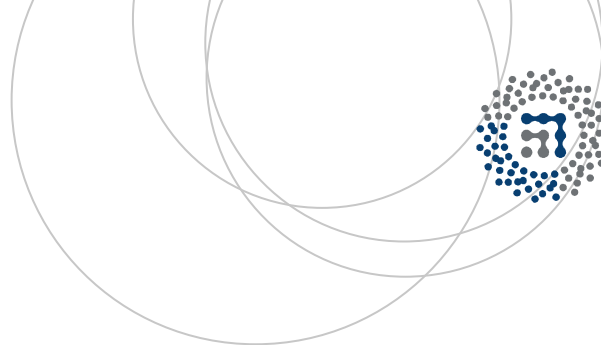


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Stability Analysis of RF amplifiers based on MIMO pole-zero identification

Creation of a Software Tool based on Vector Fitting

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Mila Esker Danoi!

Abstract

Spurious oscillations are one of the principal issues faced by microwave and RF circuit designers. The rigorous detection of instabilities or the characterization of measured spurious oscillations is still an ongoing challenge.

This project aims to create a new stability analysis CAD program that tackles this challenge. Multiple Input Multiple Output (MIMO) pole-zero identification analysis is introduced on the program as a way to create new methods to automate the stability analysis process and to help designers comprehend the obtained results and prevent incorrect interpretations. The MIMO nature of the analysis contributes to eliminate possible controllability and observability losses and helps differentiate mathematical and physical quasi-cancellations, products of overmodeling.

The created program reads Single Input Single Output (SISO) or MIMO frequency response data, and determines the corresponding continuous transfer functions with Vector Fitting. Once the transfer function is calculated, the corresponding pole/zero diagram is mapped enabling the designers to analyze the stability of an amplifier.

Three data processing methods are introduced, two of which consist of pole/zero eliminations and the latter one on determining the critical nodes of an amplifier. The first pole/zero elimination method is based on eliminating non resonant poles, whilst the second method eliminates the poles with small residue by assuming that their effect on the dynamics of a system is small or non-existent. The critical node detection is also based on the residues; the node at which the effect of a pole on the dynamics is highest is defined as the critical node.

In order to evaluate and check the efficiency of the created program, it is compared via examples with another existing commercial stability analysis tool (STAN tool).

In this report, the newly created tool is proved to be as rigorous as STAN for detecting instabilities. Additionally, it is determined that the MIMO analysis is a very profitable addition to stability analysis, since it helps to eliminate possible problems of loss of controllability, observability and overmodeling.

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Chapter 1

Introduction and Objectives

Radio Frequency (RF) is the field of high frequency systems and circuits (3 KHz -300 GHz, see Figure 1.1). Science and technology improvements, as well as a high industrial interest, have given way to many high-speed applications, such as, wireless communication, navigation, radar, and scientific equipment in this field. This project focuses mainly on the analysis of microwave circuits, more concretely on the stability analysis of power amplifiers for satellite applications and RF instrumentation.

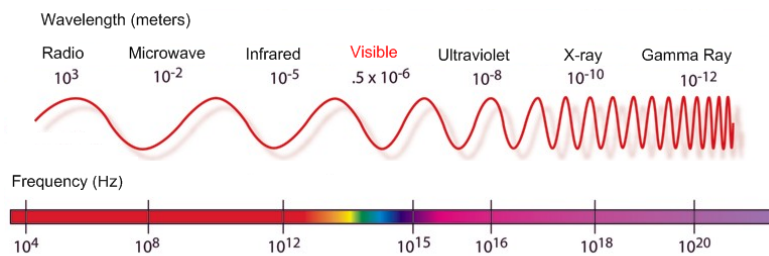


Figure 1.1: Electromagnetic Spectrum

Radio Frequency amplifiers are a key component in Communications Systems as illustrated in Figure 1.2, since most Communication Systems include Power Amplifiers (PA) as transmitters and Low Noise Amplifiers (LNA) as receivers.

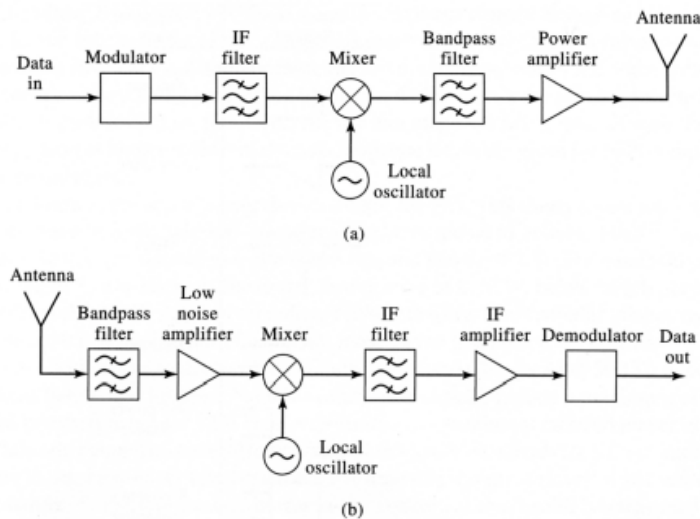


Figure 1.2: Basic Communication System Block Diagrams

Spurious oscillations are one of the principal issues faced by microwave circuit designers, especially for those who focus on RF amplifiers. These oscillations are often encountered on circuits containing multiple active devices, since these circuits are more prone to oscillate due to the presence of many feedback loops that reach the oscillation condition at a certain spurious frequency. Detection of spurious oscillations through simulation is essential to provide appropriate solutions to eliminate them with a minimum impact over the circuit performances.

Computer Aided Design (CAD) for detection of instabilities is still a challenge for RF designers. Nevertheless, a Stability Analysis Tool that tackles this challenge named the STAN tool [1] has recently been proposed. The mentioned commercial and user friendly STAN tool is a pole-zero stability analysis technique for linear and non-linear microwave circuits, based on obtaining a closed-loop Single Input Single Output (SISO) transfer function of the circuit linearized about a given steady state.

This project focuses on creating a CAD program on Matlab, to analyze the stability of DC solutions of RF amplifiers by obtaining a closed-loop SISO transfer function, like the mentioned STAN tool. But goes beyond and expands the mentioned pole-zero identification technique, by including Multiple Input Multiple Output (MIMO) analysis. The MIMO analysis is available due to the Vector Fitting algorithm [2][3][4] that formulates multiple continuous transfer function, with the same denominator, from a given set of frequency responses. MIMO analysis gives way to creating new methods to further characterize the stability of RF and microwave amplifiers and helps prevent problems of loss of controllability and observability and overmodeling.

Objectives

The principal objectives pursued by this project are the following:

- Create a program on Matlab for SISO and MIMO frequency domain response analysis using Vector Fitting.
- Create and incorporate additional pole/zero elimination methods for further automation of the stability analysis.
- Create and incorporate a method to determine critical nodes of an amplifier for subsequent stabilization approaches.
- Evaluate Vector Fitting as a methodology for the fitting of frequency domain responses with rational function approximations.
- Evaluate the benefits of MIMO identification for stability analysis of microwave amplifiers.
- Compare the created program with existing stability analysis tools.

Structure of the Report

Based on these objectives, the paper is structured as follows:

- Chapters 2, 3 and 4: The techniques and procedures used to perform the project are analyzed.
- Chapter 5: The results obtained with the designed Stability Analysis CAD tool for two microwave amplifiers are presented and discussed.
- Chapter 6: The main areas covered throughout the project are summarized and brought together.

Chapter 2

Stability in Microwave and RF Circuits

The mandatory requirement when designing an electronic system is its stability because an unstable system is generally considered useless. So, understanding the stability concept and defining it, as it is done in this chapter, is essential to design any type of electronic systems that contains active elements, linear, non-linear, variant or invariant in time.

Regarding microwave circuits and more concretely power amplifiers, carrying out a stability analysis of the design is indispensable, given that a designer must ensure that an amplifier will not oscillate under normal operating conditions. A common method to determine possible instabilities is to calculate the stability factor created by Rollet from the Scattering Parameters, as discussed on this chapter. However, this conventional method has several limitations, so, other stability analysis techniques, such as the techniques discussed on this project, are sought.

2.1 Basic Concepts on Stability

There are many existing and not necessarily equivalent definitions for stability of Linear Time-Invariant (LTI) systems, but the following three are the most employed [5]:

- A system is stable if the output for a bounded input is bounded. This Bounded Input Bounded Output (BIBO) time domain stability definition is used mainly in matters of signal processing and control theory.
- A system is stable if $\int_0^{\infty} |g(t)| dt$ is finite or equivalently if the $\lim_{x \rightarrow \infty} g(t) = 0$ condition is met, $g(t)$ being the impulse function defined on the time domain.
- A system is stable if all of the poles of the transfer function are on the left hand side (LHS) of the s plane of the Laplace transform domain. This method and information regarding poles and transfer functions is defined further in the chapter.

Thus, there is no need to calculate the complete temporal output of the system to determine its stability information. By analyzing the roots of its characteristic equation, one can determine whether the electronic system is stable or not. However, there are also other existing criteria to determine the stability or instability of a system that do not depend on the characteristic equation, such as [6]:

- Routh-Hurwitz criteria: This algebraic method provides information regarding the absolute stability of a linear time-invariant system. The principal constraint is that the coefficients of the characteristic equation must be constants. But, the method determines the number of roots on the $j\omega$ axis and on the right hand side (RHS) of the s plane.

- This semi-graphic method provides information regarding the difference between the number of poles and zeros of the closed loop transfer function that are on the RHS of the s plane. This is achieved by observing the behavior of the Nyquist graph (see Figure 2.1) of the transfer function.

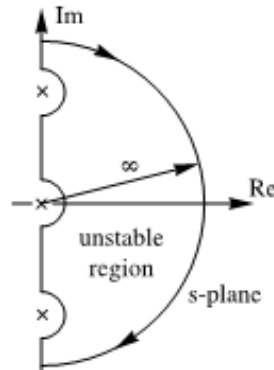


Figure 2.1: Illustration of the Nyquist Trajectory

- **Bode Diagram:** The Bode diagram is a magnitude (dB) and phase (either radians or degrees) graph of the transfer function, with frequency ω on the abscissa axis. The stability can be determined by monitoring the resonance and anti-resonance peaks of the transfer function curves.

In view of possible coexistence of various stable solutions on non-linear systems, the difference between global and local stability is to be stated and illustrated with the intuitive image of a ball on a hill [1]:

- The absolute stability is a boolean condition that specifies whether a solution is stable or not (see Figure 2.2). Additionally, a ball at the bottom of an infinite valley represents a globally stable solution, whilst a ball at the peak of a infinite hill illustrates a globally unstable system.

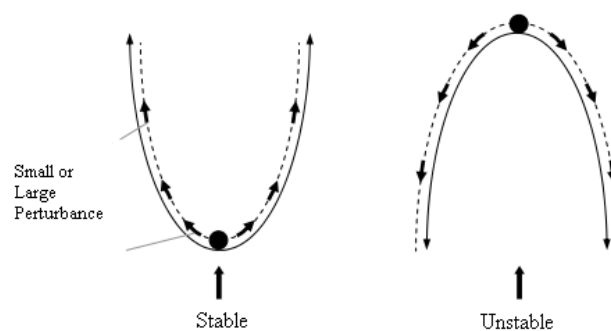


Figure 2.2: Illustration of Absolute Stability. The image on the left represents a system with a single stable solution. Whilst, the only solution of the system on the image on the right is unstable. Illustration from [1]

- Once the stability has been proven, determining the degree of stability of the solutions, that is, determining the local stability, is often suggested. This requires analyzing the effect that small perturbations such as vibrations and noise have on the solutions of the system. That is, if a small perturbation dispels the systems from the solutions or if the perturbed system returns exponentially in time back to the initial solution (see Figure 2.3).

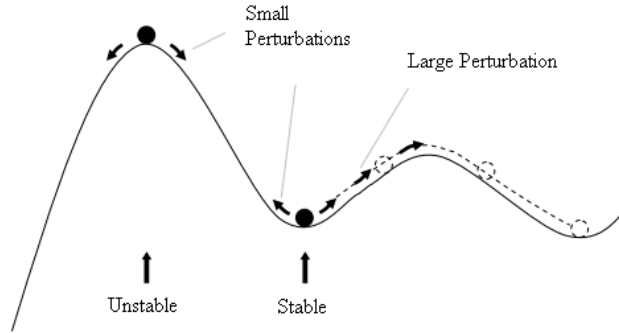


Figure 2.3: Illustration of Local Stability. A small perturbation will dispel the ball from the unstable solution on the left, but not from the stable solution on the center. A larger perturbation is required to dispel the ball from the set stable solution to another solution.

Illustration from [1]

As we see on Figure 2.3 a local stability does not imply a global stability, since a large perturbation can force the system to abandon a stable solution and relocate on a new stable solution nearby.

2.2 Stability of Systems

All systems, linear and nonlinear can be modeled mathematically by a state space representation that relates the outputs and inputs of the system through first-order differential equations as shown in equation 2.1 [7]. This method can be employed to determine the solution of the system equations and so, to analyze its stability.

$$\begin{aligned}\dot{\bar{x}}(t) &= f(\bar{x}(t), \bar{u}(t), t) \\ \bar{y}(t) &= g(\bar{x}(t), \bar{u}(t), t)\end{aligned}\tag{2.1}$$

Where \bar{x} represents the state vector, \bar{u} is the input vector or control vector and \bar{y} is the output vector.

Depending on whether a system is linear or non-linear and variant or invariant in time, the input and state variable dependencies on equation 2.1 vary.

2.2.1 Stability of Linear Systems

The equation for linear time invariable systems developed from previous equation 2.1 is the following [8]:

$$\begin{aligned}\dot{\bar{x}}(t) &= A\bar{x}(t) + B\bar{u}(t) \\ \bar{y}(t) &= C\bar{x}(t) + D\bar{u}(t)\end{aligned}\quad (2.2)$$

The output of the system for no input ($\bar{u}(t) = 0$) is the solution of the linear $\dot{\bar{x}}(t) = A\bar{x}(t)$ equation and takes the form of:

$$\bar{x}(t) = c_1 e^{\lambda_1 t} \bar{v}_1 + c_2 e^{\lambda_2 t} \bar{v}_2 + \dots + c_n e^{\lambda_n t} \bar{v}_n \quad (2.3)$$

The \bar{v}_i and λ_i parameters on equation 2.3 are the eigenvalues and eigenvectors of the A matrix, and the c_i coefficients depend on the initial conditions of the system.

Computing the Laplace transforms and solving the system for $Y(s)$, $U(s)$ and null initial conditions, the transfer function matrix $[H_{ij}]$ of a system can be defined:

$$Y(s) = [H(s)]U(s) = [C(sI - A)^{-1}B + D]U(s) \quad (2.4)$$

All elements on the transfer function matrix share the same denominator or characteristic function on equation 2.5; therefore, all the elements of the transfer function matrix provide the same poles. And so, as it is discussed in the following Chapter, it is not necessary to analyze all the elements on the transfer function matrix in order to determine the stability of a linear system.

$$Y(s) = \frac{N_{ij}(s)}{D(s)} = \frac{N_{ij}(s)}{\det[sI - A]} \quad (2.5)$$

As hinted on the previous paragraph, each element on transfer function matrix can be defined as a ratio of two polynomials, function of the complex variable $s = \sigma + j\omega$:

$$H(s) = \frac{N(s)}{D(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s^1 + b_m}{a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s^1 + a_n} \quad (2.6)$$

Moreover, for convenience, the mentioned polynomials are often factored, which leads to defining the gain of the transfer function and most importantly, its poles and zeros.

$$H(s) = \frac{N(s)}{D(s)} = K' \frac{(s - z_1)(s - z_2) \dots (s - z_{m-1})(s - z_m)}{(s - p_1)(s - p_2) \dots (s - p_{n-1})(s - p_n)} \quad (2.7)$$

The gain of the transfer function can be determined from previous equation 2.6 as $K = b_m/a_m$. On another note, the z_i elements, that is, the roots of the $N(s)$ polynomial ($N(s) = 0$), are known as zeros of the transfer function. And most importantly for stability analysis, the poles of a transfer function are defined as the roots of the $D(s)$ polynomial ($D(s) = 0$), or p_i . Additionally, since all coefficients of $N(s)$ and $D(s)$ are real, the poles and zeros of a transfer function are either real or come in complex conjugate pairs.

For further analysis, the poles and zeros of a system are often shown graphically on an s plane pole/zero plot, which is used with extent in system control and dynamic analysis.

The effect of the pole locations on the systems response can be summarized in the following Figure 2.4 and list [9]:

- The poles on the LHS or equivalently with negative real part are stable and generate components that decay in time.
- The poles on the RHS or equivalently with positive real part are unstable and generate components that grow in time.
- Imaginary poles, that is, poles with null real part, create purely oscillatory elements that do not grow or decay.
- Poles in the origin generate constant components.
- The oscillatory frequency and decay rate are determined by the distance of the pole to the origin.

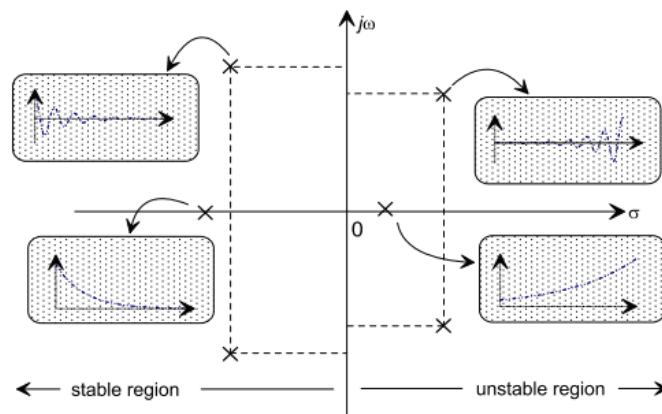


Figure 2.4: Graphic representation of a systems response depending on pole location.
Illustration from [9]

2.2.2 Stability of Non-Linear Systems

A non-linear system can be represented on the state space model as a system of differential non linear equations [8]:

$$\dot{\bar{x}} = f(\bar{x}, t) \quad (2.8)$$

The equation above has many possible solutions, including the equilibrium point solutions, periodic solutions, quasi-periodic solutions and chaotic solutions. Anyhow, this project discusses solely the equilibrium point solutions, given that the project focuses on analyzing the stability of DC solutions.

An equilibrium point is defined as a fixed point that cancels the $f(t)$ function. For example, the DC solutions (\bar{x}_{DC}) of an electronic circuit are equilibrium solutions.

$$f(\bar{x}_{DC}) = 0 \quad (2.9)$$

In order to analyze the variations on a system caused by small perturbations, we define $\bar{\xi}(t)$, a small perturbation around the DC solution and we determine the new state space model:

$$\begin{aligned} \bar{x} &= \bar{x}_{DC} + \bar{\xi}(t) \\ \dot{\bar{x}}_{DC} + \dot{\bar{\xi}}(t) &= f(\bar{x}_{DC} + \bar{\xi}(t)) \end{aligned} \quad (2.10)$$

Since the introduced perturbation is small, a Taylor expansion of $f(\bar{x}_{DC} + \bar{\xi}(t))$ can be defined. So, the previous equation can be rearranged by replacing $f(\bar{x}_{DC} + \bar{\xi}(t))$ with the first two significant terms of the calculated Taylor expansion.

$$\dot{\bar{x}}_{DC} + \dot{\bar{\xi}}(t) = f(\bar{x}_{DC}) + Jf(\bar{x}_{DC})\bar{\xi}(t) \quad (2.11)$$

The $Jf(\bar{x}_{DC})$ term is the jacobian matrix of the system evaluated on the DC solution (\bar{x}_{DC}):

$$Jf(\bar{x}_{DC}) = \left. \frac{\partial f(\bar{x})}{\partial \bar{x}} \right|_{\bar{x}_{DC}} = \begin{bmatrix} \left. \frac{\partial f_1}{\partial x_1} \right|_{\bar{x}_{DC}} & \cdots & \left. \frac{\partial f_1}{\partial x_n} \right|_{\bar{x}_{DC}} \\ \vdots & \ddots & \vdots \\ \left. \frac{\partial f_n}{\partial x_1} \right|_{\bar{x}_{DC}} & \cdots & \left. \frac{\partial f_n}{\partial x_n} \right|_{\bar{x}_{DC}} \end{bmatrix} \quad (2.12)$$

And given that \bar{x}_{DC} is a possible solution of many co-existing system solutions, the perturbed system can be described as the linear system in equation 2.2. In other words, closely around the \bar{x}_{DC} equilibrium point, the non-linear system in equation 2.10 can be approximated through a linearization of the system around the DC fixed point.

As on linear systems, the roots of the characteristic equation can be defined to determine the stability of the system. In this case the mentioned roots can be calculated from the eigenvalues of the Jacobian Matrix.

$$\det[\lambda I - Jf(\bar{x}_{DC})] = 0 \quad (2.13)$$

2.3 Stability of Small Signal RF Amplifiers

Small signal RF amplifiers can be described by the Scattering parameters $[S]$, and throughout history, different stability analysis methods have been created that employ these parameters [10].

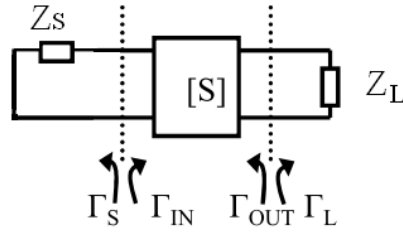


Figure 2.5: Illustration of amplifier represented with Scattering Parameters

The conventional method to analyze the stability of amplifiers proposed by Rollet consists of checking the following four conditions that, if met, prove the unconditional stability of a two-port network for any given Z_L and Z_S loads:

- $K = (1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2) / 2 |S_{12}S_{21}|$
- $|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| < 1$
- $S_{11} < 1$
- $S_{22} < 1$

Checking these four conditions is equivalent to checking whether the negative resistance condition is met.

Additionally if a transistor is unilateral ($S_{12} = 0$), one can determine the unconditional stability of an amplifier by checking only the $|S_{11}| < 1$ and $|S_{22}| < 1$ conditions.

If the conditions are not met, one can calculate the input and output stability circles that determine the border between the loads that might potentially create instabilities. This is carried out by calculating the circumferences that meet the $\Gamma_{in} = 1$ and $\Gamma_{out} = 1$ conditions, taking into account the definitions in equations 2.14 and 2.15, and plotting them on a Smith Chart.

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \quad (2.14)$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \quad (2.15)$$

Despite how effective this method is to determine the values of the loads that might destabilize the system, its main limitation is related to the fulfillment of the condition known as Rollet Proviso. The Rollet stability analysis is limited because it is only valid for systems with no inner unstable loops. In other words, one must guarantee a priori that none of the poles of the uncharged system are on the RHS of the s plane, which, for example, is very common in multistage amplifiers.

Chapter 3

Stability Analysis Methods Based on Transfer Functions

The strategy proposed by this project for stability analysis of a DC solution of microwave circuits consists of obtaining the closed loop transfer function associated to the linearization of the circuit around its DC solution for the established operating conditions as in [8].

The stability analysis based on transfer functions and frequency responses of a RF amplifier is commonly carried out following three main steps:

- *1st* Step: The first step consists of obtaining the frequency response of an amplifier on one or multiple nodes via simulation. A frequency response is defined as the output of a system in response to a frequency varying sinusoidal stimulus and it is mainly used to characterize the dynamics of a system. Simulations can be carried out on various Computer Aided Design (CAD) programs, Advanced Design Systems (ADS) for example, but valid simulation models that represent the amplifier with precision are required, as mentioned further in the chapter.
- *2nd* Step: This second step consists of identifying a transfer function that represents the frequency response obtained on the previous step. This can be accomplished following many different existing methods, of which two are discussed on this chapter, the Least Square Method and the Vector Fitting method.
- *3rd* Step: This last step is essential for the stability analysis since it is where the poles and zeros of the transfer function obtained on the previous step are processed and plotted. This step permits the detection of unstable poles and therefore spurious oscillations and often, incorporates additional information regarding its origin.

3.1 Formalism of DC Stable Solutions

As mentioned on the second chapter and following [1], it can be shown that the dynamics of a non-linear circuit around a DC solution can be approximated with the linearization of the circuit around the DC solution, obtaining the following lineal system that represents the perturbed system:

$$\dot{\bar{\xi}}(t) = A\bar{\xi}(t) \quad (3.1)$$

A represents the Jacobian matrix of the circuit evaluated on (\bar{x}_{DC}) continuous solution. Hence, the stability analysis of a DC solution can be carried about by calculating the eigenvalues (λ_i) of the Jacobian matrix A , which are equal to the roots of the characteristic equation or the poles of the system.

$$\dot{\bar{\xi}}(t) = Jf(\bar{x}_{DC}\bar{\xi}(t)) \quad (3.2)$$

To represent the system with a transfer function and prove that the poles are the eigenvalues of the Jacobian matrix as in previous subsection 2.2.2, arbitrary small signals (\bar{u}) are introduced to the system:

$$\dot{\bar{\xi}}(t) = Jf(\bar{x}_{DC}\bar{\xi}(t) + B\bar{u}(t)) \quad (3.3)$$

And the following output vector of variables is defined to complete the state space representation:

$$\bar{y}(t) = C\bar{\xi}(t) \quad (3.4)$$

Once the inputs and outputs of the system have been defined, the $[H_{ij}]$ transfer function matrix can be defined as analyzed on chapter 2. Moreover and following the proof on chapter 2, all H_{ij} elements of the matrix share the same denominator or characteristic equation $\det[\lambda I - A]$ and so, all information regarding the stability of the non-linear circuit around the DC solution is contained on the poles of any H_{ij} transfer function. Nonetheless, differences on the denominator for each H_{ij} might occur due to exact pole/zero cancellations caused by the losses of controllability or observability from the observation point. Still and as proven on chapter 2, in principle, the poles of a system can be obtained from any of the nodes of the system.

For example, by considering the simplest configuration of a single entry $\bar{u} = u$ and a single output $\bar{y} = y$ (SISO) and determining the transfer function (equation 3.5) one can determine all information concerning the stability of the DC solution.

$$H(s) = \frac{Y(s)}{U(s)} \quad (3.5)$$

One can try to ensure that the denominator of the transfer function obtained on a node has not suffered pole/zero eliminations. This can be done by considering more than one input and output (MIMO), and comparing the denominators or forcing all denominators to be equal when calculating the transfer functions for all inputs and outputs.

3.2 Obtaining SISO and MIMO Frequency Response

As mentioned on the introduction of this chapter, by following three simple steps one can obtain the SISO or MIMO frequency responses of an amplifier. This way, one can determine the characteristic equation of a system and analyze its stability.

The SISO analysis of a system consists of introducing a single input probe to the system and analyzing the obtained single output. This analysis is often linked to problems of loss of controllability and observability that depend on the node or branch at which the probe is introduced. MIMO analysis reduces this problem, since the designer no longer has to choose a single node or branch and can introduce multiple probes to the system, sequentially. Taking into consideration the characteristics of the transfer function matrix discussed in section 3.2, the best way for fitting MIMO data is to force the same characteristic equation on all the nodes. However, a less precise but valid analysis can be acquired by analyzing the multiple SISO input separately.

Throughout this section, the three basic steps are described in depth for a CAD simulation analysis. As mentioned at the beginning of the chapter, a valid simulation model is required to carry out the simulation and proceed with an accurate stability analysis. Unfortunately, amplifier designers and manufactures often struggle in finding valid simulation models, challenging the stability analysis process further.

3.2.1 Current and Voltage Probes

Introducing current or voltage probes on one or multiple nodes of a circuit gives access to the SISO or MIMO frequency responses required for the stability analysis. However, when introducing these additional sources one must ensure that they will not modify the circuit steady state under analysis. Therefore, the small-signal current sources should be inserted in parallel (Impedance Analysis) and small-signal voltage sources in series (Admittance Analysis) [11].

Additionally, when carrying out a MIMO analysis one must sequence the multiple inputs, avoiding all interferences between the multiple small-signal inputs. That is, MIMO simulations should be carried about as multiple sequenced SISO simulations.

As proven on the previous section when discussing the formalisms of stable DC solutions, the current sources and voltage sources can be introduced on any of the nodes or branches of the circuit. This is true in theory, since the characteristic equations of all the nodes of the system are equal. However, a resonance that is part of the dynamics is not always detected equally on all nodes due to the previously mentioned losses of observability and controllability [11]. The resonance is always detected on the nodes that provide the feedback loops but not always on the nodes located electrically isolated from the feedback loops. Therefore, for more precision, obtaining and analyzing MIMO frequency responses by introducing the probes in the proximity of the inputs and outputs of the transistors is recommended.

Current Probes

The frequency response or transfer function of the linearized circuit can be obtained as the impedance seen by a small-signal current source in parallel at a given node (see Figure 3.1)

[11].

$$H(j\omega) = Z^n(j\omega) = \frac{v^n(j\omega)}{i_{in}(j\omega)} \quad (3.6)$$

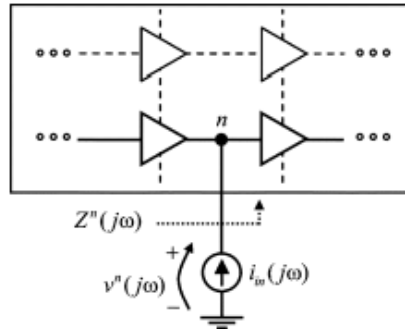


Figure 3.1: Illustration of current probe.
Illustration from [11]

Voltage Probes

Alternatively, an accurate frequency response or transfer function can also be obtained as the admittance seen by a small-signal voltage source in series at a particular branch (see Figure 3.2) [11].

$$H(j\omega) = Y^b(j\omega) = \frac{i^b(j\omega)}{v_{in}(j\omega)} \quad (3.7)$$

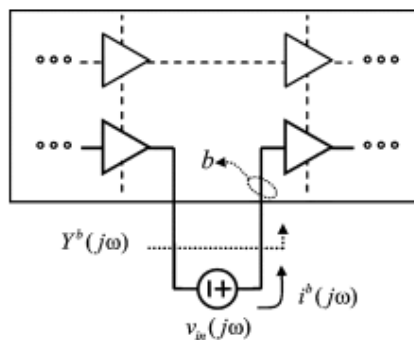


Figure 3.2: Illustration of voltage probe
Illustration from [11]

3.3 Transfer Function Identification

There are many algorithms that solve the difficult problem of formulating a continuous transfer function, ratio of polynomials in s as in equation 2.6, for a measured or calculated frequency response. These algorithms minimize the error between the measured or simulated frequency responses and the transfer function models proposed. Some of those algorithms are the maximum probability method, the recursive methods, the least-square algorithms and the genetic algorithms [11]. When choosing between the proposed methods, one must analyze the convergence of each method, the validity of the obtained transfer functions for noisy or distorted frequency responses and find the most adequate method for each application.

The least-square method is introduced in this chapter, as it is one of the most employed algorithms for identifying linear systems in absence of noise and since the stability analysis method proposed by this project is based on simulations and therefore, noise free. Subsequently, the Vector Fitting technique that employs the least-square method to identify transfer functions is discussed.

Additionally, the quality assessment of the identification for all methods is a highly critical step for our application since the order of the transfer function $H(s)$ is unknown a priori and there is no existing method to determine it [12]. The incorrect selection of the order can create unwanted undermodeling or overmodeling that should be prevented as discussed on following section 4.4.

3.3.1 Least-Square Method

The least square method is a standard data fitting procedure to determine the best fit of experimental or calculated data points. The method relies on finding a linear combination of linear or non-linear functions for a given data set $\{(x_1, y_1), (x_2, y_2), \dots, (x_m, y_m)\}$ that depends on a x variable as well as on a $\beta = (\beta_1, \beta_2, \dots, \beta_n)$ vector with dimension ($n \leq m$).

$$y = f(x, \beta) \quad (3.8)$$

As mentioned, the procedure consists of minimizing the sum of squares of the error (S), so one must define the error (r_i) and determine the β vector that meets the set error condition in equation 3.11.

$$r_i = y_i - f(x_i, \beta) \quad (i = 1, 2, \dots, m) \quad (3.9)$$

$$S = \sum_{i=1}^m r_i^2 \quad (3.10)$$

That is, one must find the (β) vector that equals the gradient of the sum of squares of the

error to zero, as follows:

$$\frac{\partial S}{\partial \beta_j} = 2 \sum_{i=1}^m r_i \frac{\partial r_i}{\partial \beta_j} = 0 \quad (j = 1, 2, \dots, m) \quad (3.11)$$

3.4 Vector Fitting Method

Vector Fitting [2][3][4] is a general methodology for the fitting of frequency domain responses with rational function approximations created by B. Gustaven. The fitting is achieved by re-locating a set of starting poles with an improved set via a scaling procedure.

This method is an interesting incorporation to the project since it fits multiple data inputs with the same set of poles, that is, the same characteristic equation. This characteristic is of great interest for precise MIMO analysis with no loss of observability and controllability.

Vector Fitting was developed by B. Gustaven and A. Semlyen in 1996 and was made available two years later as a public Matlab routine that can be found in [13]. On this cited web page one can find information about the method and a working highly useful Matlab implementation with a user manual and instructions.

This method estimates the real or complex residues (c_n) and poles (a_n), as well as the real d and h parameters of the rational function approximation of $f(s)$ in equation 3.12 in order to fit the input data. The problem is solved as a linear problem and in two steps, as described in [2], despite it being a nonlinear problem in terms of the unknowns.

$$f(s) \approx \sum_{n=1}^N \frac{c_n}{s - a_n} + d + sh \quad (3.12)$$

1st step: Pole Identification

On this first step, a set of real or complex poles (\bar{a}_n) is specified over the frequency range and an additional unknown frequency dependent function $\sigma(s)$ is created.

$$\begin{bmatrix} \sigma(s)f(s) \\ \sigma(s) \end{bmatrix} \approx \begin{bmatrix} \sum_{n=1}^N \frac{c_n}{s - \bar{a}_n} + d + sh \\ \sum_{n=1}^N \frac{\tilde{c}_n}{s - \bar{a}_n} + 1 \end{bmatrix} \quad (3.13)$$

As we can see on equation 3.13 the rational approximation of $\sigma(s)$ and $\sigma(s)f(s)$ share the same poles, and by rearranging them, an overdetermined linear problem ($Ax = b$) is obtained.

The linear equations on equations 3.14 and 3.15 can be solved as a least-square problem, determining the c_n, d, h, \tilde{c}_n unknowns:

$$(\sigma f)_{fit}(s) \approx (\sigma_{fit}(s)) f(s)$$

$$\left(\sum_{n=1}^N \frac{c_n}{s - \bar{a}_n} + d + sh \right) \approx \left(\sum_{n=1}^N \frac{\tilde{c}_n}{s - \bar{a}_n} + 1 \right) f(s) \quad (3.14)$$

A rational approximation for $f(s)$ can be obtained by rewriting the equation above as fractions:

$$(\sigma f)_{fit}(s) = h \frac{\prod_{n=1}^{N+1} (s - z_n)}{\prod_{n=1}^N (s - \bar{a}_n)} \quad \sigma_{fit}(s) = \frac{\prod_{n=1}^N (s - \tilde{z}_n)}{\prod_{n=1}^N (s - \bar{a}_n)} \quad (3.15)$$

From equations 3.14 and 3.15, one realizes that the poles of $f(s)$ are equal to the zeros of $\sigma_{fit}(s)$, and that the initial poles are eliminated from the problem. This realization gives way to a simple method to determine the poles for fitting the $f(s)$ function that consists of determining the zeros of $\sigma_{fit}(s)$.

$$f(s) = \frac{(\sigma f)_{fit}(s)}{\sigma_{fit}(s)} = h \frac{\prod_{n=1}^{N+1} (s - z_n)}{\prod_{n=1}^N (s - \tilde{z}_n)} \quad (3.16)$$

Since the $\sigma_{fit}(s)$ residues have been calculated and its poles are the chosen starting poles, the sum of partial functions of $\sigma_{fit}(s)$ 3.13 is known. So, its zeros, that is, the poles of $f(s)$ can be calculated easily by rewriting the function as fractions, as in equation 3.15. This potentially complex calculation is simplified as described in appendix A and B of [2]. The proposed method consists of determining the eigenvalues of the H matrix shown bellow.

$$H = A - b\tilde{c}^T \quad (3.17)$$

Where A contains the starting poles, b is a column vector with ones and \tilde{c}^T is a row vector containing the mentioned and calculated residues.

2nd step: Residue Identification

The residues can be calculated directly or, more accurately by solving equation 3.12 by setting the calculated $\sigma_{fit}(s)$ zeros as a_n and solving the new overdetermined linear problem ($Ax = b$) with x containing the $c_n, d,$ and h unknowns.

3.5 Pole/Zero Diagram Analysis

For a correct detection of possible spurious oscillations one must analyze the obtained transfer function. This can be done by calculating, analyzing and preferably plotting the poles and zeros of the transfer function, as discussed in subsection 2.2.2.

If poles and zeros with positive real parts are detected, the system will generate an unwanted spurious oscillations. So in order to eliminate them, one can analyze the system and its instability in depth from the additional information provided by the pole/zero diagrams of different nodes. Detecting critical nodes, one can detect possible feedback loops that are at the origin of the oscillation and modify the amplifier to eliminate them.

However, as mentioned in section 2.3, there is no method to determine the order of the transfer function before fitting the curve. This might create unwanted undermodeling or overmodeling problems that might lead to erroneous pole/zero analysis, as illustrated in the following example described on [12] and illustrated in Figure 3.3.

As we can see in Figure 3.3, the real transfer function (order 6) contains a physical unstable quasi-cancellation that is not detected if the order is inferior. Thus, a low order can fail in detecting problematic unstable poles. However, choosing a higher order might create mathematical quasi-cancellations that can be interpreted erroneously as physical quasi-cancellations leading to misguided conclusions as on the example. The overmodeling product on the example is on the right hand side, so it is problematic; however, had it been on the left hand side it would not modify the stability results. The following chapter proposes two methods to help discard products of overmodeling.

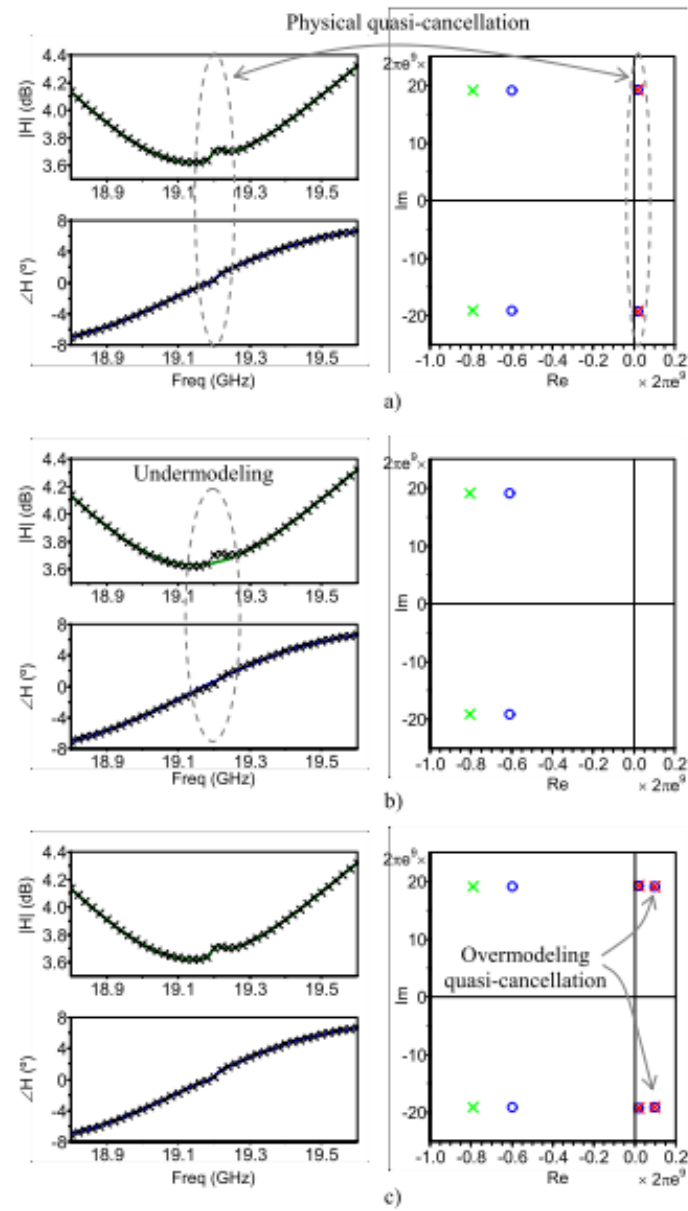


Figure 3.3: Example of identification results with order: a) 6 (correct fitting) b) 4 (undermodeling) c) 8 (overmodeling)

Left graphs represent the magnitude and phase of frequency responses, while right graphs are the corresponding pole/zero diagrams (x: poles, o: zeros).

Illustration of [12]

Chapter 4

Tools for Stability Analysis Based on Transfer Functions

Many powerful Computer Aided Design (CAD) tools based on harmonic balance, such as Advanced Design System (ADS) and Microwave Office, allow accurate simulations of microwave and RF circuits. But the detection of instabilities that potentially give rise to spurious oscillations is still a challenge.

Despite the mentioned challenging aspect, there are many stability analysis methods to help microwave and RF designers prevent instabilities, such as, the feedback factor and NDF by Platzker, the method of the envelope of the stability by Narhu and Valtonen and many more that can be found more extensively on [1].

There are also rigorous, fast and user-friendly stability analysis tools that are compatible with existing CAD software programs. Due to their high efficiency in detecting instabilities, the commercial STAN program and the new program created as a product of this project are discussed on this chapter. However, these are not the sole existing programs that attempt to help designers with the stability matter. Searching on the Internet one can find from basic Rollet factor calculators [14][15] to more complex stability analyzers as the RF-STABILITY tool [16]. This latter tool with the RFEM add-on, both designed by Dlubal, permits the designer to analyze the stability of structures by determining critical load factors and the corresponding stability modes. This complex method has similarities with the two tools discussed on this chapter since it too analyzes the stability by calculating eigenvalues.

4.1 STAN Tool

The STAN tool, created by the RF and Microwave Research Group of the Department of Electricity and Electronics of the University of the Basque Country in collaboration with CNES and AMCAD, offers a stability analysis technique, valid for small-signal and large-signal regimes. This technique is able to detect and determine the nature of oscillations, such as parametric oscillations in power amplifiers, with SISO or parametric SISO simulations. A parametric analysis consists of varying the value of an element on the simulation model and obtaining a frequency response for each parameter value on the sweep. The STAN tool reads these frequency response inputs, and plots the root locus of the system, that can give additional information of possible instabilities. The knowledge of the type of oscillation mode, and the pole/zero diagram of a given frequency response facilitates the insertion of stabilization networks, with a better balance between the required oscillation avoidance and maintaining the original circuit performances. Further reading on this technique can be found on the AMCAD Engineering webpage

[17], on the PhD Thesis by Aitziber Anakabe from the University of the Basque Country [1] *”Detección y eliminación de inestabilidades paramétricas en amplificadores de potencia para radiocomunicaciones”* and on these works [12][8].



STAN is compatible with commercial CAD tools for microwave circuit design, and its user-friendly and efficient interface gives not necessarily expert users a great ability to analyze the stability of their linear or non-linear design in an intuitive manner.

The stability analysis of a system can be carried out with STAN by following the three steps described on chapter 3 as follows:

- As a first step, the designer must obtain the SISO frequency response of the system with a CAD program; ADS will be set as a CAD example program for the project. A valid simulation model is needed to carry out this step, and current or voltage probes are to be introduced in the circuit as discussed in section 3.2. The frequency response obtained must be exported to a .txt, file to be read by STAN on the next step. A more complete frequency response will be obtained if the following points are taken into account when simulating:
 - From a theoretical point of view, all of the nodes or branches at which the current or voltage probes are introduced give the same stability information. Nevertheless, introducing the probes near the non-linearity is advised to ensure correct detection of possible instabilities and to prevent the physical pole/zero cancellations mentioned in previous section 3.4.
 - Choosing the frequency range at which the active devices present gain is advised for a correct stability analysis.
- The second step, which consists of identifying the transfer function, is automated by the STAN tool. But the designer can establish different simulation parameters in figure 4.1, depending on the characteristics of the analysis to be carried out. This automation is described in more detail in the following paragraphs.
- The results captured by STAN are displayed on graphs and tables, as in figure 4.2, for the designer to analyze.

The transfer function identification methodology of the STAN tool is based on the following principal ideas introduced in [1]:

- The fitting method employed by STAN is the least-square method, due to its high efficiency for fitting simulation data. It consists of determining the coefficients of the numerator and denominator of the transfer function by minimizing the square of the error between the fitted transfer function and the ideal curve.

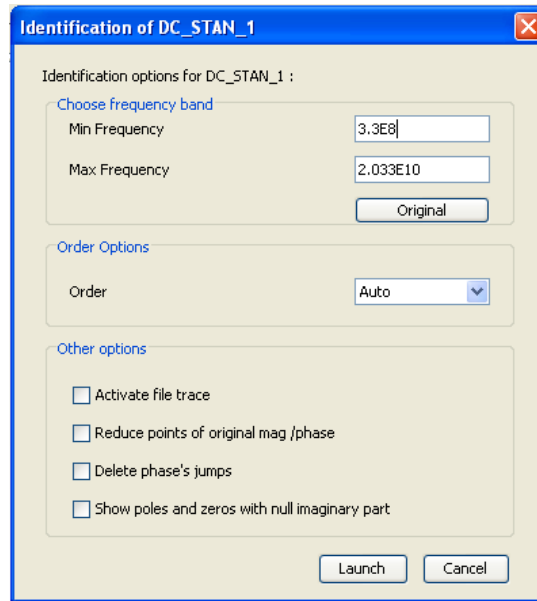


Figure 4.1: STAN Input Interface

- STAN introduces an algorithm to determine the order of the transfer function to be fitted automatically. This is done by dividing the frequency band in small fragments and fitting the small fragments by iteratively increasing the order for each fragment, until a fixed phase error is reached.
- In order to help designers understand the pole/zero diagrams displayed, STAN discards the poles and zeros, result of the transfer function identification, that are not on the frequency range of the frequency response. The reason for it being that STAN fits the data on the specified frequency range, but the obtained transfer function is not limited to that frequency range.

One can carry out a MIMO analysis with the STAN tool, by analyzing multiple SISO frequency responses independently. Unfortunately, STAN does not incorporate a method to force the same denominator for the multiple inputs, as the program discussed in the following section does. However, as it is shown on an example in Chapter 5, even though the obtained poles with MIMO in STAN are not the same for all the nodes, they maintain similar values on all nodes.

4.2 Program Based on Vector Fitting

The stability analysis tool based on Vector Fitting for Matlab described on this section is the main result and product of this project. The principal objective for creating this program was to evaluate the benefits of MIMO identification for stability analysis of microwave amplifiers. Nevertheless, the program is not limited to MIMO analysis since it also reads SISO and parametric SISO frequency response inputs, like STAN.

With the objective of describing this newly created program, this section is divided into three subsections that discuss the basic user interface, some Vector Fitting settings and other

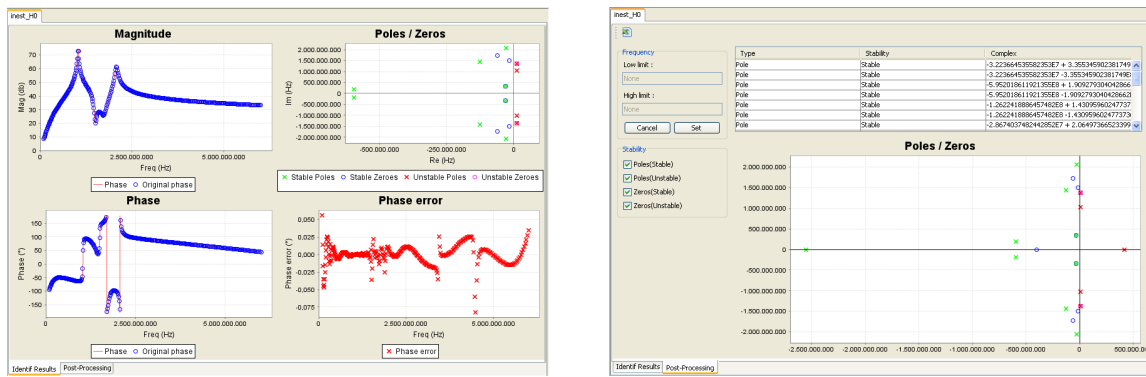


Figure 4.2: STAN Output Interface includes:

- Magnitude and phase plots to check fitting, pole/zero diagram to check instabilities and phase error graph to determine fitting
- Expanded Pole/Zero Diagram and table with output data

implemented algorithms, such as an algorithm for detection of critical nodes and two algorithms for pole/zero elimination.

4.2.1 Basic User Interface

Similarly to STAN, this program follows the three steps discussed in length in chapter 3:

- The designer must obtain the SISO or MIMO frequency response of the system with ADS. This stability analysis program is more restrictive than STAN and permits only .txt files with concatenated magnitude and phase data on the last column or magnitude/phase data separated by '/' on last column. Additionally, the program proposes two approaches for MIMO analysis:
 - The simplest approach consists of introducing current probes sequentially on different nodes and determining the impedances on each node.
 - Another procedure for a systematic stability analysis method consist of introducing a current probe on the input and output of each transistor and calculating the impedance as in the ADS schematic example in Figure 4.3.
- The second step is also automated following the Vector Fitting specifications in subsection 4.2.2, and the simulation parameters on Figure 4.4 can be specified for the stability analysis.
- The identified poles and zeros are processed taking into account the algorithms discussed on subsections 4.2.3 and 4.2.4. The results chosen by the designer are displayed on graphics and tables for the designer to analyze.

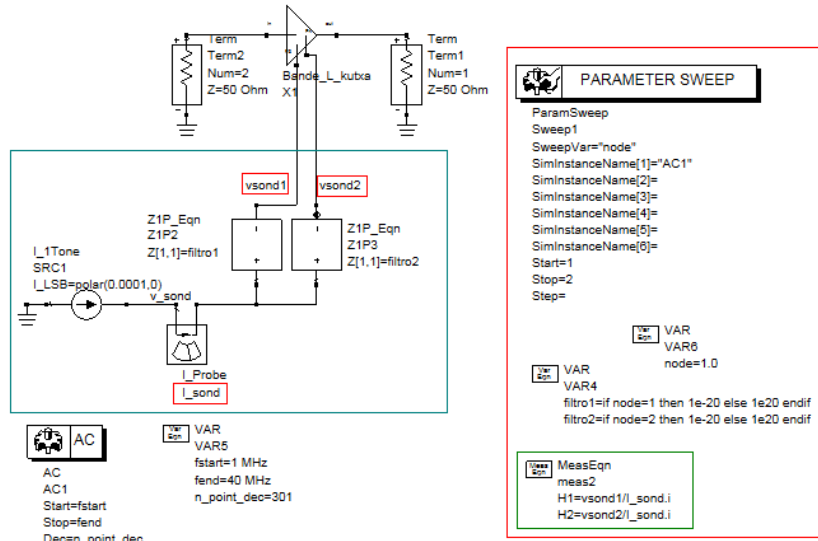


Figure 4.3: Illustration of systematic stability MIMO analysis in ADS.

The large red square enables the MIMO analysis on the area marked by the blue square. The green square specifies the impedance to be calculated on a systematic analysis of a single transistor amplifier.

4.2.2 Vector Fitting Settings

The Vector Fitting User Manual [18] and examples were used at length to establish the correct settings for this application and determine the Matlab function commands and parameters. The employed Matlab function call is:

$$[SER, poles, rmserr, fit] = vectfit3(f, s, poles, weight, opts)$$

Where the inputs are:

- f : (n, Ns) f holds the complex $(a + bj)$ $f(s)$ samples. For MIMO analysis, each input is to be stored on a new column on f and the fitting of all curves is carried out with the same poles, forcing the characteristic equation of each node to be equal.
- s : $(1, n)$ This structure holds the frequency samples (rad/s) .
- $poles$: $(1, N)$ Poles holds the initial starting poles. The author advises to choose complex conjugate poles with weak attenuation and with imaginary parts β that cover the frequency range of interest, to reduce the need for long distance re-allocations of poles. The created Matlab program allows the designer to choose between linear or logarithmic spacing of β (see figure 4.4) for faster convergence. Nevertheless, the poles should be chosen as:

$$\begin{aligned} a_n &= -\alpha + j\beta & a_{n+1} &= -\alpha - j\beta \\ \alpha &= \beta/100 \end{aligned} \tag{4.1}$$

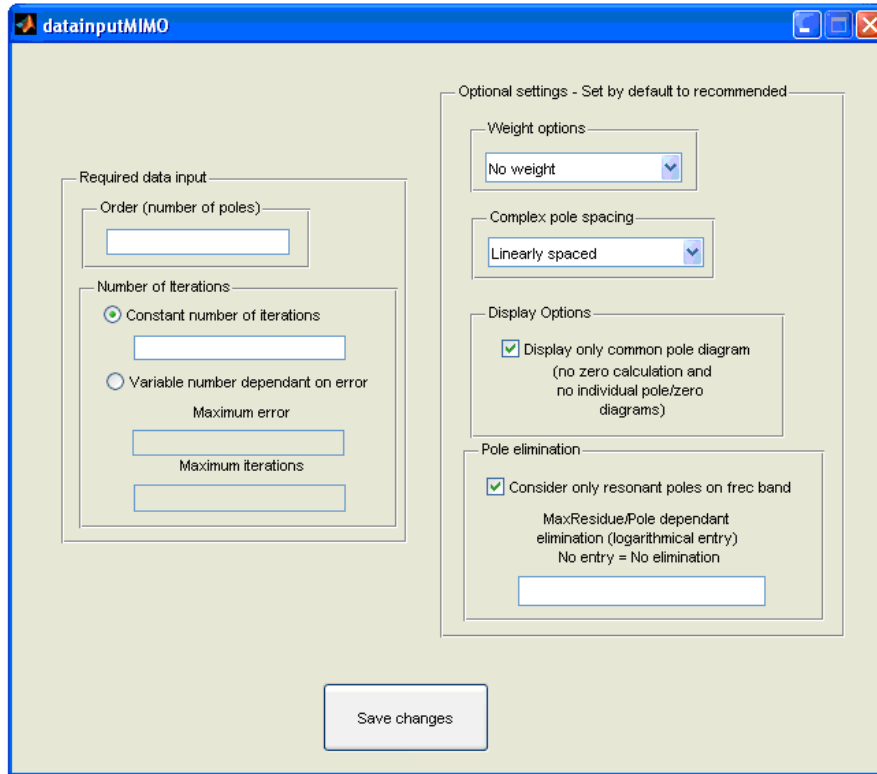


Figure 4.4: Input Interface of Program based on Vector Fitting

- Weight: (n, N_s) Weighting is a useful tool to control the accuracy of the least square problem. Three weight options are available:
 - No weight (recommended choice): $weight = 1$
 - Strong Inverse Weight (for noisy data): $weight = 1/|f|$
 - Weaker Inverse Weight (for noisy data): $weight = 1/\sqrt{|f|}$
- Opts: For this application, the main Vector Fitting initial settings are set as follows:
 - `opts.stable=0` Allow unstable poles
 - `opts.asymp=2` Do not include D and E in fitting (only include it on last iteration)
 - `opts.relax=1` Use vector fitting with relaxed non-triviality constraint
 - `opts.skippole=0` Do not skip pole identification
 - `opts.skipres=1` Do not calculate C,d,e (only on last iteration)
 - `opts.complexss=1` Create complex state space model

And the outputs of interest for this application are:

- SER structure: This structure contains the poles, and residues of the fitted $f(s)$ data
- Rmserr: The resulting RMS-error of the fitting

4.2.3 Pole/Zero Elimination

In order to discard the mathematical products of overmodeling discussed on section 3, this project proposes the following two pole-eliminating techniques:

4.2.3.1 Non Resonant Poles

Resonance is the tendency of a system to oscillate with greater amplitude at certain frequencies. At these frequencies, even small periodic perturbations, such as noise, can produce large amplitude oscillations.

The resonance in frequency domain is greater for second order systems with low damping as illustrated on the Bode plot in Figure 4.5. More concretely, for a system with complex conjugated pairs of poles, no resonance peaks are detected for damping ratios ζ greater than 0.707.

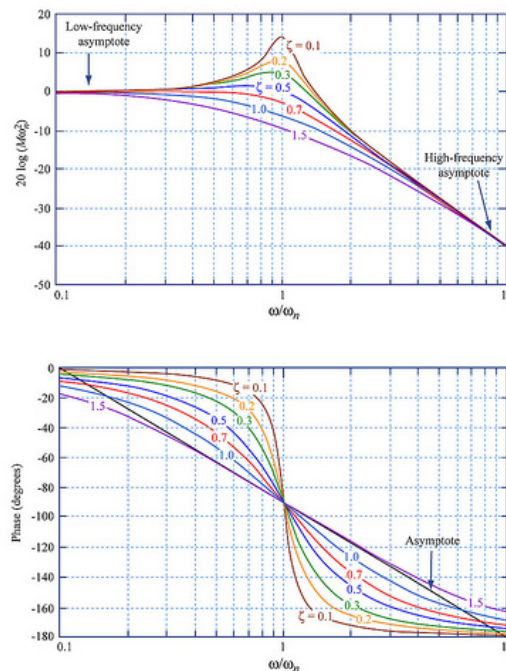


Figure 4.5: Bode plot of second order transfer functions with different dampings that illustrates the damping effect on the resonance

Non resonant poles located far from the frequency band of the frequency response data are often products of overmodeling and are not the cause of unwanted oscillations. Therefore, by calculating the damping ratio ζ of all the poles ($a + bj$) from equation 4.2 (or with the damp function in Matlab) we can discard those poles with damping ratios greater than 0.707, for they are not resonant poles and, therefore, will not create spurious oscillations.

$$\zeta = \frac{a}{\sqrt{a^2 + b^2}} \quad (4.2)$$

Additionally and following the same reasoning as in the STAN tool, the poles whose resonance is not in the frequency band are discarded. That is, only the poles with a resonance frequency (ω_r) that meets the condition in equation 4.3 are considered.

$$\omega_{min} \leq \omega_r \leq \omega_{max} \quad (4.3)$$

Where, the natural frequency (ω_n) and the resonance frequency (ω_r) are defined as:

$$\begin{aligned} \omega_n &= \sqrt{a^2 + b^2} \\ \omega_r &= \omega_n \sqrt{1 - 2\zeta^2} \end{aligned} \quad (4.4)$$

4.2.3.2 Elimination Based on Residues

Another pole elimination method is introduced in this project, to help designers understand and analyze the obtained results. This method is based on the residues and requires the input of a tolerance parameter as we see on Figure 4.4. It consists of not displaying poles with low residues, by assuming that their effect on the resulting dynamic is of little relevance.

For SISO, the elimination process is simpler, because each pole has one residue value. Since the MIMO analysis is composed of N individual transfer functions, there are N residues for each pole. So to have one residue value for each pole as in SISO and to apply the algorithm, the maximum residue for each pole is chosen, and the node at which this maximum is found is recorded for critical node detection (discussed in the following subsection).

Initially only the residues were considered as a standardization factor for the pole elimination, but a posterior analysis proved that considering a *normalized residue*, as the $|residue/pole|$ division, was more efficient and reduced the amount of unstable poles eliminated for the same user input.

The main idea behind this method is to determine the maximum *normalized residue* relation in logarithm and compare all other *normalized residue* logarithmic values with it. Depending on the tolerance parameter, which represents the margin (expressed in orders) below the maximum *normalized residue* value to be considered, more or less poles are eliminated as illustrated on Figure 4.6.

4.2.4 Detection of Critical Nodes (MIMO)

As hinted on the previous subsection, the detection of critical nodes is directly linked with the node at which the maximum residue for each pole is found. In other words, the maximum residue of a pole will be detected at the node at which the effect of the pole on the dynamics of the system is higher.

The printed stable and unstable poles output of the program incorporates the maximum residue and critical node information as follows:

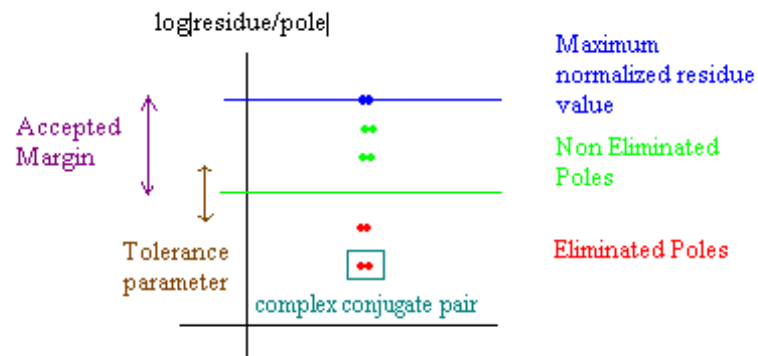


Figure 4.6: Graphic illustration of pole elimination based on residues

- Column1: Resonant Stable poles, Column2: Maximum Residue, Column3: Node corresponding to max residue
- Column1: Resonant Unstable poles, Column2: Maximum Residue, Column3: Node corresponding to max residue

For an unstable pole, detecting the node at which its effect on the dynamics is higher can give circuit designers additional information on the feedback loop and help stabilize an unstable system.

Chapter 5

Examples

The main objectives of this chapter are to prove the functionality of the Vector Fitting based program and to determine the differences between the two programs stated on Chapter 4. Furthermore, the additional pole/zero elimination methods and critical node determining method are evaluated throughout the examples. But, most importantly, the benefits of MIMO identification for stability analysis are evaluated.

To do so, the most significant RF amplifier examples analyzed with STAN and the stability analysis program based on Vector Fitting are represented in this chapter. The first section analyzes the stability of an L-band amplifier with three examples that illustrate a SISO analysis with a current probe, a Parametric SISO analysis with a voltage probe and a MIMO analysis with sequentially introduced current probes. The second section analyzes the stability of a K-band Multistage Amplifier with a more complex MIMO example.

5.1 Analysis of an L-band Amplifier

The L-band amplifier analyzed on this section was designed as a prototype for radio navigation applications, more concretely for Galileo. This L-band amplifier is based on a FET transistor of AsGa on hybrid technology [19].

It was noted and measured that the amplifier oscillated at a given low frequency (13.88MHz) for certain values of the gate resistor on the feedback loop. This prototype helps illustrate the methodology discussed throughout this paper and analyze the low frequency oscillation by plotting the pole/zero plots of the following three examples.

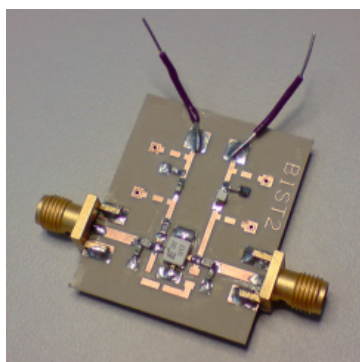


Figure 5.1: Photograph of the L-band FET transistor and the mentioned low frequency oscillation. Illustration from [19]

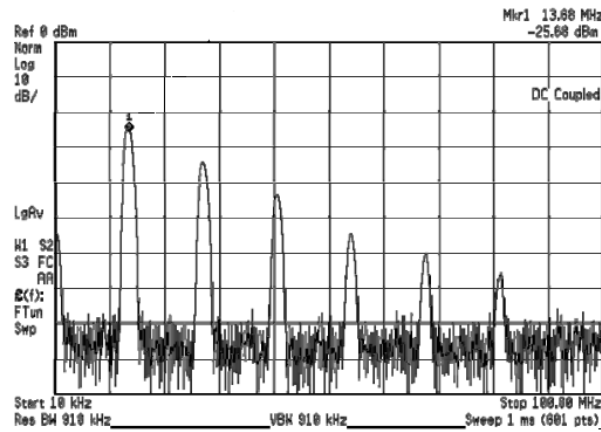


Figure 5.2: Spectrum analyzer output of the L-band FET transistor that illustrates the mentioned low frequency oscillation at 13.88MHz. Illustration from [19]

5.1.1 SISO with Current Probe

The topology of the amplifier is illustrated in Figure 5.3, including the electrical model of the transistor. A current probe is introduced on the input the transistor and the circuit is simulated in ADS with a 1MHz - 40MHz frequency range. The frequency response in Figure 5.4 is obtained and extracted to a .txt type file following the indications on the figure. To analyze the stability of the resulting frequency response, it is identified with STAN and with the VF based program.

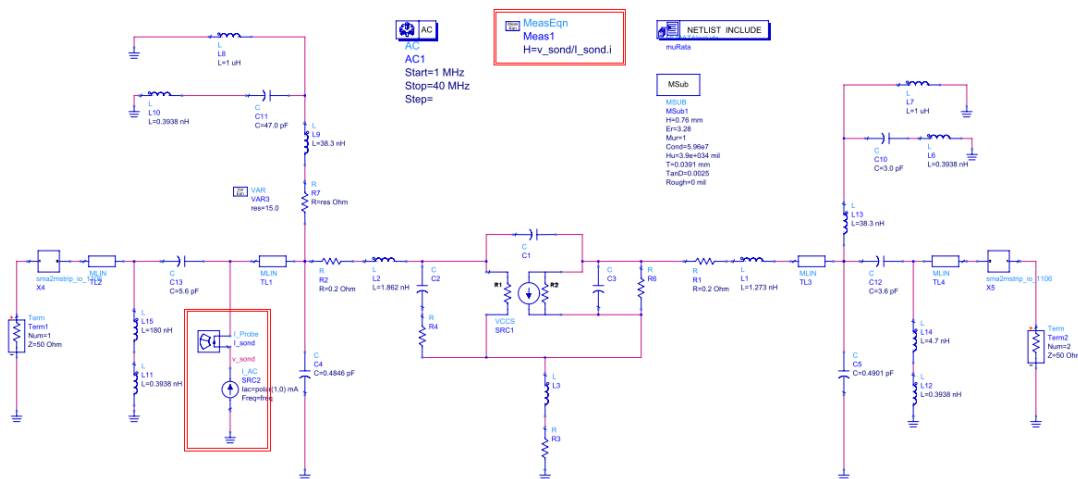


Figure 5.3: The L-band FET amplifier simulation model in ADS with a current probe on the input (red) of the transistor

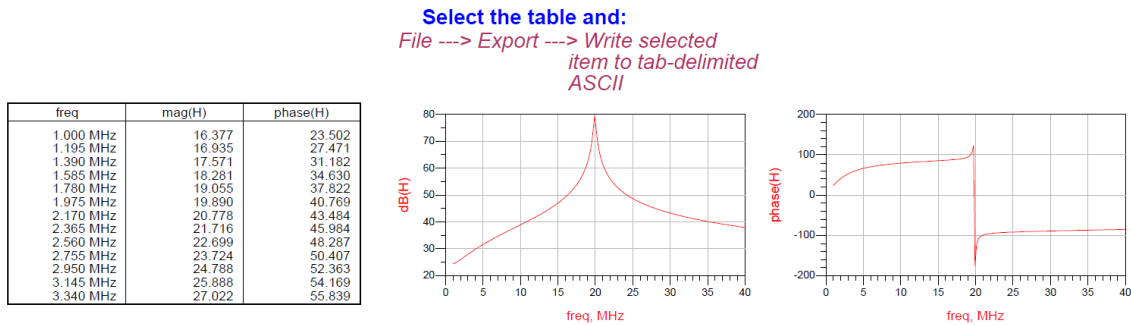


Figure 5.4: The L-band FET amplifier simulation model in ADS with a current probe on the input of the transistor

Program Based on Vector Fitting

The analysis with Vector Fitting of this example is carried out with an order 8 and iteration number set to 7. These parameters were chosen as follows:

- From orders 2 to 14 only the two unstable poles are displayed, but the fitting for order two is not precise. The rmserr error can be taken into account to ensure correct fittings and for the simulated frequency response it varies for different orders as follows:

Order	2	4	6	8	10	12	14	16
rmserr Error	3.1753	0.011561	5.5385e-005	1.7931e-006	1.5327e-007	1.7247e-008	4.8742e-009	3.3653e-009

With an order 16 results of overmodeling are displayed, 2 additional stable poles show up, that appear quasi-cancelled on the pole/zero plot. However, their residues are small, so we can eliminate these poles, products of overmodeling with tolerance parameter smaller than 26. A more graphic example of this pole/zero elimination can be found in the following MIMO examples.

- The number of iterations improves the error considerably for low order values but remains practically constant for higher values. More iterations are required for complex frequency responses, in this case, the fitting is correct with a single iteration, however, the error varies with the number of iterations as in the following table:

Iterations	1	2	3	4	5	6	7	8
rmserr Error	9.8443e-005	1.8612e-006	1.8065e-006	1.7964e-006	1.794e-006	1.7933e-006	1.7931e-006	1.7931e-006

For cases where a certain simulation error to be reached is known a priori, the program includes an algorithm that allows the designer to prevent unnecessary iterations by determining a maximum error and a maximum iteration number.

The order and iteration number on the subsequent examples will not be illustrated in such length, but a similar previous analysis has been carried out on all examples.

The magnitude and phase plots and the pole-zero diagram obtained on Matlab with the order set to 8 and iterations to 7 can be found in Figure 5.5. The top-left graphic contains

the real and fitted magnitude curves as well as a curve that indicates the deviation error of the fitting. We can visually see that the fitting is correct and can also verify from the graphic that the deviation error is very small, around 10^{-6} for all frequencies. The phase graphic on the top-right is also correctly fitted, since the real and fitted curves are completely overlapped. The bottom figure is the pole/zero diagram of the frequency response for an order 8, where two unstable complex conjugate poles are plotted. Therefore, by inserting a current probe on the input of the transistor, the instability is easily detected.

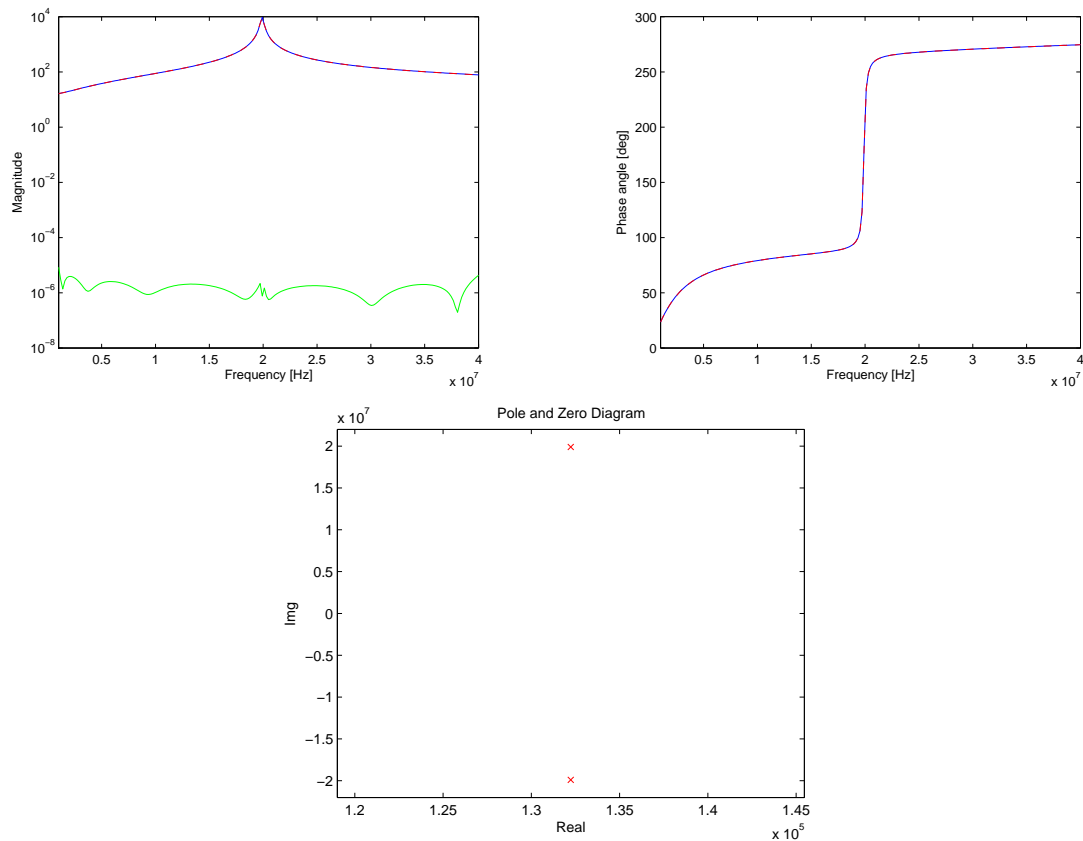


Figure 5.5: Output graphics and pole/zero diagram of the program based on Vector Fitting. On top figures the approximation is in dashed red, the initial data is in blue and the green curve is the complex deviation. On bottom figure a red x indicates an unstable pole.

STAN

The exact same output poles are obtained on STAN, with an automatic order:

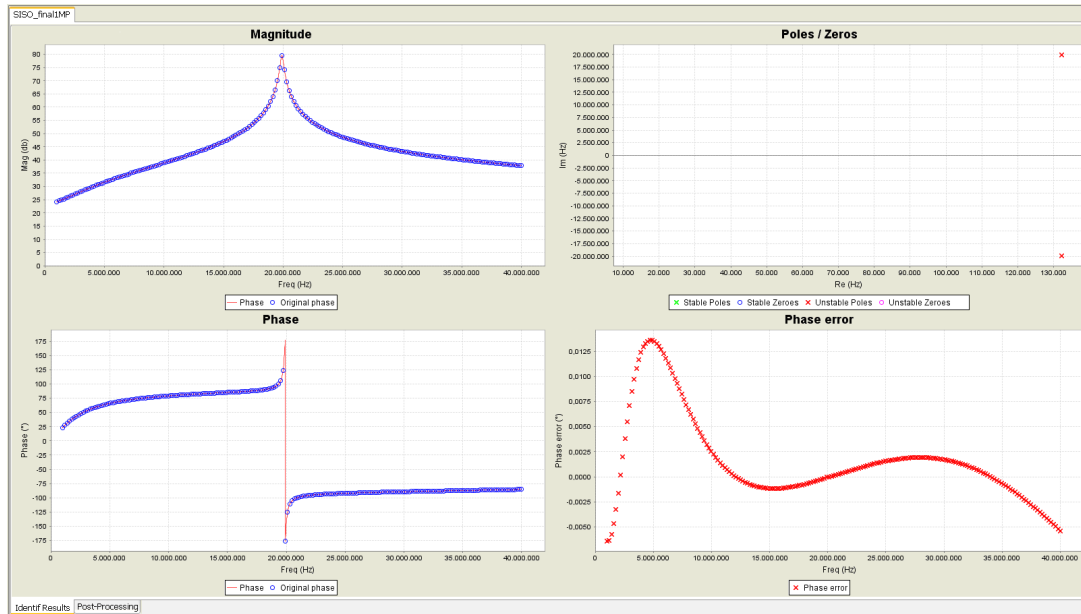


Figure 5.6: Output graphics of the STAN tool

Analysis of the Results

The obtained results on both programs are summarized in the following table:

Stable or Unstable	STAN Value	VF Value	VF Residue
Unstable	132236.1917371 +1.9898520016e7 i	1.32236230780e5 +1.9898519760229e7i	7.90153011799e+9 -4.5547327735038e+8i
Unstable	132236.1917371 -1.9898520016e7 i	1.32236230780e5 -1.9898519760229e7i	7.90153011799e+9 -4.55473277350e+8i

In this example, an unstable complex conjugate set of poles has been detected, which give way to an oscillation at 19.8MHz. We also note, that 6 of the calculated poles have been eliminated by the pole/zero elimination method based on resonances. That is, 6 of the calculated poles are non resonant, possibly results of overmodeling, but are not displayed on the final pole/zero diagram due to their low effect on the dynamics of the systems.

5.1.2 Parametric SISO with Voltage Probe

As mentioned previously, the amplifier can be stabilized increasing the value of the gate resistor. The gate resistor is the resistor connected in series at the gate of the transistor in the gate bias path. The STAN tool and VF based program can be used to determine the resistor values that stabilize this FET amplifier, as illustrated on this example. This is achieved by analyzing the SISO input of the amplifier with a parameter sweep on the critical gate resistance. These frequency responses are obtained by simulating the following ADS schematic:

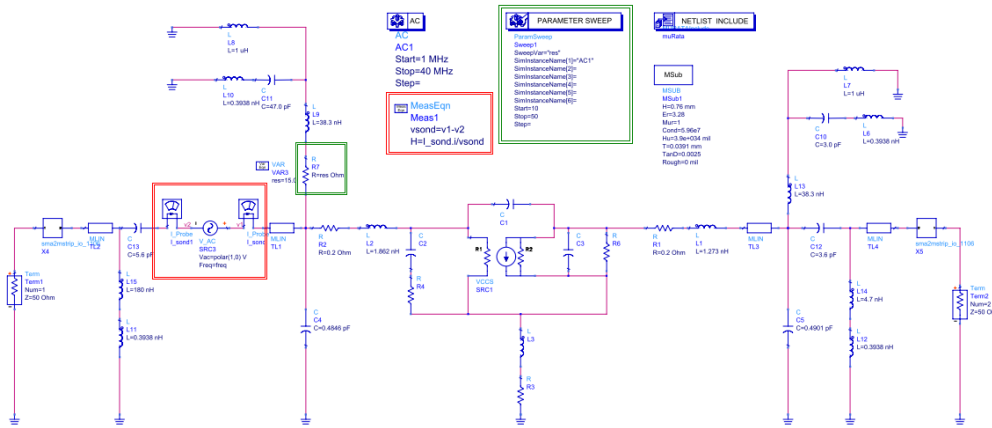


Figure 5.7: The L-band FET amplifier simulation model in ADS with a voltage probe on the input (red) of the transistor and a parameter sweep (green)

Program Based on Vector Fitting

The output graphics and diagrams in Figure 5.8 are obtained on Matlab with the same order (8) and iterations (7) as on previous example, since the exact same node is under analysis in this example:

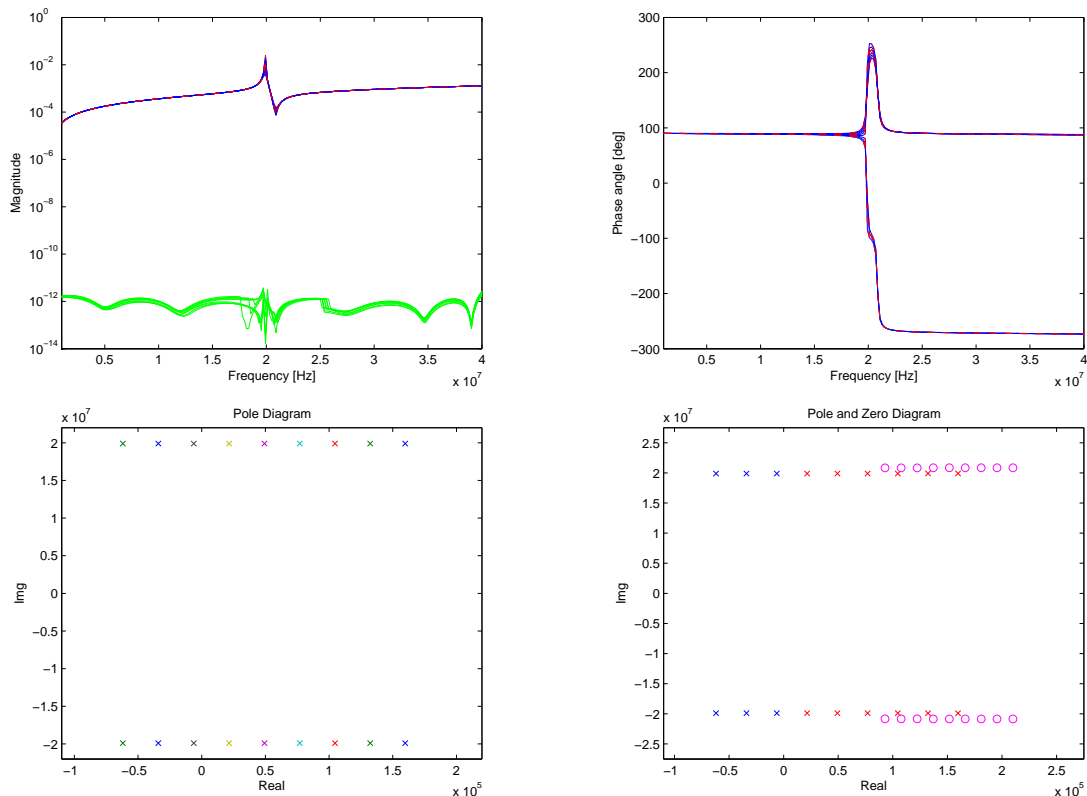


Figure 5.8: Output graphics and pole/zero diagram of the program based on Vector Fitting. On both top figures a curve is mapped for each parameter value. On bottom-left plot each color represents the poles for a parameter value.

STAN

Exactly as on the previous example, the poles calculated, for each parameter value in this case, are the same on both analysis. The pole/zero diagrams calculated with STAN are in Figure 5.9.

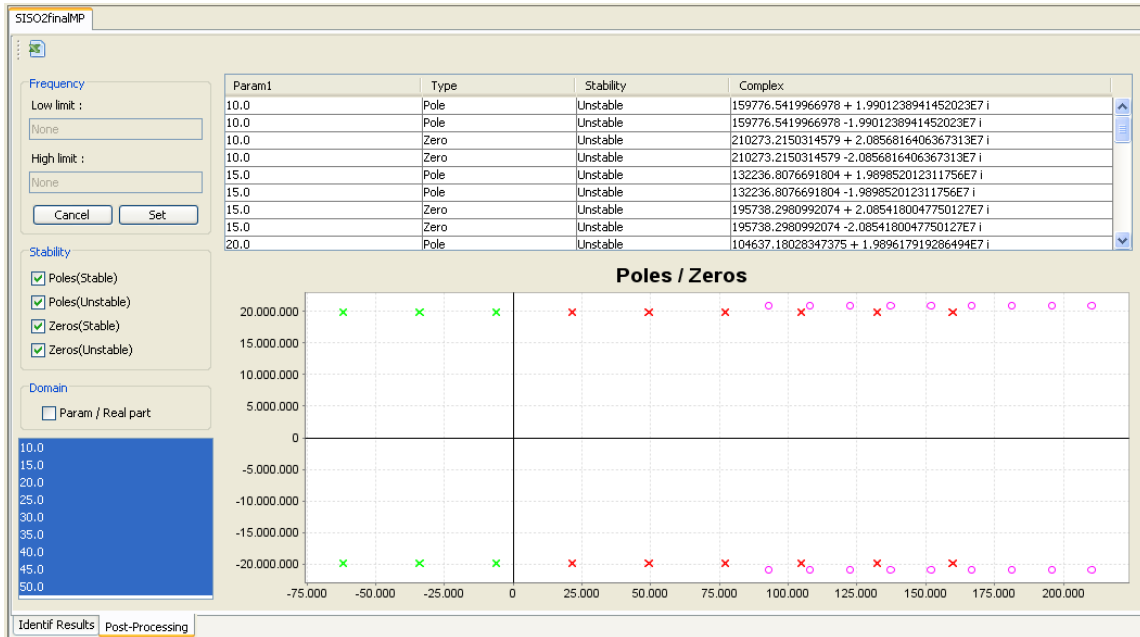


Figure 5.9: Output graphics of the STAN tool

Analysis of the Results

Once again, the obtained results on both programs are summarized on the a table to restate the similarities:

Parameter Value	S or U	STAN Value	VF Value	VF Residue
10	U	159776.541996 + 1.990123894145e7 i	1.59775600988e+5 + 1.99012383379e+7i	3.9705228484e+3 - 2.7507428916e+2i
10	U	159776.541996 - 1.990123894145e7 i	1.59775600988e+5 - 1.99012383379e+7i	3.9705228484e+3 + 2.7507428916e+2i
15	U	132236.807669 + 1.98985201231e7 i	1.32236230807e+5 + 1.98985197601e+7i	3.9690427372e+3 - 3.3052947700e+2i
15	U	132236.807669 - 1.98985201231e7 i	1.32236230807e+5 - 1.98985197601e+7i	3.9690427372e+3 + 3.3052947700e+2i
20	U	104637.1802834 + 1.98961791928e7 i	1.04636860319e+5 + 1.98961789907e+7i	3.9666890195e+3 - 3.8600999137e+2i
20	U	104637.1802834 - 1.98961791928e7 i	1.04636860319e+5 - 1.98961789907e+7i	3.9666890195e+3 + 3.8600999137e+2i
25	U	76983.399487 + 1.989421758943e7 i	7.69832434480e+4 + 1.98942174887e+7i	3.9634601312e+3 - 4.4150745172e+2i
25	U	76983.399487 - 1.989421758943e7 i	7.69832434480e+4 - 1.98942174887e+7i	3.9634601312e+3 + 4.4150745172e+2i
30	U	49281.18616061 + 1.989263670737e7 i	4.92811653597e+4 + 1.98926366666e+7i	3.9593541056e+3 - 4.9701345632e+2i
30	U	49281.18616061 - 1.989263670737e7 i	4.92811653597e+4 - 1.98926366666e+7i	3.9593541056e+3 + 4.9701345632e+2i
35	U	21536.457532 + 1.98914379046e7 i	2.15364455504e+4 + 1.98914378905e+7i	3.9543685683e+3 - 5.5251954392e+2i
35	U	21536.457532 - 1.98914379046e7 i	2.15364455504e+4 - 1.98914378905e+7i	3.9543685683e+3 + 5.5251954392e+2i
40	S	-6245.05224593 + 1.98906224895e7 i	-6.24505882755e+3 + 1.98906224795e+7i	3.9485007397e+3 - 6.0801713968e+2i
40	S	-6245.05224593 - 1.98906224895e7 i	-6.24505882755e+3 - 1.98906224795e+7i	3.9485007397e+3 + 6.0801713968e+2i
45	S	-34057.4317684 + 1.989019172818e7 i	-3.40574497465e+4 + 1.98901917051e+7i	3.9417474274e+3 - 6.6349753583e+2i
45	S	-34057.4317684 - 1.989019172818e7 i	-3.40574497465e+4 - 1.98901917051e+7i	3.9417474274e+3 + 6.6349753583e+2i
50	S	-61894.74875018 + 1.989014683535e7 i	-6.18947850383e+4 + 1.98901467903e+7i	3.9341050358e+3 - 7.1895183344e+2i
50	S	-61894.74875018 - 1.989014683535e7 i	-6.18947850383e+4 - 1.98901467903e+7i	3.9341050358e+3 + 7.1895183344e+2i

We realize on the obtained graph and script outputs that the amplifiers is destabilized for lower resistance values than 35Ω approximately, and that the frequency of the oscillation for the simulated 10, 15, 20, 25, 30 and 35 gate resistor values is approximately 20MHz.

5.1.3 MIMO Analysis with Sequential Current Probes

In this example, a MIMO stability analysis is discussed. Two current probes are sequentially introduced to the simulation model of the amplifier, one electrically isolated from the feedback loop, before a RF bypass series capacitor, and another on the output of the amplifier (on the feedback loop). The frequency responses H1 and H2 are calculated as in previous Figure 4.3 as we can see in the ADS schematic in Figure 5.10. Additionally and as a designer might do when beginning to analyze the stability of a created design, a significantly large frequency band is chosen, 1MHz - 5GHz. This example illustrates the basics of a MIMO analysis as well as how to solve and prevent some of the problems that a designer might encounter with large frequency bands.

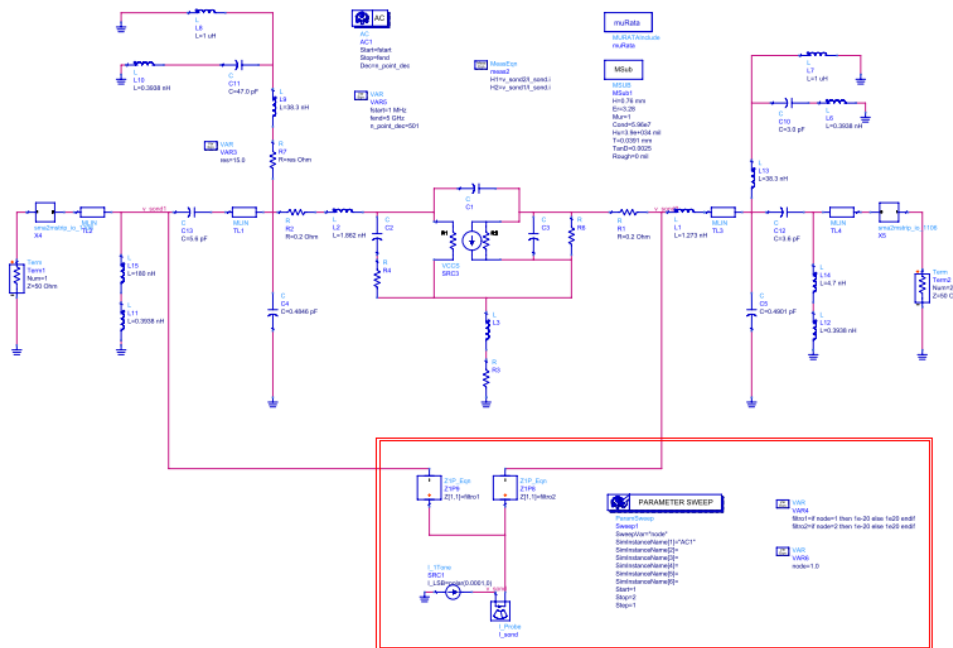


Figure 5.10: The L-band FET amplifier simulation model in ADS with multiple current probes

In order to illustrate the problem of loss of controllability and observability, a SISO analysis of the node before the RF bypass series capacitor is carried out. The order was initially set to 38, and the top two correctly fitted graphics and the leftmost pole/zero diagram in Figure 5.11 were obtained. We can see that there are many pole/zero quasi-cancellations caused by overmodeling, and that the unstable poles detected on the previous examples are not found on this pole/zero plot. The unstable poles are also not detected by increasing the order to 65. Unfortunately, analyzing the stability from the pole/zero diagram of such a high order (see bottom-right diagram in Figure 5.11) is impossible due to the excessively high number of quasi-cancellations. Therefore, even if the unstable poles were detected, no precise conclusion could have been deduced with such a high order.

After introducing the problem of loss of controllability and observability of SISO pole-zero identifications, a solution to prevent them, that is, MIMO, is illustrated. To do so, the frequency response outputs obtained from simulating 5.10 in ADS are analyzed with the VF based tool and the STAN tool.

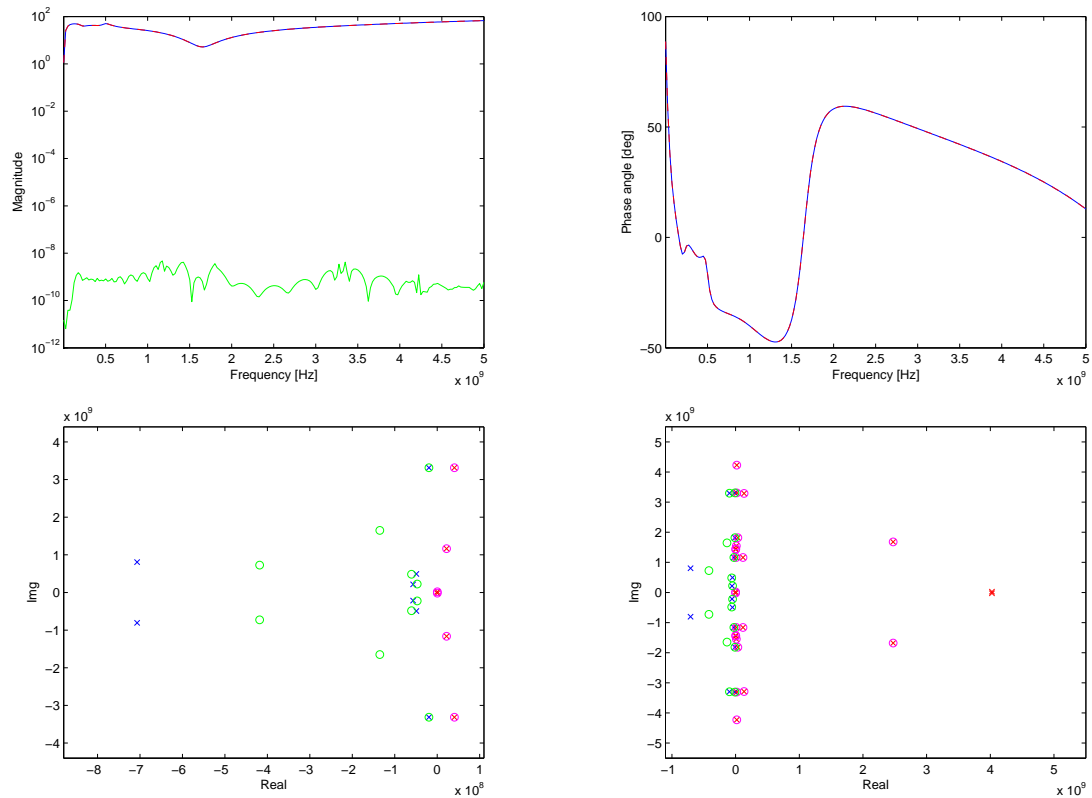


Figure 5.11: Output graphics and pole/zero diagram of the SISO analysis obtained with Vector Fitting based tool. The top two graphics and bottom-left pole/zero diagram are obtained with an order 38. To illustrate that for high orders the instability is not detected, the pole/zero diagram of an order 65 is displayed on the bottom-right subfigure.

Program Based on Vector Fitting

Since the magnitude variation of the input data is very large, for a precise setting, a very large input order is required. After many attempts to choose the precise parameters, and order 38 and 7 iterations were established and the output graphics and pole/zero diagrams in Figures 5.12 and 5.13 were obtained with the VF based program.

STAN

The same .txt file processed by STAN, which analyzes the data as a parametric SISO entry with an automatic order, results in the pole/zero diagrams on Figure 5.14.

Analysis of the Results

The main conclusion inferred from this example is that MIMO analysis is capable of detecting oscillations that might not be found when analyzing SISO frequency responses. That is, that MIMO analysis is less likely to encounter problems related to loss of observability and controllability.

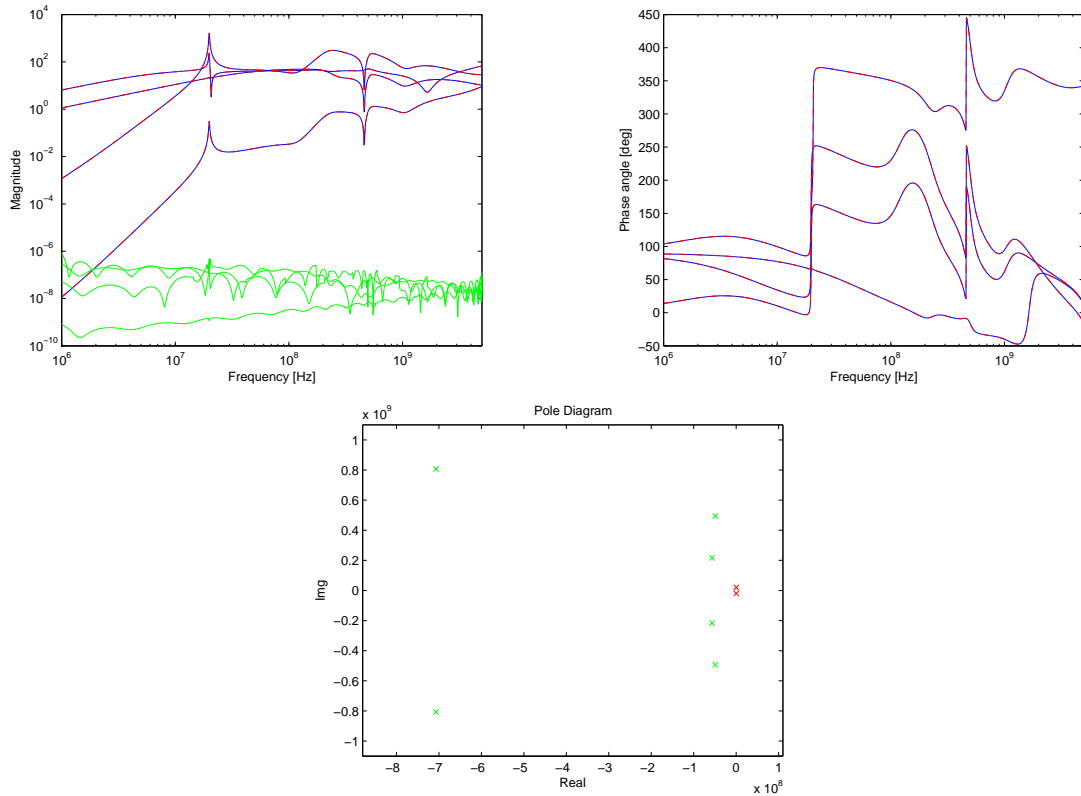


Figure 5.12: Output graphics and pole/zero diagram of the program based on Vector Fitting.

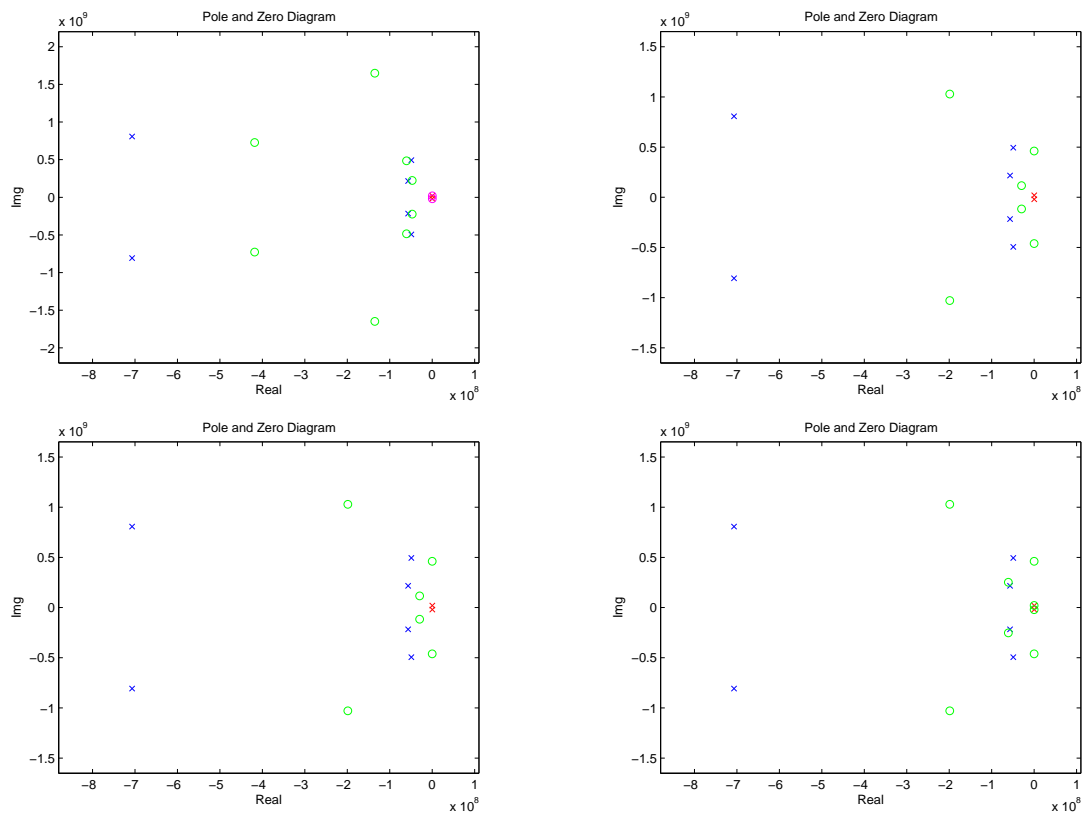


Figure 5.13: Output pole/zero diagrams of the individual nodes obtained with Vector Fitting

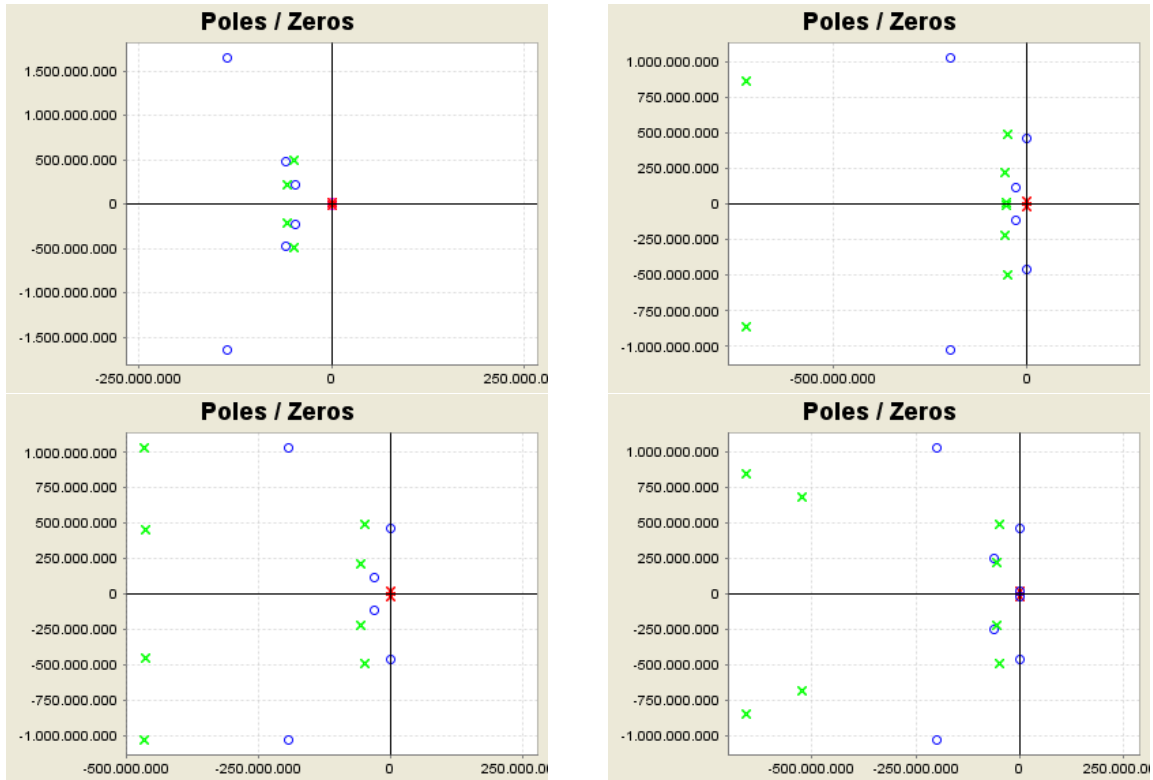


Figure 5.14: Output pole/zero diagrams of the individual nodes obtained with STAN tool.

The calculated unstable poles are detected similarly on both programs, whilst the values of the stable poles are slightly different. Therefore, due mainly to the simplicity of this example, the MIMO analysis that forces the same poles on all nodes and the MIMO analysis of independent SISO data inputs lead to the same conclusion and instability detection.

In this example, the residue obtained with the VF based tool on the Matlab script of the unstable poles is not the highest residue of the identified transfer function, but takes a considerable value. That is, the unstable pole is not the most significant for the dynamic of the system in such a large frequency band. Unfortunately, a tolerance parameter value larger than 4 eliminates its display. This example serves to illustrate that one must look at the values of the residues first to determine a correct tolerance parameter. If all residues are similar as in this example, no elimination based on residues or a high tolerance parameter is advised.

5.2 Analysis of Output Data of a K-Band Multistage Amplifier

The two-stage K-band amplifier discussed on this section is more complex than the previous amplifier. It contains more resonances, and therefore, its analysis is more challenging for stability analysis tools.

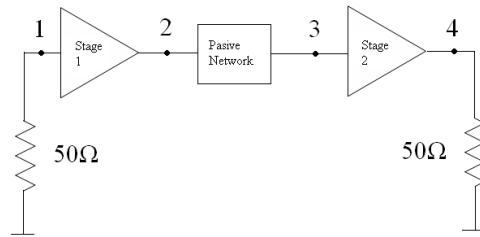


Figure 5.15: Block diagram of the K-Band Multistage Amplifier.

Due to confidentiality, no additional information on the amplifier details can be mentioned. Only the output graphics and diagrams obtained with the VF tool are displayed in order to analyze the stability of the amplifier. This section pretends to illustrate the first steps that a RF and microwave designer could follow to analyze the stability of a completely unknown amplifier and restate the problem of loss of controllability and observability.

The output data consists of multiple frequency responses obtained by introducing multiple probes in the input port of the first stage (1), between the first and second stage (2), in the input port of the second stage (3) and in the output node (4) that are illustrated in Figure 5.15.

As before and to illustrate the problem of loss of controllability and observability, the individual frequency responses of each node are analyzed independently (SISO) with the VF based tool. The pole/zero diagrams in Figure 5.16 are obtained, where the top-left diagram corresponds to input node 1, the top-right diagram to intermediate node 2, the bottom-left diagram to second stage node 3 and the bottom-right diagram to output node 4. As we see in Figure 5.16 the complex conjugate unstable poles are only detected on nodes 2 and 3, and are detected with more precision in node 3 since the unstable poles of node 2 have similar valued zeros near the unstable poles.

Once the SISO analysis has been carried out, we can proceed to analyzing the same results as MIMO with the VF based program. The MIMO analysis can start by analyzing a wide frequency range as on the example in 5.3 and establishing an excessively high order (80) and the numbers of iterations (10) to obtain the top two and bottom-left graphics in Figure 5.17. Next, a tolerance parameter can be determined by approximation from the residues obtained on a first compilation, to eliminate the products of overmodeling, that is, possible mathematical quasi-cancellations. The results of the pole-zero elimination method based on residues for a tolerance parameter of 4 are illustrated on the bottom-right diagram in Figure 5.17. With the determined tolerance parameter, we can detect the unstable complex conjugate poles with high precision.

An unstable complex conjugate pole pair at a frequency of approximately 10GHz have been identified in this analysis. If a more correct and precise analysis is wanted, a narrower frequency range around 10GHz should be analyzed. Nevertheless, on this example, very similar results are obtained with a narrower frequency band. The calculated unstable poles, their residue and the critical node for the wide band are shown in table 5.1.

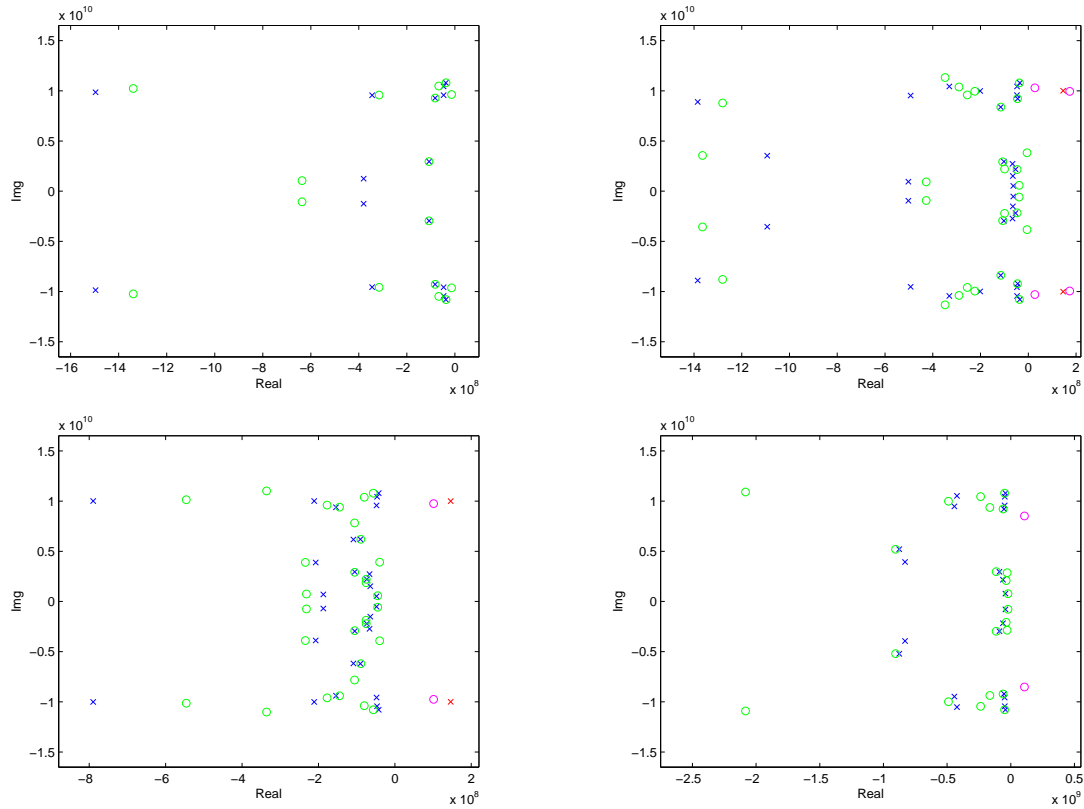


Figure 5.16: Output pole/zero diagrams of the multiple SISO Analysis on nodes 1 (top-left), 2 (top-right), 3 (bottom-left) and 4 (bottom-right) of a K-Band Multistage Amplifier obtained with VF based tool

Unstable Poles	Maximum residue	Critical node
1.457203119183668e+008 +1.000046259092223e+010i	1.643578710378006e+010 +6.632234428154502e+010i	3
1.457203119183668e+008 -1.000046259092223e+010i	1.643578710378006e+010 -6.632234428154502e+010i	3

Table 5.1

We conclude therefore, that the analyzed multistage amplifier is unstable and oscillates at a frequency of 10GHz and that the critical node is the node at the input port of the second stage. Most importantly we have illustrated a solution for loss of controllability and observability. This has been achieved by analyzing an example where the feedback loop was not detected on 2 out of 4 nodes, the input and output nodes, but was detected by analyzing the 4 frequency responses with a MIMO pole/zero identification tool.

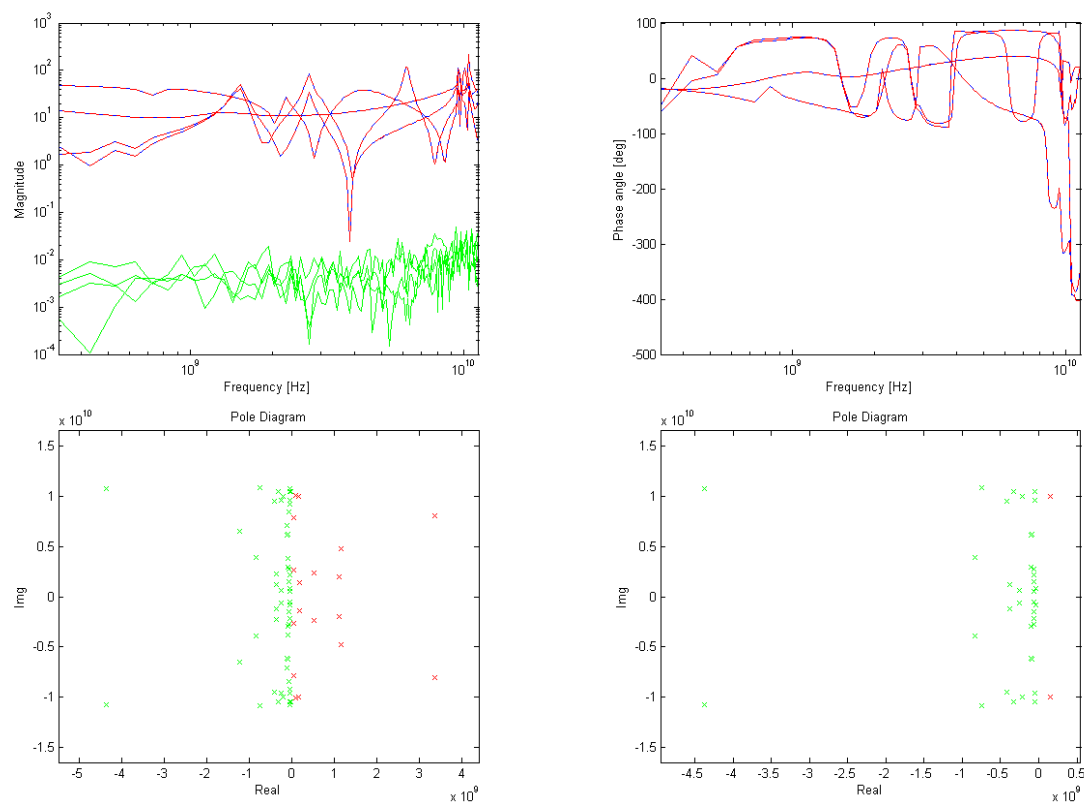


Figure 5.17: Output graphics and poles for a MIMO Analysis of the K-Band Multistage Amplifier. The top two graphics and bottom-left diagram correspond to an analysis with order 38 and no tolerance parameter. The bottom-right diagram is obtained with the same order and a tolerance parameter 4.

Chapter 6

Conclusions

The principal achievement of this project is that a rigorous SISO and MIMO pole-zero identification stability analysis CAD tool, based on Vector Fitting, that can be compared to existing commercial tools, has been created. As mentioned, the created tool introduces MIMO analysis that can improve the efficiency of pole-zero identification for stability analysis of microwave circuits and give way to automating further the stability analysis process.

By forcing the same characteristic equations on all nodes on MIMO analysis, more precise MIMO outputs are calculated. Therefore, this project demonstrates that Vector Fitting or similar fitting programs that force the same denominator when fitting multiple transfer functions are an essential incorporation for rigorous MIMO analysis. Additionally, errors due to loss of controllability and observability are prevented in MIMO, especially if the multiple probes are systematically introduced in the vicinity of the inputs and outputs of the transistors.

By incorporating MIMO, three new methods to process the obtained results were created and introduced. These methods have been proved highly efficient to help users understand and not misinterpret the output graphics and diagrams.

The simple resonant dependent pole-elimination method has been proven highly efficient for eliminating the products of overmodeling that have close to no effect on the dynamics of the system. The second pole-eliminating method based on the residues is also a very powerful asset for the program but lacks automation. As a consequence, studying the automation of the method via iterations is a way of further improving the functionality of the program. The critical node or branch detection method was determined as highly useful when evaluating the created stability analysis tool, since it gives additional information for possible subsequent stabilization approaches.

To sum up, all of the objectives stated on the first chapter of this chapter have been tackled thoroughly through the process of shaping and completing this project.

Future work to improve the created program could consist of creating methods to improve the sensibility of the identifications and reduce the need for fine frequency sweeps to avoid misinterpretations. For example, an algorithm to determine the order for fitting the frequency response data automatically would increase considerably the automation of the program. Most importantly, this report concludes by stating that further work to study the benefits of MIMO analysis for pole-zero identification will be very profitable for stability analysis of RF and microwave amplifiers.

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