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A note on collusion sustainability with optimal punishments and detection lags

A note on collusion sustainability with optimal punishments and detection lags*

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Abstract

In this note we characterize optimal punishments with detection lags when the market consists of n oligopolistic firms. We extend a previous note by Colombo and Labrecciosa (2006) [Colombo, L., and Labrecciosa, P., 2006. Optimal punishments with detection lags. Economic Letters 92, 198-201] to show how in the presence of detection lags optimal punishments fail to restore cooperation also in markets with a low number of firms.

Keywords: Optimal punishments; Detection lags; Collusion sustainability.

JEL Classification: C73; D43

1 Introduction

Strategic interaction plays an important role to determine the sustainability of cartel agreements. Friedman (1971) stressed the importance of time for collusion sustainability under trigger strategies profile. Abreu (1986, 1988) also characterized optimal punishments in supergames. He shown that a symmetric optimal penal code yields the lowest critical discount factor such that collusion can be sustained. Moreover, Abreu et al. (1986) and Fudenberg and Maskin (1986) characterize folk theorems for infinitely repeated games with discounting.

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Colombo and Labrecciosa (2006) describe in a previous note an optimal penal code with n detection lags in duopoly supergames. They attemp to show how large detection lags after deviation hinders collusion. They only provide examples with one period detection lag under Bertrand and Cournot competition.

This note characterizes collusion sustainability with detection lags in a n-firm oligopoly. We extend the result by Colombo and Labrecciosa (2006) showing how optimal punishments fail to restore cooperation for a sufficiently large detection lag. We show that there is a trade-off between the amount of detection lags and the maximum number of firms compatible with collusion sustainability.

2 The model

Consider an infinitely repeated n-firm game ($n \geq 2$) with discounting δ. Call such game $G^l(N, \infty, \delta)$ where the superscript denotes that deviation is detected after l period(s); that is, we assume that an individual deviation from any collusive agreement is detected by the rest $n-1$ firms after l period(s). Following Colombo and Labrecciosa (2006) firms agree upon the following penal code: If a deviation occurred at any given time τ, it is detected at the end of the period $\tau+l$, then firms adopt the punishment strategy symmetrically at time $\tau+l+1$; finally, they restore cooperation from period $\tau+l+2$ ownwards. If, otherwise, any firm at time $\tau+l+1$ does not join the penal code and decides not to retaliate, the punishment phase continues until the same action is taken by the n firms.

We call the best collusive strategy $s^* = (s_1^*, .., s_i^*, .., s_n^*)$, and the optimal punishment strategy $s^c = (s_1^c, .., s_i^c, .., s_n^c)$. $\pi^d(s^*)$ are deviation profits from the best collusive profits $\pi(s^*)$, $\pi(s^c)$ are the profits during the punishment phase, and $\pi^d(s^c)$ are optimal deviation profits from the punishment phase. Then, following Abreu's theorem 15 and Colombo and Labrecciosa's (2006) expressions (1) and (2), we state that,

Proposition 1 *Let (s^*, s^c) be the optimal stick-and-carrot punishment with an l period detection lag. Then the following is hold,*

$$\sum_{t=0}^{l} \delta^t \left[\pi^d(s^*) - \pi(s^*)\right] \leq \delta^{l+1}\left[\pi(s^*) - \pi(s^c)\right], \quad (1)$$

$$\sum_{t=0}^{l} \delta^t \left[\pi^d(s^c) - \pi(s^c)\right] = \sum_{t=1}^{l+1} \delta^t \left[\pi(s^*) - \pi(s^c)\right], \quad (2)$$

where l is the detection lag.[1]

[1]When $l = 0$ and the assumption that $s^* = s^m$ where s^m is the monopoly outcome, theorem 15 of Abreu (1986, p.203) is obtained. Notice also that Proposition 1 by Colombo and Labrecciosa (2006) should be applied only in the case of zero detection lags.

Proof. Consider first expression (1) by Colombo and Labrecciosa (2006). Rewritting $\sum_{t=0}^{l} \pi(s^*)$ into $\sum_{t=0}^{l} \pi(s^*) + \sum_{t=l+1}^{\infty} \pi(s^*)$ we obtain,

$$\sum_{t=0}^{l} \delta^t \left[\pi^d(s^*) - \pi(s^*)\right] \leq \sum_{t=l+1}^{\infty} \delta^t \pi(s^*) - \sum_{t=l+2}^{\infty} \pi(s^*) - \delta^{l+1}\pi(s^c),$$

where the left hand side reach the ones at our equation (1). Now, taking the terms $\sum_{t=l+1}^{\infty} \delta^t \pi(s^*) - \sum_{t=l+2}^{\infty} \pi(s^*)$ on the right hand side we obtain $\delta^{l+1}\pi(s^*)$ which jointly with $\delta^{l+1}\pi(s^c)$ fits the right hand side of equation (1) in Proposition 1. Now, consider expression (2) by Colombo and Labrecciosa (2006). After adding $-\sum_{t=1}^{l+1} \delta^t \pi(s^*)$ in both sides and reordering we obtain

$$\begin{gathered}\sum_{t=0}^{l} \delta^t \pi^d(s^c) - \sum_{t=1}^{l+1} \delta^t \pi(s^c) - \pi(s^c) + \delta^{l+1}\pi(s^c) = \\ \sum_{t=1}^{\infty} \delta^t \pi(s^*) - \sum_{t=l+2}^{\infty} \delta^t \pi(s^*) - \sum_{t=1}^{l+1} \delta^t \pi(s^c).\end{gathered}$$

With the last three terms on the left hand side we can obtain $\sum_{t=0}^{l} \delta^t \pi(s^c)$. Then, adding it to the first term we get the left hand side on equation (2) in Propostion 1. Finally, take $\sum_{t=1}^{\infty} \delta^t \pi(s^*) - \sum_{t=l+2}^{\infty} \delta^t \pi(s^*)$ on the right hand side to obtain $\sum_{t=1}^{l+1} \delta^t \pi(s^*)$. It must be added to $\sum_{t=1}^{l+1} \delta^t \pi(s^c)$ to get the desired expression on the right hand side of equation (2). This completes the proof. ■

Equation 1 is the no-defection condition. Equation 2 requires that each firm does not deviate from the punishment path after deviation is detected. If that was the case, punishment would follow until all firms agree to go along.

We now consider the game $G^l(N, \infty, \delta)$ to find how detection lags and the number of firms affect collusion sustainability. Market demand is given by $p(Q) = 1 - Q$ where $Q = \sum_{i=1}^{N} q_i$. We assume that marginal cost are zero for each firm.[2] Firms engage in quantity competition. Then, a strategy $s_i = q_i$. By using Proposition 1 and the best response function $q_i(q_{-i}^c) = \left(1 - c - (n-1)q_{-i}^c\right)/2$ where q_{-i}^c is ths punishment strategy of the remaining $n-1$ firms, equations (1) and (2) can be expressed as,

$$\begin{gathered}\sum_{t=0}^{l} \delta^t \left[\frac{(n+1)}{16n^2} - \frac{1}{4n}\right] = \delta^{l+1}\left[\frac{1}{4n} - (1 - nq_i^c)q_i^c\right], \\ \sum_{t=0}^{l} \delta^t \left[(1 - q_i(q_{-i}^c) - nq_{-i}^c)q_i(q_{-i}^c) - (1 - nq_i^c)q_i^c\right] = \sum_{t=1}^{l+1} \delta^t \left[\frac{1}{4n} - (1 - nq_i^c)q_i^c\right].\end{gathered}$$

We proceed as follows: for a given number of detection lags find the maximum number of firms compatible with a discount factor $\delta < 1$. Corollary 1 summarizes the results.

Corollary 1 *For the game $G^l(N, \infty, \delta)$ when optimal punishments à la Abreu are adopted, under Cournot competition collusion is sustainable accordingly the*

[2] This assumption allows for negative prices during the punishment phase. This is done for ease of exposition.

following trade-off between number of lags and number of firms,

Table 1. Detection lags and number of firms							
# *lags*	**1**	**2**	**3, 4**	**5, 6, 7**	**8, .., 14**	**15, 16**	**17, .., 270**
# *firms*	**9**(**.96**)	**7**(.94)	**6**(.98)	**5**(.99)	**4**(.99)	**3**(.94)	**2**(.999)

Each number in parenthesis in Table 1 is the minimum value of the discount factor such that collusion is sustainable for the largest detection lag compatible with the number of firms considered.[3] For example, with seven detection lags five firms are able to sustain collusion over time iff $\delta \in (.99, 1)$. It is shown that there is an inverse relationship between the amount of detection lags and the number of firms that reach collusion sustainability over time.

3 Concluding remarks

We consider an infinitely repeated game with discounting to extend the result by Colombo and Labrecciosa (2006). We show also that when quantity competition is considered, given a number of firms, Abreu's stick-and-carrot punishment fails to restore cooperation for a sufficient large detection lag. This result calls for attention to detect collusion in markets characterized by infrequent interaction or imperfect information. We prove that the sustainability of collusion could be difficult also if the number of firms is low.

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[3] Upon request authors send the programme which provides the results reported. Calculations have been run with the computer package *Mathematica*® by Wolfran Research.