DFAE-II WP Series

2008-02

Ana I. Saracho & José M. Usategui

Policy Coordination in an Oligopolistic Housing Market

Policy Coordination in an Oligopolistic Housing Market[*]

Ana I. Saracho and José M. Usategui
Universidad del País Vasco

Abstract

This paper analyzes the consequences of the interaction between two different levels of government (regulators) in the development of housing policy when their decisions determine the level of competition in the housing market. The analysis discusses the implications derived from a lack of coordination between a local regulator who controls the supply of land for housing development and a central regulator who decides on housing subsidies. The results suggest that lack of coordination has significant effects on prices and supply of houses, housing developers' profits, and buyers' surplus.

JEL classification codes: L 13, L 50

Keywords: Imperfect competition, housing policy coordination.

1   Introduction

In many countries local governments have considerable control over the supply of land for housing development. Sometimes local governments issue building permits and charge development fees for the right to build houses on privately owned land. In other situations, local governments own developable land (land banking) and decide on the rate at which to release it for development. Furthermore, often local governments finance a significant proportion of their expenses through development fees or the sale of sites for housing development. The revenue that local authorities obtain in this way permits them to finance public services for local residents, which are the ones that vote in elections for the local government. A recent document of the International Monetary Fund (2006), referred to Spain, states (paragraph I.B.11) that: "... local administrations derive a substantial share of their revenues from taxing real state. This creates perverse incentives to limit the supply of residential and commercial land".

In this paper we show that local authorities may prefer not to allow development of all sites available to increase their revenues from development fees or from the sale of sites. This occurs even though local authorities care also about consumer surplus in the housing market and not only about their revenues from the development or sale of sites. This behavior of local governments makes market failure important in the housing market, as they induce oligopolistic control over housing development in a local housing market and houses in different local markets may not be good substitutes for each other. The situation is analogous if local regulators require payments in kind to issue building permits. In this case the local regulator may ask the housing developers to supply public services to the community such as parks, swimming pools, day nurseries, or sports fields. The local regulator would now be concerned with value of the payment in kind that the local community would obtain.

In this context the central government realizes that with respect to the case where social surplus is maximized, too few houses would, in general, be built and the price paid by buyers would be too high. To correct market failure the central governments subsidize the purchase of houses, usually through subsidies implicit in personal income tax. This approaches the distribution of decision power on housing policy between local and central regulators in several countries (see Smith, Rosen and Fallis (1988)).

The primary advantage of this division of housing policy between higher and lower levels of government is that the central government can strongly influence the evolution of the housing market, while taking advantage of local governments' intimate knowledge of local conditions. The primary difficulty is that such a division of housing policy can lead to coordination failure and reduce social surplus. In this paper we stress that the control of the supply of land may be used by the local government to increase his revenue collection. Alternatively, as in Brueckner and Lai (1996), one could consider that land is privately owned and landowners influence the political process at the local level. These landowners would try to induce decisions of the local government that increase the revenue obtained through the sale of sites for housing development.

In our model, the local regulator increases those revenues by reducing the supply of land for housing development, thereby reducing competition among housing developers. We explore the implications of this behavior of the local regulator by constructing a housing market model where both levels of government (regulators) interact and influence the decisions of the developers and the buyers of houses. The central governments subsidize the purchase of houses in order to achieve a higher social surplus. In our analysis, subsidies to home buyers try to offset the distortions deriving from the market power induced by the behavior of the local regulator in a context where the number of sites available for housing development is, anyway, limited.

The central regulator is a central housing agency concerned with the surplus of the agents in the housing market. We implicitly suppose that there are other central agencies that worry about other markets, other objectives and other public policies. This specialization of concerns of public agencies helps us to justify our partial equilibrium approach when defining the objective function of our central housing agency. However, we also consider in the paper that the social cost of public funds may be greater than 1, so that the measure of social surplus includes the surplus loss from the distortions caused by the taxes used to finance housing subsidies.[1]

We compare the situation of the housing market when both regulators act independently and when they behave as a single (or integrated) regulator

---

[1] Some housing policies such as the subsidies implicit in personal income tax have been found to be vertically and horizontally inequitable (see Rosen (1985), and Smith, Rosen and Fallis (1988)). Distributive aspects of housing policies, which are important and may justify regulator involvement in the housing market, will not be considered in this work.

who decides on both housing subsidies and the supply of sites for housing development, to maximize social surplus. More precisely, we analyze the consequences of the lack of coordination of both regulators on the supply of houses, the supply of sites for housing development, the price of houses, the net price paid by buyers, the profits of the housing developers, and buyers' surplus. This lack of coordination may cause a perverse mix of policies: growth control of the housing market by the local regulator and subsidies by the central regulator that increase growth in the market.

In this work we explicitly examine if savings in subsidies when the two regulators act in an integrated way are high enough to allow a money transfer to the local regulator that would permit this regulator to attain the same level of his objective function than under independent regulators. Our concern with compensation of the local regulator is based on the fact that housing laws often assign the decision power on site development to the local regulator and, thereby, this regulator cannot be compelled to develop a specific number of sites.[2] We consider that the local regulator gives up his decision power on the development of sites only if he is compensated to attain the level of his objective function that would obtain under independent regulators. The question then is whether or not it is possible to compensate the local regulator when the two regulators act as an integrated regulator. The analysis provides, explicitly, realistic circumstances in which the compensation is impossible.

The study is carried out by considering a housing market where there are many price-taking buyers and a group of housing developers competing à la Cournot. However, the analysis presented is also valid if we consider that there are many identical, but geographically differentiated and independent, housing markets. Local housing markets may be approximated by districts or cities, which are far enough apart for only local supply and demand conditions to affect prices and output. In each housing market there would be a local regulator who would face the same objective function, under the same circumstances, and would behave identically. The interaction among all the agents can then be analyzed through the study of the interaction among the agents in each market, since there would not be any strategic behavior among the agents in different markets.

The rest of the paper is organized as follows: Section 2 presents the housing market model. Section 3 analyzes housing policy under two

---

[2]It is usually considered that land use regulation is decided by the local regulator (see Hanushek and Quigley (1990) and Brueckner (1990)).

independent regulators. Section 4 studies housing policy when the two regulators act in an integrated way. We call the basic model to the model studied in those sections. Section 5 extends the analysis considering two variants of the basic model: substitution effects between new and used houses and an alternative type of housing subsidy set by the central regulator. In section 6 we discuss three extensions of the previous analysis that generate new results: a social cost of public funds greater than 1, congestion effects on housing demand and density regulation. Finally, section 7 concludes and offers some suggestions for further research.

2 The Housing Market

Consider a perfect and complete information context in which the market for houses is made up by housing developers who decide quantities as in a Cournot oligopoly and by many price-taking buyers. All sites are homogeneous and each developer gets only one particular site available for housing development and builds houses on it.[3] The unit cost of production of houses, represented by c, is constant. We will comment briefly on convex production costs in section 5.

We shall initially study the market for new houses. Substitution effects between new houses and houses built in the past are assumed to be negligible.

To proceed let us denote by x_j the number of new houses on site j with $j = 1, ..., n$ where n denotes the number of sites devoted to housing development. We consider that all houses offered in the market are identical from the consumer's point of view. Let $h(x)$ be the inverse housing demand function for new houses where $x = \sum_{j=1}^{n} x_j$ is the total number of new houses.

We make the following assumption on the demand function:

Assumption 1: For any x the inverse demand function $h(x)$ is such that $h(0) > c$, $h'(x) < 0$ and the marginal revenue is decreasing, i.e.,

$$2h'(x) + h''(x)x < 0.$$

Clearly, if $h(x)$ is decreasing, concave or linear $(h''(x) \leq 0)$ and $h(0) > c$ then Assumption 1 will be satisfied.

---

[3]We consider that the extension of a site is big enough to allow several-story buildings to be built on it. Hence, a site is clearly bigger than a lot, and we may see it as closer to a subdivision.

In the following sections we discuss housing policy decisions in a context where there is interaction between two regulators: a local regulator and a central regulator. Often the local authorities control the supply of sites for housing development while the decision on housing subsidies is taken by higher levels of government, and there may be lack of coordination between the two regulators.[4] This lack of coordination between both regulators may affect the number of sites available for housing development, thereby reducing the number of housing developers and, as a result, the number of houses built.

We are going to compare, in terms of social welfare, supply of houses, the price paid for buyers, price of houses and the profits of housing developers, the case in which both regulators act independently and the case in which both act in an integrated way. In the first scenario the timing of the game is the following: In the first stage the central regulator elects the per unit subsidy she will pay each house buyer. In the second stage the local regulator decides on the number of sites for housing development. Finally, in the third stage housing developers, which compete à la Cournot, decide the number of houses to be built. In the case in which both regulators act in an integrated way the game has only two stages. In the first one the integrated regulator decides on both, the per unit subsidy and the number of sites to be developed. In the second stage housing developers decide the number of houses to be built. We start with the case of independent regulators.

3 Housing Policy Under Independent Regulators

We solve the decision problem by backward induction in order to look for a subgame perfect equilibrium.

Third stage:

Each consumer who purchases a house receives a per unit subsidy s. Hence, the relationship between producer price p and the market sale level x will be such that

$$h(x) = p - s.$$

Housing developer j, $j = 1, ..., n$, will solve the following problem (notice

---

[4]An interesting discussion on decisions in a system of hierarchical governments may be found in Hoyt and Jensen (1990).

that the housing developers have market power):

$$\max_{x_j}(h(x) + s - c)x_j$$

and the solution, $x_j(n, s)$ and $x(n, s) = \sum_{j=1}^{n} x_j(n, s)$, will satisfy

$$h'(x(n, s))x_j(n, s) + h(x(n, s)) = c - s. \tag{1}$$

From (1) it is clear that the impact of the subsidy on house building is analogous to the impact of a decrease in the unit cost of production by an amount equal to the subsidy.[5] Moreover, notice from (1) that $x_i(n, s) = x_j(n, s)$ for all $i, j \in \{1, ..., n\}$ with $i \neq j$ and, hence, the distribution of houses among sites will be symmetric. Assumption 1 and this symmetric distribution of houses among sites guarantee that the second order condition is satisfied. Adding conditions (1) for $j = 1, ..., n$ we obtain

$$n(h(x(n, s)) - c + s) + h'(x(n, s))x(n, s) = 0. \tag{2}$$

Differentiating (2) and using Assumption 1 it is not difficult to show that $x(n, s)$ increases with n and s and that $x_j(n, s)$ increases with s but decreases with n for all $n > 1$.

Second stage:

The goal of the local regulator is to maximize a weighted sum of the revenue from the sale of sites and surplus of the buyers of houses, given the per unit subsidy established by the central regulator. The demand for sites arises from the profits obtained by the sale of the houses built on sites (hence the land market is endogenous in this model). We consider that the local regulator attains a proportion t of the profits of the housing developers in a context where the local regulator owns initially all sites and sells some, or all, of them to housing developers. This t could be the result of a bargaining process, not specified in this paper, between the local regulator and the housing developers.

Alternatively, we may assume that sites are privately owned and housing developers pay proportional taxes on the profits they obtain and that the local government either collects these taxes directly or receives a percentage

---

[5]Sometimes the housing policy reduces the cost of house building directly. A subsidized reduction in the interest rate in loans to developers would be an example of this type of policy. In our analysis, s would correspond to the savings per house in interest paid.

of taxes collected by the central government.[6] Another possibility would be to consider that the local regulator charges development fees for the right to build houses on privately owned land. Notice that all these alternatives are compatible with the assumption of profit maximization by housing developers. Moreover, to simplify the analysis, we consider that those revenues obtained by the local regulator do not affect the demand for houses in a significant way because they finance expenses that do not make more valuable the ownership of a house.[7]

We consider that the objective function of the local regulator includes his revenues from the sale of sites for housing development with a weight γ and consumer surplus with a weight $1 - \gamma$, where $0 \leq \gamma \leq 1$. Let us denote by N the maximum number of sites that may be given over to housing development. So, given s, the local regulator will solve the following problem:[8]

$$\max_n \gamma t(h(x(n, s)) + s - c)x(n, s)$$
$$+(1 - \gamma) \left(\int_0^{x(n,s)} h(x)dx - h(x(n, s))x(n, s) \right)$$
$$s.t. \quad n \leq N.$$

Assuming interior solutions the first order condition of this problem implies:

$$\gamma t(h(x(n, s)) + s - c + h'(x(n, s))x(n, s)) - (1 - \gamma)h'(x(n, s))x(n, s) = 0.$$

Taking into account (2) we get:

$$\left[\gamma t(-\frac{1}{n} + 1) - (1 - \gamma) \right] h'(x(n, s))x(n, s) = 0,$$

so we may conclude that the number of sites developed by the local regulator, n^*, will be:

$$n^* = \begin{cases} \frac{\gamma t}{\gamma t + \gamma - 1} & \text{if} \quad \frac{1}{1+t-\frac{t}{N}} < \gamma \\ \\ N & \text{otherwise.} \end{cases} \tag{3}$$

The number of sites developed by the local regulator does not depend on the unit cost of production and, therefore, see (1), it does not depend on the level s of the per unit subsidy established by the central regulator. When we consider below ad valorem subsidies we obtain, instead, that the number of sites developed by the local regulator depends on the level of the subsidy. Note, however, that, as we will see in the analysis of the first stage, houses per site will change as a result of the per unit subsidy.

As consumer surplus increases with n and the profits of housing developers decrease with n, we have that n^* decreases with γ, the weight of the revenues from the sale of sites. Notice, for example, that when $\gamma = 1$ it is $n^* = 1$. The reason is that industry profits are maximized under monopoly, so the local regulator, given s, would like to get $x(1,s)$ houses built. In addition, if $\gamma t < 1 - \gamma$ then $n^* = N$. The reason is that consumer surplus increases with the number of houses built, which increases with the number of housing developers n.[9]

It has been assumed that it is the local regulator who owns land for housing development. Alternatively it could be the case that land is privately owned. In that context the analysis of a situation in which landowners dominate the political process at the local level as in Brueckner and Lai [3] would be equivalent to the assumption $\gamma = 1$ in our model. The more general specification, $\gamma \neq 1$, would imply that landowners do not quite dominate the political process at the local level as in Brueckner [2].[10]

Let us consider now the first stage.

First stage:

---

[9]Obviously, if n^* is not a natural number then the number of sites developed by the local regulator will be $\lceil n^* \rceil$ or $\lceil n^* \rceil + 1$ where $\lceil n^* \rceil$ denotes the integer part of n^*.

[10]The implications of the analysis for the case $\gamma = 1$ are also valid for a situation where there is no local regulator and all existing sites are owned by a cartel that maximizes industry profits. For a given s the cartel would like to produce $x(1,s)$. In this case, instead of an integrated regulator we would have a regulator that decides on the housing subsidy and is concerned with the compensation of the cartel for the profits that the cartel loses when the regulator's solution is established in the housing market.

The central regulator sets the subsidy that maximizes social surplus. In this work we measure social surplus as the sum of consumer (buyers of houses) surplus, producer (housing developers) surplus and revenue obtained by the local regulator from the development or sale of sites, net of the cost of subsidies given by the central regulator to home buyers. So, as the central regulator knows that the local regulator will develop n^* sites, she will solve the following problem:

$$\max_s \int_0^{x(n^*,s)} h(x)dx - cx(n^*,s).$$

The first order condition of this problem implies

$$h(x(n^*,s)) - c = 0.$$

As $h(0) > c$ and $h(x)$ is decreasing we may conclude that there is a unique value of x, x^w, such that $h(x^w) = c$. Therefore, the central regulator will set the subsidy, $s^w(n^*)$, such that $x(n^*, s^w(n^*)) = x^w$, i.e., the subsidy that induces the efficient level of house building. Taking into account (2) we can conclude that her decision with respect to housing subsidies will be

$$s^w(n^*) = -\frac{h'(x^w)x^w}{n^*}. \tag{4}$$

As x^w does not depend on the number of sites developed we may conclude that the subsidy that maximizes social welfare decreases with the number of sites developed, i.e., $\frac{ds^w}{dn} < 0$.

Maximization of social surplus requires that the buyers of houses pay a consumer price equal to the marginal cost of production c. However, we know that the housing developers receive per house sold an amount equal to the sum of the consumer price and of the subsidy set by the central regulator. Therefore, the gross profits of the developers are equal to the amount spent on housing subsidies:

$$\Pi(n^*) = (c + s^w(n^*) - c)x^w = s^w(n^*)x^w.$$

The results would remain unchanged if the decisions of the two independent regulators on subsidies and supply of sites were simultaneous instead of sequential. In that case, the decisions $n = n^*$ and $s^w = -\frac{h'(x^w)x^w}{n^*}$ constitute the unique Nash equilibrium since from (4) the subsidy selected by the central regulator maximizes social surplus given $n = n^*$, and given

any housing subsidy the local regulator chooses $n = n^*$ to maximize $\gamma t \Pi(n) + (1 - \gamma)CS$, where CS stands for consumer surplus.

4 Housing Policy Under an Integrated Regulator

Consider now an integrated regulator who decides on the number of sites to be developed, n, and on the per unit subsidy, s. From her point of view, the sale of sites generates revenues while housing subsidies are costly. However, the integrated regulator decides on n and s to maximize social surplus. The timing of the game is as follows: In the first stage the integrated regulator decides on both, n and s. In the second stage housing developers decide the number of houses to be built.

Obviously the second stage of this game is identical to the third stage of the game corresponding to housing policy under two independent regulators. So, we solve the first stage of the game.

From the analysis in Section 3 we know that the housing sale level that maximizes social surplus does not depend on n. Hence, in this context any number of sites $n \in [1, N]$ combined with the corresponding housing subsidy given by (4), with $n^* = n$, maximizes social surplus. However, as the optimal subsidy decreases with n, the cheapest way to maximize social surplus is to set $n = N$ and to choose the corresponding (low) housing subsidy.

Assume that the integrated regulator is required to maximize social surplus with the lowest cost of subsidies. In this case, she decides $n = N$ and $s = s^w(N)$. Therefore, integration of both regulators affects neither the supply of houses nor the net price paid by buyers. It does, however, reduce the price of houses, the supply of houses per site and the profits of housing developers if $n^* < N$. Coordination of housing policies between the two regulators in this context is valuable because it avoids the reduction in competition induced by the local regulator under two independent regulators. In other words, integration will foster competition in the housing market.

As we will show next, it would be cheaper to establish a housing policy that maximizes social surplus under an integrated regulator and to compensate the local regulator than to operate under two independent regulators. On one hand, we have that consumer surplus is the same under independent regulators than under an integrated regulator. On the other

hand, denote by $D(n)$ the difference between the cost of subsidies and the revenue obtained by the local regulator through housing development when n sites are developed; that is:

$$D(n) = s^w(n)x^w - t\Pi(n) = (1-t)s^w(n)x^w$$

We have,

$$\frac{dD(n)}{dn} = (1-t)x^w\frac{ds^w(n)}{dn} < 0 \text{ for all } t < 1.$$

Therefore the coordination or integration of the two regulators permits savings in subsidies that compensate for the reduction in revenue from the sale of sites (except, obviously, when under independent regulators $n^* = N$). In other words, under integration it would be possible to guarantee the local regulator the same revenue as in the case of independent regulators and to save money from the public budget as a result of the reduction in the cost of subsidies. We present an example of this compensation problem in Table 1 (see example A). Lack of integration between regulators would benefit the housing developers that get a transfer from the public budget.

5   Variants of the basic model

In this section we present two variants of the basic model analyzed in Sections 3 and 4: a model that includes substitution effects between used and new houses, and the consideration of ad valorem subsidies instead of per unit subsidies. The results obtained for the basic model apply also to these variants. However, the level of the ad valorem subsidy affects the number of sites developed.

An important property of houses is that they are a durable good. In any housing market used houses constitute an important proportion of the stock of houses available for use in a given period. Therefore, there may be significant substitution effects between used and new houses. Next we are going to study the implications derived from this fact in our analysis. Consider that there exist U used houses in the market and that the elasticity of substitution between new houses and houses built in the past is infinite. Specifically, assume that a used house is equivalent, in terms of housing services provided, to a proportion α of a new house with $0 < \alpha \leq 1$ (the

value of a used house for any buyer is a proportion α of the value of a new house for that buyer).

The inverse demand function for new houses will be $g(y) \equiv h(y + \alpha U)$ where y represents the number of new houses. Let us consider that only purchases of new houses are subsidized.[11] Expression (2) will now become:

$$n(h(y(n,s) + \alpha U) - c + s) + h'(y(n,s) + \alpha U)y(n,s) = 0. \qquad (5)$$

Under independent regulators, the local regulator, given s, will solve the following problem

$$\max_n \gamma t(h(y(n,s) + \alpha U) + s - c)y(n,s)$$
$$+(1-\gamma)\left(\int_0^{y(n,s)+\alpha U} h(x)dx - h(y(n,s) + \alpha U)y(n,s)\right)$$
$$s.t. \quad n \leq N.$$

From the first order condition of this problem we get, taking into account (5), that the local regulator will develop exactly n^* sites, where n^* is given by (3). Hence, in the first stage of the game the central regulator will select $s^{wu}(n^*)$ such that
$$h(y(n^*, s^{wu}(n^*)) + \alpha U) = c$$

If we denote by y^w the number of new houses built it will be $h(y^w + \alpha U) = c$. As $h(x^w) = c$, we have

$$y^w = x^w - \alpha U,$$

i.e., the number of new houses built will diminish by an amount equal to the value of the stock of old houses in terms of new house units (that is, the total number of houses expressed in new house units does not change).

From (5) we obtain

$$n^* s^{wu}(n^*) + h'(y^w + \alpha U)y^w = 0.$$

Hence, taking into account (4) and that $y^w = x^w - \alpha U$ we may conclude that the subsidy set by the central regulator is lower than in the case where

---

[11]Alternatively, we could consider that the purchase of a used house also receives a subsidy from the central or integrated regulator, that this subsidy is αs, where s is the subsidy received by the purchaser of a new house, and that the seller of a used house pays αs in taxes to that regulator.

substitution effects between new houses and houses built in the past are assumed to be negligible (i.e., $s^{wu}(n^*) < s^w(n^*)$). Therefore, as the number of new houses decreases with respect to this latter case but the net price paid by the buyers of new houses is still c, both, the price received by the developers of new houses ($p = c + s^{wu}(n^*)$) and their profits will decrease with respect to the case where substitution effects are negligible.[12] In addition, the budget required to attain x^w under independent regulators is lower than in that case.

The integrated regulator will decide to develop all sites available for housing development if she is required to maximize social surplus with the lowest cost of subsidies. As $\frac{ds^{wu}}{dn} < 0$, it is immediate to see that $\frac{dD(n)}{dn} < 0$ for all $t < 1$. Hence, integration of the two regulators also permits in this case savings in subsidies that compensate for the reduction in revenue from the sale of sites.

In the basic model it has been considered that the tool of the central regulator is a per unit subsidy. However, it would also be realistic to consider that the central regulator sets an ad valorem subsidy. Would this change the results obtained in the analysis? Let us consider that each buyer of a new house receives as subsidy an amount equal to $d.p$. In this case it may be shown that, with independent regulators, the total number of houses built and the price paid by buyers are the same than under a per unit subsidy. However, the price of houses, the profits of housing developers and the cost of subsidies are greater when the housing subsidy is ad valorem rather than per unit. The reason for these latter results is that with ad valorem subsidies an increase in the price implies an increase in the amount of the subsidy; then the housing developers are more interested in reducing quantity (increasing price). It is easy to obtain that with this type of housing subsidies the number of sites developed by the local regulator, n^{**}, is:

$$
n^{**} = \begin{cases} \frac{\gamma t}{\gamma t + (\gamma - 1)(1-d)} & \text{if } \frac{1-d}{1-d+t-\frac{t}{N}} < \gamma \\[2em] N & \text{otherwise.} \end{cases}
$$

Hence, we have that $\frac{dn^{**}}{dd} \leq 0$ and that $n^{**} \leq n^*$. Lastly, it may be concluded that, as in the case of per unit subsidies, the gross profits of the developers are equal to the amount spent on subsidies and the coordination or integration of the two regulators permits savings in subsidies that compensate for the

---

[12] Note that if a used house is sold, the buyer will pay αc for this house.

reduction in revenue from the sale of sites (except, obviously, when under independent regulators $n^{**} = N$).

6 Extensions

In this section we extend the previous analysis to study separately: (A) the implications of considering that the social cost of public funds is greater than 1, (B) the consequences of introducing a congestion effect on housing demand, and (C) the case where density regulation is a policy tool of the local regulator.

(A) *Social cost of public funds greater than 1.* An additional aspect which seems relevant in the analysis is that, as a consequence of the distortions caused by taxes, the social cost of public funds may be greater than 1. In this case social surplus will be given by the sum of consumer surplus and producer surplus net of the social cost of subsidies; that is,

$$\int_0^x h(y)dy - cx - (\lambda - 1)sx,$$

where λ is the marginal cost of public funds with $\lambda > 1$. Let us consider the case of independent regulators. The third and second stages of the game do not change by considering a social cost of public funds greater than 1. It is because neither the developers of new houses nor the local regulator do care about the social cost of subsidies. The level of the subsidy which maximizes social surplus when n sites are developed, $s^{w\lambda}(n)$, satisfies

$$(h(x(n, s^{w\lambda}(n) - c - (\lambda-1)s^{w\lambda}(n))\frac{\partial(x(n, s^{w\lambda}(n)))}{\partial(s^{w\lambda}(n))} = (\lambda-1)x(n, s^{w\lambda}(n)). \quad (6)$$

As from (2) we get $\frac{\partial(x(n,s))}{\partial(s)} > 0$, we may conclude that the price paid by buyers will be greater than the marginal cost of production, i.e., $h(x(n^*, s^{w\lambda}(n^*))) > c$, and therefore the profits of the housing developers will be higher than the cost of the subsidies. The reason is that the social cost of the subsidies is greater than when $\lambda = 1$ and so, the subsidy set by the central regulator, and as a result the number of houses built, will be smaller than in that situation.

Consider now the case of an integrated regulator who decides on n and s. The integrated regulator solves

$$\max_{n,s} \int_0^{x(n,s)} h(y)dy - cx(n, s) - (\lambda-1)sx(n, s)$$

$$s.t. \quad n \leq N.$$

Assuming interior solutions the first order conditions are such that

$$(h(x(n,s)) - c - (\lambda - 1)s)\frac{\partial(x(n,s))}{\partial s} = (\lambda - 1)x(n,s) \qquad (7)$$

and,

$$(h(x(n,s)) - c - (\lambda - 1)s)\frac{\partial(x(n,s))}{\partial n} = 0.$$

Obviously for $x > 0$ both equations cannot be satisfied simultaneously. As $\frac{\partial(x(n,s))}{\partial s} > 0$, from condition (7) we get $h(x(n,s)) - c - (\lambda - 1)s > 0$. Hence, as $\frac{\partial(x(n,s))}{\partial n} > 0$ the integrated regulator will decide to develop all the sites and will set the subsidy $s^{w\lambda}(N)$ given by (6). This subsidy is smaller than the subsidy corresponding to the case $\lambda = 1$, that is, $s^w(N)$ given by (4). Therefore the price paid by buyers of houses is greater than c, and the profits of the housing developers will be higher than the cost of subsidies.

When $\lambda > 1$ savings in subsidies after integration may not be enough to compensate the local regulator as the profits of the housing developers under independent regulators may be too high. Examples B and C of Table 1 illustrate this situation.[13] Hence, an observed lack of coordination between the two regulators may be due to social cost of public funds being greater than 1. In that context it may be difficult to convince the local regulator to act in an integrated way.

(B) *Congestion effects on housing demand.* We now introduce a congestion effect on housing demand and discuss its consequences. Let us denote by $g(x, x_o)$ the inverse housing demand function corresponding to a site when there are x_o houses built on that site. This demand function incorporates a congestion effect as the value for a consumer of a house on a site depends on the number of houses built on it. We assume that $\frac{\delta g}{\delta x_o} < 0$ and, thus, that an increase in the number of houses built on a site diminishes the value of a house on that site. The reason is that householders prefer sites with low rather than high densities.

---

[13]In example B savings in subsidies after integration are enough to compensate the local regulator. As in this example $\gamma < 1$, compensation of the local regulator takes into account the variation in consumer surplus when both regulators coordinate. In example C savings in subsidies after integration are not enough to compensate the local regulator when $t > 0.85$.

The main effects of congestion may be unraveled considering that g is linear. We study the case of g linear in the Appendix and present here the intuition of the effects that generalize to other functional forms of g.[14] Notice first that, for any n, $x(n, s)$ decreases with the intensity of the congestion effect as demand will be lower the higher this intensity is. This is also the reason for the decrease of the number of houses that maximizes social surplus with the size of the congestion effect. Moreover, this number increases with n as the congestion effect of an additional house for a given number of houses is smaller the higher n is and, hence, the total social marginal cost, i.e., the production cost plus the congestion cost caused by this house, decreases with the number of sites. Notice that this total social marginal cost will be exactly the net price paid by buyers.

Under independent regulators, congestion effects induce the local regulator to develop a greater number of sites as, for any supply of houses, congestion diminishes with n and, therefore, the willingness to pay for a house increases for each buyer with that number. However, the local regulator may choose $n < N$ given that he also considers the effect of the number of sites developed on the total profit of housing developers through the change in market competition. In the case of an integrated regulator she still chooses to develop all sites available for housing development as, both, competition and demand increase with the number of sites developed.

Hence, lack of coordination between the local and the central regulator decreases, in general, social surplus and the number of houses available. Moreover, the price of houses, the profits of sellers and the net price paid by buyers will be greater under independent regulators. Finally, from the analysis in the Appendix we know that the difference between the cost of subsidies and the revenue obtained through the sale of sites may increase when both regulators coordinate. If this occurs, it will be more expensive to establish under an integrated regulator a housing policy that maximizes social surplus and to compensate the local regulator, than to operate under two independent regulators.

When g is separable the analysis of the effects of congestion on the housing market is analogous to that of considering different production cost functions. From the Appendix it is clear that the assumption of a linear congestion effect is equivalent, in terms of results, to the consideration of the convex cost

---

[14]In particular, we concentrate on situations where the equilibrium in the housing market implies a symmetric distribution of houses among sites.

17

function $C(x) = cx + zx^2$, with $c > 0$ and $z > 0$, in a context where there are no congestion effects. Therefore, the analysis presented in this section is also valid to illustrate the consequences derived from the incorporation of convex cost functions in our study.

(C) *Density regulation.* In a context with congestion effects, *density regulation* may be a housing policy tool. Density regulation may be attained by limiting the height of the apartment buildings or establishing a minimum size for individual lots within sites. We will interpret density regulation as a limit by the local regulator on the maximum number of houses that may be built on a site. In order to briefly discuss the consequences of density regulation we consider that g is such that both the integrated regulator and a cartel of housing developers that owned all sites available for housing development would be interested in a symmetric distribution of houses among sites (this includes the case of g linear).

In this context, under independent regulators the local regulator decides on the maximum number of houses that can be built on a site, x_j^*, and on n. If $\gamma = 1$ he will select $n = N$ and will decide on x_j^* in such a way that $x^* = \sum_{j=1}^{n} x_j^* = N x_j^*$ is the quantity of houses that a cartel which had bought all sites would supply given s. The reasons are that for a given quantity of houses congestion is minimized when $n = N$ and that a cartel maximizes industry profits. If $\gamma < 1$ the number of houses built would be greater. Taking into account the behavior of the local regulator, the central regulator will choose the subsidy inducing the level of housing construction that maximizes social welfare when $n = N$. Although an integrated regulator would also choose $n = N$, she would permit on each site the level of production resulting from Cournot competition. Hence, the subsidy required to induce the level of housing construction that maximizes social welfare when $n = N$ will be lower under an integrated regulator than under independent regulators. However, integration would maintain the supply of houses, congestion, the net price paid by buyers, and social surplus. Therefore, the price of houses and the profits of housing developers will be lower when both regulators coordinate.

7 Conclusion

This paper has examined the interplay between two different levels of government in the development of housing policy when the regulators'

decisions determine the level of competition in the housing market. To our knowledge, there is no previous theoretical research on this problem in the literature. We have discussed the effects of a lack of coordination between a local regulator who decides on the supply of sites for housing development and a central regulator who decides on housing subsidies. The central regulator maximizes social surplus and the local regulator maximizes a weighted sum of the revenue from the sale of sites and of the consumer surplus.

We have compared the effects (on prices, supply of houses, developers' profits, and buyers' surplus) of the decisions when the two regulators act independently and when they behave as a single integrated regulator who maximizes social surplus and decides on both housing subsidies and the supply of sites. The ability to compensate the local regulator when both regulators act in a coordinated way has been explicitly studied because he will not want to give up his decision power on the development of sites if compensation is not guaranteed. We have pointed out that our analysis of the case of independent regulators is also valid for a situation where landowners influence the political process at the local level and have power to determine the supply of sites.

We have developed a model in which we analyze the interaction among all agents active in the housing market: housing developers, buyers and two different levels of government. In this model we have shown how subsidies given by the central regulator may become profits for the housing developers and how the local regulator may reduce competition in the housing market to increase his revenues. As a consequence, under an integrated regulator competition in the housing market is at least as high as under two independent regulators. Competition under two independent regulators, however, increases with some factors, as the existence of congestion effects on housing demand or increasing marginal costs of production.

Summarizing, on one hand, when the profits of the housing developers are equal to the subsidy cost, compensation is feasible because the subsidy cost is decreasing with the number of housing developers (competition in the housing market) and this number is not lower under the integrated regulator than under two independent regulators. The level of both the profits of the housing developers and the subsidy cost will depend, for example, on the elasticity of substitution between new and used houses. On the other hand, when the profits of the housing developers under independent regulators are

higher than the subsidy cost (the net price paid by buyers is higher than the unit cost of production) integration of both regulators increases social surplus. However, in this case compensation may be impossible. This occurs in our model when the social cost of public funds is greater than 1, there are congestion effects on housing demand or the marginal cost of production is increasing.

Our analysis provides a framework where other problems of regulation and competition in the housing market may be studied. One could consider, for instance, that the revenue of the local regulator comes from ad valorem taxes on houses sold in the market. Moreover, further research could examine contexts where the owners of sites compete in ways different from the one considered in this work (competition on location or quality, for instance). Finally, as houses are a durable good one may introduce time into a housing market related to the one presented in this work and study the effects of policies such as the promotion of rental housing.

Appendix: Congestion effects on housing demand with a linear inverse demand function

Let us denote by z the effect of an additional house built on a site on the willingness to pay for a house on that site. We assume that the inverse demand for houses on site j, $j = 1, ..., n$, is

$$g(x, x_j) = e - fx - zx_j,$$

where e, f and z are positive constants.

Considering that each buyer receives a per-unit subsidy s, firm j, $j = 1, ..., n$, will solve the following problem:

$$\max_{x_j}(e - fx - zx_j + s - c)x_j,$$

and the solution, $x_j(n, s)$ and $x(n, s) = \sum_{j=1}^{n} x_j(n, s)$, will satisfy

$$e - fx_j(n, s) - 2zx_j(n, s) - fx(n, s) = c - s. \tag{8}$$

From (8) we derive as solution a symmetric distribution of houses among sites and obtain[15]

---

[15] The symmetric distribution of houses among sites guarantees that the solution derived

$$x(n, s) = \frac{n(e - c + s)}{f(n + 1) + 2z}. \tag{9}$$

Observe that $x(n, s)$ increases with n and s and decreases with z.

Without loss of generality we perform the analysis of the regulators decisions considering that $\gamma = 1$ in the objective function of the local regulator. In this case under independent regulators the local regulator will solve

$$\max_{n} t(e - fx(s, n) - zx_j(s, n) + s - c)x(s, n).$$

Using (9), we obtain that the local regulator will decide $n^{*z} = 1 + \frac{2z}{f}$ (if $1 + \frac{2z}{f} < N$) and $n^{*z} = N$ (otherwise).

The total congestion effect is collected by $z \sum_{j=1}^{n} (x_j)^2$ given that the surplus of each buyer of a house on site j diminishes in zx_j by the congestion effect. Hence, when n sites are going to be developed the central regulator will solve

$$\max_{s} \int_{0}^{\frac{n(e-c+s)}{f(n+1)+2z}} (e - fy)dy - z \sum_{j=1}^{n} \left(\frac{(e - c + s)}{f(n + 1) + 2z} \right)^2 - c\frac{n(e - c + s)}{f(n + 1) + 2z}.$$

From the first order condition of this problem we obtain that the number of houses that maximizes social surplus will satisfy:

$$x^w(n) = \frac{n(e - c)}{nf + 2z}. \tag{10}$$

Observe that x^w increases with n and the optimal number of houses per site (congestion) decreases with n.

From the second stage of the game we know that $n = n^{*z}$. Then we obtain from (9) and (10) that the subsidy set by the central regulator will be:

$$s^w(n^{*z}) = \frac{f(e - c)}{n^{*z}f + 2z}. \tag{11}$$

Notice from (10) and (11) that $x^w(n^{*z})$ and $s^w(n^{*z})$ decrease with z.

Consider now that an integrated regulator decides on n and s. It is easy to show that in order to maximize social surplus the integrated regulator chooses $n = N$ and

from (8) satisfies the second-order condition.

$$s^w = \frac{f(e-c)}{Nf + 2z}.$$

Hence integration of the two regulators increases, in general, the number of houses available and social surplus, and reduces congestion, the price paid by buyers, $c + zx_j^w$, and housing prices, $c + s^w + zx_j^w$.

Finally, notice that

$$D(n) = s^w(n)x^w(n) - t\Pi(n) = (1-t)s^w(n)x^w(n) - tz\frac{(x^w)^2}{n}.$$

In this case savings in subsidies after integration may not be enough to compensate the local regulator. Clearly it is more difficult to compensate the local regulator the greater t and z are. In Table 1 we include an example, example D, of this compensation problem. In that example savings in subsidies are not enough to compensate the local regulator when $t > 0.4$.

We have obtained, in our subgame perfect equilibria, that all housing developers build the same number of houses. As we will show next, this is a requirement for the maximization of social surplus when there are congestion effects. Given x^w and x_j^w for all $j \neq i, k$, maximization of social surplus requires minimization of total congestion effects on sites i and k. The solution to the problem

$$\min_{\{x_i^w, x_k^w\}} z(x_i^w)^2 + z(x_k^w)^2$$

$$\text{s.t. } x_i^w + x_k^w = A,$$

where A is a constant, is $x_i^w = x_k^w$. As this result is valid for any pair of sites i and k we may conclude that $x_j^w = \frac{x^w}{n}$ for all $j = 1, ..., n$.

Lastly, notice that from the maximization problem of firm j we may conclude that the analysis developed in this Appendix applies to the situations where $h(x)$ is linear, there are no congestion effects, and the production cost function is $C(x_j) = cx_j + zx_j^2$ with $c > 0$ and $z > 0$.

References

[1] Brueckner, J. K., 1990, Growth Controls and Land Values in an Open City, Land Economics 66, 237-248.

[2] Brueckner, J. K., 1995, Strategic Control of Growth in a System of Cities, Journal of Public Economics 57, 393-416.

[3] Brueckner, J. K. and F. Lai, 1996, Urban Growth Controls with Residents Landowners, Regional Science and Urban Economics 26, 125-143.

[4] Hanushek, E. A. and J.M. Quigley, (1990), Commercial Land Use Regulation and Local Government Finance, American Economic Review 80 (2), 176-180.

[5] International Monetary Fund, (2006), Spain: Financial Sector Assessment Program, Technical Note on Housing Prices, Household Debt, and Financial Stability, IMF Country Report No. 06/210.

[6] Hoyt, W. H. and R. A. Jensen., 1996, Precommitment in a System of Hierarchical Governments, Regional Science and Urban Economics 26, 481-504.

[7] Rosen, H. S., 1985, Housing Subsidies: Effects on Housing Decisions, Efficiency and Equity, in: A. J. Auerbach and M. Feldstein eds., Handbook of Public Economics, vol. 1 (North Holland, Amsterdam), 345-420.

[8] Smith, L.B., K.T. Rosen and G. Fallis, 1988, Recent Developments in Economic Models of Housing Markets, Journal of Economic Literature XXVI, 29-64.

TABLE 1

	Example A $\gamma = \frac{2}{3}, t = \frac{2}{3}$	**Example B** $\gamma = \frac{2}{3}, t = \frac{2}{3}, \lambda = 1.1$	**Example C** $\gamma = 1, \lambda = 1.1$	**Example D** $\gamma = 1, \; z = 1.5$
s^w	$\frac{50}{n}$	$\frac{50(9-n)}{12n+2}$	$\frac{50(9-n)}{12n+2}$	$\frac{50}{n+3}$
x^w	50	$\frac{275n}{6n+1}$	$\frac{275n}{6n+1}$	$\frac{50n}{n+3}$

INDEPENDENT REGULATORS

n	4	4	1	4
R	416.66	264	$1543.37t$	$510.2t$
SC	625	220	1122.45	204.08
CS	1250	968	771.68	408.17

INTEGRATED REGULATOR

n	5	5	5	5
R	333.33	262.31	$393.47t$	$488.28t$
SC	500	143.08	143.08	195.31
CS	1250	983.67	983.67	488.28

Note: R: Regulator's revenue from the sale of sites, SC: Subsidy cost, CS: Consumer surplus, and it has been considered that $h(x) = 100 - x$, $c = 50$, and $N = 5$.