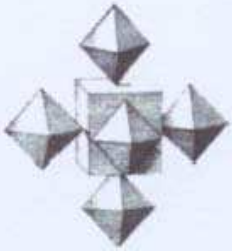

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COALITIONAL GAMES WITH VETO PLAYERS: MYOPIC AND FARSIGHTED BEHAVIOR

by

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Coalitional games with veto players: myopic and farsighted behavior

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Abstract

This paper studies an allocation procedure for coalitional games with veto players. The procedure is similar to the one presented by Dagan et al. (1997) for bankruptcy problems. According to it, a player, the proposer, makes a proposal that the remaining players must accept or reject, and conflict is solved bilaterally between the rejector and the proposer. We allow the proposer to make sequential proposals over several periods. If responders are myopic maximizers (i.e. consider each period in isolation), the only equilibrium outcome is the serial rule of Arin and Feltkamp (2012) regardless of the order of moves. If all players are farsighted, the serial rule still arises as the unique subgame perfect equilibrium outcome if the order of moves is such that stronger players respond to the proposal after weaker ones.

Keywords: veto players, bargaining, myopic behavior, serial rule.
JEL classification: C71, C72, C78, D70.

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1 Introduction

Dagan et al. (1997) introduced a noncooperative bargaining procedure for bankruptcy problems. In this procedure the player with the highest claim has a distinguished role. He makes a proposal and the remaining players accept or reject sequentially. Players who accept the proposal leave the game with their share; if a player rejects the proposal this conflict is solved bilaterally by applying a normative solution concept (a "bilateral principle" based on a bankruptcy rule) to a two-claimant bankruptcy problem in which the estate is the sum of the two proposed payoffs. They show that a large class of consistent and monotone bankruptcy rules can be obtained as the Nash equilibrium outcomes of the game. They describe this kind of procedure as consistency based: starting from a consistent solution concept, they construct extensive forms whose subgames relate to the respective reduced cooperative games and by finding the equilibrium of the extensive form they are able to provide noncooperative foundations for the consistent solution of interest.

The model above can be extended to other bargaining situations in the following way. Suppose we have a multilateral bargaining situation with one distinguished player (the most senior creditor in the bankruptcy case, the chair of a committee, the manager of a firm...). The distinguished player negotiates bilaterally with each of the other players. Negotiations are constrained by a fairness or justice principle that is commonly accepted in society and can be enforced (possibly by an external court). Players are assumed to be selfish, hence they only appeal to this principle when it is in their material interest to do so. To what extent does the bilateral principle determine the global agreement? In Dagan et al. (1997) the answer is that the bilateral principle completely determines the outcome: if a particular bankruptcy rule can be enforced in the two-player situation, the outcome is the same bankruptcy rule applied to the case of n creditors.

Dagan et al.'s paper focuses on bankruptcy games, hence their justice principles are also restricted to this class. The question arises of what the appropriate justice principle should be for general TU games. In this paper we use the (restricted) standard solution of a reduced game between the two players. The idea behind this principle is that each of the two players gains

(or loses) the same amount with respect to an alternative situation in which the two players cannot cooperate with each other (unless this would result in a negative payoff for one of the players, in which case this player gets zero).

Using this bilateral principle, Arin and Feltkamp (2007) studied the bargaining procedure in another class of games with a distinguished player, namely games with a veto player. A veto player is a player whose cooperation is essential in order for a coalition to generate value. Games with a veto player arise naturally in economic applications. Examples include a production economy with one landowner and many landless peasants (Shapley and Shubik (1967)), an innovator trading information about a technological innovation with several producers (Muto (1986), Muto et al. (1989), Driessen et al. (1992)) and hierarchical situations where a top player's permission is necessary in order for a project to be developed (Gilles et al. 1992). Arin and Feltkamp (2007) found that the equilibrium of this bargaining procedure is not always efficient: the proposer may be strictly better-off by proposing an allocation that does not exhaust the total available payoff.

In the present paper, we modify the above procedure by allowing the proposer to make a fixed number of sequential proposals, so that players can continue bargaining over the remainder if the first proposal did not exhaust the value of the grand coalition. Each period results in a partial agreement, and then a new TU game is constructed where the values of the coalitions take into account the agreements reached so far; the final outcome is the sum of all partial agreements. We assume that the number of available bargaining periods T is at least as large as the number of players n. In order to analyze this multiperiod game, we start by a simplified model in which responders behave myopically, that is, we initially assume that responders consider each period in isolation, accepting or rejecting the current proposal without anticipating the effects of their decision on future periods. The proposer is assumed to behave farsightedly, taking into account the effect of his actions on future periods and also taking into account that the responders behave myopically. We refer to this kind of strategy profile as a myopic best response equilibrium.

It turns out that all myopic best response equilibria are efficient and lead

to the same outcome, which is the serial rule of Arin and Feltkamp (2012). This solution concept is based on the idea that the strength of player i can be measured by the maximum amount a coalition can obtain without player i, denoted by d_i. Since it is impossible for any coalition to obtain a payoff above d_i without i's cooperation, player i can be viewed as having a veto right over $v(N) - d_i$. The serial rule divides $v(N)$ into segments, and each segment is equally divided between the players that have a veto right over it.

We then turn to the analysis of subgame perfect equilibrium outcomes and show that they may differ from the serial rule. The order of moves may be such that the proposer is able to hide some payoff from a stronger player with the cooperation of a weaker player: the proposal faced by the stronger player is not too favorable for the proposer so that the stronger player cannot challenge it, but later on a weak player rejects the proposal and transfers some payoff to the proposer; the weak player may have an incentive to do so because of the effect of this agreement on future periods. However, if the order of moves is such that stronger players have the last word in the sense that they respond to the proposal after weaker ones, the only subgame perfect equilibrium outcome is the serial rule. Hence, myopic and farsighted behavior of the responders lead to the same outcome in this case.

2 Preliminaries

2.1 TU games

A *cooperative n-person game in characteristic function form* is a pair (N, v), where N is a finite set of n elements and $v : 2^N \to \mathbb{R}$ is a real-valued function on the family 2^N of all subsets of N with $v(\emptyset) = 0$. Elements of N are called *players* and the real-valued function v the *characteristic function* of the game. We shall often identify the game (N, v) with its characteristic function and write v instead of (N, v). Any subset S of the player set N is called a *coalition*. The number of players in a coalition S is denoted by $|S|$. In this work we will only consider games where all coalitions have nonnegative

worth and the grand coalition is efficient, that is, $0 \leq v(S) \leq v(N)$ for all $S \subset N$.

A payoff allocation is represented by a vector $x \in \mathbb{R}^n$, where x_i is the payoff assigned by x to player i. We denote $\sum_{i \in S} x_i$ by $x(S)$. If $x(N) \leq v(N)$, x is called a *feasible allocation;* if $x(N) = v(N)$, x is called an *efficient allocation*. An efficient allocation satisfying $x_i \geq v(i)$ for all $i \in N$ is called an *imputation* and the set of imputations is denoted by $I(N, v)$. The set of nonnegative feasible allocations is denoted by $D(N, v)$ and formally defined as follows

$$D(N, v) := \left\{ x \in \mathbb{R}^N : x(N) \leq v(N) \text{ and } x_i \geq 0 \text{ for all } i \in N \right\}.$$

A *solution* ϕ on a class of games Γ is a correspondence that associates with every game (N, v) in Γ a set $\phi(N, v)$ in $\mathbb{R}^N$ such that $x(N) \leq v(N)$ for all $x \in \phi(N, v)$. This solution is called *efficient* if this inequality holds with equality. The solution is called *single-valued* if it contains a unique element for every game in the class. A single-valued solution ϕ satisfies the *aggregate monotonicity* property (Meggido, 1974) on a class of games Γ if the following holds: for all $v, w \in \Gamma$ such that $v(S) = w(S)$ for all $S \neq N$ and $v(N) < w(N)$, then $\phi_i(v) \leq \phi_i(w)$ for all $i \in N$. Increasing the value of the grand coalition never leads to a payoff decrease for any of the players.

The *core* of a game is the set of imputations that cannot be blocked by any coalition, i.e.

$$C(N, v) := \{ x \in I(v) : x(S) \geq v(S) \text{ for all } S \subseteq N \}.$$

A game with a nonempty core is called a *balanced game*. A player i is a *veto player* if $v(S) = 0$ for all coalitions where player i is not present. A game v is a *veto-rich game* if it has at least one veto player and the set of imputations is nonempty. A balanced game with at least one veto player is called a *veto balanced game*. Note that balancedness is a relatively weak property for games with a veto player, since it only requires $v(N) \geq v(S)$ for all $S \subset N$.

Given a game (N, v) and a feasible allocation x, the *excess of a coalition* S *at* x is defined as $e(S, x) := v(S) - x(S)$. Its mirror concept, the *satisfaction of a coalition* S *at* x, is defined as $f(S, x) := x(S) - v(S)$. We define $f_{ij}(x, (N, v))$ as the minimum satisfaction of a coalition that contains i but not j.

$$f_{ij}(x, (N, v)) := \min_{S: i \in S \subseteq N \setminus \{j\}} \{x(S) - v(S)\}.$$

If there is no confusion we write $f_{ij}(x)$ instead of $f_{ij}(x, (N, v))$. The higher $f_{ij}(x)$, the better i is treated by the allocation x in comparison with j. The *kernel* can be defined as the set of imputations that satisfy the following *bilateral kernel conditions*:

$$f_{ji}(x) > f_{ij}(x) \text{ implies } x_j = v(j) \text{ for all } i, j \text{ in } N.$$

Note that, if j is a veto player, $f_{ij}(x) = x_i$.[1]

Let $\theta(x)$ be the vector of all excesses at x arranged in non-increasing order of magnitude. The lexicographic order $\prec_L$ between two vectors x and y is defined by $x \prec_L y$ if there exists an index k such that $x_l = y_l$ for all $l < k$ and $x_k < y_k$ and the weak lexicographic order $\preceq_L$ by $x \preceq_L y$ if $x \prec_L y$ or $x = y$. Schmeidler (1969) introduced the *nucleolus* of a game v, denoted by $\nu(N, v)$, as the imputation that lexicographically minimizes the vector of non-increasingly ordered excesses over the set of imputations. In formula:

$$\{\nu(N, v)\} := \{x \in I(N, v) \,|\theta(x) \preceq_L \theta(y) \text{ for all } y \in I(N, v)\}.$$

For any game v with a nonempty imputation set, the nucleolus is a single-valued solution, is contained in the kernel and lies in the core provided that the core is nonempty. The kernel and the nucleolus coincide for veto rich games (see Arin and Feltkamp (1997)).

[1] An equivalent definition of the kernel is based on the mirror concept of f_{ij}, which is the surplus of i against j at x (terminology of Maschler, 1992), $s_{ij}(x) := \max_{S: i \in S \subseteq N \setminus \{j\}} \{v(S) - x(S)\}$. The kernel is the set of imputations such that $s_{ij}(x) > s_{ji}(x)$ implies $x_j = v(j)$. We found it more convenient to work with $f_{ij}(.)$ rather than $s_{ij}(.)$.

2.2 One-period bargaining (Arin and Feltkamp, 2007)

Given a veto balanced game (N, v) where player 1 is a veto player and an order on the set of the remaining players, we will define an extensive-form game associated to the TU game and denote it by $G(N, v)$. The game has n stages and in each stage only one player takes an action. In the first stage, a veto player announces a proposal x^1 that belongs to the set of feasible and nonnegative allocations of the game (N, v). In the next stages the responders accept or reject sequentially. If a player, say i, accepts the proposal x^{s-1} at stage s, he leaves the game with the payoff x_i^{s-1} and for the next stage the proposal x^s coincides with the proposal at $s-1$, that is x^{s-1}. If player i rejects the proposal, a two-person TU game is constructed with the proposer and player i. In this two-person game the value of the grand coalition is $x_1^{s-1} + x_i^{s-1}$ and the value of the singletons is obtained by applying the Davis-Maschler reduced game[2] (Davis and Maschler (1965)) given the game (N, v) and the allocation x^{s-1}. Player i will receive as payoff the restricted *standard solution* of this two-person game[3]. Once all the responders have played and consequently have received their payoffs the payoff of the proposer is also determined as x_1^n.

Formally, the resulting outcome of playing the game can be described by the following algorithm.

[2]Let (N, v) be a game, T a subset of N such that $T \neq N, \emptyset$, and x a feasible allocation. Then the *Davis-Maschler reduced game* with respect to $N \setminus T$ and x is the game $(N \setminus T, v_x^{N \setminus T})$ where

$$v_x^{N \setminus T}(S) := \begin{cases} 0 & \text{if } S = \emptyset \\ v(N) - x(T) & if \quad S = N \setminus T \\ \max\limits_{Q \subseteq T} \{v(S \cup Q) - x(Q)\} & \text{for all other } S \subset N \setminus T\,. \end{cases}$$

Note that we have defined a modified Davis-Maschler reduced game where the value of the grand coalition of the reduced game is $x(N \setminus T)$ instead of $v(N) - x(T)$. If x is efficient both reduced games coincide. See also Peleg (1986).

[3]The standard solution of a two-person TU game v gives player $i = 1, 2$ the amount $v(i) + \frac{v(1,2) - v(i) - v(j)}{2}$. The restricted standard solution coincides with the standard solution except when the standard solution gives a negative payoff to one of the players, in which case this player receives 0 and the other player receives $v(1, 2)$.

Input : a veto balanced game (N, v) with a veto player, player 1, and an order on the set of remaining players (responders).
Output : a feasible and nonnegative allocation $x^n(N, v)$.

1. Start with stage 1. Player 1 makes a feasible and nonnegative proposal x^1 (not necessarily an imputation). The superscript denotes at which stage the allocation emerges as the proposal in force.

2. In the next stage the first responder (say, player 2) says yes or no to the proposal. If he says yes he receives the payoff x_2^1, leaves the game, and $x^2 = x^1$.

   If he says no he receives the payoff[4]

$$y_2 = \max\left\{0, \frac{1}{2}\left[x_1^1 + x_2^1 - v_{x^1}(1)\right]\right\} \text{ where}$$

$$v_{x^1}(1) := \max_{1 \in S \subseteq N\setminus\{2\}} \left\{v(S) - x^1(S\setminus\{1\})\right\}$$

$$\text{Now, } x_i^2 = \begin{cases} x_1^1 + x_2^1 - y_2 & \text{for player 1} \\ y_2 & \text{for player 2} \\ x_i^1 & \text{if } i \neq 1, 2. \end{cases}$$

3. Let the stage s where responder k plays, given the allocation x^{s-1}. If he says yes he receives the payoff x_k^{s-1}, leaves the game, and $x^s = x^{s-1}$.

   If he says no he receives the payoff

$$y_k = \max\left\{0, \frac{1}{2}\left[x_1^{s-1} + x_k^{s-1} - v_{x^{s-1}}(1)\right]\right\} \text{ where}$$

$$v_{x^{s-1}}(1) = \max_{1 \in S \subseteq N\setminus\{k\}} \left\{v(S) - x^{s-1}(S\setminus\{1\})\right\}.$$

$$\text{Now, } x_i^s = \begin{cases} x_1^{s-1} + x_k^{s-1} - y_k & \text{for player 1} \\ y_k & \text{for player } k \\ x_i^{s-1} & \text{if } i \neq 1, k \end{cases}.$$

[4]Note that, since 1 is a veto player, $v_{x^s}(i) = 0$ for any proposal x^s and any player $i \neq 1$.

4. The game ends when stage n is played and we define $x^n(N,v)$ as the vector with coordinates $\left(x_j^n\right)_{j\in N}$.

In this game we assume that the conflict between the proposer and a responder is solved bilaterally. In the event of conflict, the players face a two-person TU game that shows the strength of each player given that the rest of the responders are passive. Once the game is formed the allocation proposed for the game is a normative proposal, a kind of restricted *standard solution*[5].

The set of pure strategies in this game is relatively simple. Player 1's strategy consists of the initial proposal x^1, which must be feasible and non-negative. A pure strategy for the responder who moves at stage s is a function that assigns "yes" or "no" to each possible proposal x^{s-1} and each possible history of play. Players are assumed to be selfish, hence player i seeks to maximize x_i^n.

2.3 Nash equilibrium outcomes of the one-period game

The set of *bilaterally balanced allocations for player* i is

$$F_i(N,v) := \{x \in D(N,v) : f_{ji}(x) \geq f_{ij}(x) \text{ for all } j \neq i\}$$

while the *set of optimal allocations for player* i in the set $F_i(N,v)$ is defined as follows:

$$B_i(N,v) := \underset{x\in F_i(N,v)}{\arg\max}\ x_i.$$

In the class of veto-balanced games, $F_i(N,v)$ is a nonempty and compact set for all i, thus the set $B_i(N,v)$ is nonempty.

Theorem 1 *(Arin and Feltkamp, 2007) Let (N,v) be a veto balanced TU game and let $G(N,v)$ be its associated extensive form game. Let z be a feasible and nonnegative allocation. Then z is a Nash equilibrium outcome if and only if $z \in B_1(N,v)$.*

[5] In some sense the game is a hybrid of non-cooperative and cooperative games, since the outcome in case of conflict is not obtained as the equilibrium of a non-cooperative game.

The idea behind this result is the following. As shown in Arin and Feltkamp (2007), the restricted standard solution that is applied if player i rejects a proposal in stage s results in $f_{1i}(x^s) = f_{i1}(x^s)$, unless this would mean a negative payoff for player i, in which case $f_{i1}(x^s) > f_{1i}(x^s)$ and $x_i^s = 0$. Hence, rejection of a proposal leads to a payoff redistribution between 1 and i until the bilateral kernel condition is satisfied between the two players. It is in player i's interest to reject any proposal with $f_{1i}(x^{s-1}) > f_{i1}(x^{s-1})$ and to accept all other proposals. Since player i rejects proposals with $f_{1i}(x^{s-1}) > f_{i1}(x^{s-1})$ and this rejection results in $f_{1i}(x^s) = f_{i1}(x^s)$, the proposal in force after i has the move always satisfies $f_{i1}(x^s) \geq f_{1i}(x^s)$. Subsequent actions by players moving after i can only reduce $f_{1i}(.)$, hence $f_{i1}(x^n) \geq f_{1i}(x^n)$. Conversely, player 1 can achieve any bilaterally balanced payoff vector by proposing it. Player 1 then maximizes his own payoff under the constraint that the final allocation has to be bilaterally balanced.

The nucleolus is a natural candidate to be an equilibrium outcome since it satisfies the Davis-Maschler reduced game property, and indeed the nucleolus is always in $F_1(N, v)$. However, the elements of $B_1(N, v)$ are not necessarily efficient. Furthermore, there are cases in which the set $B_1(N, v)$ contains no efficient allocations. The existence of an efficient equilibrium is not guaranteed because the nucleolus does not satisfy aggregate monotonicity for the class of veto balanced games. If (N, v) is such that decreasing the value of the grand coalition (keeping the values of other coalitions constant) never increases the nucleolus payoff for player 1, the nucleolus of the game is a Nash equilibrium outcome (Arin and Feltkamp, 2007, theorem 13).

As shown by Dagan et al. (1996) for bankruptcy games and Arin and Feltkamp (2007) for veto balanced games, the set of Nash equilibrium (NE) outcomes and the set of subgame perfect equilibrium (SPE) outcomes coincide for this bargaining procedure. This contrasts sharply with bargaining situations as simple as the ultimatum game (see Güth et al. 1982), which has a unique SPE outcome but a continuum of NE outcomes. The reason for this difference is the unavailability of incredible threats: in the ultimatum game the responder can take actions that hurt both himself and the proposer, but here any action that hurts the responder would benefit the proposer.

3 A new game: sequential proposals

3.1 The model

We extend the previous model to T periods where T is assumed to be at least as large as the number of players n. The proposer can now make T sequential proposals, and each proposal is answered by the responders as in the previous model. We will denote a generic period as t and a generic stage as s. The proposal that emerges at the end of period t and stage s is denoted by $x^{t,s}$, and the proposal that emerges at the end of period t is denoted by $x^t := x^{t,n}$. Given a veto balanced game with a proposer and an order on the set of responders we will construct an extensive form game, denoted by $G^T(N, v)$.

Formally, the resulting outcome of playing the game can be described by the following algorithm.

Input : a veto balanced game (N, v) with a veto player, player 1, as proposer, and an order on the set of the remaining players (responders) which may be different for different periods.

Output : a feasible and nonnegative allocation x.

1. Start with period 1. Given a veto balanced TU game (N, v) and the order on the set of responders corresponding to period 1, players play the game $G(N, v)$. The outcome of this period determines the veto balanced TU game for the second period, denoted by (N, v^{2,x^1}), where $v^{2,x^1}(S) := \max\{0, \min\{v(N) - x^1(N), v(S) - x^1(S)\}\}$ and x^1 is the final outcome obtained in the first period. Note that by construction, the game (N, v^{2,x^1}) is a veto balanced game where player 1 is a veto player. Then go to the next step. The superscripts in the characteristic function denote at which period and after which outcome the game is considered as the game in force. If no confusion arises we write v^2 instead of v^{2,x^1}.

2. Let the period be t ($t \leq T$) and the TU game $(N, v^{t,x^{t-1}})$. We play the game $G(N, v^{t,x^{t-1}})$ and define the veto balanced TU game (N, v^{t+1,x^t})

where $v^{t+1,x^t}(S) := \max\{0, \min\{v^t(N) - x^t(N), v^t(S) - x^t(S)\}\}$ and x^t is the final outcome obtained in period t. Then go to the next step.

3. The game ends after stage n of period T. (If at some period before T the proposer makes an efficient proposal (efficient according to the TU game underlying at this period) the game is trivial for the rest of the periods).

4. The outcome is the sum of the outcomes generated at each period, that is, $x := \sum_{t=1}^{T} x^t$.

3.2 A serial rule for veto balanced games

We now introduce a solution concept defined on the class of veto balanced games and denoted by ϕ. Somewhat surprisingly, this solution will be related to the non-cooperative game with sequential proposals.

Let (N, v) be a veto balanced game where player 1 is a veto player. Define for each player i a value d_i as follows:

$$d_i := \max_{S \subseteq N \setminus \{i\}} v(S).$$

Because 1 is a veto player, $d_1 = 0$. Let $d_{n+1} := v(N)$ and rename the remaining players according to the nondecreasing order of those values. That is, player 2 is the player with the lowest value and so on. The solution ϕ associates to each veto balanced game, (N, v), the following payoff vector:

$$\phi_l := \sum_{i=l}^{n} \frac{d_{i+1} - d_i}{i} \text{ for all } l \in \{1, ..., n\}.$$

The following example illustrates how the solution behaves.

Example 1 *Let $N = \{1, 2, 3\}$ be a set of players and consider the following 3-person veto balanced game (N, v) where*

$$v(S) = \begin{cases} 50 & \text{if } S = \{1, 2\} \\ 10 & \text{if } S = \{1, 3\} \\ 80 & \text{if } S = N \\ 0 & \text{otherwise.} \end{cases}$$

Computing the vector of d-values we get:

$$(d_1, d_2, d_3, d_4) = (0, 10, 50, 80).$$

Applying the formula,

$$\begin{array}{lllll} \phi_1 = & \frac{d_2-d_1}{1} + & \frac{d_3-d_2}{2} + & \frac{d_4-d_3}{3} & = 40 \\ \phi_2 = & & \frac{d_3-d_2}{2} + & \frac{d_4-d_3}{3} & = 30 \\ \phi_3 = & & & \frac{d_4-d_3}{3} & = 10 \end{array}$$

The formula suggests a serial rule principle (cf. Moulin and Shenker, 1992). Since it is not possible for any coalition to obtain a payoff above d_i without player i's cooperation, we can view player i as *having a right* over the amount $v(N) - d_i$. The value $v(N)$ is divided into segments ($d_2 - d_1$, $d_3 - d_2, ..., v(N) - d_n$) and each payoff segment is divided equally among the players that have a right over it.

In the class of veto balanced games, the solution ϕ satisfies some well-known properties such as nonemptiness, efficiency, anonymity and equal treatment of equals among others. It also satisfies aggregate monotonicity.[6]

The next section shows that $\phi(N, v)$ is the unique equilibrium outcome assuming that all responders act as myopic maximizers and the proposer plays optimally taking this into account.

3.3 Myopic Best Response Equilibrium

We start our analysis of the non-cooperative game with sequential proposals by assuming *myopic behavior* on the part of responders. Responders behave myopically when they act as payoff maximizers in each period without considering the effect of their actions on future periods.

Suppose all responders maximize payoffs myopically for each period and that the proposer plays optimally taking into account that the responders

[6]For a definition of those properties, see Peleg and Sudhölter (2003). It is not the aim of this paper to provide an axiomatic analysis of the solution. Arin and Feltkamp (2012) characterize the solution in the domain of veto balanced games by core selection and a monotonicity property.

are myopic maximizers. Formally, player $i \neq 1$ maximizes x_i^t at each period t whereas player 1 maximizes $\sum_{t=1}^{T} x_1^t$. We call such a strategy profile a *myopic best response equilibrium* (MBRE). We will show in this section that all MBRE lead to the same outcome, namely the serial rule.

3.3.1 MBRE and balanced proposals

The notion of *balanced proposals* will play a central role in the analysis of MBRE.

Definition 1 *Let (N, v) be a veto balanced TU game, and $G^T(N, v)$ its associated extensive form game. Given a period t, a proposal x is balanced if it is the final outcome of period t regardless of the actions of the responders.*

We will start by showing that any payoff the proposer can attain under myopic behavior of the responders can also be attained by making balanced proposals: player 1 can cut the payoff of other players until a balanced proposal is obtained at no cost to himself (lemma 2). Hence, from the proposer's point of view there is no loss of generality in restricting the analysis to balanced proposals. We will then show that the highest payoff the proposer can achieve with balanced proposals is ϕ_1. Finally, we will show that the only way in which the proposer can achieve ϕ_1 requires *all* players to get their component of the serial rule, so that the only MBRE outcome is $\phi(N, v)$.

If there is only one period in the game, myopic and farsighted behavior coincide. This means that the following lemma holds if responders behave myopically.

Lemma 1 *(Arin and Feltkamp, 2007, lemmas 2 and 3) Let (N, v) be a veto balanced TU game, and $G^T(N, v)$ its associated extensive form game. At any period t and stage s, the responder (say, i) will accept $x^{t,s-1}$ if $f_{i1}(x^{t,s-1}; v^t) > f_{1i}(x^{t,s-1}; v^t)$, and will reject it if $f_{i1}(x^{t,s-1}; v^t) < f_{1i}(x^{t,s-1}; v^t)$ in a MBRE. If $f_{i1}(x^{t,s-1}; v^t) = f_{1i}(x^{t,s-1}; v^t)$, the responder is indifferent between accepting and rejecting since both decisions lead to the same outcome. Also, the final outcome x^t of any period t is such that $f_{i1}(x^t; v^t) \geq f_{1i}(x^t; v^t)$ for all i.*

We have established that myopic behavior of the responders leads to $f_{i1}(x^t; v^t) \geq f_{1i}(x^t; v^t)$, or equivalently to $x_i^t \geq f_{1i}(x^t, v^t)$. We now show that the proposer can obtain the same payoff with balanced proposals in all such cases.

Lemma 2 *Let (N, v) be a veto balanced game. Consider the associated game with T periods $G^T(N, v)$. Let $z = \sum_1^T x^t$ be an outcome resulting from some strategy profile. Assume that the final outcome of any period t, x^t, is such that for any player i, $x_i^t \geq f_{1i}(x^t, v^t)$. Then there exists y such that $y_1 = z_1$, $y = \sum_1^T q^t$ where q^t is a balanced proposal for period t.*

Proof. If $(x^1, x^2, ..., x^T)$ is a sequence of balanced proposals the proof is done.

Suppose that $(x^1, x^2, ..., x^T)$ is not a sequence of balanced proposals. This means that for some x^t and for some $i \neq 1$ it holds that $x_i^t > f_{1i}(x^t, v^t)$ and $x_i^t > 0$. Let k be the first period where such result holds. Therefore, $(x^1, x^2, ..., x^{k-1})$ is a sequence of balanced proposals. We will construct a *balanced proposal* where the payoff of the proposer does not change.

Since $f_{i1}(x^k) = x_i^k$, by reducing the payoff of player i we can construct a new allocation y^k such that $f_{1i}(y^k) = f_{i1}(y^k)$ or $f_{1i}(y^k) < f_{i1}(y^k)$ and $y_i^k = 0$. In any case, $x_1^k = y_1^k$ and the payoff of the proposer does not change. Note also that reducing i's payoff can only lower $f_{1j}(y^k)$, so it is still the case that $f_{1j}(y^k) \leq f_{j1}(y^k)$ for all j.

Now, if there exists another player l such that $f_{1l}(y^k) < f_{l1}(y^k)$ and $y_l^k > 0$ we construct a new allocation z^k such that $f_{1l}(z^k) = f_{l1}(z^k)$ or $f_{1i}(z^k) < f_{i1}(z^k)$ and $z_i^k = 0$. Note that $z_1^k = y_1^k$. Repeating this procedure we will end up with a balanced allocation q^k.

The TU game (N, w^{k+1}) resulting after proposing q^k satisfies that $w^{k+1}(S) \geq v^{k+1}(S)$ for all $S \ni 1$. Therefore, $f_{1i}(x, w^{k+1}) \leq f_{1i}(x, v^{k+1})$ for any feasible allocation x, and $x_l^{k+1} \geq f_{1l}(x^{k+1}, w^{k+1})$ for all l.

Consider the game (N, w^{k+1}) and the payoff x^{k+1}. Suppose that $x_i^{k+1} > f_{1i}(x^{k+1})$ for some $i \neq 1$ and $x_i^{k+1} > 0$. We can repeat the same procedure of period k until we obtain a balanced allocation q^{k+1}. The procedure can be repeated until the last period of the game to obtain the sequence of balanced proposals $(x^1, x^2, ..., x^{k-1}, q^k, ..., q^T)$. ■

Some interesting properties of balanced proposals:

Lemma 3 *If x^t is a balanced proposal, any player i with $x_i^t > 0$ will be a veto player at $t+1$.*

This is because if x^t is balanced we have $f_{1i}(x^t, v^t) = x_i^t$, so that all coalitions that have a positive v^t but do not involve i have $v^t(S) < x^t(S)$. Thus, after the payoffs x^t are distributed any coalition with positive value must involve i. Note that this result requires x^t to be a balanced proposal and not merely the outcome of a MBRE. In a MBRE it may be the case that $f_{1i}(x^t, v^t) < x_i^t$, and we cannot conclude anything about the sign of $f_{1i}(x^t, v^t)$.

Balanced proposals coincide with the nucleolus (kernel) of special games. In the class of veto-rich games (games with at least one veto player and a nonempty set of imputations) the kernel and the nucleolus coincide (Arin and Feltkamp, 1997). Therefore we can define the nucleolus as

$$\nu(N,v) := \{x \in I(N,v) : \ f_{ij}(x) < f_{ji}(x) \Longrightarrow x_j = 0\}.$$

We use this alternative definition of the nucleolus in the proof of the following lemma.

Lemma 4 *Let (N,v) be a veto balanced TU game. Consider the associated game $G^T(N,v)$. Given a period t, a proposal x^t is balanced if and only if it coincides with the nucleolus of the game (N,w^t), where $w^t(S) = v^t(S)$ for all $S \neq N$ and $w^t(N) = x^t(N)$.*

Proof. Assume that x^t is a balanced proposal in period t with the game (N,v^t).

a) Let l be a responder for which $x_l^t = 0$. If whatever the response of player l the proposal does not change then $f_{1l}(x^t) \leq 0 = x_l^t = f_{l1}(x^t)$.

b) Let m be a responder for which $x_m^t > 0$. If whatever the response of player m the proposal does not change then $f_{1m}(x^t) = x_m^t = f_{m1}(x^t)$.

Therefore, the bilateral kernel conditions are satisfied for the veto player. Lemma 12 in Arin and Feltkamp (2007) shows that if the bilateral kernel

conditions are satisfied between the veto player and the rest of the players then the bilateral kernel conditions are satisfied between any pair of players.

Therefore, x^t is the kernel (nucleolus) of the game (N, w^t). The converse statement can be proven in the same way. ■

3.3.2 The serial rule can be achieved with balanced proposals

We now show that, by making balanced proposals, the proposer can secure the payoff provided by the serial rule ϕ.

Lemma 5 *Let (N, v) be a veto balanced TU game and $G^T(N, v)$ its associated extensive form game with $T \geq n$. The proposer has a sequence of balanced proposals that leads to $\phi(N, v)$.*

Proof. The sequence consists of n balanced proposals. At each period t, $(t \in \{1, ..., n\})$ consider the set $S_t = \{l : l \leq t\}$ and the proposal x^t, defined as follows:

$$x_l^t = \begin{cases} \frac{d_{t+1} - d_t}{t} & \text{for all } l \in S_t \\ 0 & \text{otherwise.} \end{cases}$$

whenever x^t is feasible and propose the 0 vector otherwise.

It can be checked immediately that in each period the proposed allocation will be the final allocation independently of the answers of the responders and independently of the order of those answers. The proposals are balanced proposals.

For example, in period 1, the proposal is $(d_2, 0, ..., 0)$. Because 1 is a veto player, $f_{i1}(.) = 0$ for all i. As for $f_{1i}(.)$, because all players other than 1 are getting 0, the coalition of minimum satisfaction of 1 against i is also the coalition of maximum $v(S)$ with $i \notin S$. Call this coalition S^*. By definition, $v(S^*) = d_i \geq d_2$ and $f_{1i}(.) = x(S^*) - v(S^*) = d_2 - d_i \leq 0$. Thus, $f_{i1}(.) \geq f_{1i}(.)$ for all i and the outcome of period 1 is $(d_2, 0, ..., 0)$ regardless of responders' behavior.

In period 2 we have a game v^2 with the property that $v^2(S) > 0$ implies $v^2(S) = v^1(S) - d_2$ for all S. Thus, player 2 is a veto player in v^2. Player 1 proposes $\left(\frac{d_3 - d_2}{2}, \frac{d_3 - d_2}{2}, 0, ..., 0\right)$. If player 2 rejects, we have $f_{12}(.) = \frac{d_3 - d_2}{2} -$

$0 = f_{21}(.)$. As for other players $i \neq 1, 2$, when computing f_{1i} we take into account that any coalition of positive value must include 1 and 2. Since players other than 1 and 2 are getting 0, the coalition 1 uses against i is $S^* \in \arg\max_{S: i \notin S} v(S)$. By definition, $v(S^*) = d_i$ and $v^2(S^*) = d_i - d_2$. Then $f_{1i}(.) = x(S^*) - v^2(S^*) = (d_3 - d_2) - (d_i - d_2) = d_3 - d_i \leq 0$.

In period 3, player 3 has become a veto player and the same process can be iterated until period n.

Therefore, this strategy of the proposer determines the total payoff of all the players, that is, the final outcome of the game $G^T(N, v)$. This final outcome coincides with the solution ϕ. ■

Remark 1 *The serial rule can also be obtained by making balanced proposals if the game has $n-1$ periods.*

This is because the proposer can combine the first two proposals in the proof of lemma 5 by proposing $(d_2 + \frac{d_3 - d_2}{2}, \frac{d_3 - d_2}{2}, 0, ..., 0)$ in the first period.

This proof, together with lemma 4, suggests a new interpretation of the serial rule. At each period the proposal coincides with the nucleolus of a veto-rich game. Formally,

Remark 2 *The serial rule is the sum of the nucleolus allocations of n auxiliary games, namely*

$$\phi(N, v) = \sum_{i=1}^{n} \nu(N, w^i)$$

where the games (N, w^t) are defined as follows: $w^1(N) = d_2$ and $w^1(S) = v(S)$ otherwise. For $i : 2, .., n$:

$$w^i(S) := \begin{cases} d_{i+1} - d_i & \text{if } S = N \\ \max\left\{0, w^{i-1}(S) - \sum_{l \in S} \nu_l(N, w^{i-1})\right\} & \text{otherwise.} \end{cases}.$$

3.3.3 The proposer cannot improve upon the serial rule

Theorem 2 *Let (N, v) be a veto balanced TU game and $G^T(N, v)$ the associated extensive form game with $T \geq n$. Let $z = \sum_1^T x^t$ be an outcome resulting from a MBRE of $G^T(N, v)$. Then $z = \phi(N, v)$.*

We have already shown that $\phi(N,v)$ can be achieved with balanced proposals. We now show that the proposer cannot improve upon ϕ. Let $z=\sum_1^T x^t$ be an outcome resulting from balanced proposals. Our objective is to show that $z_1 \geq \phi_1$ implies $z_i \geq \phi_i$ for all i. This result, together with the efficiency of the serial rule, leads to $z=\phi$ being the unique MBRE outcome. We start by establishing the result not for the original game (N,v), but for the sequence of auxiliary games (N,w^t) (lemma 8). We then check that the sum of the serial rules of the games w^t cannot exceed the serial rule of the original game (N,v) (lemma 9).

The following lemma establishes a relationship between balanced proposals in $G^T(N,v)$ and the serial rule. Suppose x^t is a balanced proposal in period t. Consider the game w^t, where $w^t(S)=\min\{v^t(S),x^t(N)\}$. The serial rule of w^t and the balanced proposal x^t do not coincide in general. However, the set of players who receive a positive payoff in x^t coincides with the set of players who receive a positive payoff according to the serial rule of w^t.[7]

Lemma 6 *Let (N,v) be a veto balanced TU game. Consider the associated game $G^T(N,v)$. Let $z=\sum_1^T x^t$ be the outcome resulting from some strategy profile with balanced proposals. Consider period t, its outcome x^t and the game (N,w^t) where $w^t(S)=\min\{v^t(S),x^t(N)\}$. Then it holds that $x_k^t>0$ if and only if $\phi_k(N,w^t)>0$.*

Proof. a) If $x_k^t>0$ we need to prove that $d_k(N,w^t)<x^t(N)$, so that the serial rule of w^t assigns a positive payoff to k.

Let $S\in \arg\max_{T\subseteq N\setminus\{k\}} v^t(T)$. Since x^t is balanced we have $f_{1k}(x^t)=x_k^t>0$ and that implies $x^t(S)>v^t(S)$ (otherwise S could have been used

[7]For example, consider the game with $N=\{1,2,3,4\}$, $v(1,2)=v(1,3)=2$, $v(1,2,3)=6$, $v(1,2,3,4)=10$ and $v(S)=0$ otherwise. The proposal $x=(2,1.5,1.5,0)$ is a balanced proposal with a total payoff distributed of 5 (and, because of lemma 4 and the uniqueness of the nucleolus, it is the only balanced proposal that distributes a total payoff of 5). The game w associated to this proposal is identical to v except that $w(1,2,3)=w(N)=5$. Its serial rule is $(3,1,1,0)$, which is different from the balanced proposal but gives a positive payoff to the same set of players.

to complain against k). Hence, $x^t(N) \geq x^t(S) > v^t(S) = d_k(v^t) = d_k(w^t)$, where the last equality follows from lemma 3.[8]

b) If $x_k^t = 0$ we need to prove that $d_k(N, w^t) = x^t(N)$. Since x^t is balanced, $f_{1k}(x^t, v^t) \leq 0$. Let P be a coalition associated to $f_{1k}(x^t, v^t)$. Because $f_{1k}(x^t, v^t) \leq 0$, $x^t(P) \leq v^t(P)$. Coalition P must contain all players receiving a positive payoff at x^t (otherwise x^t is not balanced since P can be used against any player outside P). Therefore $x^t(N) = x^t(P) \leq v^t(P)$. Because of the way w^t is defined it cannot exceed $x^t(N)$, so $x^t(N) = w^t(P) = d_k(w^t)$ and k receives 0 according to the serial rule of w^t. ■

The following lemma concerns a property of the serial rule. By definition, the serial rule is such that d_k is divided among players $\{j \in N, j < k\}$. Above d_k, player k and any player $j < k$ get the same payoff.

Lemma 7 *For any player k we have $\sum_{i\in\{1,2,...,k-1\}} \phi_i = d_k + (k-1)\phi_k$. Hence, $\sum_{i\in\{1,2,...,k-1\}} \phi_i \geq d_k + \phi_k$. The latter inequality is strict except if $k = 2$ or $\phi_k = 0$.*

The next lemma tell us that, given a strategy profile with balanced proposals, the proposer cannot get more than the serial rule of the games w^t.

Lemma 8 *Let (N, v) be a veto balanced TU game. Consider the associated game with T periods $G^T(N, v)$. Let $z = \sum_1^T x^t$ be an outcome resulting from balanced proposals. Consider period t, its outcome x^t and the game (N, w^t) where $w^t(S) = \min\{v^t(S), x^t(N)\}$. Then $x_1^t \geq \phi_1(N, w^t)$ implies $x_l^t \geq \phi_l(N, w^t)$ for all $l \in N$.*

Proof. Let T be the set of veto players in (N, w^t), and let $M = \{l_1, ..., l_m\}$ be the ordered (according to the $d-$values of (N, w^t)) set of nonveto players that have received a positive payoff at x^t. That is, $d_{l_1} \leq ... \leq d_{l_m}$.[9]

[8] Because x^t is a balanced proposal, the $d-$values of w^t coincide with the $d-$values of v^t for all players receiving a positive payoff. Any player j that is receiving a positive payoff at t will be veto at $t+1$ (lemma 3). The values $d_j(w^t)$ and $d_j(v^t)$ can only differ if $v^t(S) > x^t(N)$ for some S such that $j \notin S$, but then player j would not be veto at $t+1$.

[9] Recall that, because x^t is a balanced proposal, the $d-$values of w^t coincide with the $d-$values of v^t for all players in M.

Suppose $x_1^t \geq \phi_1(N, w^t)$. Since x^t is balanced, $x_1^t = x_i^t$ for all $i \in T$, thus if $x_1^t \geq \phi_1(N, w^t)$ it follows that $x_i^t \geq \phi_i(N, w^t)$ for all $i \in T$.

We now want to prove that $x_i^t \geq \phi_i(N, w^t)$ for all $i \in M$. We will do it by induction.

Consider the responder l_1. Since x^t is balanced we have $f_{1l_1}(x^t) = x_{l_1}^t$. If the coalition associated to f_{1l_1} has a value of 0, it follows that $x_1^t = x_{l_1}^t$ so $x_{l_1}^t \geq \phi_{l_1}(N, w^t)$. If the coalition 1 is using has a positive value, all veto players must be in it, so its payoff must be at least $|T|\phi_1(N, w^t)$, and its value (by definition of d_{l_1}) cannot exceed d_{l_1}. Hence, $f_{1l_1}(x^t) \geq |T|\phi_1(N, w^t) - d_{l_1}$. Because of lemma 7, $|T|\phi_1(N, w^t) - d_{l_1} \geq \phi_{l_1}(N, w^t)$.

Now suppose the result $x_i^t \geq \phi_i(N, w^t)$ is true for all $i \in \{l_1, ..., l_{k-1}\}$. We will prove that $x_{l_k}^t \geq \phi_{l_k}(N, w^t)$. Let S be a coalition such that $f_{1l_k}(x^t) = x^t(S) - v^t(S)$. As before, the result follows immediately if $v^t(S) = 0$. If $v^t(S) > 0$ it must be the case that $T \subseteq S$, but S need not involve all players in $\{l_1, ..., l_{k-1}\}$. Denote $\{l_1, ..., l_{k-1}\}$ by Q. We consider two cases, depending on whether $Q \subseteq S$.

If $Q \subseteq S$, we have $x_{l_k}^t = f_{1l_k}(x^t) = x^t(S) - v^t(S) \geq \sum_{i \in T \cup Q} \phi_i(N, w^t) - d_{l_k}$, where the last inequality uses the induction hypothesis. The set $T \cup Q$ contains all players with $d_i < d_{l_k}$. Hence, by lemma 7, $\sum_{i \in T \cup Q} \phi_i(N, w^t) - d_{l_k} \geq \phi_{l_k}(N, w^t)$.

If $Q \nsubseteq S$, there is a player $l_p < l_k$ such that $l_p \notin S$. Because x^t is a balanced proposal, $x_{l_p}^t = f_{1l_p}(x^t)$. Because the veto player can use S to complain against l_p, $f_{1l_p}(x^t) \leq f_{1l_k}(x^t) = x_{l_k}$, hence $x_{l_p} \leq x_{l_k}$. By the induction hypothesis, $x_{l_p} \geq \phi_{l_p}(N, w^t)$. Since $d_{l_p} \leq d_{l_k}$ we also know that $\phi_{l_p}(N, w^t) \geq \phi_{l_k}(N, w^t)$, so that

$$x_{l_k} = f_{1l_k}(x^t) \geq f_{1l_p}(x^t) = x_{l_p} \geq \phi_{l_p}(N, w^t) \geq \phi_{l_k}(N, w^t).$$

So far we have discussed the set of veto players and the set of nonveto players that are getting a positive payoff in x^t. For players in $N \backslash (T \cup M)$, we have shown in lemma 6 that $x_j^t = 0$ implies $\phi_j(N, w^t) = 0$, hence $x_j^t \geq \phi_j(N, w^t)$ for all players. ■

Corollary 1 *If $z = \sum_1^T x^t$ is an outcome resulting from balanced proposals, $x_1^t \geq \phi_1(N, w^t)$ implies $x_l^t = \phi_l(N, w^t)$ for all $l \in N$.*

This corollary follows directly from lemma 8 and the efficiency of the serial rule. Lemma 8 states that $x_1^t \geq \phi_1(N, w^t)$ implies $x_l^t \geq \phi_l(N, w^t)$ for all $l \in N$. By definition of w^t, $\sum_{l \in N} x_l^t = w^t(N)$. By the efficiency of the serial rule, $\sum_{l \in N} \phi_l(N, w^t) = w^t(N)$. Hence, the only way in which player 1 can obtain the serial rule of (N, w^t) with balanced proposals is that *all* players in the game obtain their serial rule payoff.

Finally, the sum of the serial rules of the games w^t cannot exceed the serial rule of the original game. This is due to the following property of the serial rule:

Lemma 9 *Consider the veto balanced TU game (N, v) and a finite set of positive numbers $(a_1, ..., a_k)$ such that $\sum_{l=1}^{k} a_l = v(N)$. Consider the following TU games: (N, w^1), (N, w^2), ..., (N, w^k), where*

$$w^1(S) \;:=\; \begin{cases} a_1 & \text{if } S = N \\ \min\{a_1, v(S)\} & \text{otherwise} \end{cases}$$

$$w^2(S) \;:=\; \begin{cases} a_2 & \text{if } S = N \\ \min\{a_2, \max\left[0, v(S) - \sum_{i \in S} \phi_i(N, w^1)\right]\} & \text{otherwise} \end{cases}$$

$$w^l(S) \;:=\; \begin{cases} a_l & \text{if } S = N \\ \min\{a_l, \max\left[0, v(S) - \sum_{m=1}^{l-1} \sum_{i \in S} \phi_i(N, w^m)\right]\} & \text{otherwise} \end{cases}$$

Then $\phi(N, v) = \sum_{i=1}^{k} \phi(N, w^i)$.

In the lemma, we take $v(N)$ and divide it in k positive parts, where k is a finite number. Then we compute the serial rule for each of the k games, and see that each player gets the same in total as in the serial rule of the original game.

The k games are formed as follows: $w^k(N)$ is always a_k; the other coalitions have $v(S)$ minus what has been distributed so far according to the serial rule of the previous games, unless this would be negative (in which case the value is 0) or above $w^k(N)$ (in which case the value is a_k).

The idea of the proof is that player i cannot get anything until d_i has been distributed, and from that point on i becomes veto. This happens

regardless of the way $v(N)$ is divided into k parts. For the same reason, if $\sum_{i=1}^{k} a_l < v(N)$, player 1 will get less than $\phi_1(N, v)$.

Note that lemma 9 refers to a sequence of TU games such that each game is obtained after distributing the *serial rule payoffs* for the previous game; the games w^t in lemma 8 are obtained by subtracting *balanced proposals* from w^{t-1}. It turns out that the TU games involved are identical in both cases: the sequence w^t depends only on the total amounts distributed $x^1(N), ..., x^n(N)$ (denoted by a_1, ..., a_n in lemma 9). This is because the set of players that get a positive payoff at period t is the same in both cases (lemma 6) and all these players become veto at period $t+1$ (lemma 3). Hence, any coalition with positive value at t has $w^t(S) = \min(w^{t-1}(S) - x^{t-1}(N), x^t(N))$ in both cases.

Putting the above lemmas together we can prove theorem 2. First, any payoff player 1 can achieve in a MBRE can be achieved by balanced proposals (lemma 2). Second, given that proposals are balanced, the payoff player 1 can get cannot exceed the sum of the serial rules of the games w^t (lemma 8). Since the sum of the serial rules of the games w^t cannot exceed the serial rule of the original game (lemma 9), player 1 can never get more than $\phi_1(N, v)$ in a MBRE. Also, player 1 can only get $\phi_1(N, v)$ if all other players get their serial rule payoff (corollary 1). Finally, $\phi(N, v)$ is achievable by the sequential proposals described in lemma 5.

Note that the assumption $T \geq n$ only plays a role in lemma 5. For time horizons shorter than $n-1$, all auxiliary results still hold but player 1 may not be able to achieve a payoff as high as $\phi_1(N, v)$.

As a byproduct of the analysis, we are able to compare the serial rule and the nucleolus from player 1's point of view.

Corollary 2 *Let (N, v) be a veto balanced TU game. Then $\phi_1(N, v) \geq \nu_1(N, v)$.*

Proof. In a MBRE, $\phi_1(N, v)$ coincides with the equilibrium payoff for the proposer in the game $G^T(N, v)$ when $T \geq n-1$. This equilibrium payoff is at least as large as his equilibrium payoff in the game $G^1(N, v)$, because the proposer can always wait until period T to divide the payoff. This equilibrium

payoff is in turn at least as high as $\nu_1(N, v)$, because $\nu(N, v)$ is a balanced proposal. ■

3.4 MBRE and SPE may not coincide

The next example illustrates that a MBRE need not be a subgame perfect equilibrium.

Example 2 *Let $N = \{1, 2, 3, 4, 5\}$ be the set of players and consider the following 5-person veto balanced game (N, v) where*

$$v(S) = \begin{cases} 36 & \text{if } S \in \{\{1,2,3,4\}, \{1,2,3,5\}\} \\ 31 & \text{if } S = \{1,2,4,5\} \\ 51 & \text{if } S = N \\ 0 & \text{otherwise.} \end{cases}$$

The serial rule for this game can be easily calculated given that $d_1 = d_2 = 0$, $d_3 = 31$, $d_4 = d_5 = 36$ and $d_6 = 51$. Player 1's payoff according to the serial rule is then $\phi_1(N, v) = \frac{31}{2} + \frac{36-31}{3} + \frac{51-36}{5} = 121/6$. As we know from the previous section, this is player 1's payoff in any MBRE for any order of the responders. Suppose the order of responders is 2, 3, 4, 5. The following result holds given this order: If the responders play the game optimally (not necessarily as myopic maximizers) the proposer can get a higher payoff than the one provided by the MBRE outcome. Therefore, MBRE and SPE outcomes do not necessarily coincide.

The strategy is the following: The proposer offers nothing in the first three periods. In the 4th period the proposal is: $(10, 10, 5, 0, 0)$.

The responses of players 2, 4 and 5 do not change the proposal (even if the proposal faced by player 4 and 5 is a new one resulting from a rejection of player 3). If player 3 accepts this proposal, the TU game for the last period will be:

$$w(S) = \begin{cases} 11 & \text{if } S \in \{\{1,2,3,4\}, \{1,2,3,5\}\} \\ 11 & \text{if } S = \{1,2,4,5\} \\ 26 & \text{if } S = N \\ 0 & \text{otherwise.} \end{cases}$$

In the last period, myopic and rational behavior coincide, so the outcome must be an element of $B_1(N,w)$. It can be checked that $B_1(N,w) = \{(5.5, 5.5, 0, 0, 0)\}$. Therefore, after accepting the proposal in period 4, player 3 gets a total payoff of 5.

If player 3 rejects the proposal, the outcome of the 4th period is $(15, 10, 0, 0, 0)$ and the TU game for the last period is:

$$u(S) = \begin{cases} 11 & \text{if } S \in \{\{1,2,3,4\},\{1,2,3,5\}\} \\ 6 & \text{if } S = \{1,2,4,5\} \\ 26 & \text{if } S = N \\ 0 & \text{otherwise.} \end{cases}$$

As before, in the last period myopic and rational behavior coincide and the outcome must be an element of $B_1(N,u)$. It can be checked that $B_1(N,u) = \{(5.2, 5.2, 5.2, 5.2, 5.2)\}$. Therefore, after rejecting the proposal player 3 gets a total payoff of 5.2.

Therefore, rational behavior of player 3 implies a rejection of the proposal in the 4th period. This rejection is not a myopic maximizer's behavior. After the rejection of player 3 the proposer gets a payoff of 20.2, higher than 121/6. Hence, the outcome associated to MBRE is not the outcome of a SPE.

In the example above, the proposer *finds a credible way to collude* with player 3 in order to get a higher payoff than the one obtained by player 2 (a veto player). Player 2 cannot avoid this collusion since he is responding before player 3. If he responded after player 3, collusion between players 1 and 3 would no longer be profitable. This observation turns out to be crucial as we will see in the next section.

Finally, consider the following profile of strategies: the proposer makes the sequence of proposals presented in Lemma 5 (and proposes 0 for all players off the equilibrium path) and the responders behave as myopic maximizers. This profile is a Nash equilibrium and its outcome is $\phi(N,v)$. Therefore:

Remark 3 *The MBRE outcome is a Nash equilibrium outcome. Also, if z is a Nash equilibrium outcome, $z_1 \geq \phi_1 = \sum_{i=1}^{n} \frac{d_{i+1}-d_i}{i}$.*

3.5 The serial rule as an SPE outcome

The previous example shows that, in general, myopic and farsighted (rational) behavior do not coincide. However, they do coincide when the model incorporates a requirement on the order of the responders. We will assume in theorem 3 that the order of the responders in period t is given by the nonincreasing order of the $d-$values of the game v^t. That is, the order of the responders is not completely fixed in advance and can be different for different periods. Given this order, any veto responder can secure a payoff equal to the one obtained by the proposer. This was not the case in Example 2, where player 2 is a veto responder responding before player 3.

We start by pointing out some immediate consequences of the results in section 3.3.

Suppose there is an SPE outcome z that differs from $\phi(N,v)$. If z differs from $\phi(N,v)$, $z_1 \geq \phi_1(N,v)$ (otherwise the proposer would prefer to play the strategy described in lemma 5, which is available since $T \geq n$). If responders are behaving myopically, the proposer can only achieve at least $\phi_1(N,v)$ if *all* players are getting their serial rule payoffs, that is, if $z = \phi(N,v)$ (theorem 2). Hence, z_1 is not achievable with myopic behavior of the responders, let alone with balanced proposals:

Corollary 3 *Let (N,v) be a veto balanced TU game and $G^T(N,v)$ its associated extensive form game with $T \geq n$. Let $z = \sum_1^T x^t$ be an outcome resulting from some SPE of the game $G^T(N,v)$. If z differs from $\phi(N,v)$ then z_1 cannot be achieved by making balanced proposals.*

Corollary 4 *Let (N,v) be a veto balanced TU game and $G^T(N,v)$ its associated extensive form game with $T \geq n$. Let $z = \sum_1^T x^t$ be an outcome resulting from some SPE of the game $G^T(N,v)$. If z differs from $\phi(N,v)$ then there exists at least one period t and one player p for which $f_{1p}(x^t,(N,v^t)) > x_p^t \geq 0$.*

This is because if $x_l^t \geq f_{1l}(x^t,v^t)$ for all l and t, z would be achievable under myopic behavior of the responders by proposing x^t in each period t, a contradiction.

Since myopic behavior always leads to $f_{1i}(x^t, v^t) \leq x_i^t$ for all i and all t (lemma 1), the presence of a player p for which $f_{1p}(x^t, (N, v^t)) > x_p^t$ indicates non-myopic responder behavior. This responder may be player p (nonmyopically accepting a proposal), or a responder moving after p (nonmyopically rejecting a proposal and transferring payoff to player 1). In example 2, player 3 rejects a proposal nonmyopically and as a result creates the inequality $f_{12}(x^t, v^t) > x_2^t$ at the end of period $t = 4$.

We are now ready to state our main result.

Theorem 3 *Let (N, v) be a veto balanced TU game and $G^T(N, v)$ its associated extensive form game in which $T \geq n$ and the responders move following the order of nonincreasing $d-$values of v^k. Then $\phi(N, v)$ is the outcome of any SPE.*

The rest of this section is devoted to proving this theorem. We will show in lemma 13 that, if responders move following the order of nonincreasing $d-$values of v^k, any SPE outcome z is such that the proposer can obtain z_1 by making balanced proposals. Since any SPE outcome z different from $\phi(N, v)$ would be unachievable with balanced proposals according to corollary 3, this will complete the proof.

In order to prove lemma 13 we need several auxiliary results.

We denote by $x^{t,i}$ the proposal that emerges in period t immediately after i gets the move. The following lemma establishes a property of $x^{t,i}$ that must be inherited by the final outcome in period t, x^t.

Lemma 10 *Suppose after player i responds to the proposal in period t it holds that $f_{1i}(x^{t,i}, v^t) > 0$. Then $f_{1i}(x^t, v^t) > 0$ regardless of the responses of the players moving after i.*

Proof. Suppose by contradiction that $f_{1i}(x^t, v^t) \leq 0$. This means that at the end of period t there is a coalition S^* such that $i \notin S^*$ and $v^t(S^*) \geq x^t(S^*)$. Because $f_{1i}(x^{t,i}, v^t) > 0$ immediately after i responds to the proposal, all coalitions excluding i had a positive satisfaction at that point, and in particular $v^t(S^*) < x^{t,i}(S^*)$. There must be a player h moving after i such that $h \notin S^*$ and h has received a payoff transfer from player 1 by rejecting the

proposal. At the moment of rejection by h we have $f_{1h}(x^{t,h}, v^t) = x_h^t > 0$. However, since S^* can be used by 1 to complain against h, at the end of period t we have $f_{1h}(x^t, v^t) \leq 0$. There must be another player l moving after h that has received a payoff transfer from player 1, and this player cannot be in S^*. Then this player is in the same situation as player h: he has $f_{1l}(x^{t,l}, v^t) > 0$ at the moment of rejection, but at the end of period t he has $f_{1l}(x^t, v^t) \leq 0$. Thus there must be another player moving after him that has caused this change and would himself be in the same situation as player h... but the number of players is finite. ■

Notice that lemma 10 holds for *any* strategy profile, not necessarily an equilibrium.

The next auxiliary result will allow us to compare the equilibria of games with different characteristic functions. If one of the characteristic functions is "worse" than the other (in the sense of having lower values), player 1 might still have a greater SPE payoff, but only with nonmyopic responder behavior.

Lemma 11 *Let (N, v) and (N, w) be two veto balanced games in which player 1 is a veto player. Let $w(S) \geq v(S)$ for any S. Let $G^T(N, v)$ and $G^T(N, w)$ be the associated extensive form games with T proposals. If the payoff provided to the proposer by a SPE outcome of the game $G^T(N, w)$ is strictly lower than the payoff provided to the proposer by a SPE outcome of the game $G^T(N, v)$, then the SPE outcome of $G^T(N, v)$ is such that $x_l^t < f_{1l}(x^t, v^t)$ for some l and t, which implies that at least one responder is behaving nonmyopically.*

Proof. Suppose the final payoffs are such that $x_l^t \geq f_{1l}(x^t, v^t)$ for all l and t. We can then use lemma 2 to construct a sequence of balanced proposals with the same payoff for the proposer. For any sequence y^t of balanced proposals of the game $G^T(N, v)$ it holds that $y_l^t \geq f_{1l}(y^t, w^t)$, and we can use lemma 2 again to construct a sequence of balanced proposals for the game $G^T(N, w)$ where the payoff of the proposer does not change, a contradiction. ■

The next auxiliary result provides a bound for the payoff difference between player 1 and player $i \neq 1$.

Lemma 12 *Let (N,v) be a veto balanced TU game. Consider the associated game with T periods $G^T(N,v)$. Fix a period $l \in \{1,...,T\}$ and a subgame that starts in period l (not necessarily on the equilibrium path), and label the responders according to the nondecreasing order of d-values in the game v^l. Let $y^l = \sum_l^T x^t$ be the vector of payoffs accumulated between l and T. Then $y_i^l \geq y_1^l - d_i^l$ for all $i \in \{2,...,n\}$ in any SPE of $G^T(N,v)$. Moreover, the inequality is strict if $d_i^l > d_2^l$.*

Proof. Note that, because period T is the last period of the game, myopic and rational behavior coincide, so $x_i^T \geq f_{1i}(v^T, x^T)$ for all i.

Consider player 2. Since players play myopically in period T, it must be the case that[10]

$$\begin{aligned} x_2^T &\geq f_{12}(x^T) \geq x_1^T - \left(d_2^l - \sum_l^{T-1} x_1^t \right) = \\ &= \sum_l^T x_1^t - d_2^l = y_1^l - d_2^l. \end{aligned}$$

Since $y_2^l \geq x_2^T$, it follows that $y_2^l \geq y_1^l - d_2^l$.

Now consider player $i \neq 2$. There are two possible cases, depending on whether $y_1^l \leq d_2^l$.

If $y_1^l \leq d_2^l$, the result follows immediately since

$$y_1^l - d_i^l \leq y_1^l - d_2^l \leq 0 \leq y_i^l.$$

It is also clear that the inequality is strict if $d_2^l < d_i^l$.

From now on we assume $y_1^l > d_2^l$. Note that since we have already shown that $y_2^l \geq y_1^l - d_2^l$, it follows that that $y_2^l > 0$ in this case. There are again two possible cases, depending on whether the coalition associated to $f_{1i}(x^T, v^T)$ contains 2.

[10] If S is a coalition associated to $f_{12}(x^T)$, the total payoff of S must be at least x_1^T. Also, the total value of S must be at most $d_2^l - \sum_l^{T-1} x_1^t$.

If the coalition contains 2, we have

$$\begin{aligned} y_i^l &\geq x_i^T \geq f_{1i}(x^T, v^T) \geq x_1^T + x_2^T - \left(d_i^l - \sum_l^{T-1} x_1^t - \sum_l^{T-1} x_2^t \right) = \\ &= y_1^l + y_2^l - d_i^l > y_1^l - d_i^l, \end{aligned}$$

where the last inequality follows from the fact that $y_2^l > 0$.

If the coalition does not contain 2, we have $f_{1i}(x^T) \geq f_{12}(x^T)$. Then

$$y_i^l \geq x_i^T \geq f_{1i}(x^T, v^T) \geq f_{12}(x^T, v^T) \geq y_1^l - d_2^l \geq y_1^l - d_i^l.$$

Note that the inequality is strict for $d_i^l > d_2^l$. ■

The main building block of the proof is the next lemma, which shows that, given the particular order of responders we impose, the proposer cannot do better than with balanced proposals.

Lemma 13 *Let (N, v) be a veto balanced TU game. Consider the associated game $G^T(N, v)$ in which the responders move following the order of nonincreasing $d-$values of v^k. Let $z = \sum_1^T x^t$ be an outcome resulting from some SPE of the game $G^T(N, v)$. Then the proposer can obtain z_1 by making balanced proposals.*

Proof. Suppose on the contrary that z_1 cannot be obtained with balanced proposals. By Lemma 2 we know that there is a player k and a stage t such that $f_{1k}(x^t, v^t) > x_k^t \geq 0$; otherwise the proposer can obtain z_1 with balanced proposals.

Let t be the last period[11] in which for some responder it holds that $f_{1k}(x^t, v^t) > x_k^t \geq 0$. Let k be the last responder at t for whom $f_{1k}(x^t, v^t) > x_k^t \geq 0$. We consider two cases:

a) There is a player p with $d_p^t \leq d_k^t$ such that $f_{1p}(x^t, v^t) \leq 0$. Note that $f_{1k}(x^t, v^t) > x_k^t \geq 0$ means that any coalition without player

[11] It is clear that $t < m$, since all responders behave as myopic maximizers in the last period.

k has a positive satisfaction and, in particular any coalition $S_k \in \arg\max_{S \subseteq N \setminus \{k\}} v^t(S)$. On the other hand since $f_{1p}(x^t, v^t) \leq 0$ then there exists a coalition without player p for which the satisfaction is not positive. Let S_p^* be one such coalition (it must contain k). Then we have the following two inequalities:

$$x^t(S_k) > d_k^t \text{ and } v^t(S_p^*) \geq x^t(S_p^*).$$

Combining the two inequalities we obtain

$$x^t(S_k) - x^t(S_p^*) > d_k^t - v^t(S_p^*) \geq d_k^t - d_p^t \geq 0.$$

The inequality above implies that there are players not in S_p^* receiving a positive payoff in period t.

Consider a new allocation, y^t, which is identical to x^t except that $y_i^t = 0$ for all i not in S_p^* (thus $y_i^t = x_i^t$ for all i in S_p^*). We now show that $f_{1i}(y^t, v^t) \leq y_i^t$ for all $i \in N$, so that player 1 can get the same payoff with balanced proposals by lemma 2.

For any player i it holds that $f_{1i}(y^t, v^t) \leq f_{1i}(x^t, v^t)$.

Because S_p^* can be used against any player outside S_p^*, for any player outside S_p^* it holds that $f_{1i}(y^t, v^t) \leq f_{1p}(y^t, v^t) \leq f_{1p}(x^t, v^t) \leq 0$. Thus, $f_{1i}(y^t, v^t) \leq y_i^t$ for all $i \notin S_p^*$.

Can there be a player $l \in S_p^*$ for which $f_{1l}(y^t, v^t) > y_l^t$? If so, this inequality must have existed already for x^t, since $f_{1l}(y^t, v^t) \leq f_{1l}(x^t, v^t)$ and $y_l^t = x_l^t$. Since player k is the last player satisfying the inequality for x^t, it must be the case that $d_l^t \geq d_k^t \geq d_p^t$, thus we can repeat the reasoning above with S_l and S_p^* and, given that nothing has changed for S_p^*, we would conclude that $y^t(S_l) - y^t(S_p^*) > 0$, a contradiction since all players outside S_p^* have zero payoffs. Thus, $f_{1i}(y^t, v^t) \leq y_i^t$ for all i.

Note that for this part of the proof no assumption is needed about the order in which the responders move.[12]

[12] All we need to assume in this part of the proof is that player k is the "last" player in the sense of being the player with the lowest d_i^t, not necessarily the one who moves last.

b) The second case is $f_{1l}(x^t, v^t) > 0$ for all players moving after k at t. By assumption, on the equilibrium path from $t+1$ onwards all proposals have an associated balanced proposal. We distinguish two subcases:

b1) The last player to act nonmyopically at t has accepted a proposal. This player must be player k or a player moving after k. Call this player p (p moving after k is possible if a myopic rejection by a player moving after p has restored $f_{1p} \leq x_p$).

We will show that it is not in p's interest to accept the proposal. To do this, we need to analyze two subgames: the subgame on the equilibrium path in which p accepts the proposal, and the subgame off the equilibrium path in which p rejects the proposal. We will talk about the A-path (the equilibrium path) and the R-path. Denote by $x^{A,t}$ and $x^{R,t}$ the final payoffs in period t depending on whether player p accepts or rejects the proposal. If player p rejects the proposal, we take any subgame perfect equilibrium of that subgame. Denote by $v^{A,t+1}$ and $v^{R,t+1}$ the corresponding TU games at $t+1$.

Because $f_{1l}(x^t, v^t) > 0$ for $l \in \{2, ..., p\}$, on the A-path all players in $\{2, ..., p\}$ are veto players at $t+1$.

The game $v^{R,t+1}$ is better than the game $v^{A,t+1}$ (in the sense of lemma 11). If $v^{A,t+1}(S) > 0$, coalition S must contain all players in $\{1, 2, ..., p\}$. For this kind of coalition $v^{R,t+1}(S) = v^{A,t+1}(S)$, since any payoff transfers after rejection occur between members of $\{1, ..., p\}$ (here the order of moves is essential). Thus, $v^{A,t+1}(S) \leq v^{R,t+1}(S)$ for all S.

We now show that p is also veto at $t+1$ on the R-path.

Suppose p is not veto at $t+1$ on the R-path. Then there is a coalition S_p such that $p \notin S_p$ and $v^{R,t+1}(S_p) > 0$. This can only happen if $v^{R,t}(S_p) > x^{R,t}(S_p)$, or equivalently $f_{1p}(x^{R,t}, v^t) < 0$, contradicting lemma 10.

Thus, player p is a veto player at $t+1$ regardless of whether he accepts or rejects the proposal. Given the order of moves, veto

players can secure at least the same payoff as the proposer. There is no reason for veto players to act nonmyopically because the game at $t+2$ will be the same regardless of how the payoff is distributed at $t+1$ between veto players; no payoff can go to anyone else given the order of responders. For the same reason the proposer will never make a proposal that gives another veto player more than he gets himself, so that all veto players must get the same payoff given the order of moves. Let y_1^R be player 1's payoff if p rejects the proposal (this is the payoff accumulated between periods $t+1$ and n) and y_1^A be player 1's payoff if p accepts the proposal. Because of lemma 11, the only way in which y_1^A can exceed y_1^R is if there is a nonmyopic move at $v^{A,t+1}$ that leads to $f_{1i}(x^{A,t+1}, v^{A,t+1}) > x_i^{A,t+1}$ for some i. By assumption this is not the case. Thus, it was not in p's interest to accept: rejecting would yield a higher payoff at t, and at least the same payoff in the rest of the game.

b2) The last player to act nonmyopically at t has rejected a proposal. Let p be the last player to act nonmyopically at t. This player cannot be player k because after any rejection (myopic or otherwise) it holds that $f_{1k}(.) \leq x_k^t$, and given that the remaining responders act myopically this inequality would never be reversed. Someone moving after k must have rejected a proposal nonmyopically (transferring payoff to the proposer) and created the inequality $f_{1k}(x^t, v^t) > x_k^t$, hence player p must be moving after k. We will show that it is not in p's interest to reject the proposal. To do this, we need to analyze two subgames: the subgame on the equilibrium path in which p rejects the proposal, and the subgame off the equilibrium path in which p accepts the proposal. We will talk about the R-path (the equilibrium path) and the A-path.

Because $f_{1l}(x^t, v^t) > 0$ for $l \in \{2, ..., k\}$, on the equilibrium path all players in $\{2, ..., k\}$ are veto players at $t+1$.

The game $v^{A,t+1}$ is better than the game $v^{R,t+1}$ (in the sense of lemma 11). If $v^{R,t+1}(S) > 0$, coalition S must contain all players in

$\{1, 2, ..., p\}$. For this kind of coalition $v^{R,t+1}(S) = v^{A,t+1}(S)$, since any payoff transfers on the A-path must occur between members of $\{1, 2, ..., p\}$ (again, here the order of moves is essential). Thus, $v^{R,t+1}(S) \leq v^{A,t+1}(S)$ for all S.

Suppose player p is veto also on the A-path. Then the reasoning of case b1 applies, and there is no reason for p to act nonmyopically in period t.

Now suppose player p is not veto on the A-path at $t+1$. We define $d_p^{A,t+1} := \max_{S \subseteq N \setminus \{p\}} v^{A,t+1}(S)$. Since p is not a veto player at $t+1$ on the A-path, $d_p^{A,t+1} > 0$. There is a coalition S_p such that $v^t(S_p) - x^{A,t}(S_p) = d_p^{A,t+1} > 0$. Since by assumption $f_{1p}(x^t, v^t) > 0$ on the R-path, we also have $x^{R,t}(S_p) - v^t(S_p) > 0$. From the two inequalities we get $x^{R,t}(S_p) - x^{A,t}(S_p) > d_p^{A,t+1}$.

Let α be the payoff player p transfers to player 1 when rejecting the proposal (part of this payoff may then go to other players between 2 and $p-1$ if they myopically reject a proposal). We want to show that $\alpha \geq x^{R,t}(S_p) - x^{A,t}(S_p)$, which implies $\alpha > d_p^{A,t+1}$. This is not completely obvious because part of the difference could be due to a player outside S_p myopically rejecting on the A-path.

Claim. $\alpha \geq x^{R,t}(S_p) - x^{A,t}(S_p) > d_p^{A,t+1}$.

Note that $x^{R,t}$ and $x^{A,t}$ are identical for all responders moving before p. Because of the order of moves, any payoff transfers due to a change of p's action from R to A occur within the set $\{1, 2, ..., p\}$. Denote by T the set $\{2, ..., p-1\} \setminus S_p$. We can write $x^{R,t}(S_p) + x^{R,t}(T) = x^{A,t}(S_p) + x^{A,t}(T) + \alpha$. All we need to show is that $x_h^{R,t} \geq x_h^{A,t}$ for all $h \in T$ (this is obvious if T is empty). This implies $x^{R,t}(T) \geq x^{A,t}(T)$ and hence $x^{R,t}(S_p) - x^{A,t}(S_p) \leq \alpha$. Suppose there is a player h in T that has $x_h^{R,t} < x_h^{A,t}$. Since this player is not in S_p, S_p can be used by player 1 against him.

Player h must have rejected in the A-path (if he had accepted he would have $x_h^{R,t} \geq x_h^{A,t}$)[13]. After rejecting he is left with

[13] By assumption, p is the last player to behave nonmyopically on the R-path. This leaves

$f_{1h}(.) = x_h^{A,t} > 0$. On the other hand, $v^t(S_p) - x^{A,t}(S_p) > 0$. Thus, $f_{1h}(x^{A,t}, v^t) < 0$, contradicting lemma 10.

Now we are in a position to compare payoffs on the A and R-paths and see that p prefers to accept the proposal.

Since player p is veto on the R-path, he gets y_1^R. On the A-path, he gets y_p^A, whereas the proposer gets y_1^A. Because lemma 12 applies to all subgames regardless of whether they are on the equilibrium path, $y_p^A \geq y_1^A - d_p^{A,t+1}$.

In order to have an equilibrium, p must prefer to reject the proposal, thus we need $x_p^t + y_1^R \geq x_p^t + \alpha + y_p^A > x_p^t + d_p^{A,t+1} + y_1^A - d_p^{A,t+1}$. Therefore we need $y_1^R > y_1^A$. Since the game $v^{A,t+1}$ is better than the game $v^{R,t+1}$ (in the sense of lemma 11), and by assumption $f_{1i}(x^l, v^l) > x_i^l$ never happens on the R-path from $t+1$ onwards, the inequality $y_1^R > y_1^A$ cannot hold.

■

We have shown that any SPE outcome is such that the proposer can always achieve z_1 with balanced proposals. From the previous section we know that the best outcome the proposer can achieve with balanced proposals is $\phi(N, v)$, hence this completes the proof.

As previously mentioned, the set of NE outcomes coincides with the set of SPE outcomes in the one-period bargaining game. This property does not carry over to the multi-period game. Theorem 3 rules out alternative NE outcomes supported by incredible threats on the part of the responders, but there may be incredible threats available to the proposer. For example, let $N = \{1, 2\}$, $v(N) = 20$, $v(1) = v(2) = 0$ and $T = 2$. The only SPE outcome is $(10, 10)$, but this is not the only NE outcome. Consider the following strategy combination. Player 1 proposes $(a, 0)$ in period 1. If period 1's

us with two possibilities for h on the R-path: myopic acceptance and myopic rejection. In neither case it is possible for h to get a higher payoff by accepting the proposal on the A-path.

proposal is accepted, player 1 proposes $\left(\frac{20-a}{2}, \frac{20-a}{2}\right)$ in period 2; if period 1's proposal was rejected, player 1 proposes $(0,0)$ in period 2. Player 2 accepts the proposal in period 1 provided that $x_1^{1,1} \leq a$, and behaves myopically in period 2. By accepting the proposal $(a,0)$, player 2 gets nothing in period 1 and $\frac{20-a}{2}$ in period 2; rejecting it brings $\frac{a}{2}$ in period 1 and 0 in period 2. This strategy combination is a NE provided that $a \leq 10$. It rests on an incredible threat by the proposer, since in the subgame after the proposal has been rejected it would not be in the proposer's interest to propose $(0,0)$.

4 Concluding remarks

We have provided noncooperative foundations for the serial rule $\phi(N,v)$ in veto balanced games. Our procedure is based on bilateral bargaining with an enforceable bilateral principle. Interestingly, the serial rule does not satisfy the bilateral principle in general (the nucleolus is the only efficient allocation that does), but it is the sum of n allocations, each of which satisfies the bilateral principle in the relevant game.

We have also shown that any SPE outcome of our bargaining procedure is achievable with myopic behavior of the responders if responders move by increasing strength (lemma 13). This result is independent of the number of periods. If there are at least $n-1$ periods, the only SPE outcome is the serial rule: the proposer is always able to obtain $\phi_1(N,v)$ by making balanced proposals, and the only way to obtain this payoff is if all other players get $\phi_i(N,v)$ as well. If there are fewer than $n-1$ periods, there may not be enough periods for the proposer to achieve the serial rule with balanced proposals. If z is a SPE outcome, it is still true that the proposer can obtain z_1 by making balanced proposals, hence all SPE outcomes must have the same z_1, but there may be several SPE outcomes if $z_1 < \phi_1(N,v)$.

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