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Abstract

We analyze optimal second-best emission taxes in a durable good industry under imperfect competition. The analysis is performed for three different types of emissions and for situations where the good is rented, sold or simultaneously sold and rented. We show, for durable goods that may cause pollution in a period (or in periods) different from the production period, that the expected overall emission tax and the expected total marginal environmental damage per unit produced in each period are the relevant variables to consider in the analysis of overinternalization and in the comparison of optimal emission taxes for renting, selling and renting-selling firms. Our results allow to extend some previous results in the literature to these durable goods and provide an adequate perspective on some other results (in particular, we point out the limitations of focusing only, for those durable goods, on the level and effects of the optimal emission tax in the production period).

JEL classification codes: H23, Q58, Q53, L13.

Keywords: optimal emission taxes, durable good, overinternalization, emission types, imperfect competition.

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1 Introduction

The production and consumption of durable goods can result in various types of pollution. For instance, environmental damage during the use of cars is the major contributor to air pollution in the form of smog and exhaust fumes, there are emissions such as smoke and water contamination when some durable goods are produced and solid waste at the end of a product's lifetime may also cause environmental damage. In order to make firms and consumers internalize this pollution damage, a regulator could consider imposing an emissions tax.

Over the past decade OECD countries have increased the number of environmentally related taxes imposed in order to reduce emissions. These taxes have often been imposed on the stock of the durable good in use in each period or at the end of the product's lifetime, although there are also taxes on production of polluting durable goods. For example, a number of OECD member countries (as Denmark, Germany, Ireland, Luxembourg, Sweden, United Kingdom and Cyprus) are now applying some form of CO_2 related taxes on the use of motor vehicles. These taxes are paid annually by the owners of the vehicles in order to be allowed to use their vehicles.[1] Moreover, in some markets Governments have assigned to producers the responsibility, financial and/or physical, for the treatment or disposal of their products at the end of life. For instance, the European Directive 2000/53/EC requires car manufacturers to take back end-of-life vehicles free of charge and, in most EU countries, each manufacturer has decided to launch its own program by contracting with car dismantlers and shredders, and the Directive 2002/96/EC of the European Parliament and of the Council states that producers should finance collection from collection facilities, and the treatment, recovery and disposal of waste electrical and electronic equipment.[2][3]Another example include the levy on plastic shopping bags first introduced by the Irish Government in 2002.[4]

Economic literature has studied the relationship between environmental

[1]See ACEA (2012) and OECD (2009) for an overview of the CO2-based taxation schemes implemented in some european countries.

[2]In the United States, there are currently 25 states that have state-wide waste electrical and electronic equipment legislation (See NERIC 2012). In July 2011, the Interagency Task Force created by President Obama "to prepare a national strategy for responsible electronics stewardship, including improvements to Federal procedures for managing electronic products" issued its report, now called the National Strategy for Electronics Stewardship Report.

[3]See also Ongondo, Williams and Cherrett (2011) for an analysis of the current waste electrical and electronic equipment management practices, policies and legislation in various countries and regions over the world and references considered therein.

[4]See http://plasticbags.planetark.org/gov/othercountries.cfm for an overview of fees placed on plastic bags in some countries (for a conprehensive list of many states and cities in the USA that have implemented plastic bags fees see http://plasticbaglaws.org/legislation/state-laws/).

policy and market structure. Under perfect competition, external damage is fully internalized when the per unit emission tax equals the marginal external damage. Under a monopoly, however, as first noted by Buchanan (1969) complete internalization imposes an additional social cost by further restricting the already sub-optimal monopolist's output, so the optimal emission tax is less than the marginal external damage; see also Barnett (1980). As production under imperfect competition is below the efficient level, due to firms' market power, optimal emission taxes under imperfect competition are, in general, below marginal environmental damage. This implies underinternalization of environmental damage. However, the relevant literature has also demonstrated the possibility of overinternalization when there is imperfect competition. Katsoulacos and Xepapadeas (1995) show that under a fixed-number oligopoly the optimal emission tax falls short of the marginal external damage but that with free entry, so that the market structure is determined endogenously, the optimal tax may exceed the marginal environmental damage. Simpson (1995) also shows the possibility of overinternalization under a Cournot duopoly with asymmetric costs of production, in order to redistribute output from the less efficient producer to his more efficient rival.

Other authors discuss overinternalization in a durable goods setting. For instance, Boyce and Goering (1997) derive that the optimal emission tax in the present may exceed marginal environmental damage under a monopoly that sells its product, when durability is exogenous, emissions occur during the production process and there are increasing returns to scale in production. Runkel (2002) shows that underinternalization results when there is an oligopoly of firms that rent their product, emissions occur during the production process and durability is endogenous. In a context with constant returns to scale in production and exogenous product durability, Runkel (2004) proves that the optimal waste taxes lie below the marginal environmental damage in the present and in the future, under a monopoly that sells its product, but that there may be overinternalization in the present when there is an oligopoly of firms that sell their product. He also extends the analysis of Goering and Boyce (1997) to show that overinternalization may occur under a monopoly that sells its product, with endogenous durability and constant returns to scale in production. However, their analysis of the internalization of environmental damage has centered on the relationship between the optimal emission tax in the present and marginal environmental damage in the present. But when emissions in each period are proportional to the stock of the product in use in that period or when they occur at the end of the product's lifetime, the environmental damage from a unit produced in the present is distributed throughout the lifetime of the product. Hence, a more adequate approach to the analysis of overinternalization in the present would be to compare the expected total emission tax paid per unit produced in the present and the expected overall marginal environmental damage caused by a unit produced in the present. So, an important innovative aspect in our paper is the consideration of that *overall* internalization.

When firms produce durable goods, there are several distortions from efficiency. First, producers do not take environmental damage into account when there are not emission taxes. Moreover, imperfect competition implies a distortion from efficient provision. Finally, when the good is durable and firms sell at least part of their production in the present there is a possible intertemporal distortion due to the strategic behavior of each firm to steal sales from its rivals in the present and in the future, in a context where the intertemporal consistency problem first noted by Coase (1972) applies. If the regulator uses only one instrument (emission taxes) to correct for all distortions from efficiency, the emission taxes that maximize total surplus will, therefore, be second-best optimal emission taxes.[5]

Some of the analyses of overinternalization with durable goods consider that emissions occur during the production process, as in Goering and Boyce (1997), while others center on situations where emissions occur at the end of the product's lifetime, as in Runkel (2004). Another contribution of this paper is to study whether the results on overinternalization are affected by the type of emissions: to this end, emissions that occur during the production process, emissions proportional to the stock of the durable good in use and emissions that occur at the end of the product's life are considered.

This work investigates the optimal second-best emissions taxation under imperfect competition in durable goods industries when products are sold, rented or simultaneously sold and rented. To the best of our knowledge, no such analysis has been carried out previously for firms that rent and sell their good simultaneously. However, there are markets in which there is simultaneous renting and selling of the durable good[6]. Bucovetsky and Chilton (1986) and Bulow (1986) prove that a monopolist facing the threat of entry chooses to sell part of the units supplied, instead of renting them all. Carlton and Gertner

[5] First-best tax-subsidy schemes are investigated in Runkel (1999).

[6] As indicated by Saggi and Vettas (2000), durable goods markets are primarily oligopolistic rather than monopolistic, and firms sell as well as lease goods. Examples include automobiles, house appliances, computers, copy machines, and machinery equipment.

(1989) show that, when there is no threat of entry, strategic interaction between rivals provides a reason for an oligopolist to choose to sell some of its output rather than rent it, in contrast to the behavior of a monopolist, which will choose to rent all its production. The reason for this behavior is that, when a firm sells a durable good in the present, it is depriving its rivals of current and future sales.

The purpose of this paper is thus to examine the extent to which the different practices used in the commercialization of durable goods (renting, selling and both renting-selling) and the existence of different types of emissions affects the relationship between environmental taxes imposed by a regulator and marginal external damage. We show that when firms sell their production and emissions in each period are proportional to the stock of the product in use in that period or they occur at the end of the product's lifetime, there may be overinternalization in the present. However, under those emission types the environmental damage from a unit produced in the present is distributed throughout the lifetime of the product. Hence, if the expected total emission tax paid per unit produced in the present and the expected overall marginal environmental damage caused by a unit produced in the present are compared, we show that, in all cases considered in our analysis, the expected total emission tax paid per unit produced in the present is lower than the expected overall marginal environmental damage per unit produced in the present. This result may provide an adequate perspective on some results on overinternalization in the previous durable goods literature.

We also compare the optimal emission taxes on renting firms, on selling firms and on renting-selling firms. We find that, when emissions are proportional to the stock of the durable good or when emissions occur at the end of the product's lifetime, the optimal emission tax in the first period on renting firms is higher than the optimal emission tax on selling firms. Nevertheless, we show that the expected total emission tax in the present is higher for selling firms than for renting firms, under any type of emissions. This latter result extends previous results in the literature obtained for the case where emissions occur in the production process.

The policy implications of these findings are substantial, as there are major polluting industries that produce durable goods and are highly concentrated (the car and aircraft industries, for instance). Knowing the characteristics of optimal emission taxes on durable goods industries is essential for public environmental policy. Our analysis shows that the expected overall emission tax and the expected total marginal environmental damage per unit produced in each period are the relevant variables to consider in the analysis.

The paper is organized as follows: Section 2 introduces the model. Market decisions with emissions taxes are investigated in Section 3. Section 4 presents the social optimum. In Section 5 we study if second-best optimal emissions taxes imply overinternalization and overall overinternalization of marginal environmental damage for the situations considered. Section 6 centers on the comparison of optimal emissions taxes when, in the present, firms only rent their output, when they only sell their output and, finally, when they both sell and rent their production. Finally, Section 7 concludes. All proofs are relegated to the Appendix.

2 Theoretical framework

We consider an oligopolistic industry with $n \geq 2$ identical firms that produce a homogeneous durable good. Entry into the industry is assumed to be unprofitable or unfeasible. There are two discrete periods of time: present ($t = 1$) and future ($t = 2$). We study the cases where, in the first period, firms only sell their product, firms only rent their product and firms may both sell and rent their production. Given that the second period is the last one, renting is identical to selling in that period.

We assume that, when firms only sell their output, they do not have commitment ability. The situation where firms sell their production but they can precommit to current buyers that the value of their stock of durable goods will be taken into account in future production (for instance, firms can precommit by offering best-price provisions) is analogous to the situation where firms rent their output[7].

We assume that all firms face the same production cost functions: $c_1(.)$ in the first period and $c_2(.)$ in the second period, with $c_1' > 0$, $c_1'' \geq 0$, $c_2' > 0$ and $c_2'' \geq 0$.

The inverse demand for services of the durable good is assumed to be constant over time. This inverse rental demand function for the services of the durable good in each period is $p(Q)$, where Q represents the quantity used by consumers in that period and $p'(Q) < 0$. We assume that marginal revenue is decreasing for each firm.

All agents participating in the market have perfect and complete information

[7] Bullow (1982) offers examples of markets in which renting is not feasible.

and potential users of the good have perfect foresight. We consider that there exits a perfect second hand market for the durable good. The discount factor is the same for all agents participating in the market and it is represented by $\rho\epsilon[0,1]$.

The durable good depreciates with time: only a proportion δ of the units produced during the first period can be used in the second period. We consider that durability is exogenous, so as to focus on the comparisons of results between renting, selling and renting-selling firms and on the consequences of a change in the type of emissions. However, we are well aware from since (1986) of the relevance of the choice of durability by producers in durable good markets.

The analysis proceeds in two stages. In the first stage the regulator sets emission taxes for the two periods. We consider that the regulator can commit to emission taxes and announces those taxes right at the beginning of the first period. In the second stage firms engage in quantity competition. Each firm chooses in every period its level of production and, in the case of renting-selling firms, the division of production between renting and selling in the first period, considering as given the decisions on production, renting and selling of its competitors. Firms' choices are simultaneous. The objective of each firm is to maximize its discounted sum of profits. The competition game among firms is, therefore, non-cooperative.

The solution concept used is that of a subgame perfect Nash equilibrium in pure strategies. In the cases of selling firms and of renting-selling firms each firm maximizes in each period the present discounted value of profits starting from that period. Therefore, the solutions are derived by backward induction from the last period of the second stage.

The following notation will be used for quantities at the firm level (for the corresponding quantities at the industry level we will use a Q, instead of a q, and eliminate the i subscript):

q^s_{1i}: quantity sold by firm i in the first period,

q^r_{1i}: quantity rented by firm i in the first period,

$q^s_{1i} + q^r_{1i}$: quantity produced by firm i in the first period,

q_{2i}: quantity sold (or rented) by firm i in the second period,

$q_{2i} - \delta q^r_{1i}$: quantity produced by firm i in the second period.

We consider situations where all units produced in $t = 1$ that do not depreciate are also used in $t = 2$ (this implies, for all i, that $q_{2i} \geq \delta q^r_{1i}$). The quantity of the durable good used in $t = 2$ in the market will be $Q_2 + \delta Q^s_1$.

Let us denote by p^s_1, p^r_1 and p_2, respectively, the (total) price paid by the buyer of a unit of the durable good in the first period, the (total) price paid by the renter of a unit of the durable good in the first period and the (total) price paid by the buyer (or renter) of a unit of the durable good in the second period. We have:

$$p_1^s = p(Q_1^s + Q_1^r) + \rho\delta p(\delta Q_1^s + Q_2)$$

$$p_1^r = p(Q_1^s + Q_1^r)$$

$$p_2 = p(\delta Q_1^s + Q_2),$$

Obviously, when firms only rent their product it will be $Q_1^s = 0$ and p_1^s will not be defined and when firms only sell their product it will be $Q_1^r = 0$ and p_1^r will not be defined. The possibility of arbitrage by consumers implies:

$$p_1^s - \rho\delta p_2 = p_1^r$$

If emission taxes are paid by producers, p_1^s, p_1^r and p_2 are also the market prices. Our presentation will follow this situation. However, for some types of emissions the emission taxes on sales are charged to consumers, as it often occurs when emissions are proportional to the stock of product in use.. In that case the total price paid by buyers of the durable good equals the sum of the corresponding emission tax and market price, or price received by producers. As we will show in section 3, the analysis and results in this work remain unchanged when emission taxes are paid by consumers, instead of being paid by producers.

There are three types of emissions that we consider: emissions that occur during the production process, emissions proportional to the stock of product in use in the market and emissions that occur at the end of the life of the product. We consider that environmental damage in period t, with $t = 1, 2$, is $\gamma(E_t)$, where E_t are total emissions in period t, and where $\gamma' > 0$ and $\gamma'' > 0$ for all $E_t \geq 0$. We may write:

$$E_1 = (1 - \delta + \alpha\delta)(Q_1^s + Q_1^r)$$

$$E_2 = \beta\delta(Q_1^s + Q_1^r) + Q_2 - \delta Q_1^r$$

where α and β depend on the type of emissions. If $\alpha = 1$ and $\beta = 0$ we have a situation where emissions occur during the production process. If $\alpha = 1$ and $\beta = 1$ we have a situation where emissions are proportional to the stock of product in use in the market. Finally, emissions occur at the end of the life of the product if $\alpha = 0$ and $\beta = 1$. The $(1 - \delta)(Q_1^s + Q_1^r)$ units produced in the first period that cannot be used in period 2, due to depreciation, cause emissions only in the first period under any of the three types of emissions. The $\delta(Q_1^s + Q_1^r)$ units produced in the first period that can be used in period

2 cause emissions only in period 1 when emissions occur during the production process, they cause emissions only in period 2 when emissions occur at the end of the product's life and they cause emissions in both periods when emissions are proportional to the stock of product in use. The expected emissions per unit produced in period 1 are $(1-\delta)+\alpha\delta$ in the first period and $\beta\delta$ in the second period.[8] A unit produced in the second period implies emissions equal to 1 in that period.

We assume throughout the paper that parameters and functions are such that we obtain interior solutions in each optimization problem and, therefore, non-negative quantities and prices of the durable good in each period.

3 Market decisions with emission taxes

In this section we study how emission taxes affect the market levels of production, renting and selling. Let us denote by τ_1 and τ_2, respectively, the emission tax paid in the first period per unit of emission in that period and the emission tax paid in the second period per unit of emission in that period. Hence, a unit produced in period 1 expects to pay $\beta\delta\tau_2$ in the second period and $(1-\delta+\alpha\delta)\tau_1$ in the first period. A unit produced in the second period pays τ_2 in that period.

Let π_1^i and π_2^i denote, respectively, the profits in period 1 and in period 2 of firm i, with $i=1,...,n$. The present value of firm i profits is then written as $\pi^i=\pi_1^i+\rho\pi_2^i$.

With these taxes, in period $t=2$ each active firm i, with $i=1,...,n$, solves the following problem (we present the general case with renting and selling, but we know that when firms only rent their product it will be $Q_1^s=0$ and when firms only sell their product it will be $Q_1^r=0$):

$$\max_{q_{2i}}\pi_1^i=\left[p(\delta Q_1^s+Q_2))q_{2i}-c_2(q_{2i}-\delta q_{1i}^r)-\tau_2(q_{2i}-\delta q_{1i}^r)-\beta\delta\tau_2\left(q_{1i}^s+q_{1i}^r\right)\right].$$

The first order condition of this problem is:

$$p(\delta Q_1^s+Q_2)+p'(\delta Q_1^s+Q_2)q_{2i}=c_2'(q_{2i}-\delta q_{1i}^r)+\tau_2 \qquad (1)$$

In equation (1) we have that marginal revenue for oligopolist i in $t=2$ equals total marginal cost in that period. Note that when firms sell at least part of

[8] When $\alpha=1$ the units of the good produced in $t=1$ that can also be used in $t=2$ cause emissions in the first period, and the contrary occurs when $\alpha=0$. When $\beta=1$ the units of the good produced in $t=1$ that can also be used in $t=2$ cause emissions in the second period, and the contrary occurs when $\beta=0$.

their output ($Q_1^s > 0$) they face the time inconsistency problem first noted by Coase (1972). As Coase (1972) pointed out, a durable goods monopolist that sells its output and has no commitment ability lacks any incentive to take into account the decline in the value of units previously sold that are in the hands of consumers when it decides its future output. This implies that the second period optimal production is implicitly determined by the first period production level. First period buyers realize that each firm will choose its second period production to satisfy (1). Thus, if consumers are rational, (1) becomes an "expectation constraint" on each firm. The higher is the discount factor, the more relevant is this "expectation constraint". When firms rent their output ($Q_1^s = 0$), however, they are not constrained by consumer' expectations of future production behavior since they own the entire stock of the good.

In period $t = 1$, each firm chooses the levels of sales and rentals, q_{1i}^s and q_{1i}^r, that maximize the present value of its profits. Thus, each firm i, with $i = 1, ..., n$, solves the following problem:

$$\max_{\{q_{1i}^r, q_{1i}^s\}} \pi_1^i + \rho\pi_2^i = [(p(Q_1^s + Q_1^r) - (1 - \delta + \alpha\delta)\tau_1)(q_{1i}^s + q_{1i}^r) - c_1(q_{1i}^s + q_{1i}^r)$$

$$+\rho\delta p(\delta Q_1^s + Q_2)q_{1i}^s$$

$$+\rho(p(\delta Q_1^s + Q_2)q_{2i} - c_2(q_{2i} - \delta q_{1i}^r) - \tau_2(q_{2i} - \delta q_{1i}^r) - \beta\delta\tau_2\,(q_{1i}^s + q_{1i}^r))]$$

subject to (1). Assuming interior solutions, the first order conditions of this problem are:

$$\begin{gathered} p(Q_1^s + Q_1^r) - (1 - \delta + \alpha\delta)\tau_1 + p'(Q_1^s + Q_1^r)(q_{1i}^s + q_{1i}^r) - c_1'(q_{1i}^s + q_{1i}^r) \\ +\rho\delta p(\delta Q_1^s + Q_2) + \rho\delta(\delta + \frac{dQ_2}{dq_{1i}^s})p'(\delta Q_1^s + Q_2)q_{1i}^s \\ +\rho(\delta + \frac{dQ_{2-i}}{dq_{1i}^s})p'(\delta Q_1^s + Q_2)q_{2i} - \rho\beta\delta\tau_2 = 0 \\ p(Q_1^s + Q_1^r) - (1 - \delta + \alpha\delta)\tau_1 + p'(Q_1^s + Q_1^r)(q_{1i}^s + q_{1i}^r) - c_1'(q_{1i}^s + q_{1i}^r) \\ +\rho\delta c_2'(q_{2i} - \delta q_{1i}^r) + \rho\delta\tau_2 - \rho\beta\delta\tau_2 = 0 \end{gathered} \tag{2}$$

subject to (1) (the first of these conditions is relevant when $Q_1^s > 0$ and the second is relevant when $Q_1^r > 0$; in the first condition it will be $Q_1^r = 0$ if firms only sell their output and in the second condition it will be $Q_1^s = 0$ if firms only rent their output).

Observe from (1) and (2) that there is a symmetric market solution for firms decisions. Moreover, note that conditions (1) and (2) hold also if emission taxes on sales are paid by buyers of the durable good. In this case buyers adjust their willingness to pay for the good to the emission taxes they will pay and the only change in the previous analysis is that the term $-\beta\delta\tau_2 q_{1i}^s$ will not be included in the maximization problem of $t = 2$. However, this change would not affect conditions (1) and the analysis and results in this work would remain unchanged.

4 Social optimum

We know that:

Total surplus (TS)=Consumer surplus+Profits of firms

+Taxes paid - Emissions damage.

Let us denote by Q_{1u} the quantity of the durable good used in the first period and by Q_{2u} the quantity used in the second period. The quantity of the good used in the first period is equal to the sum of the quantity sold and the quantity rented in that period, that is, Q_{1u} is also the quantity of the durable good produced in $t = 1$. The quantity used in the second period will be equal to the sum of the quantity sold (or rented) in the second period and the non-depreciated part of the quantity sold in the first period (or equal to the quantity produced in the second period plus the non-depreciated part of the quantity used in the first period). Hence, $Q_{1u} = Q_1^s + Q_1^r$ and $Q_{2u} = \delta Q_1^s + Q_2$. As the cost functions are not concave we obtain the social optimum dividing production equally among firms. With this notation we have:

$$\begin{aligned} TS = \textstyle\int_0^{Q_{1u}} p(Q)dQ + \rho \int_0^{Q_{2u}} p(Q)dQ - nc_1(\frac{Q_{1u}}{n}) - \rho nc_2(\frac{Q_{2u}-\delta Q_{1u}}{n}) \\ -\gamma((1-\delta+\alpha\delta)Q_{1u}) - \rho\gamma(Q_{2u} - (1-\beta)\delta Q_{1u}) \end{aligned} \tag{3}$$

To obtain the social optimum we solve, using (3):

$$\max_{Q_{1u}, Q_{2u}} TS$$

The first order conditions of this problem are:

$$\begin{aligned} &p(Q_{1u}) = c_1'(\tfrac{Q_{1u}}{n}) - \rho\delta c_2'(\tfrac{Q_{2u}-\delta Q_{1u}}{n}) \\ &+(1-\delta+\alpha\delta)\gamma'((1-\delta+\alpha\delta)Q_{1u}) - \rho(1-\beta)\delta\gamma'(Q_{2u} - (1-\beta)\delta Q_{1u}) \end{aligned} \tag{4}$$

and

$$p(Q_{2u}) = c_2'(\frac{Q_{2u} - \delta Q_{1u}}{n}) + \gamma'(Q_{2u} - (1-\beta)\delta Q_{1u}) \tag{5}$$

In equation (5), price in the second period equals marginal production cost in that period plus marginal environmental damage. In equation (4), (rental) price in the first period equals net expected marginal production cost in that period plus net expected marginal environmental damage from a unit produced in $t = 1$. Net expected marginal production cost refers to marginal cost in the first period net of expected marginal cost saved in the second period as, with probability δ, a unit produced in the first period will be in use during the second period and it will allow a reduction in new production in $t = 2$. The interpretation of net expected marginal environmental damage is analogous, in terms of marginal environmental damages. The (rental) price in the first period incorporates the fact that production in that period allows to save on production costs and environmental costs in the second period.

Using (5), equation (4) may be written:

$$
\begin{aligned}
&p(Q_{1u}) + \rho\delta p(Q_{2u}) = c_1'(\tfrac{Q_{1u}}{n}) \\
&+(1-\delta+\alpha\delta)\gamma'((1-\delta+\alpha\delta)Q_{1u}) + \rho\beta\delta\gamma'(Q_{2u} - (1-\beta)\delta Q_{1u})
\end{aligned}
\tag{6}
$$

In this equation, the sale price in the first period (or the rental price in the first period plus expected marginal benefits in the second period from a unit produced in the first period) equals marginal production cost in that period plus expected marginal environmental damage from a unit produced in $t = 1$.

Let Q_{1u}^* and Q_{2u}^* be the solutions of the system of equations given by (4) and (5). We consider that parameters and functions are such that $Q_{1u}^* > 0$ and $Q_{2u}^* - \delta Q_{1u}^* > 0$. From (5) we have that $p(Q_{2u}^*) > 0$. As it has been pointed out in section 2, we also assume that the cost and environmental damage functions are such that the price for rentals in the first period at the social optimum, $p(Q_{1u}^*)$ given by (4), is non-negative.[9]

Without emission taxes firms do not take into account environmental damage in their decisions and imperfect competition implies a distortion from efficient provision. Moreover, when firms sell their production, imperfect competition and the durability of the good induce each firm to behave strategically to steal sales from its rivals in the present and in the future, in a context where firms experience the commitment problems implied by the Coase conjecture.

We consider that the regulator uses only one instrument (taxes on emissions) to correct for all those distortions from efficiency. In this context, the emission taxes that maximize total surplus are, therefore, second-best optimal emission taxes. Those emission taxes induce the use of Q_{1u}^* units of the durable good in the first period and the use of Q_{2u}^* units of the durable good in the second

[9] With constant environmental damage per unit of emission in each period given by γ, with $\gamma > 0$, and constant marginal costs of production in the first and second periods given, respectively, by c_1 and c_2, we have a positive price for rentals in the first period at the social optimum if $c_1 > \delta\rho c_2$.

period, given by (4) and (5).

We are interested also in the total expected optimal emission tax per unit produced in the first period (defined as $\tau^*_{E1} = (1-\delta+\alpha\delta)\tau^*_1 + \rho\beta\delta\tau^*_2$ for emission taxes τ^*_1 and τ^*_2), since any unit of the durable good produced in the first period expects to pay that total expected emission tax. Starting from the next section let us denote the second-best optimal emission taxes by τ_1^{s*} and τ_2^{s*} when firms sell their output, by τ_1^{r*} and τ_2^{r*} when firms rent their output and by τ_1^{rs*} and τ_2^{rs*} when firms rent and sell their output.

5 Second-best emission taxes and overall underinternalization

In this section we study the second-best optimal emission taxes and we compare the optimal emission taxes with the corresponding marginal environmental damages. Following the literature we consider that there is overinternalization of the environmental damage for an optimal emission tax in a period if this tax is greater than the marginal environmental damage in that period. In the case of durable goods we know, however, that a unit produced in period 1 might also be used in period 2. As the expected total emission tax per unit produced in the first period is $(1-\delta+\alpha\delta)\tau_1 + \rho\beta\delta\tau_2$ and the expected overall marginal environmental damage caused per unit produced in period 1 is $(1-\delta+\alpha\delta)\gamma'((1-\delta+\alpha\delta)Q^*_{1u}) + \rho\beta\delta\gamma'(Q^*_{2u} - (1-\beta)\delta Q^*_{1u})$, we define:

Definition 1. *There is overall underinternalization in the first period if:*

$$(1-\delta+\alpha\delta)\tau_1+\rho\beta\delta\tau_2 < (1-\delta+\alpha\delta)\gamma'((1-\delta+\alpha\delta)Q^*_{1u})+\rho\beta\delta\gamma'(Q^*_{2u}-(1-\beta)\delta Q^*_{1u})).$$

From this definition we have that there will be overall underinternalization in the first period if there is not overinternalization in any of the two periods.

Our main result on overinternalization is:

Proposition 1. *If firms sell their output then:*

i) the optimal emission tax in the first period may imply overinternalization when emissions are proportional to the stock of the product or they occur at the end of the product's lifetime, but not when emissions occur in the production process;

ii) the optimal emission tax in the second period never implies overinternalization; and

iii) there is overall underinternalization in the first period for the three types of emissions considered.

Proof: See Appendix. ■

Proposition 1 indicates that the type of emission matters for overinternalization. We find that the optimal emission tax in the first period may be greater than marginal environmental damage in that period only if the units produced in $t = 1$ which are still in use in the second period cause environmental damage (and pay emission taxes) in this latter period.[10]

When there is overinternalization in the first period, however, we obtain in Proposition 1 that there is overall underinternalization. The excess of the expected emission tax to be paid in the second period from a unit produced in $t = 1$ over the expected marginal environmental damage of that unit in $t = 2$ compensates for the excess of the expected emission tax to be paid in the first period over expected marginal environmental damage in that period. This possibility of compensation cannot occur when emissions occur during the production process but, in that case, there is not overinternalization in the first period.[11]

For renting-selling firms and for firms that rent their output we, instead, obtain:

Proposition 2. *If firms rent their output or if firms rent and sell their output in the first period, then the optimal emission taxes in the first and second periods do not imply overinternalization for the three types of emissions considered.*

Proof: See Appendix. ■

[10] The possibility of overinternalization in the first period in a durable goods industry with selling firms and emissions that occur at the end of the product's life was already proved in Runkel (2004).

[11] As examples of overinternalization in the first period if emissions are proportional to the stock of product in the market or if emissions occur at the end of the product's life consider the following, in a context with linear demand, $p(Q) = a - bQ$, constant environmental damage per unit of emission in each period given by γ and constant marginal costs of production in the first and second periods given, respectively, by c_1 and c_2: i) if $\rho = 0.5$, $\delta = 1$, $a = 3.5$, $n = 3$, $c_1 = 2.4$, $c_2 = 0.1$, $b = 1$, $\gamma = 1$, $\alpha = 1$ and $\beta = 1$, production levels are positive, $\tau_1^{s*} = 1.1313 > \gamma$ and $(1 - \delta + \alpha\delta)\,\tau_1^{s*} + \rho\beta\delta\tau_2^{s*} = 1.256 < 1.5 = \gamma\,(1 - \delta + \alpha\delta + \rho\beta\delta)$, ii) if $\rho = 1$, $\delta = 0.5$, $a = n = 3$, $c_1 = 2.4$, $c_2 = 0.1$, $b = 1$, $\gamma = 1$, $\alpha = 0$ and $\beta = 1$, production levels are positive, $\tau_1^{s*} = 1.19792 > \gamma$ and $(1 - \delta + \alpha\delta)\,\tau_1^{s*} + \rho\beta\delta\tau_2^{s*} = 0.794 < 1 = \gamma\,(1 - \delta + \alpha\delta + \rho\beta\delta)$. As we have learned in Proposition 1-iii), there is overall underinternalization in the two situations considered in the examples.

From the proofs of Proposition 1 and 2 we have that production in each period is affected by emission taxes in both periods for the three emission types considered and for selling firms, renting firms and renting-selling firms. As a consequence, optimal emission taxes induce the social optimum through the combination of the effects on the production level in each period of emission taxes in both periods. When the goods is durable, therefore, the results on overinternalization of emission taxes cannot be explained considering only the effects of those taxes on first period production. The incentives of firms in both periods and the interaction between emission taxes in the present and in the future, and their effects on firms decisions, jointly explain the results.

Optimal emission taxes in periods 1 and 2 amend, simultaneously, for the distortion in production due to the oligopolistic market structure and for environmental damage, taking into account the implications of the durability of the good and the type of emission. If environmental damage were the only distortion in the market, the optimal emission tax in each period would be equal to marginal environmental damage. These would be the first-best emission taxes. From the expressions of the optimal emission taxes in the proof of Proposition 1, we have that under perfect competition (that is, when $n \to \infty$) the optimal emission tax in each period is equal to marginal environmental damage.

6 Comparison of optimal emission taxes

In this section we compare, for the three types of emission considered, the optimal emission taxes with selling firms, with renting firms and with renting-selling firms, when parameter values guarantee, simultaneously, interior solutions for the three market configurations.

Boyce and Goering (1997), considering a durable goods monopolist and emissions during the production process, find that the optimal emission tax on a selling monopolist in any period is higher than the optimal emission tax on a renting monopolist in the same period. We show below that their result holds also for the case where the market structure is a Cournot oligopoly. However, we prove that the optimal emission tax on selling firms in $t = 1$ is always lower than the optimal emission tax on renting firms in that period, if emissions are proportional to the stock of the product in the market or if emissions occur at the end of the life of the product, provided that parameter values guarantee interior solutions for the three market configurations. Nevertheless as, when $\beta = 1$, a unit produced in the first period may result in emissions in $t = 2$ and, as a consequence, may pay emission taxes in that period, the relevant emission tax on units produced in the first period to consider is the total expected optimal emission tax. In the next Proposition we also obtain that this total expected optimal emission tax is greater for selling firms than for renting firms.

We can prove:

Proposition 3.

i) When emissions occur in the production process we have: $\tau_1^{s*} > \tau_1^{rs*} > \tau_1^{r*}$;

ii) when emissions are proportional to the stock of the product or when they occur at the end of the product's life we have: $\tau_1^{rs*} = \tau_1^{r*} > \tau_1^{s*}$;

iii) $\tau_2^{s*} > \tau_2^{rs*} > \tau_2^{r*}$ *for the three types of emissions considered; and*

iv) $(1-\delta+\alpha\delta)\tau_1^{s*} + \rho\beta\delta\tau_2^{s*} > (1-\delta+\alpha\delta)\tau_1^{rs*} + \rho\beta\delta\tau_2^{rs*} > (1-\delta+\alpha\delta)\tau_1^{r*} + \rho\beta\delta\tau_2^{r*}$ *for the three types of emissions considered.*

Proof: See Appendix. ■

We should not expect a direct relationship between the optimal emission taxes in the first period, for renting, selling, and renting-selling firms, and the amounts of the good that would be produced without emission taxes in that period. To attain the social optimum we have to consider the two periods simultaneously and note that the emission tax in the second period interacts with the emission tax in the first period.

7 Conclusion

We have analyzed optimal emission taxes in durable goods oligopolies. We have considered emissions that occur during the production process, emissions proportional to the stock of the durable good and emissions that occur at the end of the product's lifetime. Moreover, we have studied the cases where firms only sell their product, firms only rent their product and firms may both sell and rent their production. We have shown that, although the optimal emission tax in the present may be, in some cases, greater than the marginal environmental damage from production in that period (overinternalization), the expected total emission tax paid per unit produced in the present is, in all situations analyzed, smaller than the expected marginal environmental damage caused by a unit produced in that period. We think that this latter comparison is the correct one to make, and our results allow us to conclude that there will be overall underinternalization, putting in perspective some of the results on overinternalization in the durable goods literature.

We have also compared the optimal emission taxes on selling firms, on renting firms and on renting-selling firms. We have obtained that, when emissions are proportional to the stock of the durable good or when emissions occur at the end of the product's lifetime, the optimal emission tax in the present on renting firms is higher than the optimal emission tax on selling firms. Nevertheless, we have shown that the expected total optimal emission tax per unit produced in the present is higher for selling firms than for renting firms, under any type of emissions. This latter result extends previous results in the literature obtained for the case where emissions occur in the production process.

The types of emissions analyzed in this paper may occur simultaneously. In this case we would have to add in each period the environmental damage functions corresponding to the types of emissions relevant for that period. The analysis would have to be applied to the aggregated environmental damage function obtained for those multiple emission types.

The environmental damage function in period 2 could be different from the environmental damage function in period 1. Cumulative pollution from production of all goods and services in the economy may reduce the capacity of the environment to assimilate pollution from the durable good industry in $t = 2$ or decreases in emissions in other industries may reduce the environmental damage caused by emissions in the durable good industry in that period. Some changes in the environmental damage function from one period to another may be easily incorporated in our analysis and do not modify the results that we have obtained. For instance, in a context with linear demand, $p(Q) = a - bQ$, constant marginal costs of production in the first and second periods given, respectively, by c_1 and c_2, and constant environmental damage per unit of emission given by γ in $t = 1$ and by $\kappa\gamma$ in $t = 2$, where κ is a constant different from 1, all results on overinternalization and overall underinternalization that we have obtained remain valid.[12] In this context it may be shown that, when

[12] This interpretation considers that the effect on cumulative pollution of emissions from the durable good industry that we are considering is negligible. If pollution in the second period were equal to emissions from the production, use or termination of the durable good in that period plus emissions in the first period multiplied by υ, with $0 < \upsilon$, (durable pollution) the analysis would have to proceed noting that the total emissions in period 2 would be:

$$E_2 = (\beta\delta + \nu)(Q_1^s + Q_1^r) + Q_2 - \delta Q_1^r.$$

emissions are proportional to the stock of the product or when they occur at the end of the product's life, the set of values of the parameters where there is overinternalization with selling firms in the first period gets smaller as κ increases. Hence, compared to the case where $\kappa = 1$, overinternalization with selling firms in the first period is more likely under those types of emissions if $\kappa < 1$ and it is less likely if $\kappa > 1$. The reason behind this latter result is that, when $\kappa > 1$, production in the present becomes more desirable, in terms of total surplus, than production in the future.

We have a different situation where there is non-stationarity when emissions are proportional to the stock of product in use and the units produced in period 1 that are in use in period 2 pollute more in this latter period than the units produced in the second period. Consider in this case that emissions in period 2 per unit of the good produced in period 1 are multiplied by κ, with $\kappa > 1$, while the emissions per unit produced in period 2 remain unchanged. In this case $\alpha = 1$, $\beta = 1$ and total emissions in period 2 are:

$$E_2 = \kappa\delta(Q_1^s + Q_1^r) + Q_2 - \delta Q_1^r$$

When κ increases production in the second period becomes more attractive, in terms of total surplus, than production in the first period. In a linear context analogous to the one considered in the previous paragraph it may be shown that the set of values of the parameters where there is overinternalization with selling firms in the first period gets smaller as κ increases. However, in this case pollution in the second period from units produced in the first period is specially important and, hence, the relevant comparison is between the total expected optimal emission tax per unit produced in the first period and the expected overall marginal environmental damage from production in the first period, just the comparison that we have pointed out as relevant for polluting durable goods throughout the paper.

When the goods are durable the optimal emission taxes depend on the type of emission. Moreover, the optimal emission taxes take into account that the quantities produced, sold or rented in each period are affected by emission taxes in both periods, as a consequence of the durability of the good and of the distribution in both periods of emissions from units produced in the present for some types of emissions.

In summary, from the analysis in this work we learn that, for durable goods, the expected overall emission tax per unit produced in each period is the relevant emission tax to consider. Our results allow to extend some previous results in the literature on durable goods and provide an adequate perspective on some other results (in particular, for durable goods that may cause pollution in periods different from the production period, we show the limitations of focusing only on the level and effects of the optimal emission tax in the production period).

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9 Appendix

9.1 Proof of Proposition 1

Let us present first some results that we will use in the proofs of Propositions 1 and 3. As from (1) and (2) we have that there is a symmetric market solution for firms decisions, equation (1) may be written as:

$$p(\delta Q_1^s + Q_2) + p'(\delta Q_1^s + Q_2)\frac{Q_2}{n} = c_2'(\frac{Q_2}{n} - \delta\frac{Q_1^r}{n}) + \tau_2$$

From this latter equation we have:

$$\begin{aligned}\frac{dQ_2}{dq_{1i}^s} &= \frac{dQ_2}{dQ_1^s} \\ &= -\frac{\delta p'(\delta Q_1^s+Q_2)+\delta p''(\delta Q_1^s+Q_2)\frac{Q_2}{n}}{p'(\delta Q_1^s+Q_2)+p''(\delta Q_1^s+Q_2)\frac{Q_2}{n}+\frac{1}{n}(p'(\delta Q_1^s+Q_2)-c_2''(\frac{Q_2}{n}-\delta\frac{Q_1^r}{n}))} < 0\end{aligned}$$

and

$$\delta+\frac{dQ_2}{dq_{1i}^s} = \frac{\delta\frac{1}{n}(p'(\delta Q_1^s+Q_2)-c_2''(\frac{Q_2}{n}))}{p'(\delta Q_1^s+Q_2)+p''(\delta Q_1^s+Q_2)\frac{Q_2}{n}+\frac{1}{n}(p'(\delta Q_1^s+Q_2)-c_2''(\frac{Q_2}{n}))} > 0$$

As in the symmetric solution $\frac{dQ_{2-i}}{dq_{1i}^s} = \frac{n-1}{n}\frac{dQ_2}{dq_{1i}^s}$ and $\frac{dq_{2i}}{dq_{1i}^s} = \frac{1}{n}\frac{dQ_2}{dq_{1i}^s}$, it is:

$$\delta+\frac{dQ_{2-i}}{dq_{1i}^s}$$

$$= \frac{\delta(p'(\delta Q_1^s+Q_2)+p''(\delta Q_1^s+Q_2)\frac{Q_2}{n})+\delta(p'(\delta Q_1^s+Q_2)-c_2''(\frac{Q_2}{n}-\delta\frac{Q_1^r}{n}))}{np'(\delta Q_1^s+Q_2)+np''(\delta Q_1^s+Q_2)\frac{Q_2}{n}+(p'(\delta Q_1^s+Q_2)-c_2''(\frac{Q_2}{n}-\delta\frac{Q_1^r}{n}))} > 0$$

and

$$\delta+\frac{dq_{2i}}{dq_{1i}^s}$$

$$= \frac{\delta(n-1)(p'(\delta Q_1^s+Q_2)+p''(\delta Q_1^s+Q_2)\frac{Q_2}{n})+\delta(p'(\delta Q_1^s+Q_2)-c_2''(\frac{Q_2}{n}-\delta\frac{Q_1^r}{n}))}{np'(\delta Q_1^s+Q_2)+np''(\delta Q_1^s+Q_2)\frac{Q_2}{n}+(p'(\delta Q_1^s+Q_2)-c_2''(\frac{Q_2}{n}-\delta\frac{Q_1^r}{n}))} > 0$$

Let us prove now the results in Proposition 1: From (1) and (2) we have that first order conditions of the maximization problems of selling firm i, with $i = 1, ..., n$, are:

$$p(\delta Q_1^s+Q_2)+p'(\delta Q_1^s+Q_2)q_{2i} = c_2'(q_{2i})+\tau_2 \tag{7}$$

and

$$\begin{aligned}&p(Q_1^s)-(1-\delta+\alpha\delta)\tau_1+p'(Q_1^s)q_{1i}^s-c_1'(q_{1i}^s) \\ &+\rho\delta p(\delta Q_1^s+Q_2)+\rho\delta(\delta+\tfrac{dQ_2}{dq_{1i}^s})p'(\delta Q_1^s+Q_2)q_{1i}^s \\ &+\rho(\delta+\tfrac{dQ_{2-i}}{dq_{1i}^s})p'(\delta Q_1^s+Q_2)q_{2i}-\rho\beta\delta\tau_2 = 0\end{aligned} \tag{8}$$

Hence, with optimal emission taxes τ_1^{s*} and τ_2^{s*}, we have from (7) and (8):

$$p(Q_{2u}^*)+p'(Q_{2u}^*)\frac{Q_{2u}^*-\delta Q_{1u}^*}{n} = c_2'(\frac{Q_{2u}^*-\delta Q_{1u}^*}{n})+\tau_2^{s*} \tag{9}$$

and

$$\begin{aligned}&p(Q_{1u}^*)-(1-\delta+\alpha\delta)\tau_1^{s*}+p'(Q_{1u}^*)\tfrac{Q_{1u}^*}{n}-c_1'(\tfrac{Q_{1u}^*}{n})+\rho\delta p(Q_{2u}^*) \\ &+\rho\delta(\delta+\tfrac{dQ_2}{dq_{1i}^s})p'(Q_{2u})\tfrac{Q_{1u}^*}{n}+\rho(\delta+\tfrac{dQ_{2-i}}{dq_{1i}^s})p'(Q_{2u}^*)\tfrac{Q_{2u}^*-\delta Q_{1u}^*}{n}-\rho\beta\delta\tau_2^{s*} = 0,\end{aligned} \tag{10}$$

where the derivatives $\frac{dQ_2}{dq_{1i}^s}$ and $\frac{dQ_{2-i}}{dq_{1i}^s}$ are evaluated at $\tau_1 = \tau_1^{s*}$ and $\tau_2 = \tau_2^{s*}$.

i) and ii) From (5) and (9) we may write:

$$\tau_2^{s*} = \gamma'(Q_{2u}^* - (1-\beta)\delta Q_{1u}^*) + p'(Q_{2u}^*)\frac{Q_{2u}^* - \delta Q_{1u}^*}{n} < \gamma'(Q_{2u}^* - (1-\beta)\delta Q_{1u}^*) \quad (11)$$

Hence, there is not overinternalization in $t = 2$ with selling firms. From (4), (9), (10) and (11) we have:

$$\begin{aligned}
\tau_1^{s*} &= \tfrac{1}{1-\delta+\alpha\delta}((1-\delta+\alpha\delta)\gamma'((1-\delta+\alpha\delta)Q_{1u}^*) - \rho(1-\beta)\delta\gamma'(Q_{2u}^* - (1-\beta)\delta Q_{1u}^*) \\
&+ p'(Q_{1u}^*)\tfrac{Q_{1u}^*}{n} + \rho\delta(\delta + \tfrac{dQ_2}{dq_{1i}^s})p'(Q_{2u}^*)\tfrac{Q_{1u}^*}{n} + \rho\tfrac{dQ_{2-i}}{dq_{1i}^s}p'(Q_{2u}^*)\tfrac{Q_{2u}^* - \delta Q_{1u}^*}{n} + \rho(1-\beta)\delta\tau_2^{s*}) \\
&= \gamma'((1-\delta+\alpha\delta)Q_{1u}^*) + \tfrac{1}{1-\delta+\alpha\delta}(p'(Q_{1u}^*)\tfrac{Q_{1u}^*}{n} + \rho\delta(\delta + \tfrac{dQ_2}{dq_{1i}^s})p'(Q_{2u}^*)\tfrac{Q_{1u}^*}{n} \\
&+ \rho((1-\beta)\delta + \tfrac{dQ_{2-i}}{dq_{1i}^s})p'(Q_{2u}^*)\tfrac{Q_{2u}^* - \delta Q_{1u}^*}{n})
\end{aligned} \quad (12)$$

From (12) we have that there is not overinternalization in $t = 1$ if $\beta = 0$. However, there may be overinternalization in $t = 1$ if $\beta = 1$.

Note that the procedure we use, in this Proposition and in Proposition 2, to determine the second-best optimal emission taxes is equivalent to a procedure where, first, the output levels are determined as functions of the emission taxes from conditions (1) and (2), then those output levels are inserted in the total surplus function and, finally, maximizing total surplus with respect to the emission taxes yields the second-best emission taxes.

iii) There is overall underinternalization as:

$$(1-\delta+\alpha\delta)\tau_1^{s*} + \rho\beta\delta\tau_2^{s*}$$

$$\begin{gathered}
= (1-\delta+\alpha\delta)\gamma'((1-\delta+\alpha\delta)Q_{1u}^*) + (p'(Q_{1u}^*)\tfrac{Q_{1u}^*}{n} + \rho\delta(\delta + \tfrac{dQ_2}{dq_{1i}^s})p'(Q_{2u}^*)\tfrac{Q_{1u}^*}{n} \\
+ \rho((1-\beta)\delta + \tfrac{dQ_{2-i}}{dq_{1i}^s})p'(Q_{2u}^*)\tfrac{Q_{2u}^* - \delta Q_{1u}^*}{n}) \\
+ \rho\beta\delta\gamma'(Q_{2u}^* - (1-\beta)\delta Q_{1u}^*) + \rho\beta\delta p'(Q_{2u}^*)\tfrac{Q_{2u}^* - \delta Q_{1u}^*}{n}
\end{gathered}$$

$$\begin{gathered}
= (1-\delta+\alpha\delta)\gamma'((1-\delta+\alpha\delta)Q_{1u}^*) + \rho\beta\delta\gamma'(Q_{2u}^* - (1-\beta)\delta Q_{1u}^*) \\
+ (p'(Q_{1u}^*)\tfrac{Q_{1u}^*}{n} + \rho\delta(\delta + \tfrac{dQ_2}{dq_{1i}^s})p'(Q_{2u}^*)\tfrac{Q_{1u}^*}{n} \\
+ \rho(\delta + \tfrac{dQ_{2-i}}{dq_{1i}^s})p'(Q_{2u}^*)\tfrac{Q_{2u}^* - \delta Q_{1u}^*}{n})
\end{gathered}$$

$$< (1-\delta+\alpha\delta)\gamma'((1-\delta+\alpha\delta)Q_{1u}^*) + \rho\beta\delta\gamma'(Q_{2u}^* - (1-\beta)\delta Q_{1u}^*) \blacksquare$$

9.2 Proof of Proposition 2

Let us consider first the case of firms that rent their output. From (1) and (2) we have that first order conditions of the maximization problems of renting firm i, with $i = 1, ..., n$, are:

$$p(Q_2) + p'(Q_2)q_{2i} = c_2'(q_{2i} - \delta q_{1i}^r) + \tau_2 \quad (13)$$

and

$$\begin{gathered} p(Q_1^r) - (1-\delta+\alpha\delta)\tau_1 + p'(Q_1^r)q_{1i}^r - c_1'(q_{1i}^r) \\ +\rho\delta c_2'(q_{2i} - \delta q_{1i}^r) + \rho\delta\tau_2 - \rho\beta\delta\tau_2 = 0 \end{gathered} \quad (14)$$

Hence, with optimal emission taxes, τ_1^{r*} and τ_2^{r*}, we have from (13) and (14):

$$p(Q_{2u}^*) + p'(Q_{2u}^*)\frac{Q_{2u}^*}{n} = c_2'(\frac{Q_{2u}^* - \delta Q_{1u}^*}{n}) + \tau_2^{r*} \quad (15)$$

and

$$\begin{gathered} p(Q_{1u}^*) - (1-\delta+\alpha\delta)\tau_1^{r*} + p'(Q_{1u}^*)\frac{Q_{1u}^*}{n} - c_1'(\frac{Q_{1u}^*}{n}) \\ +\rho\delta c_2'(\frac{Q_{2u}^* - \delta Q_{1u}^*}{n}) + \rho\delta\tau_2^{r*} - \rho\beta\delta\tau_2^{r*} = 0 \end{gathered} \quad (16)$$

From (5) and (15) we may write:

$$\tau_2^{r*} = \gamma'(Q_{2u}^* - (1-\beta)\delta Q_{1u}^*) + p'(Q_{2u}^*)\frac{Q_{2u}^*}{n} < \gamma'(Q_{2u}^* - (1-\beta)\delta Q_{1u}^*) \quad (17)$$

Moreover, from (4), (16) and (17) we have:

$$\begin{aligned} &\tau_1^{r*} = \tfrac{1}{1-\delta+\alpha\delta}((1-\delta+\alpha\delta)\gamma'((1-\delta+\alpha\delta)Q_{1u}^*) \\ &-\rho(1-\beta)\delta\gamma'(Q_{2u}^* - (1-\beta)\delta Q_{1u}^*) + p'(Q_{1u}^*)\tfrac{Q_{1u}^*}{n} + \rho(1-\beta)\delta\tau_2^{r*}) \\ &= \gamma'((1-\delta+\alpha\delta)Q_{1u}^*) + \tfrac{1}{1-\delta+\alpha\delta}(p'(Q_{1u}^*)\tfrac{Q_{1u}^*}{n} + \rho(1-\beta)\delta p'(Q_{2u}^*)\tfrac{Q_{2u}^*}{n}) \\ &< \gamma'((1-\delta+\alpha\delta)Q_{1u}^*) \end{aligned} \quad (18)$$

Let us obtain now the results for renting-selling firms. The first order conditions of the maximization problems of renting-selling firm i, with $i = 1, ..., n$, for interior solutions are given by equations (1) and (2). Hence, with optimal emission taxes τ_1^{rs*} and τ_2^{rs*}, we have from (1) and (2):

$$p(Q_{2u}^*) + p'(Q_{2u}^*)q_{2i} = c_2'(\frac{Q_{2u}^* - \delta Q_{1u}^*}{n}) + \tau_2^{rs*}, \quad (19)$$

$$\begin{gathered} p(Q_{1u}^*) - (1-\delta+\alpha\delta)\tau_1^{rs*} + p'(Q_{1u}^*)\frac{Q_{1u}^*}{n} - c_1'(\frac{Q_{1u}^*}{n}) \\ +\rho\delta p(Q_{2u}^*) + \rho\delta(\delta + \frac{dQ_2}{dq_{1i}^s})p'(Q_{2u}^*)q_{1i}^s \\ +\rho(\delta + \frac{dQ_{2-i}}{dq_{1i}^s})p'(Q_{2u}^*)q_{2i} - \rho\beta\delta\tau_2^{rs*} = 0 \end{gathered} \quad (20)$$

and

$$\begin{gathered} p(Q_{1u}^*) - (1-\delta+\alpha\delta)\tau_1^{rs*} + p'(Q_{1u}^*)\frac{Q_{1u}^*}{n} - c_1'(\frac{Q_{1u}^*}{n}) \\ +\rho\delta c_2'(\frac{Q_{2u}^* - \delta Q_{1u}^*}{n}) + \rho\delta\tau_2^{rs*} - \rho\beta\delta\tau_2^{rs*} = 0 \end{gathered} \quad (21)$$

From sections 2 and 4 we know that in (19), (20) and (21) it is $q_{2i} < \frac{Q_{2u}^*}{n}$.

Using (19) in (20) we may write:

$$\begin{array}{c} p(Q_{1u}^*) - (1-\delta+\alpha\delta)\tau_1^{rs*} + p'(Q_{1u}^*)\frac{Q_{1u}^*}{n} - c_1'(\frac{Q_{1u}^*}{n}) \\ +\rho\delta c_2'(\frac{Q_{2u}^*-\delta Q_{1u}^*}{n}) + \rho\delta\tau_2^{rs*} + \rho\delta(\delta + \frac{dQ_2}{dq_{1i}^s})p'(Q_{2u}^*)q_{1i}^s \\ +\rho(\frac{dQ_{2-i}}{dq_{1i}^s})p'(Q_{2u}^*)q_{2i} - \rho\beta\delta\tau_2^{rs*} = 0 \end{array} \quad (22)$$

Comparing (21) and (22) we have:

$$\delta(\delta + \frac{dQ_2}{dq_{1i}^s})q_{1i}^s + (\frac{dQ_{2-i}}{dq_{1i}^s})q_{2i} = 0 \quad (23)$$

From (5) and (19) we may write:

$$\tau_2^{rs*} = \gamma'(Q_{2u}^* - (1-\beta)\delta Q_{1u}^*) + p'(Q_{2u}^*)q_{2i} < \gamma'(Q_{2u}^* - (1-\beta)\delta Q_{1u}^*) \quad (24)$$

Hence, there is not overinternalization in $t = 2$.

From (4), (19), (21) and (24) we have:

$$\begin{array}{c} \tau_1^{rs*} = \frac{1}{1-\delta+\alpha\delta}((1-\delta+\alpha\delta)\gamma'((1-\delta+\alpha\delta)Q_{1u}^*) \\ -\rho(1-\beta)\delta\gamma'(Q_{2u}^* - (1-\beta)\delta Q_{1u}^*) + p'(Q_{1u}^*)\frac{Q_{1u}^*}{n} + \rho\delta\tau_2^{rs*} - \rho\beta\delta\tau_2^{rs*}) \end{array}$$

$$\begin{array}{c} = \frac{1}{1-\delta+\alpha\delta}((1-\delta+\alpha\delta)\gamma'((1-\delta+\alpha\delta)Q_{1u}^*) \\ +p'(Q_{1u}^*)\frac{Q_{1u}^*}{n} + \rho(1-\beta)\delta p'(Q_{2u}^*)q_{2i})) \end{array} \quad (25)$$

$$= \gamma'((1-\delta+\alpha\delta)Q_{1u}^*) + \frac{1}{1-\delta+\alpha\delta}(p'(Q_{1u}^*)\frac{Q_{1u}^*}{n} + \rho(1-\beta)\delta p'(Q_{2u}^*)q_{2i}))$$

$$< \gamma'((1-\delta+\alpha\delta)Q_{1u}^*)$$

Hence, there is not overinternalization in $t = 1$. ■

9.3 Proof of Proposition 3

i) and ii) From (12) and (25) we have:

$$\tau_1^{rs*} - \tau_1^{r*} = \frac{1}{1-\delta+\alpha\delta}\rho(1-\beta)\delta p'(Q_{2u}^*)\left(q_{2i} - \frac{Q_{2u}^*}{n}\right)$$

Hence, when $\beta = 1$ we have $\tau_1^{rs*} = \tau_1^{r*}$ and when $\beta = 0$ we have $\tau_1^{rs*} > \tau_1^{r*}$ as $\frac{Q_{2u}^*}{n} > q_{2i}$.

From (18) and (25) we also have:

$$\begin{aligned} \tau_1^{s*} - \tau_1^{rs*} &= \frac{1}{1-\delta+\alpha\delta}(\rho\delta(\delta + \frac{dQ_2}{dq_{1i}^s})p'(Q_{2u}^*)\frac{Q_{1u}^*}{n} \\ &\quad +\rho((1-\beta)\delta + \frac{dQ_{2-i}}{dq_{1i}^s})p'(Q_{2u}^*)\frac{Q_{2u}^* - \delta Q_{1u}^*}{n} - \rho(1-\beta)\delta p'(Q_{2u}^*)q_{2i}) \end{aligned}$$

Hence, when $\beta = 1$ it is, from (23) and $\delta + \frac{dq_{2i}}{dq_{1i}^s} > 0$:

$$\begin{aligned} \tau_1^{s*} - \tau_1^{rs*} &= \frac{\rho p'(Q_{2u}^*)}{1-\delta+\alpha\delta}(\delta(\delta + \frac{dQ_2}{dq_{1i}^s})\frac{Q_{1u}^*}{n} + \frac{dQ_{2-i}}{dq_{1i}^s}\frac{Q_{2u}^* - \delta Q_{1u}^*}{n}) \\ &= \frac{\rho p'(Q_{2u}^*)}{1-\delta+\alpha\delta}(\delta(\delta + \frac{dQ_2}{dq_{1i}^s})(q_{1i}^s + q_{1i}^r) + \frac{dQ_{2-i}}{dq_{1i}^s}(q_{2i} - \delta q_{1i}^r)) \\ &= \frac{\rho p'(Q_{2u}^*)\delta q_{1i}^r}{1-\delta+\alpha\delta}(\delta + \frac{dQ_2}{dq_{1i}^s} - \frac{dQ_{2-i}}{dq_{1i}^s}) = \frac{\rho p'(Q_{2u}^*)\delta q_{1i}^r}{1-\delta+\alpha\delta}(\delta + \frac{dq_{2i}}{dq_{1i}^s}) < 0 \end{aligned}$$

When $\beta = 0$ we have, from (23) and noting that $\frac{Q_{2u}^* - \delta Q_{1u}^*}{n} - q_{2i} = -\delta q_{1i}^r$:

$$\begin{aligned}
\tau_1^{s*} - \tau_1^{rs*} &= \frac{\rho p'(Q_{2u}^*)}{1-\delta+\alpha\delta}((\delta + \frac{dQ_2}{dq_{1i}^s})\frac{\delta Q_{1u}^*}{n} + (\delta + \frac{dQ_{2-i}}{dq_{1i}^s})\frac{Q_{2u}^* - \delta Q_{1u}^*}{n} - \delta q_{2i}) \\
&= \frac{\rho p'(Q_{2u}^*)}{1-\delta+\alpha\delta}(\delta(\delta + \frac{dQ_2}{dq_{1i}^s})(q_{1i}^s + q_{1i}^r) + (\delta + \frac{dQ_{2-i}}{dq_{1i}^s})\frac{Q_{2u}^* - \delta Q_{1u}^*}{n} - \delta q_{2i}) \\
&= \frac{\rho p'(Q_{2u}^*)}{1-\delta+\alpha\delta}(\delta(\delta + \frac{dQ_2}{dq_{1i}^s})q_{1i}^r + (\delta + \frac{dQ_{2-i}}{dq_{1i}^s})\frac{Q_{2u}^* - \delta Q_{1u}^*}{n} - (\delta + \frac{dQ_{2-i}}{dq_{1i}^s})q_{2i}) \\
&= \frac{\rho p'(Q_{2u}^*)}{1-\delta+\alpha\delta}(\delta(\delta + \frac{dQ_2}{dq_{1i}^s})q_{1i}^r - (\delta + \frac{dQ_{2-i}}{dq_{1i}^s})\delta q_{1i}^r) \\
&= \frac{\rho\delta p'(Q_{2u}^*)q_{1i}^r}{1-\delta+\alpha\delta}\frac{dq_{2i}}{dq_{1i}^s} > 0
\end{aligned}$$

iii) From (11), (17) and (24) we have, as $\frac{Q_{2u}^*}{n} > q_{2i} > \frac{Q_{2u}^* - \delta Q_{1u}^*}{n}$:

$$\tau_2^{s*} - \tau_2^{rs*} = p'(Q_{2u}^*)(\frac{Q_{2u}^* - \delta Q_{1u}^*}{n} - q_{2i}) > 0$$

and

$$\tau_2^{rs*} - \tau_2^{r*} = p'(Q_{2u}^*)(q_{2i} - \frac{Q_{2u}^*}{n}) > 0.$$

iv) From i), ii) and iii) we have that $(1-\delta+\alpha\delta)\tau_1^{rs*} + \rho\delta\tau_2^{rs*} - ((1-\delta+\alpha\delta)\tau_1^{r*} + \rho\delta\tau_2^{r*}) > 0$. From i) and iii) we also have that $(1-\delta+\alpha\delta)\tau_1^{s*} + \rho\delta\tau_2^{s*} - ((1-\delta+\alpha\delta)\tau_1^{rs*} + \rho\delta\tau_2^{rs*}) > 0$ when $\beta = 0$. When $\beta = 1$ we have, from (17), (18), (24) and (25), as $\frac{Q_{2u}^* - \delta Q_{1u}^*}{n} - q_{2i} = -\delta q_{1i}^r$:

$$\begin{aligned}
&(1-\delta+\alpha\delta)\tau_1^{s*} + \rho\delta\tau_2^{s*} - ((1-\delta+\alpha\delta)\tau_1^{rs*} + \rho\delta\tau_2^{rs*}) \\
= \;& \rho p'(Q_{2u}^*)\delta q_{1i}^r(\delta + \frac{dq_{2i}}{dq_{1i}^s}) + \rho\delta p'(Q_{2u}^*)(\frac{Q_{2u}^* - \delta Q_{1u}^*}{n} - q_{2i}) \\
= \;& \rho\delta p'(Q_{2u}^*)q_{1i}^r\frac{dq_{2i}}{dq_{1i}^s} > 0 \blacksquare
\end{aligned}$$