+\eta_2)} \geq \theta\frac{\left[(1+\eta_2) - \eta_1\right]\left[(1+\eta_2)^\sigma(2+\eta_2)^{1-\sigma} + \eta_1\right]}{(1+\eta_1)(1+\eta_2)\left[(1+\eta_2)^\sigma(2+\eta_2)^{1-\sigma} - \eta_1\right]}$$

$$(1+\eta_2)^\sigma(2+\eta_2)^{1-\sigma} \geq (1+\eta_2)$$

For $\sigma \in (0, 1)$ and $\eta_i > 0$ the previous inequality holds. Therefore, (B.5) holds and so unemployment level at the first union is larger than at the second one.

B.III. If there is a single union, the utility of worker i would be:

$$V_i(T, 0, 1) = \frac{\left[\theta \frac{(\eta_1)}{(1+\eta_1)}\right]^{1-\sigma}}{1-\sigma} \frac{\theta}{(1+\eta_1)} \frac{1}{T}$$

If l_2 workers change union and form union 2 the utility of a worker i would be:

$$V_i(t_2, 0, 2) = \frac{\left[\theta \frac{\eta_1 \eta_2}{(1+\eta_1)(1+\eta_2)}\right]^{1-\sigma}}{1-\sigma}$$

And the utility in the second union is larger, that is:

$$V_i(T, 0, 1) \leq V_i(t_2, 0, 2)$$

$$T > \frac{(1+\eta_2)^{1-\sigma}}{(\eta_2)^{1-\sigma}(1+\eta_1)}\theta \tag{35}$$

References

[1]   Banerji, A., 2002. "Sequencing Strategically: Wage Negotiations Under Oligopoly", *International Journal of Industrial Organization* 20, 1037-1058.

[2]   Belleflamme, P., 2000. "Stable Coalition Structures with Open Membership and Asymmetric Firms", *Games and Economic Behavior* 30, 1-21.

[3]   Blanchard, O. J., and S. Fisher, 1989. *Lectures on Macroeconomics*, The Mit Press, Cambridge.

[4]   Bloch, F., 1996. "Sequential Formation of Coalitions in Games with Externalities and Fixes Payoff Division". *Games and Economic Behavior* 14, 90-123.

[5]   Carraro, C., and Siniscalco, D., 1998. "International Environmental Agreements: Incentives and Political", *European Economic Review* 42(3-5), 561-72.

[6]   Chica, Y. and Espinosa, M. P., 2009. "Union Formation and Bargaining Rules in the Labor Market", *European Journal of Social Sciences*, 8(1), 88-106.

[7]   De la Rica, S., and Espinosa, M. P., 1997. "Testing Employment Determination in Unionised Economies as a Repeated Game", *Scottish Journal of Political Economy* 44(2), 134-52.

[8]   Dobson, P. W., (1994). "Multifirm Unions and the Incentive to Adopt Pattern Bargaining in Oligopoly", *European Economic Review* 38, 87-100.

[9]   Dunlop, J. T., 1944. Chap. III in Wage Determination under Trade Unions.

[10] Espinosa, M.P., and I. Macho-Stadler, 2003. "Endogenous Formation of Competing Partnerships with Moral Hazard", *Games and Economic Behavior* 44, 172-183.

[11] Espinosa, M. P., and Rhee, C., 1989. "Efficient Wage Bargaining as a Repeated Game", *Quarterly Journal of Economics* 104(3), 565-88.

[12] Farber, H., 1986. The Analysis of Union Behaviour, in O., Ashenfelter, and R., Layard, (eds.), Handbook of Labour Economics, Amsterdam, North-Holand.

[13] Horn, H. and A. Wolinsky, 1988. "Worker Substitutability and Patterns of Unionisation", *Economic Journal* 98, 484-497.

[14] MacDonald, I. M., and Solow, R. M., 1981. "Wage Bargaining and Employment", *American Economic Review*, p. 896-908.

[15] Manning, A., 1987a. "An Integration of Trade Union Models in a Sequential Bargaining Framework", *Economic Journal* 97, 121-139.

[16] Manning, A., 1987c. "Collective Bargaining Institutions and Efficiency", *European Economic Review* 31, 168-176.

[17] Naylor, R. A., 1995. "On the Economic Effects of Multiple Unionism", *Scandinavian Journal of Economics* 97 (1), 161-167.

[18] Nickell, S.J. and M. Andrews, 1983. "Unions, real wages and employment in Britain 1951-1979", *Oxford Economic Papers* 35, suplemento, 183-206.

[19] Oswald, A. J., 1985. "The Economic Theory of Trade Union: An Introductory Survey", *Scandinavian Journal of Economics* 87 (2), 160-193.

[20] Ray, D. and R. Vohra, 1999. "A Theory of Endogenous Coalition Structures", *Games and Economic Behavior* 26, 286-336.

[21] Salant, S. W., Switzer, S., and Reynolds, R. J., 1983. "Losses from horizontal merger: The effects of an exogenous change in industry structure on cournot-Nash equilibrium", *The Quarterly Journal of Economics*.

[22] Yi, S.-S., 1997. "Stable Coalition Structures with Externalities", *Games and Economic Behavior* 20 (2), 201-237.