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JAVIER FERNÁNDEZ-MACHO

Abstract. In a recent paper Leong and Huang [6] proposed a wavelet-correlation-based approach to test for cointegration between two time series. However, correlation and cointegration are two different concepts even when wavelet analysis is used. It is known that statistics based on nonstationary integrated variables have non-standard asymptotic distributions. However, wavelet analysis offsets the integrating order of nonstationary series so that traditional asymptotics on stationary variables suffices to ascertain the statistical properties of wavelet-based statistics. Based on this, this note shows that wavelet correlations cannot be used as a test of cointegration.

Key words: Econometric Methods; Integrated process; Spectral Analysis; Time Series Models; Unit roots; Wavelet Analysis.
JEL Classification: C22, C12.

1. Introduction

When relationships among nonstationary integrated time series are considered, distinguishing between the different, although related, concepts of correlation and cointegration still seems to be a source of confusion among empirical researchers. As Johansen [5] points out 'the important paper by Phillips [8] solved the problem of finding the asymptotic distribution of correlation and regression coefficients, when calculated from a class of nonstationary time series. Thus the problem and its solution has been known for a long time but we still find numerous examples of misunderstandings in applied and theoretical work.' He further illustrates the point with some recent examples of these misunderstandings [see also 4].

In a recent paper Leong and Huang [6] (L&H in what follows) proposed a wavelet-based approach to analyse bivariate spurious and cointegrated time series relationships. They based their approach on the empirical properties of wavelet covariances and correlations obtained via Monte Carlo simulation of some spurious and cointegrated regressions.

Wavelet analysis uses generalised differences of sufficient length that will effectively offset the integrating order of nonstationary time series. Therefore, after a fashion, it reverses theoretical analysis back to traditional asymptotics on stationary variables. This may sometimes be confusing if practitioners are not aware. In what follows, it will be shown that L&H proposal cannot in fact be used as a test of cointegration.

2. Discrete wavelet transform

In short, the discrete wavelet transform (DWT) can be thought of as an energy (variance) preserving transform that uses a pair of high-pass and low-pass filters to decompose a subsampled input series into the detail achieved by the wavelet filter and the corresponding smooth approximation. The level of decomposition can be escalated by subsampling and transforming recursively the previous approximation so that a new vector of wavelet coefficients associated with changes at an ever higher scale (lower frequency) is produced alongside a remainder associated with an even smoother approximation of the

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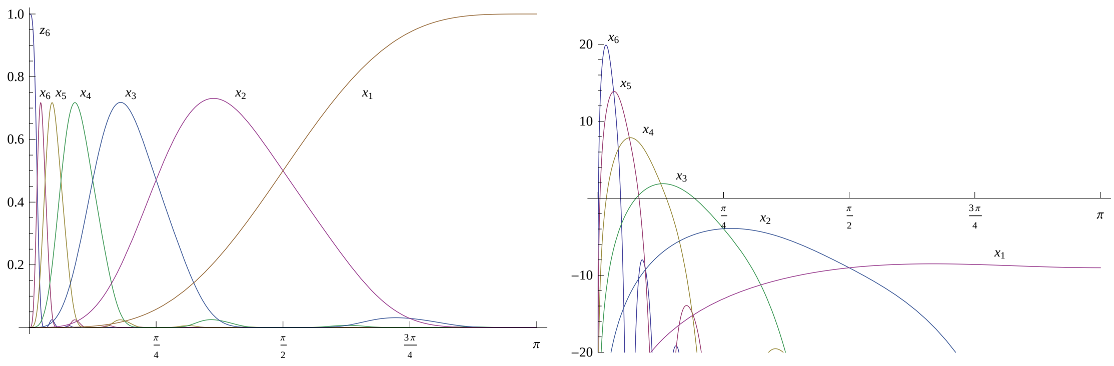

(a) D(4) MODWT filter gains.

(b) Spectral densities of wavelet decomposition of a I(1) series using the D(4) MODWT, log (dB) scale.

FIGURE 1.

original series at the new scale. Common examples are members of Daubechies [1, 2] family of compact support filters that are best described through their low-pass squared gain functions

$$|\mathfrak{G}(\lambda)|^2 = 2^{1-L/2}(1+\cos\lambda)^{L/2} \sum_{s=0}^{L/2-1} 2^{-s} \binom{L/2-1+s}{s} (1-\cos\lambda)^s, \quad \lambda \in [0,\pi],$$

where the filter length L is a positive even number [*cf.* 7, p. 105]. Furthermore, different spectral factorisations of the frequency response function $\mathfrak{G}(\lambda)$ give rise to D(L) (extremal phase or minimum delay) and LA(L) (least asymmetric) groups of Daubechies filters, of which the D(4) is probably the most popular and the one used by L&H as well as in empirical results of this note.

The DWT succinctly described above uses the least number of coefficients conveying the same amount of information as the original series. However, it can only be applied to dyadic samples whilst, in certain applications, it is convenient that the number of coefficients remains the same as the original length T at each level of decomposition. An alternative transform, the maximal overlap DWT (MODWT), is obtained by not subsampling the filtered series. Using the MODWT has a number of advantages over the DWT [7, p159], one of which is crucial here since the variance of MODWT-based estimators of wavelet covariances and correlations are, unlike DWT-based estimators, known to be invariant with respect to the, usually unknown, lag between time series [3, p253].

On the other hand, from the frequency response functions corresponding to the high and low-pass filters of a MODWT [7, p163] it can be shown that the square gains of these two filters valued at the origin are equal to 0 and 1 respectively. In particular, this means that, at any given wavelet level j, the MODWT using a wavelet filter of sufficient length will decompose an $I(d)$ series into two components associated respectively with high and low frequencies: the first one (x_j) will effectively be stationary while the second (z_j) one will retain the $I(d)$ behaviour. Figure 1(a) shows the square gains of D(4) MODWT high-pass filter (x_1) and band-pass filters ($x_2 \ldots x_6$) for the wavelet details at different levels and the low-pass filter (z_6) for the smooth reminder whilst Figure 1(b) shows the typical spectral densities resulting from filtering an I(1) series through those wavelet filters. They are all finite at all frequencies in $[0,\pi]$ and, in particular, they all can be shown to be zero at the origin.

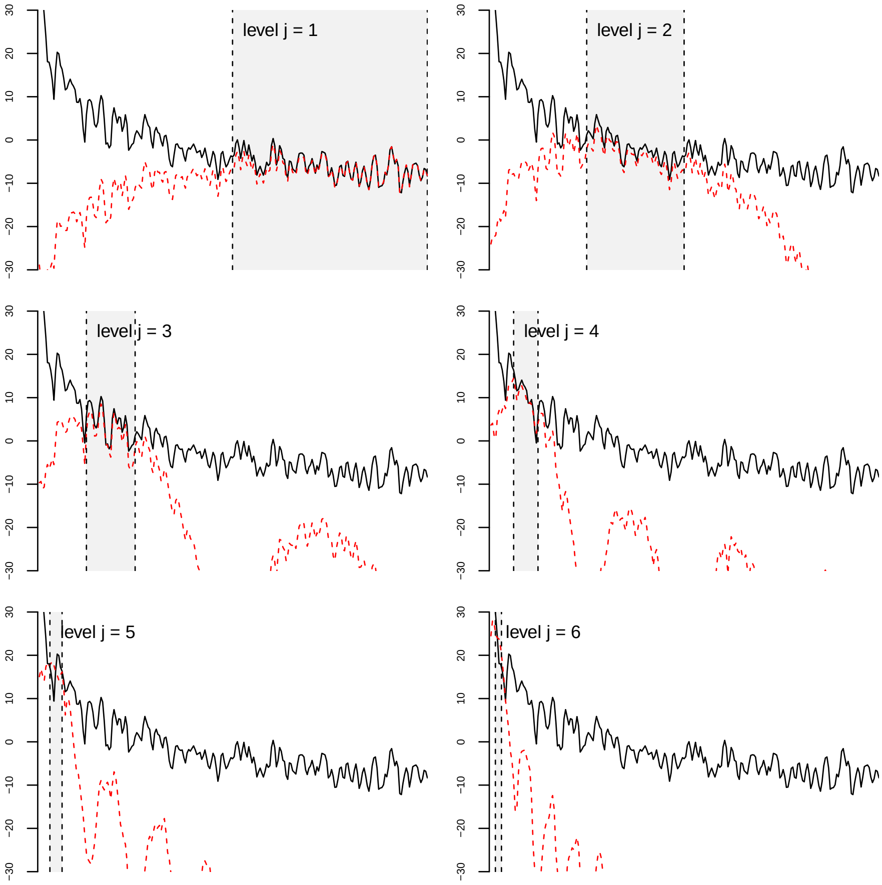

FIGURE 2. D(4) MODWT I(1) spectra: original I(1) series vs. wavelet levels, log (dB) scale, shaded area: frequencies in the range $[2^{-j}\pi, 2^{1-j}\pi)$.

As an empirical example, Figure 2 compares the spectral density (smoothed) estimates for the wavelet details of an I(1) series of length $T = 480$ with the estimated spectrum of the original series. It can be seen that as the wavelet scale $\lambda_j = 2^j$ increases the corresponding wavelet coefficients capture changes associated with frequencies in the range $[2^{-j}\pi, 2^{1-j}\pi)$ for $j = 1 \ldots J$. However, their spectra are always finite, that is, they are all stationary.

Therefore, wavelet analysis offsets the integrating order of nonstationary series and traditional asymptotics on stationary variables should suffice to ascertain the statistical properties of wavelet-based statistics.

3. Wavelet correlation and cointegration

L&H claim that "the null hypotheses of zero wavelet covariance and correlation for these series across the [wavelet] scales" can be used as a test for no-cointegration in the sense that these null hypotheses are rejected when the bivariate time series is cointegrated whilst, on the other hand, they fail to be rejected in the case of the spurious regression.

The actual meaning of "the null hypotheses" appears to signify the joint hypothesis of all possible wavelet correlations (or covariances) being equal to zero under the null. More formally,

$$H_0 : \rho_{(x,y)}(\lambda_j) = 0, \quad \forall j = 1, \ldots, J, \quad \text{(L\&H no-cointegration)},$$
$$H_a : \rho_{(x,y)}(\lambda_j) \neq 0, \quad \text{for some } j, \quad \text{(L\&H cointegration)}.$$

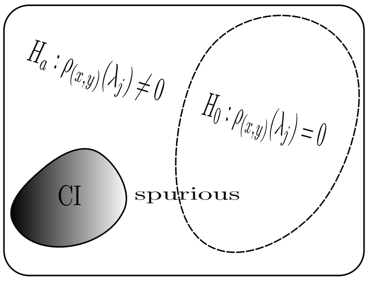

FIGURE 3. H_0 vs H_a compared with CI vs spurious regressions.

where $\rho_{(x,y)}(\lambda_j)$ denotes the wavelet correlation for scale λ_j and likewise for the test based on covariances $\gamma_{(x,y)}(\lambda_j)$.[1] However, rejecting H_0 does not necessarily means that the bivariate time series is cointegrated as claimed by L&H since, as sketched in Figure 3, both dichotomies are not equivalent.

In fact it is not difficult to construct spurious regressions whose wavelet coefficients would appear to be correlated. *e.g.* consider the single linear regression under the classical assumptions

$$y_t = \alpha + \beta x_t + u_t, \quad \beta \neq 0.$$

The variables y_t, x_t and u_t are all I(0) and there is a meaningful relationship between x_t and y_t as measured by the regression and the correlation coefficients. On the other hand, the relationships between cumulated values $\Delta^{-d}x_t$, $\Delta^{-d}y_t$, $d > 0$, are clearly spurious since $\Delta^{-d}u_t \sim I(d)$. However, wavelet coefficients are obtained as generalised differences of sufficient length that will offset the integrating order d and the corresponding wavelet correlations

$$\rho_{(\Delta^{-d}x,\Delta^{-d}y)}(\lambda_j) \neq 0, \quad \forall d > 0.$$

That is, H_0 (L&H no-cointegration) should be rejected in spite of the spuriousness of the relationship, thus contradicting L&H's claim. This is illustrated empirically in what follows.

4. Empirical results

Tables 3 to 9 in the appendix provide evidence on the mean, median and 95% c.i. of the wavelet covariance, correlation and regression coefficients together with their respective bootstrap t-statistics and percent rejections at the nominal 5% significance for all relevant wavelet levels (with d0 as the original data). A summary of relevant results regarding L&H test is given in Tables 1 and 2. The reported results were obtained through Monte Carlo simulation using 1000 replications of size $T = 120$ from the linear regression

$$y_t = \alpha + \beta x_t + u_t; \quad x_t = \delta_x x_{t-1} + \varepsilon_t,\ u_t = \delta_u u_{t-1} + \eta_t; \quad \delta_x = \mathcal{I}(x_t \sim I(1)),\ \delta_u = \mathcal{I}(\text{no-CI}),$$

with $\alpha = 10$ and $\beta = 0.0, 0.6, 6.0, 60.6$, where ε_t, η_t are uncorrelated iid N(0,1) and $\mathcal{I}(\cdot)$ is the indicator function. A D(4) wavelet filter, *i.e.* the Daubechies extremal phase compactly supported wavelet of length $L = 4$ [2], was use in the decomposition via R routines modwt and brick.wall from waveslim package version 1.7.1. The tables with $\beta = 0$ refer to independent I(0) and I(1) series (Tables 3 and 6), whilst the tables with $\beta > 0$ refer to correlated series that include cointegration (Table 5) and spurious cases

[1] They actually write the lhs of both hypotheses as $\{\rho_{(x,y)}(\lambda_j)\}_j$ to denote the "set of all wavelet correlations", which renders the expression $\{\cdots\} = 0$ rather awkward.

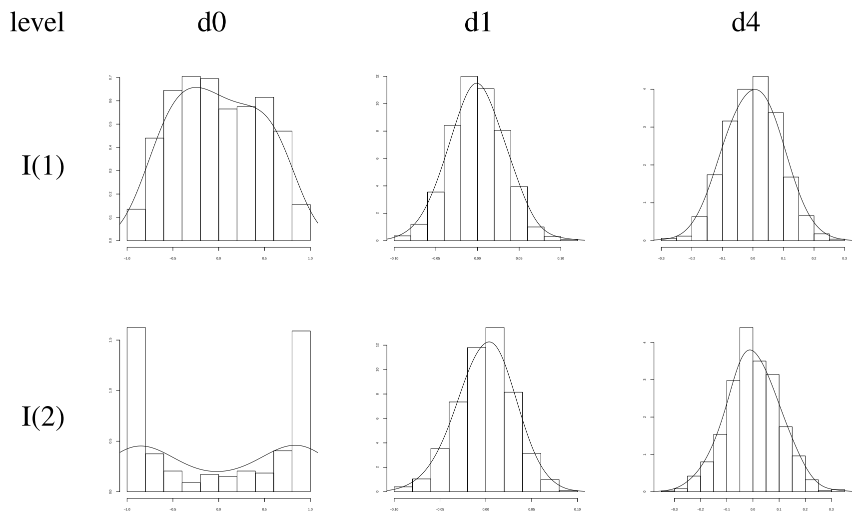

FIGURE 4. Distribution of the empirical wavelet correlation coefficient for independent random walks, I(1), and independent cumulated random walks, I(2).

TABLE 1. Summary: H_0 within 95% c.i. of empirical distribution.

		I(0)		CI(1,1)	I(1): spurious regression			
	β	0	0.6	0.6	0	0.6	6.0	60.6
covariance	d0	yes	no	no	yes	yes	**no**	**no**
	d1–d4	all	none (except d4)	none	all	**none** (except d4)	**none**	**none**
correlation	d0	yes	no	no	yes	yes	**no**	**no**
	d1–d4	all	none (except d4)	none	all	**none** (except d4)	**none**	**none**
regression coef.	d0	yes	no	no	yes	yes	**no**	**no**
	d1–d4	all	none (except d4)	none	all	**none** (except d4)	**none**	**none**

(Tables 7 to 9). We note that the only two cases considered by L&H, independent vs cointegrated I(1), do not constitute a proper dichotomy in Figure 3.

Figure 4 shows the distribution of the empirical correlation coefficient for independent random walks, I(1), and independent cumulated random walks, I(2), at different levels of the wavelet decomposition. We note that whilst the empirical distribution for the original series exhibits the inverted-U and U shapes associated with spurious cases, the wavelet correlations have the typical bell shape corresponding to stationary I(0) variables [*cf.* 9, figs. 16–18]. This is consistent with the fact that, as already mentioned, wavelet analysis offsets the integrating order of the original series and reverses to traditional asymptotics on stationary variables.

Table 1 provides a qualitative summary on whether the value set by L&H null hypothesis, *e.g.* $H_0 : \rho_{(x,y)}(\lambda_j) = 0$, is included within a 95% c.i. of the corresponding empirical distribution (incorrect exclusions as a test for no-cointegration in bold type). As expected, wavelet 95% c.i. for correlated cases exclude H_0 inclusive of spurious cases (except level d4 when the correlation is too small for this sample size, since this wavelet level is left with very few observations after elimination of coefficients affected by boundary effects), thus showing that this is not a test of cointegration. This is corroborated by Table 2 that shows t-test percent rejections of H_0 using the nominal 5% significance critical value of the *t*-Student

TABLE 2. Summary: t-test percent rejections of H_0 at 5% significance.

	β	I(0) 0	I(0) 0.6	CI(1,1) 0.6	I(1): spurious regression 0	0.6	6.0	60.6
covariance	d0	5.1	99.9	8.7	2.3	1.3	**7.7**	**8.4**
	d1–d4	5.5	70.1	78.4	5.4	**68.3**	**85.9**	**85.9**
correlation	d0	5.0	100	100	1.9	**28.9**	**100**	**100**
	d1–d4	4.8	87.9	98.3	4.5	**85.6**	**100**	**100**
regression coef.	d0	5.4	100	100	4.8	**13.1**	**100**	**100**
	d1–d4	5.1	81.3	97.7	5.2	**77.8**	**100**	**100**

distribution with $T-2$ degrees of freedom (percent rejections in spurious cases higher than 7.5% in bold type). It is shown that L&H wavelet-based test has a correct size of around 5% for independent series. However, in cases that are in fact spurious it rejects H_0 too often for it to be of any use as a test of cointegration.

In conclusion, all this shows that L&H test is just a mere test of correlation that cannot be used as a test of cointegration.

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Appendix

Table 3. Independent I(0) series, $\beta = 0.0$

level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
	covariance				t-statistic				
d0	0.002	0.001	-0.178	0.183	0.023	0.008	-1.884	2.048	5.1
d1	-0.000	-0.002	-0.111	0.119	-0.005	-0.034	-1.920	1.974	4.8
d2	0.000	0.001	-0.069	0.068	0.007	0.021	-2.008	1.981	5.3
d3	0.001	0.000	-0.048	0.055	0.060	0.016	-1.922	2.228	6.4
d4	-0.000	-0.000	-0.038	0.039	-0.021	-0.014	-2.007	1.999	5.6
	correlation				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	0.002	0.001	-0.170	0.186	0.024	0.009	-1.881	2.027	5.0
d1	-0.000	-0.004	-0.223	0.229	-0.004	-0.033	-1.963	2.089	4.9
d2	0.002	0.003	-0.270	0.271	0.012	0.023	-2.025	2.033	5.5
d3	0.012	0.003	-0.365	0.414	0.061	0.017	-1.880	2.163	5.3
d4	-0.005	-0.006	-0.551	0.572	-0.015	-0.021	-1.848	1.894	3.5
	regression coef.				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	0.002	0.001	-0.177	0.185	0.019	0.009	-1.921	2.032	5.4
d1	-0.001	-0.004	-0.229	0.233	-0.012	-0.030	-2.017	1.880	4.9
d2	0.001	0.003	-0.278	0.265	0.004	0.020	-2.043	1.899	5.4
d3	0.011	0.003	-0.389	0.439	0.053	0.014	-1.947	2.138	6.2
d4	-0.004	-0.007	-0.636	0.648	-0.010	-0.020	-1.912	1.950	4.1

Table 4. Correlated I(0) series, $\beta = 0.6$

level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
	covariance				t-statistic				
d0	0.604	0.598	0.371	0.851	5.093	5.071	3.224	7.141	99.9
d1	0.302	0.298	0.155	0.466	3.914	3.846	2.060	6.128	97.9
d2	0.153	0.150	0.071	0.253	3.421	3.383	1.511	5.556	92.5
d3	0.078	0.074	0.021	0.150	2.410	2.354	0.649	4.689	63.5
d4	0.037	0.034	-0.001	0.097	1.467	1.299	-0.046	3.853	26.4
	correlation				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	0.515	0.519	0.375	0.643	7.677	7.647	5.318	10.006	100.0
d1	0.512	0.516	0.327	0.678	5.905	5.908	3.618	8.038	99.9
d2	0.514	0.524	0.278	0.708	5.104	5.168	2.709	7.092	99.7
d3	0.515	0.529	0.188	0.767	3.513	3.590	1.245	5.418	90.9
d4	0.498	0.538	-0.026	0.841	2.133	2.270	-0.104	3.732	61.0
	regression coef.				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	0.602	0.601	0.423	0.785	6.573	6.547	4.423	8.828	100.0
d1	0.599	0.596	0.371	0.833	5.054	5.077	2.991	7.124	99.8
d2	0.601	0.603	0.322	0.865	4.365	4.384	2.205	6.350	98.6
d3	0.611	0.603	0.211	1.039	3.027	3.004	1.052	5.202	83.2
d4	0.596	0.593	-0.036	1.248	1.808	1.819	-0.105	3.864	43.4

Table 5. CI(1,1) series, $\beta = 0.6$

	covariance				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	12.414	8.611	2.175	47.768	0.906	0.682	0.166	2.982	8.7
d1	0.114	0.112	0.041	0.193	3.075	3.049	1.154	5.081	86.0
d2	0.161	0.157	0.077	0.263	3.574	3.482	1.770	5.702	95.7
d3	0.294	0.286	0.130	0.503	3.010	2.931	1.346	5.153	85.6
d4	0.564	0.525	0.162	1.178	2.039	1.889	0.563	4.345	46.3
	correlation				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	0.903	0.917	0.749	0.983	15.216	14.903	11.415	20.922	100.0
d1	0.344	0.345	0.148	0.529	3.625	3.631	1.515	5.637	93.7
d2	0.523	0.529	0.315	0.692	5.520	5.514	3.268	7.626	99.6
d3	0.755	0.767	0.549	0.887	8.759	8.763	6.038	11.335	100.0
d4	0.910	0.926	0.746	0.979	14.805	14.707	10.305	20.343	100.0
	regression coef.				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	0.599	0.599	0.544	0.656	23.982	23.261	18.300	32.538	100.0
d1	0.598	0.595	0.252	0.956	3.394	3.379	1.406	5.552	91.2
d2	0.600	0.596	0.336	0.848	4.755	4.720	2.651	6.909	99.5
d3	0.597	0.596	0.401	0.798	6.050	6.017	4.015	8.448	99.9
d4	0.600	0.600	0.432	0.771	7.028	7.008	4.879	9.452	100.0

Table 6. Independent random walks, I(1) series, $\beta = 0.0$

	covariance				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	0.511	0.492	-26.164	29.608	-0.054	0.031	-1.815	1.269	2.3
d1	-0.000	-0.001	-0.037	0.040	-0.006	-0.029	-1.946	1.910	4.7
d2	0.002	-0.000	-0.070	0.079	0.045	-0.007	-1.886	2.152	5.7
d3	0.005	0.004	-0.195	0.215	0.045	0.038	-1.873	2.063	5.1
d4	-0.007	-0.010	-0.653	0.643	-0.018	-0.028	-2.109	2.002	6.1
	correlation				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	0.028	0.046	-0.829	0.850	0.073	0.095	-1.677	1.890	1.9
d1	-0.000	-0.003	-0.199	0.208	-0.004	-0.030	-1.997	1.978	5.1
d2	0.006	-0.001	-0.265	0.299	0.043	-0.006	-1.921	2.176	5.4
d3	0.011	0.008	-0.365	0.401	0.051	0.038	-1.775	1.967	3.7
d4	-0.004	-0.012	-0.623	0.618	-0.014	-0.036	-1.916	1.912	3.8
	regression coef.				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	0.057	0.043	-1.124	1.379	0.071	0.064	-1.749	2.053	4.8
d1	-0.001	-0.003	-0.212	0.204	-0.012	-0.029	-2.090	1.900	4.9
d2	0.005	-0.001	-0.281	0.308	0.033	-0.007	-1.978	2.137	6.1
d3	0.011	0.008	-0.401	0.460	0.047	0.035	-1.844	2.132	5.0
d4	-0.005	-0.011	-0.699	0.715	-0.016	-0.029	-1.955	1.945	4.6

Table 7. Spurious I(1) series, $\beta = 0.6$

	covariance				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	12.932	8.459	-7.846	61.899	0.476	0.388	-0.429	1.702	1.3
d1	0.114	0.113	0.066	0.169	4.444	4.381	2.559	6.508	99.8
d2	0.162	0.157	0.071	0.275	3.392	3.358	1.413	5.674	92.3
d3	0.300	0.291	0.066	0.572	2.257	2.187	0.502	4.449	58.0
d4	0.558	0.500	-0.068	1.508	1.351	1.196	-0.163	3.812	23.1
	correlation				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	0.495	0.606	-0.489	0.944	1.356	1.522	-1.206	3.328	28.9
d1	0.513	0.515	0.343	0.658	6.708	6.699	4.458	8.943	100.0
d2	0.515	0.523	0.281	0.713	5.030	5.109	2.714	7.021	99.3
d3	0.516	0.530	0.165	0.771	3.297	3.379	1.089	5.177	89.5
d4	0.494	0.534	-0.087	0.859	1.921	2.085	-0.322	3.437	53.5
	regression coef.				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	0.657	0.643	-0.524	1.979	0.981	0.960	-0.833	2.953	13.1
d1	0.599	0.597	0.388	0.804	5.741	5.726	3.705	7.830	100.0
d2	0.605	0.599	0.319	0.908	4.310	4.278	2.219	6.451	98.6
d3	0.611	0.608	0.199	1.060	2.829	2.803	0.927	4.944	78.2
d4	0.595	0.589	-0.099	1.315	1.622	1.578	-0.277	3.649	34.9

Table 8. Spurious I(1) series, $\beta = 6.0$

	covariance				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	124.716	88.805	22.350	474.671	0.871	0.675	0.167	2.745	7.7
d1	1.137	1.129	0.828	1.498	6.878	6.782	5.035	9.072	100.0
d2	1.610	1.581	1.082	2.310	5.181	5.062	3.463	7.412	100.0
d3	2.962	2.881	1.533	4.861	3.410	3.337	1.779	5.598	95.1
d4	5.643	5.157	1.721	11.748	2.097	1.918	0.669	4.347	48.3
	correlation				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	0.982	0.991	0.908	0.999	37.572	36.661	20.022	63.709	100.0
d1	0.986	0.986	0.979	0.991	346.922	344.554	284.232	418.351	100.0
d2	0.986	0.987	0.977	0.993	254.213	253.123	208.133	303.542	100.0
d3	0.986	0.987	0.969	0.995	158.134	156.966	122.347	196.516	100.0
d4	0.985	0.988	0.952	0.997	83.869	83.600	55.312	115.129	100.0
	regression coef.				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	6.057	6.043	4.876	7.379	9.176	9.093	6.947	11.558	100.0
d1	5.999	5.997	5.788	6.204	57.521	57.400	49.871	66.393	100.0
d2	6.005	5.999	5.719	6.308	42.797	42.678	36.918	49.672	100.0
d3	6.011	6.008	5.599	6.460	27.862	27.758	23.506	32.554	100.0
d4	5.995	5.989	5.301	6.715	16.360	16.198	13.021	20.005	100.0

Table 9. Spurious I(1) series, $\beta = 60.6$

	covariance				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	1254.97	874.60	220.28	4799.70	0.90	0.69	0.16	2.94	8.4
d1	11.479	11.404	8.401	15.181	6.931	6.822	5.142	9.070	100.0
d2	16.250	15.934	11.043	23.325	5.212	5.106	3.536	7.387	100.0
d3	29.873	29.248	15.659	48.529	3.429	3.360	1.827	5.568	95.3
d4	57.063	52.137	18.546	118.402	2.114	1.946	0.688	4.340	48.3
	correlation				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	1.000	1.000	0.999	1.000	3530.3	3414.6	1699.3	6012.5	100.0
d1	1.000	1.000	1.000	1.000	34900.9	34825.4	28394.9	41823.6	100.0
d2	1.000	1.000	1.000	1.000	25561.6	25456.8	20925.9	30661.0	100.0
d3	1.000	1.000	1.000	1.000	15851.2	15817.9	12085.4	19752.6	100.0
d4	1.000	1.000	0.999	1.000	8359.6	8339.6	5335.8	11627.9	100.0
	regression coef.				t-statistic				
level	mean	median	95% c.i.		mean	median	95% c.i.		% reject
d0	60.657	60.643	59.476	61.979	92.036	91.697	75.521	110.554	100.0
d1	60.599	60.597	60.388	60.804	581.07	578.83	507.52	669.63	100.0
d2	60.605	60.599	60.319	60.908	431.94	430.82	376.73	497.69	100.0
d3	60.611	60.608	60.199	61.060	280.98	280.56	239.51	325.12	100.0
d4	60.595	60.589	59.901	61.315	165.38	164.88	140.26	194.53	100.0