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*Social Security, Education, Retirement and
Growth*

Social Security, Education, Retirement and Growth ^{*}

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Abstract

In this paper we analyze the effects of social security policies in an unfunded, earnings-related social security system on the incentives to education investment and voluntary retirement, on growth and on income inequality. Growth is endogenously driven by human capital investment, individuals differ in their innate (learning) ability at birth, and the pension scheme includes a minimum pension. More skilled individuals spend more on education, minimum pensions reduce low skill individuals' incentives to invest in human capital, there is no monotonic relationship between per capita growth and income inequality.

JEL classification: O40; H55; J10

Keywords: Social Security; Pay-as-you-go; Voluntary Retirement; Human Capital; Minimum and Maximum Pensions

1. Introduction

A great deal of literature has analyzed the effect of *pay-as-you-go social security* on workers' *voluntary retirement age*. The available empirical evidence suggests that, at least for the US economy, social security is relevant for retirement age issues, despite the lack of total agreement on the effect of changes in the payout from the social security program.

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[See, *e.g.*, Diamond *et al.* (1997), Coile *et al.* (2000), Fabel (1994), Fenge and Pestieau (2005), Kalemli-Ozcan (2002a).]

However, very few papers study the effect of a *minimum pension* upon workers' voluntary retirement age. Jiménez-Martín and Sánchez-Martín (2007) and Sánchez (2005) showed that in Spain *37.6 percent of the contributive old-age pensions were topped up under the minimum pension scheme in 1999, and that almost 70% of people retiring at the age of 60 were enjoying a top-up of their pensions.* According to their calculations of the role of minimum pensions in postponing early retirement, total early retirement was almost 50% larger with minimum pensions. Their analysis of the role of prohibiting borrowing from future pensions revealed very little impact on retirement incentives. Since they were considering an *exogenous growth* model, they could not, of course, analyze the effect of minimum (or maximum) pensions on growth.

Some papers have explored the impact of a *pay-as-you-go social security* system on *human capital investment incentives*, and hence on *endogenous growth* (see, *e.g.*, Echevarría and Iza (2006)). Echevarría and Iza (2006) obtained a net discouraging effect of the size of social security on human capital accumulation and retirement age. Furthermore, the relationship between the size of social security and the per capita *GDP* growth rate was mostly negative, except for very low values for the social security contribution rate. The explanation lies in the discouraging effect that social security imposes on education and, in particular, retirement age, which causes a fall in the share of the working population in the economy. However, they did not consider the effect of *minimum* or *maximum pensions* on *education* and *growth*.

In this paper we focus on the effects of the existence of a minimum pension payment in a pay-as-you-go system on human capital (education) investment incentives, and hence, on growth and income inequality. Additionally, retirement age is endogenously determined, so we also analyze the effects of pension policies on early retirement incentives.

We build up a two-period, *OLG* model economy with a pay-as-you-go social security system in which pension benefits are earnings-related and populated by *ex ante* heterogeneous individuals who differ in their innate (learning) ability.

Individuals in their first period of life choose their level of education. Those born with higher ability are expected to invest more in their education. Given that pension payments are earnings-related, the return on human capital investment is not constrained to labor income while working, but in fact extends to pensions during retirement. Therefore, when individuals choose their optimal level of education, they take into account not only the effect on future labor earnings, but also on future pension benefits. Consequently, social

security introduces an incentive for higher investment in human capital.¹

This incentive, however, might break down because the pension scheme includes a minimum pension payment. For instance, as the minimum pension increases, so does the threshold for the innate ability for which individuals end up receiving the minimum pension. Minimum pensions, therefore, have a discouraging effect on education investment for those individuals with low enough innate ability. As a by-product, growth (driven by individuals' education investment) is also influenced.

In their second period, individuals supply labor elastically (*i.e.* optimally choose their retirement age). Therefore, voluntary retirement age also depends on the incentives that the public pension system embeds: not only minimum pensions, but also penalties for early retirement which take the form of reductions in the net pension payment if retirement occurs before some *normal* retirement age. Minimum pensions work in the opposite direction as they promote early retirement.

In short, in this economy social security influences both the size of the working population in the economy and its productivity.

We calibrate the model and construct a benchmark case which fairly reproduces some stylized facts of the Spanish economy. Starting from this baseline case, we analyze the effects of changes in social security policies: changes in *i*) minimum pensions, *ii*) early retirement penalty scheme, *iii*) early retirement age, and *iv*) normal retirement age. Our main results follow:

1. Our model predicts that more skilled individuals enjoy a higher return on their investment in education and, consequently, spend more on education.
2. The existence of a minimum pension, however, may reduce low skill individuals' incentives to invest in human capital.
3. Since the per capita growth rate of the economy depends on the educational attainment of the more skilled individuals, any reform that decreases the incentives of these individuals to invest in their education will decrease the per capita growth rate. Policies that increase the minimum pension, the early retirement penalty, the minimum legal retirement age and the normal retirement age have this effect.
4. There is not a monotonic relationship between per capita growth and income inequality: increments in the minimum pension, in the early retirement penalty, in the minimum legal retirement age and in the normal retirement age lead to lower

¹ We assume that fertility and mortality are exogenous. Thus, we do not follow the current of the literature which assumes that parents care about the number of children and their well being and where parents invest in their children's human capital. [See, *e.g.*, Zhang & Zhang (2004)].

growth rates; the three first policies induce lower income inequality, although the fourth one gives rise to higher inequality.

5. Large (up to 20%) increments in minimum pension lead to no change in retirement ages for medium and most skilled individuals.
6. Increments in the early legal retirement age (up to 63) imply higher voluntary retirement ages (in a one to one relationship) and human capital investment for the least skilled workers.
7. Increments in normal retirement age (up to 68) give rise to higher voluntary retirement ages for intermediate and high skill workers, while low skill workers still leave the labor force at the minimum legal retirement age. As for per capita growth, this is the policy measure with the strongest (negative) effect: the resulting increment in the social security surplus remarkably reduces private (young) savings and physical capital accumulation, leading to a fall in incentives for education expenditure.
8. Increments in early retirement penalty cause higher voluntary retirement ages for intermediate and high skill workers, while low skill workers' is unaffected because the minimum pension is not subject to early retirement penalty.

The paper is organized as follows: Section 2 describes the economy. Section 3 characterizes the competitive equilibrium. The calibration and the corresponding numerical exercise is carried out in Section 4. Section 5 presents the conclusions. A mathematical Appendix is included at the end.

2. The economy

This economy is characterized by the behavior of three kinds of agents (households, social security and firms) which act in perfectly competitive markets for one unique (aggregate) commodity good and two production factors (physical capital and human capital). Time is discrete.

2.1. Households

At any time t this economy is populated by two overlapping generations of individuals, young and old. Individuals consume both when young and old (their first and second periods of life, respectively), and supply inelastically one unit of labor when young. In their second period, however, individuals choose their optimal leisure consumption (*i.e.* their labor supply). In this setup, higher leisure consumption is interpreted as workers choosing to retire earlier. For instance, a worker who wished to retire as late as possible

would choose the minimum leisure time legally available (maybe strictly positive). Similarly, a worker who decided to retire as early as the law allowed would choose a leisure time equal to the corresponding upper bound². Second period leisure is modeled as a continuous variable choice, bounded both above and below, so that a whole range of intermediate choices are possible. A similar setup is used in Garriga and Manresa (1999).

Additionally, individuals in their first period must choose their optimal level of education (*i.e.* human capital investment): this choice will affect not only their labor income, but also their retirement pension benefits. This is so because we assume that *i*) social security is *non-funded*, and *ii*) pensions are *earnings-related* (and, therefore, defined-benefit type).

We assume one unique source of heterogeneity among individuals. Thus, we assume that there are three types of individuals ($i = 1, 2, 3$) who differ by their innate ability, θ_i (where $\theta_1 < \theta_2 < \theta_3$): higher innate ability means higher learning ability and higher return on investment in human capital and, therefore, higher education expenditure in principle (which, in turn, implies higher economic growth). Types 1, 2 and 3 represent individuals attaining primary, secondary and college education, respectively.

This heterogeneity, of course, drives the income inequality in this economy, partially mitigated by the social security system.³ As we will see, the existence of minimum pension benefits, along with the earnings-related nature of pension benefits, might pose an incentive problem. Low-skill individuals might find it optimal to reduce their investment in education for a high enough minimum retirement pension.

The preferences of an i -th type individual born at time t are represented by the utility function

$$u(c_{y,t}^i, c_{o,t+1}^i) = \ln c_{y,t}^i + \beta (\ln c_{o,t+1}^i + \xi \ln \ell_{t+1}^i), \quad (1)$$

where $\beta \in (0, 1)$ stands for the discount factor, $c_{y,t}^i$ and $c_{o,t+1}^i$ denote first period and second period consumption (respectively), $\xi > 0$ represents the second period relative preference of leisure upon consumption, and $\ell_{t+1}^i \in [\ell_L, \ell_U]$ denotes second period leisure. We assume that second period leisure is bounded below ($\ell_L > 0$), *i.e.* workers are legally forced to retire at some time before a maximum age; and, also, bounded above ($\ell_U < 1$), *i.e.* a minimum retirement age exists.⁴ Whenever an individual choice variable is affected

² In this model one period represents 32.5 years. Assuming that individuals start their active life when they are 15 years old, the maximum amount of leisure depends on the minimum retirement age. For instance, in the U.S. it may be 0.77 (*i.e.* some individuals can retire at 55). See http://www.opm.gov/fers_election/fersh/h_fers3.htm for minimum retirement age (US Federal Employees Retirement Service).

³ Huggett *et al.* (2006), referring to the US economy, claim that “differences in learning ability account for the bulk of the variation in the present value of earnings across agents.”

⁴ We do not make any distinction between “retirement age” and “pension age”.

by two subscripts, the first one denotes the individual's age (y for young and o for old, respectively), and the second one denotes calendar time.

We assume the productivity level h_t^i this individual attains is a function of his/her innate ability, θ_i , and his/her expenditure on education, e_t^i , once normalized by the total factor productivity at time t , A_t . More precisely, we assume that

$$h_t^i = \theta_i[1 + (e_t^i/A_t)^\gamma], \quad \gamma \in (0, 1). \quad (2)$$

As in Bouzahzah, De la Croix and Docquier (2002) [*BDD* hereafter], the engine of growth of this economy will be given by the aggregate state of knowledge (or total factor productivity) in the economy. Even though our model is very close to the one in *BDD*, the way in which we separate the individual human capital level from the state of knowledge is in fact closer to Romer (1990). As in Romer (1990) we distinguish between the *private* knowledge attained by an individual who lives a finite life, h_t^i , and the *non-rival* knowledge (the state of technology) of the economy which can be accumulated indefinitely, A_t .

Why is education expenditure normalized by the state of knowledge in Eq. (2)? Consider, for instance, the balanced growth path: wage (per unit of labor) will be growing at the same rate as the total factor productivity, A_t . Therefore, the expenditure on education will increase at the same rate as A_t , and all individuals of type i will spend a constant share of their income on education, so that e_t^i/A_t and h_t^i remain constant too. This way, the *total* productivity in the production process of any of these individuals will be $h_t^i A_t$, thus growing at the same rate as A_t . Consequently, the investment in education, e_t^i , must be normalized by the total factor productivity of the economy, A_t , or measured in *efficiency* units, in order to make the model consistent.

A major difference between our model and the one in *BDD* is that investment in education comes from income rather than time. Therefore, while in the *BDD* specification the individual investment in education is bounded (since it cannot be greater than the total endowment of time), it might not be bounded in our case.

A second difference between *BDD* and our model is that we treat retirement age as endogenous: we believe that a thorough understanding of all the incentives embedded in social security systems entails considering the retirement decision as a choice variable.

The first period budget constraint is given by

$$c_{y,t}^i + s_{y,t}^i + e_t^i = w_{n,t} h_t^i A_t, \quad (3)$$

where $s_{y,t}^i$ denotes savings, $w_{n,t} \equiv (1 - \tau_t^{ss})w_t$ denotes the net of social security contribution wage rate per efficient unit, τ_t^{ss} denotes the social security contribution rate, and w_t denotes the wage rate per efficient unit. Note that the labor income of the household

depends on the total factor productivity, A_t .⁵

The second period budget constraint is given by

$$c_{o,t+1}^i = (1 + r_{t+1})s_{y,t}^i + \ell_{t+1}^i b_{t+1}^i + (1 - \ell_{t+1}^i)w_{n,t+1}h_t^i A_{t+1} + ss_{t+1}, \quad (4)$$

where $w_{n,t+1} \equiv (1 - \tau_{t+1}^{ss})w_{t+1}$, r_{t+1} denotes the interest rate between periods t and $t + 1$, b_{t+1}^i stands for the social security retirement pension benefit (per unit of time), and ss_{t+1} denotes the lump-sum transfer that old individuals receive as a result of sharing the social security surplus.⁶ Given the redistributive role played by social security, thereby it mainly transfers income from young (workers) to old, we assume that the surplus in our model is transferred to individuals in their second period. Note that both the pension benefit and the labor income that the individual is paid in his/her second period are conveniently weighted by leisure time and labor time, ℓ_{t+1}^i and $1 - \ell_{t+1}^i$, respectively.⁷

As for the retirement pension, two cases must be considered in turn: *i*) retirees whose pension benefit is the result of applying a replacement rate τ_{t+1}^{rep} to past earnings and a before-normal-age-retirement penalty, q_{t+1}^i ; and *ii*) retirees who are paid the minimum pension, b_{t+1}^{min} (so that the retirement pension ends up being earnings *unrelated*).⁸ Formally, retirement pension for an i -th type individual at time $t + 1$ is given by

$$b_{t+1}^i = \begin{cases} b_{t+1}^{\text{min}}, & \text{for } q_{t+1}^i \tau_{t+1}^{\text{rep}} w_t h_t^i A_t < b_{t+1}^{\text{min}} \\ q_{t+1}^i \tau_{t+1}^{\text{rep}} w_t h_t^i A_t, & \text{otherwise.} \end{cases} \quad (5)$$

Concerning the first case, we assume that the replacement rate applies to the average labor income obtained during the first active periods (*i.e.* at t). Note that if the economy grows at a non-zero per capita rate (that we call λ_t), for a balanced growth path to exist,

⁵ Note also that all individuals in their first period of life enter the labor market at the same time, *i.e.* regardless of the education expenditure that they make. Had we assumed a different time setting in our model, we could have assigned different ages for entering the labor market: thus, individuals attaining college education, for instance, would start working later than, say, those attaining primary school education. This point is left out in this paper.

⁶ Retirement pensions are *not* the only type of transfers that social security systems in real economies pay. For instance, Spanish social security also pays incapability, widowhood and orphanhood pensions and family benefits, representing 66.45% of total pensions in 2000-2005. [See Section 4 for details.] As an alternative, one might consider a unique consolidated budget for the social security and the government, so that (for a given government spending path), tax rates were adjusted such that budget balanced. [See, *e.g.* Sánchez-Martín (2005).]

⁷ Alternatively, one may assume that the retirement pension benefit does not depend on whether the individual is completely or partially retired, so that the pension payment is simply b_{t+1}^i . [See, *e.g.*, Garriga and Manresa (1999).]

⁸ We might have also allowed a maximum pension benefit. However, at least for the Spanish case, retirees who are paid the maximum pension represent a negligible proportion of all retirement pensions: 0.03% in December 2007. [See Spanish Social Security web page at <http://www.seg-social.es>.]

pension benefits (whether proportional to first period labor income or not) must grow at the same rate at which per capita variables (such as A_t) grow.

Concerning Eq. (5) two remarks are in order. *First*, we assume that the before-normal-age retirement penalty only applies to individuals whose retirement pensions are earnings-related. *Second*, we assume that q_{t+1}^i as a linear function *decreasing* in ℓ_{t+1}^i ,

$$q_{t+1}^i = \begin{cases} 1, & \text{if } \ell_L \leq \ell_{t+1}^i \leq \ell_N \\ 1 - \alpha_1(\ell_{t+1}^i - \ell_N), & \text{if } \ell_N < \ell_{t+1}^i \leq \ell_U \end{cases} \quad (6)$$

where $\alpha_1 \equiv (1 - \alpha_0)/(\ell_U - \ell_N)$, $\alpha_0 \in (0, 1)$, $\ell_N \in (\ell_L, \ell_U)$ denoting the leisure corresponding to the normal-retirement-age.^{9,10} Function q_{t+1}^i is graphed in Figure 1.

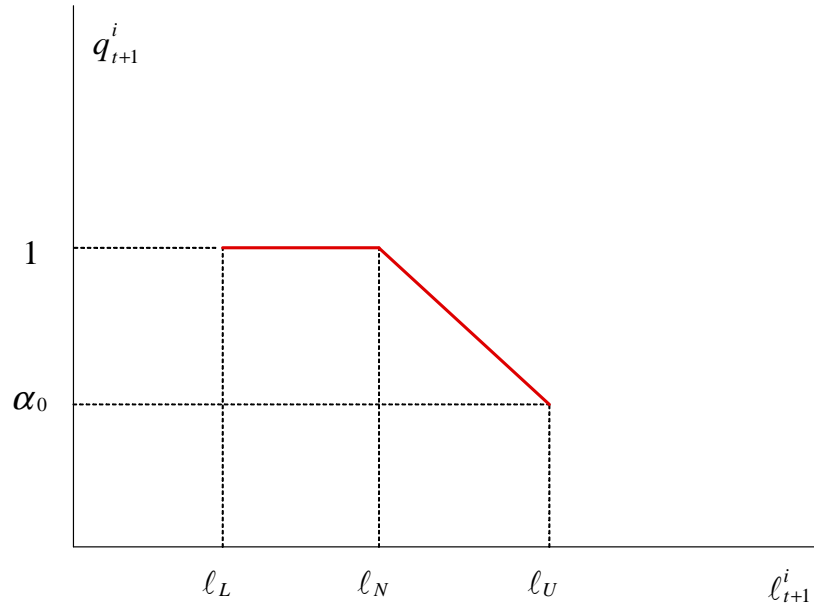


Figure 1

Thus, assuming away borrowing constraints, the problem that an i -th type individual faces can formally be expressed as the maximization of Eq. (1) with respect to first and second period consumption ($c_{y,t}^i$ and $c_{o,t+1}^i$, respectively), first period savings ($s_{y,t}^i$), second period leisure (ℓ_{t+1}^i), and optimal levels of education (e_t^i) and pension benefits (per period) (b_{t+1}^i), subject to Eqs. (2), (3), (4) and (5). Additionally, it must be the case that $\ell_L \leq \ell_{t+1}^i \leq \ell_U$. For the sake of clarity, this problem can be studied in two parts: *first*,

⁹ A similar specification was used by Jiménez-Martín & Sánchez-Martín (2004). This scheme follows the 1985 Spanish Social Security reform. The reform enacted in 1997 introduced a third interval for values of ℓ in Eq. (6). [See, *e.g.* Jiménez-Martín and Sánchez-Martín (2004), p. 54.]

¹⁰ Note that ℓ_{t+1}^i could be dropped from (5) because the utility function in (1) trivially prevents optimal ℓ_{t+1}^i from being zero.

for some generic level of leisure ℓ_{t+1}^i , we obtain the first order necessary conditions that determine the optimal e_t^i and b_{t+1}^i which arise from maximizing the difference between the sum of the discounted value of first and second period earnings (pension benefits included), minus the education expenditure.¹¹ And, *second*, we obtain the first order necessary conditions for optimal $c_{y,t}^i$, $c_{o,t+1}^i$, $s_{y,t}^i$ and ℓ_{t+1}^i . The optimal values for all choice variables are obtained, of course, by solving all f.o.n.c. simultaneously.

2.1.1. Optimal education and earnings-related retirement pension

The first order necessary condition that determines the optimal solution for education expenditure depends on whether the retirement pension that the retiree gets paid is earnings-related or not. Suppose first that the pension benefit does depend on the labor income that the individual obtained when he/she was a worker. Formally, from Eqs. (2), (3), (4) and (5), and denoting $R_{t+1} \equiv 1 + r_{t+1}$, the first order necessary condition comes from solving the following problem

$$\begin{aligned} \max_{\{e_t^i\}} NPV_1(e_t^i) &= w_{n,t} h_t^i A_t + \frac{q_{t+1}^i \tau_{t+1}^{\text{rep}} w_t h_t^i A_t \ell_{t+1}^i}{R_{t+1}} \\ &+ \frac{w_{n,t+1} h_{t+1}^i A_{t+1} (1 - \ell_{t+1}^i)}{R_{t+1}} - e_t^i, \end{aligned} \quad (7)$$

Differentiating $NPV_1(e_t^i)$ with respect to e_t^i , taking into account Eq. (2), equating to 0 and solving the first order necessary condition for e_t^i yields the solution for education expenditure¹²

$$e_{1,t}(\theta_i) = A_t \left\{ \gamma \theta_i \left[w_t \left(1 - \tau_t^{ss} + \frac{q_{t+1}^i \tau_{t+1}^{\text{rep}} \ell_{t+1}^i}{R_{t+1}} \right) + \frac{w_{n,t+1} (1 + \lambda_t) (1 - \ell_{t+1}^i)}{R_{t+1}} \right] \right\}^{\frac{1}{1-\gamma}}, \quad (8)$$

where we use the fact that $A_{t+1} \equiv (1 + \lambda_t) A_t$.¹³ When the pension benefit depends on the labor income obtained during the active period, the optimal education expenditure must be such that the marginal increase in the sum of current and (discounted) future labor income plus the marginal increase in (discounted) pension benefits must equal 1, *i.e.* the marginal cost of education. As expected, and along balanced growth paths, $e_{1,t}(\theta_i)$ grows over time (assuming that A_t grows too) and depends positively on the net wage rates per efficiency unit, $w_{n,t}$ and $w_{n,t+1}$, and the pension replacement rate, τ_{t+1}^{rep} . Of course,

¹¹ For the sake of emphasizing the economic intuition of the solutions, we break this problem into two separate cases, depending on whether pension benefits are earnings-related or not [Sec. 2.1.1 *vs.* Sec. 2.1.2, respectively].

¹² The first necessary condition is also sufficient as $NPV_1(e_t^i)$ is concave in e_t^i .

¹³ Notice the notation: $e_{1,t}(\theta_i)$ denotes the education chosen at time t by an individual of skill level θ_i whose retirement pension benefit is earnings related and higher than the minimum.

a higher discount rate $R_{t+1} - 1$ reduces the discounted value of retirement pensions and second period labor income. Therefore, it reduces the incentive to invest in education or human capital. Last but not least, $e_{1,t}(\theta_i)$ depends positively on the ability parameter θ_i : more skilled individuals enjoy a higher return on their investment in education and, consequently, are expected to spend more on education. This is a well known result in the human capital literature. [See Le Garrec (2005) and references there in.]

Note that Eq. (8) can be rewritten as

$$R_{t+1} = \frac{\partial h_t^i(\cdot)}{\partial e_t^i} A_t [w_{n,t} R_{t+1} + q_{t+1}^i \tau_{t+1}^{\text{rep}} w_t \ell_{t+1}^i + w_{n,t+1} (1 + \lambda_t) (1 - \ell_{t+1}^i)], \quad (9)$$

which has a fair interpretation: along the optimal education investment path, the returns to savings have to be equal to the returns to education.

For such an individual, from Eqs. (5) and (8), we would have that

$$e_t^i = e_{1,t}(\theta_i), \quad \text{and} \quad b_{t+1}^i = q_{t+1}^i \tau_{t+1}^{\text{rep}} w_t h_t^i A_t. \quad (10)$$

Later we characterize the range of values of θ for which Eq. (10) is the solution to Eq. (7).

Figure 2 illustrates this case:

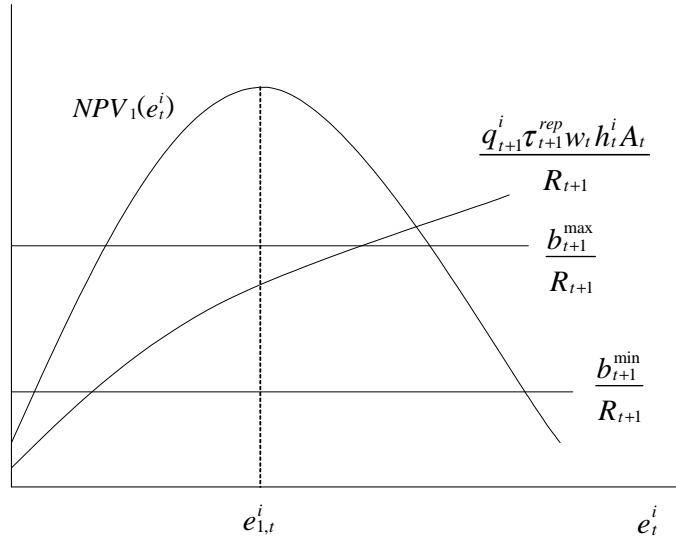


Figure 2

2.1.2. Optimal education and non-earnings-related retirement pension

The existence of a minimum retirement pension, however, might break this link. Individuals with a skill level θ_i below some lower bound $\underline{\theta}_t$ might find it optimal to get

paid just the minimum pension and invest in education accordingly (*i.e.* less, taking into account that their pension benefit would *not* depend on the education investment). In this case, the individual chooses e_t^i so as to maximize the sum of current plus (discounted) future labor income minus the education expenditure. Formally, the problem can be expressed as

$$\max_{\{e_t^i\}} NPV_2(e_t^i) = w_{n,t}h_t^iA_t + \frac{w_{n,t+1}h_t^iA_{t+1}(1 - \ell_{t+1}^i)}{R_{t+1}} - e_t^i. \quad (11)$$

Differentiating $NPV_2(e_t^i)$ with respect to e_t^i [again, taking into account Eq. (2)], equating to 0 and solving the first order necessary (and sufficient) condition for e_t^i yields the solution for education expenditure

$$e_{2,t}(\theta_i) = A_t \left\{ \gamma\theta_i \left[w_{n,t} + \frac{w_{n,t+1}(1 + \lambda_t)(1 - \ell_{t+1}^i)}{R_{t+1}} \right] \right\}^{\frac{1}{1-\gamma}}, \quad (12)$$

where, as above, we have used the fact that $A_{t+1} \equiv (1 + \lambda_t)A_t$.¹⁴ When the pension benefit does not depend on the labor income obtained during the active period, the optimal education expenditure must be such that the marginal increase in the sum of current and (discounted) future labor income must equal the marginal education cost. Note that, as long as $q_{t+1}^i\tau_{t+1}^{\text{rep}}\ell_{t+1}^i > 0$, it must be the case that $e_{1,t}(\theta_i) > e_{2,t}(\theta_i) > 0$. Thus, our model predicts that the existence of a minimum pension may reduce the incentives of low skill individuals to invest in human capital acquisition. For such an individual, from Eqs. (5) and (12), we would have that

$$e_t^i = e_{2,t}(\theta_i), \quad \text{and} \quad b_{t+1}^i = b_{t+1}^{\min}. \quad (13)$$

Note that $e_{2,t}(\theta_i)$ is increasing in θ_i . Therefore, a lower bound exists on the skill parameter θ_i , which we denote by $\underline{\theta}_t$, such that

$$e_t^i = e_{2,t}(\theta_i) \quad \text{and} \quad b_{t+1}^i = b_{t+1}^{\min} \quad \text{for all } \theta_i < \underline{\theta}_t. \quad (14)$$

Once $\theta_i \geq \underline{\theta}_t$, the retirement pension becomes earnings-related and higher than or equal to b_{t+1}^{\min} . The bound $\underline{\theta}_t$ is implicitly given by the condition $q_{t+1}^i\tau_{t+1}^{\text{rep}}w_t h_t(\underline{\theta}_t)A_t \ell_{t+1}^i = b_{t+1}^{\min}\ell_{t+1}^i$, where the left-hand-side represents the pension benefit obtained by an individual of ability level $\underline{\theta}_t$ and whose pension benefit is *earnings-related* [*i.e.* whose optimal education is given by Eq. (8)], and the right-hand-side denotes the pension benefit obtained by a retiree who

¹⁴ Notice the notation: $e_{2,t}(\theta_i)$ denotes the education chosen at time t by an individual of skill level θ_i whose retirement pension benefit is not earnings-related. As before, it can be shown that the returns on savings and on education investment coincide along the optimal path for $e_{2,t}(\theta_i)$.

is paid the minimum pension. Taking into account that [given the utility function in Eq. (1)] optimal e_{t+1}^i is *strictly positive*, the bound $\underline{\theta}_t$ is implicitly given by

$$q_{t+1}^i \tau_{t+1}^{\text{rep}} w_t h_t(\underline{\theta}_t) A_t = b_{t+1}^{\text{min}}. \quad (15)$$

Figure 3 illustrates this case:

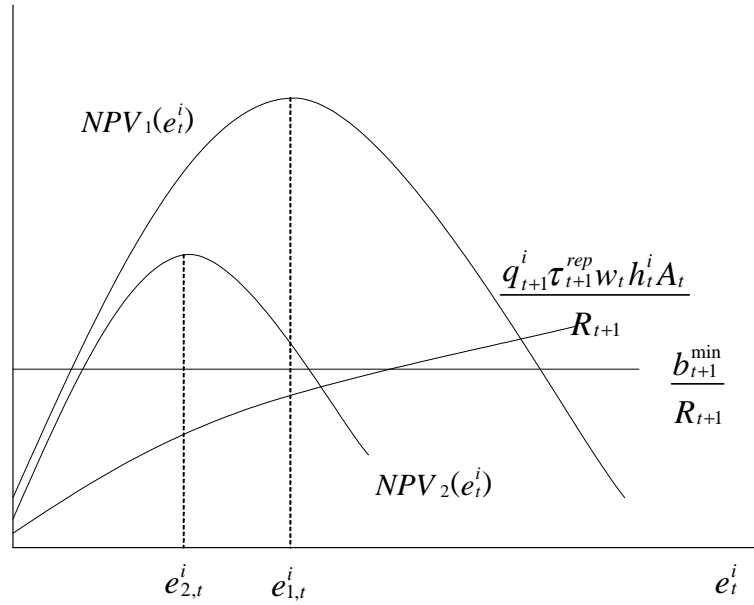


Figure 3

In short, from Eqs. (2), (5), (8), (10), (12), (14) and (15), the optimal education expenditures and retirement pension benefits are the ones shown in Table I.

TABLE I: EDUCATION & PENSION

	$\theta_i < \underline{\theta}_t$	$\underline{\theta}_t \leq \theta_i$
e_t^i :	$e_{2,t}(\theta_i)$	$e_{1,t}(\theta_i)$
b_{t+1}^i :	b_{t+1}^{min}	$q_{t+1}^i \tau_{t+1}^{\text{rep}} w_t h_t^i A_t$

For the sake of completeness, Figure 4 shows how optimal education and pension payment are related to ability parameter θ . Note the discontinuity of e_i at $\theta = \underline{\theta}$: starting at a low θ , when the learning ability parameter equals the lower bound $\underline{\theta}$, education expenditure jumps upwards.¹⁵

¹⁵ Education expenditure increases with learning ability, since the individual labor productivity is an increasing function of the individual's learning ability.

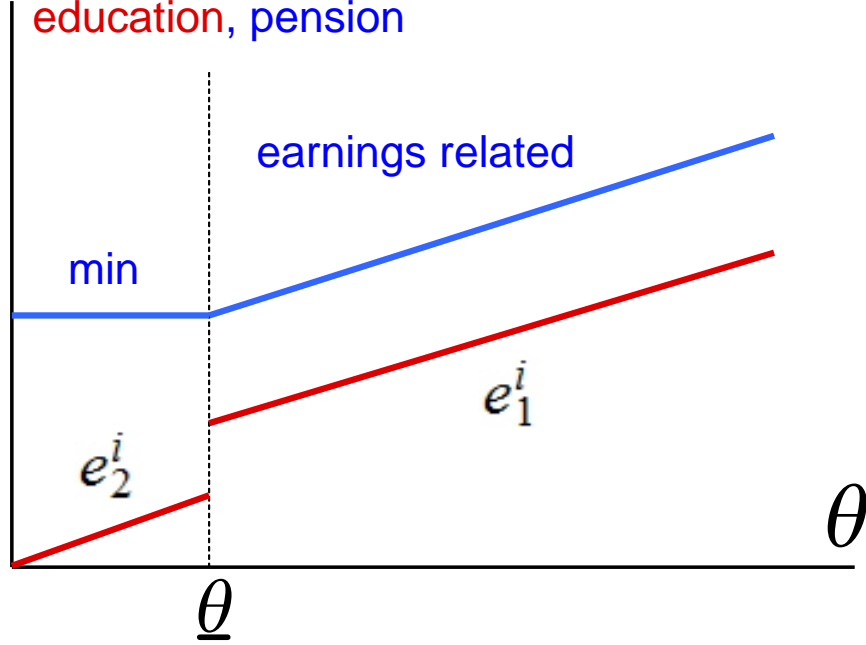


Figure 4

2.1.3. The other optimal decisions

In this section we obtain the first order necessary conditions for first and second period consumption, $c_{y,t}^i$ and $c_{o,t+1}^i$, respectively, first period savings, $s_{y,t}^i$, and second period leisure, ℓ_{t+1}^i . From Eqs. (3) and (4), and remembering that $R_{t+1} \equiv 1 + r_{t+1}$, we obtain the intertemporal budget constraint

$$\frac{ss_{t+1}}{R_{t+1}} + w_{n,t}h_t^i A_t + \frac{w_{n,t+1}h_t^i A_{t+1}(1 - \ell_{t+1}^i)}{R_{t+1}} + \frac{b_{t+1}^i \ell_{t+1}^i}{R_{t+1}} = e_t^i + c_{y,t}^i + \frac{c_{o,t+1}^i}{R_{t+1}}. \quad (16)$$

Maximizing Eq. (1) with respect to $c_{y,t}^i$, $c_{o,t+1}^i$ and ℓ_{t+1}^i , subject to Eqs. (16) and (6), and using Eq. (3), yields the following system of non-linear equations which [along with Eq. (16)] characterize the optimal interior $c_{y,t}^i$, $c_{o,t+1}^i$, ℓ_{t+1}^i and $s_{y,t}^i$:

$$\frac{1}{c_{y,t}^i} = \frac{\beta R_{t+1}}{c_{o,t+1}^i}, \quad (17)$$

$$\frac{\xi c_{o,t+1}^i}{\ell_{t+1}^i} = w_{n,t+1}h_t^i A_{t+1} - b_{t+1}^i - \ell_{t+1}^i \frac{\partial b_{t+1}^i}{\partial \ell_{t+1}^i}, \quad \text{for } \ell_L < \ell_{t+1}^i < \ell_U, \quad (18)$$

where

$$\frac{\partial b_{t+1}^i}{\partial \ell_{t+1}^i} = \begin{cases} 0, & \text{if } \ell_L < \ell_{t+1}^i < \ell_N \text{ or } b_{t+1}^i = b_{t+1}^{\min} \\ -\frac{\alpha_1 b_{t+1}^i}{q_{t+1}^i}, & \text{if } \ell_N < \ell_{t+1}^i < \ell_U \text{ and } b_{t+1}^{\min} < b_{t+1}^i \end{cases} \quad (19)$$

where h_t^i is given by Eq. (2) and, of course, the optimal e_t^i will in general depend on ℓ_{t+1}^i .¹⁶ Eq. (17) represents the standard Euler equation for optimal consumption between two consecutive periods. Eq. (18) represents the optimality condition for leisure ℓ_{t+1}^i for an *interior* solution such that $\ell_L < \ell_{t+1}^i < \ell_U$. The gain in utility from an additional unit of leisure in the second period must equal the loss in utility from consumption in the second period (the pension payment net of labor income and penalty falls).

Note that both when the retiree is paid the minimum pension and when the retirement pension is earnings-related but the retirement takes place at normal age or later, the penalty terms [q_{t+1}^i and α_1] vanish from Eq. (18). [See Eq. (19).]

Finally, Eq. (20) gives us the first period savings

$$s_{y,t}^i = w_{n,t} h_t^i A_t - c_{y,t}^i - e_t^i. \quad (20)$$

2.2. Aggregate labor force

We assume an exogenous, constant population growth rate $n \geq 0$, so that the proportions of young and old individuals are given by $\mu_y = (1+n)/(2+n)$, and $\mu_o = 1/(1+n)$, respectively. Additionally, we assume that the exogenous distribution of skills among the population is such that the proportion of individuals of type i is given by $\{\psi_i\}_{i=1}^3 \geq 0$, where and $\sum_{i=1}^3 \psi_i \equiv 1$. Denoting by P_t the total population at time t , aggregate labor force supplied is given by

$$L_t = \mu_y P_t \sum_{i=1}^3 \psi_i h_t^i + \mu_o P_t \sum_{i=1}^3 \psi_i h_{t-1}^i (1 - \ell_t^i). \quad (21)$$

The first term on the right-hand-side of Eq. (21) represents the labor force of young individuals, and the second term stands for the labor force of old individuals. Note that in this latter case the decision on retirement is crucial.

2.3. Social security

The social security budget equation at any time t is given by

$$\tau_t^{ss} w_t L_t A_t = P_t \mu_o \left[\sum_{i=1}^3 \psi_i b_t^i \ell_t^i + ss_t \right], \quad (22)$$

where the left-hand-side represents total revenue, and the right-hand-side denotes total expenditure on retirement pensions plus lump-sum transfers. Both social security revenues and payments on retirement pensions depend on *i*) the age structure of the population, *ii*) the distribution of skill levels, and *iii*) (as in the case of the aggregate labor force) the distribution of retirement ages across old individuals. Retirement pension benefits have been specified in Table I.

¹⁶ Note that $e_{1,t}(\theta_i)$ and $e_{2,t}(\theta_i)$ depend on ℓ_{t+1}^i . Therefore, $c_{y,t}^i$, $c_{o,t+1}^i$, $s_{y,t}^i$, ℓ_{t+1}^i and $e_{1,t}(\theta_i)$ [or $e_{2,t}(\theta_i)$] are chosen simultaneously. [See Eqs. (8) and (12).]

2.4. Firms

Concerning the production sector, we assume the existence of a representative, competitive firm which produces one unique output Y_t out of physical capital K_t and human capital in *efficiency units* $A_t L_t$, and which maximizes current profits. Formally, assuming Cobb-Douglas production technology, it faces the following problem

$$\max_{\{K_t, L_t\}} F K_t^\alpha (A_t L_t)^{1-\alpha} - w_t A_t L_t - (r_t + \delta) K_t, \quad (23)$$

where $F > 0$ is a scaling factor of the technology level, $\alpha \in (0, 1)$ denotes the elasticity of output with respect to physical capital, and $\delta \in (0, 1)$ stands for the physical capital depreciation rate.

The productivity of the labor force here depends on two *independent* factors: *i*) the state of knowledge of the economy, A_t , (which individuals take as given even though it is a function of type-3 individuals' education investment), and *ii*) the individuals' skills (their human capital level¹⁷), h_t^i , which in turn depend on *ii.1*) their innate ability, θ_i , and *ii.2*) their investment in education, e_t^i , which allows individuals to increase their human capital level above their innate ability.

The first order necessary (and sufficient) conditions for the program in Eq. (23) give us the factor price equations

$$w_t = (1 - \alpha) F k_t^\alpha \quad \text{and} \quad r_t + \delta = \alpha F k_t^{\alpha-1}, \quad (24)$$

where $k_t \equiv K_t / (A_t L_t)$, *i.e.* the stock of physical capital per efficient unit of labor.

2.5. Goods market equilibrium

As in Diamond (1965), the condition for equilibrium in the goods market states that the aggregate savings of the young generation at any time t must equal the stock of physical capital installed in the economy at time $t + 1$. Formally, we have that

$$\mu_y P_t \sum_{i=1}^3 \psi_i s_{y,t}^i = K_{t+1}, \quad (25)$$

where type- i young individual's savings $s_{y,t}^i$ have been specified in Eq. (20).

2.6. Growth

We assume that the total factor productivity, A_t , evolves according to the following law of motion

$$A_{t+1} = (1 + \lambda_t) A_t, \quad (26)$$

¹⁷ In a narrow sense, as in Romer (1990).

where the growth rate at time t is given by

$$\lambda_t = \rho_1 [e_t(\theta_3)/A_t]^{\rho_0} \geq 0, \quad (27)$$

for some $\rho_1 > 0$, and $\rho_0 \in (0, 1)$, and where we are assuming that only the investment in education carried out by the cleverest agents produces new knowledge. Eq. (27) implies that this growth model is not of vintage type. It is an analogous specification to the one in *BDD* with two differences: *i*) we assume heterogeneity of innate abilities, and *ii*) we allow for the possibility that the parameters in the technologies for *individual* and *social* human capital accumulation might differ (*i.e.* $\gamma \neq \rho_0$, $\rho_1 \neq 1$).

Once the model is set up, we define the equilibrium for this economy.

3. Competitive equilibrium

Definition 1. *A competitive equilibrium for this economy is a set of sequences of quantities $\{c_{y,t}^i, c_{o,t}^i, e_t^i, h_t^i, s_{y,t}^i, \ell_t^i, b_t^i\}_{t=0}^\infty$ (for $i = 1, 2, 3$), aggregate human and physical capitals and total factor productivity $\{L_t, K_t, A_t\}_{t=0}^\infty$, stock of physical capital per efficient unit of labor $\{k_t\}_{t=0}^\infty$, prices $\{w_t, r_t\}_{t=0}^\infty$, bound $\{\underline{\theta}_t\}_{t=0}^\infty$ for the skill parameter θ , social security lump-sum transfers $\{ss_t\}_{t=0}^\infty$, and growth rates for the total factor productivity $\{\lambda_t\}_{t=0}^\infty$ such that for a sequence of social security policy parameters $\{\tau_t^{ss}, \tau_t^{\text{rep}}, b_t^{\text{min}}\}_{t=0}^\infty$ and $(\ell_L, \ell_N, \ell_U, \alpha_0, \alpha_1)$, a rate of population growth n , and for initial $K_0 > 0$, $P_0 > 0$, $A_0 > 0$ and $\{e_{-1}^i \geq 0\}_{i=1}^3$ the following holds at any time t :*

i) individuals maximize utility in Eq. (1) taking prices w_t, r_t and social security policy parameters $\{\tau_t^{ss}, \tau_t^{\text{rep}}, b_t^{\text{min}}\}_{t=0}^\infty$ and $(\ell_L, \ell_N, \ell_U, \alpha_0, \alpha_1)$ and social security lump-sum transfers ss_t as given, such that Eqs. (2), (3), (4), (5), (6), (8), (12), (14), (16), (17), (18), (19), (20) hold; and where $\underline{\theta}_t$ has been defined in Eq. (15);

ii) the representative firm maximizes profits in Eq. (23) taking prices w_t, r_t as given, such that Eq. (24) holds;

iii) physical capital and labor markets clear, so that Eqs. (25) and (21), respectively, hold;¹⁸

iv) social security budget is satisfied, so that Eq. (22) holds; and

v) total factor productivity A_t grows over time according to Eqs. (26) and (27).

In order to solve the model, some (quantity) variables must be first redefined relative to efficiency units so that on a balanced growth path all these redefined variables remain constant. We have normalized the individual variables by the total factor productivity, A_t ,

¹⁸ Needless to say that, by Walras Law, Eq. (25) necessarily holds if Eqs. (3), (24), (21), (22) and (4) hold.

[which on a balanced growth path grows at a constant rate equal to λ] and the aggregate variables by the aggregate labor force in efficiency units, $A_t L_t$, [which also grows at a constant rate $(1 + \lambda)(1 + n) - 1$ on a balanced growth path]. Thus, we can rewrite the competitive equilibrium in Definition 1, in terms of variables expressed in efficiency units, as follows:

Definition 2. *A competitive equilibrium for this economy is a sequence of quantities $\{\hat{c}_{y,t}^i, \hat{c}_{o,t+1}^i, \hat{e}_t^i, h_t^i, \hat{s}_{y,t}^i, \ell_t^i, \hat{b}_t^i\}_{t=0}^\infty$ (for $i = 1, 2, 3$), stock of capital per efficiency unit $\{k_t^i\}_{t=0}^\infty$, prices $\{w_t, r_t\}_{t=0}^\infty$, bound $\{\underline{\theta}_t\}_{t=0}^\infty$ for the skill parameter θ , the social security lump-sum transfers $\{\hat{s}_t^i\}_{t=0}^\infty$ and growth rate $\{\lambda_t\}_{t=0}^\infty$ such that for social security policy parameters $\{\tau_t^{ss}, \tau_t^{\text{rep}}, b_t^{\text{min}}\}_{t=0}^\infty$ and $(\ell_L, \ell_N, \ell_U, \alpha_0, \alpha_1)$, and for initial $k_0 > 0, A_0 > 0$ and $\{\hat{e}_{-1}^i \geq 0\}_{i=1}^3$ at any time t the following holds:*

i) conditions i)-v) in Definition 1, and

ii) equations (28)-(39), where

$$h_t^i = \theta_i [1 + (\hat{e}_t^i)^\gamma], \quad (28)$$

$$\hat{c}_{y,t}^i + \hat{s}_{y,t}^i + \hat{e}_t^i = w_{n,t} h_t^i, \quad (29)$$

$$\hat{c}_{o,t+1}^i = (1 + r_{t+1}) \hat{s}_{y,t}^i + \ell_{t+1}^i \hat{b}_{t+1}^i (1 + \lambda_t) + (1 - \ell_{t+1}^i) w_{n,t+1} h_t^i (1 + \lambda_t) + \hat{s}_{t+1}^i, \quad (30)$$

where

$$\hat{e}_t^i = \begin{cases} \left\{ \gamma \theta_i \left[w_{n,t} + \frac{w_{n,t+1} (1 - \ell_{t+1}^i) (1 + \lambda_t)}{R_{t+1}} \right] \right\}^{\frac{1}{1-\gamma}}, & \text{for } \theta_i < \underline{\theta}_t, \\ \left\{ \gamma \theta_i \left[w_t \left(1 - \tau_t^{ss} + \frac{q_{t+1}^i \tau_{t+1}^{\text{rep}} \ell_{t+1}^i}{R_{t+1}} \right) + \frac{w_{n,t+1} (1 + \lambda_t) (1 - \ell_{t+1}^i)}{R_{t+1}} \right] \right\}^{\frac{1}{1-\gamma}}, & \text{for } \theta_i \geq \underline{\theta}_t \end{cases} \quad (31)$$

$$\hat{b}_{t+1}^i = \begin{cases} \hat{b}_{t+1}^{\text{min}}, & \text{for } \theta_i < \underline{\theta}_t, \\ \frac{\text{pen}_{t+1}^i \tau_{t+1}^{\text{rep}} w_t h_t^i}{1 + \lambda_t}, & \text{for } \theta_i \geq \underline{\theta}_t, \end{cases} \quad (32)$$

$$\frac{1}{\hat{c}_{y,t}^i} = \frac{\beta R_{t+1}}{\hat{c}_{o,t+1}^i}, \quad (33)$$

$$\frac{\xi \hat{c}_{o,t+1}^i}{\ell_{t+1}^i} = (1 + \lambda_t) \left[w_{n,t+1} h_t^i - \hat{b}_{t+1}^i - \ell_{t+1}^i \frac{\partial \hat{b}_{t+1}^i}{\partial \ell_{t+1}^i} \right], \quad \text{for } \ell_L < \ell_{t+1}^i < \ell_U, \quad (34)$$

where

$$\frac{\partial \hat{b}_{t+1}^i}{\partial \ell_{t+1}^i} = \begin{cases} 0, & \text{if } \ell_L < \ell_{t+1}^i < \ell_N \text{ or } \hat{b}_{t+1}^i = \hat{b}_{t+1}^{\text{min}}, \\ \frac{-\alpha_1 \hat{b}_{t+1}^i}{q_{t+1}^i}, & \text{if } \ell_N < \ell_{t+1}^i < \ell_U \text{ and } \hat{b}_{t+1}^{\text{min}} < \hat{b}_{t+1}^i. \end{cases} \quad (35)$$

$$\sum_{i=1}^3 \psi_i \hat{s}_{y,t}^i = k_{t+1} \left[(1+n) \sum_{i=1}^3 \psi_i h_{t+1}^i + \sum_{i=1}^3 \psi_i h_t^i (1 - \ell_{t+1}^i) \right] (1 + \lambda_t), \quad (36)$$

$$L_t = \mu_o P_t \left[(1+n) \sum_{i=1}^3 \psi_i h_t^i + \sum_{i=1}^3 \psi_i h_{t-1}^i (1 - \ell_t^i) \right], \quad (37)$$

$$\tau_t^{ss} w_t L_t = P_t \mu_o \left[\sum_{i=1}^3 \psi_i \hat{b}_t^i \ell_t^i + \hat{s}s_t \right], \quad (38)$$

$$\lambda_t = \rho_1 (\hat{c}_t^3)^{\rho_0} \geq 0, \quad (39)$$

where $\hat{c}_{y,t}^i \equiv c_{y,t}^i/A_t$, $\hat{s}_{y,t}^i \equiv s_{y,t}^i/A_t$, $\hat{c}_{o,t+1}^i \equiv c_{o,t+1}^i/A_t$, $\hat{b}_{t+1}^i \equiv b_{t+1}^i/A_{t+1}$, $\hat{b}_{t+1}^{\min} \equiv b_{t+1}^{\min}/A_{t+1}$, $\hat{s}s_{t+1} \equiv ss_{t+1}/A_t$, $A_{t+1} \equiv (1 + \lambda_t)A_t$, $P_{t+1} \equiv (1+n)P_t$, $\mu_o P_{t+1} \equiv \mu_y P_t$ and $k_{t+1} \equiv K_{t+1}/(L_{t+1}A_{t+1})$.

Definition 3. A stationary steady state equilibrium for this economy is a time-independent set of quantities \hat{c}_y^i , \hat{c}_o^i , \hat{e}^i , h^i , \hat{s}_y^i , ℓ^i , \hat{b}^i (for $i = 1, 2, 3$), k , time-independent factor prices w , r , time-independent bound $\underline{\theta}$ for the skill parameter θ , time-independent social security lump-sum transfer $\hat{s}s$ and time-independent growth rate λ such that for time-independent social security policy parameters τ^{ss} , τ^{rep} , b^{\min} , ℓ_L , ℓ_N , ℓ_U , α_0 and α_1 , Definition 2 holds.

The stationary steady state equilibrium can be solved as a system of simultaneous non-linear equations with the help of some numerical techniques.¹⁹ [See Appendix.]

As a by-product, our model allows us to study the redistributive role played by the social security and its eventual conflict with individual incentives to labor supply, retirement and economic growth. We focus on one particular measure of (in)equality such as Gini's index relative to the sum of discounted life-time net income, social security lump-sum transfers included

$$\frac{\hat{s}s}{R} + w_n h^i + \frac{\hat{b}^i \ell^i (1 + \lambda)}{R} + \frac{w_n h^i (1 + \lambda) (1 - \ell^i)}{R},$$

which we denote by I_G .

4. A numerical example

4.1. Calibration

The non-linearity of the model and the number of equations involved (in spite of its simple dynamic structure) prevent us from obtaining analytical results for the solution to

¹⁹ In particular, we have used GAUSS[©] 6.0.4 and the subroutine for non-linear equations NLSYS[©].

the individual problem, let alone for the general equilibrium problem, so that uniqueness must be assumed. Therefore, we have to rely on numerical analysis for which we need some basic values for preferences, technology, demographics and social security policy. Our aim when choosing values is simply to qualitatively illustrate the working and the main features of our model, but approaching to some extent certain observed figures of the Spanish economy. In order to go further and obtain more quantitatively realistic conclusions from this exercise, some very careful calibration work needs to be done.

•**Demographics.** Assuming that each of the two periods in the model represents about 32.5 years, $n = 0.27$ means that the yearly rate of population growth equals $(1+n)^{1/32.5} - 1 = 0.007$. According to *INE*²⁰, the average yearly rate of population growth between 1986 and 2006 was 0.7%.

•**Preferences.** As for preferences, the subjective discount factor is set at $\beta = 1.323$. This means that the yearly preference discount factor equals $\beta^{1/32} = 1.008$, slightly higher than others found in the literature.²¹ For instance, Conesa and Garriga (1999) set it at 0.985, and Garriga and Manresa (1999) at 0.987.

The leisure-related parameter in the utility function ξ is set equal to 0.255. This value has been chosen such that type-1 individuals choose early retirement (*i.e.* at 60), and type-2 and type-3 individuals choose to retire around normal retirement age (*i.e.* at 65), (64.09 and 64.28, respectively). Jiménez-Martín and Sánchez-Martín (2007) show that retirement hazard rate clearly exhibits two peaks: at 60 and at 65.

As for ℓ_L and ℓ_U , considering that individuals start solving their maximization problem at the age of 15, and that each period represents 32.5 years (so that their deterministic life expectancy is $80 = 2 \times 32.5 + 15$), the upper bound ℓ_U equals 0.615, which corresponds with an early retirement age of 60 years [*i.e.* $\ell_U = (80 - 60)/32.5$]. The lower bound ℓ_L is set at 0.308, thus representing a compulsory retirement age of 70 years [*i.e.* $\ell_L = (80 - 70)/32.5$].

•**Heterogeneity of individuals' innate ability.** Concerning the values of innate abilities, we normalize $\theta_1 = 1$, and we pick up the values for θ_2 and θ_3 taking into account that the higher the innate ability, the higher the educational attainment. In short, we make a one-to-one correspondence between individuals' innate abilities and their educational attainments. We set θ_2 such that the ratio of type-2 workers' hourly wage rate to that of type-1 workers fairly replicates the observed ratio of the monthly wage rate of workers with high school education to that of workers with primary school (note that we are assuming that monthly hours are the same for all workers). In particular, in

²⁰ *INE* stands for *Instituto Nacional de Estadística* (Spanish National Institute of Statistics), which can be accessed at <http://www.ine.es>

²¹ As pointed out elsewhere, a discount factor higher than one (*i.e.* a negative time preference rate) is not a problem in OLG economies. [See Ventura (1999) and Constantinides *et al.* (2002).]

Spain in 2002, this ratio equals 1.38, and θ_2 is set equal to 1.33. Analogously, to set θ_3 we consider that the ratio of the monthly wage rate of workers with college education to that of workers with high school education is the same as the observed ratio. This value was equal to 1.52 in Spain in 2002, and θ_3 is set equal to 1.94²² Furthermore, this way we are able to obtain in our benchmark case *i*) the two types of pension benefits: minimum, for type-1 individuals, and earnings-related, for type-2 and type-3 individuals; and *ii*) the two types of education expenditure: e_2 , for type-1 individuals, and e_1 , for type-2 and type-3 individuals.

As for the distribution of the skill parameter, we assume a constant intra-generational distribution that mimics the observed distribution of the workers regarding their retirement age and pension benefits.²³ In particular, we choose the value for ψ_1 such that the proportion of workers retiring at early retirement age and receiving the minimum pension is close to the observed. This implies a value for $\psi_1 = 0.20$. The value for $\psi_2 = 0.454$ is chosen such that the proportion of workers that retire at the normal retirement age and whose pension is close to the minimum pension is equal to the proportion of type-2 individuals. Trivially, the value for ψ_3 is equal to $1.0 - \psi_1 - \psi_2$.

Finally, concerning human capital production we assume that $\gamma = 0.39$.²⁴

•**Social security system.** We assume that $\hat{b}^{\min} = 0.095$. The minimum pension is chosen to replicate the ratio of monthly wage rate of type-1 individuals to minimum pension, which was around 1.0 in Spain in 2000 (see Jiménez-Martín and Sánchez-Martín (2007)).²⁵ Jiménez-Martín and Sánchez-Martín (2007) mentions that 70% of workers retiring at 60 are low-income workers who receive the minimum pension. Since in our model all type-1 individuals are homogeneous, we obtain that all type-1 individuals receive the same (the minimum) pension.

We set $\tau^{ss} = 0.283$ thus equating the observed value.²⁶ We do not consider sources of

²² These figures have also been obtained from the *INE*.

²³ These figures have been obtained from *IMSERSO*, web-page <http://www.imsersomayores.csic.es/estadisticas/informemayores>. We have used the data published in 2002.

²⁴ In this model, this value does not play an important role since we are not building a concave life-cycle labor income pattern. Nevertheless, we have used it to approach the average replacement rate to the observed value.

²⁵ In our benchmark case, this figure is equal to 0.95.

²⁶ See *Bases y Tipos de Cotización 2006* at <http://www.seg-social.es>. This has been the contribution rate for *Régimen General y Regímenes Especiales Asimilados* (General Regime and Assimilated Special Regimes), since 1995 and which is split between employers (23.6%) and employees (4.7%). Special regimes included: the self-employed, agriculture workers and “home employees”. Special regimes excluded: sea workers and coal miners. See *Anexo al Informe Económico Financiero a los Presupuestos de la Seguridad Social de 2005. Capítulo I. Cotización a la Seguridad Social, Cuadro 3*, without page number. *Ministerio de Trabajo y Asuntos Sociales. Secretaría de Estado de la Seguridad Social. Dirección General de Ordenación de la Seguridad Social.*

revenues other than contributions (such as transfers and subsidies, financial asset income or sale of real estate and financial assets).

We assume, as in Jiménez-Martín and Sánchez-Martín (2007), that for those workers whose pension benefits are earnings-related and who retire after the normal retirement age (*i.e.* at the age interval [65, 70]), their replacement rate is equal to one. However, there are individuals whose replacement rate is below one. In particular, those who retire before the normal retirement age and are penalized accordingly. Replacement rate for individual i is obtained as $\hat{b}_{t+1}^i(1 + \lambda_t)/(w_t h_t^i)$, so that the average replacement rate along balanced growth paths is given by $ARR = [(1 + \lambda)/w] \sum_i^3 \psi_i(\hat{b}^i/h^i)$.²⁷

Observed replacement rates vary depending on the life experience of workers. For an average worker, pension represents 81.2% of average earnings.²⁸ We obtain a value equal to 0.95. For the sake of comparison, Conesa and Garriga (1999) obtain 0.72. As for inequality, the Gini index equals 0.1389.

Defining the (balanced growth rate) internal rate of return for individual i , IRR^i , as that rate of return for which the sum of discounted values of his/her contributions equals the sum of discounted values of his/her pension payments, IRR^i is given by

$$0.53\tau^{ss}wh^i + \frac{0.53\tau^{ss}wh^i(1 + \lambda)(1 - \ell^i)}{1 + IRR^i} = \frac{\hat{b}^i(1 + \lambda)\ell^i}{1 + IRR^i}. \quad (40)$$

Following Jimeno and Licandro (1999), we adjust the contribution rate τ^{ss} by the coefficient 0.53, because the expenditure on retirement pensions approximately represents a 53% share of total contributive pensions.

Taking into account that 1 period in our model represents 32.5 years, an approximate measure of the annualized social security internal rate of return for individual i can be given by $irr^i = (1 + IRR^i)^{1/32.5} - 1$. We obtain that $irr^1 = 3.88\%$, $irr^2 = 2.240\%$ and $irr^3 = 2.243\%$; this yields a weighted average of 2.57%. Jimeno and Licandro (1999) claim that [depending on the number of active (contributed) years, retirement age and life expectancy] the observed values range between 3.7% and 5.03%. As expected, the internal rate of return is higher for type-1 individuals (*i.e.* those receiving the minimum pension) and lower for type-3 individuals. Note that in this economy, even though the contribution rates and replacement rates are constant, the existence of the minimum pensions means that the Social Security system is progressive.

•**The penalty parameters.** As for α_0 and α_1 in the penalty function Eq. (6), remember that $\alpha_1 \equiv (1 - \alpha_0)/(\ell_U - \ell_N)$, $\alpha_0 \in (0, 1)$, $\ell_N \in (\ell_L, \ell_U)$. Setting normal

²⁷ Note that, along balanced growth paths, the replacement rate for type-1 retirees is given by $\hat{b}^{\min}(1 + \lambda)/(wh)$, and that of type-2 and type-3 is equal to $q\tau^{\text{rep}}(1 - \tau^{ss})$.

²⁸ See *OECD* (2005), p. 172.

retirement age (NRA) equal to 65, and early retirement age (ERA) equal to 60, and an 8% penalty per year of advanced retirement, makes $\alpha_0 = 1 - 0.08 \times (NRA - ERA) = 0.6$. On the other hand, taking into account, once more, that 1 period represents 32.5 years and that individuals are assumed to become optimizing agents at 15, $\ell_N = 1 - (NRA - 47.5)/32.5 = 0.462$. As a result of the values assigned to α_0 , ℓ_U and ℓ_N , one obtains that $\alpha_1 = 2.6$.

•**Production technology parameters.** Concerning production technology, the participation of capital income in total income is set equal to $\alpha = 0.35$. Conesa and Garriga (1999) set it at 0.375 and Garriga and Manresa (1999) equal to 0.33. The depreciation rate of physical capital is set at $\delta = 1 - (1 - 0.06)^{32} = 0.862$, as in Conesa and Garriga (1999). The scaling factor F is set at 1.09.

•**Growth.** Finally, concerning growth parameters, we assume $\rho_0 = 0.25$ and $\rho_1 = 2.275$. This way we are able to replicate the observed yearly per capita growth rate of 0.02 [what implies that $\lambda = (1 + 0.02)^{32.5} - 1 = 0.903$].

Table II summarizes the parameter values for the benchmark case, and Table III compares simulated and observed values for the main magnitudes.

TABLE II. BENCHMARK CASE: PARAMETER VALUES

	Parameter	Value	Parameter	Value
Demographics	n	0.270		
Preferences	β	1.323	ξ	0.255
Ability	θ_1	1.000	Ψ_1	0.200
	θ_2	1.326	Ψ_2	0.454
	θ_3	1.941	Ψ_3	0.346
Learning	γ	0.390		
Technology	F	1.090	α	0.350
	δ	0.862		
Social Security	\widehat{b}^{\min}	0.095	α_1	2.600
	α_0	0.600	ℓ_U	0.615
	τ^{ss}	0.283	ℓ_L	0.308
	ℓ_N	0.462	τ^{rep}	1.000
Growth	ρ_0	0.250	ρ_1	2.275

KEY TO TABLE II. The parameters have already been introduced in the text.

TABLE III. BENCHMARK CASE: RESULTS

Variable	Data	Model	Variable	Data	Model
$(wh_2)/(wh_1)$	1.38	1.37	$(wh_3)/(wh_2)$	1.52	1.53
R^1	60.0	60.0	R^2	65.0	64.1
R^3	65.0	64.3	irr^1	—	3.88
irr^2	—	2.240	irr^3	—	2.243
irr	—	2.57	ARR	0.812	0.95
b^1	—	\hat{b}^{\min}	r_{annual}	—	5.4
λ_{annual}	0.02	0.02	$(wh_1)/\hat{b}^{\min 29}$	1.0	0.98
n_{annual}	0.7	0.7	I_G	0.34	0.14

KEY TO TABLE III. $(wh_2)/(wh_1)$: ratio of gross hourly wage rate of type-2 individuals to that of type-1 individuals. $(wh_3)/(wh_2)$: ratio of gross hourly wage rate of type-3 individuals to that of type-2 individuals. R^i : type- i individual's optimal retirement age. irr^i : type- i annualized internal rate of return. irr : average annualized internal rate of return. r_{annual} : annualized per cent rate of interest. b^i : type- i individual's pension benefits. ARR : average gross replacement rate. λ_{annual} : annualized per capita growth rate. n_{annual} : annualized per cent population growth rate. Our I_G is not strictly comparable to observed Gini's index of 0.34 as the latter refers to yearly gross wage income in 2002.³⁰ The rest of the notation has already been introduced in the text.

4.2. Findings

We perform some numerical exercises to see what our theoretical model predicts about the response of the economy (in terms of incentives of human capital investment, early retirement, growth, social security budget, internal rate of return and replacement rate of the pension system, and life-time income inequality) upon changes in social security policies: minimum pension, normal and early retirement ages, and annual penalty rate for early retirement. Results are summarized in Table IV.

•**Minimum pension.** Consider changes (increments) in the minimum pension while keeping the remaining parameters constant. Thus, assume that \hat{b}^{\min} is increased in a 5%, 10%, 15% and 20%.

For these exogenous changes, patterns are clear. Retirement ages do not change for any type of individuals: only type-1 workers keep on choosing retirement at early retirement age, *i.e.* even type-2 individuals retire at the same age as in the benchmark case.

²⁹ Jiménez-Martín & Sánchez (2007) show that in Spain in 2000 the minimum pension was very similar to the legislated Minimum Wage. However, in our model, we force the minimum pension close to the average wage received by the least educated individuals.

³⁰ See *Encuesta de Estructura Salarial 2002*, p. 3, Instituto Nacional de Estadística.

Since the proportion of individuals receiving the minimum pension does not change (ψ_1), and nor does the fraction of their second period in which they receive the pension (ℓ^1), an increase in \hat{b}^{\min} implies a rise in the social security pension expenses and, consequently, a fall in the per capita social security surplus, $\hat{s}s$.

Given the preferences that we are assuming, a reduction in $\hat{s}s$ (which is lump-sum transferred to workers in their second period of life, *i.e.* old individuals) gives rise to an increase in the first year savings of all individuals. However, it can be shown that type-1 workers (*i.e.* those being paid the increased minimum pension) reduce their first year savings substantially. This way, even though they represent only 20% of all workers, the net effect is a *drop* in aggregate young workers' savings. The expected result is a fall in the stock of capital per worker (in efficiency units).

If this is the case, the effect upon incentives for education expenditure is clearly understood. A lower k means not only a lower wage rate w_n (*i.e.* lower labor income in both periods: when the worker is young and when old), but also a lower discounted value of old age labor income and pension payments (as $R \equiv 1 + r$ gets higher). [See Eqs. (8) and (12).] This explains why education expenditure, \hat{e} , and individual human capital, h , fall for all types of workers, included type-3 workers, so that the per capita growth rate λ ends up falling too.

Regarding the internal rate of return, *irr*, as expected, the corresponding internal rate of return of type-1 individuals increases the most with increments of the minimum pension (around the same proportion). As for type-2 and type-3 individuals, changes in their rates of return are negligible.

The inequality index, I_G , falls: retirement ages do not change, type-1 individuals retire with higher pensions, while type-2 and type-3 individuals' pension payments (slightly) fall: these two types of workers' pensions are earnings related and, as explained above, labor incomes fall. Therefore, the dispersion of labor earnings (both when active and when retired, interpreting pensions as deferred labor earnings) must necessarily fall.

The average replacement rate, ARR , necessarily increases if the minimum pension is augmented: type-1 pensioners' goes higher, while those of type-2 and type-3 stay constant.

•**Early retirement age (ERA).** What happens if the social security policy makers decide to increase the early retirement age from 60 to 61, 62 or 63?³¹ This is accomplished in our model by reducing the upper bound ℓ_U from 0.615 to 0.585, 0.554 and 0.523, respectively. In order to keep the penalty per year for retiring before normal age constant, we change the values for α_0 and α_1 accordingly³².

³¹ See Cutler, Liebman and Smyth (2006) for an analysis of the *optimal* earliest eligibility age for retirement.

³² Note that, given our period convention, $\ell_U = 1 - (ERA - 47.5)/32.5$. Keeping an 8% penalty per year

Upon increases in ℓ_U , workers who initially (*i.e.* in the benchmark case) retire at 60 will retire later, at 61 (62 or 63, respectively): this is the case for type-1 (lower innate ability and labor income) workers. Retirement ages for type-2 and type-3 individuals remain about the same as in the benchmark case: an increase of 3 years in the *ERA* would just make these workers advance their voluntary retirements about 1.2 months.

Concerning other variables, the rise in the *ERA* implies an increase in the total social security surplus, $\hat{s}s$. Following a similar reasoning to the one that we have just seen when commenting on the effects of increments in the minimum pension, given individuals' preferences, this leads to a drop in all workers' first year savings, so that the stock of capital per worker (in efficiency units), k , goes down.

As in the case of increments in the minimum pension, the negative effect upon incentives for education expenditure for type-2 and type-3 workers is transparent: if k falls, so do the wage rate w_n and the discounted values of both second period labor income and pension payments. Thus, \hat{e}^2 and \hat{e}^3 fall. [See Eq. (8).] The case for type-1 workers, however, is completely different. Why? These workers end up postponing retirement, thereby increasing their active life-time (*i.e.* the time span along which they enjoy earnings as an increasing function of their education investment). The result is that, despite the described effects on w_n and R , the increment of $1 - \ell$ leads to higher incentives to spend on education, so that \hat{e}^1 becomes higher. [See Eq. (12).] Once we have seen the response of education investments, it becomes straightforward to see why the growth rate, λ , falls.

Regarding the internal rate of return, patterns naturally differ for type-1, on the one hand, and for type-2 and type-3 individuals, on the other. The former, first, postpone retirement and, second, their pension benefits remain constant (they are paid the minimum pension in all cases): irr^1 should necessarily fall, as it does. As for irr^2 and irr^3 , the changes that they display are negligible: given the changes in k that we have seen above, both contributions and (earnings related) pension benefits fall, and the net effect on the internal rates of return turns out to be positive. Given the signs and the magnitudes of the variations in the individual internal rates of return, it is not surprising that the aggregate, irr , ends up falling.

The inequality index in this economy falls: type-1 workers' labor income goes up and, additionally, for an extended time span; and type-2 and type-3 workers' labor incomes and pension benefits fall. Inequality must fall as it is the case.

Finally, the replacement rate for type-1 retirees remains (about) the same, while those of type-2 and type-3 fall. The case of type-1 workers is the compound result of the effects

on advanced retirement yields $\alpha_0 = 1 - 0.08 \times (NRA - ERA)$, where *NRA* denotes *Normal Retirement Age*. Finally, remembering that $\alpha_1 = (1 - \alpha_0)/(\ell_U - \ell_N)$, one recognizes that changes in *ERA* imply changes in α_0 , ℓ_U and α_1 .

on λ , w and h ; while the result for type-2 and type-3 workers is the consequence of changes in q , the (absence of) penalty for early retirement [See footnote 27.]. The effect on the aggregate ARR is, of course, a drop.

•**Normal retirement age (NRA)**. We proceed next to changes in the normal retirement age. Suppose now increases in the NRA , for instance from 65 to 66, 67 and even 68.³³ This means that ℓ_N goes down in our model from 0.462 to 0.431, 0.400 and 0.369, respectively.³⁴ As before, we keep the penalty per year at a constant 8% rate. Therefore, we change the values for α_0 and α_1 accordingly, as mentioned above.

The increase in the normal retirement age (slightly) increases the retirement age of type-2 (to 64.4, 64.7, 64.9) and type-3 workers (to 64.6, 64.9, 65.1). This is an expected result given the nature of the exogenous change and that the (*yearly*) penalty for early retirement has not changed. Type-1 individuals do not change their retirement age as they still retire at 60.0 (early retirement age) despite the big penalty that their pension benefits suffer (as α_0 becomes equal to 0.521, 0.439 and 0.360, respectively.)

As for the social security surplus, $\hat{s}s$, it increases in a remarkable way: 7.1%, 14.2% and 20.9% for $NRA = 66, 67$ and 68 , respectively. This is the natural after checking that type-1 retirees still get paid the minimum pension, but that type-2 and type-3 retirees' pension benefits suffer substantial reductions. For instance, type-3 workers' pension would drop 6%, 12.2% and 18.2%, respectively. Why do pension payments fall that much? This is so because, as we will shortly see, education expenditure, \hat{e} , (and, therefore, individual human capital, h), the wage rate, w_n , and the (absence of) penalty, q , fall. [See Eq.(5).]

If one reasons as in the case of increments in the minimum pension, this implies a drop both in all workers' first year savings and, consequently, in the stock of capital per worker, k .

What about education? Of course: all individuals (both those being paid the minimum pension and those whose pension benefits are earnings related) see their education investment incentives fall. The (net) wage rate, w_n , falls and the discount factor, R , rises and, additionally, the penalty for early retirement that type-2 and type-3 suffer remarkably increases: note that α_0 falls and, therefore, so does q . For instance, q^2 drops from 0.93 to 0.87, 0.81 and 0.75. [See, once again, Eqs. (8) and (12).] The drop in the per capita growth rate, λ , needs no further explanation.

Concerning the internal rates of return, that of type-1 individuals, irr^1 , hardly changes

³³ Due to the increase in workers' longevity, and its effects on the financial sustainability of current unfunded social security systems, Western economies have long considered raising the normal retirement age. [See, *e.g.* Blanchet *et al.* (2005).] Mastrobuoni (2006) reports empirical evidence for the U.S. economy suggesting that the mean retirement age of the affected cohorts has increased by about half as much as the increase in the NRA .

³⁴ Note that $\ell_N = 1 - (NRA - 47.5)/32.5$.

and due to variations in w , h^1 and λ : the main determinants for irr^1 , *i.e.* pension payment, \hat{b}^1 , and retirement, R^1 , (or its equivalent, leisure, ℓ^1) remain unchanged. [See Eq. (40).] Internal rates of return for type-2 and type-3 individuals, however, sharply fall in about the same proportion for these two workers: -18.2% , -39.2% and -62.3% for $NRA = 66$, 67 and 68 , respectively. The intuition for the result is straightforward if one uses the same reasoning as for type-1 workers: as we have noticed above, pension payments fall substantially and retirement is slightly postponed. Given the changes in the irr^i 's for the three kinds of workers and their distribution, the aggregate rate of return turns out to fall: -12.7% , -27.3% and -43.3% for $NRA = 66$, 67 and 68 , respectively.

Inequality falls: 0.68% (for $NRA = 66$), 1.36% (for $NRA = 67$) and 2.0 (for $NRA = 68$): first, as we have seen, education (and, thereby, labor income) falls for type-2 and type-3 individuals in slightly higher proportions than for type-1 individuals; and, second, more importantly, pension payments for type-2 and type-3 workers remarkably drop.

Finally, the aggregate replacement rate substantially falls: for instance, for $NRA = 68$, ARR drops by 14.5% . It can be shown that type-1 retirees' replacement rate stays (almost) constant, but those of type-2 and type-3 pensioners are reduced remarkably. Why so? The intuition must be found in that for the case of earnings related pensions, that rate is given by $q^i \tau^{rep}(1 - \tau^{ss})$ and, as commented on above, q^2 and q^3 become much lower.

• **Early retirement age penalty.** Suppose now, finally, changes in the early retirement age penalty, for instance from 8% to 8.5% , 9.0% and 9.5% per year. This means that α_0 decreases in our model from 0.60 to 0.575 , 0.550 and 0.525 , respectively.³⁵ In other words, if individuals *retire at the early retirement age and their pensions are earnings-related*, they will have a penalization, $1 - q$, of 42.5% , 45.0% and 47.5% , respectively. Type-1 individuals are paid the minimum pension, so that they turn out *not* to be penalized for early retirement. This explains why, despite the large drops in α_0 , they still find it optimal to retire at 60 . However, reductions in α_0 do affect type-2 and type-3 individuals' retirement decision, although marginally. For instance, type-3 workers' retirement age would rise from 64.3 to 64.5 , 64.8 and 65.0 (*i.e.* no penalty at all), respectively.

Regarding other variables, the rise in the penalty implies an increase in the total social security surplus, $\hat{s}s$. [It can be shown that even though the (per old individual) pension payment $\sum_{i=1}^3 \psi_i \hat{b}^i \ell^i$ goes up, and the young workers' (per old individual) contributions $\tau^{ss} w(1+n) \sum_{i=1}^3 \psi_i h^i$ go down, the old workers' (per old individual) contributions $\tau^{ss} w \sum_{i=1}^3 \psi_i h^i (1 - \ell^i)$ rise so much that $\hat{s}s$ ends up being higher.] [See Eqs. (37) and (38).]

Reasoning in a similar way as in previous policy experiments, given individuals' pref-

³⁵ Remember that $\alpha_0 = 1 - \pi(NRA - ERA)$, where π equals 0.08 (in the benchmark case), 0.085 , 0.09 and 0.095 , respectively, and where $NRA = 65$ and $ERA = 60$.

erences, this gives rise to a fall in all workers' first year savings and in the stock of k , so that w_n falls and $1 + r$ gets higher. Not surprisingly, education expenditure for type-1 individuals falls. [See Eq. (12).] As for type-2 and type-3 workers, things are slightly different. Even though it can be shown that the terms $q^i \tau^{\text{rep}} \ell^i$ and $(1 + \lambda)(1 - \ell^i)$ rise, the just mentioned responses of w_n and $1 + r$ lead to a decrease in the incentives for education expenditure. [See Eq. (8).] As it has become familiar by now, the negative response of the per capita growth rate, λ , is the natural result.

Concerning the internal rate of return for type-1 workers, contributions $\tau^{ss} w h^1$ fall and pension payments, $\hat{b}^{\min} \ell^1$, keep invariant, so that their internal rate of return, irr^1 , slightly rises. However, the internal rates of return for type-2 and type-3 individuals drop. Remember the argument just used above to give a rationale to the rise in \hat{s} : although the pension payments for these two types of workers, $\hat{b}^i \ell^i$, get higher and the young workers' contributions, $\tau^{ss} w h^i$, go down, the old workers' contributions $\tau^{ss} w h^i (1 - \ell^i)$ rise so much that irr^2 and irr^3 , and the aggregate irr , slightly fall.] [See Eq. (40).]

Income inequality increases: pension payments, $\hat{b}^i \ell^i$, and second period labor income, $w_n h^i (1 - \ell^i)$, for type-2 and type-3 workers become higher, while those of type-1 workers stay constant [$\ell^1 = 1$, and $\hat{b}^1 = b^{\min}$].

Lastly, the average replacement rate gets higher as all types of individuals' rates are increased: in the case of type-1 workers, both w and h^1 fall; in the case of type-2 and type-3 workers, q^2 and q^3 rise.

TABLE IV. COMPARATIVE STATICS RESULTS

	R^1	R^2	R^3	$\Delta \hat{e}^1$	$\Delta \hat{e}^2$	$\Delta \hat{e}^3$	$\Delta \lambda$	$\hat{s}s$	Δk	Δirr^1	Δirr^2	Δirr^3	Δirr	ΔI_G	ΔARR
$\Delta \hat{\delta}^{\min}$	5	60.0	64.1	64.3	-0.03	-0.04	-0.007	-0.88	-0.05	4.87	0.004	0.003	1.47	-0.42	1.08
	10	60.0	64.1	64.3	-0.06	-0.07	-0.013	-1.75	-0.09	9.48	0.007	0.006	2.87	-0.83	2.16
	15	60.0	64.1	64.3	-0.09	-0.01	-0.020	-2.63	-0.14	13.85	0.011	0.009	4.19	-1.24	3.23
	20	60.0	64.1	64.3	-0.13	-0.14	-0.027	-3.50	-0.18	18.01	0.015	0.012	5.45	-1.66	4.31
$\Delta \ell_U$	61.0	61.0	64.1	64.3	1.10	-2.26	-0.047	1.12	-0.30	-6.56	0.008	0.015	-1.97	-0.27	-0.09
	62.0	62.0	64.1	64.3	2.23	-0.52	-0.096	2.28	-0.60	-13.89	0.016	0.029	-4.18	-0.55	-0.18
	63.0	63.0	64.0	64.2	3.32	-0.77	-0.144	3.41	-0.89	-21.68	0.023	0.043	-6.52	-0.82	-0.26
$\Delta \ell_N$	66.0	60.0	64.4	64.6	-0.17	-0.85	-0.156	7.08	-0.23	0.01	-18.24	-18.09	-12.69	-0.68	-4.74
	67.0	60.0	64.7	64.9	-0.35	-1.70	-0.312	14.18	-0.46	0.02	-39.20	-38.85	-27.26	-1.36	-9.66
	68.0	60.0	64.9	65.1	-0.52	-2.50	-0.460	20.88	-0.69	0.03	-62.29	-61.66	-43.28	-2.00	-14.46
	$\Delta \alpha_0$	8.5	60.0	64.4	64.5	-0.51	-0.25	-0.05	0.23	-0.74	0.26	-0.55	-0.45	-0.27	0.15
	9.0	60.0	64.6	64.8	-0.98	-0.46	-0.09	0.29	-1.43	0.51	-0.70	-0.51	-0.28	0.31	3.21
	9.5	60.0	64.8	65.0	-1.42	-0.65	-0.12	0.21	-2.07	0.74	-0.50	-0.24	-0.05	0.47	4.90

KEY TO TABLE IV. $\Delta \hat{\delta}^{\min}$: percent change in minimum pension. $\Delta \ell_U$: new early retirement age, ERA , as a result of a change in ℓ_U . $\Delta \ell_N$: new normal retirement age, NRA , as a result of a change in ℓ_N . R^i : type- i individuals' retirement age. $\Delta \hat{e}^i$: percent change in type- i individuals' education expenditure. $\Delta \lambda$: percent change in annual per capita growth rate. Δk : percent change in capital per worker in efficiency units. $\hat{s}s$: per capita social security lump-sum transfer. Δirr^i : percent change in type- i individuals' annualized internal rate of return. Δirr : percent change in average annualized internal rate of return. ΔI_G : percent change in I_G . $\Delta \alpha_0$: new annual percent penalty for early retirement (right) as a result of a change in α_0 . ΔARR : percent change in average replacement rate.

5. Conclusions and final remarks

In this paper we have analyzed the effects of policy changes of a pay-as-you-go social security system in a two-period, OLG economy with these main features: *i*) pensions are earnings-related, *ii*) there exists a minimum pension, *iii*) social security surplus is lump-sum transferred to individuals living in their second period, *iv*) retirement age is endogenous, *v*) early retirement is penalized, *vi*) growth is driven by most skilled individuals' investment in education, and *vii*) workers differ in their innate ability.

Given that pension payments are earnings-related, when individuals choose their optimal level of education, they take into account not only the effect on future labor earnings, but also on future pension benefits. Consequently, social security introduces an incentive for higher investment in human capital. This incentive, however, partly breaks down due to the minimum pensions.

Individuals' second period labor supply is elastic. Therefore, the voluntary retirement age depends on the incentives that the public pension system embeds: not only minimum pensions, but also penalties for early retirement.

We have calibrated the model and constructed a benchmark case which fairly reproduces some stylized facts of the Spanish economy. Starting from this baseline case, we have analyzed the effects of changes in *i*) minimum pensions, *ii*) early retirement penalty scheme, and *iii*) early and normal retirement ages.

Our main conclusions follow:

(1) Our model predicts that more skilled individuals enjoy a higher return on their investment in education and, consequently, spend more on education.

(2) The existence of a minimum pension, however, may reduce low skill individuals' incentives to invest in human capital.

(3) Since the per capita growth rate of the economy depends on the educational attainment of the more skilled individuals, any reform that decreases the incentives of these individuals to invest in their education will decrease the per capita growth rate. In particular, we have found that policies that increase the minimum pension, the early retirement penalty, the minimum legal retirement age and the normal retirement age have this effect.

(4) There is no monotonic relationship between per capita growth and income inequality: increments in the minimum pension, in the early retirement penalty, in the minimum legal retirement age and in the normal retirement age lead to lower growth rates; but the three first policies induce lower income inequality, while the fourth one gives rise to higher inequality.

(5) Large (up to 20%) increments in minimum pension cause no change in retirement ages for medium and most skilled individuals.

(6) Increments in the early legal retirement age (up to 63) give rise to higher voluntary retirement ages (in a one to one relationship) and human capital investment for the least skilled workers.

(7) Increments in normal retirement age (up to 68) lead to higher voluntary retirement ages for intermediate and high skill workers, while low skill workers still leave the labor force at the minimum legal retirement age. As for per capita growth, this is the policy measure with the strongest (negative) effect: the resulting increment in the social security surplus remarkably reduces private (young) savings and physical capital accumulation, leading to a fall in incentives for education expenditure.

(8) Increments in early retirement penalty imply higher voluntary retirement ages for intermediate and high skill workers, while low skill workers' is unaffected because the minimum pension is nit subject to early retirement penalty.

6. APPENDIX

The steady state competitive equilibrium is characterized by the following equations.

From Eqs. (15) and (6) we obtain, respectively:

$$q^i \tau^{\text{rep}} w_n \underline{\theta} \{1 + [\hat{e}_1(\underline{\theta})]^\gamma\} = \hat{b}^{\text{min}}(1 + \lambda), \text{ and}$$

$$q^i = 1 - \alpha_1(\ell^i - \ell_N),$$

where $w_n = (1 - \tau^{\text{ss}})w$.

From Eq. (24), we obtain

$$w = (1 - \alpha)Fk^\alpha \quad \text{and} \quad r + \delta = \alpha Fk^{\alpha-1}.$$

From Eqs. (28)-(39), we obtain

$$h^i = \theta_i [1 + (\hat{e}^i)^\gamma],$$

$$\hat{c}_y^i + \hat{s}_y^i + \hat{e}^i = w_n h^i,$$

$$\hat{c}_o^i = (1 + r)\hat{s}_y^i + \ell^i \hat{b}^i (1 + \lambda) + (1 - \ell^i)w_n h^i (1 + \lambda) + \hat{s}s,$$

$$\hat{e}^i = \begin{cases} \left\{ \gamma \theta_i w_n \left[1 + \frac{(1 - \ell^i)(1 + \lambda)}{R} \right] \right\}^{\frac{1}{1 - \gamma}}, & \text{for } \theta_i < \underline{\theta}, \\ \left\{ \gamma \theta_i w_n \left[1 + \frac{q^i \tau^{\text{rep}} \ell^i}{R} + \frac{(1 + \lambda)(1 - \ell^i)}{R} \right] \right\}^{\frac{1}{1 - \gamma}}, & \text{for } \theta_i \geq \underline{\theta}, \end{cases}$$

where $R \equiv 1 + r$, and

$$\hat{b}^i = \begin{cases} \hat{b}^{\text{min}}, & \text{for } \theta_i < \underline{\theta}, \\ \frac{q^i \tau^{\text{rep}} w_n h^i}{1 + \lambda}, & \text{for } \theta_i \geq \underline{\theta}, \end{cases}$$

$$\hat{c}_o^i = \beta R \hat{c}_y^i,$$

$$\xi \hat{c}_o^i = \ell^i w_n h^i (1 + \lambda) - \ell^i \left[\hat{b}^i + \ell^i \frac{\partial \hat{b}^i}{\partial \ell^i} \right] (1 + \lambda), \quad \text{for } \ell_L < \ell^i < \ell_U,$$

where

$$\frac{\partial \hat{b}^i}{\partial \ell^i} = \begin{cases} 0, & \text{if } \ell_L < \ell^i < \ell_N \text{ or } \hat{b}^i = \hat{b}^{\min} \\ \frac{-\alpha_1 \hat{b}^i}{q^i}, & \text{if } \ell_N < \ell^i < \ell_U \text{ and } \hat{b}^i > \hat{b}^{\min} \end{cases}$$

and

$$\sum_{i=1}^3 \psi_i \hat{s}_y^i = k(1 + \lambda) \left[(1 + n) \sum_{i=1}^3 \psi_i h^i + \sum_{i=1}^3 \psi_i h^i (1 - \ell^i) \right],$$

$$\tau^{ss} w \left[(1 + n) \sum_{i=1}^3 \psi_i h^i + \sum_{i=1}^3 \psi_i h^i (1 - \ell^i) \right] = \sum_{i=1}^3 \psi_i \hat{b}^i \ell^i + \hat{s} \hat{s},$$

$$\lambda = \rho_1 (\hat{e}^3)^{\rho_0} \geq 0.$$

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