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Research Article

Stability of Switched Feedback Time-Varying Dynamic Systems Based on the Properties of the Gap Metric for Operators

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The stabilization of dynamic switched control systems is focused on and based on an operator-based formulation. It is assumed that the controlled object and the controller are described by sequences of closed operator pairs (L, C) on a Hilbert space H of the input and output spaces and it is related to the existence of the inverse of the resulting input-output operator being admissible and bounded. The technical mechanism addressed to get the results is the appropriate use of the fact that closed operators being sufficiently close to bounded operators, in terms of the gap metric, are also bounded. That philosophy is followed for the operators describing the input-output relations in switched feedback control systems so as to guarantee the closed-loop stabilization.

1. Introduction

Control Theory is a relevant field from the mathematical theoretical point of view as well as in many applications. What is important, in particular, is the closed-loop stabilization of dynamic system under appropriate feedback control as a minimum requirement to design a well-posed feedback system. Concerning the stabilization, the stabilization accomplishing with the properties of absolute stability is a very important issue (stabilization for whole sets of families of nonlinear controlled systems subject to nonlinear controllers satisfying Lure's-type or Popov-type inequalities) and hyperstability (the nonlinearity can be, in addition, time-varying) or its most general property of passivity. See, for instance, [1–15] and references therein. If the feed-forward controlled object is linear, then hyperstability of the whole closed-loop system requires, in addition, the positive realness of the feed-forward loop of the controlled system. See [4–9] and references therein. It is also important to maintain the stability properties with a certain tolerances to modelling errors to better describe real situations, that is, the achievement of closed-loop robust stabilization. See, for instance, [2, 16, 17] and references there in. On the other hand, the problems of closed-loop stabilization as oscillatory behaviour in switched and impulsive dynamic systems with several potential active parameterizations has been investigated in the last years with an important set of background results. See, for instance, [18–27] and references there in. In particular, it can be said that if all the parameterizations are stable and linear and possess a common Lyapunov function then the closed-loop stabilization of the switched system is possible under arbitrary switching. However, in the general case, it is needed to maintain a minimum residence time at each active parameterization before next switching or, alternatively, at certain active parameterizations, being active after a bounded whole time from its last activation. Some formulations replace the minimum residence time required for stabilization of the switched system by a sufficiently large averaged time at each stable parameterization. See, [18–25, 28–37] and references there in for a background subject coverage. Extensions have been proposed for certain classes of hybrid systems and time-delay systems. See, for instance, [21–27] and references there in. Generally speaking, most of the proposed results about the stabilization under switching rules for the various involved parameterizations have been formulated for feedback regulation controls, that is, for a closed-loop regulation system in the absence of an external reference signal.

This paper gives a formal framework for the case when both controlled system and controller are described by closed operators. In this way, the stability of the switched system is not mainly related to a feedback control law but to the switching law in-between parameterizations. The given formulation is based on the properties of the operators describing the input-output relations of the combined controlled object and its controller. In particular: (a) bounded operators are closed-operators while the converse is not true, in general, (b) linear closed operators being sufficiently close to bounded operators are also bounded, and (c) the input-output operator of a stable dynamic system has a bounded nonlinear inverse operator and vice versa for any admissible stabilizing controller, [38, 39]. The closeness between operator controlled object/controller (L, C)-parameterized pairs associated with the given switching law is characterized in terms of "smallness" of the gap metric on the Hilbert space H of inputs and outputs.

Let $(X, \|\cdot\|)$ be a Banach space and let $FP := \{P_k \in L(X) : k \in \overline{\operatorname{In} FP} \subseteq \mathbf{N}\}$ denote a nonempty set of linear bounded (then continuous) operators on X with operator norms $\|P\| = \sup\{\|Px\| : x \in X, \|x\| \leq 1\}$; for all $P \in FP$, where $\overline{\operatorname{In} FP}$ is an indicator set denoted by the notation $\overline{n} := \{1, 2, \ldots, n\} \subseteq \mathbf{N}\,(n \leq \infty)$. It will be said that a sequence of operators $SP := \{P_k\}_{k\in \operatorname{In} SP}$ where $\operatorname{In} SP \subseteq \mathbf{N}$ is the corresponding indicator set. The notation for the indicator $\overline{\operatorname{In} FP}$ refers to a finite or infinite number n of members of the operator set FP with all the subscripts from unit to n being present in the set while the notation for the indicator $\operatorname{In} SP$ (a finite or infinite numerable set of natural numbers) means they are not always consecutive natural numbers. Let us denote by $S^* = S(P_*) := \{\{P_k : P_k \in FP, P_k \to P_* \text{ as } k \to \infty\}_{k\in \operatorname{In} SP_*}\}$ the set of all convergent sequences of operators in FP which converge to some $P_* \in L(X) \cap \operatorname{cl} FP$, where $\operatorname{cl}(\cdot)$ stands for the closure. Such a convergence, in principle, is open to happen in any well-posed sense as, for instance, weak convergence, strong convergence or uniform convergence in the sense that the convergence happens in the weak, strong, or uniform topologies. Sequences of operators where convergence properties are of interest are denoted simply by $S := \{\{P_k : P_k \in FP\}_{k\in \operatorname{In} SP_*}\}$.

2. Preliminary Results on Closed and Bounded Sequences of Linear Operators

The subsequent result relies on the convergence of sequences of operators to limits so that we give the following simple result.

Theorem 2.1. $P_{*1} \neq P_{*2} \Rightarrow S(P_{*1}) \neq S(P_{*2})$*; for all* $P_{*1}, P_{*2} \in L(X) \cap \mathrm{cl}\, FP$.

Proof. $P_k \in S(P_{*1}) \Leftrightarrow P_k \to P_{*1}$ as $k \to \infty$ in the strong operator topology. Thus, for any $\varepsilon_1 > 0$, $\exists n_1 = n_1(\varepsilon_1) \in \mathbf{N}$ such that, in the strong operator topology,

$$\|P_k - P_{*1}\| := \|P_k x - P_{*1} x\|_X = \sup_{x \in X,\, \|x\|=1} \|P_k x - P_{*1} x\| < \varepsilon_1, \quad \forall k > n_1 \tag{2.1}$$

as x ranges over the unit ball in X. Now, assume that $P_k \to P_{*1}$ in the uniform operator topology as $k \to \infty$. Thus, for any $\varepsilon_2 > 0$, $\exists n_2 = n_2(\varepsilon_2) \in \mathbf{N}$ such that $\|P_k x - P_{*1} x\|_X < \varepsilon_1$; for all $k > n_2$. Since $P_{*1} \neq P_{*2}$, $\delta = \|P_{*1} - P_{*2}\| > 0$, then if $P_k \to P_{*2}$ as $k \to \infty$ in the uniform operator topology,

$$\begin{aligned} 0 < \delta - \varepsilon_1 &\leq \big|\|P_k - P_{*1}\| - \|P_{*1} - P_{*2}\|\big| \\ &= \|P_k - P_{*2}\| \leq \|P_k - P_{*2}\| = \|(P_k - P_{*1}) + (P_{*1} - P_{*2})\| < \varepsilon_2 \end{aligned} \tag{2.2}$$

for $\varepsilon_2 = \varepsilon_2(\delta) < \delta$, $\varepsilon_1 = \varepsilon_1(\delta, \varepsilon_2) > \delta - \varepsilon_2$, and $k > n := \max(n_1, n_2) = n(\varepsilon_1, \varepsilon_2, \delta)$. But the choice of $\varepsilon_1 > 0$ is arbitrary and then the above constraint fails if $0 < \varepsilon_2 \leq \delta - \varepsilon_1$ for any given $\varepsilon_1 < \delta$. Hence, $P_{*1} = P_{*2}$ is what contradicts the assumption $P_k \to P_{*2}(\neq P_{*1})$ as $k \to \infty$ in the uniform operator topology so that there is some infinite subsequence $\{P_{n_k}\}$ of $\{P_k\}$ such that $\{P_{n_k}\} \subset SP_{*1} \Rightarrow \{P_{n_k}\} \notin S(P_{*2})$. Thus, either $S(P_{*1}) \supset S(P_{*2})$ with improper set inclusion or $S(P_{*1}) \cap S(P_{*2}) = \emptyset$. (Here, we are considering the sequences as sets what is trivially consistent). By reversing the roles of $S(P_{*1})$ and $S(P_{*2})$ one concludes that $S(P_{*1}) \subset S(P_{*2})$ with improper set inclusion or $S(P_{*1}) \cap S(P_{*2}) = \emptyset$. Then, $P_{*1} \neq P_{*2} \Rightarrow S(P_{*1}) \neq S(P_{*2})$. □

Note that proof of the above result might be easily readdressed with the uniform operator topology as well with the replacements $\delta \to \delta(x) := \|P_{*1} x - P_{*2} x\|_{x \in X} \leq \delta \|x\|$ and $\|P_k x - P_{*i} x\|_{x \in X} < \varepsilon_i(x) \leq \varepsilon_i \|x\|$ for any $x \in X$. Note that $P_{*1} \neq P_{*2} \Rightarrow S(P_{*1}) \neq S(P_{*2})$ is compatible with the existence of distinct convergent sequences $S(P_{*1}), S(P_{*2})$ to either P_{*1} or to $P_{*2}(\neq P_{*1})$ which can contain common operators $P_k \in L(X)$. The above result is linked with the so-called gap metric [1, 2], as follows. If M_i $(i = 1, 2)$ are closed subspaces of X then the directed gap $\vec{\delta}(M_1, M_2)$ from M_1 to M_2 is defined by

$$\vec{\delta}(M_1, M_2) := \sup\{\inf[\|x - y\| : y \in M_2] : x \in M_1, \|x\| = 1\} = \|(I - P_2) P_1\|, \tag{2.3}$$

where P_i $(i = 1, 2)$ are the corresponding projection operators. Note that the directed gap is not symmetric, in general. The gap metric is defined by

$$\delta(M_1, M_2) := \max\left(\vec{\delta}(M_1, M_2), \vec{\delta}(M_2, M_1)\right) = \|P_1 - P_2\| = \sup_{x \in X,\, \|x\|=1} \|P_1 x - P_2 x\|, \tag{2.4}$$

where the last identity holds (see, e.g., [38]). Note that the gap metric has the symmetry property so that it is a well-posed metric. Then, the metric space (X,δ), with the gap metric δ, is a complete metric space which is also the Banach space $(X,\|\cdot\|)$, defined for the above norm, which induces the strong topology on X. We can also use the above concept to define distances between closed operators P via the gap metric. Since closed operators P on X of domain $D(P)=X$ are bounded, the gap metric is also useful to quantify the "separation" between linear bounded (then continuous) operators whose domain is the whole vector space X of the Banach space $(X,\|\cdot\|)$ and which are not necessarily projection operators. If P_1 and P_2 are closed linear operators on X, then the gap distance between them is $\delta(G(P_1),G(P_2))$, where $G(P_i)\subset X\times X$ is the graph of P_i; $i=1,2$ and we will denote such a gap distance by $\delta(P_1,P_2)$ for the sake of simplicity. Note that the linear operator $P:D(P)\subset X\to X$ is closed if its graph $G(P)$ is closed in the direct sum $X\oplus X$. The following result involves the proofs of some properties of convergent sequences on a Hilbert space X under contractive conditions for the gap metric.

Theorem 2.2. *Consider a sequence of linear closed operators $\{P_k\}_{k\in\mathbf{N}_0}$ on a Hilbert space X, where $\mathbf{N}_0$ is the set of nonnegative integers, such that P_0 is also bounded. Then, the following properties hold:*

(i) *The operators in $\{P_k\}_{k\in\mathbf{N}_0}$ are also bounded if $\delta(P_{k+1},P_{k+2})<1/\sqrt{1+\|P_{k+1}\|^2}$. Also, the sequence $\{P_k\}_{k\in\mathbf{N}_0}$ converges to a unique bounded operator P on H, which is bounded and unique and $\delta(P_{k+1},P_k)\to 0$ as $k\to\infty$, if the following constraints hold for some real sequence $\{K_k\}_{k\in\mathbf{N}_0}$:*

$$\|P_{k+1}-P_k\|\le\frac{1}{K_k}\quad \text{for } K_k\in(0,K)\subset(0,1);\ \forall k\in\mathbf{N}_0, \tag{2.5}$$

$$\delta(P_{k+2},P_{k+1})<\frac{K_k\|P_{k+1}-P_k\|}{1+\|P_{k+1}\|^2+K_k\|P_{k+1}-P_k\|\left(1+\|P_{k+1}\|^2\right)^{1/2}},\quad \forall k\in\mathbf{N}_0. \tag{2.6}$$

(ii) *Assume that P_0 is invertible with bounded inverse P_0^{-1} and $\delta(P_{k+1},P_{k+2})<1/\sqrt{1+\|P_{k+1}\|^2}$. Then, the sequence $\{P_k^{-1}\}_{k\in\mathbf{N}_0}$ exists consisting of bounded operators on X, In addition, such a sequence of inverse operators converges to a unique bounded operator P^{-1} on H, which is the bounded inverse operator of P on X in Theorem 2.2(i), if* (2.5)-(2.6) *hold.*

(iii) *If the operators in $\{P_k\}_{k\in\mathbf{N}_0}$ are linear and closed and $\delta(P_{k+1},P_{k+2})<1/\sqrt{1+\|P_{k+1}\|^2}$ then the operators in $\{P_k\}_{k\in\mathbf{N}_0}$ are also densely defined. If, in addition,* (2.5)-(2.6) *hold then the densely defined operators of the sequence $\{P_k\}_{k\in\mathbf{N}_0}$ converge to a limit operator P which is also densely defined, that is, their domains $D(P_k)$; $k\in\mathbf{N}_0$ and $D(P)$ are dense subsets of X and their images $R(P_k)$; $k\in\mathbf{N}_0$ and $R(P)$ are contained in X.*

Proof. Let $\delta(P_k,P_{k+1})$ be an abbreviated notation for $\delta(G(P_k),G(P_{k+1}))$ where the graph of P_k, is the range of the operator $\begin{bmatrix} I \\ P_k\end{bmatrix}$ defined on $D(P_k)$, that is, $G(P_k)=R\begin{bmatrix} I \\ P_k\end{bmatrix}=\{\begin{bmatrix} x \\ P_k x\end{bmatrix}: x\in D(P_k)\}\subset X\oplus X$ of the operator $\begin{bmatrix} I \\ P_k\end{bmatrix}$ defined on $D(P_k)$. Proceed by complete induction by taking any $k \in \mathbf{N}_0$ and assuming that $\{P_j\}_{j(\le k+1)\in\mathbf{N}_0}$ is bounded, $x \in D(P_{k+1})$ such that $\|\left[\begin{smallmatrix} x \\ P_{k+2}x \end{smallmatrix}\right]\| = 1$ and take

$$\begin{aligned} \delta'_{k+1} &\in \left(\delta(P_{k+1},P_{k+2}), \frac{K_k\|P_{k+1}-P_k\|}{1+\|P_{k+1}\|^2+K_k\|P_{k+1}-P_k\|\left(1+\|P_{k+1}\|^2\right)^{1/2}}\right] \\ &\subset \left(\delta(P_{k+1},P_{k+2}), \frac{1}{\left(1+\|P_{k+1}\|^2\right)^{1/2}}\right]. \end{aligned} \tag{2.7}$$

Then, there exists $\left[\begin{smallmatrix} v \\ P_{k+1}v \end{smallmatrix}\right]$ such that $\|\left[\begin{smallmatrix} v \\ P_{k+1}v \end{smallmatrix}\right] - \left[\begin{smallmatrix} x \\ P_{k+2}x \end{smallmatrix}\right]\| < \delta'_{k+1}$. Define $\widetilde{P}_{k+1} = P_{k+2} - P_{k+1}$ so that

$$\begin{aligned} \left\|\widetilde{P}_{k+1}x\right\| &= \|P_{k+2}x - P_{k+1}v - P_{k+1}(x-v)\| \\ &\le \|P_{k+2}x - P_{k+1}v\| + \|P_{k+1}\|\|x-v\| \\ &\le \delta'_{k+1}\left(1+\|P_{k+1}\|^2\right)^{1/2} \end{aligned} \tag{2.8}$$

by Schwarz's inequality. Since

$$1 = \|x\|^2 + \left\|P_{k+1}x + \widetilde{P}_{k+1}x\right\|^2 \le \left(1+\|P_{k+1}\|^2\right)\|x\|^2 + 2\|P_{k+1}\|\|x\|\left\|\widetilde{P}_{k+1}x\right\| + \left\|\widetilde{P}_{k+1}x\right\|^2. \tag{2.9}$$

one gets from (2.8)-(2.9) that

$$\begin{aligned} \left\|\widetilde{P}_{k+1}x\right\|^2 &\le \delta'^2_{k+1}\left(1+\|P_{k+1}\|^2\right) \\ &\quad\times\left[\left(1+\|P_{k+1}\|^2\right)\|x\|^2 + 2\|P_{k+1}\|\|x\|\delta'_{k+1}\left(1+\|P_{k+1}\|^2\right)^{1/2} + \left\|\widetilde{P}_{k+1}x\right\|^2\right] \end{aligned} \tag{2.10}$$

so that

$$\left\|\widetilde{P}_{k+1}x\right\| \le \frac{\delta'_{k+1}\left(1+\|P_{k+1}\|^2\right)\left[\left(1-\delta'^2_{k+1}\right)^{1/2} + \delta'_{k+1}\|P_{k+1}\|\right]}{1-\delta'_{k+1}\left(1+\|P_{k+1}\|^2\right)}\|x\| \le \frac{\delta'_{k+1}\left(1+\|P_{k+1}\|^2\right)}{1-\delta'_{k+1}\left(1+\|P_{k+1}\|^2\right)^{1/2}}\|x\|, \tag{2.11}$$

provided that $\delta(P_{k+1},P_{k+2}) < 1/\sqrt{1+\|P_{k+1}\|^2}$ is what guarantees that P_{k+2} is bounded since P_{k+1} is bounded. The above inequality is homogeneous in x then it is true for all $x \in D(P_{k+2})$. Thus, $\widetilde{P}_{k+1}$ is bounded so that it is $P_{k+2} = P_{k+1} + \widetilde{P}_{k+1}$. As a result, if $\{P_k\}_{k\in\mathbf{N}_0}$ is a sequence of closed operators with $\{P_j\}_{j(\le k)\in\mathbf{N}_0}$ being bounded then $\{P_j\}_{j(\le k+1)\in\mathbf{N}_0}$ is also bounded since P_{k+1} is bounded since $\delta(P_{k+1},P_k) < 1/\sqrt{1+\|P_k\|^2}$. Thus, if $\{P_k\}_{k\in\mathbf{N}_0}$ is a sequence of linear closed operators with bounded P_0 then $\{P_k\}_{k\in\mathbf{N}_0}$ is also a sequence of linear bounded operators. Now, note from (2.11) that $\widetilde{P}_k \to 0$ as $k \to \infty$, then there is a unique $P = \lim_{k\to\infty} P_k$ (uniqueness follows by construction), under $\delta(P_{k+1}, P_{k+2}) < 1/\sqrt{1+\|P_{k+1}\|^2}$, if

$$\left\|\widetilde{P}_{k+1}\right\| \leq \frac{\delta(P_{k+1}, P_{k+2})\left(1+\|P_{k+1}\|^2\right)}{1-\delta(P_{k+1}, P_{k+2})\left(1+\|P_{k+1}\|^2\right)^{1/2}} \leq K_k\left\|\widetilde{P}_k\right\| \tag{2.12}$$

is what holds if

$$\begin{aligned}\delta(P_{k+1}, P_{k+2}) &< \min\left(\frac{1}{\sqrt{1+\|P_{k+1}\|^2}}, \frac{K_k\left\|\widetilde{P}_k\right\|}{1+\|P_{k+1}\|^2+K_k\left\|\widetilde{P}_k\right\|\left(1+\|P_{k+1}\|^2\right)^{1/2}}\right) \\ &= \frac{K_k\left\|\widetilde{P}_k\right\|}{1+\|P_{k+1}\|^2+K_k\left\|\widetilde{P}_k\right\|\left(1+\|P_{k+1}\|^2\right)^{1/2}}.\end{aligned} \tag{2.13}$$

Thus, note that the constraint (2.5) is a sufficient condition for the necessary $\delta(P_{k+1}, P_{k+2}) < 1/\sqrt{1+\|P_{k+1}\|^2}$ to hold to guarantee a well-posed (2.11). Also, since $\{P_{ki}\}_{k\in\mathbf{N}_0}$ converges to P, $\delta(P_k, P) \to 0$ as $k \to \infty$ and there is a sufficiently large finite $k_0 \in \mathbf{N}_0$ such that $\delta(P_k, P) < 1/\sqrt{1+\|P_k\|^2}$; for all $k > k_0$, it is proven that then $P = \lim_{k\to\infty} P_k$ is bounded since P_k is bounded. Assume not, then there exists $C \in \mathbf{R}_+$ and $\widetilde{C} \in \mathbf{R}_+$ such that $\|P_k x\| \leq C\|x\|$ and $\|Py\| > (C+\widetilde{C})\|y\|$ for all $x \in D(P_k)$ and some nonzero $y \in D(P)\cap D(P_k)$ for any given $k \in \mathbf{N}_0$. Then, $\|P_k x + (Py - P_k x)\| = \|Py\| > (C+\widetilde{C})\|y\|$ and, one gets by taking $x = y$, that

$$[C + o(\|P-P_k\|)]\|x\| \geq \|P_k x\| + \|(P-P_k)x\| \geq \|P_k x + (Px - P_k x)\| = \|Px\| > \left(C+\widetilde{C}\right)\|x\| \tag{2.14}$$

leading to $o(\|P-P_k\|) > C > 0$; for all $k \in \mathbf{N}_0$ which leads to the contradiction $\lim_{k\to\infty} o(\|P - P_k\|) = 0 > C > 0$. Thus, $P = \lim_{k\to\infty} P_k$ is bounded, and then closed since linear, and $D(P_k) = D(P) = X$ since $\{P_k\}_{k\in\mathbf{N}_0}$ and P are linear, closed, bounded, and then continuous operators on X. Property (i) has been proven.

To prove Property (ii), note that if P_k^{-1} exists then $\delta(P_{k+1}^{-1}, P_k^{-1}) = \delta(P_{k+1}, P_k)$ provided that P_{k+1}^{-1} exists and is bounded and $\delta(P_k, P_{k+1}) < 1/\sqrt{1+\|P_k\|^2}$. Thus, it suffices to prove that P_{k+1} is invertible. Assume not and proceed by contradiction by assuming that $\operatorname{Ker}(P_{k+1}) \neq \{0\}$. By linearity of the operator P_k, it always exists $u, v \in D(P_k)$ with $\|u\| = 1$ such that $\begin{bmatrix} u \\ 0 \end{bmatrix} \in G(P_k)$, $\begin{bmatrix} v \\ P_{k+1}v \end{bmatrix} \in G(P_{k+1})$, and $\delta(P_k, P_{k+1}) \leq \delta'_{k+1} < 1/\sqrt{1+\|P_k\|^2}$ with $\|u-v\|^2 + \|P_{k+1}v\|^2 < \delta'^2_{k+1}$ so that:

$$1 = \|u\|^2 \leq \left(\|u-v\| + \left\|P_k^{-1}\right\|\|P_k\|v\right)^2 \leq \left(1+\left\|P_k^{-1}\right\|^2\right)\delta'^2_{k+1} < 1. \tag{2.15}$$

Then $\operatorname{Ker}(P_{k+1}) = \{0\}$ and P_{k+1} is invertible. By complete induction it follows that if $\delta(P_{k+1}, P_{k+2}) < 1/\sqrt{1+\|P_{k+1}\|^2}$ for all $k \in \mathbf{N}_0$, and P_0 is invertible with bounded inverse P_0^{-1} then $\{P_k^{-1}\}_{k\in\mathbf{N}_0}$ exists consisting of linear bounded operators. If, in addition, (2.5)-(2.6) hold then the sequences of inverse operators which is bounded and consists of sufficiently close linear operators converges to the bounded inverse linear operator P^{-1}.

To proof Property (iii), consider the inequality

$$\left\| \begin{bmatrix} v \\ P_{k+1}v \end{bmatrix} - \begin{bmatrix} x \\ P_{k+2}x \end{bmatrix} \right\| < \delta'_{k+1} \Longleftrightarrow \left[\|v-x\|^2 + \|P_{k+2}x - P_{k+1}v\|^2\right]^{1/2} < \delta'_{k+1} \Longrightarrow \|v-x\| < \delta'_{k+1} \tag{2.16}$$

for any given $k \in \mathbf{N}_0$. Therefore, $d(v, \operatorname{cl} D(P_{k+2})) < \delta'_{k+1}$, where $\operatorname{cl}(\cdot)$ stands for the closure, for the norm-induced metric on $(X, \|\cdot\|)$ and, since $1 \le (1+\|P_{k+1}\|^2)\|v\|^2$, this leads to the homogeneous inequality $d(z, \operatorname{cl} D(P_{k+2})) < \delta'_{k+1}\sqrt{1+\|P_{k+1}\|^2}\|z\|$ for any $z \in X$. Then, $\operatorname{cl} D(P_{k+2}) = X$. If $\{P_k\}_{k\in\mathbf{N}_0}$ consists of closed operators and $\{P_j\}_{j(\le k+1)\in\mathbf{N}_0}$ is a sequence of densely defined operators then $\{P_j\}_{j(\le k+2)\in\mathbf{N}_0}$ is a densely defined sequence of operators. Proceeding by complete induction, it follows that the sequence $\{P_k\}_{k\in\mathbf{N}_0}$ is densely defined and $P = \lim_{k\to\infty} P_k$ is closed by an analogous reasoning that the corresponding one used in the proof of Property (i). Furthermore, one has for large enough $k \in \mathbf{N}_0$ that

$$d(z, \operatorname{cl} D(P)) \le \frac{1+\|P_k\| + o(\|P-P_k\|)}{\sqrt{1+\|P_k\|^2}}\|z\| \tag{2.17}$$

so that $\operatorname{cl} D(P) = X$ and $P = \lim_{k\to\infty} P_k$ is densely defined. The proof is complete. □

Remark 2.3. Note that if Theorem 2.2(i) holds then $D(P_k) = D(P) = X$; for all $k \in \mathbf{N}_0$ since the sequence of linear closed operators $\{P_k\}$ and its linear closed limit operator P are bounded.

The next result extends for a sequence of convergent operators that a sequence of closed operators is also a linear sequence of operators if each of its elements are sufficiently close in terms of difference of norms, or in terms of the gap metric, to some linear operator and the above theorem holds.

Theorem 2.4. *$\{P_k\}_{k\in\mathbf{N}_0}$ is a sequence of linear operators on X if it is a sequence of closed operators on X, P_0 is linear operator, and*

$$\frac{\|P_{k+1}-P_k\|}{\left(1+\|P_k\|^2\right)^{1/2}\left[\left(1+\|P_k\|^2\right)^{1/2} + \|P_{k+1}-P_k\|\right]} \le \delta(P_{k+1}, P_k) < \frac{1}{\left(1+\|P_k\|^2\right)^{1/2}}, \quad \forall k \in \mathbf{N}_0. \tag{2.18}$$

The sequence $\{P_k\}_{k\in\mathbf{N}_0}$ *has a linear limit operator if* (2.18) *is replaced by the stronger condition:*

$$\frac{\|P_{k+1}-P_k\|}{\left(1+\|P_k\|^2\right)^{1/2}\left[\left(1+\|P_k\|^2\right)^{1/2}+\|P_{k+1}-P_k\|\right]} \\ \leq \delta(P_{k+1},P_k) < \frac{K_{k-1}\|P_k-P_{k-1}\|}{1+\|P_k\|^2+K_{k-1}\|P_k-P_{k-1}\|\left(1+\|P_k\|^2\right)^{1/2}}, \quad \forall k\in\mathbf{N} \tag{2.19}$$

for some given real sequence $\{K_k\}_{k\in\mathbf{N}_0}$ *with* $K_k\in(0,K)\subset(0,1)$ *satisfying* $K_k\geq\|P_{k+1}-P_{k+2}\|/\|P_{k+1}-P_k\|$*; for all* $k\in\mathbf{N}_0$ *for which a sufficient condition is*

$$\|P_{k+1}\|\leq\min\left(\frac{K_k\|P_k\|-\|P_{k+2}\|}{1+K_k},K_{k-1}\|P_{k-1}\|\right), \quad \forall k\in\mathbf{N}. \tag{2.20}$$

Proof. The first part related to (2.18) follows from Theorem 2.2 for δ'_{k+1} arbitrarily close to $\delta(P_k,P_{k+1})$, P_{k+1} being either bounded or densely defined if P_k is a linear closed operator on X for any given $k\in\mathbf{N}_0$. Then, either $D(P_{k+1})=X$ or $\operatorname{cl}D(P_{k+1})=X$ so that P_{k+1} is, furthermore, linear and closed since $\{P_k\}_{k\in\mathbf{N}_0}$ is closed and the property that bounded or densely defined linear operators are closed. Then, if P_0 is linear, then it follows by complete induction that $\{P_k\}_{k\in\mathbf{N}_0}$ is a sequence of linear operators. The second part of the theorem is proven by first noting that (2.19) guarantees (2.18) so that $\{P_k\}_{k\in\mathbf{N}_0}$ is still a sequence of linear operators which are also either bounded or densely defined. Furthermore, it is guaranteed from Theorem 2.2 that $\{P_k\}_{k\in\mathbf{N}_0}$ converges to a limit linear operator P either bounded or densely defined provided that (2.19) holds under the necessary condition:

$$\|P_{k+1}-P_k\|\left[\left(1+\|P_k\|^2\right)^{1/2}+K_{k-1}\|P_k-P_{k-1}\|\right] \\ \leq K_{k-1}\|P_k-P_{k-1}\|\left[\left(1+\|P_k\|^2\right)^{1/2}+\|P_{k+1}-P_k\|\right], \quad \forall k\in\mathbf{N}, \tag{2.21}$$

namely, if $K_k\geq\|P_{k+1}-P_{k+2}\|/\|P_{k+1}-P_k\|$ which is guaranteed if $K_k\geq(\|P_{k+1}\|+\|P_{k+2}\|)/|\|P_k\|-\|P_{k+1}\||\geq\|P_{k+1}-P_{k+2}\|/\|P_{k+1}-P_k\|$, that is, if $\|P_{k+1}\|\leq\min((K_k\|P_k\|-\|P_{k+2}\|)/(1+K_k),K_{k-1}\|P_{k-1}\|)$; for all $k\in\mathbf{N}$. □

3. Stability of Dynamic Systems with Eventual Switches

We now describe a closed-loop (or feedback) linear dynamic system of a separable Hilbert space H by the operator pair (L,C), formally identifying the physical closed-loop system, where L and C are operators on H describing the input-output relationships of the controlled system (sometimes, simply referred to as the "plant" to be controlled) and its controller, respectively, as follows:

$$\begin{bmatrix}u_1\\u_2\end{bmatrix}=\begin{bmatrix}I & C\\ L & -I\end{bmatrix}\begin{bmatrix}e_1\\e_2\end{bmatrix}, \tag{3.1}$$

where u_1 and u_2 and e_1 and e_2 are, respectively, externally applied (reference and noise) inputs and inputs to the controlled system and its controller, respectively. The separable Hilbert space is assumed over vector fields $\mathbf{C}^{n_\ell}$ or $\mathbf{R}^{n_\ell}$ for the input to the controlled system, of state dimension n_ℓ, and over vector fields $\mathbf{C}^{n_c}$ or $\mathbf{R}^{n_c}$ for the input to the controller, of state dimension n_c. The closed-loop dynamic system operator pair (L, C) is defined by operator $\Psi = \Psi(L, C) := \left[\begin{smallmatrix} I_{n_c} & C \\ L & -I_{n_\ell} \end{smallmatrix}\right]$, where I_m demotes the m-identity matrix, defined on the direct sum $H_e \oplus H_e$ of the extended space H_e of the Hilbert space H with itself, where $\mathbf{P}_t H_e = \mathbf{P}_t H$, $t \in \Gamma$ and $\mathbf{P}_t \neq \mathbf{I}$, subject to $\mathbf{P}_{t_1} \leq \mathbf{P}_{t_2}$ if $t_1 \leq t_2$, is a projection operator which defines the seminorm $\|x\|_t = \|\mathbf{P}_t x\|$ on H for $x \in H$ and $t \in \Gamma$. The subscript denoting the orders of identity matrices will be omitted in the following when no confusion is expected. The family $\{\|\cdot\|_t : t \in \Gamma\}$ of seminorms defines the resolution topology on H, since $\{\mathbf{P}_t : t \in \Gamma\}$ is a resolution of the identity with $x_t(\in H_e) = \mathbf{P}_t x$ being a truncation of $x \in H$; for all $t \in \Gamma$, the separation property for $x(\neq 0) \in H$, $\exists t \in \Gamma$ such that $\|x\|_t = \|\mathbf{P}_t x\| \neq 0$ and the convergence in this topology is defined as follows: $\{x_n\}$ converges to $x \in H$ if $\|x_n - x\|_t \to 0$ as $n \to \infty$; for all $t \in \Gamma$. It is said that the closed-loop system is well-posed if the internal input (e_1, e_2) is a causal function of the external input (u_1, u_2). This is equivalent to the operator $\left[\begin{smallmatrix} I & C \\ L & -I \end{smallmatrix}\right]$ to be causally invertible. If Γ is a discrete set starting at $t = 0$, all invertible operators are bounded and causally invertible. The closed-loop system (L, C) is said to be stable if $\Psi(L, C) : D(L) \oplus D(C) \to H \oplus H$ has a bounded causal inverse

$$\Psi^{-1} = \Psi^{-1}(L, C) = \begin{bmatrix} I & C \\ L & -I \end{bmatrix}^{-1} = \begin{bmatrix} (I + CL)^{-1} & C(I + LC)^{-1} \\ L(I + CL)^{-1} & -(I + LC)^{-1} \end{bmatrix} \tag{3.2}$$

defined on $H \oplus H$ such that

$$\widehat{e}_t = Q_t e_t = Q_t \mathbf{P}_t e = Q_t\left(\Psi^{-1}\mathbf{P}_t u + P_t \Phi x_0\right) = Q_t\left(\mathbf{P}_t \Psi^{-1}\mathbf{P}_t u + \mathbf{P}_t \Phi x_0\right), \quad \forall t \in \Gamma, \tag{3.3}$$

where $Q_t = (I, 0, \ldots, 0)$ is a matrix of $(n_\ell + n_c) \times (t+1)(n_\ell + n_c)$-order composed of $(t+1)(n_\ell + n_c)$ block matrices of which the first one is the $(n_\ell + n_c)$-identity matrix and the remaining ones are zero, so as to collect the current $(n_\ell + n_c)$ components of e_t in a vector $\widehat{e}_t$ of $(n_\ell + n_c)$ components. $\Phi = \Phi(L, C)$ is an operator defining the response to initial conditions from $\mathbf{C}^{n_\ell + n_c}$ to $H \oplus H$ if the operators L and C represent linear dynamic systems subject to initial conditions $x_{0L} \in \mathbf{C}^{n_\ell}$, $x_{0C} \in \mathbf{C}^{n_c}$, $e = \left[\begin{smallmatrix} e_1 \\ e_2 \end{smallmatrix}\right]$, $u = \left[\begin{smallmatrix} u_1 \\ u_2 \end{smallmatrix}\right]$, $x_0 = \left[\begin{smallmatrix} x_{0L} \\ x_{0C} \end{smallmatrix}\right]$. If Γ is a set of consecutive nonnegative integers, then (3.3) describes a causally invertible linear controlled discrete system. It can also describe a linear continuous dynamic system if Γ is formed for nonnegative real intervals of the form $\Gamma = \Gamma_c = \{[0, t_1), [t_1, t_2), \ldots\}$. The stability of the linear dynamic system is associated with the existence of a causal inverse of Ψ as follows, [38].

Theorem 3.1. *The closed-loop system (L, C) is stable if and only if*

$$G(L) \oplus G^{-1}(-C) = R\begin{bmatrix} I \\ L \end{bmatrix} \oplus \begin{bmatrix} -C \\ I \end{bmatrix} D(-C) = H \oplus H, \tag{3.4}$$

equivalently (in geometric terms), if and only if the orthogonal projection

$$\mathbf{\Lambda} = \mathbf{P}_{G^{-1}(-C)^{\perp}} \mid G(L) : G(L) \longrightarrow G^{-1}(-C)^{\perp} \tag{3.5}$$

of $G(L) \subset H \oplus H$ onto $G^{-1}(-C)^{\perp} \subset H \oplus H$ is an invertible operator, where $G(L)$ is the graph of L defined on $D(L)$ and $G^{-1}(-C)$ is the inverse graph of $(-C)$, being the subspace $\left[\begin{smallmatrix} -C \\ I \end{smallmatrix}\right] D(-C)$ of H, whose graph is $G(-C) = R\left[\begin{smallmatrix} I \\ -C \end{smallmatrix}\right]$ defined on $D(-C)$.

Note that if the closed-loop system (L, C) is stable then Ψ^{-1} and $\mathbf{\Lambda}$ are bounded operators. Assume Ψ^{-1} not bounded. Then, one can take $x_0 = 0$ and $u \in H$ with $\|u\| = 1$ such that e is unbounded, thus (L, C) is not stable. Thus, Ψ^{-1} is bounded. Assume that $\mathbf{\Lambda}$ is not bounded. Then, there is some bounded $u \in H$ such that e is unbounded from (3.3), since Ψ^{-1} is bounded so it is $\Psi^{-1}u$, so that $(e - \Psi^{-1}u)$ is unbounded. But then the external input u is unbounded from (3.1), here a contradiction, so that the operator $\mathbf{\Lambda}$ is bounded. The stability of the controlled system is now discussed under eventual switching in the parameterizations in both controlled object and its controller. To establish the particular stability properties, Theorem 3.1 is addressed together with the relevant results of Section 2. In the following, we adopt the convention that the projector $\mathbf{P}_t$ is defined for all t with $\mathbf{P}_{-t}x = 0$; for all $t \in \mathbf{R}_+$ so as to facilitate the formal presentation of some of the subsequent equations. The subsequent result, supported by Theorem 2.2, relies on the stability of a switched system with switches between several possible stable parameterizations provided that there is a convergence to one of them either in finite time (i.e., the switching process ends in finite time) or asymptotically.

Theorem 3.2. *Assume that there is a finite or infinite switching set of strictly ordered time instants $\Gamma_s = \{t_{s_0}, t_{s_1}, \ldots, t_{s_n}, \ldots\} \subseteq \Gamma_s \subseteq \Gamma = \{t_0, t_1, \ldots,\}$ with $t_{s_i} = t_k < t_{s_j}$, $t_i < t_j$ for any $j(> i)$, $n, i, j \in \mathbf{N}_0$ and some $k(\geq i) \in \mathbf{N}_0$, where $\mathbf{N}_0 = \mathbf{N} \cup \{0\}$. Consider the sequence of linear closed operators $\{\widehat{\Psi}_{t_{s_n}}\}_{n \in \mathbf{N}_0}$ on the Hilbert space H, where $\widehat{\Psi}_{t_{s_n}} = \left[\begin{smallmatrix} I & C_{t_{s_{n+1}}} \\ L_{t_{s_{n+1}}} & -I \end{smallmatrix}\right]$ such that $\widehat{\Psi}_{t_{s_0}}$ is also bounded and invertible with bounded inverse $\widehat{\Psi}^{-1}_{t_{s_0}}$, and*

$$\left\| \widetilde{\widehat{\Psi}}_{t_{s_{n-1}}} \right\|^2 < \delta^{-2}\left(\widehat{\Psi}_{t_{s_{n+1}}}, \widehat{\Psi}_{t_{s_n}}\right) - \left\| \widehat{\Psi}_{t_{s_{n-1}}} \right\|^2 - 1, \quad \forall t_{s_n} \in \Gamma_s, \ \forall n \in \mathbf{N}_0, \tag{3.6}$$

where $\widetilde{\widehat{\Psi}}_{t_{s_n}} = \left[\begin{smallmatrix} 0 & C_{t_{s_{n+1}}} - C_{t_{s_n}} \\ L_{t_{s_{n+1}}} - L_{t_{s_n}} & 0 \end{smallmatrix}\right]$. Then, the following properties hold:

(i) *The sequence $\{\widehat{\Psi}^{-1}_{t_{s_n}}\}_{n \in \mathbf{N}_0}$ exists and it consists of bounded operators on H.*

(ii) *In addition, such a sequence of inverse operators converges to a unique bounded operator $\widehat{\Psi}^{-1}$ on H, which is the bounded inverse operator of $\widehat{\Psi} = \left[\begin{smallmatrix} I & C \\ L & -I \end{smallmatrix}\right]$ on H in Theorem 2.2(i), if*

$$\left\| \widetilde{\widehat{\Psi}}_{t_{s_n}} \right\| \leq \frac{1}{K_{t_{s_n}}} \quad \text{with } K_{t_{s_n}} \in (0, K) \subset (0, 1), \ \forall n \in \mathbf{N}_0, \tag{3.7}$$

$$\delta\left(\widehat{\Psi}_{t_{s_{n+2}}}, \widehat{\Psi}_{t_{s_{n+1}}}\right) < \frac{K_k \left\| \widehat{\Psi}_{t_{s_{n+1}}} - \widehat{\Psi}_{t_{s_n}} \right\|}{1 + \left\| \widehat{\Psi}_{t_{s_{n+1}}} \right\|^2 + K_k \left\| \widehat{\Psi}_{t_{s_{n+1}}} - \widehat{\Psi}_{t_{s_n}} \right\| \left(1 + \left\| \widehat{\Psi}_{t_{s_{n+1}}} \right\|^2\right)^{1/2}}, \quad \forall n \in \mathbf{N}_0 \tag{3.8}$$

hold with the replacement $P_{(\cdot)} \to \widehat{\Psi}_{(\cdot)} = \left[\begin{smallmatrix} I & C_{(\cdot)} \\ L_{(\cdot)} & -I \end{smallmatrix}\right]$. Furthermore, the switched closed-loop system of sequence pairs of operators $\{(L_{t_{s_n}}, C_{t_{s_n}})\}_{t \in \Gamma}$, each of them being stable, is stable.

Proof. One gets from (3.3) that

$$Q_{t_{s_n}}\mathbf{P}_{t_{s_n}}e = Q_{t_{s_n}}\left(\Psi_{t_{s_n}}^{-1}\mathbf{P}_{t_{s_n}}u + \mathbf{P}_{t_{s_n}}\Phi_{t_{s_n}}x_0\right), \quad \forall t \in \Gamma_s,\ \forall n \in \mathbf{N}_0 \tag{3.9}$$

if $\operatorname{card}(\Gamma_s) \leq \chi_0$, where χ_0 is an infinite cardinal number for numerable sets, and

$$Q_{t_{s_n}}e_{t_{s_n}} = Q_{t_{s_n}}\left(\Psi_{t_{s_n}}^{-1}\mathbf{P}_{t_{s_n}}u + \mathbf{P}_{t_{s_n}}\Phi x_0\right) = Q_{t_{s_n}}\left(\Psi_{s}^{-1}\mathbf{P}_{t_{s_n}}u + \mathbf{P}_{t_{s_n}}\Phi x_0\right), \quad \forall t_{s_n} \in \Gamma_s,\ \forall n \in \mathbf{N}_0 \tag{3.10}$$

if $\operatorname{card}(\Gamma_s) = \chi_0$ provided that the above inverse operators exist, where

$$\begin{aligned} Q_{t_{s_n}}\Psi_{t_{s_n}}\mathbf{P}_{t_{s_n}}e = \Bigg(&\begin{bmatrix} I & \left(C_{t_{s_n}} - L_{t_{s_{n-1}}}\right)\mathbf{P}_{t_{s_n}} \\ \left(L_{t_{s_n}} - L_{t_{s_{n-1}}}\right)\mathbf{P}_{t_{s_n}} & -I \end{bmatrix} \\ &+ \begin{bmatrix} 0 & \sum_{i=0}^{n}\left(C_{t_{s_i}} - C_{t_{s_{i-1}}}\right)\mathbf{P}_{t_{s_i}} \\ \sum_{i=0}^{n-1}\left(L_{t_{s_i}} - L_{t_{s_{i-1}}}\right)\mathbf{P}_{t_{s_i}} & 0 \end{bmatrix}\Bigg)e, \quad \forall t_{s_n} \in \Gamma_s,\ \forall n \in \mathbf{N}_0 \end{aligned} \tag{3.11}$$

with a number of parameterization switches being $n_f := \operatorname{card}(\Gamma_s) \leq \chi_0$; for all $t_{s_n} \in \Gamma_s \cap [t_{j_{s_n}}, t_{j_{s_n}+1})$ for some two consecutive switching time instants $t_{j_{s_n}}, t_{j_{s_n}+1} \in \Gamma$ such that $t_{s_n} \in [t_{j_{s_n}}, t_{j_{s_n}+1})$. One has under zero initial conditions that

$$\begin{aligned} C_{t_{s_{n+1}}}u_{t_{s_{n+1}}} &= C_{t_{s_n}}\begin{bmatrix} I & C_{t_{s_{n+1}}} \\ L_{t_{s_{n+1}}} & -I \end{bmatrix}\mathbf{P}_{t_{s_{n+1}}}e \\ &+ C_{t_{s_{n+1}}}\begin{bmatrix} \sum_{i=0}^{n}\left[\left(\mathbf{P}_{t_{s_i}} - \mathbf{P}_{t_{s_{i-1}}}\right)e_{1t_{s_i}} + \left(C_{t_{s_i}} - C_{t_{s_{i-1}}}\right)\mathbf{P}_{t_{s_i}}e_{2t_{s_i}}\right] \\ \sum_{i=0}^{n}\left[\left(L_{t_{s_i}} - L_{t_{s_{i-1}}}\right)\mathbf{P}_{t_{s_i}}e_{1t_{s_i}} - \left(\mathbf{P}_{t_{s_i}} - \mathbf{P}_{t_{s_{i-1}}}\right)e_{2t_{s_i}}\right] \end{bmatrix}, \quad \forall t_{s_n} \in \Gamma_s, \forall n \in \mathbf{N}_0 \end{aligned} \tag{3.12}$$

provided that $\widehat{\Psi}_{t_{s_{n+1}}}^{-1} = \begin{bmatrix} I & C_{t_{s_{n+1}}} \\ L_{t_{s_{n+1}}} & -I \end{bmatrix}^{-1}$ exists. Note that

$$\widehat{\Psi}_{t_{s_{n+1}}}^{-1} = \left(\widehat{\Psi}_{t_{s_n}} + \widetilde{\widetilde{\Psi}}_{t_{s_n}}\right)^{-1} = \left(I + \widehat{\Psi}_{t_{s_n}}^{-1}\widetilde{\widetilde{\Psi}}_{t_{s_n}}\right)^{-1}\widehat{\Psi}_{t_{s_n}}^{-1}, \quad \forall t_{s_n} \in \Gamma_s, \forall n \in \mathbf{N}_0 \tag{3.13}$$

if the inverses exist, where $\widetilde{\widetilde{\Psi}}_{t_{s_n}} = \begin{bmatrix} 0 & C_{t_{s_{n+1}}} - C_{t_{s_n}} \\ L_{t_{s_{n+1}}} - L_{t_{s_n}} & 0 \end{bmatrix}$, for all $n \in \mathbf{N}_0$, and it is bounded if $\delta(\widehat{\Psi}_{t_{s_n}}, \widehat{\Psi}_{t_{s_{n+1}}}) < 1/\sqrt{1 + \|\widehat{\Psi}_{t_{s_n}}\|^2} < 1$, for all $t_{s_n} \in \Gamma_s$, for all $n \in \mathbf{N}_0$ from Theorem 2.2 with the replacements $P_{(\cdot)} \to \widehat{\Psi}_{(\cdot)} = \begin{bmatrix} I & C_{(\cdot)} \\ L_{(\cdot)} & -I \end{bmatrix}$ being what is guaranteed if $\|\widetilde{\widehat{\Psi}}t_{s_{n-1}}\|^2 < \delta^{-2}(\widehat{\Psi}_{t_{s_{n+1}}}, \widehat{\Psi}_{t_{s_n}}) - \|\widehat{\Psi}_{t_{s_{n-1}}}\|^2 - 1$; for all $t_{s_n} \in \Gamma_s$, for all $n \in \mathbf{N}_0$, since

$$1 + \left\|\widehat{\Psi}_{t_{s_n}}\right\|^2 \leq 1 + \left\|\widehat{\Psi}_{t_{s_{n-1}}}\right\|^2 + \left\|\widetilde{\widehat{\Psi}}_{t_{s_{n-1}}}\right\|^2 < \delta^{-2}\left(\widehat{\Psi}_{t_{s_{n+1}}}, \widehat{\Psi}_{t_{s_n}}\right), \quad \forall t_{s_n} \in \Gamma_s, \forall n \in \mathbf{N}_0. \tag{3.14}$$

Thus, $\{\widehat{\Psi}^{-1}(t_{n_s})\}_{n\in\mathbf{N}_0}$ is a sequence of sufficiently close operators in terms of the gap metric so that they are bounded closed operators from Theorem 2.2((i)-(ii)) with the replacements $P_{(\cdot)} \to \widehat{\Psi}_{(\cdot)} = \begin{bmatrix} I & C_{(\cdot)} \\ L_{(\cdot)} & -I \end{bmatrix}$ following complete induction. The remaining of the proof follows also by complete induction concerning the convergence of the sequence $\{\widehat{\Psi}^{-1}(t_{n_s})\}_{n\in\mathbf{N}_0}$ to a bounded closed operator follows from the convergence conditions (2.5)-(2.6) of Theorem 2.2 to get conditions (3.7)-(3.8). Thus, the operator sequence $\{Q_{t_{s_n}}\Psi_{t_{s_n}}\mathbf{P}_{t_{s_n}}\}_{n\in\mathbf{N}_0}$ of elements defined in (3.11) is also closed with an associate bounded existing sequence of bounded inverse operators which has a bounded invertible limit if $\{\widehat{\Psi}_{t_{n_s}}\}_{n\in\mathbf{N}_0}$ converges under the conditions (3.7)-(3.8). Note also from (3.11) that

$$Q_t\Psi_t\mathbf{P}_t e = \begin{bmatrix} I & (L_t - L_{t_{s_n}})\mathbf{P}_t + \sum_{i=0}^{n}\left(C_{t_{s_i}} - C_{t_{s_{i-1}}}\right)\mathbf{P}_{t_{s_i}} \\ (L_t - L_{t_{s_n}})\mathbf{P}_t + \sum_{i=0}^{n}\left(L_{t_{s_i}} - L_{t_{s_{i-1}}}\right)\mathbf{P}_{t_{s_i}} & -I \end{bmatrix} e, \quad \forall t \in [t_{s_n}, t_{s_{n+1}}) \tag{3.15}$$

if $t_{s_{n+1}} \in \Gamma_s$ exists and for all $t \in [t_{n_{sf},\infty})$ if $\operatorname{card}(\Gamma_s) = n_f < \chi_0$. Thus, $\{Q_{t_{s_n}}\Psi_{t_{s_n}}\mathbf{P}_{t_{s_n}}\}_{n\in\mathbf{N}_0}$ is bounded with a an existing finite sequence of bounded and closed inverse operators converging to a bounded limit if $\operatorname{card}(\Gamma_s) = n_f \leq \chi_0$ with $Q_t\Psi_t\mathbf{P}_t$ having also a bounded inverse; for all $t \in [t_{s_n}, t_{s_{n+1}})$ for $t_{s_{n+1}} \in \Gamma_s$ and for all $t \in [t_{n_{sf},\infty})$ if $\operatorname{card}(\Gamma_s) = n_f < \chi_0$. Then (3.4) holds and the switched system is stable from Theorem 3.1 since:

$$G(L_{t_{s_n}}) \oplus G^{-1}(-C_{t_{s_n}}) = R\begin{bmatrix} I \\ L_{t_{s_n}} \end{bmatrix} \oplus \begin{bmatrix} -C_{t_{s_n}} \\ I \end{bmatrix} D(-C_{t_{s_n}}) = (\mathbf{P}_{t_{s_n}}H) \oplus (\mathbf{P}_{t_{s_n}}H), \tag{3.16}$$

$$G(L_t) \oplus G^{-1}(-C_t) = R\begin{bmatrix} I \\ L_t \end{bmatrix} \oplus \begin{bmatrix} -C_t \\ I \end{bmatrix} D(-C_t) = (\mathbf{P}_t H) \oplus (\mathbf{P}_t H) \quad \forall t \in [t_{s_n}, t_{s_{n+1}}), \forall t_{s_{n+1}} \in \Gamma_s \tag{3.17}$$

since $\{\widehat{\Psi}_{t_{s_n}}^{-1}\}_{n\in\mathbf{N}_0}$ exists consisting of bounded operators on H, and by construction on $\mathbf{P}_{t_{s_n}}H$ for all $t_{s_n} \in \Gamma$. In addition, such a sequence of inverse operators converges to a unique bounded operator $\widehat{\Psi}^{-1}$ on H so that (3.16) implies (3.4) and, equivalently, (3.5) in Theorem 3.1. Then, the switched closed-loop system defined by the convergent sequence of operator pairs $\{(L_{t_{s_n}}, C_{t_{s_n}})\}_{t_{s_n}\in\Gamma}$ is stable. □

Remark 3.3. Note that the above result also holds if a finite time interval is removed from the analysis, that is, if there is a finite number of switches between a set of parameterizations not all being stable and after such a finite time interval the hypotheses hold. Note that (3.6) in Theorem 3.2 guarantees the existence of the inverse operator and its boundedness since they are closed and also sufficiently close (in terms of the gap metric) to each next consecutive element within such a sequence. On the other hand, the constraint (3.8) in Theorem 3.2 guarantees the convergence of the sequence of existing inverse operators to a closed operator which is also bounded so that Theorem 3.1 holds.

Remark 3.4. Note that Theorem 3.2 guarantees the stability of a switched system whose sequence of parameterizations converges to a stable configuration while all such parameterizations are stable. However, it is easy to generalize the result to two weaker conditions as follows:

(1) Not all the parameterizations $(L_{t_{s_n}}, C_{t_{s_n}})$; $t_{s_n} \in \Gamma$ are stable but Theorem 3.2 conditions are fulfilled for $t^*_{s_n} \in \Gamma^* \subset \Gamma$, where $\Gamma^* \ni t^*_{s_n} = t_{s_k} \in \Gamma$ and $\Gamma^* \ni t^*_{s_{n+1}} = t_{s_{k+\ell_k}} \in \Gamma$, with $\ell_k \le \ell < \infty$ are two consecutive marked elements of Γ^* which are not necessarily consecutive in Γ such that $(L_{t^*_{s_n}}, C_{t^*_{s_n}})$ is stable.

(2) Theorem 3.2 is fulfilled only for the subset $\Gamma_{t^*_n} \subset \Gamma$ obtained by removing a finite set $\{t^*_0, t^*_1, \ldots, t^*_{n-1}\}$ from Γ.

The fact that the convergence of the sets of parameterizations to a stable one is not required for stabilization purposes is now discussed while it is sufficient that the switched parameterized sequence has consecutive stable parameterizations of sufficiently large norms.

Theorem 3.5. *Assume the following.*

(1) *There is a switching set of infinite cardinal of strictly ordered time instants* $\Gamma_s = \{t_{s_0}, t_{s_1}, \ldots, t_{s_n}, \ldots\} \subseteq \Gamma_s \subseteq \Gamma = \{t_0, t_1, \ldots,\}$ *with* $t_{s_i} = t_k < t_{s_j}$, $t_i < t_j$ *for any* $j(> i), n, i, j \in \mathbf{N}_0$ *and some* $k(\ge i) \in \mathbf{N}_0$, *where* $\mathbf{N}_0 = \mathbf{N} \cup \{0\}$.

(2) *The sequence of linear closed operators* $\{\widehat{\Psi}_{t_{s_n}}\}_{n\in\mathbf{N}_0}$ *on the Hilbert space* H, *where* $\widehat{\Psi}_{t_{s_n}} = \begin{bmatrix} I & C_{t_{s_{n+1}}} \\ L_{t_{s_{n+1}}} & -I \end{bmatrix}$, *is subject to* $\widehat{\Psi}_{t_{s_0}}$ *being bounded and invertible with a bounded inverse* $\widehat{\Psi}^{-1}_{t_{s_0}}$, *and* $\|\widetilde{\widehat{\Psi}}_{t_{s_{n-1}}}\|^2 < \delta^{-2}(\widehat{\Psi}_{t_{s_{n+1}}}, \widehat{\Psi}_{t_{s_n}}) - \|\widehat{\Psi}_{t_{s_{n-1}}}\|^2 - 1$ *where* $\widetilde{\widehat{\Psi}}_{t_{s_n}} = \widehat{\Psi}_{t_{s_{n+1}}} \widehat{\Psi}_{t_{s_n}}$; *for all* $t_{s_n} \in \Gamma_s$, *for all* $n \in \mathbf{N}_0$.

(3) $\sum_{t_{s_n}\in\Gamma} \|\widetilde{\widehat{\Psi}}_{t_{s_n}}\| < \infty$ *where*

$$\left\|\widetilde{\widehat{\Psi}}_{t_{s_n}}\right\| = \left\|\begin{bmatrix} 0 & C_{t_{s_{n+1}}} - C_{t_{s_n}} \\ L_{t_{s_{n+1}}} - L_{t_{s_n}} & 0 \end{bmatrix}\right\| \le f_{t_{s_n}} \cdot g_{t_{s_n}}(p(t_{s_n}, t_{s_{n+1}})) \tag{3.18}$$

for some nonnegative strictly decreasing real sequence $\{f_{t_{s_n}}\}_{n\in\mathbf{N}_0}$ *and some bounded nonnegative real sequence* $\{g_{t_{s_n}}(p(t_{s_n}, t_{s_{n+1}}))\}_{n\in\mathbf{N}_0}$ *which depends on the active parameterization within* $[t_{s_n}, t_{s_{n+1}})$ *which is parameterized by a bounded function of parameters* $p(t_{s_n}, t_{s_{n+1}}) \in \mathbf{R}^{q(t_{s_n}, t_{s_{n+1}})}$.

Then, the switched closed-loop system of sequence pairs of operators $\{(L_{t_{s_n}}, C_{t_{s_n}})\}_{t\in\Gamma}$, *each of them being stable, is stable.*

Proof. From the first part of Theorem 3.2, one deduces that the sequence $\{\widehat{\Psi}^{-1}_{t_{s_n}}\}_{n\in\mathbf{N}_0}$ exists and it consists of bounded operators on H. The assumption $\sum_{t_{s_n}\in\Gamma} \|\widetilde{\widehat{\Psi}}_{t_{s_n}}\| < \infty$ implies $\|\widetilde{\widehat{\Psi}}_{t_{s_n}}\| \to 0$

as $t_{s_n} \to \infty$ since $\operatorname{card}(\Gamma_s) = \chi_0$, with $f_{t_{s_n}} \to 0$ as $t_{s_n} \to \infty$, according to (3.18). Thus, from (3.15) in the proof of Theorem 3.2, $\{Q_{t_{s_n}} \Psi_{t_{s_n}} \mathbf{P}_{t_{s_n}}\}_{n\in\mathbf{N}_0}$ is bounded, the sequence $\{\widehat{\Psi}^{-1}(t_{n_s})\}_{n\in\mathbf{N}_0}$ exists and consists of bounded and closed operators while it converges to a bounded closed operator on H and then (3.16)-(3.17) hold and the switched closed-loop system is stable. □

The condition $\|\widetilde{\widehat{\Psi}}_{t_{s_n}}\| \le f_{t_{s_n}} \cdot g_{t_{s_n}}(p(t_{s_n}, t_{s_{n+1}}))$ of (3.18) with $\sum_{t_{s_n}\in\Gamma} \|\widetilde{\widehat{\Psi}}_{t_{s_n}}\| < \infty$ in Theorem 3.5 implies that the operators describing the controlled object and controller both converge. It is not required for the parameterization, whose worst-case contribution to the norm $\|\widetilde{\widehat{\Psi}}_{t_{s_n}}\|$, given by $g_{t_{s_n}}(p(t_{s_n}, t_{s_{n+1}}))$, to converge. In real situations, the switching process can activate stable parameterizations without convergence to a particular one provided that a sufficiently large residence time $T_{\min} > 0$, such that $T_{s_n} := t_{s_{n+1}} - t_{s_n} \ge T_{\min}$, is respected at each active parameterization so that $f_{t_{s_n}} \to 0$ as $t_{s_n} \to \infty$ in (3.18).

Theorem 3.5 is extended as follows by addressing the existence of the inverses of the relevant operators for finite strips of composite operators rather that for each individual operator.

Theorem 3.6. *Assume the following.*

(1) *There is a switching set of infinite cardinal of strictly ordered time instants* $\Gamma_s = \{t_{s_0}, t_{s_1}, \dots, t_{s_n}, \dots\} \subseteq \Gamma_s \subseteq \Gamma = \{t_0, t_1, \dots,\}$ *with* $t_{s_i} = t_k < t_{s_j}$, $t_i < t_j$ *for any* $j(> i)$, $n, i, j \in \mathbf{N}_0$ *and some* $k(\ge i) \in \mathbf{N}_0$, *where* $\mathbf{N}_0 = \mathbf{N} \cup \{0\}$.

(2) *There is a switching set of infinite cardinal of marked strictly ordered time instants* $\Gamma_s^* = \{t_{s_0}^*, t_{s_1}^*, \dots, t_{s_n}^*, \dots\} \subset \Gamma_s$ *such the composite operator sequence* $\{\widehat{\Psi}(t_{s_i}^*, t_{s_{i+1}}^*)\}_{n\in\mathbf{N}_0}$ *is a composite operator defined for* $s_i^* = s_{n_i}$ *and* $s_{i+1}^* = s_{n_i+n_{\ell_i}}$ *for some* $n_i \in N_0$, $n_{\ell_i} \in \mathbf{N}_0$, *and*

$$\widehat{\Psi}(t_{s_i}^*, t_{s_{i+1}}^*) \equiv \widehat{\Psi}\left(t_{s_{n_i}}, t_{s_{n_{i+1}}}\right) := \widehat{\Psi}_{t_{s_{n_{i+\ell_{n_i}}}}} \circ \cdots \circ \widehat{\Psi}_{t_{s_{n_{i+1}}}} \circ \widehat{\Psi}_{t_{s_{n_i}}}, \tag{3.19}$$

where the operator

$$\widehat{\Psi}\left(t_{s_{n_1}}^*, t_{s_{n_2}}^*\right) = \begin{bmatrix} I & C_{t_{s_{n_1}}^*} + \sum_{j=1}^{n_2-1}\left(C_{t_{s_{n_{1+j}}}^*} - C_{t_{s_{n_{1+j-1}}}^*}\right) \\ L_{t_{s_{n_1}}^*} + \sum_{j=1}^{n_2-1}\left(L_{t_{s_{n_{1+j}}}^*} - L_{t_{s_{n_{1+j-1}}}^*}\right) & -I \end{bmatrix}, \quad \forall t_{s_n}^* \in \Gamma_s^*, \forall n \in \mathbf{N}_0 \tag{3.20}$$

consists of finite strips of linear closed operators on H *such that* $\widehat{\Psi}(t_{s_{n_0}}^*, t_{s_{n_{0+1}}}^*)$ *is bounded and invertible of bounded inverse* $\widehat{\Psi}^{-1}(t_{s_{n_0}}^*, t_{s_{n_{0+1}}}^*)$ *satisfying* $\sum_{t_{s_n}\in\Gamma} \|\widetilde{\widehat{\Psi}}(t_{s_n}^*, t_{s_{n+1}}^*)\| < \infty$, *and*

$$\left\|\widetilde{\widehat{\Psi}}\left(t_{s_{n_{1-1}}}^*, t_{s_{n_1}}^*\right)\right\|^2 < \delta^{-2}\left(\widehat{\Psi}\left(t_{s_{n_1}}^*, t_{s_{n_{1+1}}}^*\right), \widehat{\Psi}\left(t_{s_{n_{1-1}}}^*, t_{s_{n_1}}^*\right)\right) - \left\|\widehat{\Psi}\left(t_{s_{n_{1-1}}}^*, t_{s_{n_1}}^*\right)\right\|^2 - 1, \quad \forall t_{s_n}^* \in \Gamma_s^*, \forall n \in \mathbf{N}_0,$$

$$\begin{aligned}&\left\| \widetilde{\widehat{\Psi}}\left(t_{s_n}^*, t_{s_{n+1}}^*\right) \right\| \\ &\qquad \le f_{s_n}\left(t_{s_n}^*, t_{s_{n+1}}^*\right) \cdot g_{s_n}\left(t_{s_n}^*, t_{s_{n+1}}^*\right)\left(p\left(t_{s_n}^*, t_{s_{n+1}}^*\right)\right), \quad \left(t_{s_{n+1}}^* - t_{s_n}^*\right) \longrightarrow T^* < \infty \text{ as } n \longrightarrow \infty\end{aligned} \tag{3.21}$$

for some nonnegative strictly decreasing real sequence $\{f_{s_n}(t_{s_n}^*, t_{s_{n+1}}^*)\}_{n\in\mathbf{N}_0}$ *and some bounded nonnegative real sequence* $\{g_{s_n}(t_{s_n}^*, t_{s_{n+1}}^*)\}_{n\in\mathbf{N}_0}$ *which depends on the active parameterization within* $[t_{s_n}, t_{s_{n+1}})$ *which is parameterized by a bounded function of parameters* $p(t_{s_n}^*, t_{s_{n+1}}^*) \in \mathbf{R}^{q(t_{s_n}^*, t_{s_{n+1}}^*)}$.

Then, the switched closed-loop system of sequence pairs of composite operators $\{(L(t_{s_n}^*, t_{s_{n+1}}^*), C(t_{s_n}^*, t_{s_{n+1}}^*))\}_{t\in\Gamma}$, *each of them being stable, is stable.*

Proof. Linked to (3.13), let us now consider (3.19)-(3.20). The theorem hypothesis guarantee the existence of the inverse operator

$$\widehat{\Psi}^{-1}\left(t_{s_{n_1}}^*, t_{s_{n_2}}^*\right) = \begin{bmatrix} I & C_{t_{s_{n_1}}^*} + \sum_{j=1}^{n_2-1}\left(C_{t_{s_{n_{1+j}}}^*} - C_{t_{s_{n_{1+j-1}}}^*}\right) \\ L_{t_{s_{n_1}}^*} + \sum_{j=1}^{n_2-1}\left(L_{t_{s_{n_{1+j}}}^*} - L_{t_{s_{n_{1+j-1}}}^*}\right) & -I \end{bmatrix}^{-1} \tag{3.22}$$

being closed and bounded. From (3.21), the finite operators $\widehat{\Psi}(t_{s_{n_1}}^*, t_{s_{n_2}}^*)$; for all $t_{s_n}^* \in \Gamma_S^*$, for all $n \in \mathbf{N}_0$ converge to a closed invertible operator of bounded inverse. □

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