



Universidad del País Vasco    Euskal Herriko Unibertsitatea

DOKTOREGO-TESIA - *PhD THESIS*

**“KONTAKTUAREN ZENBAKIZKO  
SIMULAZIOA ARTEZKETAN”**

**“ON THE NUMERICAL MODELLING OF THE  
CONTACT IN GRINDING”**

*Presented in the Mechanical Engineering Department by*

**Juan Luis OSA AMILIBIA Jn.-k**

aurkeztua Ingeniaritza Mekanikoa Sailean

*Zuzendariak / Supervisors:*

**José Antonio SÁNCHEZ GALINDEZ Dr.**

**Naiara ORTEGA RODRÍGUEZ Dr.**

*Eibar, November 2016 Azaroa*



*El sueño de la razón produce monstruos*  
Francisco de Goya (1799)



*Nere aitari*



# Contents

I	Kontaktuaren zenbakizko simulazioa artezketan (EU)	1
II	On the numerical modelling of the contact in grinding (EN)	143





## Part I

# Kontaktuaren zenbakizko simulazioa artezketan



# Gaien Aurkibidea

<b>1. Sarrera</b>	<b>9</b>
1.1. Testuingurua . . . . .	9
1.2. Helburuak eta ekarpenak . . . . .	12
1.3. Edukiak . . . . .	13
<b>2. Aurrekariak</b>	<b>15</b>
2.1. Sarrera. Artezketa eragiketa . . . . .	17
2.1.1. Artezketa lauaren zinetika eta indarrak . . . . .	18
2.1.2. Diamantaketa . . . . .	21
2.1.3. Harriaren gainazal topografia . . . . .	25
2.2. Artezketa kontaktu-luzera . . . . .	31
2.2.1. Kontaktu-luzera ereduak . . . . .	32
2.2.2. Kontaktu-luzera neurtzeko metodo esperimentalak . . . . .	37
2.2.3. Kontaktu-luzera seinalearen analisisa . . . . .	41
2.3. Zenbakizko ereduak artezketan . . . . .	43
2.3.1. Harriaren egituraren ereduak . . . . .	44
2.3.2. Topografia-ereduak . . . . .	46
2.4. Ale urratzaile-pieza interakzioa . . . . .	51
2.4.1. Kontaktu-mekanika . . . . .	52
2.4.2. Urraduraren mekanika . . . . .	55
2.4.3. Biskoplastikotasuna . . . . .	66
2.5. Laburpena . . . . .	72
<b>3. Zenbakizko kontaktu-eredua</b>	<b>75</b>
3.1. Sistemen diseinua . . . . .	77
3.1.1. Harria-pieza kontaktuaren zenbakizko ereduak . . . . .	77
3.2. Elementu diskretuen metodoa . . . . .	78
3.2.1. DEMaren moldaketa artezketa kontaktu-eredura . . . . .	80
3.2.2. DEM ereduaren sorkuntza . . . . .	89
3.2.3. DEM ereduaren egitura . . . . .	90
3.3. Harriaren zenbakizko ereduaren eraikuntza . . . . .	91
3.3.1. Harriaren ezaugarriak . . . . .	91
3.3.2. Harriaren zenbakizko ereduak DEM bidez . . . . .	97
3.4. Zenbakizko diamantaketa . . . . .	100

3.5. Mugalde-baldintzak . . . . .	104
3.5.1. Kontaktu estatikoaren eredua . . . . .	105
3.5.2. Ebaki-gabeko txirbil-lodieraren eredua . . . . .	107
3.5.3. Indar zentrifugoa . . . . .	110
3.6. Laburpena . . . . .	110
<b>4. Kontaktuaren simulazioa artezketan</b>	<b>113</b>
4.1. Lan esperimentala . . . . .	115
4.2. DEM ereduaren prestaketa . . . . .	121
4.2.1. Habeen propietateen kalibrazioa . . . . .	121
4.2.2. Sarrera-datuak . . . . .	122
4.3. Simulazioak eta saiakuntzak . . . . .	127
4.4. Esperimentu faktorialak DEM ereduarekin . . . . .	134
4.5. Laburpena . . . . .	138
<b>5. Ondorioak</b>	<b>139</b>
5.1. Etorkizuneko lan-ildoak . . . . .	140

# Irudien Zerrenda

1.1. Indexatutako aldizkarietan <i>grinding, contact, numerical model</i> eta <i>DEM</i> hitz gakoak dituzten argitaratutako artikuluko kopurua .	10
1.2. Zenbakizko eredu termikoan lortutako gainazaleko tenperaturak .	11
2.1. Ale urratzailearen sorbatza materiala harrotzen . . . . .	17
2.2. Artezketa lauaren lan-parametro nagusiak eta indarrak . . . . .	18
2.3. Artezketaren mikro-fresaketa analogia . . . . .	19
2.4. Ebaki-gabeko txirbil-geometria alearen zeharkako profilaren arabera: triangeluarra, errektangeluarra eta zirkularra . . . . .	20
2.5. Diamantaketa finkoaren zinematika; ebaketa-sekzioaren detailea .	22
2.6. Ale haustura eta aglomeratzaile-zubi haustura . . . . .	23
2.7. Diamanteak ondoz ondoko aleetan sortutako profilak . . . . .	23
2.8. Sorbatzen arteko distantzia estatikoa eta dinamikoa . . . . .	27
2.9. Sorbatzen zorrotasunaren karakterizazioa . . . . .	27
2.10. Sorbatzen eta gainazaleko aleen batz besteko profilak; tontorren malda eta kurbatura . . . . .	29
2.11. Ordezko harria eta CBN harri lagina . . . . .	31
2.12. Kontaktu-luzeraren definizioa; deformazioak harria-pieza artean .	32
2.13. Rowe-Qi kontaktu-eredua . . . . .	36
2.14. Termopare polobakarra; isolatzailearen lodieraren eragina . . . .	38
2.15. APS metodoa; harria-pieza eta pieza-pieza aldaerak . . . . .	39
2.16. Gelditze azkarra eta adabaki-artezketa metodoak . . . . .	40
2.17. APS seinaleak kontaktu-zabaleraren arabera; ebaketa-eremuak kontaktu-luzeran; termopare eta APS seinaleak . . . . .	42
2.18. Harriaren malguki-eredu uniformea; aglomeratzaile-zubiak modelatzen dituen habe-sarea; artezketaren simulazioa DEM bidez .	45
2.19. Ale geometria aldaerak; alearen proiektzioak eta forma faktorea .	50
2.20. 3D topografia-ereduen diagrama orokorra . . . . .	52
2.21. Erdi-espazio elastikoan kontaktua esfera eta zilindro zurrunean .	53
2.22. Ale urratzailearen akzioa: igurzketak, goldaketa eta ebaketa . . .	56
2.23. Ale bakarraren frogan aldiuneko energia espezifikoa eta indarra .	58
2.24. Taborren ale higatuen igurzketak-kontaktu ereduak . . . . .	59
2.25. Konoak gainazala urratuz sortutako ildaska . . . . .	60
2.26. Ebakidura labainketa-lerroen ereduak . . . . .	60

2.27. Challengen eta Oxleyen labainketa-lerro ereduak . . . . .	61
2.28. Ebaketa-eremuak; txirbil-sorkuntza hiru dimentsiotan . . . . .	61
2.29. Txirbil-sorkuntzaren gogortasun-froga analogia . . . . .	63
2.30. Ebaketa ortogonal eraso-angelu negatiboetan . . . . .	69
3.1. Sarrerak eta irteerak artezketa-sisteman . . . . .	78
3.2. Harria-pieza kontaktua fluxu-diagrama errealitatean eta zenbakizko ingurunean . . . . .	79
3.3. Proposatutako DEM ereduaren formulazioaren egitura . . . . .	81
3.4. DE-en arteko ohiko interakzio-motak . . . . .	84
3.5. Habe elastikoa atsedenean eta kargatuta . . . . .	86
3.6. Habeen propietateen kalibratzeko prozeduraren fluxu-diagrama . . . . .	88
3.7. Simulazioak jarraitzen duen fluxu-diagrama . . . . .	91
3.8. Ohiko harrien izendapena . . . . .	92
3.9. Osagaien proportzio bolumentrikoen eragina harrien egiturari . . . . .	94
3.10. Harrien osaketa bolumentrikoaren diagrama trigonala . . . . .	95
3.11. Aglomeratzailearen tantaren froga . . . . .	96
3.12. Harriaren eta piezaren DEM ereduak . . . . .	97
3.13. Harriaren DEM ereduaren eraikitze fluxu-diagrama . . . . .	98
3.14. Harriaren DEM ereduaren eraikitze fluxu-diagrama . . . . .	99
3.15. Harriaren DEM ereduari gainazal topografia aplikatzeko pausoak . . . . .	101
3.16. Ale-dentsitate metatua sakonera erradialean . . . . .	102
3.17. Chenen topografia ereduaren aplikazioa era diskretuan . . . . .	103
3.18. Kontaktu-detekzioa KE ereduari . . . . .	106
3.19. ETL ereduaren oinarriak . . . . .	108
3.20. Kontaktu-detekzioa ETL ereduari . . . . .	109
4.1. Diamantearen profilaren zeharkako neurketa . . . . .	116
4.2. Artezketa-endarren neurketa-katea . . . . .	117
4.3. Neurrira egindako ordezkari harria eta lagina . . . . .	117
4.4. Topografia 3D neurketa; zeharkako SEM irudia . . . . .	118
4.5. Sorbatz-erradioaren neurketa 2D profila aztertuz . . . . .	119
4.6. APS saiakuntza prestaketa . . . . .	120
4.7. APS seinale adibideak irtengunearen zabaleraren arabera . . . . .	121
4.8. Habeen elastikotasun moduluen kalibrazioa graduarekiko . . . . .	122
4.9. DEM ereduaren altueraren azterketa FEM bidez . . . . .	124
4.10. Artezketa-endar normal espezifikoa . . . . .	126
4.11. Bermazal-kurba esperimentalak eta DEM ereduari sortutakoak . . . . .	127
4.12. KE eta ETL ereduari aldagaien garapena simulazioan zehar . . . . .	129
4.13. Kontaktu-luzera neurketak eta zenbakizko emaitzak . . . . .	130
4.14. Harriaren deformazioaren adibidea . . . . .	133

# Taulen Zerrenda

2.1. Arteketa kontaktu-luzera eredu nagusien bilduma . . . . .	33
2.2. APS saiakuntza-baldintzak literaturan . . . . .	40
4.1. Saiakuntza-baldintzak eta baliabideak . . . . .	115
4.2. DEM ereduaren sarrera-datuak . . . . .	123
4.3. Kontakto-luzera, sarketa maximo, zimurtasun-faktore eta ale- densitate emaitza esperimentalak eta zenbakizkoak . . . . .	131
4.4. Mailen esleipena eta simulazioetako emaitzak . . . . .	136
4.5. ANOVA taulak: kontaktu-luzera, sarketa maximoa eta ale-den- tsitatea . . . . .	137





# 1. Atala

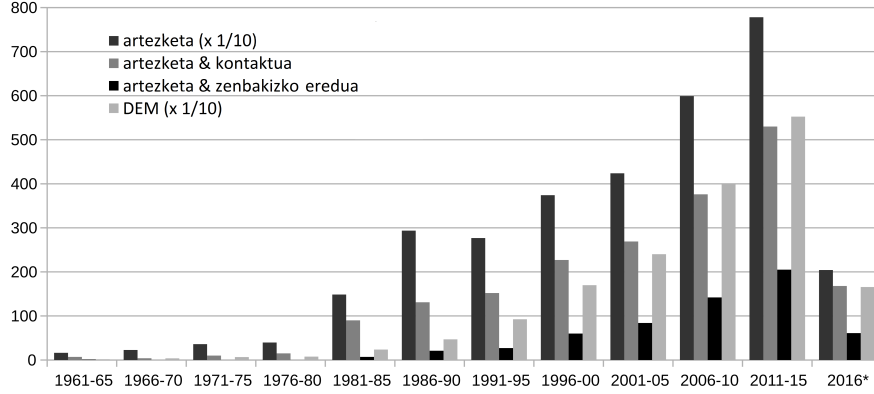
## Sarrera

### 1.1. Testuingurua

Artezketa dirauten fabrikazio-prozesu zaharretakoa izan arren, gaur egungo elementu mekanikoen fabrikazioan pisu nabarmena izaten jarraitzen du. Bere eraldaketaren arrakasta, akabera eta doitasun handiko piezen ekoizpenean ezinbesteko prozesu bihurtzean datza. Besteak beste, aeronautika eta autogintza sektoreek izan duten garapenaren erantzule izateaz gain, puntako errobotika eta mikro-ekoizpen prozesuetan ere nagusitasunez gailendu da.

Teknologiaren aurrerapenak fenomeno fisikoen oinarritzko ikerketetan oinarritu behar du. Horregatik, ez da harriztekoa azken 30 urteetan ikertzaileen eskutik artezketak jaso duen arreta, batez ere prozesuaren zenbakizko ereduen eraikuntzan eta simulazioan [KBH<sup>+</sup>02, BAG<sup>+</sup>06, DWB09]. Ikerkuntza-arloen artean, harriaren eta piezaren arteko kontaktuaren azterketak hirurogeigarren hamarkadaren hasieran hasi ziren, hurbilketa enpiriko eta erdi-enpirikoen eskutik. Ikertzaile nagusiek azpimarratu dute kontaktu-eremuak artezketan duen garrantzia [Sha96, MRDI04, Mal08]. Kontaktu-eremuaren definizioa prozesuen analisi guztien abiapuntua da. Material harroketa, higadura eta bero-sorkuntza fenomenoak kontaktu-gainazalean gertatzen dira. Harriaren egitura granularrak eta gainazal-topografia konplexuak eragiketaren modelatzea zailtzen du. Horregatik da kontaktua ikertzaile askoren jomuga artezketaren arloan, nahiz eta bere behaketa korapilatsua izan. Azken 56 urteetan *azterketa*, *zenbakizko eredu* eta *kontaktua* hitzak jasotzen dituzten ikerkuntza-artikuluen goranzko kopuruak, gaiak duen garrantzia azpimarratzen du (1.1. Ir.).

Gaur egungo harriak, harroketaz arduratzen diren ale urratzailez osatutako diseinuzko konglomeratu artifizialak dira. Hauskorragoa den aglomeratzaileak eusten ditu aleak, tartetean poroak eratuz. Kontaktu-luzerak eta -zabalerak definitzen dute kontaktu-azalera. Ohiko artezketa eragiketetan harriaren zabalera  $b_s$  konstante mantentzen denez, kontaktu-eremua zuzenean kontaktu-luzerarekin definitu ohi da. Artezketa-indarren eta ale-mailan dauden kontaktu-baldintzen ondorioz, harria eta pieza deformatu, eta beraien arteko kontaktu-luzera geo-



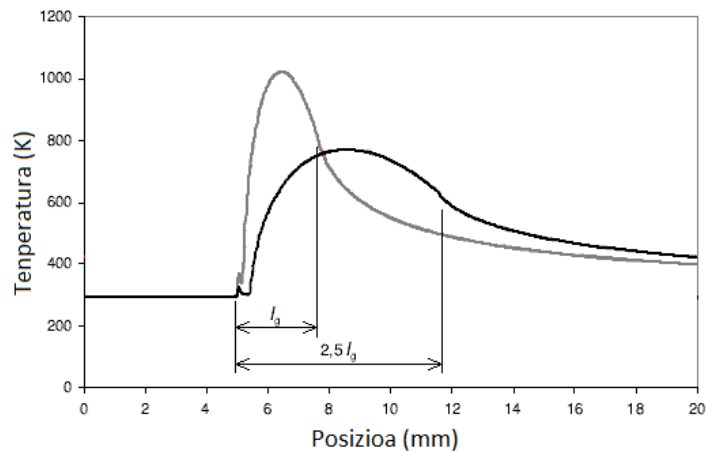
**1.1. Irudia:** Indextatutako aldizkarietan *grinding*, *contact*, *numerical model* eta *discrete element method (DEM)* hitz gakoak dituzten argitaratutako artikulu kopurua 1961. urtetik aurrera (\* 2016-11-01 arte; iturria: Scopus)

metrikoki ondorioztatutakoa baino luzeagoa da. kontaktu-luzeraren neurketa esperimentala zaila da, ia ezinezkoa industria-inguruetan.

Artezketa azterketa denek  $l_c$  kontaktu-luzeraren garrantzia azpimarratzen dute. Harria eta piezaren arteko kontaktu-luzera, alderdi termikotik nahiz mekanikotik aztertuz, arteztutako piezaren gainazalaren kalitatean eragiten duen faktore garrantzitsuenetakoa da. Kontaktuluzerak beroaren eta indarren banaketa-eremua definitzen duenez, zuzenki eragiten du piezara doan energia fluxuaren intentsitatean, lortutako tenperatura maximoan eta harriaren higaduran. Argi dago ale urratzaileen higadurak eta piezaren tenperaturak kontaktu-luzeraren eragina jasaten dutela.

Kontaktu-eremuaren ahalik eta hurbilpen zehatzenak erabiltzea ezinbestekoa da prozesuak simulatzeko orduan. Artezketa eredu termikoak kalte termikoa aurreikusteko erreminta interesgarriak dira. Orain dela gutxi arte, ikerlariek harria zurruntzat hartu dute deformazioak arbuatuz. Hainbat ikerlariek suposizio hori oker dagoela erakutsi dute [QRM01, MRDI04, Mal08, Han06, Row10, Pom10].  $l_g$  kontaktu-luzera geometrikoa (harri zurruna) erabiltzen duten eredu termikoek aurreikusitako tenperaturak altuagoak dira esperimentalki neurtutakoak baino. Adibidez, kontaktu-luzera  $l_g$  halako bi ( $l_c \approx 2 \cdot l_g$ ) den egoera batean,  $l_c$ -ren ordez  $l_g$  erabiliz gero, adierazpen analitikoek % 41 gain-tenperatura aurreikusten dute [MRDI04]. Antzeko emaitzak lortzen dira artezketa eredu termikoetan  $l_g$  sarrera-datu gisa erabiliz. Adibidez, 1.2. Irudiak zenbakizko eredu batek aurreikusitako piezaren gainazaleko tenperaturak aurkezten ditu. Bi kasuetan bero kantitatea berdina izanik, kontaktu-luzerak bero-fluxua edo intentsitatea aldatu eta, ondorioz, tenperatura maximoetan alde agertzen da.

Kontaktu-luzerak higadurarekin ere lotura estua dauka. Adhesio-higadura, kontaktuan dauden bi gorputzen konposaketaren, hozgarriaren eta mugalde-



**1.2. Irudia:** Zenbakizko eredu termikoan lortutako gainazaleko tenperaturak kontaktu-luzeraren funtzioan [Pom10]

baldintzen menpe dagoen fenomeno konplexua da. Archard-en arabera [Arc53], baldintza egonkorrean labaintzen diren material konbinaketa jakinarentzat, erlazioatuta daude adhesio-higadura tasa, indar normala, kontaktu-azalera erreal eta labainketa-distantzia. Verkerk-ek Archarden adierazpena harriaren higadura estimatzeko moldatzen du, indentazio erlazioak kontaktu-azalera erreal ordezkatuz [Ver77]. Nahiz eta Archarden adierazpenak ale-haustura eta -askatzea kontuan ez izan, artezketa-tasaren maldarekin bat egiten du birdiamantatzepuntuaren aurretik dagoen higadura konstanteko fasean.

Artezketa-indarrek ere kontaktu-luzerarekin lotura dute [Ono61]. Kontaktuluzerak indarrarekiko erlazio koadratikoa dauka.

Kontaktuluzera neurtzean lan esperimental arindu eta ezagutza teknologikoan sakondu nahian, zenbakizko simulazioak ikerketa-aukera berriak irekitzen ditu. Oro har, kontaktuaren analisisian hiru faktore nagusitzen dira: kontaktuan dauden gorputzen elastikotasuna, gainazalen prestaketa eta kontaktubaldintzak. Lan hau material metalikoen artezketara mugatuko da. Beraz, pieza homogenea eta isotropoa da. Harriaren ezaugarriak, berriz, guztiz kontrakoak dira: solido ez-jarraitu heterogeneoa da. Ondorioz, harriaren portaera mekanikoa deskribatzeko ingurumen jarraituen mekanikako ohiko zenbakizko erreminten eta hurbilpen analitikoaren erabilera zailtzen da. Piezaren gainazalzimurtasuna harriarena baino askoz ere txikiagoa denez, gainazal leuntzat har daiteke. Harriaren gainazalari dagokionez, topografia oso zimurra du. Ale-dentsitateak gainazalaren irekitasuna definitzen du eta artezketaren eraginkortasunean eragin zuzena dauka. Harriaren gainazala diamantaketa bidez prestatzen da. Ale-haustura eta -askatzea dira diamantaketa-mekanismo nagusiak. Diamantaketa-baldintzek mekanismoen intentsitatean eragiten dute, gainazal ireki bigunak edo itxi zurrinak sortuz.

Gorputzen elastikotasunaz eta zurruntasunaz gain, kontaktu-baldintzek eragin zuzena dute kontaktuan. Ohikoa izan da artezketan harriaren eta piezaren arteko kontaktua elastikotzat jotzea. Hala ere, urratzaileek materiala abiadura bizian eta tenperatura altuetan harrotzen dutenez, benetako fenomenotik urrun dago kontaktu elastikoa.

Teknikaren egoera berrikustean, elementu diskretuen metodoak (*discrete element method*, DEM) artezketaren simulazioan duen potentziala ikusi da. DEM-aren formulazio irekiak simulatu nahi den fenomeno fisikoari moldatzeko malgutasun ikaragarria eskaintzen du. Bere ausazko natura berezitu dela eta, harriaren ezaugarriak erreproduzitzeko erreminta egokia da. Gainazal zimurren arteko kontaktua simulatzeaz gain [JM12], harriaren egituraren eredu bat ere aurkeztu da [LYZW15a, LYZW15b]. 1.1. Irudian, DEM ereduaren erabilerak literaturan izan duen progresio esponontziala ere erakusten da.

Laburbilduz, harriaren morfologiaren eta zurruntasunaren portaera islatu, diamantaketak sortutako gainazal topografia deskribatu eta ale-pieza arteko kontaktu-baldintzak kontuan hartu behar ditu kontaktu-eredu idealak.

## 1.2. Helburuak eta ekarpenak

Tesi honen helburua, artezketa lauan kontaktuak dituen ondorioak aztertzea da, harriaren deflexioan, ale mailako indarretan eta itxurazko kontaktu-luzeran arreta ipiniz. Helburua, harriaren eta piezaren arteko kontaktua modelatuz eramango da aurrera, zehazki harriaren DEM zenbakizko ereduak garatuz. Eredua, beiratzatutako aluminazko harriekin altzairuen artezketa lauaren esparrura mugatuko da; etorkizunean bere erabilera beste harrietara eta artezketa-prozesuetara zabaltzeko aukera izango da. Harriaren DEM ereduak morfologia granular eta zurruntasun ezaugarriak biltzeaz gain, gainazal topografia sortzen du diamantaketa-baldintzen arabera. Tesian bi kontaktu-baldintza aztertzen dira. Alde batetik, Rowe-Qi ereduaren agertzen den harriaren eta planoaren arteko kontaktu estatiko zimurra simulatzen da [RMQZ93]. Gainazal zimurren arteko kontaktu-eredu gisa duen interesaz gain, bi ikertzaile horiek proposatutako kontaktu-arkuaren hurbilpena eta zimurtasun-faktore enpirikoa ebaluatze-ko erabiliko da. Eta bestetik, ebaki-gabeko txirbil-lodiera hurbilpenak, kontaktuan dauden ale urratzaileetako ebaketa-indarra estimatzen du txirbil-lodieraren funtzioan, zuzenean kontaktu-luzera lortuz. Bietan materialaren portaera biskoplastikotzat hartzen da. Mugalde-baldintzek indar zentrifugoaren eragina ere jasotzen dute.

Lan honen helburuak zehaztuz, honako erronka hauei erantzuten zaie tesian:

- harriaren gorputzaren portaera mekanikoa modelatzea, bere morfologia granularra eta elastikotasuna DEM eredu bidez erreproduzitzen,
- harriaren topografia aztertuz, ebaketa-sorbatzak karakterizatzea eta ale-dentsitatea neurtzea,
- harriaren zenbakizko ereduaren gainazal topografia islatzea diamantaketa-baldintzen arabera,

- alearen eta piezaren arteko kontaktu-baldintza bereziak zehaztea,
  - ale-pieza kontaktua zurrun-plastiko-perfektutzat hartzea,
  - ale-pieza kontaktuan piezaren materialaren portaera biskoplastikotzat jotzea,
  - Rowe-Qi kontaktu-ereduan deskribatutako kontaktu zimurraren modelizazioa, harriaren eta planoaren arteko kontaktu estatikoaren ereduak garatuz,
  - alearen aldiuneko txirbil-lodiera kalkulatzeko, artezketa ebaketa-sakonera kontuan hartuz,
- harriaren DEM ereduak, gainazal topografia eta ale-pieza arteko kontaktu-baldintzak konbinatzen dituen artezketa kontaktu-eredua garatzea, eta azkenik,
- zenbakizko ereduak esperimentalki balioztatzea, eta, aldagaiek kontaktuan duten eraginaren azterketa saiakuntzen diseinu faktorial bidez.

### 1.3. Edukiak

Lana bost ataletan banatuta dago. Bigarren atalak tesiak jorratuko dituen gaiei buruz jakintzak biltzen ditu. Arreta berezia ipini da kontaktu-luzera ereduetan, harriaren gainazal topografian eta kontaktu-mekanikan. Ezagutza hauek garatu den ereduaren oinarriak osatzen dituzte. Hirugarren atalak kontaktu-eredua sortzeko metodologia aurkezten du. Atala, harria-pieza kontaktua sistema tribologikoaren aldagaiak identifikatuz hasten da. Ondoren, DEM formulazioa moldatzen da harria eta kontaktu-baldintzak simulatu ahal izateko. Harriaren gorputzaren zenbakizko ereduaren eraikuntza, harriaren izendapenaren eta dimentsioen arabera egiten da. Diamantaketak sortutako topografia, harriaren zenbakizko ereduari nola aplikatzen den ere azaltzen da. Eredua osatzeko, ereduaren mugalde-baldintzak, kontaktu-detekzio algoritmoa eta kontaktu-legea zehazten dira eredu bakoitzarentzat. Laugarren atalak DEM ereduak aplikatzen du aurrez esperimentalki frogatutako baldintzetan. Simulazio emaitzek kontaktu-baldintzei eta harriaren deformazioari buruz informazio interesgarria eskaintzen du. Atalak, DEM ereduarekin egindako saiakuntzen diseinua ere aurkezten du, aldagai bakoitzak kontaktuan dituen eraginak ebaluatzeko. Azkenik, bostgarren atalak garatutako lanaren ondorioak biltzen ditu. Irekitako bidea etorkizunean jarraitzeko lan-ildoak ere proposatzen dira.



## 2. Atala

# Aurrekariak

*Aurreko mende hasieratik gaur arteko artezketarekin erlazionatutako makina ikerketa-lan aurkeztu dira. Atal honek, tesiarekin erlazio zuzena duten lan azpimarragarrienak biltzen ditu, kontaktu-ereduaren oinarri eta abiapuntu izan direnak.*

*Atala artezketa eta eragiketaren zinematika azaltzen hasten da. Diamantaketak sortzen du harriaren gainazala, eta topografiaren ezaugarriek (irekitasuna eta zorrotasuna) prozesuaren emaitzetan eragin zuzena dute. Orain arte proposatu diren kontaktu-luzera estimatzeko ereduak arreta berezia jaso dute. Kontaktu-ereduen berrikusketak kontaktu-eremuan eragiten duten aldagai garrantzitsuenak identifikatzen ditu: harriaren elastikotasuna eta harriaren gainazal topografia. Gehienek ale urratzaile-pieza kontaktua elastikotzat hartzen dute, nahiz eta errealitatean aleak materiala deformatu eta harrotu ere egiten duen. Horien artean ez dago zenbakizko eredurik, enpirikoak edo erdi-analitikoak dira guztiak. Horregatik berrikusi dira beste helburu batekin garatutako harriaren zenbakizko ereduak. Horien artean, interesgarriak dira harriaren morfologia eta elastikotasuna modelatzea helburu duten ereduak. Diamantaketak sortutako topografia deskribatzen duten eredu zinematikoek ere berrikuspen zabala jaso dute. Garatu nahi den kontaktu-eredua bi eredu mota horien arteko konbinaketa da: harriaren morfologiari topografia aplikatzen zaio. Kontaktu-mekanika kontzeptuak ere berrikuspenean sartu dira. Horiek ezinbestekoak dira ale urratzaileen eta piezaren arteko kontaktu-baldintzak definitzerakoan, arreta kontaktu-indarretan ipinita. Ale bakarraren akzioa igurzketa-, urradura- eta harroketa-mekanismoez osatzen da eta, horietan oinarrituz, ale bakarraren indar-ereduak garatu dira. Artezketan sorbatzek materiala abiadura bizian desplazatzen dute aldi berean beroa sortuz. Baldintza horietan metalek portaera biskoplastikoa erakusten dute.*

*Bildutako informazioa harriaren egituraren ereduak eraikitzen, diamantaketak sortutako topografia sortzen eta ale-pieza arteko kontaktu- eta mugaldebaldintzak finkatzen erabili da.*





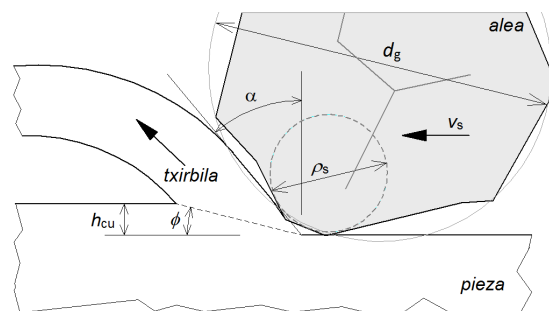
## 2.1. Sarrera. Artezketa eragiketa

Teknologia mekanikoaren barnean, prozesu urratzaileak txirbil-harroketa familia handiaren parte dira [Klo09]. Prozesu urratzaileak osatzen dituzte besteak beste, azterketa, leunketa, lapeaketa eta lixaketa. Prozesu horien ezaugarri komuna materiala harrotzeko mekanismoa da: urradura. Prozesu urratzaileei buruzko azterketa gehienak deterministak izan dira, zientzian baino esperientzian oinarritutakoak. Prozesu urratzaileak, teknologia ez, artea direla esan ohi da. Hala ere, prozesu hauek gaur egungo doitasun eta akabera fineko piezen ekoizpen mekanikoan giltzarri dira.

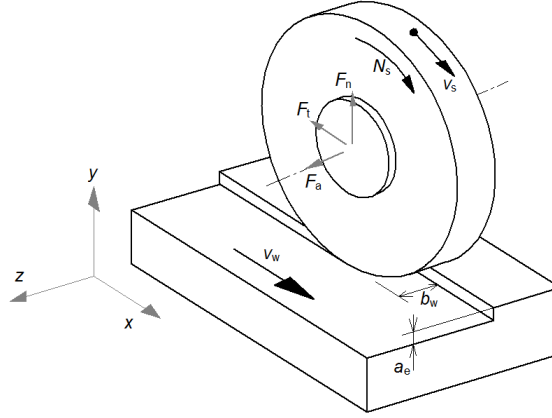
Artezketan erreminta harria da, aglomeratzailez eta ale urratzailez osatutako konglomeratu porotsua. Bi osagai horien konbinaketak harri aldaera anitz sortzen ditu. Txirbil-harroketa ale urratzaileen ardura da. Ale bakoitza neurri mikroskopikoa duen ebaketa-sorbatzat har daiteke. 2.1. Irudian ale bat lanean ikusten da. Alearen sorbatzak  $\alpha$  jaulkitze-angelu oso negatiboa dauka,  $-60^\circ$  eta  $-70^\circ$  artean [Sha96]. Horregatik, eragiketaren efizientzia energetikoa txikia da. Sortutako beroak pieza gainazalean tenperatura altuak eragiten ditu, kaltetzera ere irits daitekeelarik. Sorbatzak  $v_s$  ebaketa-abiadura azkarretan egiten du lan ( $> 20$  m/s) eta sakonera txikietan ( $h_{cu} < 10$   $\mu\text{m}$ ), abiadura handiko mekanizazioaren antzeko baldintzetan alegia. Horrela, artezketak gaurdaino bere esparru propioa mantendu du material gogorren eta hauskorren mekanizazioan, torneaketak eta fresaketak aplikazio mugatua duten eremuetan. Abiadura handian mekanizatzeko azken joeren aurrean lehiakortasuna ez galtzeko, artezketaren garapen berriak bultzatu dira. Horrela, *grind hardening*, *creep-feed* eta *quick-point* aldaerak azaldu dira.

Piezaren geometriaren arabera sailkatzen dira oinarritzko artezketa-eragiketak: artezketa lauak gainazal lauak lortzen ditu eta zilindrikoak, berriz, gainazal zilindrikoak. Profil-artezketan harriak piezaren profila hartzen du.

Harriaren gainazalak bi higadura mota jasaten ditu: ebaketa-sorbatzen higadurak sortutako higadura-planoak (*wear flat*) eta aglomeratzaile-zubien apurketek eragindako ale-askatzeak. Diamantaketak berreskuratzen ditu gainazalaren ebaketa gaitasunak, gainazaleko urratzaile geruza finak harrotuz.



2.1. Irudia: Ale urratzailearen sorbatza materiala harrotzen



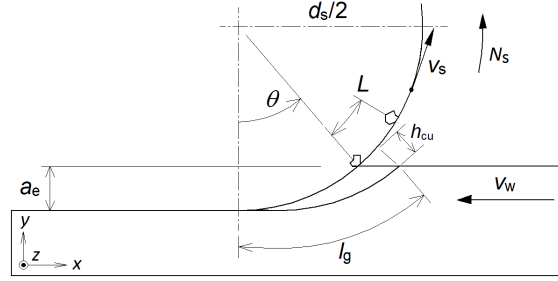
**2.2. Irudia:** Artezketa lauaren lan-parametro nagusiak eta artezketa-indarrak [MRDI04]

### 2.1.1. Artezketa lauaren zinematika eta indarrak

Ale baten ibilbidea aztertuz, analisi zinematikoak ebaki-gabeko txirbilaren geometria eta interakzio kopurua zehazten ditu. Artezketan, aldi berean, hainbat ale urratzailek parte hartzen dute. Aleek geometria ezberdina dutenez, halakoa izaten da harrotzen duten material kantitatea. Ondorioz, eskala makroskopikoan eta mikroskopikoan egindako azterketak kalkulu estatistikoetan oinarritzen dira.

2.2. Irudiak artezketa lauaren zinematika erakusten du. Harriak  $z$  ardatzaren inguruan biratzen du  $v_s$  ebaketa-abiaduran. Makinaren orgak aurrera eta atzera mugiarazten du pieza  $v_w$  aitzinamendu-abiaduran.  $a_e$  ebaketa-sakonera, harriak piezan harrotzen duen material kantitatea definitzen du.

Harria eta piezaren arteko elkarrekintzan indarra agertzen da. Indarra hiru osagaietan bana daiteke (2.2. Irudia):  $F_t$  ebaketa-indarra edo indar tangentziala,  $F_n$  indar normala eta  $F_a$  indar axiala. Osagai bakoitzak bere eran eragiten du eragiketean.  $F_t$  ebaketa-indarrak harriarekiko tangentean eta piezaren noranzkoan eragiten du. Bera da potentzia xahutzaile nagusia.  $F_n$  indar normala harriaren gainazalarekiko perpendikularra da. Indar normala tangentziala baino handiagoa izan ohi da. Indar normalaren ondorioz, makinaren ardatz nagusia deformatu, eta  $a$  makinaren kontrolean ezarritakoa baino txikiagoa izaten da  $a_e$  ebaketa-sakonera. Bukaeran sakonerarik gabeko txinparta-itxalketa iraganaldi gehigarriek lortzen dute desiratutako material geruza harrotzea.  $k_m$  makina-harria-pieza sistemaren zurruntasunaren portaera linealtzat hartuz, 2.1 Ekuazioak  $a_e$  ebaketa-sakonera eraginkorra estimatzen du. Hala ere, elementu guztiek ez dute portaera lineala lan-baldintza guztietan [MRDI04]. Azkenik, indar axiala  $F_a$  profil-artezketan edo harriak alboko mugimendua duenean agertzen da.



**2.3. Irudia:** Artezketaren mikro-fresaketa analogia: ondoz-ondoko bi aleen ibilbidea eta ebaki-gabeko txirbilaren geometria [Ald1914]

$$a_e = a - \frac{F_n}{k_m} \quad (2.1)$$

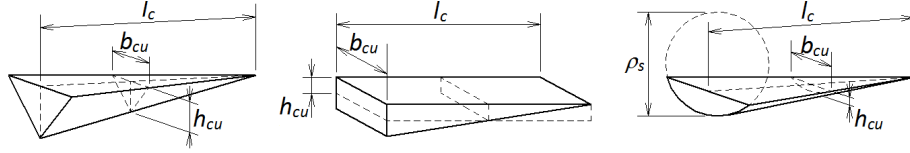
Alor energetikoan, material harroketa are eta eraginkorragoa da ahalik eta energia kantitate txikienarekin egiten bada. Eraginkortasuna ebaluatzeko,  $E_s$  energia espezifiko erabiltzen da alderantzizko neurri gisa.  $E_s$ -k harrotutako bolumen unitateko xahututako energia adierazten du. Artezketara ekarriz, mekanizazio-potentziaren eta denbora unitateko harrotutako material bolumenaren arteko zatiketak definitzen du  $E_s$ . 2.2 Ekuazioak  $E_s$  estimatzen du  $F_t$  indar tangentialaren,  $v_s$  ebaketa-abiaduraren,  $b_w$  piezaren zabaleraren,  $a_e$  ebaketa-sakonkeraren eta  $v_w$  aitzinamendu-abiaduraren funtzioan [MRDI04].

$$E_s = \frac{F_t \cdot v_s}{b_w \cdot a_e \cdot v_w} \quad (2.2)$$

Mikro-fresaketaren analogiak artezketa ulertarazten laguntzen du. Mikro-fresaketaren analogiak ale urratzaile bakoitza ebaketa-sorbaztat hartzen du [Ald1914]. Era honetan, sorbatz bati egindako analisia, harri osora estrapolatzen da. 2.3. Irudiak artezketaren mikro-fresaketa analogia aurkezten du. Irudiak bata-bestearen segidan dauden bi ale urratzailek piezaren gainazalean egiten dituzten ibilbide zirkularrak erakusten ditu. Aleak ibilbidean zeharkatzen duen material bolumenaren geometriari ebaki-gabeko txirbil deritzo. Jasaten dituzten deformazio eta tenperatura handien ondorioz, txirbilek errealtatean geometria oso ezberdina dute [Mal08]. Aleak zeharkatzen duen bolumenetik zati bat piezatik askatu ere ez da egiten, alboetara desplazatuz [ÖC12].

Ondoz-ondoko bi aleen zinematikaren azterketak, txirbil ebaki-gabearen geometria eta kontaktu-luzera definitzen ditu. Kontakuzko luzera (edo txirbil-luzera), harria eta pieza, biak zurruntzat hartuz ondorioztatzen da. Lortutako balioari  $l_g$  kontaktu-luzera geometrikoa deritzo eta soilik  $d_s$  harriaren diametroaren eta  $a_e$  ebaketa-sakonkeraren funtzioan dago (2.3 Ek., 2.3.).

$$l_g = \overline{AB} = \sqrt{d_s \cdot a_e} \quad (2.3)$$



**2.4. Irudia:** Ebaki-gabeko txirbil-geometria alearen zeharkako profilaren arabera: triangeluarra, errektangeluarra eta zirkularra [MRDI04]

Txirbil-lodiera aldagai garrantzitsua da ale mailako artezketa-indarrak eta higadura-portaera aztertzean. Ondoz-ondoko bi aleen ibilbidea aztertuz (2.3. Irudia), 2.4 Ekuazioak ale baten  $h_{cu}$  ebaki-gabeko txirbil-lodiera kontaktu-luzerari zehar definitzen du. 2.5 Ekuazioak, berriz, txirbil-lodiera maximoa  $h_{cu,max}$  estimatzen du  $L$  aleen arteko distantziaren funtzioan. Ohiko adierazpen honen inplizituki txirbil-zabalera handia barneratzen du eta, ondorioz, oso txirbil-lodiera finak aurreikusten ditu. [Row10].

Backer *et al.* [BMS52] arazoa gainditzen dute, aleen sorbatzek zeharkako profila triangeluartzat hartuz eta  $L$  gainazaleko  $G$  ale-dentsitatearekin hurbilduz. Horrela,  $r_{cu}$  ratioa definitzen dute, ebaki-gabeko txirbilaren zabalera eta lodieraren arteko erlazioa ( $b_{cu}/h_{cu}$ ) (2.6 Ek.). Ale urratzaileak piezaren gainean utzitako ildasken zeharkako profilak neurtuz, txirbil-zabalera lodiera baino 8-15 aldiz handiagoa dela baieztatzen dute. 2.6) Ekuazioa beste ale profiletara ere molda daiteke, hala nola, txirbil errektangeluarrera [Mal08]. 2.7 Ekuazioak hurbilpen hori aurkezten du,  $r_{cu}$   $b_{cu}$  txirbil-zabalerarekin berdinduz. Era berean, 2.8 Ekuazioak, berriz,  $h_{cu,max}$  adierazpena sorbatz zirkularrentzat egokitzen du [CR96]. Lan honetan, adierazpen horretako ale diametroa, zorrotasuna deskribatzen duen  $\rho_s$  sorbatz-erradioagatik ordezkatu da (ikusi 2.1.3. Atala). 2.4. Irudiak ebaki-gabeko txirbil-geometriak alearen profilaren (triangeluarra, errektangeluarra eta zirkularra) arabera erakusten ditu.

$$h_{cu} = L \frac{v_w}{v_s} \sin \theta \quad (2.4)$$

$$h_{cu,max} = 2L \frac{v_w}{v_s} \sqrt{\frac{a_e}{d_s}} \quad (2.5)$$

$$h_{cu,max} = \sqrt{\frac{6v_w}{Grv_s}} \sqrt{\frac{a_e}{d_s}} \quad (2.6)$$

$$h_{cu,max} = \frac{2v_w}{Gr_{cu}v_s} \sqrt{\frac{a_e}{d_s}} \quad (2.7)$$

$$h_{cu,max} = \left[ \frac{15v_w}{8Grv_s} \sqrt{\frac{a_e}{2\rho_s d_s}} \right]^{\frac{2}{3}} \quad (2.8)$$

Formula hauetan,  $v_w$  eta  $v_s$  aitzinamendu- eta ebaketa-abiadurak dira,  $\theta$  posizio angelua,  $a_e$  ebaketa-sakonera eta  $d_s$  harriaren diametroa. Txirbil-lodieraren aldaketa lineala da kontaktu-luzeran zehar, zerotik balio maximora ( $h_{cu,max}$ ). Ale-profil triangeluarrak zirkularrak baino txirbil lodiagoak aurreikusten ditu.

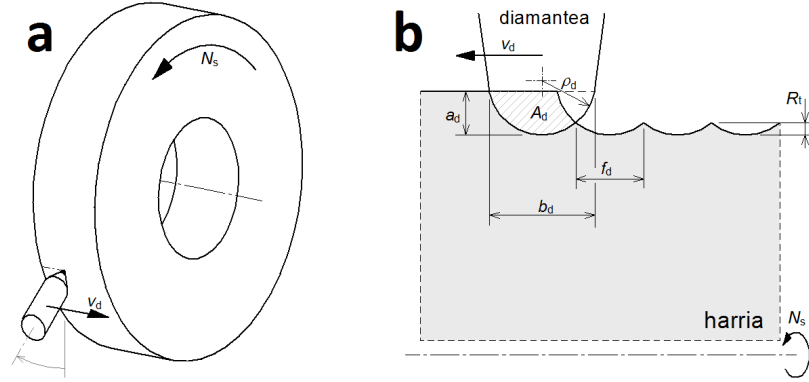
Rowek bere liburuan [Row10]  $h_{cu}$  adierazpenak moldatzen ditu harriaren deflexioa kontuan hartzeko. Kontaktulu-geometria erabilera zuzentzen du kontaktu-luzera ratioa ( $l_c/l_g$ ) ekuazioetan barneratuz; horrek  $h_{cu}$  murrizten du.

### 2.1.2. Diamantaketa

Azterketa-harria jardunean higituz doa. Gainera, piezatik harrotutako txirbilek harriaren gainazaleko hutsuneak betetzen dituzte, txirbila sortzeko beharrezkoa den lekua murriztuz. Harriaren gainazalaren prestaketa bi eragiketen esku dago, harriaren geometria (*truing*) eta aleen ebakitzeko ahalmena (diamantaketa, *dressing*) berreskuratzeko. Ingelesez bi eragiketa horiek bereizten diren arren, euskarak, ohiko aplikazioetan biak aldi berean egiten direnez, biak diamantaketa izendatzen ditu. Diamantaketa erreminta diamantezkoa izan ohi da, baina boro nitruro kubikoa (CBN), silizio karburoa eta burdinurtua ere erabiltzen dira. Diamanteak naturalak edo sintetikoak, eta, kristal-bakarrak edo kristal-anitzak izan daitezke.

Diamantaketa aldaerak zinematikaren arabera sailkatzen dira: diamantaketa finkoa eta birakaria. Disko birakaria finkoa baino eraginkorragoa da. Diamantaketa birakarian, diamanteek harrian eragindako talka-kopuru handiak aitzinamendu azkarragoak ahalbidetzen ditu. Diamante finkoak punta-bakarrak edo -anitzak izan daitezke. Aurreko arrazoi berdinetatik, aitzinamendu azkarragoak erabili daitezke diamante punta-anitzekin punta-bakarrekin baino. Aldiz, diamante punta-bakarrekin diamantatzean, parametro zinematikoak eta diamantearen geometria ezagunak dira eta kontrolpean daude. Horrek oinarritzko diamantaketa-mekanismoen azterketa ahalbidetzen du, ikertzaileen arreta irabaziz. Gainera, diamante punta-bakarra oso erabilia da industrian. Horregatik tesi honetan garatutako ereduak diamantaketa finko punta-bakarrera mugatu da. Jarrantutako metodologia, aurrerago diamante anitzetara eta birakarietara moldatu ahal dira.

2.5. Irudiak diamante punta-bakarrekin diamantaketa finkoaren zinematika erakusten du. Mugimenduak torneaketaren antzekoak dira: aldagai nagusiak  $a_d$  diamantaketa-sakonera eta  $f_d$  diamantaketa-aitzinamendua dira. 2.9 Ekuazioak  $f_d$  definitzen du, 2.10 Ekuazioak, berriz,  $u_d$  teilakapen-ratioa [Mal08]. Ekuazio hauetan  $d_s$  harriaren diametroa da,  $v_d$  diamantaketa abiadura axiala eta  $b_d$  harriaren eta diamantearen kontaktu-zabalera  $a_d$  sakonera.  $u_d$ -ren eta  $a_d$ -ren konbinaketak diamantaketaren intentsitatea adierazten du, eta intentsitateak, lortutako gainazalaren irekitasuna.  $u_d$  balio txikiek (< 2-3) gainazal irekiak lortzen dituzte; balio altuek, berriz, ( $u_d > 7-10$ ) gainazal itxiak. Nolanahi ere, balio horiek diamantearen erradioaren eta ale-neurriaren menpekoak ere badira.



**2.5. Irudia:** Bi irudiek diamantaketa finkoaren zinematika erakusten dute: (a) harria biratzen eta diamantea zeharka mugitzen; (b) ebaketa-sekzioaren detailea [MRDI04]

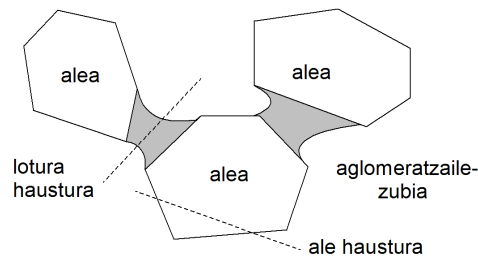
$$f_d = \frac{\pi d_s v_d}{v_s} \quad (2.9)$$

$$u_d = \frac{b_d}{f_d} \quad (2.10)$$

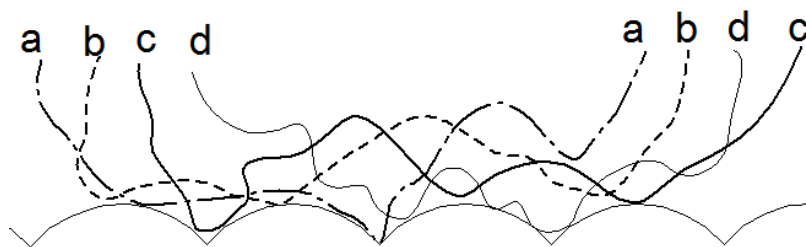
### Diamantaketa-mekanismoak

Marruskadura dinamikoak eta urratzaile-aglomeratzaile haustura hauskorrak diamantaketaren analisia zailtzen dute. Literaturan, adostasuna da nagusi diamantaketa-mekanismoak eta honen emaitzak deskribatzean. Egile gehienek [PA54, Vic70, MC71, Bha74, Che95, Hes03, MRDI04, KL08] 2.6. Irudiak azaltzen dituen bi mekanismo nagusiak aipatzen dituzte: eragindako pitzaduren ondorioz urratzaile haustura, eta ale osoak askatzen dituen aglomeratzaile-zubien haustura. Klockek eta Linkek [KL08] bigarrena bitan banatzen dute: alea urratzaile-aglomeratzaile interfazetik askatzea eta aglomeratzaile-zubiaren haustura. Nolanahi ere ondorioa berdina da: alea askatzen da. Diamantearen talka-sekzioak, -kopuruak eta -intentsitateak gainazaleko ale urratzaileen forma eta sorbatzen egoera baldintzatzen dute [BC95].

Pahlitzsch-ek eta Brunswick-ek [PA54], eragiketa sinplifikatu nahian, diamanteak urratzaileak era harikorrean mozten dituela proposatzen dute. Haztagailuz egindako topografia neurketetan, diamantearen ibilbidearen aztarnarik ezin da antzeman. Piezaren gainazalean, berriz, helizearen arrastoak zimurtasunean islatzen dira [BCP72],  $f_d$  diamantaketa-aitzinamenduaren funtzioan. 2.7. Irudiak diamantaketaren ondorioak erakusten ditu. Emaitza, haustura handi eta mailakatze (*levelling*) efektuen konbinaketa da. Oliveira *et al.* [OBF10]



**2.6. Irudia:** Ale haustura eta aglomeratzaile-zubi haustura [Mal08]



**2.7. Irudia:** Diamanteak ondoko ondoko hainbat planotan kokatutako aleetan sortutako profilak [CR96]

fenomeno horretaz baliatzen dira, piezaren gainazalean zimurtasun-testura jakinak lortzeko. Horretarako, diamantea kitzikatzaile elektromekaniko bati lotu, eta zenbakizko kontrolaren seinaleak patroia jakin batzuk sortzen ditu harriaren gainazalean.

Malkin-ek eta Cook-ek, harriaren higaduraren azterketa zabalaren barnean, diamantaketan jasotako partikulen neurri-banaketa aztertu dute [MC71, Mal08]. Partikula denek dute aleak baino neurri txikiagoa eta, pisu proportzioan, tamaina handienekoak dira urratzaile harroketaren erantzule nagusia. Horrela, diamantaketa bi etapetan gertatzen dela ondorioztatzen dute: ale urratzaileak apurtuz doaz eta, neurri kritiko bat lortzean, aglomeratzailetik askatzen dira. Gradu gogorragoko harrietan ale haustura gehiago bultzatzen da, loturak gogorragoak baitira. Ale-neurri txikietan berriz, lotura-zubietatik askatzea ohikoagoa da, urratzaileak zailtasun handiagoa baitauka.

Bi etapen prozesuak inspiratuta, diamantaketan aleak askatzeko duen probabilitate-eredua garatzen dute. Abiapuntua, aglomeratzailetik askatutako urratzaile proportzioa da, hau da, gainazalaren irekitasuna. Ereduak diamantaketak sortutako gainazalaren irekitasuna estimatzen du. Horretarako bi parametro berri proposatzen ditu:  $d_b$  batz besteko askatze ale-neurria eta  $B$  aglomeratzailetik askatutako urratzaile proportzioa pisuan.  $d_b$ -k diamantaketak sortutako kaltearen sakonera islatzen du. 2.11 Ekuazioak bi kontzeptuak erlazionatzen ditu, non  $d_g$  batz besteko ale-neurria den. Erro kubikoa, eredu hiru dimen-

tsiotan garatzen delako azaltzen da.  $G_o$  ale-dentsitate maximoa, plano batek harria moztean zeharkatzen duen ale kopurua da. 2.12 Ekuazioak  $G_o$  estimatzen du  $V_g$  urratzaile frakzioaren funtzioan. Diamantearentzat eskuragarri ez dauden aleak aglomeratzaile hausturagatik askatu direla pentsatuz, 2.13 Ekuazioak  $G_{\text{dyn}}$  ale-dentsitate dinamikoaren eta  $G_o$  maximoaren arteko erlazioa definitzen du. Ekuazioak  $G_{\text{dyn}}/G_o$  erlazioa, dimentsio batean diamantagarria den luzeraren eta ale-neurriaren arteko erlazioarekin estrapolatzen du. 2.11 Ekuazioa 2.13 Ekuazioan ordezkatzuz,  $G_{\text{dyn}}$ ,  $G_o$  eta  $B$  aldagaien arteko erlazioa lortzen da.

$$d_b = B^{1/3} d_g \quad (2.11)$$

$$G_o = \frac{6V_g}{\pi d_g^2} \quad (2.12)$$

$$\frac{G_{\text{dyn}}}{G_o} = \frac{d_g - d_b}{d_g} = 1 - B^{1/3} \quad (2.13)$$

Berez,  $G_{\text{dyn}}$  diamantatzeko eskuragarri dauden aleen sakoneran ( $d_g - d_b$ ) neurtu beharko litzateke. Aldiz, Malkinek eta Cookek  $G_{\text{dyn}}$  balioa arteztean aleen higadura-planoen isla kopurua zenbatuz hurbiltzen dute, nahiz eta harria piezarekin kontaktuan deformatu. Diamantaketa-partikulen neurri banaketaren azterketatik  $d_b$  nola ondorioztatzen den ez dute zehazten. Hala ere, eredia topografia neurketa soilarekin aplikatu daiteke, diamantaketa-partikulen neurri-banaketa azterketen beharrik gabe. Nolanahi ere, diamantaketa-partikulen neurri-banaketa azterketa bereziki interesgarria da diamantaketa-mekanismoen analisirako.

Chen-ek bere tesian [Che95], ale-askatzea diamantaketa-indarren ikuspuntutik aztertzen du. Alea askatzeko, diamantaketa-indarrak lotura-zubien erresistentzia gainditu behar du, harriaren graduarekin lotura duena. Froga esperimentalen bidez ondorioztatzen du  $F_d$  diamantaketa-indarrak bi osagai dituela: haustura-indarra  $F_f$  eta marruskadura-indarra  $F_r$ . 2.14 Ekuazioak Chenen eredia aurkezten du, non  $\sigma_s$  harriaren erresistentzia,  $A_d$  diamantaketa-sekzioa,  $a_d$  sakonera eta  $f_d$  aitzinamendua diren (2.5. Ir.).

$$F_d = F_f + F_r = \sigma_d \cdot A_d + F_r = \sigma_d \cdot a_e \cdot f_d + F_r \quad (2.14)$$

$\sigma_s$  harriaren erresistentzia,  $F_d$  neurketa esperimentalen maldak aztertuz estimatzen da. Aleak iraungo du, baldin eta  $F_d$ -k alearen euste indarra  $F_g$  (2.15 Ek.) gainditzen ez badu.  $A_g$  harriaren irauten duen alearen sekzioa da. Ereduek ale-askatze frakzio txikiak lortzen ditu.  $\sigma_s$  definitzea erraza izan arren, ereduak ez dauka erlazioz gainazal topografiarekin.

$$F_g = \sigma_s \cdot A_g \quad (2.15)$$

Torrance-k eta Badger-rek [TB00] diamantaketa-iraganaldia iraungo duen ale frakzioaren  $f$  eredia garatzen dute ale-dentsitatea  $G_{\text{dyn}}$  estimatzeko (2.16 Ek.). Beraien ereduak diamantaketa-baldintzak ( $a_d$  diamantaketa-sakonera,  $f_d$



aitzinamendua eta  $\rho_d$  diamantearen puntaren erradioa) eta harriaren ezaugarri mekanikoak ( $F_o$  aglomeratzaile-zubien erresistentzia karakteristikoa eta  $\sigma_c$  urratzailearen zapaltze-erresistentzia) erabiltzen ditu. Osoagoa da eta faktore bakoitzaren pisuak diamantaketaren intentsitatea definitzen du. Hala ere, zaila da aglomeratzailearen eta urratzailearen erresistentzia ezaugarriak definitzea. Gainera, eredu honek ere ez dauka gainazal topografiarekin erlaziorik.

$$f = \frac{42\sigma_c d_g f_d}{F_o \sqrt{\rho_d}} \quad (2.16)$$

### 2.1.3. Harriaren gainazal topografia

Harriaren topografia, gainazaleko geometria eta ezaugarriak deskribatzen dituen arloa da. Diamantearen eta geruza urratzailearen arteko interakzioaren ondorio da gainazal topografia.

Makro- eta mikro-topografia, bakoitza bere ikuspuntutik, biak dira garrantzitsuak. Harriaren oinarriko geometriak (normalki zilindrikoa) makro-topografiarekin dauka lotura eta piezan sortutako gainazalaren desbideratzetan eragiten du. Mikro-topografia, berriz, artezketaren eraginkortasunarekin lotzen da; besteak beste, piezaren gainazalaren zimurtasunean, xahututako energian, harroketa-ratioetan, zorrotasunean eta harriaren bizitzan eragiten du. Mikro-topografiak zehazten ditu sorbatzen zorrotasuna eta banaketa (gainazal irekitasuna edo aleen arteko distantzia eta sakonera). Harriaren topografiak honako faktore hauen eragina jasotzen du: ale-neurria, harriaren egitura (urratzaile frakzio bolumetrikoa), diamantaketa-baldintzak eta urratzaileak arteztean jasandako higadura. Harrien ezaugarrien konbinaketak anitzak dira eta, ondorioz, haien gainazalek ere aniztasun hori islatzen dute. Topografia deskribatzen duten aldagaiak eta ezaugarriak 2.3.2. Atalean deskribatzen dira, topografia-ereduekin batera.

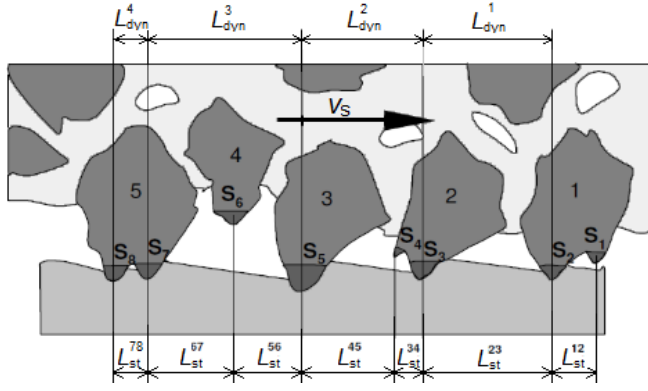
### Topografia neurketa teknikak

Neurketa-teknikek lotura zuzena dute neurtu beharreko parametroekin eta karakterizatu nahi den gainazalaren konplexutasunarekin. Aldiro gaiari buruzko berrikusketa-artikuluak argitaratu izanak, harriaren topografia neurtzeko metodoek duten interesa erakusten du [Ver77, Sha96, MRDI04, Dar13]. Gainazalak antzemateko metodoak neurketa zuzenak eta ez-zuzenak multzotan sailkatzen dira [MRDI04]. Metodo zuzenak, harriaren gainazala zuzenean behatzen duten metodo fisikoak dira: haztatze-teknikak, sistema optikoak eta elektronikoak. Metodo ez-zuzenek, topografiaren ondorioak era ez zuzenean aztertzen dituzte, indarrak, potentzia, tenperaturak edo zarata neurtuz. Era berean, neurketa-teknikak estatikoak edo dinamikoak izan daitezke. Estatikoetan, harria geldidagoela egiten da neurketa, profilaren trazak antzemanaz (haztatzea, mikroskopia optikoa edo elektronikoa, ikatz-papera). Dinamikoek, berriz, soilik artezketan parte hartzen duten aleak kontatzen dituzte. Dinamikoetan harria hutsean

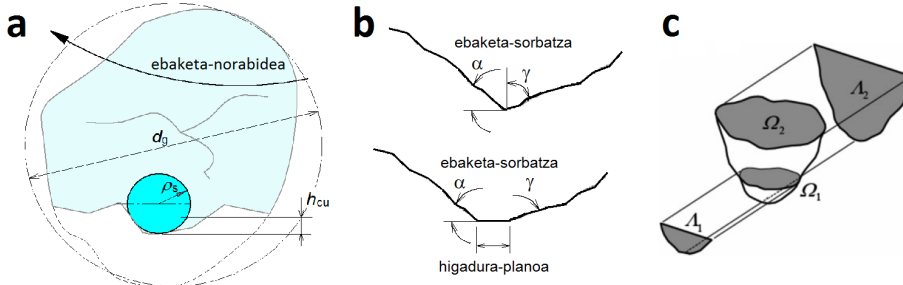
biraka (emisio akustikoak), artezten (termoparea, bizar-xafla) edo arteztu ondoren geldi dagoela (argi islatua higadura-planotan) neurtzen da.

Topografia, parametro bakarraren edo hainbat parametroren bidez karakterizatu daiteke, ereduaren dimentsioarekin bat eginez (1D, 2D eta 3D). Parametro bakarrak, adibidez ale-dentsitateak, zorrotasunak edo higadura-plano ehunekoak (*wear flat*), gainazalari buruzko informazio interesgarria eskaintzen du. Neurketa-metodo ez-zuzen dinamikoek gainazalaren irekitasuna ebaluatzen dute, ale-dentsitatea definituz. 2D eta 3D neurketek topografia era deskriptiboagoan azaltzen dute: 2D-ak zeharkako ebaketaren profila atzematen du; 3D-an, berriz, topografia gainazal-funtzio batez erreproduzitzen da, atzemandako datuak interpolatuz. 2D eta 3D gainazal-neurketak 1D parametroak definitzeko erabiltzen dira, hala nola, ale-dentsitatea, sorbatzen sakonera eta zorrotasuna. Dimentsio fraktalen bidez ere deskribatu daitezke harrien gainazalak [Lia95]. Hala ere, dimentsio fraktalak ez dauka erlazio zuzenik urratzaileen ezaugarriekin eta, hortaz, ez du gainazalaren artezteko ahalmena adierazten [HS95].

Verkerk-ek, CIRPek lankidetzan egindako lanean [Ver77], ordura arteko topografia neurketa-metodoak biltzen eta alderatzen ditu. Lan haren ondorioetako bat da, ale berdinari dagozkion sorbatzak edo oso gertu dauden aleen artean daudenak, sorbatz bakartzat har daitezkeela. Ebaketa-sorbatzak hain gertu egotean, beraien artean ez dago txirbila sortzeko beharrezkoa den tartea. Horrela, sorbatz-dentsitatean arreta ipini beharrean, soilik ale-dentsitatea kontuan hartzea gomendatzen du. Era horretan, ale urratzaileak material harroketan duten parte-hartzearen arabera sailkatzen ditu: ale estatikoak eta ale dinamikoak (edo zinematikoak, edo aktiboak). Ale estatikoak, tontorraren sakoneraren arabera definitzen dira. Aldiz, tontor arteko distantziak eta aleen deflexioak baldintzatzen dute ale dinamikoen parte-hartzea harroketan. 2.8. Irudiak ale estatikoen eta dinamikoen arteko aldea agertzen du [Klo09]:  $v_s/v_w$  abiadura-erlazioaren arabera, ale bereko edo oso gertu dauden sorbatzek ez dute lekurik txirbila sortzeko. Irudiak ez du aleen deflexioaren efektua deskribatzen. Haztatze 2D profil neurketak nagusitzen dira  $L_{st}$  aleen arteko bataz besteko distantzia estatikoa definitzean.  $L_{st}$ , eredu enpirikoak aplikatuz,  $G_{st}$  ale-dentsitate estatikoa estimatzeko erabiltzen da. Ikatz-paper teknikak ere  $G_{st}$  neurtzen du, diaman-tatu berria den harria ikatz-paper gainean biratzean utzitako arrasto kopuruak kontatuz. Hala ere, ikatz-paperarekin ezin da  $G_{st}$  sakonerarekiko zehaztasunez atzeman. Bestalde, higadura-planoen kontaketa eta bizar-xafla metodo dinamikoak dira parametro dinamikoak neurtzeko metodo nagusiak. Bizar-xafla metodoan, xafla estu bat arteztean aleek sortutako  $N_c$  indar-tontorrek kontatzen dira. Horrela,  $L_{dyn}$  (2.17 Ek.) eta  $G_{dyn}$  (2.18 Ek.) estimatzen dira  $N_c$  alderatzean  $v_s$  ebaketa-abiadurarekin eta  $t_c$  kontaktu-denborarekin [BS74]. Ekuazio horietan  $b_s$  bizar-xaflaren (edo piezaren) zabalera da. Metodoa, ale-pieza kontaktua detektatzean seinale bat igortzen duen edozein metodorekin aplikatu daiteke, hala nola, aplikatutako potentzial-iturriarekin (*applied power source*, APS). Artezketan sortutako txirbilak bilduz eta kontatuz ere  $G_{dyn}$  estimatu ahalko litzateke, baina piezarekin kontaktuan dauden ale guztiek ez dute materiala harrotzen.



**2.8. Irudia:** Sorbatzen arteko distantzia estatikoaren eta dinamikoaren ( $L_{st}$ ,  $L_{dyn}$ ) definizioa abiadura-erlazioaren ( $v_s/v_w$ ) arabera [Klo09]



**2.9. Irudia:** Sorbatzen zorrotasunaren karakterizazioa: (a) sorbatzaren hurbilpena  $\rho_s$  erradioko zirkunskribatutako zirkunferentziarekin [Mas52], (b) geometria jakina duen sorbatzaren analogia ebaketa-angeluen bidez [Klo09], eta, (c) proiektio horizontalen eta bertikalen arteko erlazioak [DPS02]

$$L_{dyn} = \frac{t_c v_s}{N_c} \quad (2.17)$$

$$G_{dyn} = \frac{N_c}{t_c v_s b_s} \quad (2.18)$$

Zorrotasunak, sorbatzek edo tontorren geometriak materiala harrotzeko duten gaitasuna ebaluatzen du, eta higadurarekin ere lotura du. Tontorrek eta horiek gainazalean duten banaketak urraduran duten ekarpena, artezketaren alderdi iluna, baina interesgarria osatzen dute, nagusiki erakusten duten aldakortasunagatik. Hainbat hurbilpen proposatu dira ebaketa-sorbatzen zorrotasuna karakterizatzeko (2.9. Ir.); besteak beste, biribiltasun gradua edo sorbatzerradioa, ebaketa-angeluak, eta, kono-doikuntza analisisa edo proiektio-ratioa.

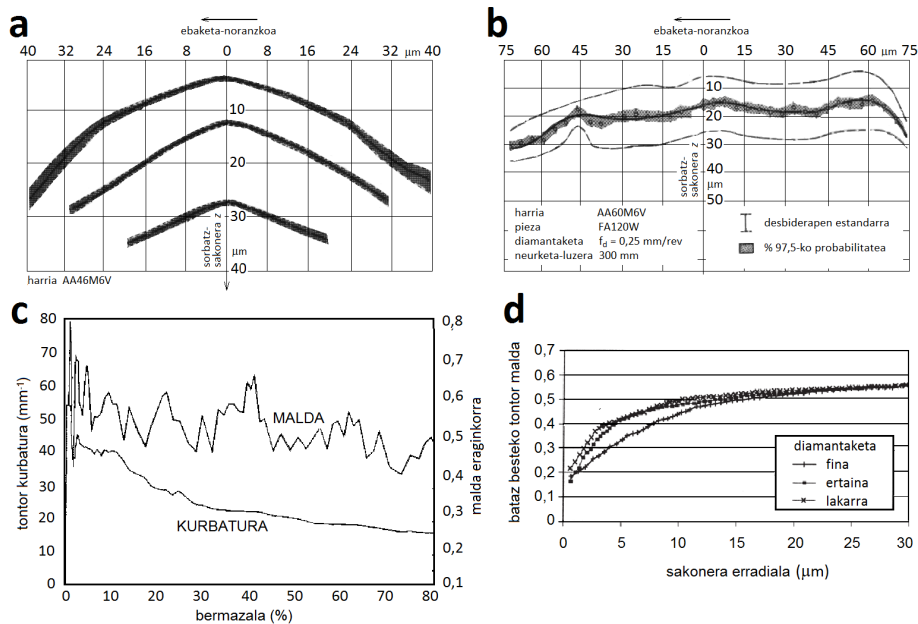
Lehen biak literaturan aipatuak izan dira diamantatutako ohiko harrietan zorrotasuna definitzerakoan. Sorbatz-erradioak zorrotasuna, tontorrean zirkunskribatutako zirkuluaren  $\rho_s$  erradioarekin adierazten du (2.9.(a) Ir.) [Mas52].  $\rho_s$ -k higadura islatu dezake ale-neurriarekiko biribiltasun-graduarekin, tontorraren eskalarekin eta urradura-sakonerarekin. Diamantatutako harrietan tontorraren aukeraketa begi bistakoa da, irtenena; baina tontorraren erradioa zehazteko irizpideak lausoak dira. Zorrotasunaren bi definizio proposatu dira: alearen eta sorbatzaren diametro erlazioa ( $d_g/(2\rho_s)$ ) [BT93], eta, ebaki-gabeko txirbilodieraren eta zorrotasun erradioaren arteko zatiketa ( $h_{cu}/\rho_s$ ) [Klo09].

Sorbatza geometria jakina balu bezala ere deskriba daiteke, torneaketan eta fresaketan antzera. Horrela, jaulkitze- eta azpijan-angeluak ebaketa-sorbatza definitzen dute (2.9.(b) Ir.). Arteketa harrietan,  $\alpha$  jaulkitze-angeluren batz besteko balioa [Klo09] edo malda [TB00] nahikoa da zorrotasuna karakterizatzeko. Higaduraren efektua ere eredura gehitu daiteke higadura-planoa gehituz. Ohikoak dira jaulkitze-angelu oso negatiboak arteketan ( $> 60^\circ$ ). Angeluen definizioan ere ez dago adostasunik erreferentzia sakonera aukeratzean.

Kono-doikuntza analisisan, proiektzio horizontalaren eta bertikalaren arteko ratioak ( $\Lambda/\Omega$ ) definitzen du zorrotasuna (2.9.(c) Ir.). Balio hori sakonerarekiko eta alearen orientazioarekiko aldatzen da. Teknika hau arrakastaz erabili da ale urratzaile osoen zorrotasuna karakterizatzeko, CBN eta diamante aleentzat. Aldiz, ez da aplikatu diamantatutako alumina harrietan [DPS02]. Diamantatutako gainazalek patroi jakin bat jarraitzen dutenez, ikerketa-ildo interesgarria izango litzateke kono-doikuntza analisisa beiratzatutako alumina harrietara zabaltzea.

Nahiz eta zorrotasuna arteketa-errendimenduan parametro garrantzitsua izan, bere neurketa esperimentalak gutxi deskribatu da literaturan. Lal-ek eta Shaw-ek [LS75]  $\rho_s$  neurtzen dute, ale bakarrak sortutako arrastoaren zeharkako zabalera-sakonera erlazioa aztertuz.  $\rho_s$ -n sakonerak soilik eragiten duela baieztatzen dute. Bestalde, aitzinamendu- eta ebaketa-abiadurek, pieza materialak eta hozgarriak ez dute eragin nabarmenik  $\rho_s$ -n.  $\rho_s$  balio altuak antzematen dituzte, 75 - 500  $\mu\text{m}$  artean. Ziur aski balio altu horiek, ale bakarraren frogak diamantatutako aleen ezaugarriak erreproduzitzen ez dituelako agertzen dira.

$\rho_s$  eta sorbatz-malda, haztatze 2D profilak aztertuz ere neurtu dira [KL75, BT93]. König-ek eta Lortz-ek [KL75] harriaren perimetroaren bosten bat haztatzen dute, neurketa-luzera adierazgarria lortuz. Ez dute zorrotasuna zuzenean definitzen, baina ebaketa-sorbatzen eta gainazaleko aleen batz besteko profilak aurkezten dituzte (2.10.(a,b) Ir.). Sorbatz-profilek  $\rho_s$  eta  $\alpha$  erraz estimatzea ahalbidetzen dute; ale-profilek, berriz, gainazal laua erakusten dute diamantearekin izandako elkarrekintzaren ondorioz (ikusi 2.1.2. Atala). Brenner-ek eta Torrancek [BT93] gainazalaren hainbat 2D profil aztertuz, sakonerarekiko sorbatzen maldak eta kurbadurak ( $1/\rho_s$ ) neurtzen dituzte. Emaitzak, arteketa-indarretan zorrotasunak duen eragina aztertzeke erabiltzen dira. Indar-frogen ondoren, harria azetonan bustita pasarazten da azetato laminen gainetik gainazal topografia erreplika lortzeko. Ondoren, azetato laminetan profila haztatzen da. Maldaren balioak bermazal-ratioarekin aldatzen dira. Kurbadura maximoak, aldiz, tontorretan antzematen dira, sakoneran barneratzean txikitzen de-



**2.10. Irudia:** (a) Ebaketa-sorbatzen eta (b) gainazaleko aleen batz besteko profilak [KL75]; (c) arteztu ondoren behatutako tontorren malda eta kurbatura bermazal-ratioaren funtzioan [BT93], eta, (d) tontor malda diamantaketa-baldintzen funtzioan [BT00]

larik (2.10.(c) Ir.). Beste lan batzuk ere, malda sakonerarekin erlazionatzen dute haztagailuz 2D profil neurketa zuzenak eginaz [Sha96, BT00, TB00]. 2.10.(d) Irudiak aurkezten ditu maldaren aldaketak sakonerarekiko eta diamantaketa-baldintzekiko [BT00]. Hiru diamantaketa-baldintzek hasieran antzeko malda kamutsa erakusten dute. Baina nahikoa da azaleko sakoneran barneratzea, maldetan hazkuntza bortitzak antzemateko. Diamantaketa larriaren malda zertxobait handiagoa da sakonera txikietan, ale-dentsitatearekin batera, material harroketa errazten duena. Sakonera batetik aurrera ( $12 \mu\text{m}$ ) hiru baldintzek bat egiten dute malda konstantea bihurtuz. Neurketa guzti horiek ausazko 2D profil neurketetatik datoz, haztagailua tontorrekin lerrokatuta egon beharrik gabe.

Inork ez ditu konparatu ale bakarraren frogen arrastoen [LS75] eta 2D profilen [BT93]  $\rho_s$  neurketen emaitzak. Lehenak alearen ondorioak neurtzen ditu arrastoaren zabaleraren arabera, bigarrenak berriz zuzenean gainazaleko 2D profilak ebaketa-noranzkoan.

3D topografia neurketak (mikroskopia optikoa, 3D haztatzea, ekortze mikroskopia elektronikoa (*scanning electron microscope*, SEM), berriz, harriaren gainazal zatiak behatzen ditu, nabardura ia guztiak jasoaz [DWB13]. Eskaintzen duen informazio gordina, batez ere 1D parametroak definitzeko eta 2D zeharkako profilak lortzeko erabiltzen da. Kontaktu-zurruntasunarekin lotura

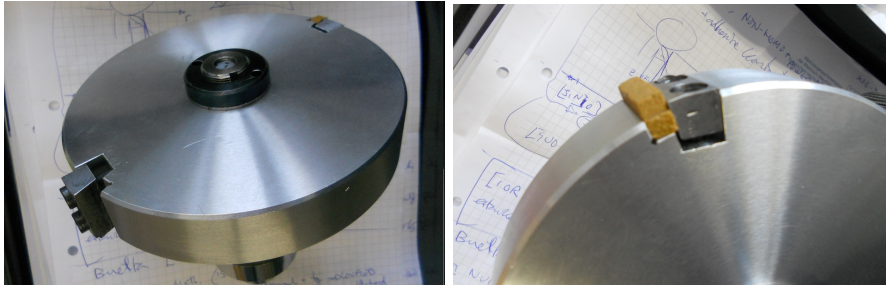
estua duten gainazalaren bermazal-kurbak ere eskaintzen dituzte. Informazio gordina piezaren gainazal zimurtasuna aurreikusten duten eredu zinematikoen sarrera ere bada [WZ98]. Harriaren gainazaleko hutsuneek eta malda pikoek 3D neurketetan arazoak sortzen dituzte. Gainera, mikroskopia optiko fokukideak argi islatzeekin arazoak ditu, helburu den gainazaletik kanpo dauden puntuak antzemanaz. Datu-tratamendu softwareak konpontzen ditu arazo horiek, interpolazioz 3D gainazala sortuz. Beharrezkoa den lagin tamaina txikiak eta harriaren gainazal irekiak, SEM teknikaren zabalpena mugatzen du artezketa arloan.

Kontaktu-gabeko teknika optikoen artean autofocus sistemak, argi zuriko interferometroak eta mikroskopia fokukideak daude. Inasaki-k [Ina96] objektuan islatutako argiaren intentsitatean oinarritutako autofocus teknika erabiltzen du. Horrela, foku-distantziak sorbatzen altuera definitzen du. Yan-en eta Darafonen lantaldeek [YRJZ11, DWB13] argi zurizko interferometroarekin neurtzen dute harriaren gainazala. Argi zuriaren islapenaren eta erreferentzia mugikorren arteko interferentzia patroia aztertuz zehazten da puntu baten altuera. Zhang-ek *et al.* [ZXF06] mikroskopia optiko fokukidea aplikatzen dute, leku ezberdinetatik hartutako eremu berdinen bi argazki alderatuz. Puntuen altuera triangelaketa bidez kalkulatu da. Gainazalak gutxienez % 15-eko islapena izan behar du, baina gehiegizko islapenak ere arazoak sortzen ditu. Ezaugarri nagusiak bereizmen bikaina (0,1 nm) eta sakonera tarte zabalak dira.

Ohikoa da, kontaktu-gabeko tekniken neurketa-bolumen txikia dela eta, gainazalaren erreplikak edo lagin prestaketa suntsitzaileak behar izatea. Darafonek *et al.* [DWB13] gainazala harriaren diametro osoan neurtzea lortu dute neurri-egindako tresneriarekin. Lagina handia denez, lortutako emaitzak esanguratsuak dira, hala nola, bermazal-kurba, ale-dentsitatea eta ale arteko distantzia fidelki atzemanaz. Teknika ez da sentikorra azaleko sakoneretan, nahiz eta behe-pasuko iragazkia aplikatu. Hala ere, lortutako emaitzek harriaren gainazal-topografiaren ausazko natura islatzen dute.

Gainazalaren erreplikak islatze arazoak saihesten ditu. Baina errepliken erabilera nagusiki neurri arazoek edo gainazal-aldaketak monitorizatzeko harria makinatik askatu ezinak bultzatzen dute [MRDI04]. Erreplikaren materiala konposatu plastikoa edo metal biguna izan ohi da. Erreplika materiala konposatu plastikoa bada, harrian aztertu nahi den eremuan aplikatzen da. Behin onduta, aztertu nahi den gainazalaren negatiboa lortzen da. Negatiboa molde gisa erabiliz, positiboa lortu daiteke antzeko konposatuarekin. Bai erreplika negatiboa, baita positiboa ere, biak haztagailuz edo mikroskopia optikoz aztertu daitezke lagin neurriaren arazoak saihestuz. Harriekin arrakastaz honako konposatu plastikoak aplikatu dira: urean oinarritakoak [Eck96], epoxi erretxinak [Geo79] eta bi kautxuzko konposatuak [CR04]. Erreplika materiala bigunegia bada ezin da haztagailuz neurtu. Ale neurria 80 baino handiagoa bada, gainazaleko hutsuneak ere handiak dira eta erreplika puskatzen da ateratzean. Berun biguna gainazalean kontra sakatuz ere erreplikak lortzen dira [BBR80]. Aplikazio indarraren eta denboraren kontrol zorrotza eskatzen du teknika horrek.

Aacheneko RWTHn diamantatutako CBN harri-laginak lortzeko ordezkoko harria edo *dummy wheel* teknika garatu da. 2.11. Irudiak ordezkoko harriaren ar-



**2.11. Irudia:** Ordezko harria edo *Dummy wheel* eta CBN harri lagina (Iturria: RWTH Aachen)

gazkiak erakusten ditu. Laginen neurria egokia da edozein neurketa-tresnetan behaketa zuzenean egiteko, SEM barne. Teknika honen desabantaila artezten aritu diren laginak ezin direla lortu da, hau da, higadura eta txirbilak ezin dira aztertu. Hala ere, ez dago higadura atzeman dezakeen erreplika teknikarik.

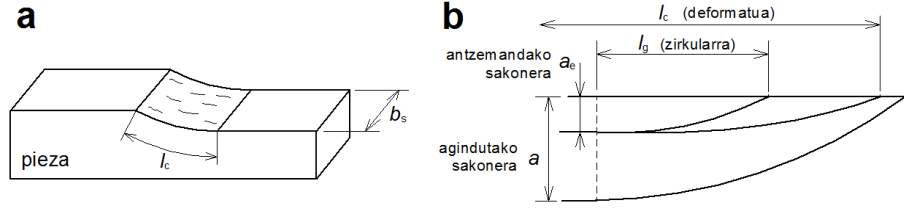
## 2.2. Artezketa kontaktu-luzera

Sarreran azaldu da artezketa lauan,  $b_s$  harriaren zabalerak eta  $l_c$  kontaktu-luzerak definitzen dutela kontaktu-azalera (2.12.(a) Ir.). Ia kasu guztietan  $b_s$  konstante mantentzen denez, kontaktu-eremua zuzenean kontaktu-luzeraren baitan definitzen da. Zinematika berrikustean,  $l_g$  kontaktu-luzera geometrikoa harria eta pieza, gainazalak leuntzat eta solidoak zurruntzat hartuz ondorioztatu da (2.3 Ek.). Hala ere, froga esperimentalek  $l_c$ ,  $l_g$  baino 1,5 eta 3 aldiz luzeagoa dela erakutsi dute.

Artezketan, aleen eta piezaren arteko interakzioan sortutako indarrek harria-pieza bikotea deformatzen dute, kontaktu-luzera handituz. 2.12.(b) Irudiak kontaktu-eremuan biltzen diren efektuak erakusten ditu. Alde batetik, harria eusten duen makinaren burua makurduraren ondorioz deformatu eta, ondorioz,  $a_e$  iraganaldi sakonera eraginkorra  $a$  agindutakoa baino txikiagoa da. Horrek kontaktu-luzera zuzenki murrizten du. Bestalde, jasandako indarren ondorioz harria era elastikoan deformatzen da kontaktu-luzera luzatuz. Brown-ek *et al.* [BSS71] piezaren deformazio elastiko berreskuragarriak eta deformazio plastiko iraunkorrak bereizten dituzte, harriak indar-banaketa berdina jasaten baitu. Hiruen konbinaketak itxurazko kontaktu-luzera handitzen du.

Artezketan kontaktua ez-konformea da, eta gainera, soilik materiala harrotzen duten sorbatzak daude kontaktuan. Beraz, itxurazko kontaktu-azalera da 2.12.(a) Irudiak definitzen duen eremua. Artezketa arloan hainbat egilek itxurazko  $l_c$  kontaktu-luzerari *kontakto-luzera erreala* deritzote, nahiz eta tribologia ikuspuntutik izendapen hori oker egon. Lan honetan  $l_c$  sinpleki *kontakto-luzera* deituko da.

Kontaktu-luzera neurketa esperimentaletatik, honako faktoreak nabarmendu



**2.12. Irudia:** (a) Kontaktu-luzeraren definizioa artezketan [MRDI04]; (b) deformazioak harria-pieza interfasean [BSS71]

dituzte hainbat egilek [GW88, ZV92, QRM97b]:

- artezketa lehorrean kontaktu-luzera motzagoa da hozgarria erabiliz baino, marruskadura jaisten baita,
- aitzinamendu-abiadura azkarragoek kontaktu-luzera handitzen dute indar normalen handitzearen ondorioz,
- ebaketa-abiadura azkarragoek kontaktu-luzera arinki laburtzen dute,
- harriaren diametroa handitzeak kontaktua luzatzen du,
- ebaketa-sakonera, artezketa-indarrak eta tenperatura handitzeak (hirurak erlazionatuak daude)  $l_c/l_g$  ratioa handitzen du, eta,
- harriaren gainazala leuntzean (diamantaketa-baldintza finak, ale-neurri txikia, higadura, kamustea edo kargatzea), indar normala handitu arren, kontaktu-luzera murrizten da.

### 2.2.1. Kontaktu-luzera ereduak

Gaur egun arte argitaratutako kontaktu-luzera ereduak, ikuspuntu enpirikotik eta erdi-analitikotik heldu dute arazoa. Eredu enpirikoak dira sinpleenak [ZV92, STH83, ZV92]. Lindsay-ek [Lin71] malguki-eredua proposatzen du. Eredu erdi-analitikoak [SS74, KS81, ZSNW93a] partzialki Hertzzen mekanikan oinarritzen dira: gainazal leunak eta kontaktu elastikoa. Eredu erdi-analitiko batzuk pausu bat aurrera ematen dute kontaktua bi eskaletan aztertuz [BSS71, RMQZ93, HWCC11]: alea-pieza eta harria-pieza eskalak. 2.1. taulak artezketa kontaktu-luzera eredu nagusiak biltzen ditu.

Maris-ek [Mar77] aurkeztu du lehen eredu enpirikoa,  $l_c$ -ren adierazpena  $a_e$  ebaketa-sakonerearekiko eta  $v_s/v_w$  abiadura-ratioarekiko doituz. Ereduak aplikazio-eremu mugatua dauka, ez baititu topografiaren ezaugarriak eta harriaren zurruntasunak kontuan hartzen. Salje-ren *et al.* [STH83] ereduak, ebaketa-sakonera gain, piezaren  $R_a$  gainazal zimurtasuna erabiltzen du, harriaren topografia ezaugarriak inplizituki barneratuak dituena. Hala ere, eredu horrek harriaren elastikotasuna arbuatzen du. Zhou-k eta Van Lutterwelt-ek [ZV92]



2.1. Taula: Artezketa kontaktu-luzera eredu nagusien bilduma

Eredua	$l_c/l_g$	Kontaktu-luzera $l_c$	Oharrak
Maris [Mar77]	2	$\left(\frac{v_s}{v_w}\right)^{C_1} e^{C_2 \ln a_e} \sqrt{a_e d_s}$	Empirikoa
Zhou, Van Lutter-welt [ZV92]	2 - 3	$C_1 + \sqrt{a_e d_s}$	Empirikoa
Salje <i>et al.</i> [STH83]	1,1 - 1,4	$\sqrt{1 + \frac{R_a}{a_e}}$	Gainazal zimurtasuna
Lindsay, Hahn [Lin71]	1,33	$10^{-6} \sqrt[3]{\frac{d_s^2 F_n}{d_g^2 (1,33HL + 2,25L - 8)}}$	Malguki elastikoen eredua
Sauer, Shaw [SS74]	1,3 - 1,7	$\sqrt{1 + 4 \cdot \frac{E^* F_n + (\delta - h_{cu})}{a_e}} = \sqrt{a + C_1 \frac{F_n^{C_2}}{a_e^{C_3}}}$	Hertz, $h_{cu}$ txiribil-lodiera eta $\delta$ indentazioa
Kumar, Shaw [KS81]	1,1 - 1,2	$\left(1 + \frac{C_1 (1 + \nu_s) d_e F_n}{E_s l_g a_e}\right)^{1/2} \cdot \left(1 + \frac{2 F_n l_n \ln(l_g/2)}{\pi E_s a_e}\right)^{-1/2}$	Hertzen mekanika
Zhang <i>et al.</i> [ZSNW93a]	1,5 - 2,7	$R_d \arccos\left(1 - \frac{a_e}{R_d}\right)$ non $R_d = d_e \left(1 + C_1 \cdot \frac{(1 - \nu_s^2) F_n}{E_s a_e}\right)$	Hertzen mekanika, emaitza iteratiboa
Brown <i>et al.</i> [BSS71]	2 - 2,9	$2 \left(\frac{9(E^*)^2}{8d_g}\right)^{1/3} \left(\frac{F_n}{C_{stat} l_c}\right)^{1/3} + 1,6 \left(\frac{E^* F_n}{a_e}\right)^{1/2}$	Hertzen mekanika, kontaktua bi eskaletan
Rowe <i>et al.</i> [RMQZ93]	1,5 - 3	$\left(a_e d_s + \frac{8 F_n F_r d_s}{\pi E^*}\right)^{1/2}$	Kontaktua bi eskaletan, GT eredua
Horn <i>et al.</i> [HWCC11]	1,4 - 2,2	$\left(a_e d_s + \frac{8 F_n d_s}{\pi C_1 E^*}\right)^{1/2}$	Rowe-Qi eredua kontaktu elastoplastikoarekin

oraindik ere eredu sinpleagoa proposatzen dute: kontaktu-luzera konstante esperimental baten eta kontaktu-luzera geometrikoaren arteko batura da. Ereduaren emaitzak neurketetan lortutakoekin ongi doitzen dira eta interesgarria gertatzen da makina-harri-pieza konbinazio jakinarekin erabiltzeko. Kontaktuluzeraren neurketan dagoen ziurgabetasunari buruzko gogoeta interesgarria gehitzen dute haien lanean.

Lindsay-ek [Lin71] malguki eredu aurkezten du. Gainazaleko ale bakoitzari malguki bat dagokio, eta, horien zurruntasuna  $HL$  harriaren graduaren eta  $SL$  harriaren egituraren funtzioan dago. Ereduak ale-neurria eta indar normala hartzen ditu kontuan, baina ebaketa-sakonera arbuatzen du. Lindsayen ereduak harriaren elastikotasunak eta topografiak kontaktuan duten garrantzia azpimarratzen du.

Literaturan hainbat eredu analitiko-enpiriko aurkitu dira. Zhang-ek *et al.* [ZSNW93a] kontaktu-azaleraren handitzearen ardura bereziki harriaren deformazio elastikoari esleitzen diote. Deformazio-kurba eliptikotzat hartuz, deflexio-kurbaren funtzioa bitan diferentziatzen dute kontaktu-luzera lortzeko. Parametro esperimental bakar batek topografia eta beste parametroen eragina doitzen du, froga bakarrarekin definitzen dena.

Shawek, Sauer-rekin [SS74] eta Kumar-rekin [KS81] batera, Hertzen kontaktumekanika oinarritzen diren adierazpenak proposatu dituzte. Haientzat, harria gainazal leuna duen zilindro homogeneo jarraitua da eta kontaktua elastikotzat jotzen dute. Biak koefiziente enpirikoez baliatzen dira ereduaren emaitzak neurketekin doitzeko. Biek harriaren elastikotasun modulua, indar normala eta ebaketa-sakonera barneratzen dituzte adierazpenetan. Aipagarria da Sauerrek batz besteko  $h_{cu}$  txirbil-lodiera eta  $\delta$  alea piezan barneratutako sakonera erabiltzea bere ereduaren, nahiz eta hiru konstante esperimental behar dituen horien balioak hurbiltzeko. Kumarren ereduarekin lortutako emaitzak kontaktu-luzera geometrikoa baino % 15 luzeagoak dira, motzegiak emaitza esperimentalekin konparatuz.

Brownek *et al.* [BSS71] eta Rowe eta Qi bikoteak [RMQZ93] kontaktua bi eskaletan aztertzen dute efektuak konbinatuz: alde batetik, banako ale urratzailearen eta piezaren arteko kontaktua eta, bestetik, harriaren eta piezaren arteko deflexioa. Era horretan, harriaren eta piezaren zurruntasunak kontuan hartzeaz gain, harriaren topografiaren eragina ere barneratzen du. Brownek [BSS71] harriaren elastikotasun modulua, ale-neurriak, ebaketa-sakonera eta ale-dentsitateak osatzen duten iteratu beharreko adierazpen konplexua garatzen du. Ez du koefiziente enpirikorik behar baina lortzen dituen emaitzak neurketa esperimentaletan lortutakoak baino motzagoak dira, ziur aski kontaktua elastikotzat hartzearen ondorioz. Rowek eta Qik [RMQZ93] Brownen eredu moldatzen dute, Greenwood-en eta Tripp-en (GT) [GT67] gainazal zimurren arteko kontaktu-eredua inplementatuz eta kontaktu-arkuen azterketa geometrikoa gehituz. Harriaren topografia  $R_r$  zimurtasun faktorean barneratzen da, esperimentalki neurtzen dena.

Roweren eta Qiren formulazioan oinarrituz, Horng-ek *et al.* [HWCC11] haiek garatutako mikro-kontaktua analisia aplikatzen dute. Ereduak zimur esferikoak eta gainazalaren arteko kontaktua elastoplastikotzat hartzen dute. Nahiz eta

Hornen eredia gaurdainoko osotuena izan,  $R_r$  zimurtasun faktore esperimentalaren neurketaren beharra mantentzen du eta lortzen duen adierazpena konplexuegia bere erabilpena zabaltzeko.

Laburbilduz, honako ezaugarri hauek dira kontaktuan garrantzitsuenak, hau da, gehien errepikatzen direnak berrikusitako eredueta: topografia parametroak [BSS71, RMQZ93, HWCC11], ebaketa-sakonera [BSS71, SS74, Mar77, KS81, STH83, ZSNW93a, RMQZ93, HWCC11], indar normala [Lin71, BSS71, SS74, KS81, ZSNW93a, RMQZ93, HWCC11], eta, harriaren eta piezaren elastikotasuna [Lin71, BSS71, SS74, KS81, ZSNW93a, RMQZ93, HWCC11]. Horietaz gain, abiadura erlazioa [Mar77], piezaren zimurtasuna [STH83] eta ebaki-gabeko txirbil-lodiera [SS74] ere agertu dira. Kontuan hartu ez diren faktoreak koefiziente enpirikoetan barneratzen dira eredu guztietan, [BSS71, Lin71] salbu.

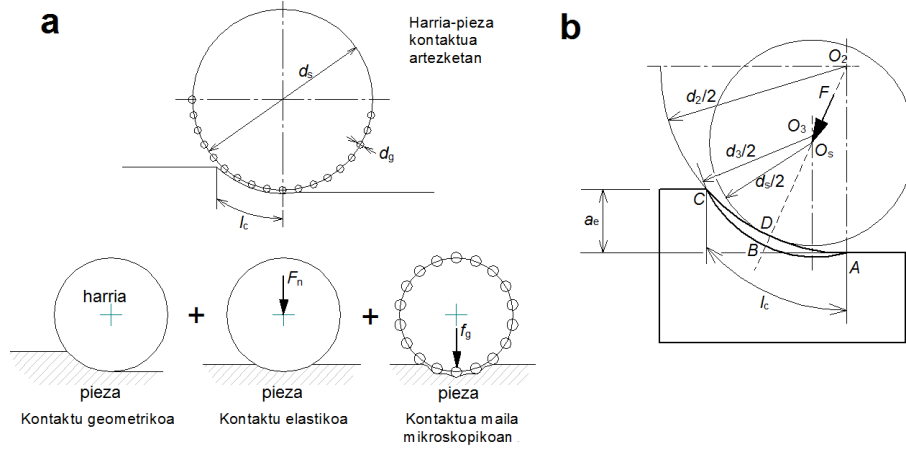
### Rowe-ren eta Qi-ren kontaktu-luzera eredia

Ziur aski, Rowek eta Qik (RQ) aurkeztutako eredia da arteketa kontaktu-eredurik onartuena eta zabalduena. Eredua hainbat argitalpenetan aurkezten dute, tartean [RMQZ93, QRM97b]. Abiapuntua Brownen eredia da [BSS71]: kontaktu-luzera, kontaktu-luzera geometrikoaren eta harria-planoa kontaktu estatikoaren arteko batura koadratikoa da. Baina kontaktu estatikoaren terminoa, Greendwood-Tripp (GT) gainazal zimurren arteko kontaktu-ereduarekin kalkulatu dute [GT67]. GT ereduak zimur esferikoen eta planoaren arteko kontaktu elastikoak estatistikoki lantzen ditu. RQ ereduak, arkuen azterketa geometrikoaren bidez hartzen du kontuan ebaketa-sakonerearen eragina. Horrela, kontaktu-luzeraren kalkulua hiru zati ditu (2.13.(a) Ir.): harria zurruntzat hartuz ebaketa-sakonera sortutako kontaktu-luzera geometrikoa, harriaren eta pieza planoaren arteko kontaktu elastiko leuna, eta, aleen eta piezaren arteko mikro-kontaktu elastikoak.

2.13.(b) Irudiak kontaktu-arkuen azterketa geometrikoa aurkezten du. Kontaktua elastikotzat eta gainazalak leuntzat hartuz, harriak  $d_s$  eta piezak  $d_2$  diametroak dituzte karga gabe daudenean. Biak kontaktuan jartzean, kontaktukurbak  $d_3$  diametroa hartzen du. Karga pean  $l_c$  kontaktu-luzerak  $ABC$  kurbarekin bat egiten du. Arteketa bat-batean etenez gero, piezak atzera bere  $d_2$  kurba berreskuratuko luke. Orduan,  $A$  puntua  $ADC$  kurbarekiko tangentea da eta  $a_e$  ebaketa-sakonera eraginkorra. Alde batetik, Hertzen kontaktu-mekanikaren arabera [Her1882], zilindroen arteko kontaktu-luzera  $l_c$  (2.19 Ek.), non  $F'_n$  luzera unitateko indar normala eta  $E^*$  elastikotasun modulu baliokidea diren. 2.20 Ekuazioak  $E^*$ -ren kalkulua azaltzen du harria eta piezaren Poissonen moduluen ( $\nu_s$ ,  $\nu_w$ ) eta elastikotasun moduluen ( $E_s$ ,  $E_w$ ) funtzioan.

$$l_c = \sqrt{\frac{8F'_n d_s}{E^*}} \quad (2.19)$$

$$\frac{1}{E^*} = \frac{1 - \nu_s}{E_s} + \frac{1 - \nu_w}{E_w} \quad (2.20)$$



**2.13. Irudia:** Harriaren eta piezaren arteko deflexio elastikoaren azterketa: (a) hiru efektuen konbinaketa eta (b) kontaktu-arkuen azterketa geometrikoa [RMQZ93]

Artezketaindarrak harriaren diametroa  $d_s$ -tik  $d_3$ -ra handitzen du. 2.21 Ekuazioak harria-pieza bikotearen diametro erlazioa adierazten du, Herten gainazal zilindriko ahurren erlazioan oinarrituz. 2.13.(b) Irudia aztertuz, kontaktu-luzera piezaren  $d_2$ -rekiko denez, honako balioa hartzen du (2.22 Ek.):

$$d_3 = \frac{d_2 d_s}{d_2 - d_s} \quad (2.21)$$

$$d_2 = \frac{l_c^2}{a_e} \quad (2.22)$$

2.19, 2.21 eta 2.22 Ekuazioak konbinatuz, eta, deflexioa elastikotzat eta gainazala leuntzat hartuz, harriaren eta piezaren arteko kontaktu-luzeraren adierazpena lortzen da (2.23 Ek.):

$$l_c = \left( a_e d_s + \frac{8 F_n' d_s}{\pi E^*} \right)^{1/2} \quad (2.23)$$

2.23 Ekuazioa berriidatziz 2.24 Ekuazioa lortzen da.  $l_g$  ebaketa-sakoneraren geometriarekin erlazionatutako kontaktu-luzera da eta  $l_f$ , berriz, indarraren ondorioz sortutako deformazioarekin.

$$l_c^2 = l_g^2 + l_f^2 \quad (2.24)$$

Indar tangentialaren eragina arbuaitzen dute Johnsonen liburuan oinarrituz [Joh87]. Lortzen duten adierazpenak Brownek [BSS71] garatutakoarekin bat

egiten du; aldaketa  $l_f$  terminoaren tratamenduan dator.  $l_f$ -k, Hertzen kontaktu-mekanikan oinarrituz, bi gainazal leun zilindrikoen arteko kontaktu elastikoa adierazten du. Rowek eta Qik  $l_f$  terminoa GT ereduaren gainazal zimurren arteko kontaktu elastikoagatik ordezkatzeko dute. GT ereduak gainazal zimurren arteko kontaktu elastikoa bi eskaletan aztertzen du: alde batetik, bi gorputzen kontaktu elastikoa gainazalak leuntzat hartuz (makro-eskala) eta, bestetik, laztasun esferikoaren arteko kontaktua (mikro-eskala). Bietan Hertzen kontaktu-mekanika aplikatzen da. GT ereduak laztasunen sakoneraren banaketa normalizatutaz jotzen du, arazoa estatistikoki landuz. GT ereduak lortzen duen adierazpenak,  $l_f$  gorputzen makro-eskalako deformazioa  $R_r$  zimurtasun faktoreaz biderkatzen du.  $R_r$  faktoreak gainazaleko topografiaren ezaugarriak barneratzen ditu. Horrela, RQ ereduak honako adierazpen trinkoa hartzen du (2.25 Ek.):

$$l_c^2 = l_g^2 + (R_r l_f)^2 = a_e d_s + \frac{8R_r^2 F_n' d_s}{\pi E^*} \quad (2.25)$$

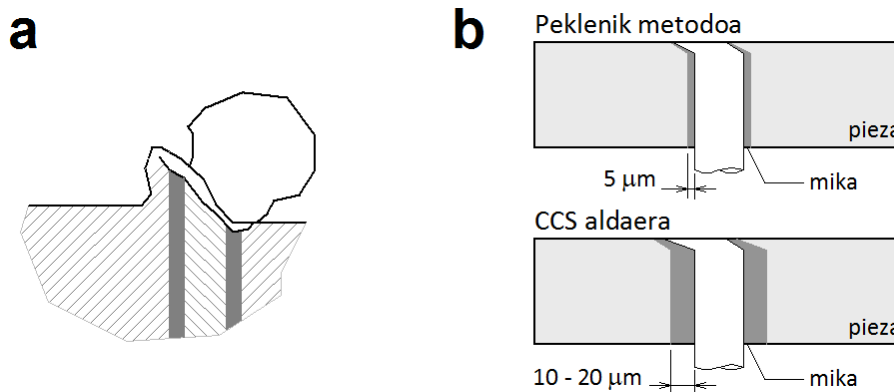
Praktikan,  $R_r$  zimurtasun faktorea esperimentalki definitzen da harri, diamantaketa-baldintza eta pieza konbinazio bakoitzarentzat. Eta hor dago gakoa: bere abantaila nagusia adierazpenaren aplikazio erraza bada ere, bere emaitzen zehaztasuna neurtzen zaila den  $R_r$  faktore enpirikoaren menpe dago. Egileen arabera,  $R_r$ -ren balioa 7 eta 23 artean aurkitzen da [QRM97b]. RQ ereduak GT eredu aplikazio-eremutik kanpo aplikatzen du, harriaren gainazala zimurregia baita. Gainera, RQ ereduak alea-pieza artean kontaktua elastikotzat jotzen du, errealitatean kontaktu-presioek plastikotasun-baldintzak sortzen dituztenean [MRDI04]. Hala ere, RQ ereduak neurketa esperimentalekin korrelazio onak lortzen ditu, harriaren gorputzaren deformazioa proportzionala baita eremu elastikoan eta plastikoan, eta, topografiaren ziurgabetasunak  $R_r$  zimurtasun parametroak biltzen dituelako.

Hasieratik, tesi honen lan-ildoetako bat RQ ereduaren  $l_f$  kontaktu-luzera zimurra zenbakiz simulatzea izan da.

### 2.2.2. Kontakto-luzera neurtzeko metodo esperimentalak

Artezketa kontaktu-luzera neurtzea eta interpretatzea benetan da konplexua. Gaurdaino, bost teknika aurkeztu dira literaturan: termoparea [Pek57], aplikatutako tentsio-iturria (*applied power source*, APS) [ZV92], bat-bateko gelditzea (*quick-stop*) [SS74, BWW77], adabaki-artezketa (*patch grinding*) [BSS71, KS81] eta, azkena, emisio akustikoak (*acoustic emissions*, AE) [BKW13]. Teknika guztiek dute berezko ziurgabetasun maila bat, neurketen dispertsioari (< % 11) eta emaitzen interpretazioaren zailtasunari lotua [GW88, ZV92, QRM97b, QRM01].

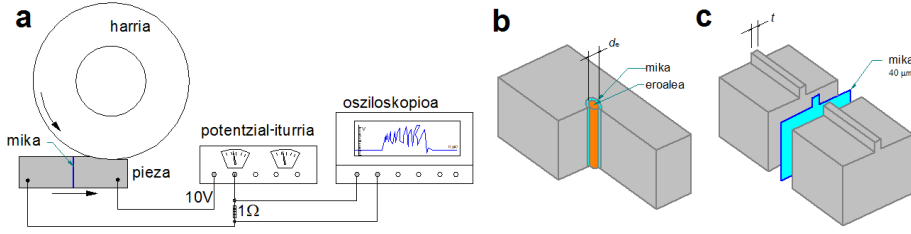
Termopare konbentzionalek, bi eroaleen lotunean tenperatura aldaketak sortutako potentzial elektrikoan oinarritzen dira. Tentsio-tenperatura erlazioa kalibratuz, tenperatura era ez zuzenean atzematen da. Ohiko termopareak ezin dira arteztu eta, beraz, ezin dute tenperatura zuzenean gainazalean neurtu. Gainera, duten inertzia termikoa dela eta, erantzuna ez da behar bezain azkarra eta ohiko lan-abiadura baino mantsoago egin behar da froga [PSO<sup>+</sup>12].



**2.14. Irudia:** (a) Termopare polobakarra: platinozko hari eroalea piezan barneratuta dago eta ale urratzailea zeharkatzean piezarekin lotura beroa sortzen da [Pek57]; (b) isolatzailearen lodierak neurketetan eragiten du: Pekleniken metodoa eta kontaktu egoera kritikoa (CCS) aldaera [GW88, ZV92]

Peklenik-ek [Pek57] konpontzen du termopareen arazoa, termopare polobakar artezgarria garatuz. Piezan zeharka zulatu eta bertatik isolatutako titaniozko haria pasaratzen da. Elektrodoek pieza eta haria anplifikadorearekin konektatzen dute, era berean osziloskopio bati konektatua dagoena. Zirkuitua irekia dago, arik eta ale urratzaile bat gaineratik pasatzean lotura beroa (*hot junction*) sortzen duen arte (2.14.(a) Ir.). Temperatura igoerak eragindako indar elektromagnetikoaren ondorioz (*induced electromagnetic force, EMF*), ale bakoitzaren urratze- eta harrotze-akzioei tentsio-pultsu bat dagokio osziloskopioan. Gaur egun titanioa isolatutako harian ez ezik, xafra eran ere erabiltzen da. Pieza eta platinozko elektrodoaren prestaketaz gain, desabantaila nagusia termoparearen kalibrazio konplexua da. Platinozko elektrodoaren eta isolatzailearen lodierak eragina dute termoparearen erantzunean. Lan-baldintzen arabera, neurketa desorekatza ere iris daiteke. Platinozko hariaren eta piezaren arteko kontaktua sakonegi zabalduz gero, lotura beroaren sekzioa handiegia da, ale bakanen antzematea temperatura-tontor eran galaraziz. Horregatik, isolatzaile lodiera txikiegiak lotura bero iraunkorra sortzea ekar dezake neurketa izorratuz.

Verkerkek eta Pekelharingek [VP75] alde batetik, Gu-k eta Wager-ek [GW88] bestetik, isolatzailearen lodierak EMF seinalean duen eragina aztertu dute. Lotura beroaren jarraitasuna aztertuz, isolatzailearen bi egoera definitzen dira [GW88]: kontaktu egoera kritikoa (*critical contact state, CCS*) eta kontaktu eza egoera (*no contact state, NCS*) (2.14.(b) Ir.). Lotura beroa muga baliora edo CCS egoerara doitzen da, isolatzailearen lodierarekin jokaturik. Horrela, aleek sortutako kontaktu-seinalea erregulatzen da, kontaktu-eremuaren ikuspegi garbiagoa eskainiz. Hala ere, NCS aldaerak kontaktu-luzerarekin erlazioa duen lotura berorik gabeko seinalea lortzen du, nahiz eta tenperaturak neurtzeko baliagarria ez izan. CCS eta NCS teknikekin neurtutako kontaktu-luzerak



**2.15. Irudia:** (a) APS metodoaren zirkuitu elektrikoa: potenzial-iturria eta osziloskopioa elektrodoei konektatuak; APS metodoaren bi aldaerak: (b) potenzial-iturria piezari eta hari isolatuari konektatua, eta (c) potenzial-iturria bi pieza zatiei konektatua, mika xaflak isolatzen dituenak [ZV92]

antzekoak dira.

Termopare polobakarrean inspiratuta, Zhouk *et al.* [ZV92] aplikatutako potenzial-iturriaren metodoa edo *applied power source* (APS) garatu dute. Termoparearen efektu termoelektikoaz baliatu beharrean, zuzenean potenzial elektriko bati konektatzen da zirkuitua. Arteztean zirkuitua itxi eta osziloskopio batek potenzial aldaketa monitorizatzen du denboran zehar. Seinalearen iraupena aztertuz kontaktu-luzera lortzen da. Metodo honen bidez lortutako kontaktu-luzerak, kontaktu-luzera geometrikoa baino 1,4 - 3 aldiz handiagoak dira. [GW88, ZV92] metodoaren bi aldaera aurkeztu dituzte: lehenengoan, isolatutako hari eroale batek pieza zeharkatzen du; bigarrenagoan, berriz, mika xaflak isolatzen du bitan zatitako pieza. 2.15. Irudiak APS metodoaren eske-ma eta bi aldaerak erakusten ditu. APS-ren tentsio-denbora seinalea aztertuz kontaktu-denbora  $t_c$  zehazten da. 2.26 Ekuazioak bi aldaerarako kontaktu-luzera kalkulatzeko adierazpena azaltzen du, aitzinamendu-abiaduraren  $v_w$  eta kontaktu-denboraren  $t_c$  funtzioan. Era berean,  $G_{dyn}$  ale-dentsitate dinamikoa (2.18 Ek.) eta  $L_{dyn}$  aleen arteko batz besteko distantzia (2.17 Ek.) estimatzeko ere gai da,  $N_c$  kontaktu-kopurua,  $v_s$  ebaketa-abiadura eta  $t_c$  erlazioatuz.

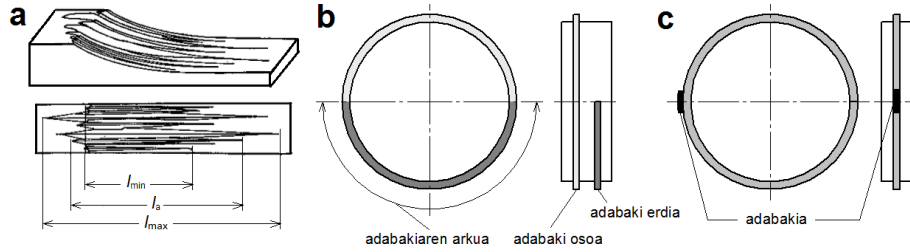
$$l_c = v_w \cdot t_c \quad (2.26)$$

2.2. Taulak literaturan argitaratu diren APS saiakuntza-baldintzak eta emaitzak biltzen ditu. Besteak beste,  $b$  elektrodo zabalera,  $s$  isolatzaile lodiera,  $v_s$  ebaketa-abiadura,  $v_w$  aitzinamendu-abiadura,  $V$  tentsioa eta lortutako  $l_c/l_g$  kontaktu-luzera arrazoa aurkezten dira.

Kontaktu-luzera neurketa metodoen bigarren familiak neurketa-printzipioak partekatzen ditu: harria artezten dagoenean, bat-batean pieza bat-batean erretiratzeko datza. Alde batetik, Sauerrek eta Shawek [SS74] gailu mekaniko bidez eta, bestetik, Brownnek *et al.* [BWW77] leherketa bidez, gelditze azkarra edo *quick-stop* metodoa garatu dute. Piezak, deformatutako harriaren profilarrekin geratzen da. Piezaren gainazalean harriak utzitako aztarnak aztertuz, kontaktu-

## 2.2. Taula: APS saiakuntza-baldintzak literaturan

	$b$ ( $\mu\text{m}$ )	$s$ ( $\mu\text{m}$ )	$v_s$ (m/s)	$v_w$ (m/s)	$V$ (V)	$l_c/l_g$
Gu, Wager [GW88]	100	10 - 40	10	0,17 - 0,25	6	2 - 2,5
Zhou, Van Lutterwelt [ZV92]	-	(wire, sheet)	22	0,25	-	1,4 - 2,2
Qi <i>et al.</i> [QRM97a, QRM97b]	200	10 - 40	30	0,17 - 0,25	1,2	1,7 - 3 (5,5)



**2.16. Irudia:** (a) *Quick-stop* metodoaren bidez lortutako harriaren arrastoak piezaren gainazalean; kontaktu-luzera maximoaren  $l_{\max}$  eta bataz bestekoaren  $l_a$  definizioa [ZV92]; (b) Brownek, Saitok eta Shawek erabilitako *patch grinding* metodoan erabilitako harria [BSS71]; (c) Kumarrek eta Shawek *patch grinding* metodoaren bigarren aldaeran erabilitako harria [KS81]

luzera zuzenean neurtzen da (2.16.(a) Ir.). Egileek piezaren berreskuratze elastikoa kontuan hartzen dute aztarnaren kurba aztertzean. Metodoak sinpleak badirudite ere, neurketen interpretazioa zaila da, kontaktu-hasiera atzematea ez baita erraza.

Neurketa metodoen hirugarren familia, adabaki edo *patch grinding* metodoa da. Harriaren gainazalean sortutako adabakiak piezaren gainazalean egindako aztarnak aztertzen dira kontaktu-luzera definitzeko. Adabakiaren geometriaren arabera, metodoaren bi aldaera proposatu dira. Shawek, oraingoan Brownekin eta Saitorekin batera [BSS71] garatutako metodoan, harriari bi irtengune egiten zaizkio: batek perimetro osoa biltzen du eta bigarrenak berriz erdia (2.16.(b) Ir.). Harri horrekin artztean piezaren gainazalean sortutako sarrera-irteera aztarnen profilak ibilbide trokoidal teorikoakin alderatzen dira. Atzemandako kurbatura-erradio handiagoek, harriak deformazio elastikoa jasaten duela frogatzen dute. Kontaktuzuzera, deflexio eta indar balioak Hertzzen kontaktu teoriarekin ondorioztatzen dituzte. Metodo horrekin neurtutako kontaktu-luzerak, kontaktu-luzera geometrikoa baino 1,9 eta 3 aldiz luzeagoak dira. Metodoak adabakiaren sarrera-irteeran errealitatean gertatzen ez diren baldintza latza-goak jasateaz gain, auto-eragindako bibrazioak ere sortarazten ditu. Deformazio aldaketaren neurketa aldiz erraza da haztatze bidez.

Berriz ere Shawek, oraingoan Kumarrekin batera, adabaki metodoari beste buelta bat ematen dio [KS81]. 2.16.(c) Irudian agertzen den adabakia sor-



tzen eta diamantatzen da. Hemen ere, harria artezterakoan sortutako aztarnen profila neurtzen da. *Quick-stop* metodoarekin lortutako emaitzekin alderatuz kontaktu-luzerak % 25 luzeagoak dira. Aurrekoak bezala, metodoak zalantzak sortzen ditu, sarrerako eta irteerako aleen lan-baldintzak ohikoak baino gogorragoak baitira eta horiek utzitako aztarna da justu neurtzen dena.

Babel-ek *et al.* proposatu dute metodo berritzaileena, emisio akustikoak (AE) baliatuz [BKW13]. Horretarako pieza artezten da, sarreran eta irteeran piezaren ertzak txafanekin eta gabe prestatuz. Sarrera-irteeretan agertzen diren zaraten tontorrak konparatuz, horien luzera kontaktu-luzerarekin bat egiten duela aurkitu dute. Nahiz eta metodoa sinplea iruditu, bere konplexutasuna zarata-uhinen iraztean eta interpretazioan dago.

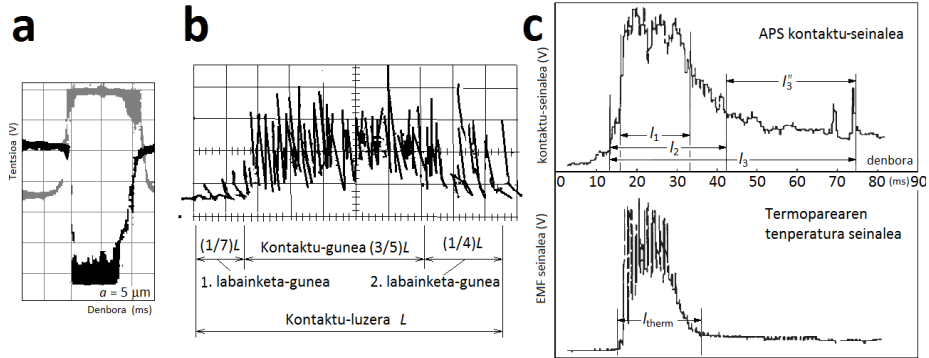
Termopare polobakar eta APS teknikak gailendu dira kontaktu-luzeran neurketetan. *Quick-stop* eta *patch grinding* teknikak ez dira literaturan gehiago agertu, ziur aski beraien prestaketa konplexuak eta interpretazio subjektiboak eraginda. Agertu berri den AE metodoa oraindik ez da literaturan aipatua izan  $l_c$ -n neurketetan, baina etorkizun itxaropentsua dauka prozesuaren monitorizazioan.

### 2.2.3. Kontakto-luzera seinalearen analisisa

Egile denek eztabaidatzen dute kontaktu-luzera neurtzean dauden ziurgabetasunei buruz, erabilitako neurketa-teknika edozein dela ere. Neurketa-baldintza egonkorretan ere, harriaren gainazaleko aleen ausazko banaketak kontaktu-luzeran % 11-arterko aldaketak sortzen ditu.

Zhouek eta Van Lutterweltek [ZV92] kontaktu-luzeraren neurketa esperimentalei buruzko gogoeta interesgarria gehitzen dute haien lanean. Neurketa-baldintzen arabera,  $l_a$  batz besteko kontaktu-luzeraren eta  $l_{max}$  kontaktu-luzera maximoaren artean bereizten dituzte emaitzak (2.16.(a) Ir.). APS bi aldaerek, pieza-pieza eta haria-pieza (2.15. Ir.), elektrodoen zabalera baldintzatzen dute, eta horiek lortutako seinaleak  $l_a$ -rekin eta  $l_{max}$ -rekin erlazionatzen dituzte. Kontakto-zabalera handitzeak (pieza, pieza) ale gehiago eskuragarri jartzen ditu. 2.17.(a) Irudiak bi aldaerekin baldintza berdinetan lortutako tentsiodenbora seinaleak erakusten ditu: beranzko seinaleak (haria-pieza, 2.15.(b) Ir.)  $l_a$ -rekin bat egiten du, eta goranzkoak (pieza-pieza, 2.15.(c) Ir.)  $l_{max}$ -rekin. Horrela, kontakto-luzera neurketa puntualak motzagoak dira esposizio zabalak erabiltzean baino.  $l_{max}$ -ren eta  $l_a$ -ren arteko aldea % 22 - % 55 artean dago. Fenomeno horrek egileen artean kontakto-luzera neurketetan dauden aldeak azal ditzake, neurketak irakurtzean egindako interpretazio ezberdinen ondorioz. Qirrentzat *et al.* [QRM97b] berriz, igurzketa-eremuak erdiko puntua da  $l_a$ , tontor kontzentrazio eremuaren bukaera eta muturreko tontorraren artean. Irizpide horrekin,  $l_{max}$   $l_a$  baino % 20 luzeagoa da gutxi gorabehera.

Egile batzuk igurzketa-, uradura- eta ebaketa-eremuak identifikatzen dituzte (ikus 2.4.2. Atala) neurketa-seinaletan eta artezketa arrastoetan. Guk eta Wagerrek [GW88] CCS egoeran polo bakarreko termoparearen seinalean hiru eremuak identifikatzen dituzte, Kops-en eta Shawen ondorioekin bat egiten dutenak [KS82]. 2.17.(b) Irudiak hiru eremuak erakusten ditu. Igurzketa eremuan,



**2.17. Irudia:** (a) Kontakto-zabaleraren arabera APS seinaleak [ZV92]: kontakto-luzera maximoa  $l_{\max}$  (goranzko uhina) eta batz bestekoa  $l_a$  (beheranzko uhina); (b) eremuak kontakto-luzeran (CCS) [GW88]; (c) APS eta termopare polobakar seinaleen konparaketa [QRM97a]

alea piezarekin kontaktuan jartzean, alearen lehen erreakzioa piezaren gainazalean makurtzea, biratzea eta labaintzea da egileen arabera. Indar normala behar-beste baliore iristean eta metala nahikoa biguntzean, alea piezari barneratu, eta materiala desplazatzen eta harrotzen hasten da. Ebaketa-eremuan, tontorren magnitudea eta dentsitatea aipagarriak dira. Horrek piezaren gainazaleko gogortasuna erorarazten du, ale asko sakon sartuta materiala harrotzen dutelarik. Bigarren igurzketaren eremuan, temperatura seinalearen amaierako tontorrak desagertzen jotzen dute. Aleak ateratzen doaz eta tontor irtenenak soilik sortzen dute lotura beroa.

Qik *et al.* [QRM97a, QRM97b] APS seinalea beste ikuspuntu batetik irakurtzen dute. Hiru eremuak identifikatzen dituzte baina gainjarriak daude, errealitatean bezala. 2.17.(c) Irudiak haien interpretazioa aurkezten du APS seinalearentzat (goiko grafikoa): harroketa-eremua ( $l_1$ ) goldaketa-eremuaren ( $l_2$ ) barnean dago, eta beraz igurzketaren eremuaren barnean ( $l_3$ ). Eremuen artean seinalearen intentsitatearen murrizketa txirbil-lodiera eta ale-dentsitate txikiagoek eragiten dutela ondorioztatzen dute.

Hainbat taldek [ZFG85, GW88, QRM97a] termopare polobakar eta APS metodoak konparatu dituzte. Bi metodoek ez-kontaktu egoera partekatzen dute, baina seinalearen iturrian ezberdintzen dira: alearen akzioak EMF seinalea eragiten du termopare polobakarrean, eta APSan, berriz, aleak isolatzailea zeharkatzean zirkuitua itxi, eta aplikatutako tentsioa antzematen du. CCS eta NCS seinaleak lausoak dira, zarata baino zerbait handiagoak. APSak, berriz, seinale indartsuak lortzen ditu. [GW88] arabera, APS eta CCS bidez lortutako emaitzak antzekoak dira, [ZFG85] berriz % 10-eko aldeak antzematen ditu. Bestalde, [QRM97a] polo bakarreko seinalea lausoa dela eta igurzketaren eremua ( $l'_3$ ) sumatzeko ezgai dela deritzo. Era horretan, termoparearen seinaleak APSrenak baino hiru aldiz txikiagoak direla baieztatzen du (2.17.(c) Ir.).

## 2.3. Zenbakizko ereduak artezketan

Orain arte ez da artezketa kontaktu-luzera modelizatzeko helburua duen zenbakizko eredurik aurkeztu. Hala ere, beste helburu batzuk dituzten hainbat zenbakizko eredu sortu dira. Horien artean eredu termikoek kalte termikoa aurreikusten dute, harriaren egitura ereduak bere zurruntasuna aztertzen dute eta, azkenik, gainazal topografia-ereduek harri-pieza arteko mugimendu erlatiboak aplikatuz, piezaren gainazal zimurtasuna eta indarrak aurreikusten dituzte.

Eredu termikoetan elementu finituen (FEM) eta diferentzia finituen metodoak (FDM) nagusitzen dira [BAG<sup>+</sup>06, DWB09]. Lan gutxi batzuk harriaren bibrazio moduak aztertzen dituzten FEM bidez. FEM gorputz heterogeneoen portaera mekanikoa simulatzeko ere gai bada. Bere desabantaila nagusia harriaren egitura porotsua fidelki deskribatzeko zailtasunak dira. Gainera, materialen propietateen erlazio eratzailen doitzea eta kontaktu-baldintzak zehaztea konplexua izango litzateke. Modelatze eskala-anitzak (*multi-scale modelling*) ereduaren neurria murriztuko luke [KBCS09]. Hala ere, gaurdaino ez da harriaren egituraren FEM egiturarik aurkeztu.

Aacheneko RWTHk [BMKW11] eta Zhangek *et al.* [ZGS<sup>+</sup>11], talde bakoitzak bere aldetik oso antzeko zenbakizko ereduak sortu dituzte. Bien helburua berdina da: era fidelean harriaren morfologiaren, egituraren eta topografiaren 3D marrazketa-eredua sortzea. Zehazki, beiratzatutako CBN harrietan ipini dute arreta. Aleen geometria eta dimentsioak parametrizatuak daude eta, astintze teknikaren bidez, uniformeki banatuak dauden elementuak ausazko posizioan eta orientazioan kokatzen dira. Aleak bolumenean homogeneoki banatuak daudela ziurtatzen dira, aleen arteko distantziak [BMKW11] edo dentsitate pilatua [ZGS<sup>+</sup>11] aztertuz. Aglomeratzailea, harriaren egitura erreferentziatzen hartuz, aleen ingurua estaltzen duen geruza borobildua da. Horrela, poro borobilduak lortzen dira errealitatean bezala. [BMKW11] ez da gainazal topografia garatzera iritsi. Zhangek bere horretan uzten du gainazala eta berarekin egiten dituzten simulazio zinematikoak. Diamantaketa kontuan ez hartzearen erabakia CBN urratzaileen erabilerak justifikatzen du. Etorkizunean, sortzen duten 3D geometria FEM bidez harriaren zurruntasuna simulatzeko lan-ildoak aipatzen dute biek, baina aukera hori ez dute aurrera eraman. Sortu dituzten aglomeratzaile-poro geometria borobildu konplexuak eta homogeneizazio arazoak, FEMaren aplikazioa zailtzen dute. Edonola ere, bi marrazketa-eredu horiek harriaren egitura heterogeneoa fidelki deskribatzen dute. Zhangen eredu matematikoak, Li-k *et al.* [LYZW15a, LYZW15b] aurkeztutako elementu diskretuen ereduaren garapena bultzatu du.

Tungsteno karburoaren artezketa simulatzeko, Zhangek *et al.* [ZYF<sup>+</sup>15] diamanteko harri elektroestaldua ez-porotsuaren FEM ereduak garatu dute *DEFORM* softwarearekin. Ale urratzaileek eta aglomeratzaileak pieza bakarra osatzen dute ereduaren, diamantearen propietate elastikoak soilik kontuan hartuz. Aldiz, piezaren portaera biskoplastikotzat jotzen dute.

Atal honek harriaren egitura eta topografia-ereduetan ipini du arreta. Ideia bi eredu motak konbinatzea da kontaktu-luzeraren zenbakizko ereduak eraikitze-ko.

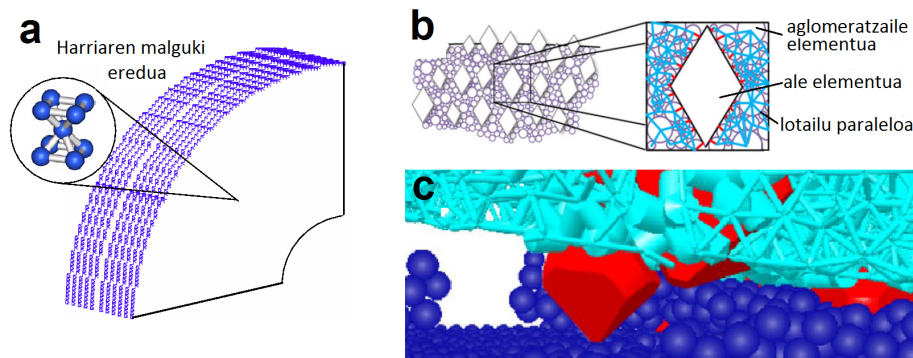
### 2.3.1. Harriaren egituraren ereduak

Harriaren egiturak eta morfologiak eragin zuzena daukate arteztean harriak duen portaeran. Harriaren egitura ereduaren erroka harriaren natura granularra deskribatzea da. Ale urratzaileek, aglomeratzaile-zubiek eta poroek gorputz heterogeneo ez-jarraitua osatzen dute. Osatzen duten mikro-egituraren eta haien arteko interakzioen arabera, eskala batetik aurrera konbinaketak material bakarra balitz bezala lan egiten du. Horrela, materialen propietate mekanikoak eskala makroskopikoan azaleratzen dira, gorputz homogeen eta isotropo osatuz [CJR13]. Harriaren egitura ereduaren helburua, ale urratzaileek eta aglomeratzaile-zubiek osatzen duten 3D sarearen portaera mekanikoa erreproduzitzea da.

Besteak beste, sinplifikazio geometriko nabarmenak dituzten ereduak proposatu dira harriak modelatzeko. Moser-ek [Mos80] proposatu du mota honetako lehen ereduak, ondorengoak inspiratuz. Berak harriaren egitura idealizatzen du, ale urratzailez eta barra zilindrikoz osatutako egitura kubiko erregularra erabiliz. Hessel-ek [Hes03], antzeko planteamendua jarraituz, diamantezko beiratzatutako harrien diamantaketa eta bere ondorioak simulatzen ditu. Aglomeratzaile-zubiek jasandako tentsio maximoak kalkulatu eta askatzen diren ale frakzioa kalkulatzeko erabiltzen du. Yamada-k *et al.* [YMLM11] makina-harria-pieza sistemaren zurruntasuna aztertzen dute erretxinazko harrientzat (2.18.(a) Ir.). Akats larria egiten dute, gainazaleko ale-dentsitatea, aleen arteko distantzia aldatuz doitzen baitute.

Oro har, Moserren ereduaren oinarritutako harriaren egituraren eredu uniformeek antzeko gabeziak dituzte. Morfologiari dagokionez, harriei eginiko hiru dimentsioko mikro-tomografia analisisiek, eredu uniformeak errealtatetik oso urrun daudela erakusten dute [BMKW11]. Haien konfigurazio erregularrak gorputz anisotropoak sortzen ditu. Gainera, aleak lotzeko malgukien erabilerak ez du bat egiten errealtatean aglomeratzaile-zubiek duten portaerarekin. Aglomeratzaile-zubiek aleen artean biraketa erlatiboa ere eragozten dute, hau da, karga axialez gain, makurdura- eta bihurtura-momentuak ere jasaten dituzte.

Lik *et al.* [LYZW15a, LYZW15b] elementu diskretuen metodoan (*discrete element method*, DEM) oinarritutako harriaren lehen ereduak aurkeztu dute bi artikuluetan. Ereduak, alde batetik, harriaren zurruntasuna, erresistentzia eta aleen arteko indar-kateak aztertzen ditu; eta, bestetik, harria-pieza arteko mugimendu erlatiboa kontuan hartuz, piezan lortutako gainazal-zimurtasuna aurreikusten du. Oktaedro geometria duten elementu diskretuek (*discrete element*, DE) ordezkatzeko erabiltzen dituzte ale urratzaileak. Ereduak, uniformeki banatutako DEak ausaz posizioz eta orientazioz astinduz eraikitzen da, besteak beste [CR96, KJL97, Dar13] antzera. Aglomeratzaile-zubiak geometrikoki eta frakzio bolumetrikoan fidelki erreproduzitzen saiatzen dira. Horretarako, paraleloan antolatutako habe elastiko sare konplexu erredundantea garatzen dute. Horretarako, hainbat aglomeratzaile DE esferiko kokatzen dituzte urratzailea modelatzen duten DE oktaedroen artean (2.18.(b) Ir.). Konektatzen dituzten DE-en arabera, urratzaile-aglomeratzaile habeak eta aglomeratzaile-aglomeratzaile habeak daude. Habeen erradioa arbitrarioki aplikatzen da: lotzen dituen DE-en



**2.18. Irudia:** (a) Harriaren malguki-eredu uniformea [YMLM11]; (b) aglomeratzaile-zubiak geometrikoki modelatzen dituen habe-sistema paralelo erredundantea [LYZW15a]; (c) artezketaren zenbakizko simulazioa oktaedro eta pieza elementuz osatutako DEM ereduarekin [LYZW15a]

erradioen frakzioaren eta dispersio osagai normalaren batura.

Makurdura- eta konpresio-saiakuntza esperimentalak eta zenbakizko simulazioak alderatuz egin da habeen propietate mekanikoen kalibrazioa. Saiakuntzetan lortutako karga maximoa eta zurruntasuna izan dira erreferentziak konparaketetan. Habeen haustura parametroak eta habea sortzeko DE-en arteko distantzia maximoa (koordinazio zenbakia edo DEko habe kopurua (*coordination number*, CN)) izan dira kalibrazio-aldagai nagusiak. Eredua, konpresioan harriaren haustura hauskorra simulatzeko erabiltzen da. Saiakuntzan emisio akustikoen (EA) seinalea ere monitorizatzen dute. Habea sortzeko luzera maximoa zehazteko, zarata-pultsu kopurua DEM simulazioan apurtutako habe kopururekin alderatzen dute. Konparaketa hori oso ausarta da, zarata-tontor bakoitzaren esanahiak ez baitu bat egiten aglomeratzaile-zubi bat osatzen duten hainbat habeetako baten hausturarekin. Gainera, material hauskorren konpresio erresistentzia deformazio-abiadura handiagoan erresistentzia handitzen da. Zurruntasuna (edo elastikotasuna) saiakuntzetako konpresio karga-denbora kurbekin doitu da. Denbora adierazgarria da erresistentziarekiko, baina ez zurruntasunarekiko, deformazioarekin lotura duena. Harriek ebaketa-abiadura azkarretan egiten dute lan, urratzaile-aglomeratzaile konposatuaren erresistentzia nabarmenki handituz. Habe-sare erredundantean sortzen diren indar-kateak ere aztertzen dituzte. Emaitzek tentsioek norabide tangenzialean jokatzen papera azpimarratzen dute.

Habiez gain, ereduak Hertz-Mindlin kontaktu-algoritmoa erabiltzen du eredu sortzeko. Algoritmo hori, partikula askeak edo solido trinkoak modelatzeko erabili ohi da. Eredua sortzeko erabilitako astintze-metodoaren ondorioz, DE-en arteko distantziak handiak dira eta, beraz, algoritmoak apenas egiten du lan oktaedro-elementuen artean, bai ordea aglomeratzaile-elementuetan. Octaedro-DEak ikutzen ez direnez, geometria horren erabilera estetikoa da, ez dauka

zurruntasunean edo egituran eraginik. Hala ere, interesgarria litzateke elementu horien erabilera harriaren fabrikazioan egiten den ale trinkotzearen simulazioan. Hala ere, eredia sortzeko astintze teknikak ez du horretarako balio. DE esferikoen multzoen edo *cluster*ren erabilera ere proposatzen dute ausazko ale-geometria modelatzeko. Horrela, aleen aspektu-erlazioa ere kontuan hartuko litzateke, trinkotzean garrantzizko faktorea dena.

Eredu zinematikoak piezaren gainazal-zimurtasuna simulatzen du. Horretarako, diamantaketa mespretxatzen dute, nahiz eta beiratzatutako silizio karburo harria modelatu. Horregatik, zenbakizko emaitzek eta neurketa esperimentalek ez dute guztiz bat egiten. 2.18.(c) Irudiak simulazioaren istant bat harrapatzen du, non habe inguratutako DE urratzaile batek DE esferikoz osatutako piezaren gainazala zeharkatzen duen. Ale bakoitzak jasandako ebaketa-indarra indarren neurketa esperimentaletako tontorretako balioak aztertuz estimatzen dira. Horrela, alearen piezarekiko posizioa (edo txirbil-lodiera) mespretxatzen da, gainjarritako indar-tontorrez gain. Pieza DE esferikoz osatutako prisma errektangeluar bat da. DE horien arteko baldintzak ez dira azaltzen. Egileek ereduak materialaren goldaketa modelatzen duela baieztatzen dute, baina ez dute argitzen materiala harrotzen den ala ez.

Nakayama-k [NBS71] gainazaleko aleen zurruntasuna aztertzeko lan esperimental nekeza aurrera eramaten du. Hertzen mekanikan oinarrituz, alearen deflexioa estimatzeko adierazpena garatzen du jasaten duen indarraren arabera. Chenek eta Rowek [CR96] eta Hecker-ek *et al.* [HRL03] bere topografia-ereduetan aplikatu dute. Horrela, eredu hauek harriaren gainazalaren zurruntasuna kontuan hartzen dute, baina ez harriaren gorputzarena. Hala ere hurbilpen sinplifikatzaile interesgarria da.

Zhangek *et al.* [ZSNW93b] zaila ikusten dute ale bakarraren frogen eredu analitikoaren aplikazioa artezketa-ereduetan (ikusi 2.4.2. Atala), alearen deskripzio zehatza behar baita aleak zenbat urratzen, goldatzen eta harrotzen duen zehazteko. Horregatik, malguki elastikoz osatutako eredu iteratiboa proposatzen dute, artezketa-indarren eta deformazio elastikoen arteko oreka bilatuz. Ebaketa-indarren banaketa kontaktu-ereduan zehar da ereduaren emaitza. Ereduak topografiaren eragina mespretxatzen du, kontaktu-zurruntasunean garrantzizko faktorea dena.

### 2.3.2. Topografia-ereduak

Diamantaketa bidez sortutako topografiak eragin zuzena dauka artezketan. Topografia-ereduek, 2.1.3. Atalean deskribatutako harriaren gainazalaren zenbait ezaugarri erabiltzen dituzte gainazal birtuala eraikitzeko. Eredu zinematikoek gainazal horri harriaren eta piezaren arteko mugimendu erlatiboak aplikatzen dizkie, horrela, piezaren gainazal zimurtasuna, indarrak edo ebaki-gabeko txirbil-lodiera aurreikusi ahal dute. Eredu zinematiko gehienek gainazala zurruntzat hartzen dute. Ondorioz, ez dira artezketa kontaktu-azalera simulatzeko gai.

Doman-en *et al.* berrikusketan [DWB06], harriaren topografia-ereduak bat, bi eta hiru dimentsiotakoetan (1D, 2D, 3D) sailkatzen dituzte. 1D ereduak zimurtasuna edo ale-dentsitatea gisako parametroen bidez karakterizatzen dute

harriaren gainazala. 2D eredueta, harriaren profila geometrikoki deskribatzen da. Horrela, diamantaketaren ondorioz lortutako ale-neurri posizioa eta tontor-altuera banaketak aztertzeke gai dira. 3D ereduak, berriz, ale-kokapena eta geometria 3D gainazal eran deskribatzen dute.

1D topografia-ereduek nagusiki, gainazaleko ale-dentsitatea aurreikusi ohi dute. 2.1.3. Atalean azaldu da, gainazaleko aleak estatikoak eta dinamikoak (edo zinematikoak, edo aktiboak) izan daitezkeela material harroketan duten parte-hartzearen arabera. Tönshoff *et al.* [TPIP92] ohartzen dira gainazaleko ale-dentsitate estatikoa eta dinamikoa aurreikusten dituzten 1D eredu enpirikoek lau aldagai komun dituztela: ebaketa-sorbatzaren forma, abiadura-ratioa, ebaketa-sakonera eta ale-neurria. Aldagai horiek beste artezketa-parametro batzuen adierazpenetan agertzeak (ebaki-gabeko txirbil-lodiera, artezketa-indar, piezaren gainazal akabera), duten garrantzia azpimarratzen du.

Zenbakizko eredu topografikoak gero eta sofistikatuagoak dira, eta harriaren ausazko topografia islatzeke gai dira. Berrikusketak 2D eta 3D eredueta ipini du arreta, harriaren gainazala geometrikoki deskribatzen baitute. Eredu hauek hiru familia nagusitan sailkatzen dira: eredu mekanizistak (ezaugarri fisikoetan oinarrituak) [CR96, KJL97, WZ98, WZ98, Heg00, CL00, GWW02, NB05, CP08, BMKW11, ZGS<sup>+</sup>11, AK12, CGL<sup>+</sup>13, Dar13, LWBG13, ZYF<sup>+</sup>15], eredu enpirikoak (ereduaren koefizienteak datu esperimentalekin doitzen dira) [Ina96] eta eredu probabilitistikoak (gertatzeko probabilitatea ebaluatzen da) [McA64, YA84, TB00, HK03, NB05, AR05, JGB<sup>+</sup>13]. Eredu mekanistikoek eta enpirikoek harriaren gainazalaren zenbait ezaugarrietatik abiatuz, harriaren gainazal birtuala sortzen dute. Parametro geometrikoak, hala nola ale-neurria eta posizioa, ausaz edo estatistikoki tratatzen dira. Aldiz, hurbilpen probabilitistikoek ikuspuntu determinista hau alboratzen dute, eta ale-pieza interakzioa harriaren gainazalaren karakterizazio estatistikoa landuz aztertzen da. Ebaki-gabeko txirbil-lodiera ereduak probabilitate dentsitate-funtzio normala edo Rayleigh-en funtzioa aplikatzen dute artezketa-indarrak eta piezaren gainazal-akabera aurreikusteko. Rayleighen funtzioa  $\beta$  parametro bakarrekin definitzen da, zehazteke zaila dena eta zentzu fisiko argirik ez duena [YA84, HL03, AR10].

2.1.2. Atalean azaldu da, ale- eta lotura-hausturak direla diamantaketa-mekanismo nagusiak. Diamantaketa-baldintzek mekanismoen intentsitatea doitzen dute, gainazal ireki-bigunak edo itxi-zurrinak sortuz. Topografia-ereduek gai izan behar du diamantaketa-baldintzen arabera harriaren gainazala erreproduzitzeko. Diamantaketak, espazioan ale-posizioarekin batera, definitzen ditu tontor-posizioa eta sorbatzen zorrotasuna.

Topografia-eredu denek ez dute diamantaketa kontuan hartzen. Kasu batzuetan arrazoitua dago, super-urratzaileak modelatzen baitituzte. Ale-dentsitatea harriaren morfologiatik azaleratzen da [ZGS<sup>+</sup>11], edo topografia-parametroak zuzenean inplementatzen dira, hala nola, ale-dentsitatea [KJL97, ZYF<sup>+</sup>15]. Badira neurketa esperimentaletan lortutako topografia zuzenean ereduak aplikatzen dutenak [Ina96, WZ98]. Beste batzuk, diamantaketa alboratzen dute harriaren morfologian oinarrituz, akats nabarmena dena [AR05, LYZW15b].

Koshy-ren *et al.* [KJL97] ekarpen nagusia gainazal parametroen tratamendu estatistikoa izan da, 1D parametroetatik abiatuz 2D eredia eraikiz. Horretarako astintze edo *shaking* teknika erabiltzen dute eremu granular birtual bat sortzeko. Aleak ordezkatzeko dituzten esferak, disposizio uniformetik abiatuz mugiarazten dira ausazko posizio parametroak aplikatuz. Esferen artean elkar-sartzerik ez dagoela ziurtatu arte errepikatzen da prozesua. Diamantaketaren eraginik ez dauka, soilik aleen kokapenaren probabilitatea aztertzen da. Zhouek eta Xi-k [ZX02] eta Cao-k *et al.* [CGL<sup>+</sup>13] 3D eredu sinpleak eraikitzen dituzte, plano batean sare uniforme bateko puntuetan banaketa normala jarraitzen duten altuerak esleituz. Horrela piramidez osatutako gainazal zimurra lortzen dute. Ale-askatzea eta diamanteak utzitako helizea arbuiatzen dute. Nguyen-ek eta Butler-ek [NB05] zenbakizko gainazala sortzen dute, ausazko eremua Johnsonen transformatuaz ordezkatzuz. Neurketa esperimentaletatik doitzen dira funtzioaren parametroak, ale-dentsitate edo protrusio-altuera antzeko parametro fisikoen beharrik gabe.

Jiang-en *et al.* [JGB<sup>+</sup>13] eredu probabilistikoak, diamante punta-bakarraren mugimenduak eta urratzaileen higadura barneratzen ditu era sinplean, ale-haustura eta askatzea arbuiatuz. Diamantaketa uniforme ale bakoitzean bere posizioarekiko aplikatzen denez, ez harriaren diametroarekiko, ereduak ausazko emaitza onak lortzen ditu. Aurich-ek eta Kirsch-ek [AK12] tontorrek harriaren erreferentziazko diametroan moztu dituzte, diamantaketa eta higadura modelatzeko CBN geruza-bakar harrian.

Chenen eta Roweren [CR96] eta Torrancen eta Badgerren [TB00] ereduak dira diamantaketaren efektua barneratzen dituzten artean osatuenak. Chenen 2D ereduak diamantaketa aplikatzen du, esferaz osatutako gainazala ausazko funtzio sinusoidal batekin moztuz. Funtzio sinusoidal jarraituak  $y_p$  bi osagai dauzka: diamantearen ibilbide helikoidala gainazala era harikorrean mozten duena  $y_d$ , eta aleen haustura hauskorra jasotzen duen ausazko funtzio osagarria  $y_f$ . 2.27 Ekuazioak aurkezten du diamantaketaren funtzio sinusoidala.  $y_d$  osagaiaren funtzioa ( $f(x)$ ), diamantearen profilarren eta  $x$  posizio axialaren menpe dago.  $h_f$  haustura osagaiaren anplitudea da,  $\omega$  ausazko maiztasuna eta  $\alpha$  ausazko hasierako angelua.  $h_f$  haustura anplitudea diamante-alearen intersektzio-azalerarekiko  $A_d$  eta teilakapen-ratioarekiko  $u_d$  proportzionala da, eta aitzinamenduarekiko  $f_d$  alderantziz proportzionala (2.28 Ek.). Hegeman-ek [Heg00] antzeko funtzio sinusoidala erabiltzen du ale elipsoidalen parametroak definitzeko, eta Darafonek [Dar13] Chenen eredia 3D domeinura zabaltzen du.

$$y_p = y_d + y_f = f(x) + h_f[\sin(\omega x + \alpha) + 1] \quad (2.27)$$

$$h_f = \frac{A_d \cdot u_d}{4f_d} \quad (2.28)$$

Torrancek eta Badgerrek [TB00] gainazala diamantatzen dute, aleak iraganaldia irauteko duen probabilitatea kontuan hartuz. Eredu probabilistikoa da eta ez du gainazal birtualik preseski sortzen. Inklinatutako hainbat segmentu zuzenek ordezkatzeko dituzte ale eta aglomeratzaile hausturak. Horien maldak



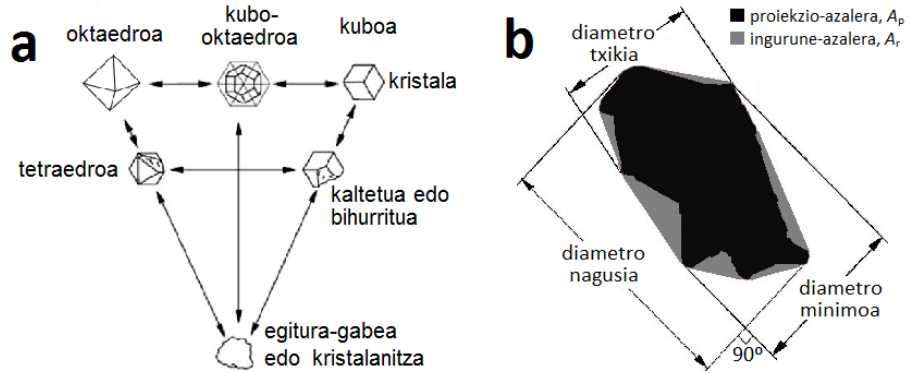
ausaz banatzen dira balio maximoaren eta minimoaren artean, mendizerra itxurako profila lortuz. Gong-ek *et al.* [GWW02] Chenen diamantaketa-eredua moldatzen dute, zuzenez osatutako ausazko funtzio jarraitua arbitrarioki definitutako tartean aplikatuz. Hala ere, diamantaketa alearen posizioarekiko aplikatzen dute, [JGB<sup>+</sup>13]-ek bezala.

Tontor-altuerak lantzeko hainbat banaketa agertu dira literaturan, besteak beste, Gaussiarra [WZ98, ZX02, HK03, CP08, AK12, CGL<sup>+</sup>13], Weibull [TB00] eta ausazkoa [CR96, KJL97, GWW02, NB05, Dar13, LWBG13, ZYF<sup>+</sup>15]. Zhou-ek, Houek eta Caok zuzenean alearen diametroaren  $d_g$  desbiderapen estandarra erabiltzen dute tontor-altuera definitzeko, inongo zerikusirik ez duen parameetroa. Koshiren ereduari tontor-altuera  $0,1d_g$  tarte arbitrarioan dago. Nguyenek, topografia-neurketa esperimentaletatik ondorioztatzen du, gainazaleko tontor-altuerak banaketa Gaussiarra ez duela jarraitzen.

2.1.2. Atalean ikusi da, diamantaketa-baldintzek eragiketaren intentsitatean eragiten dutela eta, ondorioz, askatutako ale frakzioan. Chenek eta Torrancek [CR96, TB00] soilik hartzen dute kontuan ale-askatzea. Alearen iraupena erabakitzen da esperimentalki zehaztutako indar mugaren arabera (2.1.2. Atala). Inork ez du Malkinen ale-askatze ereduak aplikatu (2.11, 2.12 eta 2.13 Ek.), baina topografia-eredu batean ezartzeak ez luke arazorik izango.

Ebaketa-sorbatzen zorrotasunaren analisia ale urratzaileen morfologia analisiarekin hasten da [JD11]. 2.19.(a) Irudiak oinarritzko hiru ale geometriak aurkezten ditu: kuboak, oktaedroak eta amorfoak. Urratzaileek hiru horien arteko geometria izan ohi dute. Diamanteak eta CBNak oktaedro-kubo arteko geometria hartzen dute, parametrizazioa errazten duena [WZ98, BMKW11, ZGS<sup>+</sup>11, AK12, LYZW15b, ZYF<sup>+</sup>15]. Aluminak egitura kristal-bakarra edo -anitza har dezake, fabrikatzean gehitutako gehigarrien eta labe-baldintzen arabera [MRDI04]. Zapaltze-prozesuak eta kristal-egiturak aleen geometria eta eragindako kaltea definitzen dute. Alumina kristal-bakarretatik abiatuz, aurpegidun edo lamina itxurako kristalak lortzen dira; alumina kristal-anitzetik, berriaz, geometria kubiko (*blocky*), angeluarra (tetraedro) eta puntadunak (luzeak). Nahiz eta egitura eta konposizioa partekatatu, alearen geometriak bere portaera (birrinkortasuna, zorrotasuna, higadura) aldatzen du. Ale urratzaileen zorrotasuna ebaluatzeko, alearen proiektzioaren ganbiltasuna eta aspektu-erlazioa aztertzen dira (2.19.(b) Ir.). Hala ere, parametro hauek duten esanahia galtzen dute diamantatu ondoren beiratzatutako harri konbentzionaletan. Ondorioz, super-urratzaileetan eta aleak era askean erabiltzen dituzten prozesu urratzaileetan aplikatzen dira nagusiki [DPS02, JD11]. Ale geometriak harriaren fabrikazioan aleen trinkotzean edo egituraren irekitasunean eragiten du: aspektu-erlazio handiagoko aleen erabilerak, arteztean auto-zorrotzketak bultzatzeaz gain, harriaren egituraren muga teknologikoak zabaltzen ditu.

Hiru eredu mota identifikatu dira ale urratzaileen eta ebaketa-sorbatzen errepresentazioaren arabera: (1) ale urratzaile ereduak [KJL97, WZ98, HK03, CP08, BMKW11, ZGS<sup>+</sup>11, AK12, LYZW15b, ZYF<sup>+</sup>15], (2) ebaketa-sorbatz ereduak [Ina96, Heg00, CL00, ZX02, CGL<sup>+</sup>13], eta (3) ale urratzaile-sorbatz eredu konbinatuak [CR96, TB00, GWW02, AR05, JGB<sup>+</sup>13, Dar13, LWBG13]. Ale urratzaile ereduak (1) ebaki-gabeko elementuz osatzen dira, ebaketa-sorbatzen pa-



**2.19. Irudia:** (a) Aleen geometria aldaerak [JD11]; (b) alearen proiektzioak eta forma faktorea, bere geometria eta ebakitzeko gaitasunak aztertzeko [DPS02]

pera hartzen dutenak. Beraz, diamantaketaren ondorioak arbuizaten dituzte. Aleek hainbat geometria har dezakete, hala nola esfera [KJL97, HK03], piramidea [CP08], tetraedroa [BMKW11, AK12], hexaedroa [ZYP<sup>+</sup>15] eta oktaedroa [BMKW11, ZGS<sup>+</sup>11, AK12, LYZW15b]. Guztiek kontuan hartzen dute aleen orientazioa, esferikoek ezik. Ebaketa-sorbatzen ereduek (2) sorbatza besterik ez dute modelatzen, aleak mesprextatuz. Konoa eta kono-enborra [Ina96, CL00], piramidea [ZX02, CGL<sup>+</sup>13] edo elipsoidea [Heg00] dira erabiltzen dituzten geometriak. Azkenik, (3) eredu batzuk lehenik harriaren egitura granularra esferaz sortzen dute, ondoren sorbatzak diamantaketaren arabera kokatuz. Batzuk aleaskatzea ere kontuan hartzen dute. [CR96, LWBG13, Dar13] sorbatz esferikoak, [TB00, GWW02, JGB<sup>+</sup>13] piramidalak eta [LWBG13] konoak erabili dira. Sorbatz eta ale-sorbatz ereduek harriaren zorrotasuna islatzen dute, ale ereduetan, berriz, erlazioa ez da hain zuzena, ale-geometriarekin ere jokatu baitaiteke. Ale-sorbatz eredu konbinatuek, topografiaz gain, harriaren egitura ere modelatzen dute (geruza bat edo hainbat).

Sorbatzen geometriak zorrotasuna islatzen du. 2.1.3. Atalean azaldu dira zorrotasuna adierazteko erak (2.9. Ir.): sorbatzaren malda eta sorbatzaren erradioa  $\rho_s$ . Sorbatzaren maldari dagokionez, Badgerrek eta Torrancek [BT00, TB00] jaulkitze-angelua piramideen malda definitzeko erabiltzen dute; Inasakik eta Cooper-rek [Ina96, CL00], berriz, konoen malda zehazteko. Higadura, tontorra moztuz [AK12, JGB<sup>+</sup>13] edo malda handituz modelatzen da.

Bestalde, sorbatza  $\rho_s$  erradioko zirkunskribatutako zirkulu batez ere karakterizatu daiteke. Aleen jaulkitze-angelua oso negatiboa denez, sorbatza esferatzen hartzea onargarria da. Gainera, alearen geometria etengabe aldatuz doa higaduraren eta ale-hausturen ondorioz, eta  $\rho_s$  moldatu daiteke gainazaleko aleen higadura-maila adierazteko. Besteak beste, [CR96, Dar13]  $\rho_s$  erabiltzen dute sorbatzen zorrotasuna adierazteko. Egile batzuk zuzenean alearen batz beste-ko diametroa  $d_g$  erabiltzen dute, akats nabarmena dena [KJL97, HK03, HRL03]. Ebaki-gabeko txirbil-lodiera eta  $\rho_s$  berdintzea ere zalantzazkoa da [AR05].

Eredu zinematikoak oro har, zurrinak dira, [CR96, HRL03] ezik. Azken bi horiek, Nakayamaren lana ere duan barneratzen dute, gainazaleko aleetan malgukiak ezarriz. Horrela, gainazal-zurruntasuna simulatzen dute, baina ez harriaren gorputzaren zurruntasun erreala. Aurichek eta Kirschek [AK12] FEM-zinematika eredu akoplatua garatzen dute CBN harri geruza-bakarrarentzat, harriaren deflexioa eta tenperatura kontuan hartzen dituen.

### 3D Topografia-ereduen egitura orokorra

Domanek *et al.* [DWB06] topografia-ereduen berrikuspenean, ordura arteko eredu ezaugarri komunak bilduz, 3D topografia-eredu orokorrak izan behar dituen ezaugarriak eta egitura deskribatzen ditu. 2.20. Irudiak topografia-ereduek jarraitu beharreko diagrama aurkeztzen du. Egiturak bi adar nagusi ditu: lehengoak diamantatu gabeko harriaren topologia eraikitzen du eta, bigarrenean, diamantaketa aplikatzen zaio.

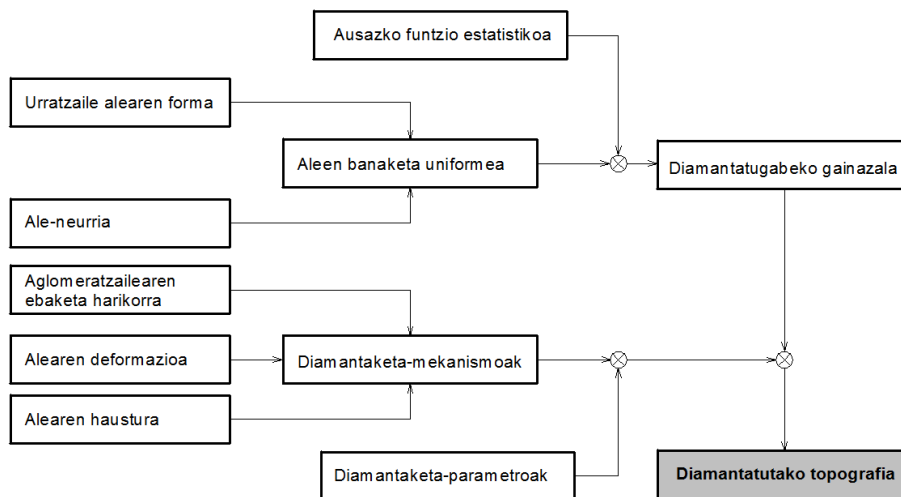
Diamantatu gabeko topologiak aleen geometria, neurria eta posizioa lantzen ditu. Normalki aleek geometria esferikoa dute, geometria sinpleena baita; ondoren, diamantaketak sorbatzak sortuko dizkie aleei. Nolanahi ere, ale-geometria konplexuagoak ere erabil daitezke, hala nola elipsoideak edota parametrizaturako poliedroak. Aleak uniformeki banatu ondoren, ausazko posizio aldaketa batek topologia egokia lortzen du. Harriaren dentsitatearen arabera, banaketa uniformeak *bcc* edo *fcc* zelula-egiturak erabil ditzake.

Bigarren zatiak gainazal-topografiaren sorrerarekin erlazionatua dago. Diamantaketa-mekanismoek emaitza baldintzatzen dute: ale-haustura eta ale-askatzea. Diamanteak deskribatzen duen mugimendu helikoidala gainazalean islatzen da, zeharkatzen dituen aleak moztzen baititu. Horrez gain, aleek haustura hauskorra jasaten dute diamantaketa-baldintzen funtzioan. Eta ale-askatzea, berriz, alearen iraupena erresistentzia edo tamaina irizpideak aplikatuz erabakitzen da.

## 2.4. Ale urratzaile-pieza interakzioa

Harria-piezaren kontaktua modelatzeko, beharrezkoa da kontaktu-baldintzak definitzea. Kontaktua erreala harriaren gainazaleko tontorren eta piezaren artean gertatzen da. Kontaktua hain puntualak izanik, indar intentsitatea handia da eta aleak materiala goldatzen eta harrotzen du abiadura bizian. Banako kontaktuetan sortutako indarren baturak artezketa-indarren erresultantea osatzen du. Indar erresultantea itxurazko kontaktu-azaleran banatua dagoela dirudi.

Atal honek, kontaktu-mekanikaren oinarriak eta material harroketaren teoria berrikusten ditu, gainazaleko aleen eta piezaren arteko interakzioan sortutako indarrak definitzeko. Berrikusketak ebaketa-baldintzetan materialaren propietate aldaketak ere jasotzen ditu, deformazio-abiadura eta tenperatura handitan harrotzen baita materiala. Baldintza hauetan, metalek portaera biskoplastikoa erakusten dute. Berrikusketak altzairuen artezketara mugatu da.



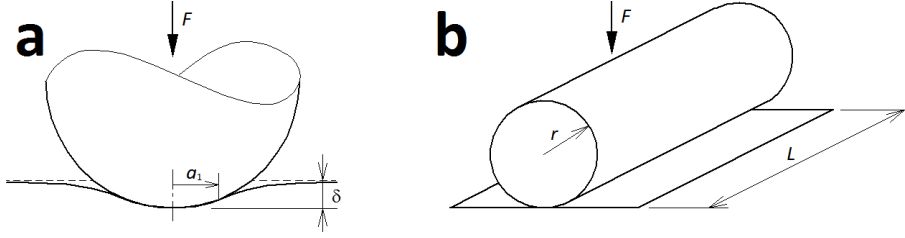
2.20. Irudia: 3D topografia-ereduen diagrama orokorra [DWB06]

### 2.4.1. Kontaktu-mekanika

Kontaktu-mekanikaren sorrera Heimrich Rudolf Hertz gazteari sor zaio. 1881ean neguko oporretan Alpeetan zela, elur-ekaitz batek etxean sartuta egotera behartu, eta kontaktu-mekanikaren oinarri den *'On the contact of elastic solids'* artikulua idatzi zuen [Her1882]. Lan horretan, eremu magnetikoen eta kontaktu elastikoan sortzen diren tentsioen artean analogia proposatzen du, azken hauen adierazpen analitikoak ondorioztatuz. Hertzek deblauki egindako ekarpenek, harrabots handia sortu zuten ordura arte gogortasuna ikuspuntu mekanizistatik lantzen zuten ingeniari komunitatean. Hortik aurrera, kontaktu-mekanikak kasu konplexuagoak (adhesioa, absortzioa, marruskadura...) aztertu ditu hainbat ikuspuntutatik, eredu enpirikoak, grafikoak, analitikoak eta zenbakizkoak proposatuz. Gaur egun, kontaktu-mekanika tribologia jakintza-arlo zabalaren parte da.

Adhesio, marruskadura, lubrifikazio edo higadura fenomenoak aztertzean, ezinbestekoa da gainazalen definizio eta modelaketa zuzena planteatzea. Adibidez, bi gorputzen artean sortzen diren adhesio eta marruskadura indarrak, haien arteko kontaktu-eremuan atomoek duten gertutasunak sortzen ditu. Kontaktua azalerak eta geometriak tarteko gainazalaren zurruntasunean, konduktibitate termikoan eta elektrikoetan eragina dauka, eta azpi-gainazalean zabalduko deformazio elastoplastikoen menpe daude [PHMR05].

Gainazal ez-konformeak dituzten gorputzen artean, lehen kontaktua puntu edo lerro batean gertatzen da [Joh87]. Kontaktua oso txikia da gorputzen dimentsioekin konparatuz. Egoera horietan, kontaktu-tentsioa eremu txikian kontzentratzen da, gorputzek jasandako kanpo indarrek sortutako tentsioekiko independentea delarik. Hertzek fenomeno hori deskribatu zuen, hona-



**2.21. Irudia:** Erdi-espazio elastikoan kontaktua: (a) esfera zurruna eta (b) zilindro zurruna [Joh87]

koa baieztatuz: *'Gure arreta kontaktu ingurura muga dezakegu, bertan tentsioak nabarmenki handiagoak baitira beste edozein puntutan baino eta, ondorioz, gorputzen beste puntuetan aplikatutako indarren ondorio txikiak dira'*. Kontrako muturrean, kontaktu konformeetan, kontaktu-eremua bi gorputzen dimentsioen antzekoa izanik, kontaktu tentsioak gorputzek jasaten dituztenen parekoak dira.

Bi gainazal leun adhesio eta marruskadurarik gabe kontaktuan ipintzean, aplikatutako indar normalaren arabera aldatuz doa egoera. Arazoa indentadorea zurruntzat edo plano zurruntzat hartuz aztertu daiteke. Lehenengo kasua aztertuen bihurtu dute gogortasun-saiakuntzarekin dituen antzekotasunek. Hertzek [Her1882] egoera elastikoan kontaktuaren garapen analitikoak garatzen du beirazko lentine leunketarako. Besteak beste, erdi-espazio elastikoan indarpenetrazio erlazioa ( $F - \delta$ ) definitu du esferarentzat (2.29 Ek.) eta zilindroarentzat (2.30 Ek.). 2.21. Irudiak bi kasuak aurkezten ditu.

$$F = \frac{4}{3} E^* R^{1/2} \delta^{3/2} \quad (2.29)$$

$$F = \frac{\pi}{4} E^* L \delta \quad (2.30)$$

Errealitatean indentadorea ere elastikoa da eta horren eragina  $E^*$  elastikotasun-modulu baliokidea erabiliz konpontzen da (ikusi 2.20 Ek.). Ekuaziotan  $R$  erradioa da eta  $L$  zilindroaren luzera da. Zilindroaren kasuan  $F - \delta$  erlazioa lineala da, zilindroaren  $R$  erradioa edozein dela ere.

Indar normala handitzean, bietako gorputz bat plastikoki deformatzen hasten da. Esfera adibidetzat hartuz, Hertz araber tentsio ebakitzailer maximoa erdi-espazioan gainazaletik  $0,47a_1$  distantziara dago,  $a_1$  kontaktu-azaleraren erradioa izanik. Isurpen-irizpidea betetzean  $\sigma > \sigma_y$ , deformazio plastikoa puntu horretan hasten da. Horrela, gainazaleko bataz besteko presioa  $1,1\sigma_y$  denean, gainazaletik  $0,47a_1$  distantziara materiala egoera plastikoan sartzen da. Esperimentalki frogatu da, deformazio plastikoan bolumena konstante mantentzen dela [PW72]. Praktikan, bi gainazalen arteko kontaktua, deformazio elastiko eta plastikoaren arteko konbinaketa da.

Karga normala handitzean, deformazio plastiko pean dagoen eremua zabalduz doa gainazalera iritsi arte [Joh87]. Puntu horretan kontaktu-azalera orain-

dik txikia da:  $a_1$  kontaktu-erradioa esferaren erradioaren % 1-era ez da iristen. Kontaktu-presioa  $3\sigma_y$  igo da eta, karga gehituz gero ere, konstante mantentzen da. Plastikotasun kontaktu-presioak osatzen du gogortasun-saiakuntzaren oinarria. Indentadorearen geometria aldatuz antzeko emaitzak lortzen dira, eta plastikoki deformatzen den solidoa (erdi-espazioa edo indentadorea) zein den ere berdin du. Horrela, kontaktu-azalera kargarekiko proportzionala dela ondorioztatzen da. Kontaktu egoera honi kontaktu *zurrun-plastiko-perfektua* deritzo, eta deformazio elastikoa arbuatzen du. Artezketan, ale-pieza artean kontaktu-presioek erraz gainditzen dute isurpen-tentsioa, eta kontaktua zurrun-plastiko-perfektutzat har daiteke. 2.2.1. Atalean ikusitako ia kontaktu-eredu denek deformazio plastikoa arbuatzen dute.

Gainazal zimurren arteko kontaktua aztertzean, orain arteko ondorioak laztasun (*asperity*) eskalan aplika daitezke: bai portaera elastikoa, baita kontaktu zurrun-plastiko-perfektua ere [Joh87]. Nolanahi ere, gainazal errealak ez daude uniformeki banatutako laztasunez osatuta. Laztasunen erradioa, altuera eta posizioa banaketa estatistiko jakin bat jarraitzen dute. Gainazal errealetan karga handituz doan heinean, laztasun bakoitzeko kontaktu-azalera handitzeaz gain, laztasun berriak ipintzen dira kontaktuan.

Archard [Arc57] eta Greenwood-Williamson (GW) bikotea [GW66] gainazal zimurren kontaktu eredu aitzindariak izan dira. Archardek gainazal zimurren arteko marruskadura aztertzean, kontaktu-azalera errealak indar normalarekiko erlazio lineala mantentzen duela ohartzen da, bietan, eremu elastikoan nahiz plastikoa. GW eredu ezagunak laztasunak esferikotzat hartzen ditu, horien altuerak banaketa normala jarraitzen dutelarik. Laztasun bakoitzean Hertzten teoria eremu elastikoan aplikatzen da era independentean. Greenwoodek eta Tripp-ek (GT) [GT67], GW eredu moldatzen dute gainazal zimur kurbatuen arteko kontaktu elastikoa aztertzeko. GT eredu ere laztasun esferikoen altuerak banaketa normala dute. Hain zuzen ere, GT eredu da Rowek eta Qik aukuratu dutena harria eta planoaren arteko kontaktu zimurra estimatzeko. Bush *et al.* [BGT75] GW eredu osotu dute, altuera aldaketaz gain, laztasunen erradio eta eliptikotasun aldakorrak barneratuz.

GWn oinarritutako izaera elastoplastikoa duten eredu berriagoak ere proposatu dira. Chang-ek *et al.* [CEB87], gainazal-zimurtasuna esferik osatutako laztasunak eta bolumen kontserbazioa kontuan hartuz, eredu eremu elastoplastikora zabaltzen dute. Zhao-k *et al.* [ZMC00], berriz, aurrekoen akatsa zuzentzen du, propietate direkzionalak dauzkan zimurtasuna modelatuz. Gainera, ereduak deformazio elastikotik guztiz plastikorako trantsizioa leuntzen du. Bere laztasunak ere esferikoak dira. Horngen eredu [Hor98], laztasunak eliptikoak dira eta kontaktua, berriz, elastoplastikoa. Horrela, ereduak gainazal zimur anisotropoak sortzen ditu elipseekin. Eredu guzti horietan laztasunen arteko interakzioak arbuatzen dira, laztasunak bata bestearengandik urrun daudela argudiatuz. Hipotesi hori motz gera daiteke, kontaktu plastikoen ondorioz, deformazio handiek laztasunak biltzen badituzte.

Edozein geometriatako azalera laztasunen arteko kontaktua 3Dn aztertzeko metodorik errazena, dimentsio murrizpen metodoa (*method of dimensionality reduction*, MDR) edo oinarri elastikoen metodoa (*elastic foundation model*) da

[Joh87]. Teknika horrek, gainazaletako bat era diskretuan malguki elastikoz, elastoplastikoz edo biskoelastikoz ordezkatzeko du. Bi gainazalen posizio funtzioak alderatuz malguki bakoitzaren desplazamendua zehazten da eta, desplazamenduarekin, puntu horretako indarra. Metodo sinplea bai, baina malgukietako zurruntasuna zehaztean agertzen dira zailtasunak. Tesi honek proposatzen duen etorkizuneko ikerketa-ildoetako bat, kontaktu-luzera eta harriaren deflexioa aurreikusten duen artezketaren MDR ereduaren garatzea da. Malgukien zurruntasuna garatu den DEM ereduarekin estima daiteke, eta gainazal topografia, berriz, harriaren bermazal-kurbarekin kontaktu probabilitatea ebaluatuz.

Elementu finituen bidezko analisiak aurreko ereduaren desabantailak gainditzeko hurbilpen sendoa da. Francis [Fra82] izan da FEM gainazal zimur planoen arteko kontaktu elastikoa aztertzeke erabiltzen aurrena. Kucharski-k *et al.* [KKPK94] pausu bat aurrera ematen dute, kontaktua elastoplastikotzat hartuz. Kogut-ek eta Etsion-ek [KE02] kontaktu elastoplastikoa esfera zimur eta planoaren artean simulatzen dute.

Bestalde, Mandelbrot-en [Man83] azterketa esperimentalek gainazalen izaera fraktala azalatu du. Laztasun bakoitzak laztasun txikiagoak barneratzen ditu behatutako eskala aldatzean. Eredu hibridoak proposatu dira gainazal fraktalak kontaktuan aplikatzeko. Komvopoulos-en eta Ye-n [KY01] FEM ereduak, eskala fraktal bateragarria duten gainazal zimur elastikoen kontaktua aztertzen dute. Hala ere, FEM eredu horiek solido jarraituen mekanikarentzat daude formulatuak, eta zaila da harriaren heterogenotasunera moldatzen. Jerier-ek eta Molinari-k [JM12], bi gainazal lauen arteko kontaktu zimurra modelatzen dute, elementu diskretuz osatutako gainazal fraktalaren eta gainazal lauaren artean.

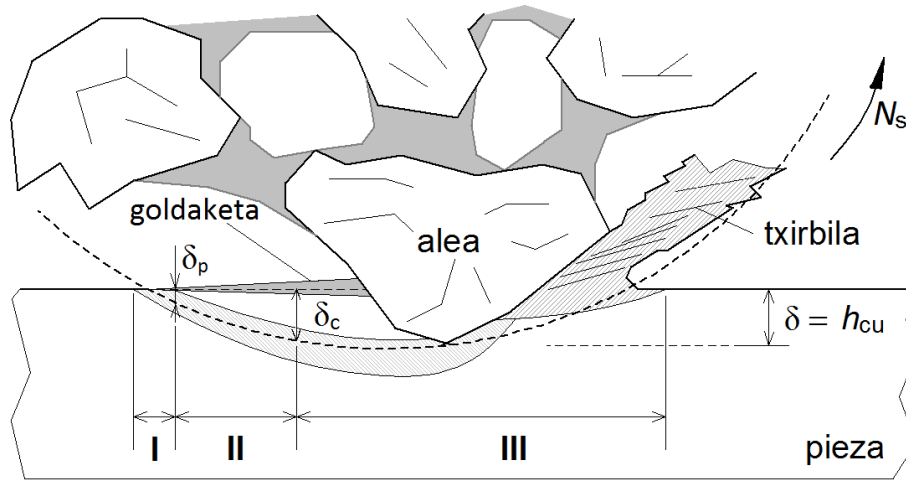
Interesgarria da orain arte gainazal zimurretarako aipatu diren teoriak eta ereduak ere, Archarden kontaktu karga-azalera erlazio lineala betetzen dutela.

### 2.4.2. Urraduraren mekanika

Harriaren mikroegitura konplexuak sorbatzen definizio geometrikoa zailtzen du. Artezketak, aldiunear piezarekin kontaktuan dagoen ale bakoitzak harrotutako materialaren batura da. Hainbat ikertzaile jardun dira interakzioak ale eskalan aztertzen, ikuspuntu esperimentalak, analitikoak eta zenbakizkoak landuz. Atal honek lehenik, lan esperimentalek lortu dituzten ondorioak biltzen ditu. Ondoren, ale bakarraren frogak modelatzen duten eredu analitikoak eta zenbakizkoak aurkezten ditu.

#### Sorbatzaren ekintza. Ale bakarraren frogak

*Material harroketaren mikro-mekanika* jakintza-arloa, ohiko metal-ebaketa arlotik eratorri da. Geometria jakineko sorbatzak dituzten prozesuekin alderatzean jada ohartu ziren artezketan harroketa-mekanismoak ezberdinak direla [BMS52]. Arteztean, txirbil txikiak ebaketa-abiadura azkarragoetan sortzen dira. Jaulkitze-angelu negatiboek eta txirbil-lodiera finek, gainera, energiaren banaketan aldaketak dakartzate.



**2.22. Irudia:** Ale urratzailearen akzioa: (I) igurzketa, deformazio elastikoa; (II) goldaketa (urradura), deformazio elastikoa eta plastikoa; (III) goldaketa eta material harroketa, deformazio elastikoa eta plastikoa, eta txirbil sorrera [Hah62, Klo09] (eskalaz kanpo)

Literaturan, ale urratzailearen akzioa sarritan aztertu da ale bakarraren frogaren bidez. Artezketan, harriaren eta piezaren arteko mugimendu erlatiboaren ondorioz, alearen ekintza ibilbidearekiko baldintzatua dago. Sorbatzaren geometria kamutsagatik, txirbila nekez sortzen da hasieran. Sorbatzaren  $\delta$  sartzea (edo  $h_{cu}$  ebaki-gabeko txirbil-lodiera) aldatuz doa aleak kontaktu-arkuan duen posizioarekiko; era berean, kontaktu-baldintzak aldatuz doaz.  $\delta$ -ren aldaketak Hahn-en ale bakarraren material harroketa eredu inspiratu du [Hah62], ondorengo egileek onartua izan dena [MRDI04, Mal08, Klo09]. Eredu horrek hiru fase bereizten ditu: igurzketa, goldaketa (edo urradura) eta ebaketa, eta 2.22. Irudiak aurkezten ditu. Jiangek *et al.* [JGB<sup>+</sup>13] laugarren bat gehitzen ere dute, hasierako ez-kontaktu egoera, sorbatzen altuera eta posizio irregularren ondorioz.

Hahnek artezketan agertzen diren energia espezifikoko balio altuak, ale-pieza arteko igurzketa faseari egosten dizkio. Igurzketa, alea eta piezaren gainazalaren artean deformazio iraunkorrik gabeko kontaktu elastikoa da.  $\delta$  penetrazioa,  $\delta_p$  goldaketa sakonera-muga baino txikiagoa da igurzketan (2.22.(I) Ir.). Marruskaduraren ondorioz sortutako beroa energia galeratzat hartzen da. Kopsek eta Shawek [KS82], argi infragorriarekin, ale urratzaileak silize-beira urratzean duen erreakzioa aztertzen dute. Labaintza eta astintze faseak identifikatzen dituzte alea beiran sartzen hasi aurretik. Altzairua adina gogorra den beiran behatutakoak, artezketan agertzea ere espero da. Orduan, harriaren gainazaleko alearen ibilbide zirkularrean piezarekin kontaktu egitean, bere lehen erreakzioak deflexioa, biraketa eta labaintzea dira; piezaren materiala nahikoa biguntzean



eta alean eragindako indar normalak beharrezko balioa gainditzean, alea piezan sartzen hasten da.

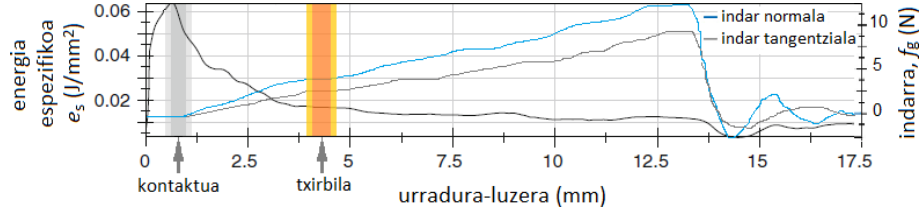
Behin alea piezan barneratuta,  $\delta_p < \delta \leq \delta_c$  sakonera tartean goldaketa gertatzen da, deformazio elastikoen eta plastikoen konbinaketa (2.22.(II) Ir.). Fase honetan, piezaren materialak alearen aurrean pilatzeko joera dauka; alboetarantz desplazatzen denak, berriz, gandorrak sortzen ditu. Berreskuratze elastikoa arbuigarria da, sortzen den ildaska nagusiki aleak alboetarantz goldatutako materialaren emaitza da. Hala ere, goldaketa ez da eraginkorra, materialik ez baita harrotzen.

Marruskadura-indarra handia izanik ere, txirbila ez da sortzen hasten arik eta sorbatzak  $\delta_c$  sakonera gainditu duen arte. Puntu horretatik aurrera, deformazio elastikoaz eta plastikoaz gain, txirbila sortzen da (2.22.(III) Ir.).

Klockek [Klo09]  $\delta$ -k eta  $\delta_c$ -k materialaren harroketan jokatzan duten papera nabarmentzen du. Txirbila soilik  $\delta$ -ren frakzio batek sortzen duela proposatzen du, eta  $h_{cu,eff}$  txirbil-lodiera eraginkorra definitzen du. Baieztapen honek ez du bat egiten eredu zinematikoez [DWB06] eta ale bakarraren froga esperimentalez [LS75, ÖC12, TFX<sup>+</sup>15] deskribatutakoarekin. Horien arabera, aleak bere ibilbidean zeharkatzen duen material dena garbitzen du: frakzio bat txirbila bidez harrotzen da, gainontzekoa alboetarantz desplazatzen delarik.

Marruskadura-baldintzek,  $v_s$  ebaketa-abiadurak eta materialaren isurpen-baldintzek eragin zuzena dute txirbil-sorkuntzan eta  $\delta_c$ -n. Marruskadurak materialak sorbatzaren azpitik isurtzeko duen gaitasuna definitzen du, beraz, marruskadura handitzeak  $\delta_c$  txikitzen du. Alderantziz, lubrifikazioa hobetuz, kontaktua luzatu eta, ondorioz, materiala plastikoki deformatzeko joera bultzatzen da  $\delta_c$  handituz. Beraz, lubrifikazio hobekuntzek harroketa eraginkortasuna murrizten dute. Sorbatzen zorrotasunak ere  $\delta_c$ -n eragiten du. Normalki, zorrotasun-parametro eskasek eta jaulkitze-angelu negatiboez materialaren deformazioa bultzatzen dute,  $\delta_c$  sakonera handituz [MTOO89]. Temperatura igoez materiala harikorra bihurtzen dute,  $\delta_c$  handituz. Deformazio plastikoez eta deformazio-abiadura handiek, berriz,  $\delta_c$  murrizten dute deformaziotatikogogortze efektuaren ondorioz.

Königek eta Lortzek [KL75]  $\delta_c$  sakonera-ren balioa 1 eta 1,6  $\mu\text{m}$  artean kuantifikatzen dute harriaren gainazaleko 2D profil-neurketak aztertuz (ale-neurria 54) eta soilik parametro zinematikoak kontuan hartuz. Txirbila sortu ahal izateko aleen arteko distantzia minimoa  $L_c$  ere estimatzen dute, 400 - 500  $\mu\text{m}$  artean, bataz besteko ale-diametroa baino handiagoa dena ( $d_g = 280 \mu\text{m}$ ). Beraz, ale berdineko sorbatzek ez dute txirbilik sortzen, eta piezarekin kontaktuan dauden sorbatz gehienek ezta ere. 10  $\mu\text{m}$  arteko txirbil-lodiera maximoak  $h_{cu,max}$  aurreikusten dituzte, analitikoki estimatutakoa baino ia 30 aldiz lodiagoa (2.5 Ek.). Tian-ek *et al.* [TFX<sup>+</sup>15] neurria diseinatutako ale bakarraren froga garatzen dute harroketa-mekanismoak aztertzeko ebaketa-abiaduraren  $v_s$  eta  $h_{cu}$ -ren funtzioan. Emaitzek harroketa-mekanismoetan aldaketa nabaritzen dute 100 m/s-tan:  $\delta_c v_s$ -rekiko aldatzen da, 1,4  $\mu\text{m}$  20 m/s-tan, 0,6  $\mu\text{m}$  minimoa 100 m/s-tan, eta 1,6  $\mu\text{m}$  165 m/s-tan. Hau da, 100 m/s azpitik  $v_s$  handitzean, deformazio-abiaduraren pisuak temperatura igoez baino garrantzia gehiago hartuz doa, materialaren plastikotasuna eta  $\delta_c$  murriztuz, eta txirbilaren sorrera erraztuz.



**2.23. Irudia:** Ale bakarraren froga:  $e_s$  aldiuneko energia espezifikoa eta  $f_g$  ebaketa-indarra kontaktu-luzeran zehar [RMK15, RKM15]

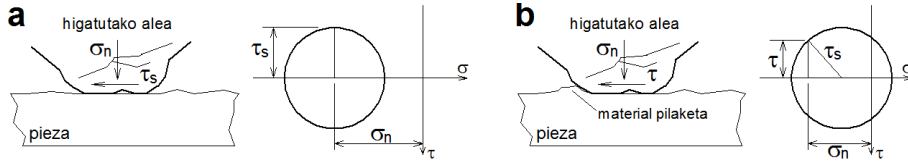
Bestalde,  $v_s$  100 m/s-tik 165 m/s-ra igotzean temperaturagatiko biguntzea indar-tzen da. Antzeko inflexio-puntuak aurkitu dituzte indar eta pilatze-ratioetan. Caik *et al.* [CFJG02]  $v_s$ -k  $\mu$  indar-arrazoian duen eragina aztertzen dute ale bakarraren frogarekin:  $v_s$  azkartzean  $\mu$  murrizten da. Urratzaile materialak ere  $\mu$ -rengan eragiten du.

Rasim-ek *et al.* [RMK15, RKM15] ale bakarraren frogan  $e_s$  aldiuneko energia espezifikoa eta  $f_g$  indarrak neurtzen dituzte. Txirbila sortzeko momentua abiadura handiko bideo-kameraz zehazten dute. 2.23. Irudiak aurkezten ditu haien emaitzak:  $f_g$  ia linealki handituz doa;  $e_s$ , berriz, balio maximoarekin hasten da, eta bere balioa azkar erortzen da bere balioaren seiren batera.  $e_s$  maximoaren iturria igurzketa-eraginkortasun eza da. Igurzketa-eremua motza da, eta goldaketa hastean erorketaren malda leuntzen da. Behin txirbila sortuta,  $e_s$ -ren balioa konstante mantentzen da.

2.2.2. Atalean ikusi da, termoparez eginiko temperatura-neurketetan ebaketa-eremuak ere bereizten direla [Pek57, KS82, GW88, ZV92, QRM97b]. Lehendabizi, tontor gutxi batzuk nabaritzen dira, igurzketa-eremuarekin bat egiten duena. Goldaketa- eta ebaketa-eremuak batera agertzen dira, temperatura eta tontor-dentsitatea biziki handituz (ikusi 2.17.(c) Ir.). Puntu honetan, piezaren materiala azkar biguntzen da materiala erraz harrotuz. Bukaeran, tontorrak desagertuz doaz, finagoak dira eta, irtenen dauden aleek, materiala beroa eta ahulduta topatzen dute. Azken ale horien lan-baldintzak ezin dira ale bakarraren frogarekin aztertu.

Kannapan-ek eta Malkinek [KM72] igurzketa, goldaketa eta ebaketa mekanismoei dagokien energia aztertu dute. Artezketan xahututako energia, hiru osagai horien batura da. Igurzketa higadurarekiko erlazonatuta dago, eta higadura indar tangenzialarekin. Ebaketa-energia txirbilak fusio-temperatura lortzeko energiarekin estimatzen dute. Gainontzeko energia goldaketari esleitzen zaio. Energia banaketak energia minimoaren printzipioa jarraitzen du: uneoro energia minimoa behar duen mekanismoa gailentzen da. Ebaketak, goldaketak eta igurzketa bako bako energia gutxiago behar du, beraz sakonera muga gainditzen denean txirbila sortuko da. Alearen sakonera  $\delta_p$  ebaketa-sakonera minimoa baino txikiagoa bada, txirbila sortzeko energia infinitua da. Artezketaren energia espezifikoa neurri efektua, goldaketarekin estuki lotuta dago.

Malkinek eta Cookek [MC71], aleen higadura aztertzean, ebaketa eta labain-



**2.24. Irudia:** Taborren ale higatuen igurzketakontaktuko ereduak (a) alearen puntan material pilaketa gabe [BT39] eta (b) alearen puntan material pilaketarekin (*junction growth*) [Tab59]

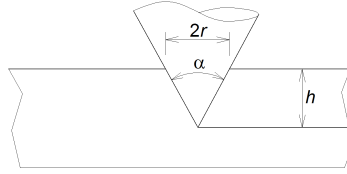
keta osagaietan banatzen dute indar tangenziala. Labainketa osagaia marruskadura eta higadura-planoen (*wear flats*) funtzioan dago. Aleak jasandako indar tangenzialekin eta normalekin aglomeratzaileko tentsioak estimatzen ditu. Horrela, alea askatzeko probabilitatea zehazten duen ereduak aurkezten du.

### Eredu analitikoak

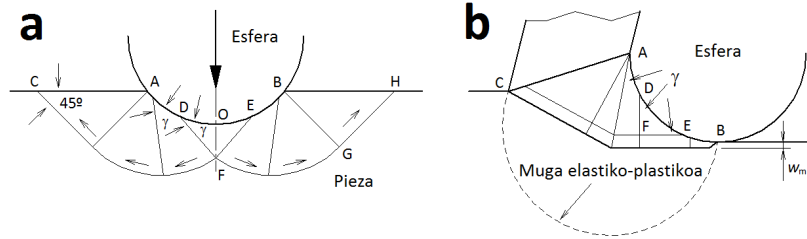
Urradura-eredu analitikoek oinarritzko mekanismoak aztertzen dituzte: igurzketak, urradura eta ebaketa. Oinarritzko hiru mekanismoak artezketan urratzaile-pieza interakzioan agertzen direnekin bat egiten dute, urradura goldaketaren baliokidea baita. Artezketan, material harroketak urradura prozesuetan gertatzen diren marruskadura eta higadura fenomenoaren antz handia dauka. Aleko ebaketa-sakonera txikiak dira eta ale askok piezaren gainazala urratzen dute txirbilik sortu gabe. Oinarritzko urradura-eredu bakoitzak hipotesi ezberdinetatik abiatzen da, hala nola, Von Misesen isurpen-irizpidea, ebakidura labainketa-lerroak edo ebaketa ortogonalaren teoria. Oinarritzko hurbilpen horiek, urraduran parte hartzen duten aldagaiak identifikatu eta bakoitzaren ekarpena zehazten dute. Bestalde, gogortasun-saiakuntzarekin eta mikro-fresaketarekin analogiak ere proposatu dira ale urratzailearen akzioa deskribatzeko.

Igurzketa kontaktua hurbilpena Tabor-ren lan klasikoetan oinarritzen da. Oinarritzko adhesioan [BT39], ale azalean sakonera txikian eta horizontalean desplazatzen da (2.24.(a) Ir.). Isurpen-baldintzak Mohr-en zirkulua erabiliz definitzen dira eta muturrean tentsio ebakitzailak ez dute isurpen-tentsioa gaintzen. Taborrek ereduak moldatzen du [Tab59], errealitatean muturrean agertzen den pilatze efektua (*junction growth*) barneratzeko (2.24.(b) Ir.). Alearen puntan pilatutako materialaren erruz, indar tangenziala handitzen dela konturatu da, indar normalean aldaketarik sumatu gabe. Horrela, pilatze eremuan konpresio tentsio normalak azpian baino txikiagoak dira. Bigarren ereduak  $\mu$  marruskadura koefiziente handiagoak estimatzen ditu, balio errealek gehiago hurbilduz.

Urradura-kontaktuari dagokionez (goldaketa), Rabinobitz-ek [Rab65] 2.25. Irudiko konoak gainazala urratzean indarrak aztertzen ditu. Arazoa bi ikuspuntutatik lantzen du: piezaren gogortasuna eta tentsio ebakitzailak. Emaitzak antzekoak dira: marruskadura koefizientea konoaren maldaren funtzioan dago. Kono angelu handiek marruskadura murrizten dute, higatutako aleetan



**2.25. Irudia:** Konoak sortutako ildaska goldaketaren ondorioz [Rab65]

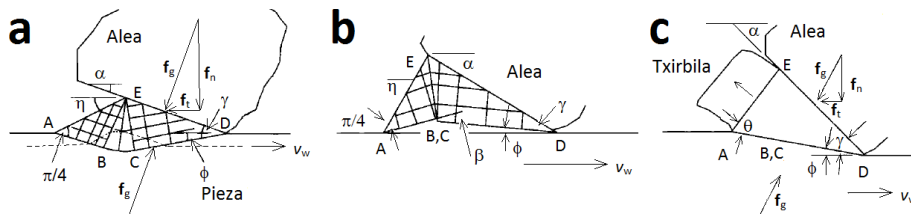


**2.26. Irudia:** (a) Indentazioaren analisia ebakidura labainketa-lerroen bidez [Joh87]; (b) ale urratzaileak arteztean sortutako labainketa-lerroen eremu sinplifikatua [Lor79]

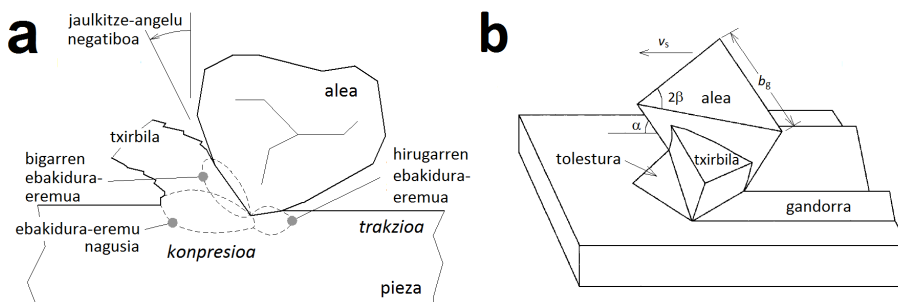
gertatzen den antzera. Hau da, ale zorrotzetan ezik, goldaketa gailentzen da deformazio mekanismoen artean.

Ebakidura labainketa-lerroek (*shear slip-lines*) materialen isurpena era grafikoan azaltzen dute. Labainketa-lerroak soilik tentsio lauko kasuetan aplikatu daitezke, baina baldintza hori betetzen ez den kasuetan ere lagungarriak dira fenomeno fisikoa azaltzeko. Morhen zirkuluan oinarritzen dira. Labainketa-lerroek tentsio ebakitzailerdinak dituzten puntuak lotzen dituzte. Isurpen plastikoan gainazal librearekin  $45^\circ$ tan ebakitzen dira; alearen gainazalekin berriz  $\gamma$  marruskadura-angeluarekin mozten dira. Tomlenov-en indentazio soluzioak [Joh87], azpialdean *D-E-F* eremu hila aurreikusten du (2.26.(a) Ir.). Eremu horrek indentadorearekin batera lan egiten du.

Lortzek [Lor79] esferaren urradura-eredua (goldaketa) aurkezten du. Materiala alearen aurrean pilatu egiten da eta azpian, berriz, deformatu (2.26.(b) Ir.). Challen-ek eta Oxley-k [CO79] artezketarako baliagarriak diren hiru labainketa-lerro eredu aurkeztu dituzte. Lehena, higadura gabeko urradura uhin-eredua da (2.27.(a) Ir.), bigarrena urradura uhin-eredua higadurarekin (2.27.(b) Ir.) eta hirugarrena, berriz, txirbil sorkuntza (2.27.(c) Ir.). Lehenengoak marruskadura koefizienteak estimatzeko erreminta baliagarria dela frogatu du. Bigarrenean, materiala etengabe pilatzen da alearen puntan, jarraitasun baldintzak hautsiz. Txirbil-sorkuntza ereduak energia minimoaren printzipioa jarraitzen du. Horren arabera, txirbil-sorkuntza urradurari beti gailenduko zaio gutxieneko baldintzak betetzean. *D* erpinaren azpian konpresio tentsioak uzten ditu marruskadura koefiziente altuak. *AD* labainketa-lerroan zehar, talka-uhin bat barreiatzen da



**2.27. Irudia:** Challengen eta Oxleyen labainketa-lerro ereduak: (a) urradura uhin-eredua higadura gabe, (b) urradura uhin-eredua higadurarekin, eta, (c) txirbil-sorkuntza ereduak [CO79]



**2.28. Irudia:** (a) Ebaketa-eremuak [MRDI04], (b) txirbil-sorkuntza hiru dimentsiotan [WX92]

txirbila askatuz. Urraduran lubrikazioa hobetzeak indar tangenziala murrizten du; txirbil-sorkuntzan, berriz, alderantzizkoa gertatzen dela erakusten du ereduak.

Alearen material harroketa metalen ebaketaren teoriarekin ere aztertu daiteke. Artezketa-indarrak, aleak materialean sortzen dituen tentsio ebakitzailen menpe daude. Metalen ebaketaren teorian hiru ebaketa-eremu deskribatzen dira (2.28.(a) Ir.). Ebaketa-eremu nagusia ebakidura-eremuari dagokio, alearen puntaren eta piezaren gainazalaren artean. Ideialki, eremu hau planotzat hartzen da eta  $\phi$  ebakidura-angeluaz definitzen da. Bigarren ebaketa-eremua txirbila eta alearen jaulkitze-aurregiaren artean agertzen da. Marruskadura da hor nagusi. Hirugarren eremua, berriz, alearen azpian kokatzen da, azpijan-aurregian. Bertan urradura nagusitzen da. Bigarren eremuan, ebakidura deformazio-abiadura eremu nagusian baino azkarragoa da, temperatura lokal handiak sortuz eta higaduran zuzenki eraginez. Aleek jaulkitze-angelu oso negatiboak dituzte, konpresio-eremu plastikoak sortuz alearen aurrean. Alea behin pasata, trakzio tentsioetara trantsizio azkarra gertatzen da materialean.

Williams-ek eta Xie-k [WX92] piramide geometria duen alearen ebaketa zeiharra aztertzen dute. Aleak  $\alpha$  jaulkitze-angelua dauka, eta  $2\beta$  angelu die-droa oinarrian. Ebaketa zeiharra baimenduz gero, txirbila  $\alpha$  txikietan sortzen

da. 2.28.(b) Irudiak erakusten du alearen aurrealdean materiala pilatu, alboetara bultzatu eta txirbila sortzen dela. Gandorrez inguratutako ildaska uzten du aleak bere atzean. Goldaketa eta harroketa arteko trantsizioa  $\alpha = 6^\circ - 12^\circ$  artean aurreikusten du ereduak, 2D azterketetan lortutakoak baino balio txikiagoak.

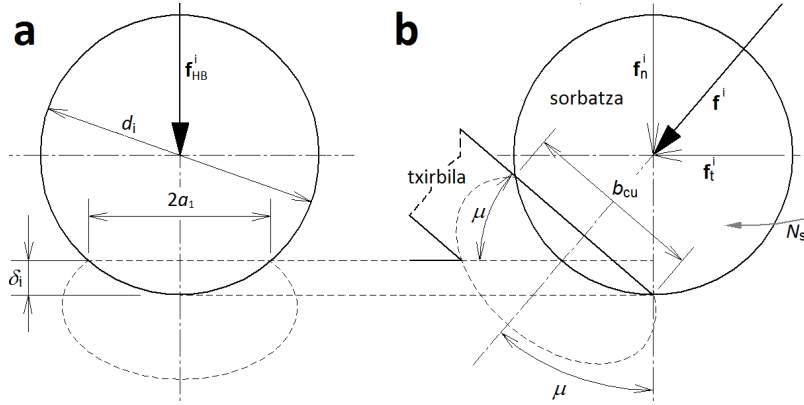
Torrancek, Brener-rekin eta Badgerrekin batera [TB96, BT00], labainketa-lerroen [CO79] eta ebaketa zeharraren [WX92] ereduak alderatzen ditu sarrera datu berdinar erabiliz. Topografia-neurketen bidez lortzen dira malda eta alentsitate sarrera-datuak. Bien emaitzak neurketa esperimentaletan neurtutakoekin bat egiten dute, ebaketa zeharrarekin lortutakoak zerbait hobeagoak direlarik.

Oinarrizko eredu analitikoek gain, ale urratzaileen akzioa gogortasun-saiakuntzarekin eta mikro-fresaketarekin alderatu da. Ale mailan, txirbil harroketa mekanismoak mikro-estrusioarekin antzekotasunak ditu. Mekanismo horrek material bolumen handi bat plastikotasunera eramatea dakar, material kantitate txiki bat txirbil eran ihesten delarik. Fenomeno honetan inspiratuta, Shawek [Sha71, Sha96], artezketan alearen akzioa indentazioaren analogiarekin deskribatzea proposatzen du. Bere esanetan, aleak jasandako indarrak gogortasun-saiakuntzan agertzen direnen natura berekoak dira. 2.29.(a) Irudiak Brinell frogaren eskema erakusten du, non piezak  $\mathbf{f}_{\text{HB}}^i$  indarra eusten duen  $\pi a_1^2$  azaleran,  $a_1$  kontaktu-azaleraren erradioa izanik. Bestalde, 2.29.(b) Irudiak Shawen txirbil-sorkuntzaren gogortasun-frogaren eredu aurkezten du. Ale urratzailearen sorbatza  $\rho_s$  erradioko esferak ordezkutzen du (ikus 2.3.2. Atala). Gogortasun-frogan indentadorearen azpiko eremu plastikoa guztiz konfinatua dagoen bitartean, sorbatzaren azpiko eremu plastikoa aske da inklinatutako kargaren ondorioz. Alea horizontalki desplazatzean materiala eremu plastikotik gorantz ihesten da, aurrean pilatuz.  $\delta_i$  sakonera nahikoa bada, txirbila sortzen da. Horrela, Shawek txirbil-sorkuntza baldintza horiek estrusioarekin alderatzen ditu.

2.31 Ekuazioak Brinell frogan  $\mathbf{f}^i$  indarra estimatzen du (2.29.(a) Ir.).  $H$  piezaren Brinell gogortasuna da.  $C'$  murrizketa-faktorea, bataz besteko kontaktu-presioaren eta piezaren isurpen-tentsioaren arteko ratioa da. Kasu gehienetan  $C'$ -ren balioa 3 inguruan dabil. Indentazio-indarra ale bakarraren analogiara ekartzean (2.29.(b) Ir.), Shawek  $2a_1$  kontaktu-azaleraren diametroa  $b_{\text{cu}}^i$  ale-pieza arteko kontaktu-zabaleragatik ordezkutzen du, eta,  $H$  Brinell gogortasuna kontaktu zurrun-plastiko-perfektuko presioagatik, hau da, hiru aldiz isurpen-tentsioa ( $3\sigma_y$ ). Horrela, bi adierazpenek 2.4.1. Atalean ikusitako kontaktu zurrun-plastiko-perfektuaren baldintzak ere betetzen ditu.

$$\mathbf{f}^i = \frac{1}{4} \pi a_1^2 H \frac{C'}{3} \quad (2.31)$$

Chenek eta Rowek [CR96] eta Heckerrek [HRL03], Shawen indentazioaren analogia barneratzen dute haien eredu zinematikoetan. Horrela, piezaren gainazal zimurtasuna simulatzeaz gain, indarrak ere aurreikusten dituzte. Gogortasun-frogaren analogiaren abantaila nagusia bere sinpletasunean datza:



**2.29. Irudia:** Txirbil-sorkuntzaren gogortasun-froga analogia: (a) gogortasun froga indentadore esferikoarekin; (b)  $\rho_s$  erradioko sorbatzaren indentazioa eta indar erresultantea  $\mathbf{f}^i$  [Sha71, Sha96]

indarraren adierazpena soilik sorbatzaren erradioaren eta ebaketa-sakoneraren funtzioan dago, eta ongi deskribatzen du ale urratzailearen akzioa.

Ebaki-gabeko txirbil-lodierak  $h_{cu}$  eta sorbatz-erradioak  $\rho_s$  artezketan magnitude orden berekoak dira. Horrela, jaulkitze-angelu eraginkorrak oso negatiboak dira eta esferaren hipotesia onargarria da. Shawen arabera, materialaren desplazamendu- eta ebakidura-baldintzek artezketan energia espezifiko bereziki altuak eragiten dituzte. Gogortasun- eta ale bakarraren frogetan, bietan ez dute antzik ez deformazio plastikoek, ez deformazio-abiadurek, ezta tenperatura-baldintzek ere. Hau da, isurpen-baldintzak gogortasun-frogan giro-tenperaturan eta baldintza ia-estatikoetan gertatzen dira, eta, ale urratzaileak, aldiz, abiadura bizian materiala urratzean. Heckerrek, tenperaturaren eta abiaduraren eraginak kontuan hartzeko, indarren adierazpenari  $C_1$  faktore zuzentzailea biderkatzen dio.  $C_1$  emaitza esperimentalekin doitzen du. Beste konponbidea, isurpen-tentsioa definitzeko materialaren portaera biskoplastikotzat jotzea da. Ale bakarraren froga simulatzen duten zenbakizko ereduaren antzera, aldiuneko isurpen-tentsio eraginkorra eredu eratzaile batez estima daiteke. Analogiak beste arazo bat du: igurzketa-, goldaketa- eta ebaketa-mekanismoak ez ditu bereizten. Beraz, gogortasun-frogaren analogiak artezketaren neurri efektua arbuiatzen du, energia espezifikoa handiagoa baita azaleko sakoneretan igurzketa- eta goldaketa-aren ondorioz.

Artezketa fresaketarekin alderatu dutenak ere badaude. Aldenen analisi zinematikoan inspiratuta [Ald1914], egile batzuk mikro-fresaketaren analogia haratago eramanez ale mailako indarrak aztertzeko [Ono61, Rub72]. Hurbilpen hauek ale urratzaile bakoitzaren akzioa sorbatzat hartzen dute. Bestalde, alean indarra estimatzeko Kienzle-k eta Victorrek [KV54] torneaketarako garatutako indar-eredua erabiltzen dutenak ere badaude [Wer78, AK12]. Changek eta Wang-ek [CW08], Wang-ek fresaketarentzat aurrez garatutako eredu analitikoak

[WZ02] ale bakoiztean aplikatzen dute energia espezifikoa funtzioan.

2.32 Ekuazioak Ono-k alean eragiten den aldiuneko indar tangenziala  $\mathbf{f}_t^i$  estimatzeko proposatzen duen adierazpena aurkezten du. Ekuazioan,  $k_s$  indar espezifikoa da,  $A_c$  aldiuneko ebaketa-sekzioa eta  $\theta$  alearen posizio-angelua.  $A_c$  aldiuneko ebaketa-sakonerarekiko eta sorbatzaren geometriarekiko aldatzen da. Onok  $k_s$  enpirikoki definitzen du 2.33 Ekuazioaren potentzia-funtzioarekin, non  $C_1$  eta  $C_2$  konstanteak diren. Horrela,  $k_s$ -k bat egiten du  $e_s$  artezketa energia espezifikoarekin.

$$\mathbf{f}_t^i(\theta) = k_s A_c(\theta) \quad (2.32)$$

$$k_s = C_1 A_c^{-C_2} \quad (2.33)$$

Aldiz, Kienzle-Werner hurbilpenak (2.34 Ek.) antzeko adierazpena proposatzen du,  $C_3$  esponentea  $A_c$ -n aplikatuz. Bi egileek adierazpenak garatzen dituzte, lan-abiaduren eta ebaketa-sakoneraren eta harriaren diametroaren menpeko funtzioak lortuz. Bi adierazpenak oso antzekoak dira. Chenek [Che95] bi ereduak konbinatzen ditu esponenteen eragina aztertzeko. Alde batetik, indarra ale-dentsitatearen eta kontaktu-luzeraren menpe dago eta, bestetik, sakonera nagusitzen da. Horrela ondorioztatzen du artezketa-indarra marruskadura- eta ebaketa-indarren konbinaketa dela, neurri-efektuarekin bat datorren deskripzioa.

$$\mathbf{f}_t^i(\theta) = k_s A_c(\theta)^{C_3} \quad (2.34)$$

Changek eta Wangek [CW08] indar tangenziala eta ebaketa-sekzioa linealki erlazionatzen dute artezketa energia espezifikoarekin (2.32 Ekuazioan  $k_s \rightarrow e_s$ ). Hurbilpen honek urradura-mekanismoak eta neurri-efektua guztiz arbuizatzen ditu.

Laburbilduz, aleak piezarekin duen interakzioan igurzketa, goldaketa eta ebaketa gertatzen dira baldintzen arabera. Igurzketa, urradura eta ebaketa oinarrizko mekanismoen eredu analitikoak erabili dira artezketan ale bakarrak duen portaera aztertzeko. Horiek fenomeno errealarekin antzekotasun erlatiboa erakutsi dute. Nahiz eta hiru mekanismoak aldi berean deskribatzeko gai ez izan, bakoitzari buruz informazio interesgarria eskaintzen dute. Horietako bi, topografia-ereduekin konbinatuz, artezketa-indarrak aurreikusteko erabili dira; ebakidura labainketa-lerroak eta zeharkako ebaketa. Neurketa esperimentalekin konparatuz, biek emaitza onargarriak lortzen dituzte, nahiz eta alearen geometriaren eta marruskadura parametroen definizioa zaila izan. Ale bakarraren akzioa deskribatzeko analogiak ere proposatu dira gogortasun-frogarekin eta mikro-fresaketarekin. Biak errazak dira indar-ereduetan inplementatzen. Gogortasun-frogaren analogiak deformazio-abiadura eta tenperatura altuak arbuizatzen ditu, beraz, isurpen-tentsioak zuzenketa behar du. Mikro-fresaketa eredu, berriz, sinplea da, baina beharrezkoa da aurrez konstante enpirikoak esperimentalki definitzea.



### Zenbakizko ereduak

Txirbil-sorkuntza, ez soilik kualitatiboki, kuantitatiboki ere deskribatzeko, hainbat zenbakizko eredu proposatu dira. Eredu analitikoek ez dituzte kontuan hartzen parametro garrantzitsu guztiak, hala nola, deformazio-abiadura, tenperatura banaketa, edo higadura-portaera. Zenbakizko erremintek ikuspuntu osoagoa eskaintzen dute. Hainbat FEM hurbilpen daude, gehienak 2D, idealizatutako ale-geometriak modelatzen dituztenak [DWB09]. Era berean, dinamika molekularra (*molecular dynamics*, MD) gisako eredu atomistikoaren erabilerak azaleratu egin du materialaren portaera eta harroketa-mekanismoak eskala mikroskopikoan [MRDI04, DWB06, BAG<sup>+</sup>06]. Ale bakarraren zenbakizko eredu gehienak ohiko metalen ebaketa ortogonalaren kontzeptuetan oinarritzen dira.

Domanek *et al.* [DWB09] ale bakarraren FEM ereduak, erabilitako materialaren eredu eratzailearen arabera sailkatzen dituzte. Horrela, materialaren portaera elastikoa, elastoplastikoa, termoplastikoa edo hauskorra (zeraamika) izan daiteke. Eredu eratzaile elastikoak, pieza materiala Hookeartzat hartzen du; eredu elastoplastikoak, berriz, elastikotasunaz gain, plastikotasuna ere kontuan hartzen du. Material termoplastikoak, aurrekoari tenperaturaren efektua gehitzen dio. Azkenik, material zeraamikoaren ereduak mikro-pitzaduren sorkuntza, batura eta zabalkuntza deskribatzen ditu, eta kaltearen pilaketak erresistentzian duen eragina simulatzeko gai da.

Erroketako bat txirbil-sorkuntzaren modelaketa izan da. Klockek *et al.* [KBH<sup>+</sup>02], formulazio esplizitua duen *DEFORM* software komertzialean oinarrituz, 2D FEM ereduak sortzen dute. Ereduak txirbil-sorkuntza simulatzen du sarrera-datuaren artezketa-parametro errealek erabiliz, hala nola, ebaketa-sakonera, -abiadura eta ale-geometria. 1700 °C tenperatura maximoak lortzen dira, baina ereduak ez du deformazio-abiaduraren eragina jasotzen. Ondorengo ereduak gabezia hori konpontzen dute, piezaren portaera biskoplastikoa modelatuz. Ohbuchi-ren eta Obikawa-ren [OO05] 2D ereduak txirbil jarraitu zerrera-erakoak simulatzen ditu. Ereduak deformazio plastikoaren, deformazio-abiaduren, tenperaturen eta isurpen-tentsioen eremuak marrazten ditu, sakoneraren eta jaulkitze-angeluaren funtzioan. Ereduak txirbila sortzeko  $\delta_c$  sakonera minimoa ere kalkulatu du isurpen-baldintzen arabera (ebaketa-sakonera, isurpen-tentsioak, tenperatura). Park-ek eta Liang-ek [PL09] ere ale bakarraren frogak modelatzen dute. Materialaren portaera biskoplastikoa modelatzeko Johnson-Cook (JC) eredu eratzailea erabiltzen dute (2.4.3. Atala). 2D eredu honek harri osoaren deflexioa eta bero iturria ere kontuan hartzen ditu.

Domanek *et al.* [DBW09] ale bakarraren frogaren lehen 3D FEM ereduak garatu dute. 2 mm diametroa duen esferak modelatzen du ale urratzailea. Igurzketa-urradura arteko trantsizioa erosio teknikaz bereizten dute, deformazio plastiko handiak jasan dituzten elementuak ezabatuz. Materialaren propietate biskoplastikoak JC ereduak aplikatuz deskribatzen dira. Nodo-nodo zigor algoritmoa erabili beharrean, segmentuetan oinarritutako kontaktu-algoritmoa garatzen dute simulazioaren egonkortasuna hobetuz. Ereduak igurzketa eta urradura faseak modelatzen ditu, eta bien arteko  $\delta_c$  trantsizio-sakonera 3  $\mu\text{m}$ -tan zehazten du. Sakonera horrek zentzua du, esferaren erradio handiaren ondorioz.

Eredua esperimentalki balioztatzen dute, zenbakizko emaitzekin korrelazio onak lortuz. Anderson-ek *et al.* [AWB11] Domanen lana jarraitzen dute, 0,5 mm diametroa duen esferaren akzioa modelatuz eta frogatuz. Ereduak formulazio Lagrangearra eta Eulertarra nahasten ditu 3D eredu berean, JC ereduaz gain. simulatzean. Emaitzak neurketa esperimentalekin alderatzen dituzte. Besteak beste, ereduak  $\delta_p$  goldaketa- eta  $\delta_c$  ebaketa-sakonera kritikoak zehazten ditu, eta baita fase bakoitzari dagokion energia espezifikoak ere.  $\delta_p$  balioak 0,06 - 1  $\mu\text{m}$  artean daude eta  $\delta_c$  balioak, berriz, 3 - 5,3  $\mu\text{m}$  artean. Balio horiek altuak dira, ziur aski alearen diametro handiagatik.

Rasimen *et al.* [RKM15] ereduak perfekzioa urratzen du, esperimentalki neurtutako ale-geometria amorfoa zenbakizko ereduari modelatuz, eta emaitzak ale berdinarekin egindako neurketa esperimentalekin alderatuz. Korrelazio onak lortzen dira pilatutako eta harrotutako material frakzioen artean, baina simulatutako indarrak behatutakoen erdiak dira. Tenperaturak termopare polobakarrarekin neurtzen dituzte. Ebaketa-sekzioa handituz doan heinean tenperatura igoerak nabaritzen dituzte. Akbari-k *et al.* [ABW16] ale bakarraren froga modelatzen dute diamante-titanio bikotearentzat eta hauek ere eredu esperimentalki balioztatzen dute. Ebaketa-abiadura mantsok dira (0,8 m/s), tenperaturak kamera infragorri bidez neurtzen baitituzte. Hala ere, tenperatura maximoak ebakidura-eremuan agertzen dira, termografia bidez behatu ezin direnak.

Bestalde, artezketarekin erlazioatutako MD ereduak material harroketa eskala nanometrikoan (< 4 nm) simulatzen dute, ohiko artezketa-sakonera mikrometrikoetatik urrun. MDren arazoak azaleratzen dira mikro-topografia, materialaren portaera eta interakzioak deskribatzeko potentzial-funtzioen definitzerakoan eta beharrezko CPU-kostua 3D eredu txikiena ere ebazterakoan. FEM-MD eredu hibridoek aukera ematen dute neurri handiagoko ereduak simulatzeko. FEM domeinu jarraituak energia elastikoa soilik pilatzen du, eta DEM domeinu diskretuan, berriz, aztertutako diren interakzio atomistikoak simulatzen dira. Eredu hauetan, zailtasunak bien arteko interfazeko baldintzak definitzean agertzen dira. Ereduen neurria murrizteko beste aukera, partikula mesoskopikoak edo atomo multzoak erabiltzea da [KD08, JDCI16].

### 2.4.3. Biskoplastikotasuna

Artezketan, ebaketa-sorbatzak abiadura ia-balistikoetan (20 - 40 - 60 m/s eta gehiago) zeharkatzen du pieza. Abiadura horietan deformazio plastikoak eta deformazio-abiadurak materiala zurruntzen dute; ebakidura-eremuan, aldiz, materiala ahultzen duen bero itzela sortzen da [Mal08]. Baldintza hauetan materialaren portaera mekanikoa guztiz aldatzen da, aldi berean erresistentzia handituz eta isurpena erraztuz. Baldintza hauetan, piezak portaera biskoplastiatikoa erakusten du.

Biskoplastikotasunaren teoria eremu jarraituen mekanikaren parte da eta soldoan abiadurarekiko menpeko portaera inelastikoa deskribatzen du. Abiadurarekiko menpekotasunak, materialaren deformazioa kargen aplikazio abiaduraren menpekoko dela esan nahi du [LC94]. Deformazio-abiaduraren funtzioan dagoen

tentsio-maila gaindituz gero, materialak deformazio iraunkorrak jasaten ditu. Metaletan biskoplastikotasuna, ale eskalan behatutako dislokazioen ondorio da. Mekanismoa fusio-tenperaturaren herena gainditzean nagusitzen da.

Materialen propietate biskoplastikoak karakterizatzeko honako frogak egiten dira: gogortze-frogak tentsio edo deformazio konstantean, *creep* frogak karga konstantean eta tentsio-lasaitzea luzapen konstantean. Materialaren isurpen-muga gainditzean, isurpen-egoera mantentzeko tentsioa handitu behar da, hau da, erresistentzia handitzen doa. Fenomeno horri deformaziogatikiko gogortzea edo *strain hardening* deritzo. Abiadurarekiko menpekoak ez diren materialen gogortze-kurben (tentsio-deformazio diagramak) antzekoak dira, baina deformazio berdinarekin abiadura handiagoan, tentsioak handiagoak dira. Isurpena edo *creep* solido batek tentsio eta temperatura konstante pean erakusten duen deformazio-portaera da. Deformazioak iraunkorrak dira. Lasaitze-frogetan deformazioa konstante mantenduz, tentsioa neurtzen da denboran zehar. Froga hauek biskositatea karakterizatzen dute, tentsioaren eta deformazio-abiaduraren artean dagoen erlazioa definituz.

Biskoplastikotasuna modelizatzeko bi hurbilpen mota daude [LC94]. Gaintentsio edo deformazio-abiadura plastikotasun ereduetan, tentsioak gaindi dezake abiadurarekiko menpeko ez den isurpen-tentsioaren eta denboraren funtzioan lasaitzen da. Deformazio-abiadura aldaketak simulatzeko gai dira. Perzyna-ren formulazioa da ezagunena [Per66].

Bigarren eredu familiak isurpen-tentsioa definitzean, deformazio-abiadura ere kontuan hartzen du. Isurpen-tentsio ereduaren helburua, Von Mises edo Tresca hutsegite-irizpideen antzera, temperatura eta deformazio-abiadura jakin batzuetarako  $\sigma_y$  isurpen-tentsio eraginkorra estimatzea da. Hainbat isurpen-tentsio eredu enpiriko eta erdi-enpiriko proposatu dira. Azken hauek dislokazioen dinamikan oinarritzen dira eta talken azterketa da nagusiki haien erabilpen arloa. Hurrengo Atalak aurkezten duen Johnson-Cook eredu eratzailea eredu-talde honen partaide da.

### Johnson-Cook eredu eratzailea

Jonhsonek eta Cookek [JC83] Estatu Batuetako armadarentzat garatutako eredu 1983an ezagutzera eman zuten kongresu batean Holandan. Ordudanik eredu eratzaileen artean erreferentzia bihurtu da, parametroak definitzeko dituen erraztasunagatik eta zenbakizko konputazioan inplementatzeko eskaintzen dituen abantailengatik. Johnson-Cook (JC) eredu guztiz enpirikoa da. 2.35 Ekuazioak  $\sigma_y$  isurpen-tentsio efektiboa estimatzeko adierazpena aurkezten du,

$$\sigma_y(\varepsilon_p, \dot{\varepsilon}_p, T) = [A + B\varepsilon_p^n] \left[ 1 + C \ln \frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_o} \right] \left[ 1 - \left( \frac{T - T_o}{T_m - T_o} \right)^m \right] \quad (2.35)$$

non  $\varepsilon_p$  deformazio plastikoa,  $\dot{\varepsilon}_p$ ,  $\dot{\varepsilon}_o$  uneko eta erreferentziatzeko deformazio-abiadurak, eta  $T$ ,  $T_o$  eta  $T_m$  uneko, giro- eta fusio-tenperaturak diren.  $A$ ,  $B$ ,  $n$ ,  $C$  eta  $m$  enpirikoki definitzen diren material konstanteak dira.

Kortxete arteko lehen adierazpenak, isurpen-tentsioa deformazioarekiko adierazten du erreferentziazko baldintzetan.  $A$  isurpen-tentsioa da;  $B$ -k eta  $n$ -k deformazio-gogortzearen efektuak biltzen dituzte. Konstante horien balioak bai trakzio-frogetatik, baita bihurtura-frogetatik ere lor daitezke. Kortxete arteko bigarren adierazpenak deformazio-abiaduraren eragina barneratzen du.  $C$  erreferentziazko tenperaturaren trakzio- edo bihurtura-frogak hainbat abiaduratan neurtuz definitzen da. Azkenik, hirugarren kortxete arteko adierazpenak tenperaturaren eragina jasotzen du.  $m$  esponentea, deformazio-abiadura konstante mantenduz, hainbat tenperaturetan egindako frogen bidez doitzen da.

JC ereduak, FEM gisako zenbakizko erremintetan erraz barneratzen da, horiek barnean  $\varepsilon_p$ ,  $\dot{\varepsilon}_p$  eta  $T$  aldagaiekin lan egiten baitute. Hainbat egilek aplikatu dute JC ereduak artezketan, piezak portaera biskoplastikoa duela; nagusiki, ale bakarraren froga zenbakizko ereduarekin simulatzean [DBW09, AWB11, RMK15, ABLW16]. Ding-ek *et al.* [DLL15] JC eta fase-transformazio ereduak batera aplikatzen dituzte maraging altzairuen artezketan modelatzen duen FEM eredu termikoan. Ohbuchik eta Obikawak [OO05] eredu eratzaila propioa garatzen dute, JC ereduaren antzekoa.

Hala ere, FEMen ez bezala, eredu mekanistiko batean alea-pieza interakzioan era analitikoan JC ereduak aplikatzea ez da hain berehalakoa. Deformazio- eta tenperatura-baldintzak aldatuz doaz kontaktu-arkuan zehar ebaketa-sakoneren funtzioan. Hurrengo Atalak ale mailan aldagai biskoplastikoen definizioaz eztabaidatzen du.

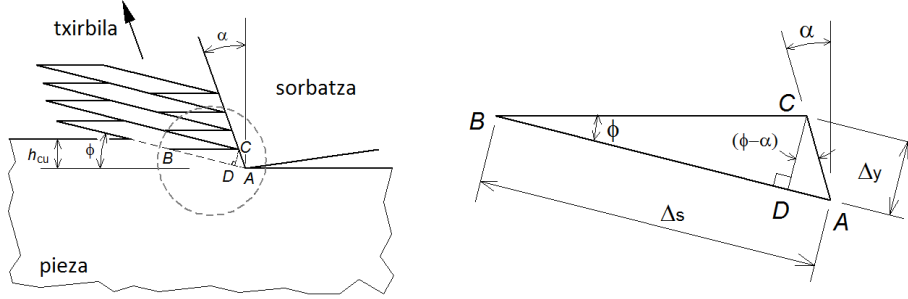
### Deformazioei eta tenperaturei buruzko eztabaida

Esan berri da,  $\varepsilon_p$ ,  $\dot{\varepsilon}_p$  eta  $T$  aurrez ezezagunak direla eredu zinematikoetan. Parametro horien hurbilpena, ikuspuntu numerikotik, analitikotik eta esperimentaletik landu daiteke. Aldi berean hainbat aleen akzioak hiru aldagaiekin duen eragina zehazteko arazoa ere gainditi behar da, tenperaturei zuzenki eragiten diona.

2.4.2. Atalak ale bakarraren froga modelatzen duten zenbakizko ereduak aipatzen ditu.  $\varepsilon_p$ ,  $\dot{\varepsilon}_p$  eta  $T$  eremuak ebaketa-baldintzen funtzioan estimatzeko gai dira. Lortzen dituzten oinarriko emaitzek parte hartzen duten aldagaien eragina eta garrantzia ebaluatzeko gaitasuna erakutsi dute. Bestalde, zaila da lortutako emaitzak esperimentalki balioztatzea. Horretaz gain, tenperatura estimatzean ez dute inguruko aleen eragina jasotzen.

Deformazioen eta tenperaturen neurketa esperimentala erronka handia da artezketan. Ebaketa ortogonalean, ebaketa bat-batean geratzeak (*quick-stop*)  $\varepsilon_p$  deformazio-plastikoen behaketa ahalbidetzen du, erreminta bat-batean askatuz eta ebaketa-eremua izoztuz. Froga ale bakarrera moldatzea konplexua izango litzateke, sorbatzen geometria irregularren eta goldaketak alboetara desplazatutako materialaren ondorioz. Bestalde,  $\phi$  ebakidura-planoaren posizio-angelua (2.30. Ir.) eta  $\dot{\varepsilon}_p$  deformazio-abiadurak ezin dira esperimentalki neurtu.

Eredu analitikoek ebakidura-eremua idealizatzen dute. Material harroketaren teoriaren oinarrietan dago txirbilaren bat-bateko sorrera ebakidura planoan. 2.30. Irudiak ebaketa-ortogonalaren aldagai geometrikoak aurkezten di-



**2.30. Irudia:** Ebaketa ortogonal eraso-angulu negatiboetan; azterketa geometrikoa ebakidura planoan [OY89]

tu. Ale urratzaileen sorbatzek  $\alpha$  jaulkitze-angulu aldakor oso negatiboa dute.  $\phi$  ebakidura-anguluak definitzen duen ebakidura-planoan gertatzen da ebaketa. Eugene Merchant-ek [Mer45] metal harroketan dauden erlazio garrantzitsuenetakoa proposatu zuen 1945. urtean. Adierazpena, ebaketa ortogonalean eragiten duten indarren analisitik ondorioztatzen du. Bere adierazpenak,  $\alpha$  eta  $\beta$  marruskadura-angelua  $\phi$ -rekin erlazionatzen ditu (2.36 Ek.),  $\phi$ -ren balioak ebakidura-energia minimizatzen duen posizioa hartzen duelarik. Adierazpenak materialaren ebakidura-erresistentzia ebakidura-planoan konstantetzat hartzen du, deformazio-abiaduren eta tenperaturen eraginik jaso gabe. Horregatik, adierazpen zehatzat baino hurbilpentzat hartzen da. Merchanten adierazpenaren hainbat moldaketa proposatu dira, hasierako hipotesiak aldatuz edo konstante empirikoekin doitzuz [Sch00]. Nolanahi ere, horien emaitzak ere hurbilpenak dira. 2.37 Ekuazioak  $\beta$  marruskadura-angelua estimatzen du,  $\mu$  indar-arrazoiaren eta  $\alpha$ -ren funtzioan.

$$\phi = 45^\circ + \frac{\alpha}{2} - \frac{\beta}{2} \quad (2.36)$$

$$\beta = \arctan(1/\mu) + \alpha \quad (2.37)$$

Ebakidura-planoko egoera ebakidura hustzat hartuz, 2.38 Ekuazioak  $\varepsilon$  deformazio unitarioa kalkulatu du. Bestalde,  $\varepsilon$  deformazio-abiaduraren definizioa ez da hain zuzena  $\Delta y$ -ren zehazgabetasunaren ondorioz (2.39 Ek., 2.30. Ir.). Oxleyk [OY89], behaketa esperimentaletan oinarrituz, 2.40 Ekuazio empirikoa aurkeztu du, non  $C_1$  materialaren konstantea den. 2.39 Ekuazioan  $v_s$  ebaketa-abiadura da eta  $h_{cu}$ , berriz, ebaki-gabeko txirbil-lodiera (edo sakonera). Deformazio-abiaduraren balioak arteketan oso altuak dira, ia talketan agertzen diren adinakoak ( $10^5 - 10^7 \text{s}^{-1}$ ) [RKM15].

$$\varepsilon = \cot \phi + \tan(\phi - \alpha) \quad (2.38)$$

$$\dot{\epsilon} = \left( \frac{\Delta s}{\Delta t} \right) \cdot \frac{1}{\Delta y} = \frac{\cos \alpha}{\cos(\phi - \alpha)} \cdot \frac{v_s}{\Delta y} \quad (2.39)$$

$$\dot{\epsilon} = C_1 \frac{v_s \cos \alpha}{\cos(\phi - \alpha)} \frac{\sin \phi}{h_{cu}} \quad (2.40)$$

Artezketaren tenperaturei dagokienez, harroketa-baldintza desegokiek piezaren gainazalean sortutako kalteak hainbat ikertzaileen interesa piztu du, eskuragarri dagoen literatura zabalak islatzen duena. Marinescu-k *et al.* [MRDI04] tenperaturak aurreikusteko zenbakizko ereduaren eta metodo analitikoaren berrikusketaz gain, neurketa-teknikak ere biltzen dituzte. Artezketan erraz gainditzen dira harrotutako materiala urtzeko beharrezkoak diren energiak; aleak eta txirbilak ia materialaren fusio-tenperaturetara iristen dira [Mal08]. Ale bakarraren zenbakizko simulazioek aurreko baieztapena konfirmatzen dute, 1700 K tenperaturak aurreikusiz [KBH<sup>+</sup>02]. Gainontzeko energia piezan, ale urratzaileetan eta hozgarrian barreiatzen da. Tenperatura maximoak jaulkitze-aurpegiaren aurkitzen dira txirbila labaintzean sortutako marruskadura-indarren ondorioz. Ausaz abiadura bizian gertatzen diren interakzio kopuru itzelak, tenperaturen neurketa zuzena zailtzen du artezketan. Ale bakarraren frogan, hozketa biziki azkarra da: milisegundo bakarrean tenperatura maximoa laudenera erortzen da behin alea pasata [USN96]. Termopare polobakarra, erradiazio pirometroa, infragorri kamera (*infra-red*, IR) eta analisi metalografikoa dira tenperaturak atzemateko neurketa-teknika nagusiak. Egitura metalografikoan aldaketak ezik, metodo guztiek bai harri osoarekin, baita ale bakarrarekin ere neurketa ahalbidetzen dute [USN96, RKM15, ABLW16]. Biak neurtuta ere, zaila izango litzateke posizio jakin batean tenperatura zehaztea ebaketa-sakonerearen arabera, erabilitako neurketa-teknika edozein dela ere.

Termopare polobakarrak harri osoarekin egindako neurketek, elektrodoa zeharkatzen dituzten aleek sortutako tenperatura-tontorrak biltzen ditu eta, gainera, baita inguruko aleek sortutako beroaren eragina ere (hondo-tenperatura). Froga berdina ale bakarrarekin, berriz, tenperatura-tontor bakarra nabaritzen du. Lehenengoan, ale bakoitzaren ebaketa-sakonera ezezaguna da; bigarrenengoan, aldiz, ezaguna. Behatutako tenperatura, hiru ebakidura-eremuetakoaren (nagusia, bigarrena eta hirugarrena) bataz bestekoa da (2.28.(a) Ir.). Termopare finagoan ( $\leq 25\mu\text{m}$ ) tenperatura neurketa lokalizatuagoak eta erantzundebora azkarragoa dituzte. Urratzailea eroalea bada, adibidez CBN, lotura-beroa pieza eta ale bakarraren artean sortu ahal izango litzateke. Disposizio hori literaturan aurkitu da torneaketari lotua, eta erraz aplikatu ahal izango litzateke CBN ale bakarraren frogan. Pirometroak piezan ikusgai dauden gainazal-tenperatura atzematen du. Doitze konplexuak eta tenperatura neurgarri minimoak pirometroen aplikazio eremua murrizten dute. Aldiz, IR sensoreak sinpleak dira, erantzun azkarra dute tenperatura-muga minimorik gabe [ABLW16]. Tenperatura-banaketak piezaren azpi-gainazalean, sakonera gogortasun aldaketak eta mikro-egiturak aztertuz lortu daitezke. Horretarako, beharrezkoa da aurrez piezaren materialaren gogortasuna tenperaturaren arabera kalibratzea, tenperatura-tarte bakoitzarentzat laginak sortuz. Ondorioz,

azken teknika honek prestatze-lan zabala eskatzen du.

$T$  temperaturaren balioa analitikoki ere hurbildu daiteke. Ebaketan sortutako beroaren frakzio bat soilik doa piezara;  $R_w$ -ri piezaren partizio-ratio deritzo. Urratzailearen eta piezaren materialak, energia espezifikokoak, hozgarriak eta kontaktu-luzerak eragiten dute  $R_w$ -n. Carslaw-ek eta Jaeger-ek [CJ59] bero-iturri mugikorren teoriaren oinarriak finkatu zituzten. Artezketan, Besselen funtzioa betetzen duen lerro bero-iturri mugikorra, denborarekiko eta sakonera-erekiko integratzen da. Artezketan erabiltzen diren ohiko bero-iturriek banaketa uniforme edo trianguluarra dute. Ekuazio diferentzial horien emaitzak tauletatik edota matematika-software bidez lortzen dira. Integraletan muga-balioak ezarriz, temperaturak kontaktu-luzeran zehar estimatzen dira. 2.41 Ekuazioak bero-iturri trianguluarren emaitza aurkezten du zehar denboraren funtzioan,  $t = 0$  kontaktu hasieratik  $t = t_c (= l_c/v_w)$  kontaktu-arkuaren bukaera arte:

$$T = \frac{4q_o}{\beta\sqrt{\pi}} t^{1/2} \left(1 - \frac{2t}{3t_c}\right) \quad \text{non } \beta = \sqrt{(k\rho c)_w} \quad (2.41)$$

$$q_o = \frac{F_t v_s}{l_c b_w} \quad (2.42)$$

non  $l_c$  kontaktu-luzera eta  $v_w$  piezaren aitzinamendu-abiadura diren.  $q_o$  azalera unitateko bero-fluxua da (2.42 Ek.).  $\beta$ -k piezaren materialaren propietate termikoak adierazten ditu,  $k$  konduktibitatea,  $\rho$  dentsitatea eta  $c$  bero espezifikoa direlarik. Temperatura-banaketa uniformeak eta trianguluarrek antzeko  $T_{\max}$  balioak aurreikusten dituzte; bere posizioa da aldatzen dena. Esperimentalki neurtutako temperatura grafikoak interpretatuz, artezketeta-baldintzei dagokien bero-iturri banaketa egokiena hautatzen da. Bero-iturriaren banaketak kontaktu-presioa ere islatzen du. 2.43 Ekuazioak  $T_{\max}$  estimatzen du, non  $C_1$  Peclet zenbakiaren menpekota den konstantea den (2.44 Ek.).  $C_1$ -en balioa 0,76 - 1,06 artean dago. Piezaren materialaren propietate termikoek ( $k$  konduktibitate termikoa,  $\rho$  dentsitatea,  $c$  bero espezifikoa) definitzen dute  $\alpha$  difusibitatea. Deskribatu den hurbilpen analitikoak gutxi gorabeherako balioak lortzen ditu; bere fidagarritasuna  $q_o$ ,  $R_w$  eta  $l_c$  balioen zehaztasunaren baitan dago.

$$T_{\max} = C_1 \cdot R_w \cdot \frac{q_o}{\beta} \cdot \sqrt{\frac{l_c}{v_w}} \quad (2.43)$$

$$Pe = \frac{v_w l_c}{4\alpha} = \frac{v_w l_c \rho c}{4k} \quad (2.44)$$

Eredu enpirikoen bidez ere aurreikusi daitezke temperaturak. Ebaketa-baldintza iraunkorretan (i.e. torneaketa) temperaturak estimatzen dituzten ereduak, oinarrian 2.45 Ekuazioak aurkeztutako egitura partekatzen dute [WG06]. Adierazpen horretan,  $T$  piezak lortutako temperatura da,  $C_1$ ,  $C_2$  eta  $C_3$  esperimentalki zehaztutako konstanteak dira,  $v_s$  ebaketa-abiadura da eta  $a$  ebaketa-sakonera. Eredu horiek kualitatiboki zuzenak dira, eta erraz inplementatuko lirake sorbatz geometria eta sakonera ezagunak diren ereduan. Hala ere, korrelazio eskasa erakusten dute artezketaren antzerako txirbil-sorkuntza prozesu ez-jarraituetan.

$$T = C_1 v_s^{C_2} A_c^{C_3} \quad (2.45)$$

Laburbilduz,  $\varepsilon$ ,  $\dot{\varepsilon}$  eta  $T$  aldatuz doaz alearen ibilbidean zehar piezarekin kontaktuan (igurzketak, goldaketa eta ebaketa). Ideialki, desiragarria da  $\varepsilon$ ,  $\dot{\varepsilon}$  eta  $T$  alearen sakonerarekiko ezagutzea, datu horiek JC eredu eratzailan  $\sigma_y$  isurpen-tentsioa estimatzen erabiltzeko. Horrela, kontaktuan dagoen ale bakoitzak eragiten duen ebaketa-indarra estimatu ahal da. Parametro horien neurketak esperimentalak konplexuak dira. Ale bakarraren zenbakizko ereduak soilik eskaintzen dute hirurak aldi berean estimatzeko aukera, nahiz eta lortutako emaitzen balioztapena froga esperimental konplexuen esku egon. Bestalde, parametroen hurbilpen zabarra lortu daiteke analitikoki, hipotesi sinplifikagarriak onartuz. Denbora errealean sorbatzaren lan-baldintzak definitzeko eredu analitikoaren inplementazioak, harri osoa kontaktuan simulatzean ereduaren konplexutasuna eta konputazio-kostuak handituz. Gainera,  $T$  kontaktu-arkuan aleak duen posizioaren arabera aurreikusiko litzateke, ez ebaketa-sakoneraren funtzioan. Bestalde, ia ez dago ale mailan aplikatzeko egokia den eredu enpirikorik, ziur aski, duen aplikazio praktikoa ezagatik.

## 2.5. Laburpena

Atal honek, artezketari buruz argitaratu diren jakintza zientifiko garrantzitsuenak eta teknologia aurrerapenak biltzen ditu. Bildutako informazioak tesiaren helburuak aurrera eramatea ahalbidetuko dute, hau da, harriaren eta piezaren arteko kontaktuaren zenbakizko ereduaren garapena.

Berrikuspenak artezketan harria-pieza kontaktuan eragiten duten faktore nagusiak identifikatu ditu. Gaurdaino aurkeztu diren kontaktu-ereduak osatzen dituzten faktore nagusiak dira harriaren eta piezaren elastikotasuna, harriaren gainazal topografia eta lan-parametroak ( $a_e$  ebaketa-sakonera,  $F'_n$  indar normala) (2.2.1. Atala). Harriaren elastikotasuna garrantzitsua da; baina, piezaren materialaren portaera elastikotzat hartzeak, aleen goldaketa- eta harroketa-mekanismoak arbuiatzea dakar. Harriaren gainazala diamantaketa bidez prestatzen da (2.1.2. Atala), kontaktu-azterketetan arreta berezia duen faktorea. Azkenik,  $a_e$  eta  $F'_n$  artezketa-abiaduren ( $v_s$ ,  $v_w$ ), gainazal topografiaren, piezaren materialaren eta makinaren zurruntasunaren menpe daude. RQ ereduak gainontzeko kontaktu-ereduengandik nabarmentzen da (2.2.1. Atala). Kontaktuaren tratamendua bi eskaletan (alea-pieza eta harria-pieza) eta kontaktu-arkuen konposizioa, edonola ere, originalak eta inteligenteak dira. Beraz, harriaren ereduak bere zurruntasuna eta morfologia granularra modelatzeaz gain, diamantaketa-baldintzen arabera gainazal topografia sortu, eta ereduak ale-pieza arteko kontaktu-baldintzak barneratu behar ditu.

Hainbat teknika daude kontaktu-luzera esperimentalki neurtzeko: termopare polobakarra, gelditze azkarra, adabaki-artezketa eta emisio akustikoak. Termoparearen aldaera batek, APSak, teknika zehatza eta sendoa dela frogatu du (2.2.2. Atala).



Harriaren eredia sortzeko (2.3.1. Atala), FEM antzerako zenbakizko erremintek konplexutasun handiak erakusten dituzte bi materialez osatutako gorputz porotsua modelatzean. Bestalde, malguki-eredu uniformeez gorputz anisotropikoak sortzen dituzte, aleen artean biraketa erlatiboa baimenduz. Eredu zinematikoak, berriz, zurrunkak izan ohi dira, haien helburua piezaren gainazaleko zimurtasuna aurreikustea baita. Hala ere, harriaren egitura granularra eta topografia sortzeko hainbat soluzio eskaintzen dituzte. Azkenik, aurkeztu berri diren DEM ereduak harriaren morfologia eta zurruntasuna modelatzeko duten potentziala erakutsi dute. Ondorioz, azken hau aukeratu da harria-pieza kontaktua simulatzeko.

Harriaren gainazal topografia (2.1.3. eta 2.3.2. Atalak) definitzen duten aldagaiak ale-dentsitatea, ale-altuera eta sorbatzen zorrotasuna dira. Diamantaketak gainazala sortzen du eta, ondorioz, topografiak bere eragina jasotzen du (2.1.2. Atala). Nahiz eta eraginkorrenak ale-anitzeko diamantatzaile finkoak eta birakariak izan, diamante punta-bakarrak diamantaketa aldagaiek eta -mekanismoek lortutako gainazalarekin erlazionatzea errazten du. Ale eta aglomeratzaile hausturak dira diamantaketa-mekanismo nagusiak. Diamantaketa-baldintzen intentsitateak mekanismo bakoitzaren papera definitzen du, ale-dentsitatean eragitean gainazal irekiagoa edo itxiagoa sortuz. Malkinek eta Cookek proposatutako ereduak ale-askatzea aurreikusten dutenen artean nabarmentzen da, diamantaketa sortutako urratzaile partikulen neurria gainazal irekitasunarekin erlazionatuz. Bi lanek soilik barneratzen dute diamantaketa-baldintzen eragina topografia ereduetan, Chen-Rowe eta Torrance-Badger bikoteenak; eta bien artean, Chenen ereduak da diamantaketa-zinematika kontuan hartzen duen bakarra. Ebaketa-sorbatzen zorrotasuna adierazteko, jaulkitze-angelua edo malda (sorbatz piramidal eta konikoak), edo sorbatz-erradioa (sorbatz esferikoak) erabiltzen dira. Biak antzera neurtzen dira, harriaren gainazalaren edo ale bakarrak utzitako ildaskatzen 2D profilak aztertuz. 3D topografia-neurketa teknikak bi familietan sailkatzen dira: optikoak eta haztagailu bidez (2.1.3. Atala).

Harriaren egituraren eredia eraiki eta gainazal topografia sortzeaz gain, ezinbestekoa da ale-pieza interakzioa era egokian deskribatzea. Kontaktu-ereduan, alearen eta piezaren artean kontaktua detektatzean, kontaktu-baldintzen araberako erreakzio indarra aplikatu behar zaio dagokion aleari. 2.4.2. Atalak ale urratzaileen ekintza kontaktu-luzeran zehar berrikusten du eta urradurari buruz aurkeztu diren lan esperimentalen, analitikoaren eta zenbakizkoen ondorioak biltzen ditu. Artezketaen helburua material harroketa denez, portaera elastikoaren hipotesia hasieratik baztertu da. Oinarrizko eredu analitikoekin (labainketa-lerroak eta ebaketa ortogonalak) lan egitea zaila da, hainbat aldagai ezezagunak (marruskadura), zehaztugabekoak (sorbatz-geometria) edo aldiuneko material propietateak baitira. Shawen gogortasun-frogaren analogiak aldiuneko ale-indarraren kalkulua asko sinplifikatzen du, erreakzioa soilik aldiuneko sakoneraren funtzioan baitago. Eredu horrek kontaktua zurrun-plastiko-perfektutzat hartzen du. Bestalde, ebaketa-sorbatzak deformazio-abiadura azkar eta tenperatura altuetan zeharkatzen du piezaren gainazala. Baldintza horietan piezak portaera biskoplastikoa erakusten du (2.4.3. Atala).

Atal honetan berrikusitako kontzeptuak izan dira ondorengo atalean deskribatutako ereduaren zimenduak.

### 3. Atala

## Zenbakizko kontaktu-eredua

*Atal honek harriaren eta piezaren arteko kontaktua modelatzeko proposatu den DEM eredua aurkezten du.*

*Ereduaren eraikuntza kontaktuaren sistema tribologikoa definitzen hasten da. Sistemaren helburua eragiketan eragina duten aldagaiak sistematikoki identifikatzea da, haien arteko interakzio posibleak eta irteeran duten pisua aztertuz. Artezketan, hainbat aldagaiek ebaketa-sorbatz askotan duten interakzioek zehazten dituzte mikro-ebaketa baldintzak. Berrikuspenak kontaktuaren analisisian gainazal topografiak eta harriaren elastikotasunak duten garrantzia azpimarratu du. Aldiz, orain arteko kontaktu-ereduek ale-pieza interakzioari bizkarra eman diote.*

*Ondoren, harria-pieza kontaktua simulatzeko DEM formulazioan egin diren moldaketak azaltzen dira. Berrikusketan ikusi da DEMaren ezaugarriak ingurune granularrak deskribatzeko oso aproposak direla. Elementu diskretuen arteko interakzioak, kontaktu-detekzioa eta integrazio-algoritmoak zehazten dira. DEM programaren egitura eta kalkulu-begiztak ere aurkezten dira.*

*Atala zenbakizko DEM eredua eraikitzeke metodologia deskribatzen bukatzen du: harriaren DEM ereduaren eraikuntza, gainazal topografiaren sorkuntza eta mugalde-baldintzak. Harriaren egituraren zenbakizko ereduak, harriaren morfologia granularra deskribatzeaz gain, bere portaera mekanikoa islatzeko gai izan behar du. Harriaren gainazal topografia diamantaketaren ondorio zuzena da. Behin DEM eredua eraikita, heldu da mugalde-baldintzak zehazteko garaia. Bertan, gainazaleko aleen eta piezaren arteko kontaktu-detekzio algoritmoa eta kontaktu-legea definitzen dira. Mugalde-baldintzen bi aldaera aplikatu dira DEM ereduaren: lehenengoak RQ ereduaren kontaktu zimurraren terminoa modelatzen du; bigarrenak, berriz, zuzenean artezketa mugalde-baldintzak erreproduzitzen ditu, uneoro aleek duten ebaketa-sakoneraren arabera.*



### 3.1. Sistemen diseinua

Sistema, hainbat sarrera ( $\{X\}$  taldea) eta irteera ( $\{Y\}$  taldea) dituen kaxa beltzat har daiteke. Sistemaren transferentzia-funtzioak definitzen ditu bi taldeen arteko erlazioak (3.1 Ek.),

$$X \longrightarrow Y \quad (3.1)$$

Artezketa sistema irekia da, babestu-gabea eta iraungikorra. Sarrerak eta irteerak mugimenduetan, materialetan, indarretan eta deflexiotan bana daitezke. Gainera, eragiketean trabak agertzen dira, hala nola bibrazioak, kasu batzuetan kontrolagarri, baina ez beti saihesgarriak. Sistemaren irteera batzuk galeratzat har daitezke, besteak beste, zarata, higadura eta bero-sorkuntza.

Harria-pieza kontaktuan, ikuspuntu mekanikotik, presioa itxurazko azalera aplikatua dagoela dirudi, nahiz eta azalera erreala asko ere txikiagoa izan [MRDI04]. Indarrak piezarekin kontaktuan dauden sorbatzetan biltzen dira, presio handiko puntuak eratuz. Horren ondorioz, piezaren gainazalak deformazio elastikoak eta plastikoak jasaten ditu. Sortutako gainazal berrian, marruskadurarekin eta material harroketarekin batera, absortzio, adsortzio, kimisortzio eta korrosio prozesuak agertzen dira. Azken ondorio horiek ez dute eragin zuzenik kontaktu-luzeran, baina bai zeharka, indarretan eta tenperaturetan.

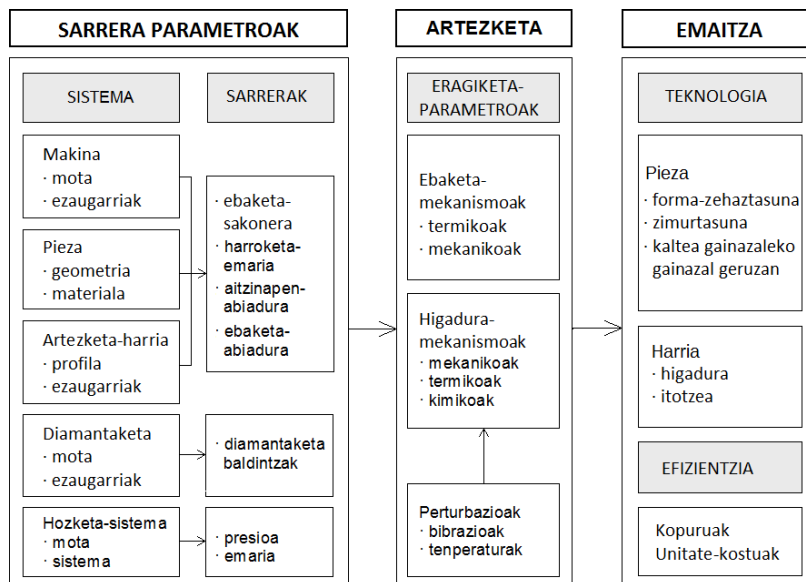
Honako faktore hauek dira kontaktu-mekanikan garrantzitsuenak:

- bi gorputzen propietate mekanikoak (isurpen-tentsioa, elastikotasun modulua eta Poissonen modulua),
- gorputzen gainazalen zimurtasuna, eta,
- parte hartzen duten indarrak.

Artezketan, harriaren eta piezaren arteko mugimendu erlatiboa gehitzen da zerrendara. Faktore bakoitza banan bana aztertu behar da. Harria bi materialez osatutako konglomeratu heterogeneo porotsua da. Bere gainazalak zimurtasun handia dauka eta diamantaketa bidez prestatzen da. Pieza berriz, solido homogeneo jarraitua da. Bere gainazala harriarenarekin konparatuz leuntzat har daiteke. Artezketa-indarrak eta bi gorputzen arteko mugimendu erlatiboa makinari esker agertzen dira. 3.1. Irudiak faktore bakoitzaren aldagaiak eta irteerak biltzen ditu.

#### 3.1.1. Harria-pieza kontaktua zenbakizko eredua

Lan honetan garatu den harriaren eta piezaren arteko zenbakizko kontaktu-ereduak, kontuan hartu ditu 3.1. Irudiko faktoreek kontaktu-luzeran eta elkarren artean dituzten eraginak. Ereduek harriaren izaera heterogeneoa erreproduzitu, gainazal topografia inplementatu eta makinaren lan parametroak mugaldebaldintzetan islatu behar ditu. Ereduek 3.2. Irudiko fluxu-diagrama jarraituko du. Errealitatean harria, propietate mekaniko jakin batzuk dituen ale urratzailez eta aglomeratzailez osatutako konglomeratua da. Harriaren gainazala



3.1. Irudia: Sarrerak eta irteerak artezketa-sisteman [Klo09]

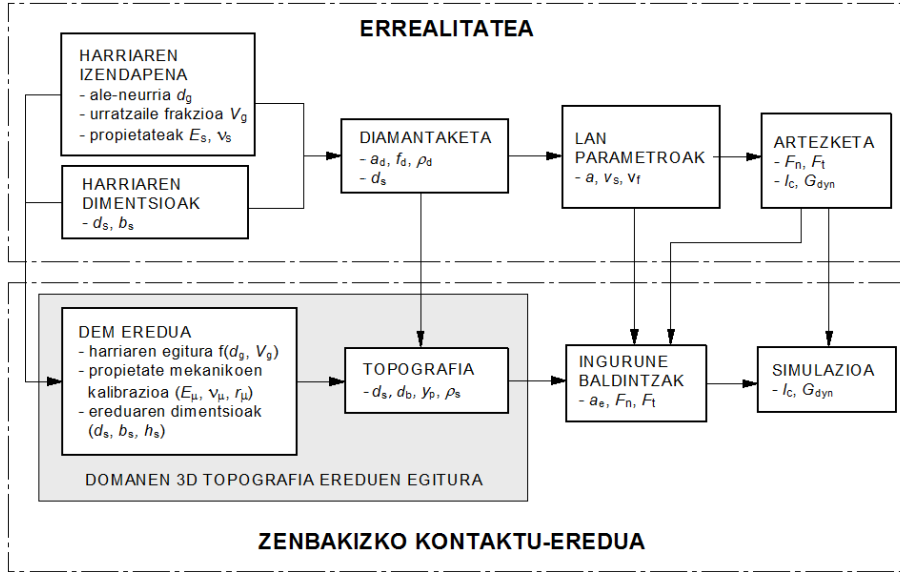
diamantatu, lan-parametroak ezarri eta eragiketa abiatzen da. Zenbakizko ingurunean, kontaktu-ereduak fluxu-diagrama paraleloa jarraitzen du: lehendabizi harriaren eredua sortzen da bere ezaugarrien arabera, gainazal topografia aplikatzen zaio diamantaketa-baldintzen arabera, eta mugaldea eta kontaktu-baldintzak ezarri ondoren, simulazioa abian jartzen da.

Garatu den DEM ereduak, Domanek *et al.* [DWB06] deskribatutako 3D topografia-ereduen egitura orokorra jarraitzen du eredua eta gainazala eraikitzean (2.3.2. Atala). Are gehiago, DEM ereduak egitura orokorra hobetzen du, harriari elastikotasuna gehituz eta aleen arteko interakzioa kontuan hartuz. 2.20. Irudiak aurkezten duen Domanen egitura orokorra, 3.2. Irudian grisez nabarmendutako azalera barneratua dago.

Aurrerantzean tesiak ere fluxu-diagramaren egitura bera jarraituko du.

### 3.2. Elementu diskretuen metodoa

Materialen mekanikan erabiltzen diren zenbakizko metodoak bi familietan sailkatzen dira: metodo diskretuak eta metodo jarraituak. Metodo diskretuak mekanika Newtondarrean oinarritzen dira, eta elementuen arteko interakzioak formulazio esplizitu bidez adierazten dituzte. Modelatzen duen fenomeno fisiko partikula solteak edo gorputz solidoak izan daiteke. Metodo jarraituek bolumenezko domeinuak ingurune jarraitutzat hartzen dituzte, hipotesi egokia hainbat arazo mota modelatzeko, baina harria bezalako gorputz heterogeneo granularretan aplikatzeko zaila.



**3.2. Irudia:** Harria-pieza kontaktuaren fluxu-diagrama errealitatean eta zenbakizko ingurunean

Metodo diskretuak, partikula-kopuru handi baten mugimenduak konputatzen dituzten zenbakizko metodoa dira [CJR13, JDCI16]. Partikulen eskala nanometrikoa edo handiagoa izan daiteke. Metodo diskretuek, elementuen arteko interakzioak funtzio erregularren bidez (jarraituak eta aski diferentziagarriak) deskribatzen dituzte, hutsuneak, abiadura erlatiboak eta erreakzio-indarrak uztertuz. Metodo diskretuen artean, dinamika molekularra (*molecular dynamics*, MD) eta elementu diskretuen metodoa (*discrete element method*, DEM) dira ezagunenak. MDk atomoen ibilbideak simulatzen ditu, haien arteko interakzioak barne. Aplikaturako formulazio fisikoak atomoen arteko interakzio-indarrak definitzen ditu. Atomo-kopurua itzela da eta, ondorioz, beharrezkoa da kontaktu detekzio-algoritmo eraginkorra erabiltzea. Denboraren diskretizazioak zenbakizko simulazioa ahalbidetzen du: lehendabizi, atomoen arteko interakzioak identifikatzen dira elementu bakoitzean eragiten duten interakzio-indarrak kalkulatzeko; eta, azelerazioa behin lortuta, bitan integratzen da partikula bakoitzak hurrengo iterazioan izango duen abiadura eta posizioa lortzeko. Integrazio-eskema esplizitu erabilienak Verlet abiadura- eta posizio-eskemak, igel-jauzia (*leap-frog*) eta iragarle-zuzentzailea (*predictor-corrector*) dira. MD arrakastaz aplikatu da ale bakarraren frogaren modelatzean oso azaleko ebaketa-sakoneratan ( $< 4$  nm) [MRDI04, BAG<sup>+</sup>06, OC10, MSKM15]. Ondorioz, MD ez da egokia artezketan ohikoak diren eskala mikrometrikokoak simulatzeko (0 - 10  $\mu\text{m}$ ).

MDk inspiratu du DEM, baina mikro- eta meso-escalan aplikatzen da. Do-

meinuaren eskalaz gain, MDkin alderatuz, DEMak partikulen orientazioa ere kontrolatu ahal du eta, partikula esferikoez gain, poliedroak ere erabili daitezke. Cundall-ek eta Strack-ek argitaratu zuten DEMaren erreferentziatzko lana [CS79], arroka sedimentarioen azterketa geoteknikoan aplikatuz. Bere aplikazio-eremua zabalduz joan da beste arazoetara, besteak beste, higadura modelatzen [FIB07], gainazal fraktal zimurren arteko kontaktua [JM12], gorputz heterogeneoetan kalteak aztertzen, adibidez hormigoian edo arroketan [HDD04], beiraren leunketan [IBNC08], apar polimerikoetan [VBIL10], eta baita karbono-zuntzez indartutako polimeroetan ere [IGI<sup>+</sup>10]. Dagoeneko DEM harriaren egitura modelatzen ere erabili da [LYZW15a, LYZW15b]. Elementu diskretu (*discrete element*, DE) batek modelatu dezake, bai material granular baten partikula askea (hautsa, hondarra, harrixkak, artoa...), baita bolumen adierazgarri elementu (*representative volume element*, RVE) baten osagai bat ere. Azken honek gorputz baten izaera jarraitua modelatzen du fenomeno fisikoaren eskalaren arabera. DEMaren ausazko natura da bere abantaila indartsuenetakoa. Eredu granularretan sortzen diren indar-sareak DEM bidez behatu daitezke. Partikula txiki askorekin ezinezkoa da neurketa horiek esperimenteralki frogatzea. Hala ere, DEMak behar konputazional handiak ditu, bere aplikazioa simulazio denborak eta elementu kopuruak mugatzen dutelarik.

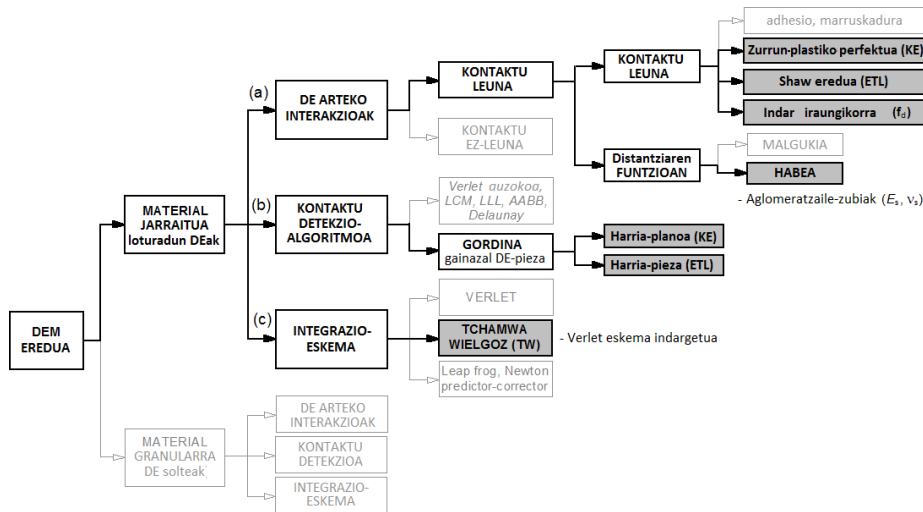
Elementuen arteko kontaktuak tratatzeko moduek bi metodo diskretu aldaera ekarri dituzte. Kontaktu-dinamika leunak, esfera zurrunen arteko interakzioak elastikotzat hartzen ditu. Hurbilpen honek integrazio-eskema esplizituak dakartza, MDn eta DEMen antzera, kasu dinamiko azkarrak aztertzeko egokiak. Bestalde, kontaktu-dinamika ez-leunak denbora integrazio-eskema implizituak dakartza, fenomeno estatikoak simulatzeko egokiagoak direnak [MP88].

Literatura berrikusketan hainbat aukera aztertu ondoren (2. Gaia) eta behin sistemen diseinuarekin aldagai nagusiak zehaztuta (3.1. Atala), DEM izan da harriaren egitura modelatzeko identifikatu den zenbakizko erreminta egokiena. Formulazioan duen malgutasunak, elementuen arteko erlazioak neurrira definitzeko aukera emanez. Horrek DEM arteketan kontaktua era naturalean simulatzea errazten du. Atal honetan, neurrira egindako harria-pieza kontaktu-eredua eraikitzeke DEM formulazioan egin diren moldaketak deskribatzen ditu.

### 3.2.1. DEMaren moldaketa artezketa kontaktu-eredura

Lehen komentatu bezala, kontaktu-ereduak aleen eta piezaren arteko interakzioa simulatzeko, harriaren morfologia granularra, portaera mekanikoa, gainazal topografia eta kontaktu-baldintzak erreproduzitu behar ditu. Harriaren morfologiak biltzen ditu ale urratzaileak, aglomeratzaile-zubiak eta biek eratzen duten egitura. Bestalde, kontaktu-baldintzen artean, harriaren barnean ale-ale interakzioak eta harria-pieza interfazeen ale-pieza interakzioak bereizten dira. Harriaren gainazal topografiak, berriz, diamantaketaren ondorioen ñabardurak jaso behar ditu. DEM formulazioaren malgutasuna da duen abantailarik nagusienetakoa, aipatu diren ezaugarriekin neurrira diseinatutako eredua eraikitzea baimentzen duena. Abiapuntua, DE batek ale urratzaile bat ordezkatzeko duela pentsatzea da. Horrela, harria makina DE-ez osatzen da. Pieza, berriz, DE





### 3.3. Irudia: Proposatutako DEM ereduaren formulazioaren egitura

errektangeluar bakarrak modelatzen du.

3.3. Irudiak neurrira diseinatutako DEM kontaktu-ereduaren egituraren fluxu-diagrama aurkezten du. Modelatu nahi den fenomeno fisikoaren arabera, DEM formulazioa material jarraitua (adibidez, hormigoia portaera mekanikoa) edo material granular soltea (adibidez, arto-toberaren hustuketa) simulatzeko moldatu daiteke. Harria, izaera solidoa izanik, material jarraitu tratamendua jasoko du. Nahiz eta material heterogeneoa izan, bere portaera jarraitua azaleratzen da, ale urratzaile-aglomeratzaile-poro nahasketak bat eginda lan egitean.

DEM formulazioak bi aldaeretan, hiru oinarri nagusi dauzka komunean: (a) DE-en arteko erlazioak, (b) kontaktu detekzio-algoritmoa eta (c) integrazio-eskema (3.3. Ir.). Aukeren arteko konbinaketak soluzio anitzak eskaintzen dienez, arlo askotako fenomeno fisikoak modelatzeko gai da.

Elementuen arteko interakzioak (a) bi talde nagusitan sailkatzen dira: kontaktu leuna eta ez-leuna. Kontakto leuna hurbilpenak DE zurrunik erabiltzen ditu, ez-leunak, berriz, DE deformagarriak. Harria osatzen duten ale urratzaileak zurruntzat har daitezke duten gogortasunagatik. Piezaren materiala, berriz, era plastikoan deformatzen da, batzuetan materiala harrotzeraino. Hala ere, DEM formulazioaren malgutasunak kontaktu-baldintzak interakzio motari moldatzea ahalbidetzen du. Horrela, nahiz eta kontaktu leunak DE zurrunik erabili, DE-en arteko interakzio-baldintzak molda daitezke, kontaktu (elastikoa, Hertziarra, adhesioa, marruskadura), lotura (malgukia, habea) edo distantziarekiko menpeko (eremu elektromagnetiko, bero kondukzioa, barne-kohesioa) tratamendua emanez. Gainera, DE zurrunen erabilerak abantailak ditu fenomeno dinamikoak modelatzean.

Artezketa kontaktu-ereduari dagokionez, DE-en artean bi interakzio-mota daude: harriaren barneko DE-en artekoa, eta harriaren gainazaleko DE-en eta piezaren artekoa. Lehenengoentzat, harria osatzen duten DE-en artean, habe elastikoak aukeratu dira. Aglomeratzaileak ale urratzaileen artean lotura fisikoak sortzen ditu, karga axialez eta ebakitzzaileez gain, makurdura- eta bihurtura-momentuak ere jasaten dituztenak. Horrela, habeek aglomeratzaile-zubien papera fidelki betetzen dute. Habeek eta aglomeratzaile-zubiek ez bezala, kontaktu-baldintzek (Hertz, adhesioa...) eta malgukiek ez dute DE-en arteko biraketa erlatiboa eragozten. Bigarrenengoentzat, gainazaleko DE-en eta piezaren arteko interakzioa, neurrira egindako kontaktu-legea garatu da bertako baldintza bereziak modelatzeko. Kontaktulegeak indar iraungikorren osagaia ere barneratzen du, simulazioa indargetzeko.

Kontaktulege-algoritmo (b) egokiaren aukeraketa garrantzitsua da, handiak baitira DEMak dituen konputazio-eskakizunak. Hainbat algoritmo proposatu dira kontaktu-analisi gordina saihesteko, hau da, DE bakoitzaren posizio erlatiboa beste DE guztiekiko aztertzea. Ezagunen artean daude Verlet-auzokoen lista, itsatsitako zelularen metodoa (*linked cell method*) eta Delaunay triangulaketa. Hala ere, haben erabilerak harria osatzen duten DE-en arteko kontaktu detekzio beharrak ezeztatzen ditu, DE-en arteko loturak finkoak eta ezagunak baitira. Hau oso onuragarria da, DE guztiak bat ezik (pieza) harriari baitagokio. Gainazaleko DE-pieza kontaktuari dagokionez, pieza ordezkatzeko DE errektangeluar bakarraren erabilerak asko murrizten du kontaktu detekzio lana. Neurrira egindako bi kontaktu detekzio-algoritmo garatu dira, 3.5. Atalean sakonki azaltzen direnak.

DEM formulazioaren hirugarren oinarria (c) integrazio-eskema da. Simulazioan, iterazio bakoitzean programak zenbakizko integrazioa burutzen du, hurrengo iterazioan DE bakoitzak izango dituen posizio eta orientazio berriak kalkulatzeko. Hainbat integrazio-eskema proposatu dira, besteak beste, abiadura-eta posizio-Verlet, igel-jauzia (*leap frog*) eta aurrikusle-zuzentzaile (*predictor-corrector*) eskemak. Verlet integrazio-eskema da erabiliena. Simulazio ia-estatikoetan konbergentzia hobetzeko, Tchamwa-Wielgosz eskemak zenbakizko moteltze-faktorea gehitzen dio ohiko Verlet hurbilpenari [MGR09].

Jarraian datozen Atalek kontzeptu hauek garatzen dituzte artezketa kontaktu-ereduaren ikuspuntutik.

### Elementu diskretua

DEMak normalki elementu esferikoak erabiltzen ditu, baina beste geometriak ere erabil ditzake: elipsoidala [MI10], esfera-anitzak eta poliedroak [HWKES11]. DE esferikoen definizioa da sinpleena, bere erabilerak ereduaren sorkuntza eta kontaktu detekzioa errazten dituelarik. Lik *et al.* [LYZW15b], ale urratzaile amorfoen geometria modelatzeko DE esfera-anitzak erabiltzea proposatzen dute, nahiz eta ideia aurrera ez eraman. Horren ordez, moztutako oktaedro elementua garatzen dute, esferoidearen eta oktaedroaren arteko intersekzio Boolearra. Aleen geometriak erlazio zuzena du trinkotzean eta, beraz, harriaren egituraren irekitasunean. Literatura-berrikuspenean (2.3.2. Atala) azaldu da alumina

urratzaileek geometria kubikoa, angeluarra eta puntaduna dutela. Geometria kubikoaren eta angeluarraren aspektu-erlazioak batetik hurbil daude; puntadunek, berriz, dimentsio bat beste biak baino luzeagoa dute. Harria osatzen duten DE-en arteko kontaktuak arbuigarriak dira, DEak lotzen dituen habesareak ematen baitio zurruntasuna harriari. Beraz, elipsoideen edo oktaedroen erabilera ez dago arrazoitua harriaren egitura modelatzeko. Harrien fabrikazioan trinkotze-prozesua simulatzeko aldiz bai, aleen geometriak garrantzia du. Geometria jakina duten urratzaileak (CBN, diamantea) tetraedro eta oktaedro nahasketekin modelatu daitezke; ale amorfo puntadunak simulatzeko, aldiz, nahikoa da DE esfera-bikoitz (edo esfera-anitz) elementuak erabiltzea. Bestalde, aluminazko harrietan diamantaketak gainazal topografia eraldatzen du, baita DEM ereduaren ere, eta, beraz, DE geometriak ez du garrantzirik harriaren gainazalean.

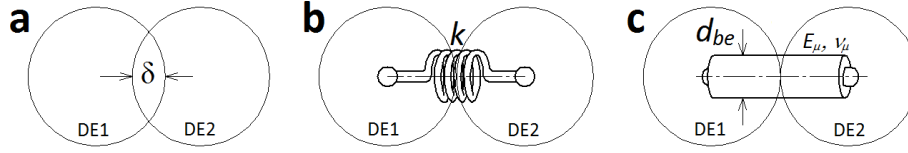
Elementuen geometriak soilik ereduaren sorkuntzan eragina duenez, arrazoizkoa da urratzaileak DE esferikoez modelatzea ale kubikoentzat eta angeluarrentzat. Tesi honetan, DE esferikoak aukeratu dira, froga esperimentaletan erabilitako harriak urratzaile angeluarrez osatzen baitira. Ale geometria puntaduna (luzatua) modelatzeko, eredia DE esfera-bikoitzera eguneratu beharko litzateke. DEM bidez harrien trinkotzearen simulazioa ale geometriaren eta aspektu-erlazioaren arabera tesi honek proposatzen duen etorkizuneko ikerketatildotako bat da.

DE esferikoa 3D espazioan honako aldagai hauek definitzen dute:  $d_{DE}$  diametroak,  $\rho_{DE}$  dentsitateak,  $(x, y, z)$  posizio-koordinatuak eta, modelatzen ari den fenomeno fisikoaren arabera, orientazioa ere elementuaren formulazioan gehitzen da.

Orientazioa, Euler-angeluen, biraketa-matrizeen, koaternioien edo aurrekoak konbinatuz zehazten da.  $\mathbf{q}$  koaternioi unitarioen erabilera ohikoa da DE-en orientazioa eta abiadura angeluarra definitzeko, konputazioan eskaintzen duten zehaztasunagatik eta sinpletasunagatik [ACI15]. Verlet integrazio-eskeman era errazean inplementatzen dira, koaternioien zenbakizko integrazioa ahalbidetuz. Koaternioiek lau aldagai eskalarren bidez orientazioa definitzen dute 3D espazioan. Zenbaki konplexuen beste bi dimentsio irudikarietara zabalkuntza adierazten dute, horrela, dimentsio erreal bat eta beste hiru dimentsio irudikariz osatzen dira. Koaternioien erabilera zabaldu du orientazioak zehazteko duen sinpletasunak, besteak beste, 3D ordenagailu grafikoetan edo plano kristalografikoen definizioan.

Biak konbinatuz, DE markoa osatzen da: posizio-koordinatu Cartesiarrak eta orientazio koaternioi unitarioa. Horrela, DE markoak posizioa eta orientazioa aldi berean definitzen ditu.

Piezari dagokionez, DE errektangeluar prismatiko bakarrak modelatzen du. Piezaren gainazala erreferentzia da DE-en sakonera kalkulatzeko. Iterazio bakoitzean harriaren gainazaleko elementuen eta piezaren posizioak konparatuz, kontaktua detektatzeko erabiltzen da. DE bakarra erabiltzeak kontaktu detekzioa izugarri errazten du. Ale-pieza kontaktuan, piezaren materialaren portaera biskoplastikotzat jo da, kontaktu-baldintzak moldatuz.



**3.4. Irudia:** DE-en arteko ohiko interakzio-motak: (a) kontaktu- edo potentzial-lotura ( $f(\delta)$ ), (b) malgukia eta (c) habea [ACI15]

### Elementu diskretuen arteko interakzioak

Elementuen arteko interakzioak indar, momentu eta, batzuetan, energia potentzialek gobernatzen dituzte. Horrela, interakzioak hiru taldetan sailkatzen dira: kontaktu-loturak, potentzial-loturak eta lotura kohesiboak (malgukia edo habea). 3.4. Irudiak DE-en arteko interakzio-motak aurkezten ditu.

Harriaren gainazaleko elementuen eta piezaren arteko interakzioa, ohiko kontaktu-loturekin egin daiteke, hau da mekanika Hertziarra eta Coulomben marruskadura, indar iraungikorrarekin batera. Hala ere, ale urratzaile eta piezaren arteko kontaktu-baldintza bereziak simulatzeko, neurrira egindako formulazioa garatu da, beti ere indar iraungikorra gehituz. Bi kontaktu-baldintza definitu dira, hartutako hipotesiak islatzeko. Lehenak, RQ kontaktu-luzera ereduaren kontaktu zimurraren terminoa modelatzen du [RMQZ93], baldintzak kontaktu elastikotik zurrun-plastiko-perfektura eguneratuz [Joh87]. Horrela, eredu honek harria eta plano baten arteko kontaktu estatikoa (KE) modelatzen du. Piezaren materialaren propietateak ebaketa-baldintzetara moldatu dira JC eredu eratzailera aplikatuz. Bigarren ereduak, berriz, Shawek proposatutako gogortasun-saiakuntzaren txirbil-sorkuntza analogia erabiltzen du [Sha71]. Kasu horretan, ereduak uneoro estimatzen ditu kontaktuan dauden DE-en ebaketa-sakonerak edo txirbil-lodierak, eta, datu horiek Showeren ereduari aplikatuz, DE bakoitzean ebaketa-indarrak kalkulatu dira. Bietan aplikatutako formulazioak sakonki azaltzen dira 3.5. Atalean. Era horretan, DEM kontaktu-eredua ale-pieza arteko baldintza berezietara moldatzen da.

Eredu dinamikoa denez, indar iraungikorra edo moteltze-indarra agertzen da bi DE-en arteko interakzioan. Indar iraungikorra  $\mathbf{f}_d$  ( $\mathbf{f}_{d1}$ ,  $\mathbf{f}_{d2}$ ) bi DE-en abiadura erlatiboarekiko  $\dot{x}$  proportzionala da eta kontaktu-planoarekiko perpendikularra (3.2 Ek.).  $c$  moteltze-koefizientea da, eta 3.3 Ekuazioarekin kalkulatu da.  $\alpha$  moteltze-faktorea  $[0, 1]$  da;  $\alpha_c$ , berriz, masa-malguki sistemaren moteltze kritikoa da (3.4 Ek.), beraz,  $c$ -ren balioa  $[0, \alpha_c]$  tartean dago (3.3 Ek.). 3.4 Ekuazioak  $\alpha_c$  kalkulatu du, non  $K_c$  kontaktuan dauden bi DE-en zurruntasunik handiena den eta  $M_{12}$  bi DE-en arteko masa baliokidea (3.5 Ek.).  $m_{DE1}$  eta  $m_{DE2}$  DE1 eta DE2 elementuen masak dira hurrenez hurren. 3.6 Ekuazioak  $\dot{x}$  kalkulatu du;  $\mathbf{v}_1$  eta  $\mathbf{v}_2$  bakoitzaren abiadurak dira eta  $\mathbf{n}_{21}$  kontaktu-planoarekiko normala den bektore unitarioa.  $\mathbf{f}_d$  indar iraungikorra bi DE-en erreakzio indarrei gehitzen da.

$$\mathbf{f}_d = \mathbf{f}_{d1} = -\mathbf{f}_{d2} = c \dot{x} \mathbf{n}_{21} \quad (3.2)$$

$$c = \alpha \cdot \alpha_c \quad (3.3)$$

$$\alpha_c = 2\sqrt{K_c \cdot M_{12}} \quad (3.4)$$

$$M_{12} = \frac{m_{DE1} m_{DE2}}{m_{DE1} + m_{DE2}} \quad (3.5)$$

$$\dot{x} = (\mathbf{v}_2 - \mathbf{v}_1) \cdot \mathbf{n}_{21} \quad (3.6)$$

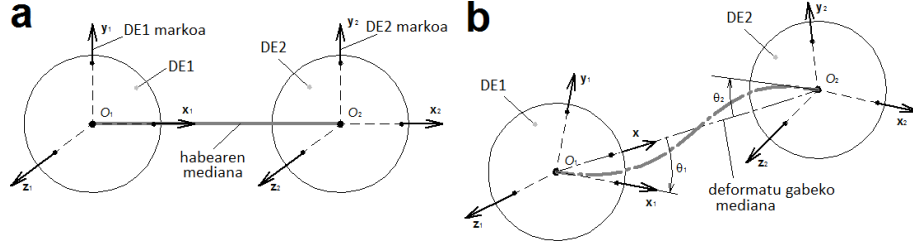
Harria osatzen duten DE-en interakzioei dagokionez, DE bakoitzak ale urratzaile bat ordezkatzan du. Aglomeratzaileak ale urratzaileekin batera 3D egitura zurruna osatzen dute; bien konbinaketak RVE eskala gainditzean portaera jarraitua erakusten du [CJR13]. Unitate minimoa ale-neurriak eta urratzaile frakzio bolumetrikokoak definitzen dute. Interakzio horiek lotura kohesiboen bidez fidelki modelatu daitezke: malgukiak (3.4.(b) Ir.) eta habeak (3.4.(c) Ir.). Habeak, malgukiak baino egokiagoak dira, aleen arteko biraketari erresistentzia ipintzen baitiote, karga axialez gain, makurdura- eta bihurtura-momentuak baititutze. Habeen haustura-tentsioa definituz gero, pitzadura-analisia ere ahalbidetzen dute. Nahiz eta aspektu hori tesi honen helburuetan ez egon, harriaren DEM ereduari habe hauskorren implementazioa ale-askatzea aztertzeke, bai arteztean, baita diamantaketan ere, lan honek proposatzen duen etorkizuneko ikerkuntza-ildo interesgarrietako bat da.

DEMen erabiltzen diren habeak Euler-Bernoulli habe teoriaran oinarritzen dira. Hauek, Timoshenko habe teoriak ez bezala, indar ebakitzalearen efektua arbuatzen dute habearen deflexioa aztertzean [Tim53]. Sinplifikazio horren eragina are eta nabarmenagoa da habearen aspektu-erlazioa (luzera-altuera) txikietan. DEM ereduko habearen aspektu-erlazioa 3 baino handiagoa da, akatsa % 2,4 baino txikiagoa delarik. Bernoulli habeak zurrunagoak dira. Nola nahi ere, habearen propietate mekanikoak definitzean (kalibratzea) efektua barneratua dagoenez, ondorioak ezerezean geratzen dira. Konkretuki, lan honetan [CWKH08, AICN12, ACI15] garatutako habe formulazioa erabili da.

Hurrengo Atalak habearen formulazioa eta propietate mekanikoen kalibratze-prozedura azaltzen ditu.

### Habeak

Habe elastikoak bi geometria parametrok ( $L_\mu$  hasierako luzera eta  $d_{be}$  diametroa) eta bi propietate mekanikok ( $E_\mu$  elastikotasun-modulua eta  $\nu_\mu$  Poissonen modulua) definitzen dituzte. Habeek sekzio zirkularra dute; haien diametroa  $d_{be}$  habeak lotzen dituen bi elementuen diametroen ( $d_1, d_2$ ) batz bestekoari  $r_\mu$  murrizpen-faktorea biderkatuz definitzen da (3.7 Ek.). DE-en diametroen barioak aldakorrek direnez, habearen diametroa izendatzeko  $r_\mu$  murrizpen-faktorea erabiltzen da. Habeek ez dute masarik, masa elementu diskretuei dagokie.  $L_\mu$



**3.5. Irudia:** Habe elastikoa (a) atsedenean eta (b) kargatuta [AICN12, ACI15]

habeak lotzen dituen elementuen arteko hasierako distantzia da. Iterazio bakoitzean, DE-en posizio eta orientazio aldaketak hasierakoekiko kalkulatu dira. Bi elementuen arteko luzera eta orientazio aldaketa erlatiboek finkatzen dituzte haien arteko indarrak (axialak eta ebakitzailak) eta momentuak (makurdura eta bihurtura) [AICN12, ACI15].

$$d_{be} = r_{\mu} \left( \frac{d_1 + d_2}{2} \right) \quad (3.7)$$

Hasieran, habeak kargarik gabe daude (3.5.(a) Ir.) eta posizio hori hartzen da erreferentziatzat.  $F_1(O_1, \mathbf{x}_1, \mathbf{y}_1, \mathbf{z}_1)$  eta  $F_2(O_2, \mathbf{x}_2, \mathbf{y}_2, \mathbf{z}_2)$  DE koordenatu-sistemak posizionatzen dira  $\mathbf{x}_1$  eta  $\mathbf{x}_2$  habearen ardatzarekin ( $\overline{O_1O_2}$ ) lerrotatuak daude. Habearen muturrak  $O_1$  eta  $O_2$  DE zentroyei lotuak daude. Medianak ( $\overline{O_1O_2}$ ) ordezkaten du habeak. 3.5.(b) Irudiak habeak kargatuta erakusten du, bi elementuen arteko desplazamendu erlatiboaren ondorioz. Erreakzio-indarrak eta momentuak habearen koordenatu-sistema lokalarekiko  $F(0, \mathbf{x}, \mathbf{y}, \mathbf{z})$  adierazten dira. 3.8 Ekuazioak koordenatu-sistema lokalaren bektore unitarioak definitzen ditu ( $\mathbf{x}, \mathbf{y}, \mathbf{z}$ ).

$$\mathbf{x} = \frac{\overrightarrow{O_1O_2}}{|\overrightarrow{O_1O_2}|}, \quad \mathbf{y} = \mathbf{x} \times \mathbf{x}_1 \quad \text{eta} \quad \mathbf{z} = \mathbf{x} \times \mathbf{y} \quad (3.8)$$

Orientazio aldaketa angelu axialarekiko eta tangenzialekoko definitzen dira ( $\theta_1(\theta_{1x}, \theta_{1y}, \theta_{1z}), \theta_2(\theta_{2x}, \theta_{2y}, \theta_{2z})$ ) hasierako koordenatu-sistemekiko ( $F_1, F_2$ ). Ondorioz, erreakzio-indarrak ( $\mathbf{f}_{DE1}, \mathbf{f}_{DE2}$ ) eta -momentuak ( $\mathbf{m}_{DE1}, \mathbf{m}_{DE2}$ ) DE1 eta DE2 elementuetan 3.9, 3.10, 3.11 eta 3.12 Ekuazioen bidez kalkulatu dira hurrenez hurren, non  $L_{\mu}$  eta  $\Delta L_{\mu}$  hasierako habearen luzera eta luzera-aldaketa,  $S_{\mu}$  habearen zeharkako azalera,  $I_{\mu}$ ,  $I_{p\mu}$  habearen inertzia-momentua eta inertzia-momentu polarra,  $E_{\mu}$  elastikotasun modulua, eta  $G_{\mu}$  ebakidura elastikotasun modulua diren.

$$\mathbf{f}_{DE1} = E_{\mu} S_{\mu} \frac{\Delta L_{\mu}}{L_{\mu}} \mathbf{x} - \frac{6E_{\mu} I_{\mu}}{L_{\mu}^2} ((\theta_{2z} + \theta_{1z})\mathbf{y} + (\theta_{2y} + \theta_{1y})\mathbf{z}) \quad (3.9)$$

$$\mathbf{f}_{DE2} = E_{\mu} S_{\mu} \frac{\Delta L_{\mu}}{L_{\mu}} \mathbf{x} + \frac{6E_{\mu} I_{\mu}}{L_{\mu}^2} ((\theta_{2z} + \theta_{1z})\mathbf{y} - (\theta_{2y} + \theta_{1y})\mathbf{z}) \quad (3.10)$$

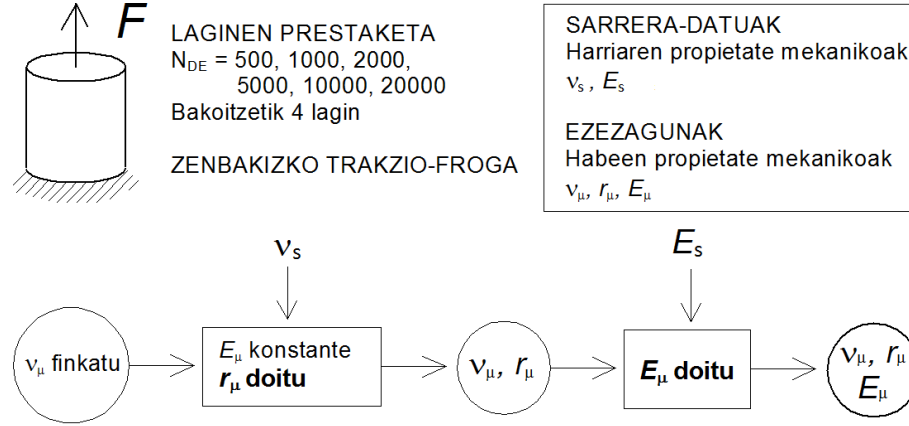
$$\mathbf{m}_{\text{DE1}} = \frac{G_{\mu} I_{\text{P}\mu}}{L_{\mu}} (\theta_{2x} - \theta_{1x}) \mathbf{x} - \frac{2E_{\mu} I_{\mu}}{L_{\mu}} ((\theta_{2y} + 2\theta_{1y}) \mathbf{y} - (\theta_{2z} + 2\theta_{1z}) \mathbf{z}) \quad (3.11)$$

$$\mathbf{m}_{\text{DE2}} = -\frac{G_{\mu} I_{\text{P}\mu}}{L_{\mu}} (\theta_{2x} - \theta_{1x}) \mathbf{x} - \frac{2E_{\mu} I_{\mu}}{L_{\mu}} ((2\theta_{2y} + \theta_{1y}) \mathbf{y} - (2\theta_{2z} + \theta_{1z}) \mathbf{z}) \quad (3.12)$$

Elementu finituen metodoan ez bezala, solido jarraituen mekanika legeak ezin dira DEM formulazioan zuzenean aplikatu. Urratzaileak eta aglomeratzai-leak propietate mekaniko ezberdinak dituzte eta, gainera, harriaren gorputza porotsua da. Harriaren gorputzaren propietate mekanikoak,  $E_s$  elastikotasun-modulua eta  $\nu_s$  Poissonen modulua, DEM ereduaren propietateen ondorioz eskala makroskopikoan azalartzen diren propietateak dira. Horrela, habeen propietate mekanikoek, eskala mikroskopikoan, modelizatutako materialaren portaera makro-mekanikoa definitzen dute. Fenomeno horri homogeneousazio deritzo [CJR13]. 3.3.1. Atalak azaltzen du harriaren gradu hizkiaren eta  $E_s$ -ren arteko erlazioa. Lik *et al.* [LYZW15b] ez bezala, aglomeratzaile frakzio bolumetrikokoak ez dauka zertan bat etorri beharrik habeen bolumenarekin, habeen zeregina ereduari zurruntasuna ematea baita, ez aglomeratzaile-zubien geometriaren erreproduzio fidela egitea. Horrela, soberako haberik gabe eredia sinpleagoa da eta konputazio-denbora murrizten da.

Propietate mekanikoen neurketa esperimentalak egiten den antzera, zenbakizko trakzio-frogen bidez kalibratzen dira habeen mikro-eskalako propietateak. André-k *et al.* deskribatzen dute habek kalibratzeko prozedura [AICN12]. Zenbakizko trakzio-frogetan lagin zilindrikoei mutur bat finkatu eta bestean indarra aplikatzen da. Simulazioak luzeratako eta alboko deformazioak monitorizatzen ditu, makro-eskalan DEM laginak izan duen elastikotasun modulua eta Poissonen modulua kalkulatu ahal izateko. Mikro-eskalan habearen hiru parametro dira doitu beharrekoak ( $\nu_{\mu}$ ,  $r_{\mu}$ ,  $E_{\mu}$ ) materialaren makro-eskalako bi propietateekiko ( $E_s$ ,  $\nu_s$ ). Kalibrazioan, dimentsio berdineko sei eredu tamaina erabili dira DE kopuruaren arabera (500, 1000, 2000, 5000, 10000 eta 20000 DE). DEM eredu tamaina bakoitzerako lau lagin prestatu dira. DEM ereduaren sorkuntza ausazkoa denez, lau laginak ezberdinak dira eta emaitzak lau emaitzen batz bestekoak dira. Emaitzak grafikoki aztertuz, mikro-eskalako habeen parametroak erregresio bidez doitzen dira banan bana.

Habeen propietateen kalibrazioaren fluxu-diagrama azaltzen du 3.6. Irudiak. Habearen  $E_{\mu}$ ,  $r_{\mu}$  eta  $\nu_{\mu}$  parametroak, bakoitzak bere eran eragiten dute materialaren  $E_s$  eta  $\nu_s$  makro-propietateetan. Andrék elkarreragin horiek aztertzen ditu saiakuntzen diseinua (*design of experiments*, DOE) erabiliz [AICN12]. Lagin neurri guztietarako,  $\nu_{\mu}$  Poissonen moduluak makro-propietateetan apenas eraginik duela ohartzen da. Hori kontuan hartuta, kalibrazioa bi pausutan egiteko prozedura finkatzen du. Lehenengo pausuan,  $\nu_{\mu}$  habeen Poissonen modulua  $\nu_s$  harriaren Poissonen moduluarekin finkatzen da, aldagai bat aurreztuz.  $E_{\mu}$  finko mantenduz,  $r_{\mu}$  habeen erradioa doitzen da, harriaren  $\nu_s$  Poissonen modulua lortu arte.  $r_{\mu}$  finko dela,  $E_{\mu}$  aldatzeak eragin mesprezagarria dauka  $\nu_s$ -n.



**3.6. Irudia:** Habeen propietateen kalibratzeko prozeduraren fluxu-diagrama [AICN12]

Bigarren pausuan,  $\nu_\mu$  eta  $r_\mu$  finkatuta, habearen  $E_\mu$  elastikotasun-modulua doitzen da materialaren  $E_s$  elastikotasun-modulua lortu arte. Horrela, habeai  $E_\mu$ ,  $r_\mu$  eta  $\nu_\mu$  balioak ezarri gero, materialak  $E_s$  eta  $\nu_s$  propietateak erakusten ditu makro-escalan. Mikro-escalako propietateak ereduaren dimentsioekiko edo urratzaile frakzioarekiko independenteak dira, hau da, graduaren arabera soilik aplikatzen dira. Propietate horiek DEM ereduera eraiki ondoren eta simulazioa hasi aurretik aplikatzen dira.

### Tchamwa-Wielgoz (TW) integrazio-eskema

Tchmwa-Wielgoz (TW) eskemak Verlet integrazio-eskema moldatzen du, DE bakoitzaren posizio eta orientazio berriak konputatzean zenbakizko moteltzea gehituz. TW eskema, maiztasun handien iraungipena bultzatzen duen zentrokanpoko integrazio-eskema esplizitua da. Simulazio ia-estatikoetan, sistemak emaitza estatikoaren inguruan bibratzen du. Iraungipena  $\beta$  zenbakizko moteltze-faktorearekin kontrolatzen da. Denborarekiko bigarren deribatua ukitzen denez, simulazioaren konbergentzia hobetzen da.

DE bakoitzaren posizio eta orientazio berriak kalkulatzeko, Verlet-abiadura eskemak azelerazioaren eta azelerazio angeluarraren (koaternioi bidez) zenbakizko integrazioa egiten du.  $\beta$ -ren erabilera zenbakizko moteltzea gehitzen du  $\Delta t$  denbora-gehikuntza balio kritikora hurbiltzean. Mahéo *et al.* [MGR09]  $\beta = 1, 3$  balioak emaitza estatikoen konbergentzia hobetzen duela baieztatzen dute.

$\ddot{\mathbf{x}}_{n+1}$  azelerazio berria dinamikaren oinarriko printzipioak aplikatuz kalkulatu da. DEaren grabitate-zentroan aplikatutako indarren erresultantea  $\mathbf{f}_{DE}$ , soilik kalkulatuakiko posizio berriaren  $\mathbf{x}(t)$  menpe dago. 3.13 Ekuazioak elementuaren azelerazio berriaren  $\ddot{\mathbf{x}}_{n+1}$  kalkulua aurkezten du, non  $m_{DE}$  elementuaren masa den.



$$\ddot{\mathbf{x}}_{n+1} = \frac{\mathbf{f}_{\text{DE}}}{m_{\text{DE}}} \quad (3.13)$$

( $n$ ) pausoan DE baten posizio, abiadura eta azelerazio osagaiak ( $\mathbf{x}_n, \dot{\mathbf{x}}_n, \ddot{\mathbf{x}}_n$ ) eta ( $n+1$ ) pausoan azelerazioa  $\ddot{\mathbf{x}}_{n+1}$  ezagututa, 3.14 eta 3.15 Ekuazioek ( $n+1$ ) pausoko abiadura eta posizioa kalkulatzeko dituzte. Era berean, ( $n$ ) pausoan DE baten posizio, abiadura eta azelerazio koaternioiak ( $\mathbf{q}_n, \dot{\mathbf{q}}_n, \ddot{\mathbf{q}}_n$ ) eta ( $n+1$ ) pausoan azelerazio koaternioia  $\ddot{\mathbf{q}}_{n+1}$  ezagututa, 3.16 eta 3.17 Ekuazioek ( $n+1$ ) pausoko abiadura eta posizio koaternioiak kalkulatzeko dituzte.

$$\dot{\mathbf{x}}_{n+1} = \dot{\mathbf{x}}_n + \frac{\Delta t}{2} (\ddot{\mathbf{x}}_n + \ddot{\mathbf{x}}_{n+1}) \quad (3.14)$$

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \Delta t \dot{\mathbf{x}}_{n+1} + \beta \frac{\Delta t^2}{2} \ddot{\mathbf{x}}_n \quad (3.15)$$

$$\dot{\mathbf{q}}_{n+1} = \dot{\mathbf{q}}_n + \frac{\Delta t}{2} (\ddot{\mathbf{q}}_n + \ddot{\mathbf{q}}_{n+1}) \quad (3.16)$$

$$\mathbf{q}_{n+1} = \mathbf{q}_n + \Delta t \dot{\mathbf{q}}_{n+1} + \beta \frac{\Delta t^2}{2} \ddot{\mathbf{q}}_n \quad (3.17)$$

Behin posizio berriko  $\mathbf{q}$  koaternioia eta  $\dot{\mathbf{q}}$  abiadura-koaternioia lortuta, koaternioientzako Eulerren erlazioak hurrengo pausoko  $\ddot{\mathbf{q}}$  azelerazio angeluarraren koaternioia lortzen da.

### 3.2.2. DEM ereduaren sorkuntza

DEM formulazioaz gain, DEM ereduaren simulatu nahi den fenomeno fisikoarekiko eraiki behar da. Partikula solteen modelaketa DE banaketa uniformetik abiatu ohi da. Batzuetan, aurre-kokatze aldia beharrezkoa da (i.e. grabitatearen ondorioz DE-en erorketa), simulazioa DE-en ausazko banaketatik has dadin.

Gorputz jarraituak modelatzean, fenomeno fisikoaren deskribatzeko kasu batzuetan egokia da DEak uniformeki antolatzea. Hala ere, solido anisotropoak sortzen dira, eta portaera mekanikoa deskribatzeko beharrezkoa da DE-en ausazkotetasuna eta homogeneitatea portaera isotropoa deskribatu ahal izateko. Arazoa, uniformeki banatutako DEak astinduz edo zenbakizko sinterizazioa erabiliz konpontzen da. Astintze-teknikak, lehendabizi DEak era uniformearen kokatzen ditu ereduaren domeinuan. DE dentsitatea unitate-gelaxka kubiko sinpleak edo gorputzean zentratuak erabiliz doitu daitezke. Ausazkotze algoritmoak DE bakoitza birkokatzen du, haien artean elkar-sartzea saihestuz. Astintze-teknika erraz erabili da artezketan topografia-ereduetan [DWB06].

Zenbakizko sinterizazioak, aldiz, domeinu itxia DE-er ausaz beteaz sortzen du DEM ereduaren. Andrék *et al.* [AICN12, ACI15] ereduaren hiru pausutan sortzeko prozedura proposatu dute. Pareta elastikoak dituen bolumen itxiak osatzen du domeinu diskretua. Lehenengo pausuan, ereduaren geometria duen domeinu itxia DE-er betetzen da libreaki eta ausaz. Elementu diskretuek banaketa uniformearen edo normalean jarraitzen duen ausazko diametroa dute. Etapa, ausaz

elementu diskretu berri gehitzeko lekurik ez dagoenean bukatzen da. Bigarren etapa prozesu iteratiboa da, lehen pausuan ipinitako DEak birkokatzen dira, elementu berriak gehitu ahal izateko. DE-en artean elkar-sartze txiki bat baimentzen da. Lekua gutxitzen den heinean, DE-en energia zinetikoaren batura gutxituz doa. Energia zinetikoak aurrez ezarritako muga minimora iristean bukatzen da bigarren etapa. Hirugarren etapan zuzentzen dira DE-en arteko elkar-sartzeak, domeinuaren pareten zurruntasuna erlaxatuz.

Astintze-teknika sinpleagoa eta azkarragoa da zenbakizko sinterizazioa baino. Hala ere, astintze bidez sortutako ereduak banaketa uniformearen trazak erakusten ditu propietate mekanikoetan. Zenbakizko sinterizazioak ez du horrelako arazorik, eraikuntza guttiz ausazkoa baita hasieratik. Tesia idazteko garaian, zenbakizko sinterizazioa oinarritzko geometria duten eredueta soilik aplikatu ahal da, hala nola, zilindro, esfera edo kutxa. Astintze-teknika, berriz, ordenagailuz sortutako 3D geometria konplexuetan ere aplikatu ahal da.

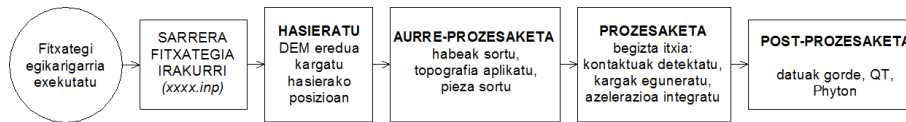
Harriak geometria sinplea duenez, zilindroa, zenbakizko sinterizazioa aukeratu da DEM eredu eraikitze. Hala ere, harriaren dimentsioak ale urratzaileen tamaina baino askoz handiagoak dira; ereduaren neurria murrizteko, zabalera txikiko diskoa altuera batean moztu da.

### 3.2.3. DEM ereduaren egitura

DEM kontaktu-eredua *granular object oriented* (GranOO) paketearekin aurrera eramana da, Bordeleko ENSAMgo *Institut de Mécanique et d'Ingénierie* (I2M) erakundeak garatutako zenbakizko erreminta. GranOO ez da software exekutagarria, DEM simulazioak garatzeko C++ liburutegi multzoa baizik. Liburutegiak erabiliz, DEM eredu sortu eta programa konpilatu ondoren, simulazioa abiarazten da. GranOO hiru liburutegik osatzen dute: geometria erremintak (ardatz kartesiarrak, koaternioiak, bektoreak, matrizeak...), DEM objektuak (elementu diskretua, malgukia, habea, kontaktua, multzoa...) eta konputazio-arkitektura (*plugIn*, XML irakurgailua, STL liburutegia...). GranOOren formulazioa bateragarria da fisika-anitzekin (mekanika, termikoa, elektrikoa) eta FEM-DEM eredu hibridoak ere onartzen ditu.

DEM simulazioa hasi aurretik, DEM eredu eraikitzen da zenbakizko sinterizazioaren prozedura jarraituz. 3.3.2. Atalean zehazten dira DEM eredu eraikitze sarrera-datuak eta harriaren morfologia lortzeko prozeduran egin den moldaketa. Batz besteko DE diametroa eta diametroaren banaketa, ale urratzaileen batz besteko neurriarekin eta urratzaile frakzio bolumetrikorekin doitzen dira DEM eredu eraikitzean. Harriaren DEM eredu DEak habez lotuz osatzen da eta, behin sortuta, gainazal topografia moldatzen da diamantaketa-baldintzen funtzioan.

Simulazioak jarraitzen dituen pausoak XML sarrera-fitxategian zehazten dira. Fitxategi horrek banan-bana deitzen ditu *plugIn*ak, funtzio jakin bat betetzen duten C++ lengoian idatzitako programa zatiak. GranOOk erabiltzaileak idatzitako *plugIn*en konpilazioa baimentzen du, eredu fenomeno fisikoari moldatzeko gaitasuna areagotuz.



3.7. Irudia: Simulazioak jarraitzen duen fluxu-diagrama [And12]

DEM simulazioak ohiko zenbakizko erreminten egitura partekatzen du: aurre-prozesaketa, prozesaketa eta post-prozesaketa. 3.7. Irudiak simulazio baten jarraitzen duen fluxu-diagrama erakusten du. Lehendabizi, sarrera-fitxategia irakurtzen da (*.inp*): lerroak banan-bana exekutatu dira zehaztutako *plugIn*ari deituz. Aurre-prozesaketan, aurrez sortutako harriaren DEM eredia kargatu eta simulazio-baldintzak zehazten dira (hasierako abiadura, kanpo-indarrak, baimendutako desplazamenduak...). Prozesaketan, simulazioa begizta itxian sartzen da. Iterazio bakoitzean, Newtonen dinamikaren oinarriko printzipioak aplikatuz, DE bakoitzaren posizioa eta DE-en arteko interakzioak kontuan hartuz, DE bakoitzean eragiten duten kanpo-indarrak definitzen dira. Indarren oreka aztertuz, DEaren azelerazioa kalkulatu eta, integrazio bidez, hurrengo iterazioan DEak izango dituen abiadura eta posizioa kalkulatu dira. Iterazio arteko denbora-tartea, konbergentzia eta egonkortasuna bermatzen duen balio handiena izatea komeni da konputazio-denbora osoa optimizatzeko.

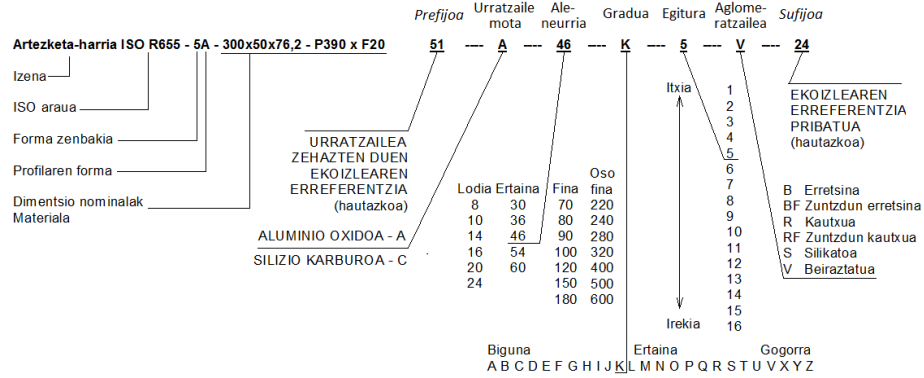
Simulazioa, ezarritako iterazio-kopuru maximoa iristean bukatzen da, edota eskuz geratzen da, simulazioa egonkortu dela ikustean. Azkenik, post-prozesaketan aztertutako izango diren emaitzak biltzen eta gordetzen ditu programak.

### 3.3. Harriaren zenbakizko ereduaren eraikuntza

Atal honetan, zenbakizko DEM eredia eraikitzekeo pausak banan-bana deskribatzen dira. Horretarako lehendabizi harriaren osagaien eta haien konbinaketatik eratorritako egitura deskribatzen da. Ondoren konbinaketa hori DEM bidez erreproduzitzen da bi alderdi kontuan hartuz: osagaien banaketaren deskribapena (morfologia) eta propietate mekanikoen transferentzia.

#### 3.3.1. Harriaren ezaugarriak

Harriaren fabrikazioaren (urratzaile-aglomeratzaile nahasketa, trinkotzea eta labaldia) ondorioa da harriaren egitura hauskorra. Harriaren osaketaren oinarriko ezagutza beharrezkoa da haren eredia eraikitzekeo. Nolanahi ere harriaren osaketa irekia da, harri ekoizleek izendapen berdinarekin pean bi harri aldaera izan ditzakete, osagaiak eta ezagutzak sekretuan gordeaz. Harriaren zurruntasuna zeharka graduak definitzen du eta parametro garrantzitsua da harriaren portaeran. Harriaren zenbakizko DEM ereduak guzti hau modelizatzeko gai izan behar du.



3.8. Irudia: Ohiko harrien izendapena [Mal08]

Harriaren DEM ereduren abiapuntua, harriaren dimentsioak eta izendapena dira. Harria osatzen duten ale urratzaile, aglomeratzaile eta poro proportzioak konbinaketa posible anitz eskaintzen ditu, aplikazio bakoitzaren beharretara moldatuz. Azkenik, harrien propietateak neurtzeko teknikak azaltzen dira, gehienak neurri garatuak eta etengabe hobetuak direnak.

### Harriaren izendapena

Harriaren portaera, urratzaile-aglomeratzaile ezaugarrien eta biek harrian duten itxurazko bolumenaren kontzentrazioan menpe dago. Harrien izendapena nazio eta nazioarteko erakundeek adostutako arauak erregulatzen dute, besteak beste, ISO R655:1999 araua. Izendapenak bi zati nagusi ditu: lehenik harriaren geometria eta dimentsioak definitzen dira eta, ondoren, osagaiak eta ezaugarri mekanikoak. 3.8. Irudiak harrien izendapenen adibidea dakar.

Izendapenaren lehen zatian, harriaren forma zenbakiz definitzen da; profilak, berriz, hizki larriz. Neurri nominalak hurrenkera jakin bat dute.

Izendapenaren bigarren zatian, ekoizleak erabil dezakeen hautazko zenbakia-ekin hasten da; normalki erabili den urratzaile mota adierazi ohi da. Hurrengo hizkiak material urratzaile familia adierazten du. Ohiko urratzaileak alumina edo korindioa (A), eta silizio karburoa (C) dira; super-urratzaileak berriz diamantea (D) eta boro nitruro kubikoa (B).

Hizkiaren eskuineko zenbakiak, ale neurria zehazteko erabilitako galbahearen sare-zenbakia adierazten du. ISO 8486-1:1996 arauak azaltzen du ale-neurria eta ale-neurri banaketa nola definitu. Harria, ondoz-ondo dauden ale-neurrien nahasketa da, eta nahasketaren proportzioak egitura eta gogortasun propietate zehatzak lorrarazten ditu. Alearen batz besteko neurria  $d_g$  galbahearen harrien arteko distantziarekin bat egiten du eta 3.18 Ekuazioarekin formularekin estimatu daiteke [Mal08],  $M$  galbahearen sare-zenbakia delarik. [KJL97] ale-neurriaren banaketa normaltzat jotzen dute, ondorengo egileek jarraitu duten hipotesia [HK03, Dar13, JGB<sup>+</sup>13, LWBG13, LYZW15b]. Ikuspuntu horrek, ale-

neurri bakarra hartzen du kontuan, neurri ezberdineko urratzaileen nahasketa arbuaiatuz. Hala ere, harri errealetan erabilitako ale nahasketaren ale-neurriak banaketa uniformetik hurbilago dagoela erakusten du [MC71].

DEMak bi banaketa-motak onartzen ditu, normala eta uniforme. Urratzaile neurri zenbaki jakin batek banaketa uniforme jarraitzen du, bi galbaheen artean banatutako ale urratzailez osatua baitago. Bestalde, harria osatzen duten ale urratzaileak bizpahiru ale-neurriren nahasketa da. Eta horrela bai, uniformeki banatutako populazioen nahasketak banaketa normala osatzen du. Hala ere, harri-ekoizleak ez dira ekoizpenaren datu teknologikoak zabaltzearen zale. Tesi honetan, banaketa uniforme aukeratu da harriaren eredia eraikitzeko. Nahastutako ale-neurriak bata-bestearen alboan daudenez, nahasketaren desbiderapen estandarrak gutxi gorabehera banaketa uniformearen tartearekin bat egiten dute. Beraz, lortutako DEM ereduak antzekoak izango lirakeke eta banaketa uniforme ustearen akatsa txikia da. Beti ere, ekoizleen nahasketa datu errealak izanez gero, lan honetan aurkeztutako DEM ereduaz arazorik gabe aplikatu ahal dira.

3.19 Ekuazioak ale-neurriaren  $\mu_g$  banaketaren heina kalkulatu du;  $d_g^1$  eta  $d_g^2$  erabili diren galbaheen ale-neurri maximoa eta minimoa dira.

$$d_g = \frac{15,2}{M} \quad (3.18)$$

$$\mu_g = \frac{d_g^1 - d_g^2}{2} \quad (3.19)$$

Hurrengo hizkiak harriaren gradua edo gogortasuna adierazten du. Hizki larriz A-tik eta Z-ra izendatzen da: A maila bigunenetik Z gogorrenera. Harriaren gogortasunak alea harritik askatzeko erresistentzia adierazten du, aglomeratzaile-zubien adhesioa eta erresistentzia neurtuz. Parametro horren kontrola, besteak beste porositatea kontrolatuz lortzen da [Mal08].

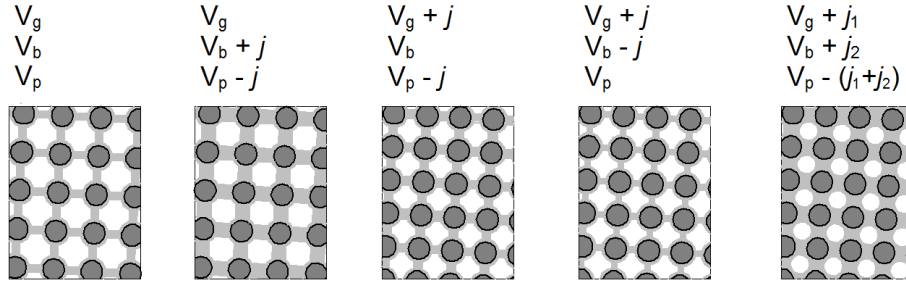
Egitura zenbakiak harriaren irekitasuna adierazten du. Urratzaile proportzioaren bidez definitzen da eta zenbaki batez izendatzen da: 0-tik (egitura itxia) 14-ra (egitura irekia). Malkinek [Mal08]  $S$  egitura-zenbakiaren funtzioan urratzaile frakzioa estimatzeko 3.20 Ekuazioa eskaintzen du.

$$V_g = 2(32 - S) \quad (3.20)$$

Hizki batek definitzen du aglomeratzaile mota: (V) beiratzatua, (R) erretxina eta (M) metalikoa. Azkenik, eragiketa-abiadura maximoa zenbakiz definitzen da. Datu hori koloredun banda batek ere adierazten da harriaren etiketan.

### Harrien osaketa

Osoetasunak, urratzaile, aglomeratzaile eta poro edukiak definitzen du harriaren ezaugarriak eta portaera. Aleen, aglomeratzailearen eta porositatearen arteko erlazio bolumetrikoko graduaren eta egituraren definizioan parte hartzen dute. Harrien ekoizleek osagai bakoitzaren bolumen frakzioaren ( $V_g$  ale urratzaile,  $V_b$  aglomeratzaile eta  $V_p$  poro frakzioak) eta pisuaren kontrol zorrotza eramatzen

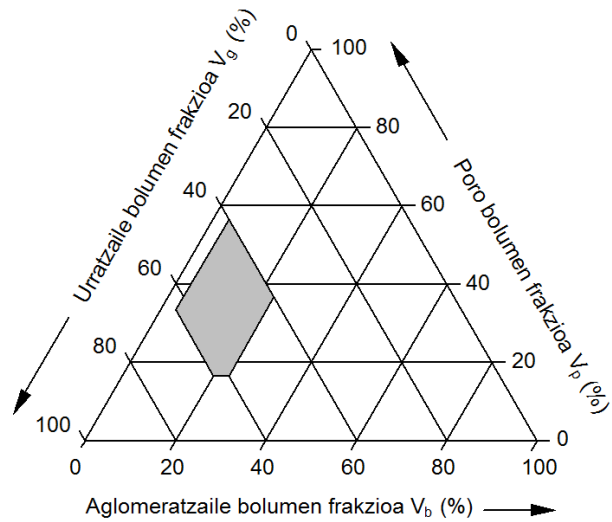


**3.9. Irudia:** Urratzaile, aglomeratzaile eta poro proportzio bolumetrikoen eragina harrien egituran [Klo09]

dute. Hiru frakzioen batura bat da. 3.9. Irudiak urratzaile, aglomeratzaile eta porositatearen proportzio bolumetrikoen aldaketek sortutako egituretan duten eragina erakusten du [Klo09].

Osagaien arteko proportzioek muga teknologikoak dituzte, bai ekoizpenean, baita erresistentzian ere. 3.10. Irudiak  $V_g$ ,  $V_b$  eta  $V_p$  proportzio bolumetrikoen grafiko trigonala erakusten du [Klo09]. Grisez nabarmendutako eremuak ohiko harrien osaketa erakusten du. 3.10. Irudiko diagrama trigonalak informazio interesgarria eskaintzen du, batez ere proportzio berdineko lerroetan aldaketak aztertzean. Poroen neurria eta forma, ale urratzaileen neurriaren eta formaren funtzioan daude. Ale urratzaile proportzioa handitu daiteke (>% 60) ale-neurri txikiagoarekin nahastuz. Alderantziz, ale bolumen proportzioa txikitu daiteke (<% 30), aleen forma kontrolatuz. Trinkotze minimoa ale arteko kontaktu minimoak zehazten du. Ale urratzaileen pilaketa osoa ez bada, aleen arteko tartekak handitu, eta ale artean aglomeratzaile zubiak luzatzen dira egitura ahulduz. Behin urratzaile bolumena definituta, gainontzeko bolumena aglomeratzailearen eta porositatearen artean banatzen da. Porositate altuak hozgarriari sarrera eman eta txirbilari irteera errazten dio artezketa burutzean. Aglomeratzailekopurua gehituz lotura-zubiak indartzen dira, harriaren gogortasuna handituz eta, era berean, porositatea murriztuz.

Beira-aglomeratzaileak, ale urratzaileekin batera tenperatura altuetan hautsen nahasketa (beira, karea, flux kimikoak) sinterizatuz lortzen dira. Hautsak eta urratzaileak, urarekin edo destrinarekin nahasten dira. Masa horrek berdean forma mantendu ahal izateko erresistentzia nahikoa dauka. Aurreformari su ematen zaio labeko ingurune kontrolatuan. Tenperatura altuek, ale urratzaile eta aglomeratzailearen arteko oxidoen difusioa errazten dute, lotura kimikoa eta fisikoa lortuz. Beirak lotura-zubiak sortzen ditu, kareak aurreformari berdean erresistentzia ematen dio manipulatu ahal izateko, eta fluxek aglomeratzaileak urratzailea bustitzea edo estaltzea errazten du, gainazal-tentsio egokia sortuz. 3.11. Irudiak aglomeratzailearen tantaren frogaren laginak erakusten ditu.



**3.10. Irudia:** Harrien osaketa bolumentrikoaren diagrama trigonala; eremu grisak muga teknologikoak zehazten ditu [Klo09]



**3.11. Irudia:** Aglomeratzailearen tantaren froga: malda jakin bateko gainazalean aglomeratzailearen jariakortasuna neurtzen da (iturria: Unesa)

### Propietate mekanikoen egiaztapena

Harri-ekoizleek harri guztiak frogatzen dituzte gain-abiaduran (1,2 – 1,5 aldiz lan-abiadura) dagokien arauaren arabera. Indar zentrifugoak harrian gaintentsioak sortzen ditu, alde batetik, dilatatzeko eta zurruntasuna handituz, eta bestetik, ebaketa-abiadura handitzean aglomeratzailearen erresistentzia ere handitzen da [Smi86]. Ekoizpenean eragindako pitzadura posibleak aurrez antzematea da gain-abiadura frogaren helburua. Zurruntasun gehikuntzak kontaktu-luzeran ere eragiten du. Efektua nabarmenagoa da erretxina aglomeratzailea darabilten harrietan eta abiadura handiko artezketan. Harria  $d_s$  diametroko diskotzat hartuz,  $d$  barne diametroa eta  $v_s$  abiadura periferikoa bada, 3.21 Ekuazioak biraketak zuloan sortzen duen tentsio maximoa adierazten du;  $\rho$  dentsitatea da eta  $\nu$ , berriz, Poissonen modulua. Tentsioa, abiadura periferikoarekiko oso sentikorra da karratura baitago.

$$\sigma_{\max} = \rho \left( \frac{3 + \nu}{4} \right) \left[ 1 + \left( \frac{1 - \nu}{3 + \nu} \right) \left( \frac{d}{d_s} \right)^2 \right] v_s^2 \quad (3.21)$$

Propietate mekanikoak ere neurtzen dira: ale urratzaileak eta harriak hainbat egiaztatze-froga gaintitu behar dituzte. Harrien materiala harrotzeko gaitasuna ale urratzaileen esku dago. Harriaren gorputzak biraketa transmititzeaz gain, aleak eusteko eta artezketaren indarrak jasateko gaitasunak behar ditu. Horietaz gain porositate kontrolatua dute hozgarria xurgatu ahal izateko. Ezauzgarri horiek neurtzeko berariazko frogak garatu dira. Harriak berak, diseinuzko material artifizial gisa, urratzaile eta aglomeratzaile konbinaketak ematen dioten zurruntasuna, porositatea eta ebaketa-propietateak dauzka.

Gogortasun- (Knoop eta Vickers) eta birrinkortasun-frogak dira urratzaileei egiten zaizkien saiakuntza nagusiak. Harriaren graduari edo gogortasunari dagokionez, lehen definizioaren arabera ebaluatzen zen, harriak ale bat askatzeko erakusten duen erresistentzia ebaluatuz. Decneut-ek *et al.* [DPS70] gogortasuna neurtzeko hogeita hiru metodo baino gehiago zenbatu zituzten, besteak beste, urradura-froga, zulaketa konikoa, makurdura-froga edo hondar-granailaketa. Gaur egun, orokorki onartzen da gogortasuna harriaren  $E_s$  elastikotasun moduluaz definitzea [KF87]: gradua alea askatzeko erresistentziarekin erlazionatua badago, orduan gogortasuna ere elastikotasunarekin erlazioa dauka. Harriaren elastikotasunak, graduaren definizioaz gain, kontaktuan ere parte-hartze zuzena dauka.

Oro har, elastikotasuna trazio- edo konpresio-frogetan neurtzen da. Baina harria material hauskor heterogeneoa izaki, bere portaera abiadurarekiko oso menpekoa da. Erresonantzia eta ultrasoinu frogak,  $E_s$  neurtzen duten teknika azkarrak eta ez suntsikorrak dira. Erresonantzia-analisiak, piezaren maiztasun naturala neurtzen du; ultrasoinuek, aldiz, soinuak gorputza zeharkatzean duen barreiatze-abiadura neurtzen dute. Nahiz eta bien emaitzak bata bestearen alderantzikoak izan, erresonantzia-analisia gailendu da. Ultrasoinuen neurketak beharrezkoa duen murgildutako ingurunea da arrazoia [Smi86]. 3.22 Ekuazioak  $E_s$  kalkulatzeko duen harriaren oinarriko bi-nodal makurdura-moduarekin erla-



zionatuz. Ekuazioak, maiztasun naturala ( $f$ ), harriaren dimentsioak ( $d_s$  kanpo-eta  $d$  barne- diametroak,  $b_s$  zabalera), propietate mekanikoak ( $\nu_s$  Poissonen modulua) eta dentsitatea ( $\rho$ ) erlazionatzen ditu.  $\nu_s$  ere erresonantzia-analisi bidez zehazten da, trakzio eta bihurtura bibrazio-moduak aztertuz.  $\nu_s$  konstantetzat har daiteke harri mota jakin batentzat tarte zabal batean [DPS70].

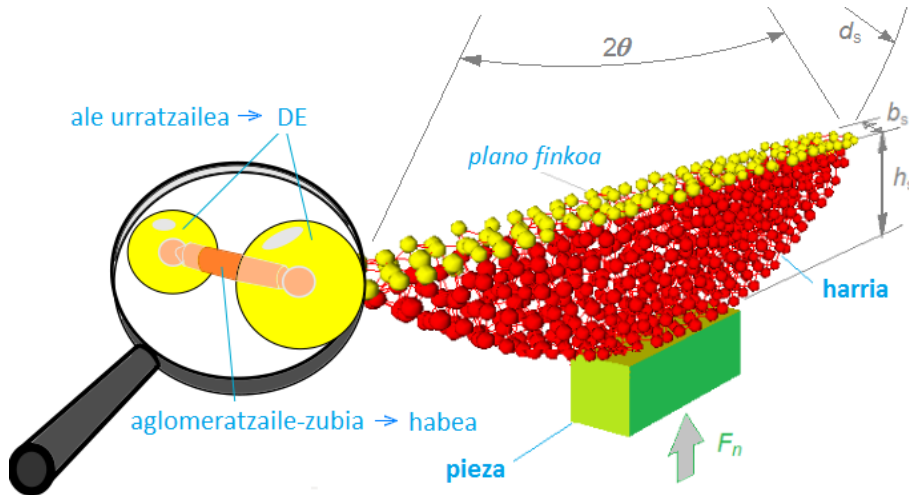
$$E_s = 1,074 \frac{(1 - \nu_s^2) f^2 d_s^4 \rho}{(1 - (d/d_s)^2)^2 b_s^2} \cdot 10^{-9} \quad (3.22)$$

### 3.3.2. Harriaren zenbakizko eredia DEM bidez

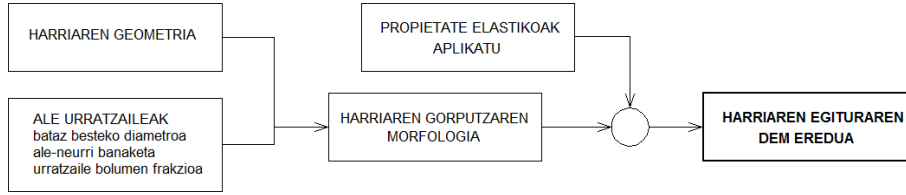
3.12. Irudiak aurkezten du lan honetan garatu den harriaren eta piezaren DEM eredia, eta bietan aplikatutako mugalde-baldintzak. Harria osatzen duten DEak habe sare batez lotuak daude, aglomeratzaile-zubien papera dutenak.

Orain erronka, ezaugarrietatik abiatuz, harriaren egituraren deskripzio morfologikoa eta mekanikoa lortzea da. 3.13. Irudiak harriaren zenbakizko DEM eredia eraikitzeke eman beharreko pausoak aurkezten dira. 3.2.3. Atalean DEM eredia eraikitzeke azaldu den zenbakizko sinterizazio prozedura, harriaren ezaugarri bereziak islatu ahal izateko moldatu da. Behin harriaren eredia sortuta, kalibrazio bidez doitutako propietate mekanikoak habeei transferitzen zaizkie. Azkenik, piezaren DE errektangeluarra multzoari gehitzen zaio .

Harriaren ezaugarriak 3.3.1. Atalean izendatzen dira. Harriaren DEM eredia eraikitzeke beharrezkoak diren parametroak, geometrikoak (harriaren morfologia granularra deskribatzeko) eta propietate mekanikoak (elastikotasuna) dira. Parametro geometrikoak  $d_g$  batz besteko ale-neurria eta egitura zenbakia ( $V_g$  urratzaile frakzio bolumetrikoa) dira, eta propietate mekanikoak graduak adierazten ditu,  $E_s$  elastikotasun eta  $\nu_s$  Poissonen moduluekin dituen erlazioarekin.



3.12. Irudia: Harriaren eta piezaren DEM eredia (eskalaz kanpo)



**3.13. Irudia:** Harriaren zenbakizko DEM eredia eraikitzeke fluxu-diagrama

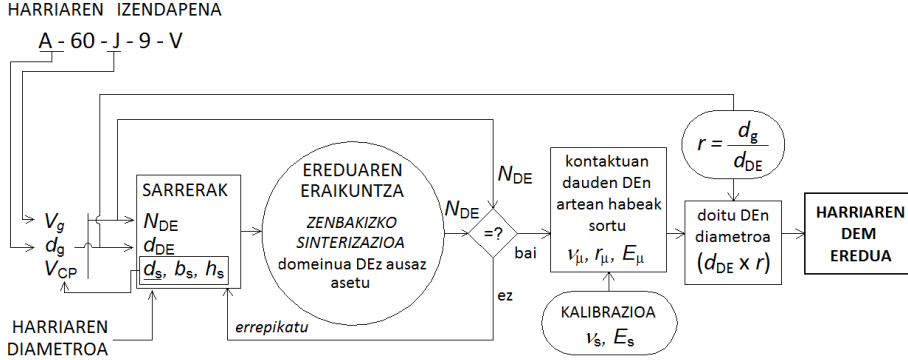
Izendapenak zehazten ditu datu horiek. Urratzaile eta aglomeratzaile motaren eraginak graduak jasotzen ditu, beraz, ez dira sarrera-datuak DEM eredian. Era berean, Lik *et al.* ez bezala, aglomeratzaile eta porositate frakzio bolumetrikoak ez daude esplizituki modelatuak DEM eredian. Helburua ez da urratzaile-aglomeratzaile-poro konbinaketaren morfologia modelatzea, harriaren egitura granularren portaera mekanikoa baizik, DE-habe sarea baliatuz. Azkenik, harriaren geometria ezagutzea ere beharrezkoa da DEM ereduaren erakuntzan; artezketa lauan bereziki harriaren  $d_s$  kanpo-diametroa da funtsezko dimentsioa.

Horietatik, honako hauek dira zenbakizko sinterizazio bidez harriaren DEM eredia sortzeko sarrera-datuak: (a) ereduaren dimentsioak, (b) DE bataz besteko diametroa eta (c) urratzaile frakzio bolumetrikoa.

Ereduaren dimentsioak (a) harriaren geometriaren menpe daude. Artezketa lauan, harriak zilindrikoak dira, eta  $d_s$  kanpo diametroarekin eta  $b_s$  zabalarekin definitzen da haien geometria. Konputazio-kostuak ereduaren DE kopuruarekiko erlazio karratua dauka. Kostua murrizteko, harri osoa ez, ale batzuen zabalera duen xafra-disko bat modelizatzen da. Ereduak aspektu-erlazio handia dauka, zabalera estua duelako. Horren ondorioz norabide axialean isotropo izateari utziko dio, beste bi norabideetan aldiz isotropia mantentzen du. Efektua RVEaren homogeneousazioa guztiz lortzen ez duelako gertatzen da, eta habeen propietateen kalibrazioa eredu neurri ezberdinekin egitean islatzen da. Horrek tentsioetan eta deformazioetan ez du eraginik izango, baina bai norabide axialeko deformazioari, hau da, Poissonen moduluari. Ereduaren helburuari, kontaktu-luzeraren kalkuluari, eragiten ez dionez, ereduak zabalera estua izatea onargarria da.

Bestalde, gainazal ez-konformearen arteko kontaktuan, tentsioak kontaktu-eremuan kontzentratzen dira eta, beraz, baita deformazioak ere [Her1882]. Harriaren gorputza ez da deformatzen eta, ondorioz, nahikoa da gainazalarekin kontaktuan dagoen diskoaren zatia modelizatzea. Hau da, diskoa  $h_s$  distantziara batetara moztuko da, kontaktu-baldintzetan ia eraginik sortu gabe.  $h_s$  mozte altuera egokia, harri osoaren deformazioan eragiten ez duen distantziarik txikiena da.  $h_s$  distantzia FEM bidez estimatu da, distantzia horren gainetik tentsioak balio maximoaren bat ehunen baldintza ezarriz. Horrela, DEM ereduaren dimentsioak  $d_s$ ,  $b_s$  eta  $h_s$  dira (3.12. Ir.).

DEM eredian,  $d_g$  bataz besteko ale-neurriak  $d_{DE}$  elementu diskretuen bataz



3.14. Irudia: Harriaren DEM eredia eraikitzeke fluxu-diagrama

besteko diametroarekin bat egiten du. Horrela, 3.18 eta 3.19 Ekuazioek,  $d_{DE}$  DE erreferentziako diametroa eta  $\mu_g$  DE-en diametro dispersioa kalkulaten dituzte  $M$  galbahe zenbakian oinarrituz. DEM ereduaren DE dentsitateak, harriaren  $V_g$  urratzaile frakzio bolumetrikorekin ere bat egin behar du.  $S$  egitura zenbakiak adierazten du urratzaile frakzioa eta 3.20 Ekuazioarekin estimatzen da  $V_g$ .

Harriaren DEM eredia eraikitzeke fluxu-diagrama erakusten du 3.14. Irudiak. Lehendabizi, (1) ereduaren dimentsioak ( $d_s, b_s, h_s$ ) definitzen dira (3.12. Ir.). Ereduren dimentsioekin, DEM domeinuaren bolumena  $V_{DE}$  kalkulaten da. Biak,  $d_g$  batuz besteko ale-neurria eta  $V_g$  urratzaile frakzio bolumetrikoa aldi berean doitu behar dira. Horretarako, DEM domeinua betetzeko  $N_{DE}$  DE kopurua hartzen da erreferentziatuz. (2)  $V_{DE}$  ereduaren bolumena,  $V_g$  urratzaile frakzioa eta  $v_g$  ale urratzaile baten batuz besteko bolumena erlazionatuz, 3.23 Ekuazioak DEM ereduak harriaren urratzaile frakzioaren baldintza betetzeko  $N_{DE}$  elementu kopurua definitzen du. 3.24 Ekuazioarekin lortzen da  $\theta$  arkuaren angelu-erdia (rad) (3.12. Ir.). (3) Orain DEM eredia eraiki daiteke domeinuaren dimentsioekin eta  $N_{DE}$  DE kopurua kontsignatuz hartuta. Beharrezko  $N_{DE}$  lortu dela ziurtatu ondoren, (4) habeak sortzen dira kontaktuan dauden DE-en artean. Habei kalibrazio prozesuan lortutako propietate mekanikoak ezartzen zaizkie. Zenbakizko sinterizazioak DEM domeinua DE-az asetzen du, lortzen den  $d_{DE}$  DE batuz besteko diametroa desiratutako  $d_g$  baino handiagoa delarik. (5) Efektu hori, DE bakoitzaren diametroa  $r$  murrizpen-faktoreaz biderkatuz konpontzen da. 3.25 Ekuazioak  $r$  murrizpen-faktorea definitzen du,  $d_g$ -ren balioa  $d_{DE}$ -rekin alderatuz. Hauaxe da DEM ereduak harriaren egitura fidela deskribatzeko gakoa,  $d_g$  eta  $V_g$  aldi berean doitzea lortzen duena.

$$N_{DE} = V_{DE} V_g \frac{1}{v_g} = \left[ \frac{d_s^2 b_s}{4} \left( \frac{\pi \theta}{360} - \frac{\sin \theta}{2} \right) \right] V_g \left( \frac{6}{\pi d_g^3} \right) \quad (3.23)$$

$$\theta = 2 \arccos \left( 1 - \frac{h_s}{d_s} \right) \quad (3.24)$$

$$r = \frac{d_g}{d_{DE}} \quad (3.25)$$

Azkenik, pieza gehitzen da DEM ereduan. 3.2.1. Atalean azaldu da, pieza DE errektangeluar bakarrarekin modelatzen dela (3.12. Ir.). Elementu berezi honek gainazal laua simulatzen du. Piezaren mugimenduak desplazamendu bertikale-ra mugatzen dira ( $y$  ardatza), harriarekiko norabide erradialean. Biraketak ere galaraziak ditu. Ohiko DEak bezala, DE errektangeluarra ere zurruna da, baina kontaktu-legearen arabera penetrazioa baimentzen du. Gainazaleko DE baten eta piezaren artean elkar-sartzea detektatuz gero, penetrazioaren arabera erreakzio-indarra aplikatzen da bai piezan, baita alean ere (3.5. Atala).

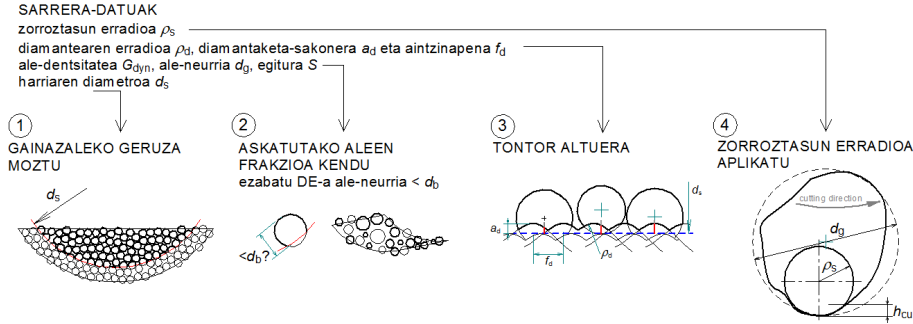
### 3.4. Zenbakizko diamantaketa

Harriaren gainazalaren topografia diamantaketaren ondorio zuzena da. Atal honetan diamantaketak sortutako gainazal topografia aplikatuko zaio 3.3. Atalean eraiki den harriaren DEM ereduari.

Topografia-ereduen berrikuspenean (2.3.2. Atala), topografia definitzen duten ezaugarri nagusiak identifikatu dira. Horien artean ale-neurria, ale-dentsitatea, tontor-altuera eta zorrotasuna daude. Ezaugarri horiek komunak dira diamantaketa finko eta birakarirako. Hala ere, 2.1.2. Atalean azaldu da diamante punta-bakar finkoen erabilerak, parametro zinematikoak eta ezaugarri geometrikoak lortutako topografiarekin erlazioatzea ahalbidetzen duela. Horregatik, tesi honen hedadura diamante punta-bakar finkoen erabilerara mugatu da. Ereduaren hedapena diamante punta-anitzetara eta birakarietara zabaltzea, tesi honek egiteke uzten duen atazetako bat da. Hedapena, aurrez aipatutako diamantaketa-ezaugarriak (ale-dentsitatea, tontor altuera eta sorbatzen zorrotasuna) gainazaleko aleetan ausaz aplikatuz lortuko da.

Diamanteak gainazala torneatzean harriaren gainazal topografia sortzen du. DEM eredura topografia transferitzeko diamantaketak lana nola egiten duen jakin behar da. Berrikuspenean aleen eta lotura-zubien hausturak diamantaketa-mekanismo nagusiak direla ikusi da. Nahiz eta *a priori* ausazko gainazala dirudien, diamantaketak tontor-patroi jakin bat sortzen du. Diamantearen profilak gainazalean bere ibilbidearen aztarna helikoidala uzten du. Urratzailearen haustura hauskorrak bihurtzen du harriaren gainazala ausazkoa. Bestalde, industria-inguruetan onartua dago diamantaketa larrian harriaren gainazala irekiagoa eta bigunagoa lortzen dela, aldiz diamantaketa-baldintza finek gainazal itxiak eta gogorrak sortzen dituzte. Soilik ale irtenenek egingo dute kontaktu piezarekin, gainazala goldatuz eta, baldintzak betez gero, materiala ere harrotuz. Ale berdineko ebaketa-sorbatzek ez dute materiala harrotzeko behar besteko leku fisikorik. Ebaketa-sorbatzen zorrotasuna sorbatz-erradioaren edo -angeluen bidez adierazi daiteke. Horrek sorbatz-geometria baldintzatzen du: esfera, konoa edo piramidea.

Kontzeptu horietan oinarrituz, topografia lau pausotan aplikatzen da harriaren DEM ereduan: (1)  $d_s$  diametroan gainazala moztu, (2) diamantaketan as-



### 3.15. Irudia: Harriaren DEM ereduari gainazal topografia aplikatzeko pausoak

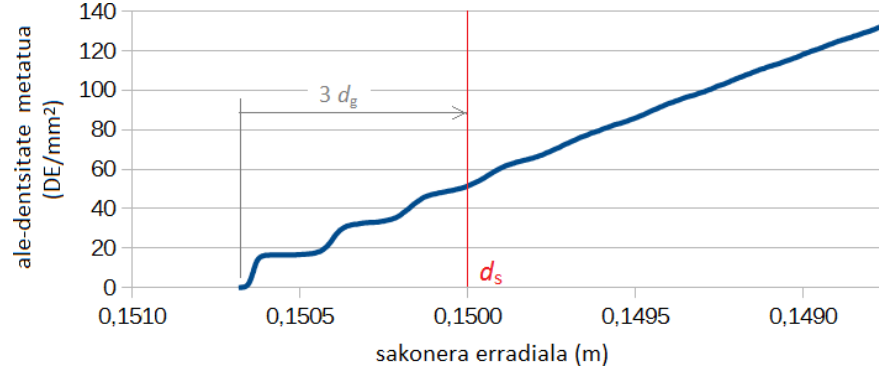
katutako ale frakzioa ezabatu, (3) ebaketa-sorbatzen sakonera posizioa definitu, eta (4) sorbatz-geometria aplikatu. 3.15. Irudiak harriaren gainazal topografia aplikatzeko fluxu-diagrama aurkezten du.

Lehen pausoak, gainazala  $d_s$  harriaren diametro nominalera uzten du, DE geruza fin bat ezabatuz. Aleak harrian ausaz kokatuak egoteak, ale kontzentrazioa sakoneran linealki handitzea dakar [McA64]. DEM eredu sortzean, nahiz eta zenbakizko sinterizazio prozedurak domeinuan DE-en banaketa homogenea eta ausazkoa ziurtatu, domeinuko pareta elastikoen inguruan DEak pilatzen dira, lokalki elementu dentsitatea handituz. 3.16. Irudiak azaltzen du elementuen pilaketa efektua, sakonerarekiko DE dentsitate metatuan pilaketa mailakatuak agertuz. Pilaketa efektua, DEM ereduaren gainazaletik hiru aldiz DE diametro sakoneran duen DE geruza kenduz konpontzen da. Horrek DEM eredu eraikitzean sarrera-datueta, harriaren diametroa zerbait handiagoa erabiltzea ( $d_s \pm 3d_g$ ) dakar.

Lehen pausua aplikatzean, DEM ereduaren eraikuntzan ale-pilaketa konpontzen da eta gainazala diametro nominalean topografia aplikatzeko prest uzten da: gainazaleko DEaren zentroa  $d_s$  erreferentziazko diametrotik kanpo geratuz gero, elementua ezabatu egiten da.

Bigarren pausuan, harriaren gainazaletik diamantaketaren ondorioz askatutako aleak ezabatzen dira. Kontuan hartzen da ale irtenenak direla askatzeko probabilitate gehien dutenak. Horretarako 2.1.2. Atalean azaldu den Malkinen diamantaketan askatutako partikulen eredu erabiliko da [MC71, Mal08]. Bertan, aleak askatzean duten batz besteko diametroa  $d_b$  harriaren gainazalaren irekitasunarekin erlazionatzen du. Gainazaleko  $G_{dyn}$  ale-dentsitate dinamiko eta  $G_o$  ale-dentsitate teorikoa erlazionatuz,  $d_b$  estimatzen da 2.11, 2.12 eta 2.13 Ekuazioekin. DEM ereduaren,  $d_s$  diametroan mozten duten gainazaleko DE-en neurria  $d_b$  baino txikiagoa bada, ezabatua da.

Orain DEM ereduaren gainazala lantzeko prest dago. Lehendabizi, gainazaleko DE bakoitzaren  $y_p$  tontorraren sakonera definitzen da eta, ondoren, sorbatz-zorroztasuna aplikatzen da tontor bakoitzean. Verkerken arabera, gainazaleko



**3.16. Irudia:** Ale-dentsitate metatua sakonera erradialean: DE-en pilaketa DEM ereduaren paretetan eta  $d_s$  harriaren erreferentziako diametroaren definizioa

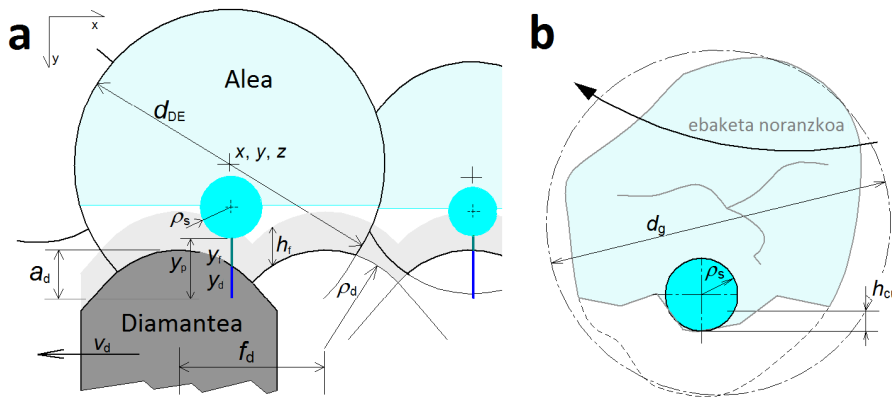
ale bakoitzak sorbatz erabilgarri bakarria dauka [Ver77]. DEM erudian, Chenen funtzio sinusoidala [CR96] era diskretuan aplikatzen da DE-en zentroetan  $y_p$  tontorren sakonera erradialak definitzeko (3.17.(a) Ir.). Tontorrek DE-en zentroekin lerrokatuak daudela jotzen da. Chenen diamantaketa eta Malkinen ale-askatze ereduaren konbinaketak, diamante punta-bakar finkoen diamantaketa-mekanismoak biltzen ditu: helize helikoidal harikorra, ausazko ale-haustura eta ale-askatzea.

$y_p$  tontor sakonera kalkulatzeko erabiliko den 3.26 Ekuazioak, 2.27 Ekuazioa gogorarazten du, honek ere  $y_d$  helizearen osagai harikorraren eta  $y_f$  haustura hauskorraren efektuak konbinatzen baititu.  $y_d$  osagai harikorra (3.27 Ek.),  $f_d$  diamantaketa-aitzinamenduarekin jauzi diskretuetan mugitzen den zirkunferentzia-funtzioak zehazten du. 3.27 Ekuazioan,  $\rho_d$  diamantearen erradiora,  $x_c$  eta  $y_c$  diamantearen aldiuneko posizioa,  $x$  tontorraren (edo DE zentroaren) posizioa norabide axialean, eta  $a_d$  diamantaketa-sakonera da.  $x_c$ ,  $f_d$  aitzinamenduarekiko proportzionala den zenbaki diskretua da, eta bere balioa DEaren zentroaren posizioarekiko gertuen dagoen balioa da. Eta 3.28 Ekuazioak  $y_f$  osagaia definitzen du  $h_f$ -ko anplitudea duen hausturaren  $s_n$  ausazko frakzioa izanik.  $h_f$  haustura anplitudea 2.28 Ekuazioak kalkulatzeko da. Honela, Chen-Malkin ereduaren konbinaketak, ale-dentsitatea azalarazten da sakonera barneratzean.

$$y_p(x) = y_d(x) + y_f \quad (3.26)$$

$$y_d(x) = \sqrt{\rho_d^2 - (x - x_c)^2} - y_c \text{ non } x_c = \mathbb{Z} \left( \frac{x}{f_d} \right) \text{ eta } y_c = a_d - \rho_d \quad (3.27)$$

$$y_f = s_n h_f \quad (3.28)$$



**3.17. Irudia:** (a) Tontorra harriaren  $d_s$  kanpo-diametroarekiko  $y_p$  sakoneran kokatzen da, bertikalean DEaren zentroarekin lerrotatuta;  $y_p$  diamantearen ebaketa harikorraren  $y_d$  eta haustura hauskorraren  $y_f$  osagaien batura da [CR96]; (b) diamantaketa haustitako ale urratzailea sorbatz-erradioaren definizioarekin [Mas52, Klo09]

Bukatzeko, tontorraren edo sorbatzaren geometria definitzea falta da. Sorbatzaren geometriak alearen zorroztasun egoera islatu behar du. Berrikuspenean ikusi da ohiko urratzaileen topografia-eruedetan  $\rho_s$  sorbatz-erradioa (esfera) eta sorbatzaren malda (konoa edo piramidea) nagusitu direla zorroztasuna adierazteko (2.3.2. Atala). Biak antzera neurtzen dira eta biak DEM eruedaren topografian erraz implementatu ahal dira. Sorbatzen maldak sakonerarekin aldatuta handiak dituzenez [TB00], esfera aukeratu da, erradioa  $\rho_s$ -rekin berdinduz (3.17.(b) Ir.).  $\rho_s$ -k higaduragatik zorroztasun aldaketak ere islatu ahal ditu. Harriaren higaduraren modelaketa, sorbatz-zorroztasuna eta ale-askatzea erlazioztatuz, tesi honek proposatzen duen etorkizuneko ikerketa-ildoetako bat da.

$\rho_s$  neurtzeko Königek eta Lortzek erabilitako prozedura [KL75] eguneratu da, gaur egungo 3D topografia neurketen abantailez baliatzeko. 3D topografia neurketetan tontor irtenenak identifikatu eta haien 2D profilak lortu dira ebaketa-norabidean software bidez. Ale-pieza kontaktua azalekoa denez, mikra gutxi batzuetako sakoneran ( $5 \mu\text{m}$ ) hurbildutako zirkunferentziaren erradioa da  $\rho_s$ . Prozedura honek 2D profilak beti aleen tontorretan aztertzea ahalbidetzen du.

Laburbilduz, atal honetan deskribatu diren pausuak jarraituz DEM eruedaren gainazalak diamantaketaren ondorioak islatzeko gai da, artezketaren ausazko natura mantenduz. Erreferentziatzko gainazal batetik abiatuz, lehenik askatu diren aleak ezabatzen dira. Ondoren geratzen diren aleen tontor-sakonera definitzen da erreferentziatzko gainazalarekiko. Ale bakoitzak ebaketa-sorbatz bakarra dauka. Ebaketa-sorbatzak geometria esferikoa du,  $\rho_s$  sorbatz-erradioak zorroztasuna adierazten duelarik. Era honetan, ale-dentsitatea sakonerarekiko azaleratzen da, harrietan gertatzen den antzera.

### 3.5. Mugalde-baldintzak

Behin harriaren DEM eredua eraiki eta diamantaketak sortutako topografia sortu ondoren, heldu da mugalde-baldintzak gehitu eta simulazioa martxan jartzeko ordua. Tartean, artezketa lan-baldintzak ezarri, ale-pieza kontaktua detektatu eta bien interakzioan dauden baldintzak erreproduzitu behar dira.

Atal hau ereduaren mugalde-baldintzak eta kargak aurkezten hasten da. 3.12. Irudiak ereduaren aplikatutako kanpo-indarrak eta mugimendu-murrizpenak erakusten ditu: piezan esperimentalki neurtutako indar normala aplikatzen da, harriaren ebaketa-planoa berriz finko dago. Harriaren ereduaren alboko partetik batek ere, harriaren ardatzarekiko norabidean desplazamendua galarazia dauka, gilbordura eragozteko. Iterazio bakoitzean, harriaren gainazaleko DE-en eta piezaren posizioak alderatzen dira, erreakzio-indarra aplikatuz elkar-sartzea detektatuz gero. Simulazioan zehar, sartze sakonera maximoa, kontaktu-indar erresultantea eta kontaktu-luzera monitorizatzen dira. Sistema dinamiko modeldua denez, iterazioak (edo denbora) aurrerantz doazen heinean piezaren posizioak emaitzarantz jotzen du, orekara iristean aleetako erreakzio-indar guztien baturak eta piezan aplikatutako indar normalak bat egiten dutelarik. Egonkortasun dinamikoa bermatzeko, indar normala zerotik linealki handitzen da, indar normal nominalera iristean konstante bihurtuz.

Sarreran komentatu da, artezketa-kontaktua bi baldintzetan modelatuko dela, hau da, bi mugalde-baldintza multzo aplikatzen dira DEM eredu berdinari: kontaktu estatikoa (KE) eta ebaki-gabeko txirbil-lodiera (ETL) mugalde-baldintzak. Bakoitzak bere kontaktu-detekzio algoritmoa eta kontaktu-legea dauka. Bietan simulazioa kontaktu-gabe-ia-ukitzen (harria-pieza distantzia  $< 10^{-16}$  m) posizioan hasten da, konputazio-denbora ahalik eta gehien optimizatzeko eta hasierako kolpeak edo urruntzeak saihesteko. Harriak ez du biratzen. Hala ere, indar zentrifugoa harriaren ale bakoitzean aplikatu da norabide erradialean kanporantz. Indar zentrifugoaren eragina bi mugalde-baldintza multzoetan (KE eta ETL) barneratu da (3.5.3. Atala).

KE ereduak harriaren eta planoaren (pieza) arteko kontaktu estatikoa modelatzen du. Kontaku estatikoaren emaitzak, RQ ereduaren  $l_f$  kontaktu-luzera zimurra terminoari dagokion (2.2.1. Atala) eta, horrela,  $R_r$  zimurtasun-parametroa estimatu daiteke. Bestalde, ETL ereduak ale-pieza arteko ebaketa-indarra kalkulatu du,  $a_e$  ebaketa-sakonera kontaktu-detekzio algoritmoan barneratuz. Shawen txirbil-sorkuntzaren gogortasun eredu (2.4.2. Atala) aukeratu da ale mailako ebaketa-indarrak estimatzeko.

KE eta ETL ereduetan, kontaktu-baldintzak zurrun-plastiko-perfektuak direla jotzen da, eta piezaren materialak portaera biskoplastikoa erakusten du. Kontaku-presioek erraz gainditzen dute eremu elastikoa eta, beraz, kontaktua zurrun-plastiko-perfektutat har daiteke (2.4.1. Atala). Tentsio ebakitzailen labaindura-lerroetatik ondorioztatua eta behaketa esperimentalek berretsia, gainazaleko bataz besteko kontaktu-presioa eremu plastikoan materialaren isurpen-tentsioaren hirukoitza da gutxi gorabehera ( $\approx 3\sigma_y$ ).

Hala ere, deformazio plastiko eta deformazio-abiadura handiek materialaren erresistentzia igotzen dute, eta eragindako tenperatura altuek, aldiz, bigundu.

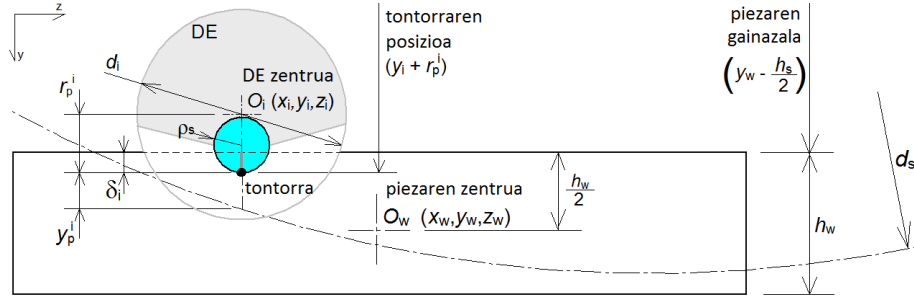


Biskoplastikotasunak materialen portaera inelastikoa deskribatzen du deforma-zioen eta tenperaturen funtzioan. Besteak beste, Johnson-Cook (JC) eredu eratzailleak  $\sigma_y$  isurpen-tentsio erabilgarria estimatzen du [JC83], aurrez azaldu diren adierazpen zurrun-plastiko-perfektuetan aplikatzen dena. 2.35 Ekuazioak  $\sigma_y$  estimatzen du  $\varepsilon_p$  deformazio plastikoaren,  $\dot{\varepsilon}_p$  deformazio-abiaduraren eta  $T$  tenperaturaren funtzioan. Hiru parametro horiek alearen ibilbidean zehar aldatuz doaz, urradura-mekanismoen aldaketarekin batera (igurzketa, goldaketa eta ebaketa). 2.4.3. Atalean eztabaidatu da,  $\phi$  ebakidura-planoaren angelua,  $\varepsilon_p$ ,  $\dot{\varepsilon}_p$  eta  $T$  xehetasunez soilik zenbakizko simulazio bidez estimatu daitezkeela. Parametro horien ebaketa-sakonerarekiko balioztapen esperimentalak konplexua da. Hurbilpen analitikoak, berriz, tenperaturaren estimazio zabarra eskaintzen du kontaktu-luzeran posizioarekiko, ez ebaketa-sakonerarekiko. KE ereduaren kontaktu-luzera zimurra (harria-planoa kontaktua) artezketa kontaktu-luzerarekin zerikusirik ez duenez, hurbilpen analitikoak ez luke zentzurik izango. Hurbilpen analitikoa ETL ereduaren inplementatzeak, berriz, konputazio eskakizunak handituko lituzke, iterazio bakoitzean ale bakoitzarentzat  $\sigma_y$ -ren balioa estimatzeko beharrak bultzatuta. Tesi honetan garatutako ereduak  $\varepsilon_p$ ,  $\dot{\varepsilon}_p$  eta  $T$  konstantetzat jotzen ditu kontaktu-luzeran zehar bi mugalde-baldintza aldaeretan. Hurbilketa zabar horrek sortutako akatsa txikiagoa da piezaren materialaren portaera biskoplastikoa arbuiazeak ekarriko lukeenak baino.

Ondorengo Atalek, KE eta ETL ereduaren mugalde- eta kontaktu-baldintzak deskribatzen dituzte. Kontaktu-detekzio algoritmoa eta kontaktu-legeak (kontaktua identifikatuz gero erreakzio-indarraren definizioa) dira bi hurbilpenen arteko ezberdintasun nagusiak. Kontaktu-detekzio algoritmoa prozesagailuan barneratzen da: iterazio bakoitzean hartzen du parte, gainazal elementuen posizioa piezarenarekiko aztertuz. Kontaktu-legeak, kontaktua detektatzen den bakoitzean kalkulatu du erreakzioa, indarra DEaren eta piezaren indar-orekara gehituz.

### 3.5.1. Kontaktu estatikoaren (KE) eredia

2.2.1. Atalean Rowek eta Qik [RMQZ93] garatutako eredia aurkeztu da. Haien arabera,  $l_c$  kontaktu-luzera bi osagaien batura koadratikoa da:  $l_g$  kontaktu-luzera geometrikoa, harria-pieza gainazal leun zurruntzat hartuz, eta,  $l_f$  kontaktu-luzera zimurra, harriaren eta gainazal lauaren artean (ikusi 2.25 Ek.).  $l_g$  osagaia geometria aztertuz kalkulatu da,  $a_e$  ebaketa-sakonera kontuan hartuta.  $l_f$  osagaiak aldiz, harria eta plano baten kontaktu estatikoaren emaitza izanik, ez du harria-pieza kontaktua zuzenean aztertzen. KE ereduaren helburua, beraz, harriaren DEM eredia  $l_f$  harriaren eta piezaren kontaktu estatiko zimurra simulatzeko erabiltzea da, ez artezketa kontaktu-luzera. RQ eredia, besteak beste,  $R_f$  zimurtasun faktore empirikoaren funtzioan dago, faktore askoren eraginak biltzen dituena. KE eredia ere baliagarria da  $R_f$  estimatzeko. KE eredia berez, bere horretan, gainazal zimurren arteko kontaktua modelatzeko hurbilpen berritzaile eta malgua da, materialari portaera elastikoa, elastoplastikoa edo zurrun-plastiko-perfektua egotziz.



**3.18. Irudia:** Kontaktu-detekzioa: elkar-sartzea harriaren gainazaleko elementu baten eta piezaren gainazalaren artean

### Kontaktu-detekzioa

Iterazio bakoitzean, gainazaleko DE bakoitzaren tontorraren posizioa, piezaren gainazalarenarekin alderatzen da. KE eredian, gainazaleko DE baten eta piezaren arteko kontaktua antzematen da, baldin eta  $\delta_i$  elkar-sartzea baieztatzen bada bien posizioak alderatzean. Prozedura gainazaleko ale bakoitzarentzat iterazio bakoitzean errepikatzen du detekzio algoritmoak. Harria geldirik dago, beraz alearen eta piezaren elkarguneak moztutako esferaren geometria dauka.

3.18. Irudiak KE eredian kontaktuak nola antzematen diren azaltzen du. 3.29 Ekuazioak kontaktu-detekzioaren adierazpena aurkezten du.  $\delta_i$  DE-pieza arteko elkar-sartzea,  $(y_i + r_p^i)$  tontorraren eta  $(y_w - h_w/2)$  piezaren gainazalaren posizio bertikalen arteko diferentzia da.  $r_p^i$  tontorraren eta DE zentroaren arteko distantzia da,  $(x_i, y_i, z_i)$  eta  $(x_w, y_w, z_w)$  elementuaren eta piezaren zentroen posizio koordenatuak dira hurrenez hurren, eta  $h_w$  piezaren altuera da. Elkar-sartze positiboak ( $\delta_i > 0$ ) kontaktua baieztatzen du.  $r_p^i$  distantzia eredia eraikitzean finkatzen da, topografia aplikatzean, gainazaleko ale bakoitzari  $y_p^i$  tontor-sakonera ezarriz  $d_s$  harriaren diametroarekiko.

$$\delta_i = (y_i + r_p^i) - \left( y_w - \frac{h_w}{2} \right) \quad (3.29)$$

### Kontaktu-legea

Behin kontaktua baieztatuta, aurkako noranzkoa duten kontaktu-indar normal  $f_n^i$  bera aplikatzen zaie piezari eta elementu diskretuari. Kontaktu-indarra, inguruko elementuen interakzio indarrekin batera, hurrengo iterazioan elementuak izango dituen posizio, abiadura eta azelerazioa kalkulatzeko erabiltzen da. Kontaktu-indar normala,  $\delta_i$  elkar-sartzearen eta bien arteko abiadura erlatiboaren funtzioan dago. Indarren norabideak ardatz bertikalarekin bat egiten du. 3.30 Ekuazioak definitzen ditu norbanako ale baten kontaktu-indarraren osagaiak.

$$\mathbf{f}_n^i = \mathbf{f}_c + \mathbf{f}_d \quad (3.30)$$

Lehenengo osagaia,  $\mathbf{f}_c$ , erreakzio kontaktu-indarrari dagokio. Kasu honetan, aleen tontorren geometria esfera da. 3.31 Ekuazioak  $\mathbf{f}_c$  kontaktu erreakzioa sorbatz esferikoaren eta gainazal lauarentzat moldatzen du.  $\mathbf{f}_c$  eta  $\mathbf{n}$  bektore unitario bertikala lerrotatuta daude, eta  $\delta_i$  elementua eta piezaren arteko elkar-sartzea da. Nahiz eta DEak  $d_g$  diametroko esferak izan,  $\rho_s$  erradioko esferak modelatzen ditu sorbatzak (3.17. Ir.). Harria geldi dago, ondorioz, sorbatzek piezan utzitako aztarnek horien alderantzizko geometria dute.

$$\mathbf{f}_c = 3\sigma_y\pi \left[ \rho_s \sin \left( \arccos \left( \frac{\rho_s - \delta_i}{\rho_s} \right) \right) \right]^2 \cdot \mathbf{n} \quad (3.31)$$

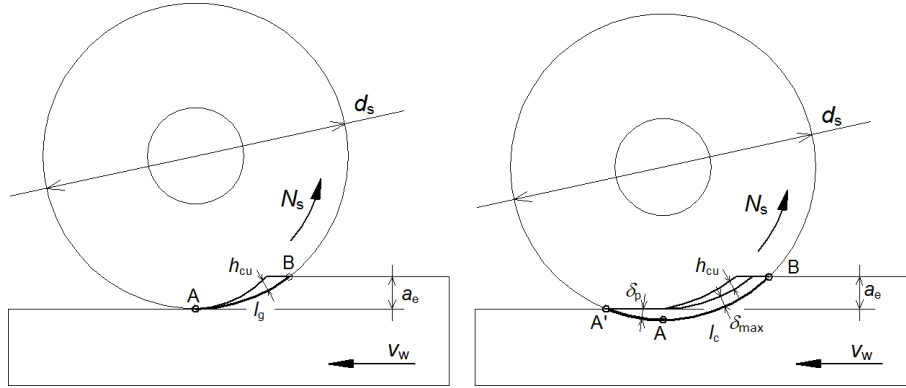
3.31 Ekuazioak kontaktua zurrun-plastiko-perfektutzat jotzen duenez, batz besteko kontaktu-presioa  $\sigma_y$  isurpen-tentsioaren hirukoitza da (2.4.1. Atala). Ereduak kontaktu estatikoa simulatu arren, materialaren portaera biskoplastikoa modelatzeko ere balio du adierazpenak: nahikoa da  $\sigma_y$  materialaren isurpen-tentsioa JC ereduak tentsio baliokideagatik ordezkatzera (2.35 Ek.).

3.30 Ekuazioaren bigarren osagaiak,  $\mathbf{f}_d$ , sartze-abiadura erlatiboarekin erlacionatuta dagoen indar iraungitzailea islatzen du. Moteltze-indarra 3.2 Ekuazioa jarraituz kalkulatzen da (3.2.1. Atala), non  $M_{iw}$  masa baliokidea 3.5 Ekuazioarekin kalkulatzen den.

### 3.5.2. Ebaki-gabeko txirbil-lodieraren (ETL) eredua

Izena, ebaki-gabeko txirbil-lodiera (ETL), ereduaren lan egiteko eragatik dator: ebaki-gabeko txirbil-lodieraren funtzioan kalkulatzen da ale bakoitzak jasaten duen indarra. Horrela, harria materiala harrotzen dagoen istant bat simulatzen da. KE eta ETL ereduak, DEM eredu eta hasierako mugalde-baldintza (3.12. Ir.) berdinak partekatzen dituzte. Baina ETL erudian, harriari artezketa-indar normala eta tangenziala, biak aplikatzen zaizkio. Ereduaren mugalde-baldintzek,  $a_e$  ebaketa-sakonera ere kontuan hartzen dutenez, lortzen den emaitza zuzenean  $l_c$  kontaktu-luzera da. Horrela, elementu bakoitzaren txirbil-lodiera posizioaren araberkoa izatea lortzen da. Harriak ez du biratzen: aleetan eragindako ebaketa-indarren erresultanteak piezan aplikatutako indarrekin bat egiten du. Shawen txirbil-sorkuntzaren gogortasun-analogian erabiltzen da alean erreakzio-indarra estimatzeko [Sha71, Sha96], piezaren materialaren portaera biskoplastikotzat hartuz.

3.19. Irudiak ereduaren funtsa azaltzen du. 3.19.(a) Irudian,  $A$ - $B$  arkuak  $l_g$  kontaktu-luzera geometrikoa grafikoki definitzen du, harria eta pieza gorputz zurruntzat hartuz. Kontaktuaren hasierak,  $A$  puntuak, harriaren azpiko koadrantearekin bat egiten du.  $h_{cu}$  ebaki-gabeko txirbil-lodiera bata bestearen segidan dauden bi aleen ibilbideek definitzen dute, eta  $h_{cu}$  linealki handituz doa kontaktu-luzeran zehar. Aldiz, harriaren deformazioaren ondorioz, 3.19.(b) Irudian kontaktua  $A'$  punturaino luzatzen da. Deformazioaren ondorioz, harriapieza arteko kontaktu-kurba erreala ezezaguna da.

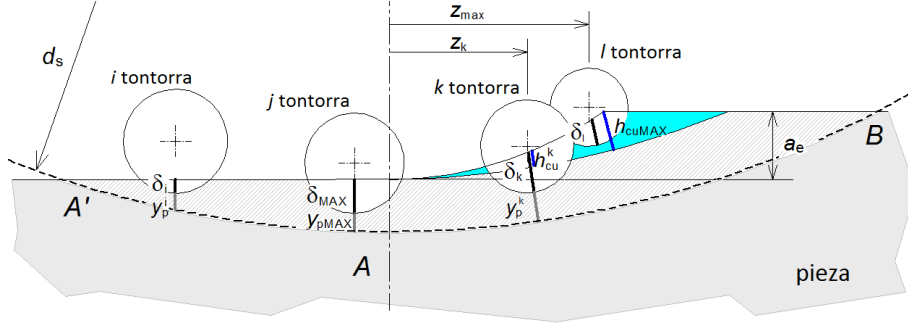


**3.19. Irudia:** ETL ereduaren oinarriak: (a) Kontaktu-luzera geometrikoa eta ebaki-gabeko txirbil-lodiera; (b) deformazio plastikoaren eta ebaki-gabeko txirbil-lodieraren konbinazioa ( $\delta_{max} + h_{cu}^i$ )

ETL ereduak barneratzen du, alde batean ( $A-A'$ ), aleak piezaren gainazala era plastikoan goldatzen duela; eta bestean ( $A-B$ ), aurrez aleak eragindako deformazio plastikoaren ondorioz lortutako sakonerari, posizio-arkuan dagokion ebaki-gabeko txirbil-lodiera gehitzen zaiola. Horrela, bi aldeetan alearen sakonekarak benetako  $h_{cu}$ -rekin bat egiten du, erreakzio-indarra estimatzeko erabiltzen dena. Deformazio elastikoen eragina arbuigarria da deformazio plastikoekin parekatuz. Aurreko aleek utzitako ildaskak eta gandorrek arbuia egin dira elkar-sartzearen definizioan. Aurreko aleen efektua barneratzea, tesiak egiteke utzi duen atazetako bat da.

### Kontaktu-detekzioa

Aurreko hariari jarraituz, bi kontaktu-eremu izateak kontaktuak detektatzeko bi algoritmoen beharra dakar. 3.20. Irudiko  $A'-A$  segmentuan, KE ereduko  $\delta_1$  elkar-sartzea bezala kalkulatzen da (3.18. Ir., 3.29 Ek.).  $A-B$  segmentuan berriz (3.20. Ir.), parte hartzen duten elementuak, piezaren gainazaletik  $a_e$  distantziara aurkitzen dira. Harria-pieza bikotearen kontaktuaren kurbatura ezezaguna denez,  $A-B$  segmentuan  $y_p$  tontor-sakonera hartzen da erreferentzia flotatzailatzat. 3.32 Ekuazioak erakusten du  $A-B$  segmentuan dauden aleen  $\delta_k$  elkar-sartzea,  $A'-A$  segmentuko  $\delta_{max}$  elkar-sartze maximoa eta dagokion  $h_{cu}^k$  ebaki-gabeko txirbil-lodieraren arteko batura dela.  $\delta_k$  positiboak kontaktua konfirmatzen du.  $h_{cu}$  kontaktu-arkuan zehar linealki aldatzen denez, bere balioa  $h_{cu,max}$  ebaki-gabeko txirbil-lodiera maximoaren (2.5 Ek.),  $z_k$  elementuaren posizioaren eta antzemandako  $z_{max}$  urrunen dagoen kontaktuaren posizioaren funtzioan dago (3.33 Ek.). Horrela,  $A'-A-B$  kontaktu-arku osoa aztertuz, kontaktu-indarrak profil trianguluar irregularrak hartzen du, errealitatean gertatzen den antzera.



**3.20. Irudia:** Kontaktu-detekzioa  $A'-A$  eta  $A-B$  eremuetan; zirkuluek sorbatzak adierazten dituzte

$$\delta_k = \left( \left( \delta_{\max}^{A'-A} + y_{\max}^p \right) - y_p^k \right) + h_{cu}^k \quad (3.32)$$

$$h_{cu}^k = h_{cu,\max} \frac{z_k}{z_{\max}} \quad (3.33)$$

### Kontaktu-legea

$\mathbf{f}_n^i$  kontaktu-indar normala bi eruedetan, KE eta ETL,  $\mathbf{f}_c^i$  erreakzio- eta  $\mathbf{f}_d^i$  moteltze-indarren batura da (3.30 Ek.), biek  $\mathbf{f}_d$  osagai iraungikorra partekatzen dutelarik (3.2 Ek.). Literatura-berrikusketan 2.4.2. Atalean aurkeztu den Shawen txirbil-sorkuntzaren gogortasun eredua aukeratu da aleko erreakzioa estimatzeko [Sha71, Sha96]. Gogortasun-froga izaki, Shawek erreakzioaren adierazpena kontaktua zurrun-plastiko-perfektutzat hartuz ondorioztatzen du (2.29. Ir.). Hala ere, deformazio-baldintzek eta tenperaturek materialaren propietate mekanikoak aldatzen dituzte, materialak portaera biskoplastikoa erakusten duelarik. Shawek garatutako 2.31 Ekuazioa moldatu da  $\mathbf{f}_c$  kontaktu-indar normala estimatzeko, gogortasuna erabili beharrean, JC eredu eratzaillearekin estimatutako  $\sigma_y$  isurpen-tentsio eraginkorragatik ordezkatur. 3.34 Ekuazioak  $\mathbf{f}_c$  kontaktu-indar normala kalkulatzeko kontaktu-detekzio algoritmoarekin kalkulaturako  $\delta_i$  edo  $\delta_k$  elkar-sartzearen (3.29 Ek., 3.32 Ek.),  $\mu$  indar-arrazioren eta JC ereduarekin lortutako  $\sigma_y$  isurpen-tentsio eraginkorraren funtzioan. Era honetan, 3.34 Ekuazioak,  $\mathbf{f}_d$  indar iraungikorrarekin batera (3.2 Ek.),  $\mathbf{f}_n^i$  kontaktu-indar normala (3.30 Ek.) kalkulatzeko dute; eta, azkenik, 3.35 Ekuazioarekin,  $\mathbf{f}_t^i$  kontaktu-indar tangentziala  $\mu$ -ren funtzioan.

$$\mathbf{f}_c = 3\sigma_y \pi \left[ \rho_s \sin \left( \arccos \left( \frac{\rho_s - \delta_i}{\rho_s} \right) \right) \right]^2 \cdot \sin(\arctan \mu) \cdot \mathbf{n} \quad (3.34)$$

$$\mathbf{f}_t^i = \mu \cdot \mathbf{f}_n^i \quad (3.35)$$

### 3.5.3. Indar zentrifugoa

Indar zentrifugoa, gorputz birakarietan agertzen den itxurazko indarra da. Newtonen zehaztu zuen 3.36 Ekuazioa, partikula batean eragiten duen  $\mathbf{f}_z^i$  indar zentrifugoa kalkulatzeko.  $\mathbf{f}_z^i$ -n eragiten dute,  $m_i$  partikularen masak,  $\omega$  abiadura anguluarrak eta  $r$  biraketa-zentroarekiko distantziak.  $\omega$  eta  $\mathbf{r}$  bektoreak dira. 3.37 Ekuazioak indar zentrifugoaren  $f_z$  modulua kalkulatu du. Adierazpenak biraketa-abiaduraren garrantzia azpimarratzen du. Indar zentrifugoak harria erradialki zabaltzen du, zurruntasuna handituz.

$$\mathbf{f}_z^i = m_i \omega \times (\omega \times \mathbf{r}) \quad (3.36)$$

$$f_z^i = m_i \cdot \omega^2 \cdot r \quad (3.37)$$

$\mathbf{f}_z$  indar zentrifugoa ereduko DE bakoitzean norabide erradialean aplikatzen da, bere  $m_i$  masaren,  $N_s$  biraketa-abiaduraren eta ardatzera distantziarekiko; azken hori elementuaren  $y_i$ ,  $z_i$  posizio koordenatuekin kalkulatu.  $f_z^i$  indar zentrifugoaren modulua 3.38 Ekuazioarekin zehazten da. Indar zentrifugoaren eragina bietan, KE eta ETL ereduetan, aplikatzen da.

$$f_z^i = m_i \left( \frac{\pi N_s}{30} \right)^2 \sqrt{y_i^2 + z_i^2} \quad (3.38)$$

## 3.6. Laburpena

Elementu diskretuen metodoa (DEM) aukeratu da harriaren morfologia eta egitura modelatzeko. DEM formulazioa fenomeno fisikora moldatzeko gaitasuna dauka. Helburua, harriaren gainazal topografia islatzeko gai den harriaren zenbakizko eredua sortzea da. Eredua artezketa lauan harriaren eta piezaren arteko kontaktua simulatzeko erabili da.

Lehendabizi, kontaktuaren sistema tribologikoa zehaztu da artezketaren ikuspuntutik, literatura-berrikuspenean agertu diren faktoreen eta aldagaien arabera. Harriaren morfologia eta zurruntasuna, gainazal topografia eta ale-pieza kontaktu-baldintzak dira kontaktu-ereduak landu beharko dituen aspektu nagusiak.

Ondoren, fenomeno fisikora moldatzeko DEM formulazioan egin diren aukerak deskribatu dira. Ale urratzaile bakoitza DE batekin eta aglomeratzaile-zubi bakoitza habe batekin ordezkatzuz modelatu da harriaren gorputz heterogeneoa. Horrela, DEM ereduak harriaren gorputz heterogeneo granular porotsua era naturalean islatzen du. Bestalde, pieza DE errektangeluar bakar batek modelatzen du. DEM formulazioak hiru oinarri ditu: kontaktu-detekzio algoritmoa, DE-en arteko interakzioak eta integrazio-eskema. Harria osatzen duten DEak habez lotuak daudenez, kontaktu-detekzio algoritmoaren beharrik ez dago. Aldiz, pieza DE bakarrak ordezkatzeko, harriaren gainazaleko DE-en eta piezaren gainazalaren arteko kontaktu-detekzioa errazten du. Beraz, neurri egindako

kontakto-detekzio algoritmo gordina garatu da, gainazal topografiaren ezau-garrieti eta kontakto-baldintzei egokituta. Habeek gobernatzen dituzte harriko DE-en arteko interakzioak, haien arteko desplazamenduen eta biraketa erlatibo-en menpe daudenak. Habeen propietate mekanikoak kalibratzen dira harriaren propietateekiko. Gainazaleko DE-en eta piezaren arteko interakzioa, berriz, deskribatu nahi den fenomeno fisikoari moldatutako kontakto-legearen baitan dago. Bi kontakto-baldintza modelatu dira harriaren DEM eredu berdinarantzatzat. Lehenak, RQ ereduko kontakto-luzera zimurraren baldintzak modelatzen ditu, harriaren eta planoaren kontakto estatikoa simulatuz. Kontakto-baldintza elastikoak eguneratu dira gainera: kontaktua zurrun-plastiko-perfektutzat hartu da eta materialaren portaera biskoplastikotzat jo da. Bigarren hurbilpenak, berriz, artezketaren istant bat modelatzen du, ebaketa-sakonera kontuan hartzen du eta ale-pieza ebaketa-indarra txirbil-sorkuntzaren gogortasun analogia erabiltzen du. Tchamwa-Wielgoz da aukeratutako integrazio-eskema, Verlet eske-mari egonkortasuna mantentzen laguntzen duen zenbakizko moteltzea gehitzen diona.

Artezketaren DEM kontakto-ereduaren eraikuntzak hiru pauso nagusi ditu: harriaren DEM ereduaren sorkuntza, gainazal topografia aplikatzea eta mugalde-baldintzen definizioa.

Harriaren DEM ereduaren zenbakizko sinterizazio prozedura jarraituz eraikitzen da. DE neurria eta DE frakzio bolumetrikoa, biak aldi berean doitu behar dira. Ereduaren neurria murrizteko, disko zati fin batek modelatzen du harri zilindrikoa. Tentsioak eta deformazioak kontakto-eremuan kokatzen direnez, diskoa altuera seguru batean mozten da, ereduaren neurria are gehiago murrizteko. Habeek ematen diote zurruntasuna harriaren gorputzari. Habeen propietate mekanikoek kalibratze beharra dute, denek batera lan egitean harriaren propietateak lortzeko.

Diamantaketak prestatzen du harriaren gainazala, era hauskorrean aleak hautsiz eta askatuz. Sortu berri den topografiak diamantaketa-baldintzen eragina jasotzen du. Zenbakizko gainazala lau pausotan sortzen da. Lehendabizi, gainazaleko DE geruza harriaren diametroan mozten da, eredu sortzean agertzen den DE pilaketa efektua ezabatuz. Ondoren, Malkinen ale-askatze eredu aplikatzen da askatu aurreko ale-neurriaren arabera. Verkerken arabera, gainazaleko ale urratzaileek sorbatz eraginkor bakarra dute. Chenen eredu era diskretuan aplikatzen da sorbatzen posizioa zehazteko. Azkenik, sorbatzen zorroztasuna esfera bidez modelatzen da.

Mugalde-baldintzak aurre-prozesaketan aplikatzen dira; besteak beste, harriaren DEM ereduaren kargatu, topografia aplikatu, harriaren ebaketa-planoa finkatu eta artezketaren indar normala piezari ezarri. Simulazio-begiztan sartzean, iterazio bakoitzean kontakto-detekzio eta erreakzio-indar algoritmoek gainazaleko DE-en eta piezaren posizio erlatiboak aldatuz, kontaktua dagoen erabaki, eta baldintzen arabera erreakzio-indarra ezartzen dute. Simulazioa amaitzen da, DEtako erreakzio-indarren erresultanteak eta piezaren aplikatutako indarrak bat egiten dutenean. Lehen aipatutako bi aldaerak garatu dira: Rowe-Qi kontakto-ereduaren kontakto-luzera zimurra harriaren eta planoaren artean; eta ebaki-gabeko txirbil-lodiera modelatzen duena ebaketa-sakonera kontuan

hartuz. Biek kontaktua zurrun-plastiko-perfektutzat eta materialaren portaera biskoplastikotzat jotzen dute.



## 4. Atala

# Kontaktuaren simulazioa artezketan

*Gai honetan, 3. Gaiaren aurkeztutako zenbakizko ereduak erabiltzen da harriaren portaera kontaktuan aztertzeko. Hainbat froga esperimental egin dira zenbakizko simulazioen baldintzetan. Lan esperimentalak, simulazioetako beharrezko sarrera-datuak lortzeaz gain, simulazioen helburu diren kontaktu-luzerak ere neurtu ditu. Zenbakizko ereduak aurreikusitako eta emaitza esperimentalen alderaketak hainbat ondorio eskaini ditu.*

*Simulazioak martxan jarri aurretik, habeen propietate mekanikoak kalibratzen dira harriaren propietateekiko doituz. DEM ereduaren ausazko natura saihesteko, baldintza berdinetan lau eredu sortu dira frogatutako harri mota bakoitzeko. Haien gainazalari zenbakizko diamantaketa aplikatu ondoren, ereduak prest dira simulazioetarako. Aurre-prozesaketan, esperimentalki neurtutako indarrak eta dagozkien murrizketak aplikatzen zaizkio ereduari. Prozesaketan, iterazio bakoitzean DE bakoitzaren posizio berria kalkulatu da kontaktu-baldintzen eta inguruko DE-en eragina aztertuz. Azkenik, behin simulazioa amaituta, monitorizatutako datuak aztertzen dira. Bi ereduak estimatzen dute kontaktu-luzera; KE ereduak, gainera,  $R_x$  zimurtasun-faktorea; eta, ETL ereduak ale-dentsitatea, aleen desplazamendua eta aleko indarrak. ETL ereduaren emaitzak KE ereduarekin lortutakoak baino gehiago hurbiltzen dira neurketetan lortutakoengana.*

*Atalak esperimentuen diseinu faktoriala ere badakar, sarrera-parametroek ereduaren irteeren aldatetetan duten pisua aztertzen duena. Bi mailako taula ortogonal erabili da saiakuntzen diseinua (design of experiments, DOE) aurrera eramateko. Saiakuntza horiek faktore bakoitzaren eragina sistemaren erantzunean ebaluatzen dute, haien arteko interakzio posibleak barne. Zurruntasun eta topografia parametroen aldatetako harriaren portaeran eragin nabarmena dutela erakutsi dute.*



## 4.1. Lan esperimentalak

Atal honetan lan baldintzak deskribatu eta emaitza esperimentalak aurkezten dira. Beiratzatutako aluminazko bi harri (A46I8V eta A60K7V) erabili dira neurketa esperimentaletan, hiru diamantaketa-baldintzapean (fina (*fine*, F), ertaina (*medium*, M) eta larria (*coarse*, C)), eta bi *a* ezarritako ebaketa-sakonera (10  $\mu\text{m}$  eta 20  $\mu\text{m}$ ). Saiakuntza guztiak hozgarri gabe egin dira 'GER SC 60/40' gainazal lauak artezteko makinan.

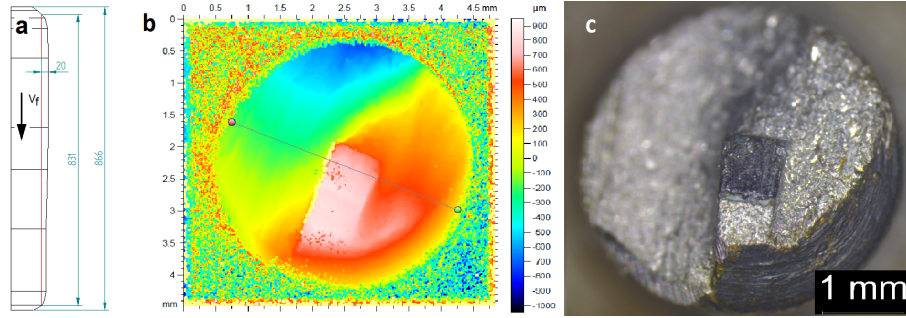
Hiru saiakuntza-mota erabili dira kontaktu-ereduaren sarrera-datuak zehazteko: (a) artezketa-endarren neurketak (indarrak eta ebaketa-sakonera), (b) topografia neurketak (sorbatz-erradioa), eta (c) aplikatutako potentzial-iturria, APS (ale-dentsitatea eta ale arteko batatz besteko distantzia). APSk kontaktu-luzerak ere neurtzen ditu, zenbakizko emaitzekin alderatzen direnak. 4.1. Taulak biltzen ditu saiakuntza-baldintzak eta erabili diren baliabideak.

4.1. Taula: Saiakuntza-baldintzak eta baliabideak

<b>Harriak</b>			
Neurriak 300x76,2x30 mm	A46I8V	A60K7V	
Urratzaile frakzio bolumetrikoa, $V_g$	48 %	50 %	
Bataz besteko ale-neurria, $d_g$ (mm)	0,330	0,253	
Youngen modulua, $E_s$ (GPa)	32	43	
Poissonen modulua, $\nu_s$	0,2	0,2	
Dentsitatea ( $\text{kg/m}^3$ )	1854	2096	
<b>Diamantaketa-baldintzak</b>			
Larria (C)	$a_d = 20 \mu\text{m}$	$f_d = 0,8 \text{ mm/rev}$	$u_d = 1$
Ertaina (M)	$a_d = 20 \mu\text{m}$	$f_d = 0,4 \text{ mm/rev}$	$u_d = 2$
Fina (F)	$a_d = 20 \mu\text{m}$	$f_d = 0,2 \text{ mm/rev}$	$u_d = 4$
Orratz erako diamantea		$b_d = 0,8 \text{ mm}$	$\rho_d = 4,06 \text{ mm}$
<b>(a) Indar-neurketak</b>			
Dinamometroa: Kistler 9257B		500 Hz, indar-tartea $\pm 1000 \text{ N}$	
Lau neurketa baldintzako	$a = 10 - 20 \mu\text{m}$	$v_s = 24 \text{ m/s}$	$v_w = 0,5 \text{ m/s}$
<b>(b) Topografia</b>			
	Mikroskopia fokukidea		Leica DCM3D
<b>(c) Ale-dentsitatea, ale arteko distantzia eta kontaktu-luzera</b>			
Applied power source (APS)	Osziloskopia, Lecroy waveRunner 104Xi		
12 neurketa baldintzako	Mika lodiera, $s = 40 \mu\text{m}$		
Ebaketa-sakonera, $a = 5-10-20 \mu\text{m}$	Tentsioa 10 V		
Neurketa-zabalera, $b = 120 - 500 \mu\text{m}$	Maiztasuna, 250 MHz		

Erabili den artezketa-makinak biraketa-abiadura konstantean egiten du lan ( $N_s = 1500 \text{ rpm}$ ). Zenbakizko kontrolak gidatzen ditu makinaren mugimenduak. Ardatz nagusiak biraketa-abiadura konstantean biratzen du eta orga nagusia eragingailu hidraulikoz mugitzen da. Saiakuntza guztietan ebaketa- eta aitzinamendu-abiadurak ( $v_s$ ,  $v_w$ ) konstanteak dira, 24 m/s eta 0,5 m/s hurrenez hurren.

Beiratzatutako aluminazko bi harri, A46I8V eta A60K7V, erabili dira saiakuntzetan. Lehena, alumina zurizkoa da; bigarrena, berriz, alumina arrosazkoa. Harri bikote honek ale neurriaren eta harriaren gogortasunaren eraginen azterketa ahalbidetzen du. Pieza 34CrMo4 erreminta-altzairuzko bloke errektange-

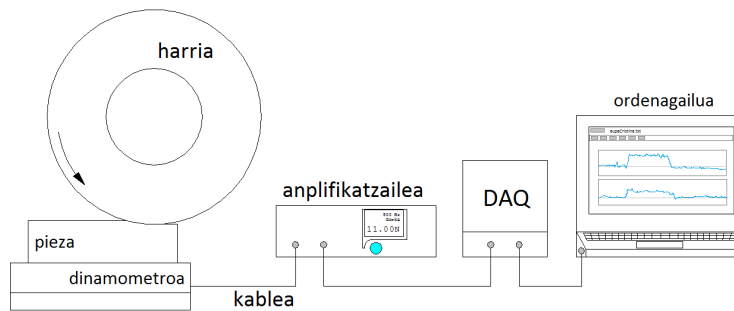


**4.1. Irudia:** (a) Diamantearen profilaren zeharkako neurketa; (b) mikroskopio fokukidearekin lortutako eremu-irudia, eta, (b) diamantearen goitik argazkia

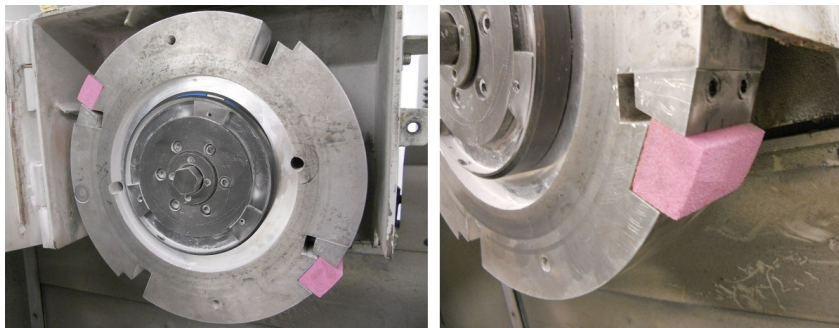
luarra da, 55 HRC gogortasuna du, eta, tenplatua eta irautua izan da.

Hiru diamantaketa-baldintzapean prestatu dira harriak. Diamante-hornitzailearen (*Winter*) gomendioak jarraitu dira diamantaketa-baldintzak aukeratzean:  $f_d$  diamantaketa-aitzinamenduarekin jokatu,  $a_d$  diamantaketa-sakonera mantendu da konstante ( $20 \mu\text{m}$ ). Aukeratutako diamantea, orratz erako diamante punta-bakar sintetiko kristal-anitza da. Diamante honen abantaila nagusia, behin egoera iraunkorrera iristean lan-bizitzan zehar profila mantentzen duela da. Hau da, higadurak apenas aldatzen du bere geometria. Kontra du, berriz, bere profil ia laua. Diamantearen profila lau aldiz neurtu da saiakuntzetan. Diamantearen puntaren batz besteko erradioa  $\rho_d = 4,06 \text{ mm}$  izan da,  $a_d$  baino bi maila handiagoa. Diamantearen zabalera  $b_d = 0,83 \text{ mm}$ -takoa da. 4.1.(a) Irudiak diamantearen profilaren neurketa erakusten du; 4.1.(b) Irudiak 3D mikroskopio fokukidearen behaketa, eta, 4.1.(c) Irudiak goitik hartutako diamantearen argazkia.

Artezketa-ingar experimentalak sarrera-datuak dira kontaktu-ereduan. Simulazioetan, indarra da erreferentzia balioa, ale-pieza interakzioen erresultanteak piezan aplikatutako indarrarekin bat egitean.  $a_e$  ebaketa-sakonera eraginkorra  $k_m$  makina-erreminta-pieza sistemaren zurruntasunarekin estimatu da (2.1 Ek.) [MRDI04]. Hainbat neurketa esperimentalek,  $k_m = 0,24 \text{ N}/\mu\text{m}$  batz besteko balioa eman dute frogatutako baldintzetan (ebaketa-zabalera  $b_w = 10 \text{ mm}$ ). Nahiz eta DEM ereduak zabalera ezberdina izan, indarrak neurtu diren baldintzetan aplikatu da  $a_e$  simulazioetan koherentzia mantentzeko.  $a_e$  sarrera-datua da ETL eredu; KE ereduak ez du erabiltzen, bai ordea RQ eredu kontaktuzko geometrikoa estimatzean. Artezketa-ingarrik bi  $a$  ezarritako ebaketa-sakonerekin neurtu dira ( $a = 10 - 20 \mu\text{m}$ ). Dagokion baldintzetan diamantatu ondoren, lau artezketa-iraganaldi eman dira, harriaren gainazalaren hasierako egoera iragankorra saihesteko. Indar-neurketa bakoitzaren ondoren, beste sei iraganaldi eman dira txinparta-itxaltzea gauzatzeko eta hurrengo indar-neurketa posizio fidagarri batetik abiatzeko. Guztira, lau indar-neurketa egin dira harri-motako eta diamantaketa-baldintzako. 4.2. Irudiak



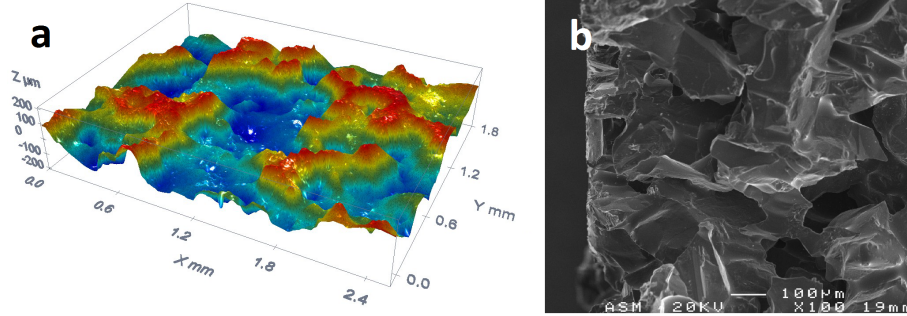
4.2. Irudia: Artezketa-indarren neurketa-katea

4.3. Irudia: Neurritan egindako ordezeko harria edo *dummy wheel* eta harriaren lagina bertan lotuta

indarren neurketa-katea erakusten du.

Harriaren gainazal topografia ere, mikroskopia optiko fokukidearekin neurtu da. Horretarako, 2.1.3. Atalean ikusitako ordezeko harria (*dummy wheel*) erabili da (ikus 2.11. Ir.), neurri txikiko diamantatutako harri laginak lortzeko. 20x20x30 mm duten harri-laginak 4.1. Taulan zehaztutako baldintzetan diamantatzen dira (4.3. Ir.). Gainazal topografiaren neurketa zuzenek, errepiken beharrik gabe, lortutako emaitzen fidagarritasuna ziurtatzen du. Hala ere, lehen artezketa-iraganaldietan gainazalak duen egoera iragankorra ez da eragozten, segurtasun arazoak direla eta, laginak ez batira artezten ipini.

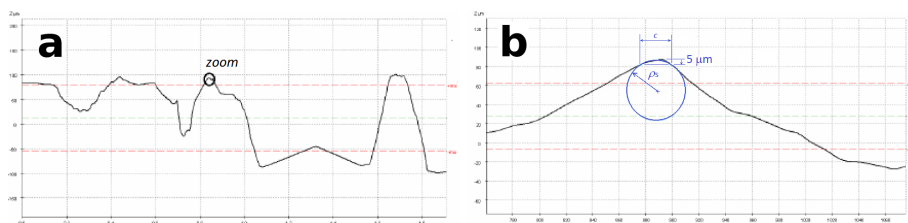
4.4.(a) Irudiak harriaren gainazalaren 3D topografia neurketa erakusten du. Tontorrak altuera berdineko planoan pilatuta daude 30  $\mu\text{m}$  tartean, diamantearen mozketa ekintza frogatzen duena. Diamanteak bidean topatzen duen urratzailea harrotzen du, eta aleak era hauskorrean hausten dira, batzuk askatu ere egiten direlarik. Hutsune urdinak, askatutako aleek utzitako lekuei dagokie. Ale-dentsitatea, ale urratzaileen hausturaren eta askatzearen konbinaketatik azaleratzen da. Ondorio horiek DEM ereduaren zehazkizko diamantaketa bidez modelatzen dira, Malkin-Chen ereduak konbinatuz ale-askatze, diamante-



**4.4. Irudia:** (a) Harriaren gainazal topografiaren 3D neurketa mikroskopio fokukidea erabiliz; (b) harriaren gainazaleko zeharkako bistaren SEM irudia (A60K8V, diamantaketa larria (C))

tearen ebaketa harikor helikoidala eta ale-haustura hauskorak konbinatuz (Section 3.4.). Diamanteak utzitako helizea ezin da antzeman 3D topografia neurketetan. 4.4.(b) Irudian, SEM bidez lortutako argazki ikusgarriak harriaren gainazala zeharka erakusten du. Irudian gainazala, literalki, moztuta agertzen da.

DEM erudian,  $\rho_s$  sorbatz-erradioak karakterizatzen du ale urratzaileen zorrotasuna. Brenerrek eta Torrancek [BT93] definitutako prozedura moldatuz egin dira  $\rho_s$ -ren neurketak. 3D topografia neurketak erabili dira aleen zorrotasuna zehazteko (4.4.(a) Ir.). 3D topografia neurketek, alde batetik, tontor irtenak identifikatzen ditu, eta beste aldetik, 2D profila edozein ebaketa-planotan eskuragarri ipintzen du. Horrela, 3D neurketetan ebaketa-planoa tontorren parean kokatuz, sorbatzaren 2D profilak ebaketaren zentzuan lortu dira, eta ez ausazko sekzioetan 2D neurketetan bezala. Sorbatz-erradioa definitzeko erreferentziako sakoneraren aukeraketa lausoa da literaturan: grafikoki sakoneraren funtzioan adierazten da, edota ez da zehazten. Aleek oso azalean egiten dute lan, horregatik tontorrekiko 5  $\mu\text{m}$  sakonera hartu da erreferentziatzat zirkunferentzia eraikitzeke. 4.4.(a) Irudiko 3D gainazaletik abiatuz, sorbatzen parean 2D profilak lortzen dira (4.5.(a) Ir.), eta horien eskala handituz,  $\rho_s$  zehazten da tontorretik 5  $\mu\text{m}$  sakoneran (4.5.(b) Ir.). Gutxienez 12 tontor aztertu dira diamantaketa-baldintzako eta harri mota<sup>o</sup>ko.  $\rho_s$  antzeko balioak lortu dira diamantaketa-baldintza eta harriarekiko loturarik gabe (29  $\mu\text{m}$  A46I8V harrian eta 27  $\mu\text{m}$  A60K7V harrian). Diamantaketa-baldintzek eragiketaren intentsitatea baldintzatzen dute, ale-askatzearekin lotuta duena. Ale-hausturak, berriz, diamante kamutsarekin azaleko sakoneretan antzeko baldintzetan gertatzen dira. Azkenak sortzen ditu sorbatzak eta, beraz, diamantaketa-baldintzak direnak direla, zentzuzkoa da antzeko zorrotasunak lortzea. Balio horiek bat egiten dute [KL75, BT93] lortutakoekin (16 - 35  $\mu\text{m}$ ), antzeko eran 2D profilak neurtzen lortuak izan direnak. [LS75], berriz,  $\rho_s = 75 - 500 \mu\text{m}$  arteko balioak lortzen ditu, baina piezan ale bakarraren frogek utzitako aztarnak neurtuz.



**4.5. Irudia:**  $\rho_s$  sorbatz-erradioaren neurketa 2D profila aztertuz: (a) tontorra zeharkatzen duen 2 mm luzerako 2D profila ebaketaren norabidean; (b) tontor baten zooma eta  $\rho_s$ -ren definizioa

Aplikaturako potentzial-iturria edo APS teknika  $G_{\text{dyn}}$  ale-dentsitatea,  $L_{\text{dyn}}$  ale arteko batz besteko distantzia eta  $l_c$  kontaktu-luzera neurtzeko erabili da, bere sinpletasunagatik eta zehaztasunagatik (ikus 2.2.2. Atala). 4.6. Irudiak erakusten du saiakuntzaren prestaketa. Mika-orri batez banatutako pieza zatituaren APS aldaera aukeratu da (2.15.(c) Ir.). Bi pieza lotzen dira orga nagusian. Lehenengoa, diamantaketaren efektu iragankorrak saihesteko, harriari sei artezketa-iraganaldi emateko erabiltzen da. Ondoren, harria prest dago saiakuntzetarako. Bigarren pieza, mika-orriak isolatutako bi zatitan banatuta dago; zati bakoitza elektrodo bati konektatua dago. Zhouen [ZFG85] teknika moldatu da, piezari  $b$  zabalera duen luzeratako irtengunea prestatuz. Irtengunea profil-proiektorean neurtzen da makinan bermatu aurretik. Ale bat irtengunearen gaitetik pasatzean, mikak banatutako bi pieza zatiak instant batez konektatu, zirkuitua itxi, eta osziloskopioan tentsio-pultsu bat sumatzen da. Ale guztiek osatutako seinalea aztertuz,  $t_c$  kontaktuaren iraupena zehazten da. 2.26 Ekua-zioarekin  $l_c$  kontaktu-luzera estimatzen da. Era berean,  $G_{\text{dyn}}$  ale-dentsitatea lortzen da,  $N_c$  tentsio-pultsu (edo kontaktu) kopurua kontaktu-azalera zehar (2.18 Ek.). Aldiz,  $L_{\text{dyn}}$  ale arteko distantzia kalkulatzeko,  $N_c$  kontaktu-azaleraekin erlazionatu beharrez, kontaktu-luzerarekin erlazionatzen du (2.17 Ek.), bizarxaffaren metodoa moldatuz [BS74].

Besteak beste, APS metodoa  $b$  neurketa-zabaleraren,  $s$  isolatzaile-lodieraren eta  $V$  tentsioaren menpe dago.  $b$ -k kontaktuaren eskuragarritasunean eragiten du,  $s$ -k kontaktuak intentsitatearen arabera diskriminatzen ditu, eta  $V$ -k seinalearen intentsitatea definitzen du. 2.2. Taulak biltzen ditu argitaratutako saiakuntzen datuak. CCS metodoak isolatzaile lodiera maximoa doitzen du [GW88] kontaktuak diskriminatu eta neurketaren egonkortasuna bermatzeko. Bestalde, potentzial iturria kenduta dagoenean, isolatutako piezaren bi zatiak arteztu ondoren polimetroarekin erresistentzia neurtzean, zirkuitua itxia dagoela antzemana da. Baldintza berdinetan potentzial-iturria aplikatuta arteztu ondoren berri, zirkuitua irekia dago. Horrek tentsioak aleek sortutako mikro-kontaktuetan barrena pasatzean erresistentziak sortutako beroak loturak deuseztatzen dituela adierazten du. Beraz, isolatzaile lodia ( $s = 40 \mu\text{m}$ ) eta tentsio handia ( $V = 10 \text{ V}$ ) konbinatuz lortu dira seinale argienak.  $b > 0,6 \text{ mm}$  denean, kontaktu-kopuru

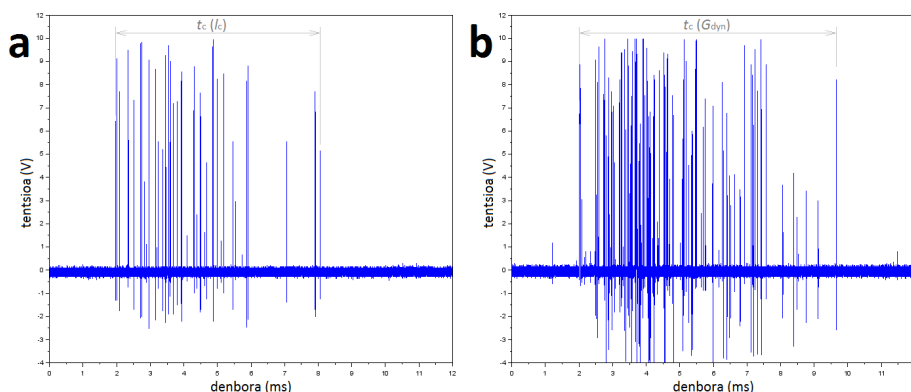


4.6. Irudia: APS saiakuntza prestaketa

handiak seinale jarraitua lortzen du, ezegonkortzera ere iristen delarik zirkuitua itxita geratuz. Teknikak azterketa sakonagoa behar du, bai aldagaiek seinalean duten eragina zehazteko, eta baita beste teknika batekin emaitzak alderatzeko ere.

APS neurketak bi irtengune-zabalerekin egin dira, kontaktu-luzera maximoak eta batz bestekoak lortuz (2.16.(a) Ir.). 4.7. Irudiak baldintza berdinetan neurtutako tentsio-denbora (V-ms) APS seinaleen bi adibide aurkezten ditu. DEM ereduaren emaitzak, batz besteko kontaktu-luzerarekin bat egiten dute, ereduaren zabalera estuagatik eta, batez ere, simulazioak kontaktuaren istant bat jasotzen duelako, ez APSk bezala, kontaktua  $t_c$  denbora tartean. Horregatik, zabalera estuko irtengunearekin ( $b = 0,12$  mm) egindako APS  $l_c$  neurketak kontaktu-eredua balioztatzeko erabili dira (4.7.(a) Ir.). Seinale berdina erabili da  $L_{dyn}$  ale arteko batz besteko distantzia estimatzeko. Breckerrek eta Shawek [BS74]  $L_{dyn}$  esperimentalki neurtzean, bizar-xaflaren zabalera neurketan duen eraginaz eztabaidatzen dute: zabalera (kasu honetan  $b$ ) sorbatzak sortutako urradurarena baino zabalagoa izan behar duela ondorioztatzen dute. Ale-neurria irtengunearen zabalera baino bi edo hiru aldiz handiagoa denez, ez dago teilakatze arazorik. Bestalde, irtengune zabala duen piezak ( $b = 0,5$  mm) azalera handiagoa estaltzen du, ale-dentsitatea definitzeko egokiagoa dena (4.7.(b) Ir.). 12 neurketa egin dira froga baldintzako. APS neurketak hiru ezarritako ebaketa-sakoneretan ( $a = 5 - 10 - 20$   $\mu\text{m}$ ) egin dira, eta lortutako datuekin  $l_c - a$  eta  $G_{dyn} - a$  erregresio-kurbak lortu dira. Makinaren ardatzaren deflexioa arbuia da, ebakera-zabalera hain estuetan sortzen diren indarrak eskasak baitira.





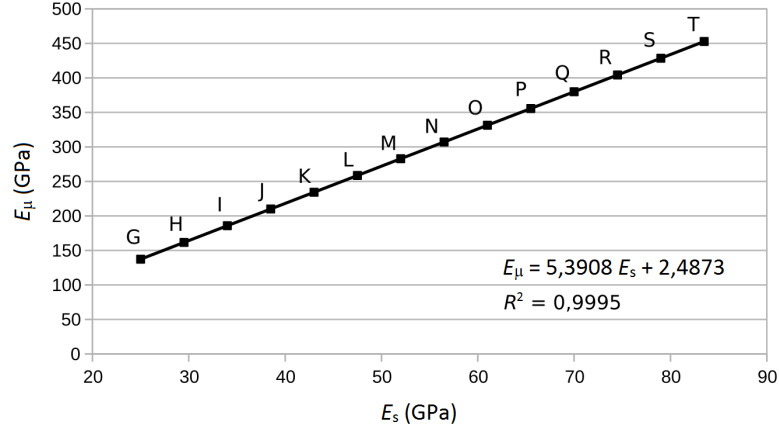
**4.7. Irudia:** APS seinale adibideak irtengunearen zabaleraren arabera: (a)  $b = 0,12$  mm eta (b)  $b = 0,5$  mm (A46I8V harria,  $a = 5 \mu\text{m}$ , diamantaketa fina (F))

## 4.2. DEM ereduaren prestaketa

Simulazioak abian jarri aurretik, haben propietate mekanikoak kalibratzen dira, ereduaren zurruntasuna harriarenari doitzeko. Froga-baldintza bakoitzerako, simulazioak lau DEM ereduakin errepikatzen dira. Ereduen eraikuntza eta gainazal topografia ausazkoak direnez, kasu bakoitzaren emaitza lau simulazioen batuz bestekoa da, errealitatean artezketa aztertzean bezala. Horrela, sarrera-datu berdinekin lau DEM eredu sortu dira harri-diamantaketa-baldintza konbinazio bakoitzarentzat 3.3. eta 3.4. Ataletan deskribatutako prozedurak jarraituz. Harriaren izendapena eta dimentsioak dira harrien DEM ereduak eraikitze-ko abiapuntua. Ondoren, gainazal topografia aplikatzen da ereduaren gainazalean diamantaketa-baldintzen, ale-dentsitatearen eta sorbatz-erradioaren arabera. Mugalde-baldintzak (3.5. Atala) simulazioa hasi aurretik ezartzen dira, bai KE, baita ETL aldaeretan. Hasieran, esperimentalki neurtutako indar normala piezan arrapala eran aplikatzen da, bere malda indar nominalaren 1/1000 delarik. Behin indar nominalera iritsita, indarra konstante mantentzen da. Arrapalak egonkortasuna bermatzen du simulazioaren hasieran, harriak eta piezak kontaktu hartzean. Simulazioan zehar, aldagai nagusiak monitorizatzen eta gordetzen dira, ondoren ereduaren portaera aztertu ahal izateko.

### 4.2.1. Haben propietateen kalibrazioa

Haben propietate mekanikoen kalibrazioa, harrien gradu guztientzat egin da 3.2.1. Atalean deskribatutako prozedura jarraituz. Kalibrazioaren emaitzak baliagarriak dira ale-neurri eta egitura zenbakientzat, erabilitako prozedurak propietateak homogeneizatzen baititu. Haben Poissonen modula  $\nu_\mu = 0,2$  finkatuz (harriarena),  $r_\mu = 0,538$  diametroaren murrizpen-faktorea lortu da harriaren  $\nu_s = 0,2$  doitzeko [KF87]. Kalibrazioaren bigarren pausua hainbat harri



**4.8. Irudia:** Habeen propietate mekanikoen kalibrazioa:  $E_s$  harriaren eta  $E_\mu$  habeen elastikotasun moduluaren, eta graduaren arteko erlazioa ( $\nu_\mu = 0,2$ ,  $r_\mu = 0,538$ )

gradutan aplikatuz, 4.8. Irudiak erregresio-kurba lortu da, makro- eta mikro-eskalako elastikotasun moduluak ( $E_s$ ,  $E_\mu$ ) erlazionatzen dituenak. Lortutako kurbak zuzenaren funtzioarekin bat egiten 0,9995  $R^2$ -arekin, beraz,  $E_s$ - $E_\mu$  erlazioa linealtzat har daiteke. Gradua eta elastikotasun moduluaren arteko korrelazioa Hernaniko Unesa harri-ekoizleari sor zaio.

Saiakuntzetan erabilitako harrien kalibrazio-parametroei dagokienez, biek dituzte Poissonen modulu ( $\nu_\mu = 0,2$ ) eta murrizpen-faktore ( $r_\mu = 0,538$ ) komunak. A46I8V harria 34 GPa-tako  $E_s$ -ari, habeetan  $E_\mu = 185,5$  GPa dagokio. A60K7V harriak, berriz,  $E_s = 43$  GPa elastikotasunari habeetan  $E_\mu = 234,3$  GPa dagokio.

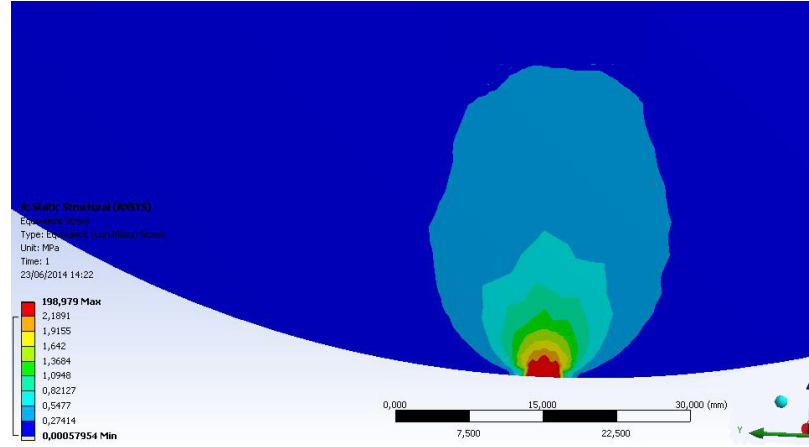
#### 4.2.2. Sarrera-datuak

4.2. Taulak biltzen ditu harriaren DEM ereduaren erakitze beharrezkoak diren sarrera-datuak eta simulazio-parametroak. Haien balioak lan esperimental bidez (artezketa-indarrak, ale-dentsitatea), simulazio bidez (habeen propietateen kalibrazioa) eta adierazpen analitikoekin (ebaki-gabeko txirbil-lodiera) lortu dira.

Harriaren DEM ereduaren erakitze dimentsioak, morfologia granularra eta propietate mekanikoak ezagutu behar dira.  $d_s$  diametroa,  $b_s$  zabalera eta  $h_s$  altuera dira DEM ereduaren neurriak (3.12. Ir.).  $d_s$ -k harriaren diametroarekin bat egiten du. Hala ere, ereduaren sortzean zenbakizko sinterizazioaren ondorioz pareta elastikoetan DEak pilatzen direnez (3.16. Ir.), ereduaren erakitzean diametroa zerbait handiagotzen da, hiru aldiz ale-neurria ( $d_s + 3d_g$ ).  $d_s$  izendatua topografia aplikatzean lortzen da.

4.2. Taula: DEM ereduaren sarrera-datuak

<b>Harriak</b>	A46I8V		A60K7V			
Ereduaren neurriak, $d_s, b_s, h_s$ (mm)	300 x 2,24 x 5,1		300 x 1,7 x 5,1			
Urratzaile frakzio bolumetrikoa, $V_g$	48%		50%			
DE diametroa, $d_{DE}$ (mm)	0,330		0,253			
DE murrizpen-faktorea, $r$	0,921		0,944			
Poissonen modulua, $\nu_s$	0,2		0,2			
Habe murrizpen-faktorea, $r_\mu$	0,538		0,538			
Habe elastikotasun modulua, $E_\mu$ (GPa)	185,5		234,3			
Dentsitatea (kg/m <sup>3</sup> )	1854		2096			
DE kopurua	11000		19100			
<b>Diamantaketa-baldintzak</b>	$a_d = 20 \mu\text{m}$	$b_d = 0,8 \text{ mm}$	$\rho_d = 4,06 \text{ mm}$			
Larria (C)	$h_f = 20 \mu\text{m}$	$f_d = 0,8 \frac{\text{mm}}{\text{rev}}$	$u_d = 1$			
Ertaina (M)	$h_f = 10 \mu\text{m}$	$f_d = 0,4 \frac{\text{mm}}{\text{rev}}$	$u_d = 2$			
Fina (F)	$h_f = 5 \mu\text{m}$	$f_d = 0,2 \frac{\text{mm}}{\text{rev}}$	$u_d = 4$			
<b>Sorbatz zorroztasuna</b>	A46I8V		A60K7V			
Sorbatz-erradioa, $\rho_s$ ( $\mu\text{m}$ )	29		27			
Desbiderapen estandarra, $\sigma_p$ ( $\mu\text{m}$ )	4,6		3,8			
	<b>F</b>	<b>M</b>	<b>C</b>	<b>F</b>	<b>M</b>	<b>C</b>
Ale askatze neurria, $d_b$ ( $\mu\text{m}$ )	233	243	259	178	188	199
<b>Txirbil-lodiera max.</b> , $h_{cu,max}$ ( $\mu\text{m}$ )						
$a = 10 \mu\text{m}$	0,34	0,39	0,51	0,31	0,34	0,4
$a = 20 \mu\text{m}$	0,52	0,58	0,7	0,48	0,48	0,54
<b>Indarrak</b>	$a = 10 \mu\text{m}$					
Indar-arrazoia, $\mu$	0,53	0,55	0,49	0,49	0,5	0,49
Indar normal espezifikoa, $F'_n$ (N/mm)	11,2	9,3	4,2	19,3	16,4	10,5
	$a = 20 \mu\text{m}$					
Indar-arrazoia, $\mu$	0,49	0,45	0,44	0,49	0,45	0,47
Indar normal espezifikoa, $F'_n$ (N/mm)	7,8	6,9	5,0	15,8	14,4	13,3
<b>Piezaren propietate biskoplastikoak</b>						
34CrMo4 erreminta-altzairua	Gogortasuna 55 HRc		$\sigma_y = 1815 \text{ MPa}$			
	$E_w = 210 \text{ GPa}$		$\nu_w = 0,3$			
JC koefizienteak [JC83]	$A = 1815 \text{ MPa}$	$B = 477 \text{ MPa}$	$C = 0,012$			
	$n = 0,18$		$m = 1,0$			
Deformazio eta tenperatura parametroak	$\varepsilon = 3,1$		$\dot{\varepsilon} = 3,6 \cdot 10^6 \text{ s}^{-1}$			
	$T_o = 300 \text{ K}$	$T_m = 1764 \text{ K}$	$T = 950 \text{ K}$			



**4.9. Irudia:** Ereduaren  $h_s$  altuera zehazteko harriaren eta piezaren arteko kontaktuaren azterketa FEM bidez

Ereduaren  $b_s$  zabalerari dagokionez, harriaren tamainaren eta kontaktuan eskuragarri dauden DE kopuruaren arteko konpromisoa da. Gutxienez lau ale ilara modelatzen dira. Baina ale arteko distantzia  $d_g$  ale-neurria baino handiagoa denez,  $b_s$  zabalera 6 - 7 aldiz  $d_g$  izatea erabaki da. 4.2. Taulak erakusten du  $b_s$  2,24 mm dela A46I8V harriarentzat eta 1,7 mm A60K7V-rentzat. Biek mantentzen dute 6,8 zabalera/DE diametro erlazioa ( $b_s/d_g$ ). Ereduaren DE kopuru onargarri maximoak baldintzatu du erlazioa: erlazio txikiagoak, eredu tamaina txikiagotzen du eta, beraz, konputazioa azkartu; bestalde, erlazio handiagoak azalera gehiago estaltzen du eta hobeto jasotzen ditu topografiaren nabardurak. Ale-neurriak eta -dentsitateak bi ereduaren zabalera ezberdintzen du, baina ( $b_s/d_g$ ) erlazio berdina erabiltzeak bien arteko emaitzak erkigarriak egiten ditu.

Azkenik,  $h_s$  ereduaren altuera FEM bidez definitzen da (3.3.2. Atala). Esperimentalki behatu den indar maximoa aplikatu zaio  $d_s$  harriaren diametroa duen zilindrori, Von Mises tentsioa maximoarekiko 1/100 da  $h_s = 5,1$  mm distantziaz aurrera. 4.9. Irudiak  $h_s$  definitzeko FEM simulazio-emaitzak aurkezten ditu. Kolore-eskala moldatu da tentsio-eremuak nabarmentzeko. FEM ereduak kontaktu Hertziarra simulatzen duenez (elastikoa eta leuna), kontaktu-azalera txikia da eta, ondorioz, kontaktu-eremuan tentsio-kontzentrazioa handia da. Efektu horrek  $h_s$ -n zehaztapenean segurtasun-tartea handitzen du.

DEM ereduaren morfologia granularra  $d_{DE}$  ( $= d_g$ ) batz besteko DE diametroak eta  $V_g$  urratzaile frakzio bolumetrikoak definitzen dute. 3.18, 3.20 eta 3.23 Ekuazioek  $d_g$ ,  $V_g$  eta ereduak izan behar duen  $N_{DE}$  DE kopuruak zehazten dituzte hurrenez hurren (4.2. Taula). Horrela, ereduaren dimentsioekin eta parametro morfologikoekin DEM ereduera eraikitzen da.

Ondoren, habeak sortu eta dagokien propietate mekanikoak aplikatzen zaizkie ( $E_\mu, \nu_\mu, r_\mu$ ). Habea sortzen da kontaktuan dauden DE-en artean, DE diame-

troa murriztu aurretik. Behar baino habe diametro handiagoa uzten da ( $r_\mu/r$ ), ondoren DE diametro murrizpenak konpentsatzeko. Harri gradu guztientzako kalibrazio-parametroak 4.2.1. Atalean aurkeztu dira.

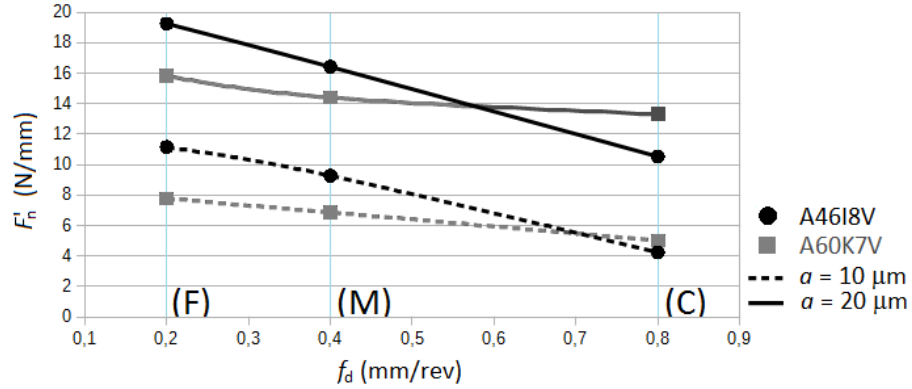
Behin DEM ereduera eraikita, zenbakizko diamantaketa aplikatzeko prest dago. Gainazal topografia aurre-prozesaketan aplikatzen da, 3.4. Atalean deskribatutako prozedura jarraituz. Diamantearen ebaketa harikorra (helizea)  $f_d$  diamantaketa-aitzinamenduaren eta  $\rho_d$  diamantearen erradioaren menpe dago. Diamantaketa-sakonera berdina da hiru diamantaketa-baldintzetan ( $a_d = 20 \mu\text{m}$ ).  $d_b$  askatutako partikulen batz besteko neurria, esperimentalki neurtutako ale-dentsitatea  $a = 20 \mu\text{m}$  sakoneran (4.2. Taula, 2.12 eta 2.13 Ek.).  $h_f$  haustura osagaiaren anplitudea 2.28 Ekuazioarekin definitzen da. Behin sorbatzen sakonera ausaz definituta,  $\rho_s$  sorbatz-erradioa ere aplikatzen zaio bakoitzari.  $\rho_s$ -k  $\sigma_\rho$  desbiderapen estandarra duen banaketa normala jarraitzen du.

$h_{\text{cu,max}}$  ebaki-gabeko txirbil-lodiera maximoa 2.5 Ekuazioarekin kalkulaten da. 2.17 Ekuazioak, berriz, aurreko ekuazioan sarrera-datu den  $L_{\text{dyn}}$  segidako bi aleen arteko batz besteko distantzia estimatzen du APS neurketak aztertuz.  $h_{\text{cu}}$  estimatzean  $L_{\text{dyn}}$ -en oinarritutako adierazpena  $G_{\text{dyn}}$ -etan oinarritutako ordez erabiltzeak,  $h_{\text{cu,max}}$  balio txikiak estimatzen ditu.

4.2. Taulak esperimentalki neurtutako luzera unitateko  $F'_n$  indar normalen batz besteko balioak eta  $\mu$  indar-arrazoiak biltzen ditu. 4.10. Irudiak, berriz,  $F'_n$ -k aurkezten ditu diamantaketa-aitzinamenduaren eta ebaketa-sakonerearen funtzioan. Grafikoak, diamantaketa finak (F) indar handiagoak sortzen dituela erakusten du. Efektu hori gainazal topografia itxiagoa duelako gertatzen da. Bestalde, diamantaketa finak zimurtasun balio hobeak lortzen ditu.  $\mu$ -ri dagokionez, balioak zerbait handiagoak dira A46I8V harriarentzat (0,49 - 0,55), A60K7V harriarentzat baino (0,44 - 0,5). DEM ereduaren, esperimentalki neurtutako  $F_n$  piezan aplikatzen da, eta simulazioaren konbergentzia maila aztertzeke erreferentzia balioa da. Indar zentrifugoa  $f_z^i$  DE bakoitzean aplikatzen da posizio erradialaren eta biraketa-abiaduraren  $N_s$  funtzioan (3.38 Ek.). Kanpoaldeko DE batentzat  $v_s = 24 \text{ m/s}$  abiaduran,  $f_z$  0,0021 N dira 46 ale-neurriarekin, eta 0,0011 N 60 neurriarekin. Ale-kopuru andanatan eragindako  $\Sigma f_z$  indarren erresultanteak  $f_z$ -ren efektua azaleratzen du.  $v_s$  bikoiztuz gero,  $f_z$  laukoizten da.

Piezaren propietate mekanikoak zehaztean, KE eta ETL ereduak materialaren portaera biskoplastikotzat jotzen dute (2.4.3. Atala). 3.5. Atalean azaldu da, beraien balioa konstantetzat hartuko dela kontaktu-arkuan zehar.  $\sigma_y$  isurpententsio eraginkorraren hurbilpen zabarra egin da JC eredu eratzaillearekin (2.35 Ek.). Pieza 34CrMo4 erreminta-altzairuzkoa da, tenplatua eta irautua. JC ereduak  $\sigma_y$  estimatzen du  $\varepsilon_p$  deformazio plastikoaren,  $\dot{\varepsilon}_p$  eta  $\dot{\varepsilon}_o$  uneko eta erreferentziako deformazio-abiaduraren, eta,  $T$ ,  $T_o$  eta  $T_m$  uneko, giro eta fusio tenperaturen funtzioan. Johnsonek eta Cookek [JC83] ereduaren aurkeztean hainbat materialen konstanteak eskaintzen dituzte. Horien artean S5 erreminta-altzairua dago, 34CrMo4-ren antzeko konposizio eta ezaugarriak dituena. S5 altzairuaren material-konstanteak erabili dira 34CrMo4-ren portaera deskribatzeko.

Material harroketa teoriaren berrikusketak, besteak beste deformazio eta deformazio-abiadura baldintzak deskribatzen ditu ebaketan (2.4.3. Atala). Ale

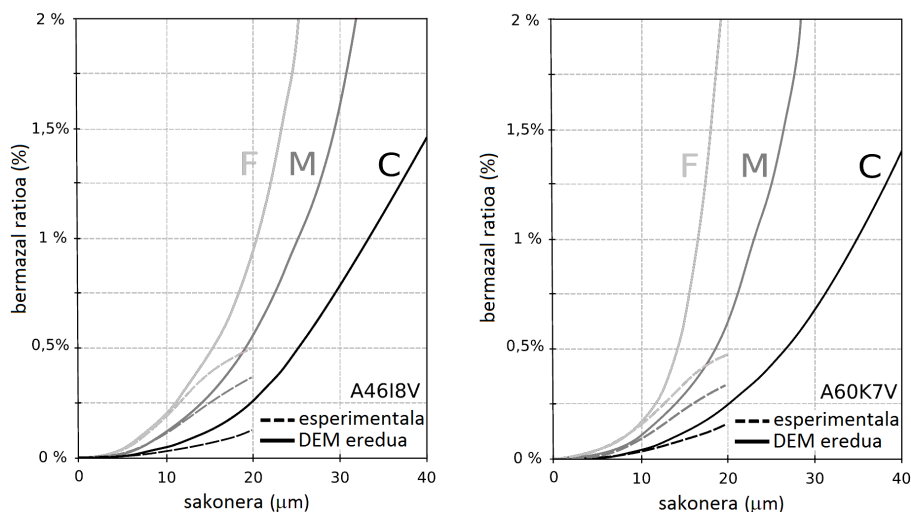


**4.10. Irudia:** Artezketan indar normal espezifikoak harri mota, ebaketa-sakonera eta diamantaketa-aitzinamenduarekiko; (F), (M) eta (C) diamantaketa-baldintzei dagokie

urratzaile baten sorbatzak  $h_{cu} = 4 \mu\text{m}$  ebaketa-sakonera [KL75]  $v_s = 24 \text{ m/s}$  ebaketa-abiaduran,  $\mu = 0,5$  indar-arrazoiarekin eta  $\alpha = -60^\circ$  jaulkitze-angulu negatiboarekin [Sha96], Ernsts-Merchant adierazpenak (2.36 Ek.)  $\theta = 13,3^\circ$  ebakidura-angelua estimatzen du.  $C_1 = 10,6$  bada [OY89], eta, 2.38 eta 2.39 Ekuazioak aplikatuz,  $\varepsilon_p = 3,1$  deformazio plastikoa eta  $\dot{\varepsilon}_p = 3,6 \cdot 10^6 \text{ s}^{-1}$  deformazio-abiadura lortzen dira. Balio hauek baldintza ia-balistikoei dagokie.

34CrMo4 altzairuaren fusio-tenperatura  $T_m = 1764 \text{ K}$  da eta  $T_o = 300 \text{ K}$  hartu da giro-tenperaturatzat. Ebaketa-eremuan  $T = 950 \text{ K}$  tenperatura konstantea aukeratu da. Zenbakizko simulazioek 800 K eta 900 K arteko tenperaturak estimatzen dituzte ebakidura-planoan [OO05], eta, altzairuen fase-aldaketa puntuan erresistentzia erorketa dago [TFX<sup>+</sup>15]. Tenperatura tartea literaturan argitaratutako termopare polobakarrarekin egindako neurketa esperimentalen emaitzek baieztatzen dute [LVLL06, DKD<sup>+</sup>10, PSO<sup>+</sup>12, RKM15]. 2.43, 2.44 eta 2.42 Ekuazioek kontaktu-azalera analitikoki estimatzen dute tenperatura maximoa, bero-iturri mugikorra triangeluartzat hartuz. Honako aldagaiak erabiliz ( $q_o = 90 \cdot 10^6 \text{ W/m}^2$ ,  $R_w = 0,9$  (hozgarrigabe),  $k = 23,5 \text{ W/m K}$ ,  $\rho = 7860 \text{ kg/m}^3$ ,  $c = 515 \text{ J/kg K}$ ,  $C_1 = 1,06$ ,  $l_c = 0,004 \text{ m}$ ), 2.43 Ekuazioak 1060 K tenperatura maximoa estimatzen du, 950 K tenperatura konstantearen gainetik.

Azkenik, material-konstanteak, deformazio-baldintzak eta tenperaturak JC ereduan ordezkatuz (2.35 Ek.),  $\sigma_y = 810 \text{ MPa}$  isurpen-tentsio eraginkorra lortzen da.  $\sigma_y$  balio horrek bat egiten du [OO05] argitaratutako balioekin. FEM eredu horrek 1000 MPa  $\sigma_y$  lortzen du  $a_e = 1 \mu\text{m}$  ebaketa-sakonera, 700 - 900 MPa  $a_e = 4,6 \mu\text{m}$  sakonera eta 450 - 1000 MPa  $a_e = 100 \mu\text{m}$  sakonera ( $\alpha = -45^\circ$ ,  $v_s = 30 \text{ m/s}$ ).  $\sigma_y$  konstantetzat jotzeak, azaleko sakoneretan indar txikiagoak sortzen ditu. Edozein kasutan, eremu horretan indarrak txikiagoak direnez, oro har eragindako akatsa onargarria da.



**4.11. Irudia:** Bermazal-kurba esperimentalak eta DEM eredian sortutakoak, harriaren (A46I8V, A60K7V) eta diamantaketa-baldintzen arabera (F, M, C)

### 4.3. Simulazioak eta saiakuntzak

Atal honek lan esperimentaleko eta simulazioetako emaitzak biltzen eta alderatzen ditu.

Sortutako topografiaren hurbilpen egokia beharrezkoa da artezketan kontaktua simulatzean. DEM ereduaren gainazal topografia aurre-prozesaketan sortzen da. 4.11. Irudiak esperimentalki behatutako bermazal-kurbak (lerro jarraituak), DEM eredian sortutako bermazal-kurbekin alderatzen ditu (lerro ez-jarraituak). Bigarrenak, DEM ereduak sortutako bermazal-kurbak gainazal sorbatzak soilik hartzen ditu kontuan. Bi kurben gainjarpenak nahiko ongi bat egiten dute 10 μm sakonera arte; hortik aurrera, behatutako bermazal-ratioa nabarmenki igotzen da. Efektu hori, Chenen eredia era diskretuan aplikatzearen ondorioa da, aleko sorbatz bakarrean aplikatzen baita. Hala ere, sorbatzen sarketa maximoa piezan 10 μm inguruan dabilenez, kontaktua modelatzeko Chenen ereduaren aplikazioa era diskretuan hurbilpen zuzena da. Diamantaketa larriak (C) bermazal-ratio txikiagoak lortzen ditu finak (F) baino, artezketaren intentsitatean eta banaketan eragina duenak.

Simulazio batek bi eta sei egun artean irauten du ereduaren tamainaren arabera Intel Xeon E5-2680 @2,80GHz prozesagailuan. Iterazio bakoitzean, testu fitxategian gordetzen dira  $N_c$  kontaktuan dauden gainazaleko DE kopurua,  $\delta_{max}$  sarketa maximoa,  $l_c$  kontaktu-luzera eta piezak jasotzen duen indar normalaren erresultantea. Simulazioan zehar, grafikoak sarri marrazten dira konbergentzia maila aztertzeko. Egoera egonkorrera iritsi bada, simulazioa gerarazten da. Indar normal erresultantea erreferentziatuz hartuz, emaitzaren iterazio zehatza

aurkitzen da, eta bertako ezaugarrien balioak izango dira simulazioaren emaitzak.

4.12. Irudiak simulazioetan sentsoreak lortzen dituen grafikoan adibide bana erakusten du; (a) KE eredia eta (b) ETL eredia. Goiko-ezkerreko grafikoak  $N_c$ -ri dagokie; ETL ereduak ale-dentsitatea estimatzeko erabiltzen du (2.18 Ek.). Goiko-eskuineko grafikoek  $\delta_{\max}$ -n garapena erakusten du. Gainazalean eskuragarri dauden sorbatzekin batera,  $\delta_{\max}$ -k kontaktuaren intentsitatearen ideia ematen du. Beheko-ezkerreko grafikoek  $l_c$  adierazten dute: lerroek ereduaren alde bakoitzean kontaktuan dagoen DE-en posizioa adierazten dute eta, beraz, kontaktu-luzera bien arteko distantzia da (ikus baita ere 4.14. Ir.). KE eredian, distantzia hori simetrikoa da harriaren ardatz bertikalarekiko (4.12.(a) Ir.), eta asimetrikoa, berriz, ETL eredian (4.12.(b) Ir.). ETL ereduak  $a_e$  ebaketa-sakonera kontuan hartzen duenez, materiala dagoen aldean kontaktua luzeagoa da laua dagoen aldean baino. Azkenik, beheko-eskuineko grafikoek kontaktuan dauden aleen  $\Sigma f_n$  indar normalaren erresultantea monitorizatzen dute. Erresultantearen balioak iterazioz iterazio piezatan aplikatua dagoen indarraren balio nominalerantz jotzen du, harriaren portaera egonkortuz. Horregatik,  $F_n$  da erreferentzia balioa simulazioan. Oro har, KE ereduak azkarrago lortzen du konbergentzia ETL ereduak baino. ETL eredian  $A-B$  zatian (ikus 3.20. Ir.) DE bat kontaktuan sartzean, zero baino handiagoa den txirbil-lodierarekin hasten baita kontaktua.

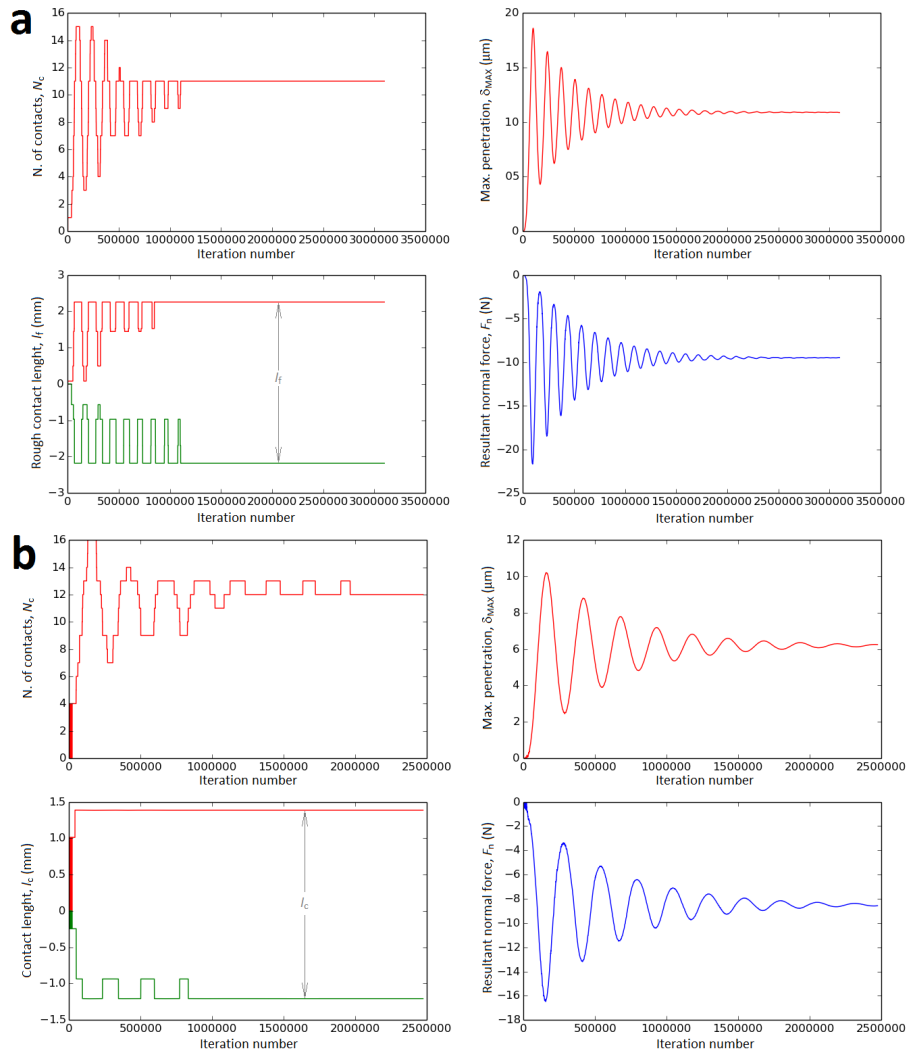
4.13. Irudiak lortutako kontaktu-luzeren saiakuntzetako (grisean) eta simulazioetako (puntuak) desbiderapen estandarrak era grafikoan aurkezten ditu, harri eta diamantaketa-baldintza bakoitzarentzat. Lehenik eta behin, aipagarria da bietan agertzen den dispersioa, bai emaitza esperimentaletan, baita simulazioetan ere. DEM ereduak artezketaren ausazko natura urratu du eta, ondorioz, emaitza guztiak batz besteko balioaren eta desbiderapen estandarraren bidez adierazten dira. Oro har, simulazioetan dispersioa estuagoa da saiakuntzetan behatutakoa baino. A60K7V harriak dispersio txikiago erakusten du, ziur aski ale-neurria txikiagoa delako.

Kontaktu-luzera esperimental-geometrikoa arrazoia ( $l_c/l_g$ ) 1,85 eta 2,52 artekoa da A46I8V harrian, eta, 1,92 eta 2,2 artekoa A60K7V harrian; balio hauek bat datoz literaturan argitaratutakoekin [MRDI04]. 4.3. Taulak,  $l_c$  emaitza berdinak biltzeaz gain,  $\delta_{\max}$  sarketa maximoak,  $R_r$  zimurtasun-faktoreak (KE), eta  $G_{\text{dyn}}$  ale-dentsitateak (ETL) ere biltzen ditu.

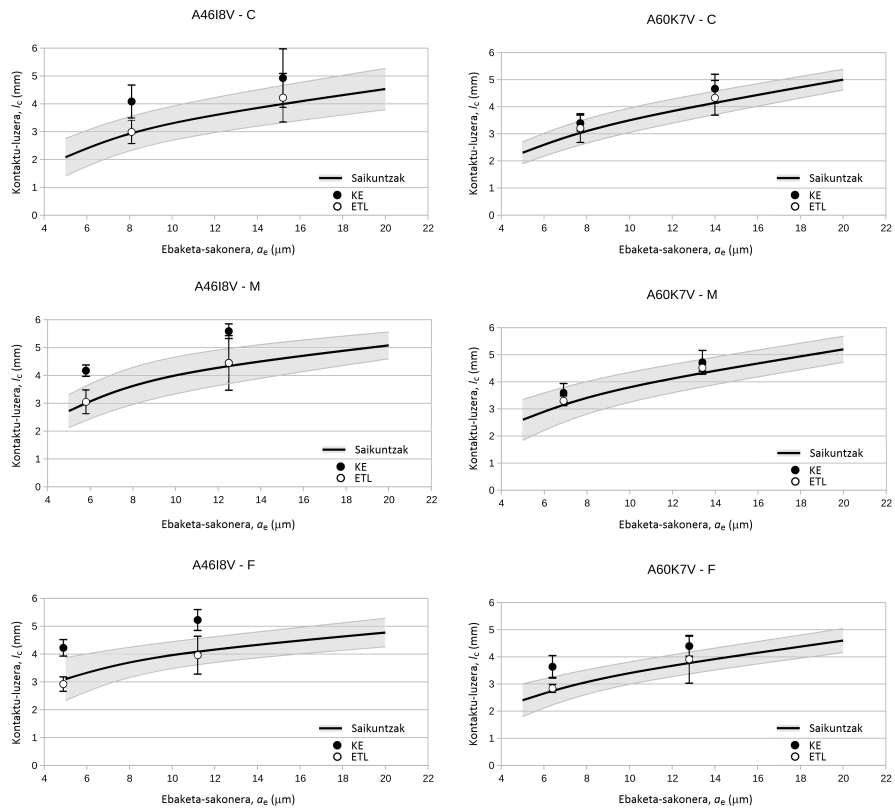
KE ereduak lortutako  $l_f$  kontaktu-luzera zimurretik abiatuz, RQ eredia aplikatuz  $l_c$  kalkulatu da. KE-RQ ereduaren konbinaketarekin lortutako  $l_c$ -k esperimentalki behatutakoak baino luzeagoak izan dira. Hala ere, RQ ereduaren emaitzek  $l_c$  erregresioen joerak jarraitzen dituzte.  $R_r$  zimurtasun-faktore balioak baxuak izan dira saiakuntzetan eta simulazioetan. Zenbakizko  $R_r$  balioak esperimentalak baino handiagoak izan dira, batez ere A46I8V harrian (< % 56).

$R_r$  faktoreetan topografia baldintza berdinetan ere aldeak daude. Ziur aski emaitzek ez dute bat egiten, KE ereduko kontaktu-baldintzak ezberdinak direlako (elastikoa beharrez, kontaktu zurrun-plastiko-perfektua eta materialaren portaera biskoplastikoa). Nolanahi ere, kontaktua elastikotzat hartuz hasieran egindako simulazioetan  $l_f$  oso motzak lortu dira ( $l_f < 1$  mm). Agian KE-RQ





**4.12. Irudia:** Kontaktu-simulazioan aldagaien monitorizazio-grafikoen adibideak: (a) KE eredu eta (b) ETL eredu (A46I8V,  $a = 10 \mu\text{m}$ ,  $C$  diamantaketa). Bietan, goi-ekker grafikoan  $N_c$  kontaktuan dagoen DE kopurua, goi-eskuin grafikoan  $\delta_{\max}$  ( $\mu\text{m}$ ) sarketa maximoa, behe-ekker grafikoan  $l_c$  (mm) kontaktu-luzera eta behe-eskuin grafikoan kontaktuan dauden DE-en  $\Sigma f_n$  (N) indar normal erresultantea; grafiko guztietan ardatz horizontalak iterazio zenbakia (edo denbora) adierazten du



**4.13. Irudia:** Kontaktu-luzera neurketa esperimentalak eta simulazioetako emaitzak, harriaren (A46I8V ezkerrean, A60K7V eskuinean) eta diamantaketa-baldintzen (C, M, F) arabera

**4.3. Taula:**  $l_c$  kontaktu-luzera,  $\delta_{\max}$  sarketa maximo,  $R_r$  zimurtasun-faktore eta  $G_{\text{dyn}}$  ale-dentsitate emaitza esperimentalak eta zenbakizkoak

diamant.-a	Emaitza esperimentalak						KE - RQ						ETL					
	$F_N^1$ ( $\frac{N}{mm}$ )	$\mu$	$l_c$ (mm)	$l_g$ (mm)	$R_r$	$G_{\text{dyn}}$ ( $\frac{\text{ale}}{\text{mm}^2}$ )	$l_f$ (mm)	$l_c$ (mm)	$e\%$	$\delta_{\max}$ ( $\mu\text{m}$ )	$R_r$	$e\%$	$l_c$ (mm)	$e\%$	$\delta_{\max}$ ( $\mu\text{m}$ )	$N_c$	$G_{\text{dyn}}$ ( $\frac{\text{ale}}{\text{mm}^2}$ )	$e\%$
C-10	4,2	0,49	2,88	1,56	4,6	1,28	3,77	4,08	42%	8,88	7,2	56%	2,75	-4,6%	6,35	9,8	1,35	5,2%
C-20	10,5	0,49	4,14	2,14	4,3	1,58	4,44	4,93	19%	16,44	5,4	25%	4,09	-1,1%	7,01	18,2	1,68	6,2%
M-10	9,3	0,55	2,96	1,32	3,4	2,05	3,96	4,17	41%	11,39	5,1	49%	3,13	5,8%	4,78	18,4	2,22	8,6%
M-20	16,4	0,5	4,45	1,94	3,9	2,37	5,24	5,59	26%	16,23	5,1	31%	4,24	-4,7%	6,87	23,4	2,1	-11%
F-10	11,2	0,53	3,08	1,22	3,3	2,62	4,04	4,22	37%	9,15	4,8	43%	2,98	-3,2%	4,28	22,6	2,87	9,5%
F-20	19,3	0,49	4,12	1,84	3,3	2,93	4,89	5,22	27%	11,54	4,4	33%	4,06	-1,4%	6,01	30,4	2,83	-3,5%

diamant.-a	Emaitza esperimentalak						KE - RQ						ETL					
	$F_N^1$ ( $\frac{N}{mm}$ )	$\mu$	$l_c$ (mm)	$l_g$ (mm)	$R_r$	$G_{\text{dyn}}$ ( $\frac{\text{ale}}{\text{mm}^2}$ )	$l_f$ (mm)	$l_c$ (mm)	$e\%$	$\delta_{\max}$ ( $\mu\text{m}$ )	$R_r$	$e\%$	$l_c$ (mm)	$e\%$	$\delta_{\max}$ ( $\mu\text{m}$ )	$N_c$	$G_{\text{dyn}}$ ( $\frac{\text{ale}}{\text{mm}^2}$ )	$e\%$
C-10	5,0	0,44	2,99	1,52	5	1,83	2,91	3,39	13%	10,3	5,7	13%	2,81	-6,1%	6,5	11,5	1,86	1,5%
C-20	13,3	0,47	4,24	2,05	4,4	2,38	4,2	4,66	10%	15,8	5	13%	4,08	-3,7%	12,7	20,2	2,26	-5,6%
M-10	6,9	0,45	3,1	1,44	4,6	2,17	3,17	3,6	16%	8,3	5,3	15%	3,27	5,4%	5,6	15,7	2,19	1,1%
M-20	14,4	0,5	4,43	2,01	4,5	3	4,27	4,72	6%	13,9	4,9	8%	4,22	-4,7%	7,2	25,7	2,77	-7,6%
F-10	7,8	0,49	2,71	1,39	3,6	2,83	3,23	3,64	34%	7,1	5,1	38%	2,84	4,6%	3,6	19,5	3,13	10%
F-20	15,8	0,49	3,84	1,96	3,6	3,23	3,93	4,4	14%	11,5	4,3	19%	4,07	5,9%	6,5	28,5	3,19	-1,4%

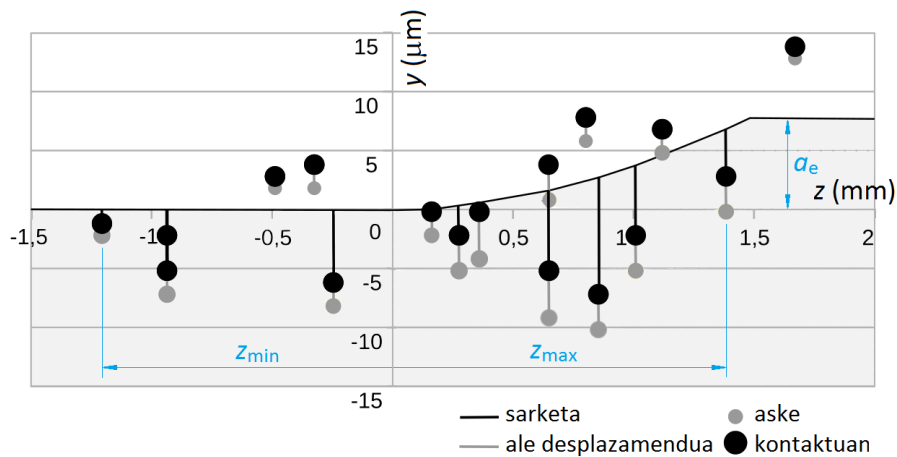
konbinaketa  $l_c$  eta  $R_r$  estimatzeko erreminta baliagarria izango litzateke,  $\sigma_y$  isurpen-tentsio eraginkorraren balioa emaitza esperimentaletara doitzuz gero.

ETL ereduaren  $l_c$  emaitzak saiakuntzetan lortutakoetatik hurbilago daude (< % 6). Bi harrietan, diamantaketa-baldintzen arabera,  $l_c$ -k ez du joera garbirik erakusten. Harriaren graduak, indar zentrifugoak eta sorbatz-erradioak antzekoak dira bi harrietan, diamantaketa-baldintza edozein dela ere. Orduan, parte hartzen duten aldagaiak  $F'_n$  indar normal espezifikoa,  $a_e$  ebaketa-sakonera eta  $G_{\text{dyn}}$  ale-dentsitatea dira.  $G_{\text{dyn}}$  harriaren ezaugarrien ( $d_g$  ale-neurria eta  $V_g$  urratzaile frakzio bolumetrikoa) eta diamantaketa-baldintzen ( $d_b$  askatutako partikula neurria,  $\rho_d$  diamantearen erradioa eta  $u_d$  teilakapen-ratio) menpe dago. Ale-dentsitateak eta tontor-sakonerak bermazal-ratioa definitzen dute, hau da, sakonera erradialarekiko euste-azalera.

Diamantaketa-baldintza finetan (F), artezketa-indarrak handiagoak dira eta, ondorioz,  $a_e$  txikiagoa da. Aldiz, diamantaketa ertainak (M)  $l_c$  luzeagoak lortzen ditu finak (F) baino aztertutako aldagai tartean. [MRDI04] diamantaketak kontaktu-zurruntasuna baldintzatzen duela baieztatzen dute, bermazal-ratioarekin lotura zuzena duena. Diamantaketa larriak (C),  $G_{\text{dyn}}$  txikiak eta  $\delta_{\text{max}}$  sakonenak lortzen ditu  $a_e$  handienekin. Aleko ebaketa-indarra sarketarekin esponentzialki erlazionatua dago, beraz, ale gutxiagok karga gehiago jasaten dute sakonago sartuz. Diamantaketa finaren (F)  $G_{\text{dyn}}$  handiagoek eta, beraz, bermazal-ratioak piezan sarketa murrizten dute. Diamantaketa ertainaren (M)  $a_e$ - $G_{\text{dyn}}$  konbinaketak bi harrietan  $l_c$  luzeenak lortzen ditu. Honek frogatzen du kontaktu-zurruntasunaren eta bermazal-ratioaren arteko erlazioa, eta, diamantaketarena ere.

DEM ereduak harriaren deformazioari buruz ere informazio interesgarria eskaintzen du. KE ereduak  $\delta_{\text{max}}$  balio handiagoak lortzen ditu ETLk baino, ardatz bertikalarekiko indar-banaketa simetrikoa baita harriaren bi aldeetan. ETL ereduak indar-banaketa asimetrikoa erakusten du txirbil-lodieraren funtzioan, indar-banaketa errearen antza hartzen duelarik. Batzaz besteko  $\delta_{\text{max}}$  ETL ereduan 6  $\mu\text{m}$  dira, Königen emaitzekin bat datozenak [KL75]. 13  $\mu\text{m}$  sakonera behin agertzen da A46K7V harriari (C) diamantaketarekin eta  $a = 20$   $\mu\text{m}$  ebaketa-sakonerarekin, ziur aski karga ale-dentsitate baxuko azalera txikian kontzentratzearen ondorioz.

ETL ereduak gainazaleko aleen desplazamendua ere aztertzeko gai da. Lortzen dituen ale desplazamenduen balioak (5  $\mu\text{m}$  inguruan) literaturan aurkitutakoekin bat egiten dute [NBS71, Sai90, Sha96]. Ale-desplazamendu maximoa 3,7 - 9,1  $\mu\text{m}$  artekoa da A46I8V harriari, eta 2,9 - 6,6  $\mu\text{m}$  artekoa A60K7V harriari. 4.14. Irudiak simulazio batean gainazaleko aleen desplazamendu-emaitzak aurkezten ditu adibide gisa (A46I8V harria,  $a = 10$   $\mu\text{m}$ , C diamantaketa). Sorbatzetan  $F'_n - G_{\text{dyn}}$  erlazioak definitzen du indar-intentsitatea, baina aleen desplazamenduak DEko habe kopuruaren eta alboko DEkiko posizioaren menpe ere badaude. 4.14. Irudiak ere kontaktu-luzeraren definizioaren esanahi geometrikoa erakusten du:  $l_c$  ebaketa-norabidean kontaktuan dauden aleen arteko distantzia luzeena da, hau da,  $z_{\text{max}}$ -ren eta  $z_{\text{min}}$ -ren batura. Gainazaleko DE batzuk ez dira piezarekin kontaktuan jartzen, baina habe-sareak bultzatuta desplazatzen dira. Marra beltzak piezaren gainazala adierazten du, eta sorbatzen



**4.14. Irudia:** Harriaren deformazioaren adibidea: gainazaleko sorbatzen posizioa aske eta kontaktuan (A60K7V,  $a = 10 \mu\text{m}$ , (C) diamantaketa)

posizio erlatiboak (zirkulu beltzak) sarketa definitzen du. Harriaren deformazio elastikoak (aleen desplazamenduak) kontaktu-luzera luzatzen du.

KEk ez bezala, ETL ereduak ale-dentsitatea ere aurreikusten du, kontaktu-luzera zuzenean modelatzen baitu. DEM ereduak instant bat simulatzen duenez,  $G_{\text{dyn}}$ -n kalkulua zuzenean  $N_c$  kontaktuan dauden ale kopurua, zati kontaktu-azalera ( $l_c \cdot b_s$ ) da. Zenbakizko  $G_{\text{dyn}}$  emaitzak neurtutakoekin ez datoz bat ( $< \% 10$ ). Diferentziak ez dira proportzionalak diamantaketa-baldintza berdinetan. Neurketa esperimentaletan  $G_{\text{dyn}}$ -en dispersioa ere handia izan da ( $< 0,41 \text{ ale/mm}^2$ ).

ETL ereduak kontaktu-eremuan indarrei buruzko informazioa ere eskaintzen du, besteak beste, aleak jasandako bataz besteko indarra eta indar maximoa. Aleko bataz besteko indarra 1 - 1,5 N artekoa da A46I8V harrian, eta 0,7 - 1,1 N artekoa A60K7V harrian. Balio honek ale-dentsitateak material harroketan duen papera azpimarratzen du. Aleko indar maximoa, berriz, 2,8 N-era iristen da A46I8V harrian, eta 4,6 N-ra A60K7V harrian. Balio hauek interesgarriak dira aglomeratzailean tentsioak aztertzeko. Bataz besteko kontaktu-presioa ere estimatu daiteke: 1,5 - 4,7 MPa artean A46I8V harrian, eta 1,8 - 3,9 MPa artean A60K7V harrian. Hala ere, bataz besteko presioak goldaketa eta ebaketa gauzatzeko beharrezkoak diren presioak, itxurazko azalera banatuta ezkututzen ditu; beraz, balio horiek kontuz erabili behar dira.

$\sigma_y$  isurpen-tentsio eraginkorra konstantetzat hartzeak eragin zuzena du simulazioen emaitzetan. Ale mailan, erreakzio indarrak  $\sigma_y$ -rekiko proportzionalak dira. Indarrak eta sorbatzaren posizioak (diamantaketa-baldintzen ondorioz), ale-dentsitatearekin batera, alearen sarketa piezan definitzen dute. Ondorioz, beharrezkoa da  $\sigma_y$ -ren hurbilpen ona DEM ereduarekin emaitza fidagarriak lortu ahal izateko. 4.2.2. Atalean eztabaidatu da,  $\sigma_y$ -ren aldaketa sakonerarekiko

ez dela oso maldatsua, eta  $\sigma_y = 810$  MPa balioa hurbilpen egokia izan da frogatutako lan-baldintzetarako. Edonola ere, ezinbestekoa da materialaren portaera biskoplastikoa kontuan hartzea ale-pieza interakzioa modelatzean.  $\sigma_y$  konstantearen sinplifikazioa gainditzea, tesi honek egiteke uzten duen etorkizuneko ikerketa-ildoetako bat da. Erronka, ikuspuntu analitikotik heldu daiteke, deformazioen eta tenperaturen adierazpen analitikoak kontaktu-legean inplematuz; horrela,  $\sigma_y$  balioak kontaktu-baldintzekiko uneoro egokituko dira (2.4.3. Atala). Beste aukera, Shawen gogortasun ereduaren ordez, mikro-fresaketa ereduagatik ordezkatzea da; indarra ale mailako energia espezifikoarekin hurbilduko litzateke (2.4.2. Atala). Horrela, JC ereduaren erabilera saihesten da, baina energia parametroak definitzeko ale bakarraren frogen beharra dakar.

Harriaren gradua, indar zentrifugoa eta sorbatz-erradioa saiakuntzetan konstante mantendu diren parametroak izan dira. Harria gero gogorragoa, gero eta zurrunagoa da. Indar zentrifugoak handitzean, harriak gradu handiagoa balu bezala portatzen da, eragindako trakzio tentsioen eta abiadura azkarrak aglomeratzailearen portaeran eragiten duen hobekuntzaren ondorioz. Sorbatz zorrotzagoek materiala errazago harrotzen dute, piezan errazago sartuz (ebaketa-sakonera handitzen da) eta, ondorioz, kontaktu-luzera luzatuz. Faktore hauen eragina hurrengo Atalean landutako saiakuntzen diseinuan aztertzen da.

#### 4.4. Esperimentu faktorialak DEM ereduarekin

Ia hogeita-hamar aldagai dituen eredu baten fidagarritasuna, aldagaien magnitudeen definizio zuzenaren gain dago. Artezketa lauaren zenbakizko kontaktu-ereduak, kontaktu-luzeraz eta ale-dentsitateaz gain, aldagai bakoitzaren eragina ebaluatzeko erreminta eraginkorra da. Saiakuntzen diseinuaren (*Design of experiments*, DOE) bidez, aldagaien aldaketek sistemaren erantzunean eta haien artean duten elkar-eragina aztertu nahi da.

Diseinu faktorial osoak faktore maila bakoitza beste faktoreen maila guztiekin alderatzen du. Bata bestearen eragina identifikatzeko metodarik eraginkorrena da, baina esfortzu gehien eskatzen duena da. DOEa diseinatzean, taula ortogonalek esfortzua nabarmenki murrizten dute; aztertutako faktoreen maila bakoitza besteekin era orekatuan alderatuko direla ziurtatzen dute. Faktoreen arteko elkarrekintza posibleak identifikatzeko ere gai bada, efektuen esangurara baieztatzean hartutako akatsa aztertuz eta ebaluazioaren konfiantza mugak zehaztuz [Mon08]. Azterketa zenbakizko kontaktu-ereduan soilik arreta ipiniko duenez, inguruneke zarata soilik DEM ereduaren ausazko naturaren baitan egongo da. Ez dago kontaktu-luzera optimorik edo helbururik, nahiz eta zeharka  $l_c$  kalte termikoarekin eta piezaren akaberarekin erlazioa izan. KE eredia gainazal zimurren arteko kontaktu-eredu interesgarria izan arren, artezketarekin erlazio zuzenik ez dauka eta DOEtik kanpo utzi da.

Kontaktu-ereduak hainbat sarrera-faktore dauzka. Literatura berrikusketak eta DEM kontaktu-ereduaren garapenak (2. eta 3. Atalak) artezketan harri-pieza kontaktuan eragiten duten faktore nagusiak identifikatu dituzte: harriaren zurruntasuna ( $E_s$  harriaren elastikotasun modulua eta  $\nu_s$  ebaketa-abiadura),

gainazal topografia ( $\rho_s$  sorbatz-erradioa, diamantaketa-baldintzak,  $d_b$  askatutako aleen batuz besteko neurria,  $d_g$  ale-neurria), ( $F_n$ ,  $F_t$ ) artezketa-indarrak,  $a_e$  ebaketa-sakonera eta piezaren  $\sigma_y$  isurpen-tentsio eraginkorra. Beraz, bederatzi faktore identifikatu dira diseinu faktorialerako hautagai bezala.

Bederatzi faktoreko diseinu faktorialak diseinuaren konplexutasuna handitzen du.  $F_n$  eta  $a_e$  zuzenki erlazionatuta daude makina-ardatz-harria sistemaren zurruntasunaren funtzioan; beraz, bikotea faktore bakartzat har daiteke. Indarra kontaktuan dauden aleetan banatzen da: indar handiagoa, itxurazko kontaktu-azalera handiagoa dakar, eta, beraz, ale gehiago daude kontaktuan. Indar banaketan eta kontaktu-baldintzetan arreta ipintzeko, indarra (eta ondorioz ebaketa-sakonera) konstante mantendu da DOEan. Bestalde,  $\sigma_y$  konstantea eta ale mailako kontaktu-erreakzioarekiko proportzionala da. Horregatik,  $\sigma_y$  ere konstante mantendu da DOEan. Horrela, gainontzeko faktoreen eragina sentikortasun eta zehaztasun handiagoarekin ebaluatzen da.

Egile guztiek nabarmendu dute harriaren elastikotasun moduluen eragina kontaktu-luzeran.  $v_s$  handitzeak, harria luzarazi eta trakzio-tentsioek zurruntasuna aldatzen dute  $f_z$  indar zentrifugoaren ondorioz. Gainera, ereduaren balioztapenean  $f_z$ -ren eragina ez da aztertu. Gainazal topografiari dagokionez, diamantaketa-baldintzekin zuzenki lotua dago. Saiakuntzetan  $f_d$  diamantaketa-aitzinamendua izan da erreferentziazko aldagaia diamantaketa-baldintzak definitzean,  $a_d$  diamantaketa-sakonera eta  $\rho_s$  diamantearen erradioa konstanteak baitira (4.1. Taula).  $f_d$ -k sorbatzen sakoneran ( $h_f$ ,  $y_p$ ) eragiten du.  $d_b$  askatze ale-neurriak, berriz, gainazalaren irekitasuna definitzen du.  $f_d$ -k eta  $d_b$ -k gainazalean eskuragarri dagoen sorbatz-kopurua baldintzatzen dute. Biak diamantaketa-baldintzek elkarlotu eta menpekoak izan arren, biak DOEan aztertu dira, bakoitzak kontaktuan duen ekarpena aztertzeko.  $\rho_s$  sorbatz-erradioa ere topografia faktorea da, baina ez dirudi diamantaketa-baldintzen eragina jasoenik frogatutako baldintzetan. Horregatik,  $\rho_s$  aldagai independente gisa hartu da DOEan.  $d_g$  ale-neurriak zuzenki eragiten du urratzaileen trinkotzean eta, beraz, baita gainazaleko ale-dentsitatean ere.

Era honetan, diseinu faktorialerako aukeratutako faktoreak hizki larriz izendatu dira. Sei faktore oraindik asko dira, baina egokia bi mailako taula ortogonalarentzat  $L_8(2^7)$ . Horrela, zortzi saiakuntza (guztira 7 askatasun-gradu (AG)) egin dira sei aldaketa-iturri aztertzeko, faktore bakoitza bi mailatan ebaluatu delarik (1 AG). Horrela, soberan dagoen AGa erreari esleitu zaio (aldakortasuna taldeen artean). Saiakuntza baldintza bakoitzarentzat lau DEM eredu simulatzen dira ereduaren ausazko natura kontuan hartzeko.

Mailak definitzean, posible den heinean, ohiko erabilpen-tarteetako mugak aukeratu dira. Harri komertzialetan erabili ohi diren graduak G eta T artean daude. Gradua, dagokion  $E_s$  elastikotasun moduluagatik ordezkatu da DOEan.  $f_z$ -ren eragina 20 m/s eta 40 m/s arteko ebaketa-abiaduretan aztertu da. Literaturan, sorbatz-erradioak 5  $\mu\text{m}$ -tik 90  $\mu\text{m}$ -ra (eta gehiago) agertzen da, neurketa-prozeduran definizio ezagatik. 30  $\mu\text{m}$  sorbatz-erradioa dagoeneko ale nahiko kamustzat hartuz, 10  $\mu\text{m}$   $\rho_s$  aukeratu da sorbatz zorrotzat [Klo09]. Diamantaketa-baldintzek eta ale-neurriek bat egiten dute saiakuntzetan erabilitakoekin (ikusi 4.1. Taula). Azkenik,  $d_b$  askatze ale-neurriaren mugak saiakun-

#### 4.4. Taula: Mailen esleipena eta simulazioetako emaitzak

Froga zb.	A	B	e	C	D	E	F	Konbinaketa	$l_c$ (mm)	$\delta_{\max}$ ( $\mu\text{m}$ )	$G_{\text{dyn}}$ ( $\frac{\text{grit}}{\text{mm}^2}$ )
1	0	0	0	0	0	0	0	$A_0B_0C_0D_0E_0F_0$	4,39	13,59	2,46
2	0	0	0	1	1	1	1	$A_0B_0C_1D_1E_1F_1$	4,66	14,48	2,64
3	0	1	1	0	0	1	1	$A_0B_1C_0D_0E_1F_1$	3,97	13,16	2,71
4	0	1	1	1	1	0	0	$A_0B_1C_1D_1E_0F_0$	4,69	16,67	2,51
5	1	0	1	0	1	0	1	$A_1B_0C_0D_1E_0F_1$	4,82	19,15	3,54
6	1	0	1	1	0	1	0	$A_1B_0C_1D_0E_1F_0$	4,63	15,93	1,59
7	1	1	0	0	1	1	0	$A_1B_1C_0D_1E_1F_0$	4,97	19,8	1,7
8	1	1	0	1	0	0	1	$A_1B_1C_1D_0E_0F_1$	4,01	11,57	3,2
marka	a	b	ab	c	ac	bc	abc				

A - elastikotasun modulua, $E_s$	(G) 25 GPa	(T) 83,5 GPa
haben elastikotasun modulua, $E_\mu$	$A_0 = 137$ GPa	$A_1 = 452,7$ GPa
B - ebaketa-abiadura, $v_s$	$B_0 = 20$ m/s	$B_1 = 40$ m/s
C - sorbatz-erradioa, $\rho_s$	$C_0 = 10$ $\mu\text{m}$	$C_1 = 30$ $\mu\text{m}$
D - diamantaketa-aitzinamendua, $f_d$	$D_0 = 0,2 \frac{\text{mm}}{\text{rev}}$ (F)	$D_1 = 0,8 \frac{\text{mm}}{\text{rev}}$ (C)
E - askatze ale-neurria, $d_b$ (%)	$E_0 = 0,67$	$E_1 = 0,78$
F - ale-neurria, $d_g$	$F_0 = 253$ $\mu\text{m}$	$F_1 = 330$ $\mu\text{m}$

Faktore konstanteak:

indar normal, $F'_n = 15,8$ N/mm	indar-arrazoia, $\mu = 0,5$
ebaketa-sakonera, $a_e = 12,8$ $\mu\text{m}$	txirbil-lodiera max., $h_{\text{cu}} = 0,43$ $\mu\text{m}$
isurpen-tentsio eraginkorra, $\sigma_y = 810$ MPa	

tzetan behatutako balio maximoa eta minimoa hartu dira.  $d_b$ , ale-neurriaren (%  $d_g$ ) frakzio gisa definitu da. Horrela,  $d_b$  bi ale-neurrietan erraz aplikatu daiteke. Indarra, ebaketa-sakonera, ebaki-gabeko txirbil lodiera eta isurpen-tentsio eraginkorra konstante mantendu dira diseinu faktorialean.

4.4. Taulak, diseinu faktorialean aztertu diren faktore eta maila konbinazioak esleitzen ditu. Hiru aldagai aztertu ditu DOEak:  $l_c$  kontaktu-luzera,  $\delta_{\max}$  sarketa maximoa eta  $G_{\text{dyn}}$  ale-dentsitatea. 4.5. Taulak aldagai horien bariantza-analisiaren (*analysis of variance*, ANOVA) emaitzak aurkezten ditu.

ANOVAk, aldaketa-iturriek ereduaren eraginari buruz informazio interresgarria eskaintzen du. Aurreko Atalean komentatu da DEM ereduak artezketaren ausazko natura islatzen duela. Diseinu faktorialak ere ausazkotan hori jasotzen du ereduagandik. Adibidez, DOE saiakuntzetan,  $l_c$  emaitzen desbiderapen estandarra 0,12 - 0,57 mm artean egon da, tarte nabarmena indarra eta sakonera konstante mantendu direla kontuan hartuta.

4.5.(a) Taulak  $l_c$  kontaktu-luzeraren ANOVA analisia aurkezten du. Topografiak paper nabarmena dauka, batez ere  $f_d$  aitzinamenduak (edo sorbatz-sakonera) eta  $d_g$  ale-neurriak % 5 esangura-mailarekin. Harritzekoa da  $\rho_s$  sorbatz-erradioak  $l_c$ -n duen eragina aipagarria ez izatea.  $E_s$  harriaren elasti-



**4.5. Taula:** ANOVA taulak: (a)  $l_c$  kontaktu-luzera, (b)  $\delta_{\max}$  sarketa maximoa, eta, (c)  $G_{\text{dyn}}$  ale-dentsitatea

<b>(a) Kontaktu-luzera, <math>l_c</math></b>					
Aldaketa- iturria	Karratuen batura	AG	Desbiderapen koadratikoa	$F_o$	$P(F(1, 1) \geq F_o) < \alpha$
A ( $E_s$ )	0,06590	1	0,11872	112,09	$P(39, 86 \geq F_o) < 0, 1$
B ( $v_s$ )	0,09162	1	0,05971	56,33	$P(39, 86 \geq F_o) < 0, 1$
e (error)	0,00106	1	0,00106		
C ( $\rho_s$ )	0,00369	1	0,00369	3,48	$P(39, 86 < F_o) > 0, 1$
D ( $f_d$ )	0,57155	1	0,57155	538,57	$P(161, 45 \geq F_o) < 0, 05$
E ( $d_b$ )	0,01360	1	0,01360	12,81	$P(39, 86 < F_o) > 0, 1$
F ( $d_g$ )	0,18681	1	0,18681	176,03	$P(161, 45 \geq F_o) < 0, 05$
Guztira	0,93	7			

<b>(b) Sarketa maximoa, <math>\delta_{\max}</math></b>					
Aldaketa- iturria	Karratuen batura	AG	Desbiderapen koadratikoa	$F_o$	$P(F(1, 1) \geq F_o) < \alpha$
A ( $E_s$ )	9,15	1	9,15	8618	$P(4052, 2 \geq F_o) < 0, 001$
B ( $v_s$ )	0,47	1	0,47	446	$P(161, 45 \geq F_o) < 0, 05$
e (error)	3,72	1	3,72		
C ( $\rho_s$ )	6,22	1	6,22	5856	$P(4052, 2 \geq F_o) < 0, 001$
D ( $f_d$ )	31,41	1	31,41	29597	$P(4052, 2 \geq F_o) < 0, 001$
E ( $d_b$ )	0,71	1	0,71	667	$P(647, 79 \geq F_o) < 0, 025$
F ( $d_g$ )	7,30	1	7,30	6880	$P(4052, 2 \geq F_o) < 0, 001$
Guztira	58,97	7			

<b>(c) Ale-dentsitatea, <math>G_{\text{dyn}}</math></b>					
Aldaketa- iturria	Karratuen batura	AG	Desbiderapen koadratikoa	$F_o$	$P(F(1, 1) \geq F_o) < \alpha$
A ( $E_s$ )	0,0113	1	0,0113	11	$P(39, 86 > F_o) < 0, 1$
B ( $v_s$ )	0,0015	1	0,0015	1	$P(39, 86 < F_o) > 0, 1$
e (error)	0,0151	1	0,0151		
C ( $\rho_s$ )	0,0265	1	0,0265	25	$P(39, 86 < F_o) > 0, 1$
D ( $f_d$ )	0,0231	1	0,0231	22	$P(39, 86 < F_o) > 0, 1$
E ( $d_b$ )	1,1754	1	1,1754	1108	$P(647, 79 \geq F_o) < 0, 025$
F ( $d_g$ )	1,8422	1	1,8422	1736	$P(647, 79 \geq F_o) < 0, 025$
Total	3,1	7			

kotasun modulua (edo gradua) eta  $v_s$  ebaketa-abiadura adierazgarriak dira % 10 probabilitate mailarekin.

4.5.(b) Taulak, berriz,  $\delta_{\max}$  sarketa maximoan aldaketa-iturri denek eragina dutela erakusten du, bereziki topografia iturriek ( $f_d$ ,  $\rho_s$ ,  $d_g$ ) eta  $E_s$  harriaren elastikotasunak. Honek azal dezake  $G_{\text{dyn}}$  zenbakizko emaitzetan izandako dispersioa. DOEak aztertu ez arren,  $\sigma_y$  isurpen-tentsio eraginkorra ere  $\delta_{\max}$ -ren aldaketa-iturri argia da eta, ondorioz, baita  $G_{\text{dyn}}$ -rena eta  $l_c$ -rena ere.

Azkenik, 4.5.(c) Taulak  $G_{\text{dyn}}$  ale-dentsitatean topografia faktoreak nagusi direla erakusten du: soilik  $d_g$  eta  $d_b$  eragiten dute  $G_{\text{dyn}}$ -n.  $G_{\text{dyn}}$ -k ale eskuragarritasunarekin lotura estua dauka, eta horrek azaltzen du  $d_g$ -ren eta  $d_b$ -ren garrantzia.

Diseinu faktorialaren ondorioak emaitza esperimentaletakoekin bat egiten dute. Baldintza ezberdinak direnez ezin dira zuzenean alderatu.  $F'_n$  eta  $h_{\text{cu,max}}$  gisako parametroak konstantetzat hartu dira DOEan, eta bestalde,  $v_s$  eta  $\rho_s$  ia konstanteak dira saiakuntza esperimentaletan. Frogatutako harrietan  $E_s$  balioak ere ez dira oso ezberdinak ( $I$  eta  $K$  graduak), eta ondorioz, bere eragina ez da nabaritu behatutako  $l_c$ -tan. Aldiz, topografia parametroak ( $f_d$  eta  $d_g$ ) badute eragina  $l_c$ -n, ANOVAk aurreikusi bezala.  $G_{\text{dyn}}$ -n ere eragina dute  $d_b$ -k eta  $d_g$ -k, DOEan eta saiakuntzetan faktore garrantzitsuak direla erakutsi dute.

## 4.5. Laburpena

Atal honetan, kontaktuan eragiten dituzten aldagaiak aztertu dira artezketan duten eragina zehazteko. Horretarako DEM ereduak aplikatu da laborategian egindako saiakuntza baldintza berdinetan. Lan esperimentalak biak eskuratu dituzte: simulazioetarako beharrezkoak diren sarrera-datuak, eta ereduaren emaitza diren kontaktu-luzerak. Harriaren DEM ereduak dimentsioak eta izendapena jarraituz eraiki dira, eta gainazal topografia haien gainean sortu da diamantaketa-baldintzen eta topografia-parametroen arabera. KE eta ETL ereduak artezketabaldintza berdinetan simulatu dira. Azkenik, simulazioz lortutako emaitzak esperimentalki lortutakoekin alderatu dira.

Emaitzek erakutsi dute, KE era RQ ereduaren konbinaketak, ez duela emaitza esperimentalekin bat egiten: simulatutako kontaktu-luzerak behatutakoak baino luzeagoak dira. Arrazoiak, erabilitako kontaktu-baldintzek doikuntza beharko luketela kontaktu-luzera esperimentalekin bat egiteko, edota, kontaktu-arkuen hurbilpena ez dela zuzena, indarren banaketa ez baita simetrikoa kontaktu-eremuan.

ETL ereduak, berriz, emaitza esperimentalekin bat egiten dute % 6 aldearekin, harriaren zurruntasunak eta topografiak kontaktuan eragindako joerak islatuz. DEM ereduak kontaktuan dauden aleen desplazamenduak ( $< 9 \mu\text{m}$ ) eta indar-banaketa buruzko informazioa ere eskaintzen du. Eta batez ere, DEM ereduak harriaren egitura konplexuaren ausazko natura erreproduzitzeko gai izan da.

DOEri dagokionez,  $L_8$  diseinu faktorialak, kontaktu-baldintzetan harriaren zurruntasunak eta topografia parametroek duten garrantzia nabarmendu du.

## 5. Atala

# Ondorioak

Lan honetan, artezketa lauan harria-pieza arteko kontaktua modelatzeko metodologia aurkeztu da DEMen oinarrituz. DE batek ale urratzailea modelatzen du eta habe batek, berriz, aglomeratzaile-zubia; DE eta habe guztien multzoak harriaren 3D egitura sortzen du. DEM ereduak, ondo ezagunak diren oinarritzko artezketa-ereduak integratzen ditu harriaren eredu sintetikoan. Gainazal topografia, Malkinen eta Chenen ereduak konbinatuz lortzen da: Malkinen ereduak ale-askatzearen ondorioz harriaren irekitasuna zehazten du; Chenen ereduak era diskretuan aplikatzen da, helizea eta haustura hauskorrak konbinatuz, eta azkenik, ale bakoitzari sorbatz esferiko bakarra esleitzen zaio. Bi mugalde-baldintza erabiltzen dira ale-pieza artean kontaktu-baldintzak ezartzean: kontaktu estati-ko ereduak, RQ ereduaren kontaktu-luzera zimur osagaia modelatzen duena; eta ebaki-gabeko txirbil-lodiera ereduak, ale bakoitzeko ebaketa-indarra estimatzeko Shawen gogortasun analogia erabiltzen duena. Piezaren materialak ale urratzaileekin interakzioan duen portaera biskoplastikotzat jo da. Ereduak arteztean, harriaren portaera islatzeaz gain, kontaktuaren garapenaren nabardurak azalera- ratu ditu.

Elementu diskretuen metodoak harria modelatzeko erreminta egokia dela erakutsi du. Harriaren DEM ereduak ausazko morfologia granularra eta zurruntasuna erreproduzitzeko gai da. Chenen topografia eta Malkinen ale-askatze eredu- en konbinaketak harriaren gainazal topografia eratzen dute, sorbatzak sakoneran banatuz. Bien konbinaketak diamantaketaren trazak artezketa-indarretan islatzeko sentikortasuna baduela erakutsi du.

KE-RQ ereduak kontaktu-luzera handiegiak estimatu dituzte. Isurpen-tentsioaren balioa emaitza esperimentalekiko doiketa beharko luke. Hala ere, KE ereduak bere baitan gainazal zimurren arteko kontaktua simulatzeko eredu berri- tzailea da. ETL ereduak, berriz, emaitza esperimentalekin ehuneko seian egiten du bat neurketa esperimentalekin. Ereduak gainazal topografiaren eragina jasotzen du, eta, indar normalaren, ebaketa-sakoneraren eta ale-dentsitatearen efektu konbinatua nabarmetzen du. Shawen gogortasun analogiak sorbatzeta- ko ebaketa-indarrak era sinplean estimatzen ditu. Hala ere, ereduaren arrakasta materialaren portaera biskoplastikoaren eskutik etorri da, aurreko kontaktu-ere-

duek arbuia izan dutena. Guztiaren gainetik, harriaren DEM ereduaren eta gainazal topografiaren konbinaketa harriaren ausazko natura deskribatzeko gaitasunagatik nabarmentzen da, emaitzen dispersioan islatu dena.

DEM formulazioaren malgutasunak eredia beste helburuetara moldatzeko aukerak irekitzen ditu. Besteak beste, piezaren gainazaleko gainazal zimurtasuna, artezketa-indarrak, indar zentrifugoaren eragina, edo artezketaren analisi termikoa harria ere kontuan hartuz. Azkenik, egilea harritua dago, nolatan artezketa bezalako eragiketa konplexua zenbakizko eredu bidez modelatu daitekeen, beti ere eredu ez hain sinplearekin.

## 5.1. Etorkizuneko lan-ildoak

Honako hauek dira lantzen jarraitu beharreko puntuak eredia guztiz osatzeko,

- aztertu  $v_s$  ebaketa-abiadurak eta, ondorioz,  $f_z$  indar zentrifugoak dituzten eraginak, kontaktu-luzeran gain, harriaren zurruntasunean, dilatazioan, eta aglomeratzailearen portaeran,
- ebaluatu hozgarrien papera kontaktu-eremuan,
- gainditu  $\sigma_y$  isurpen-tentsio eraginkorra konstantearen erabilpena:
  - JC eredia ale bakoitzaren baldintzen arabera uneoro kalkulatu, deformazioak eta tenperaturak analitikoki estimatuz alean aldiuneko sakonerarekiko eta kontaktu-arkuan duen posizioarekiko, edo,
  - alean ebaketa-indarra estimatzeko, Shawen gogortasun-frogaren analogia erabili beharrean, mikro-fresaketaren analogia inplementatuz eta, horrela, portaera biskoplastikoaren hipotesia saihestuz,
- ereduaren aplikazio eremua diamante birakarietara eta punta-anitz finkoetara hedatzea, zenbakizko diamantaketa prozedura moldatuz, eta,
- kontuan hartzea aurrez pasatako aleek utzitako ildaskak, aleak alboetan pilatutako material gandorak barne.

DEM formulazioaren malgutasunak harriaren DEM eredia beste helburu batzuetara egokitzea ahalbidetzen du, hala nola,

- aplikazio-eremuak zabaltzea, eredia artezketa zilindriko, aurretiko artezketa eta profil-artezketa aldaeretara moldatuz,
- harria biraraztea, artezketaren eredu osoa garatuz; besteak beste, harriaren zurruntasuna, artezketa-indarrak, kontaktu-luzera eta piezan lortutako zimurtasuna modelatuko lituzke,
- harriaren higadura modelatzea, sorbatzen zorrotasunaren aldaketak ale askatzean duen eragina aztertuz, habeen formulazioa aldatuz hauskorrak bihurtzeko,

- aurreko proposamenaren bidea zabalduz, diamantaketa mekanismoen modelaketa, ale-hausturaz gain, ale-askatzean arreta ipiniz,
- topografia bermazal-ratioaren eta kontaktu probabilitate-dentsitatearen funtzioan adieraziz, antzeko oinarriak dituen kontaktu-eredu determinista eta esplizitua garatzea, zeinak kontaktu-luzera zuzenean kalkulatu duen,
- harrien fabrikazioan erabiltzen den ale urratzaileen trinkotze-prozesua simulatzea esfera-anitzeko elementu konposatuak erabiliz, egitura zenbakia eta urratzaile frakzio bolumetrikoa estimatzeko, eta,
- DE-en artean interakzio termikoak gehituz formulazioan, artezketa-energiaren banaketa simulatzea; horrela piezaren, txirbilaren, hozgarriaren eta, batez ere, harriaren arteko partizio-ratioak estimatuko lirateke.



## Part II

# On the numerical modelling of the contact in grinding





# Contents

<b>1</b>	<b>Introduction</b>	<b>151</b>
1.1	Introduction . . . . .	151
1.2	Objectives and contributions . . . . .	154
1.3	Contents . . . . .	155
<b>2</b>	<b>Review of the state of the art</b>	<b>157</b>
2.1	Introduction to the grinding process . . . . .	159
2.1.1	Kinematics and forces in surface grinding . . . . .	160
2.1.2	The dressing . . . . .	163
2.1.3	Topography of the grinding wheel surface . . . . .	166
2.2	Grinding contact length . . . . .	172
2.2.1	Contact length models . . . . .	173
2.2.2	Contact length measurement methods . . . . .	178
2.2.3	Analysis of the contact length measurement signal . . . . .	182
2.3	Numerical models in Grinding . . . . .	183
2.3.1	Structure models of the grinding wheel . . . . .	184
2.3.2	Topography models . . . . .	187
2.4	Abrasive grit-workpiece interaction . . . . .	192
2.4.1	Contact mechanics . . . . .	192
2.4.2	Interactions at grit level . . . . .	195
2.4.3	Viscoplasticity . . . . .	206
2.5	Summary . . . . .	212
<b>3</b>	<b>The numerical contact model</b>	<b>215</b>
3.1	System design . . . . .	217
3.1.1	Numerical model of the contact in grinding . . . . .	217
3.2	The discrete element method . . . . .	218
3.2.1	Adaptation of DEM to the contact model . . . . .	220
3.2.2	Creation of the DEM model . . . . .	229
3.2.3	Structure of the DEM model . . . . .	230
3.3	Grinding wheel DEM model . . . . .	231
3.3.1	Characteristics of the grinding wheel . . . . .	231
3.3.2	The DEM model of the Grinding wheel . . . . .	236
3.4	Numerical dressing . . . . .	239

3.5	Boundary conditions . . . . .	242
3.5.1	Static contact model . . . . .	244
3.5.2	Uncut chip thickness contact model . . . . .	245
3.5.3	Centrifugal force . . . . .	248
3.6	Summary . . . . .	248
<b>4</b>	<b>Simulation of the contact in grinding</b>	<b>251</b>
4.1	Experimental work . . . . .	253
4.2	DEM model set-up . . . . .	258
4.2.1	Beam properties calibration . . . . .	259
4.2.2	Input data . . . . .	259
4.3	Experimental and simulation results . . . . .	264
4.4	Factorial experiments with the DEM model . . . . .	271
4.5	Summary . . . . .	275
<b>5</b>	<b>Conclusions</b>	<b>277</b>
5.1	Future work . . . . .	278

# List of Figures

1.1	Number of articles published in indexed journals that contain the words <i>grinding</i> , <i>contact</i> , <i>numerical model</i> and <i>DEM</i> . . . . .	152
1.2	Numerical simulation of temperatures on the workpiece surface .	153
2.1	Abrasive grit removing material . . . . .	159
2.2	Main parameters in surface grinding and grinding forces . . . . .	160
2.3	Micro-milling analogy of grinding . . . . .	161
2.4	Grain shape assumptions: triangular, rectangular and circular . .	162
2.5	Kinematics of stationary dressing; detail of the cutting section .	164
2.6	Abrasive grit breakage and bonding fracture . . . . .	164
2.7	Surface created by the dresser on consecutive grains . . . . .	165
2.8	Static and dynamic distances between cutting edges . . . . .	168
2.9	Characterisation of cutting edge sharpness . . . . .	169
2.10	Average profiles of the cutting edge and the grain; peak slope and curvature regarding the bearing ratio and dressing . . . . .	170
2.11	The dummy wheel and a CBN wheel specimen . . . . .	172
2.12	Definition of contact length; deformations at the interface . . . .	173
2.13	The Rowe-Qi contact model . . . . .	177
2.14	Single-pole thermocouple; influence of isolation thickness . . . . .	179
2.15	Electric circuit of the APS method . . . . .	180
2.16	Traces left by the wheel; patch grinding method . . . . .	181
2.17	APS signals; regions on the contact length; thermocouple and APS signals . . . . .	183
2.18	Uniform spring model of the wheel; redundant beam network that models a bonding bridge; DEM simulation of grinding . . . . .	186
2.19	Abrasive grit geometries; grit projections and shape factor . . . .	190
2.20	Framework of a general 3D topography model . . . . .	192
2.21	Rigid sphere and cylinder contact with an elastic half-space . . .	193
2.22	Action of an abrasive grit: rubbing, ploughing and cutting . . . .	196
2.23	Single-grit test: instantaneous specific energy and force . . . . .	198
2.24	Tabor's rubbing contact models . . . . .	200
2.25	Groove created by a cone . . . . .	200
2.26	Slip-line field proposed for indentation and a grit in grinding . .	200
2.27	Slip-line models of Challen and Oxley . . . . .	201

2.28	Main, secondary and tertiary cutting zones; chip formation in 3D	201
2.29	Analogy of the hardness model of chip formation	203
2.30	Orthogonal cutting with negative rake angles	209
3.1	Input and outputs of the grinding system	218
3.2	Flow chart of the contact in grinding	219
3.3	DEM formulation of the grinding contact model	221
3.4	Contact conditions between DEs: contact, spring and beam	224
3.5	Elastic beam relaxed and loaded	226
3.6	Flow chart of the calibration of beam properties	228
3.7	Flow chart of a simulation	230
3.8	Designation of grinding wheels	231
3.9	Effect of the variation of proportions on the structure	233
3.10	Trigonal graph of the composition of grinding wheels	234
3.11	Drop test of the binder	235
3.12	Grinding wheel and workpiece DEM model	236
3.13	Flow chart of the construction of the grinding wheel DEM model	237
3.14	Construction of the grinding wheel DEM model	238
3.15	Implementation of the topography on the DEM model	240
3.16	Cumulative grain density through radial depth	240
3.17	Location of the peak of a grit; grit dressed and sharpness radius	242
3.18	Contact detection between a surface DE and the workpiece	245
3.19	Foundations of the UTC model	246
3.20	Contact detection on $A'-A$ and $A-B$ segments	247
4.1	Dresser profile measurement	254
4.2	Set-up of the grinding forces measurement	255
4.3	Dummy wheel and wheel specimen	255
4.4	3D topography measurement; a transverse view taken by SEM	256
4.5	Example of sharpness radius measurement	256
4.6	The APS set-up	257
4.7	APS signal examples regarding protuberance width	258
4.8	Calibration of beam Young's modulus according to the grade	261
4.9	Definition of the model height by FEM	262
4.10	Specific grinding forces regarding the dressing feed	263
4.11	Experimental and numerical bearing ratio curves	265
4.12	Examples of the simulation outputs	266
4.13	Experimental and numerical contact lengths	268
4.14	Example of wheel deformation unloaded and loaded	270

# List of Tables

2.1	Summary of the main contact length models published . . . . .	175
2.2	APS test conditions in the literature . . . . .	181
4.1	Testing equipment and conditions . . . . .	253
4.2	DEM model input data . . . . .	260
4.3	Experimental and numerical results of contact length, maximum penetration, roughness factor and dynamic grain density . . . . .	267
4.4	Allocation of levels and simulation results . . . . .	273
4.5	ANOVA tables for contact length, maximum penetration and grain density . . . . .	274



# Chapter 1

## Introduction

### 1.1 Introduction

Although grinding is probably one of the oldest manufacturing processes, it still plays a relevant role in the modern manufacturing. The reason is that grinding has been upgraded to a finishing process for high quality and precision mechanical components. Grinding is the key to understand the development and continuous improvements in the automotive and aeronautical industries, as well as the creation of the latest robotics and micro-manufacturing sectors.

Technological progress must be based on a deeper scientific knowledge of the process. Thus, unsurprisingly, the past 40 years the grinding process has been the target of an extensive research, especially by modelling and simulation [KBH<sup>+</sup>02, BAG<sup>+</sup>06, DWB09]. The analysis of the contact between the grinding wheel and the workpiece started in the early 60's, faced by empirical and semi-analytical approaches. The main authorities in grinding remark the role of the contact region in the grinding operation [Sha96, MRDI04, Mal08]. The definition of the contact area is the starting point of any process analysis. Aspects such as material removal and heat generation take place in the contact region. The granular structure of the grinding wheel and the complex surface topography make difficult the modelling of the operation. That is the reason of the interest of researchers in the characterisation of the contact in grinding, although its observation is complex. The increasing number of papers published about *grinding*, *numerical model* and *contact* the last 56 years remarks its importance (Fig. 1.1).

Current grinding wheels are man-made conglomerates of hard abrasive grits, which do the cutting, held in a weaker bonding matrix. The contact area is defined by the contact width and the contact length. In most of the grinding processes, the contact width remains constant, so the contact area is directly characterised by the contact length. Due to the grinding forces and cutting conditions at grit level, the wheel and workpiece deform, and the contact length is larger than the deduced geometrically. The experimental measurement of the

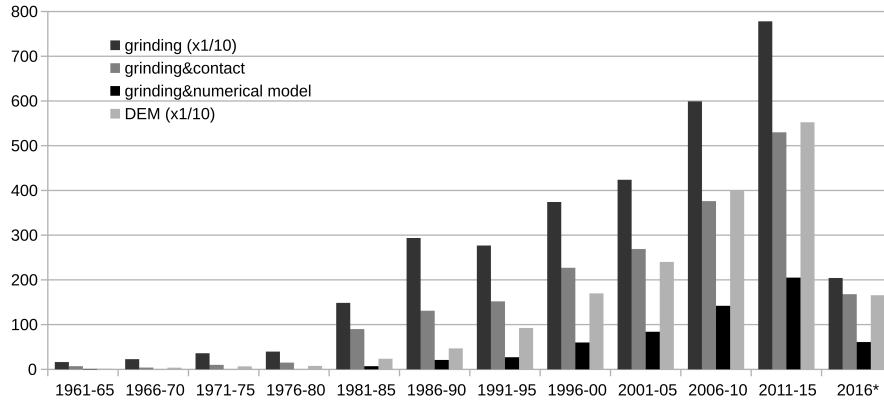


Figure 1.1: Number of articles published in indexed journals since 1961 that contain the words *grinding*, *contact*, *numerical model* and *discrete element method (DEM)* (\*) up to 01-11-2016; source: Scopus)

contact zone is complex and it is hardly measured in industrial environments.

The importance of the contact length  $l_c$  in grinding is highlighted in any grinding analysis. An accurate estimation of the contact zone is required to obtain reliable results in process simulation. For example, the grinding thermal models are valuable tools to predict the thermal damage on workpiece. The temperatures obtained by analytical approaches and numerical simulation depend closely on the heat source intensity, which is directly related to the contact area. If the geometrical contact length  $l_g$  (assuming rigid wheel) is used instead of  $l_c$  for the contact length, an error is introduced in the heat-flux. For example, for a grinding condition where the contact length  $l_c \approx 2 \cdot l_g$ , the analytical approach using  $l_g$  instead of  $l_c$  predicts a temperature which is over-estimated by 41%. Similar outputs are obtained setting  $l_g$  as an input in numerical thermal models. Many researchers have remarked that the  $l_g$  approach is incorrect [QRM01, MRDI04, Mal08, Han06, Pom10]. Experimental measurements agree this assertion. Figure 1.2 shows the relationship between the contact length and the surface temperature on the workpiece [Pom10]. This phenomenon occurs due to the application of the same heat quantity on a smaller area, which increases the heat intensity.

The contact length is closely related to wear too. Adhesion wear depends on the composition of the bodies in contact, the cutting fluid and the environment conditions. According to Archard [Arc53], under stable conditions for a given combination of materials, the rate of adhesive wear is related with the normal force, real contact area and the sliding distance. Verkerk uses the expression of Archard to estimate the wear of the wheel, replacing the real contact area with the indentation ratio [Ver77]. Even though it does not take into account the fracture and break-out of grains, the predicted slope of the grinding ratio before the dressing point coincides with experimental results.

The grinding cutting force is also related to the contact length in a squared



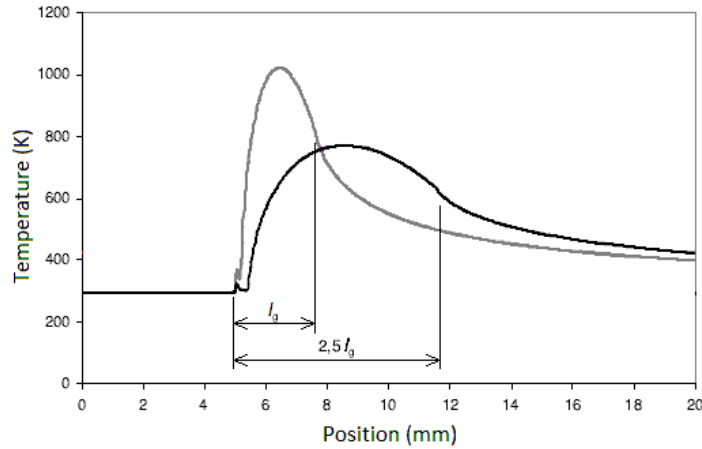


Figure 1.2: Numerical simulation of temperatures on the workpiece surface regarding the contact length [Pom10]

relationship, together with the cutting speed and the specific grinding force [Ono61].

In order to avoid costly experimental work on contact length measurements and to progress on a deeper scientific knowledge of the process, the numerical simulation of the contact in grinding opens a promising research line. Three factors stand out in the analysis of two surfaces in contact: the stiffness of the bodies, the conditioning of the surfaces and the contact conditions. This thesis is going to focus on the grinding of metallic materials, mainly steels. So, the workpiece material is homogeneous, continuous and isotropic. The grinding wheel is quite the opposite: a porous, granular and heterogeneous body. Therefore, common numerical tools and analytical approaches would require a complex formulation to model the grinding wheel body. Regarding surface roughness, the workpiece has a good surface roughness that can be assumed smooth. On the contrary, the grinding wheel has a very rough surface governed by abrasive grit size and structure openness. In addition, the surface of the grinding wheel is generated by dressing, which has a direct impact on the performance. Grain and bond fracture are the main dressing mechanisms. Dressing conditions modify the aggressiveness of dressing mechanisms, creating an open softer or a close stiffer wheel surface.

Beside the elasticity and surface roughness of the bodies, contact conditions play an important role on the contact. It is a common practice in grinding to consider elastic the contact between the grinding wheel and the workpiece. But the high strain rates and temperatures at the cutting edge-workpiece interaction produce high contact pressures, which displace and remove material from the workpiece surface. So the elastic contact assumption offers a poor approach to the contact phenomena in grinding.

In the survey of the state of the art, the discrete element method (DEM) ap-

pears as a promising numerical tool to model the grinding wheel. Beside the use of DEM for the contact modelling of rough surfaces [JM12], some attempts have been presented to model the grinding wheel structure [LYZW15a, LYZW15b]. Its open formulation and random granular nature makes DEM very appropriate to model grinding wheel features. Figure 1.1 also shows the exponential progression of the publication of DEM models in the literature.

Summarising, the ideal contact model would be the one that reproduces the structure and stiffness of the wheel, combined with an accurate topography description. In addition, it should take into account the actual contact conditions in the contact spots.

## 1.2 Objectives and contributions

The objective of this thesis is to analyse the consequences of the contact in surface grinding, studying the grinding wheel deflection, forces at grit level and the apparent contact area. The objective is carried out modelling the contact in surface grinding of steels by means of the discrete element method. The DEM model of the grinding wheel will have the features of wheel granular morphology, stiffness and topography regarding the dressing conditions. Two contact conditions are analysed in this thesis. On the one hand, it is modelled the rough contact term of the Rowe-Qi contact model [RMQZ93]. Beside its interest as a rough surface contact model, it will be used to evaluate the geometrical arc approach and experimentally defined roughness factor. On the other hand, the uncut chip thickness contact model estimates the force on the surface grains in contact according to the penetration in the workpiece, modelling directly the contact zone. The material behaviour is assumed viscoplastic instead of elastic. The effect of the centrifugal force is also included in the model.

The following challenges are faced on the PhD thesis,

- the modelling of the mechanical behaviour of grinding wheel body, reproducing its granular morphology and elasticity by means of the discrete element method,
- the measurement of the wheel topography and the characterisation of the cutting edges,
- the implementation of the surface topography on the grinding wheel numerical model according to the dressing conditions,
- the definition of the individual grit force regarding grain penetration,
- the assumption of viscoplastic material behaviour in the modelling of the contact,
- the application of rigid-perfectly-plastic contact conditions at grit level,
- the simulation of the rough contact term of the Rowe-Qi model, modelling the static contact between the grinding wheel and a plane,

- the definition of the boundary conditions in the contact area, taking into account the depth of cut and the uncut chip thickness,
- the development of a grinding contact model, that integrates the DEM model of the wheel, the surface topography, the viscoplastic material behaviour and the depth of cut, and,
- the validation of the numerical contact model experimentally, and, the analysis of the influence of each variable on the grinding contact.

### 1.3 Contents

The thesis is divided in five Chapters. The second Chapter reviews the most relevant works that constitute the grounds for the development of the numerical contact model of surface grinding. Special attention is paid on contact length models, surface topography models and contact mechanics. The third Chapter introduces the methodology followed to develop the numerical contact model. The Chapter starts identifying the parameters that take part on the grinding wheel-workpiece tribological system. Then, the formulation of DEM is adapted to model the structure of the grinding wheel and the contact conditions. Once the DEM model of the grinding wheel body is built, the surface topography is applied on the surface of the grinding wheel numerical model regarding dressing conditions. The boundary conditions, the contact detection and contact laws are defined, completing the contact model. The fourth Chapter applies the DEM model on the simulation of experimental tests. The simulation results reveal valuable information about the contact area and wheel deflection. Finally, the fifth Chapter compiles the conclusions inferred along the thesis. The last Chapter also proposes the future working areas to continue the research line opened by the grinding wheel DEM model developed in the thesis.



## Chapter 2

# Review of the state of the art

*Many papers about grinding have been published since the beginning of the XXth century. The most remarkable works related to the objectives constitute the foundations of the contact model presented in this thesis.*

*The Chapter starts introducing the grinding process and the kinematics of the operation. The dressing process generates the surface of the wheel. Then, the main contact length models are reviewed. The review identifies the main factors that have influence on the contact zone: the wheel elasticity and the surface topography. Most of them consider elastic the contact between grits and the workpiece, when actually the material is displaced and removed. None of them is numerical, they are either empirical or semi-analytical. That is the reason to quote the numerical models applied on grinding developed for different purposes: the models that reproduce grinding wheel morphology and stiffness, and topography models that describe the surface generated by dressing. The aimed model is a combination of both. Contact mechanics are included in the review, which have been disregarded in the contact length models. They are required to define contact conditions between abrasive grits and workpiece. Single-grit action involves rubbing, scratching and cutting mechanisms, which are described by the single grit models.*

*The compiled information is used to create the model of the wheel structure, apply the surface topography created by dressing and set the contact conditions between the cutting edges the workpiece.*



## 2.1 Introduction to the grinding process

Precision machining with geometrically undefined cutting edges is a key production engineering technology of high efficiency, security and machining quality [Klo09]. Grinding belongs to the group to this group, together with processes such as honing, lapping and free abrasive grinding. Machining is carried out when irregular grains of hard materials interact with the work material. Most of the studies about abrasive processes are deterministic, based on the experience more than science. It is said that they are not a technology but an art.

In grinding, the tool is the grinding wheel, a porous conglomerate of abrasive grits and bonding material. These two components form several combinations, adaptable to a wide range of applications. Abrasive grits are responsible of the material removal and the bonding material brings cohesion to the mass. Each grain acts as a microscopic cutting edge. Fig. 2.1 shows a single-grit in action. The grain edge has a very negative  $\alpha$  rake angle, around  $-60^\circ$  and  $-70^\circ$  [Sha96]. The cutting edges work at high cutting speeds ( $> 20$  m/s) and shallow depths per grain ( $< 10$   $\mu\text{m}$ ), like in high speed machining. This is the secret of grinding for the machining of hard materials. Grinding is widely used in the machining of hard and fragile materials, where turning and milling have limited application.

Regarding workpiece geometry, surface (flat) and cylindrical grinding are the basic grinding operations. In profile grinding, the grinding wheel takes the profile of the workpiece.

The wheel surface suffers two kinds of wear: the wear of cutting edges creating wear flats and the release of whole grains due to bonding breakage. The cutting ability of the surface is recovered by the dressing process, which removes a thin layer of abrasive.

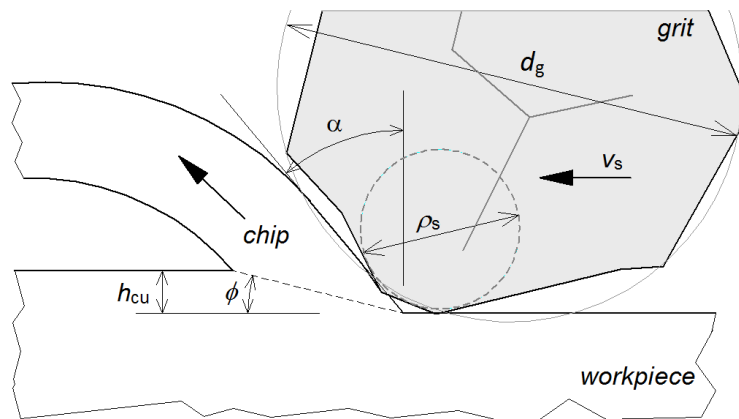


Figure 2.1: The cutting edge of an abrasive grit removing material from workpiece surface

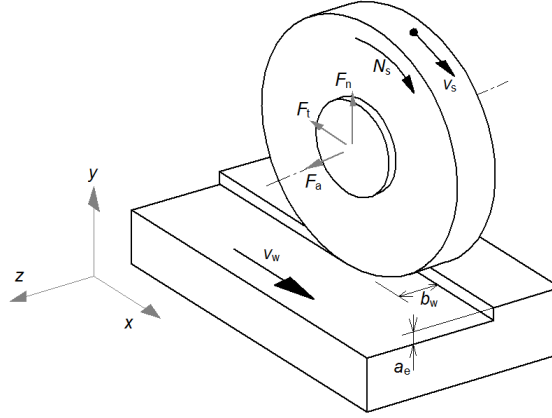


Figure 2.2: Main kinematic parameters in surface grinding and grinding forces [MRDI04]

### 2.1.1 Kinematics and forces in surface grinding

The kinematic analysis of a grit trajectory allows the study of the contact length, duration and interaction number between grits and workpiece surface. Each grit has a different geometry, so the material removed by each one differs too. Therefore, the studies about grinding made in the micro- and macroscopic scale are based on statistical calculus.

Figure 2.2 shows the relative movements of the grinding wheel and the workpiece in surface grinding. The wheel rotates around the  $z$  axis at  $v_s$  cutting speed. The workpiece advances linearly at  $v_w$  work speed. The radial depth of cut  $a_e$  controls the removed material quantity from the workpiece surface, together with axial depth of cut  $b_w$ . The deflection of the machine spindle due to grinding forces reduces the set depth of cut  $a$  on the machine. The spark-out with extra passes without depth at the end leaves the surface in the desired depth as the machine spindle relaxes.

In the interaction between an abrasive grit and the workpiece appears a force. This force can be decomposed in three components (Fig. 2.2): the tangential or cutting force  $F_t$ , the normal force  $F_n$  and the axial force  $F_a$ .  $F_t$  is the main power consumer and is responsible for the material ploughing and chip generation. Usually  $F_n$  is higher than  $F_t$  and tries to keep the set depth of cut  $a$ , deforming the workpiece-wheel-spindle set at the same time. Assuming that the system stiffness  $k_m$  has a linear behaviour, the actual depth of cut  $a_e$  is estimated with Equation 2.1 [MRDI04].  $F_a$  appears in profile grinding or when the wheel performs a lateral movement.

$$a_e = a - \frac{F_n}{k_m} \quad (2.1)$$

In energy terms, a machining process is efficient if material is removed quickly with low energy consumption. The specific energy  $e_s$  is widely used as an inverse



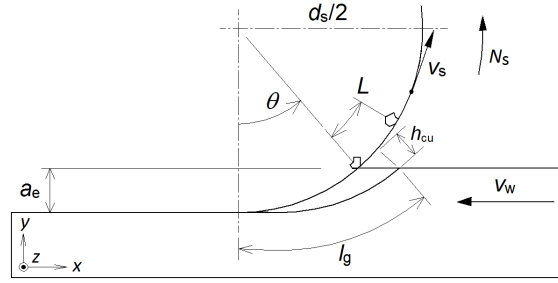


Figure 2.3: Micro-milling analogy of grinding: kinematic analysis of two consecutive grains and the geometry of the uncut chip [Ald1914]

measure of efficiency and it is defined as the energy consumed per volume unit. In grinding,  $e_s$  is the ratio of machining power to removal rate, obtaining the same ratio. Equation 2.2 estimates  $e_s$  regarding the tangential force  $F_t$ , the cutting speed  $v_s$ , the work width  $b_w$ , the depth of cut  $a_e$  and the work speed  $v_w$  [MRDI04].

$$e_s = \frac{F_t v_s}{b_w a_e v_w} \quad (2.2)$$

The micro-milling analogy of grinding leads the understanding of the process [Ald1914]. Taking each grain as a cutting edge, the behaviour of the whole wheel is deduced studying its movements. Figure 2.3 presents the micro-milling analogy approach. The figure shows the circular trajectory of a grit on the workpiece surface. The uncut chip is defined as the material volume that the grain goes through. Actual chips have a totally different geometry, due to the high strains and temperatures that are exposed [Mal08]. Only a fraction of the intersected volume is removed from workpiece surface, and the rest is displaced to the sides [ÖC12].

The study of the kinematics of two consecutive grits allows the definition of the geometry of the uncut chip and contact zone. The contact length (or the length of the uncut chip) is inferred assuming that both, wheel and workpiece, are rigid. The obtained value is called geometrical contact length  $l_g$  and it only depends on the wheel diameter  $d_s$  and depth of cut  $a_e$  (Eq. 2.3, Fig. 2.3).

$$l_g = \overline{AB} = \sqrt{d_s a_e} \quad (2.3)$$

The study of grinding forces and wear behaviour requires the estimation of the chip thickness. The uncut chip thickness can be viewed as the grain penetration into the workpiece and in that sense is the grain depth of cut. Analysing the trajectory of two consecutive grits separated by the distance  $L$  (Fig. 2.3), the uncut chip thickness  $h_{cu}$  removed by a single-grit can be estimated with Equation 2.4. The maximum uncut chip thickness  $h_{cu, \max}$  corresponds to the thickness on the maximum position angle  $\theta_{\max}$  (Eq. 2.5). However, the large

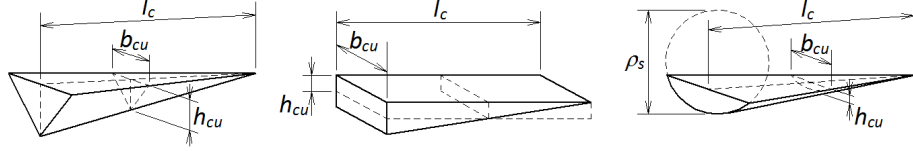


Figure 2.4: Grain shape assumptions: triangular, rectangular and circular [MRDI04]

value of chip width inferred by the classical grain spacing expression, leads to extremely thin uncut chips [Row10].

Backer *et al.* [BMS52] overcome the problem assuming a triangular cutting edge geometry and modifying the  $L$  approach with the surface grain density  $G$ . Thus, they define the ratio  $r_{cu}$  as the relationship between the uncut chip width and thickness ( $b_{cu}/h_{cu}$ ) (Eq. 2.6). By measuring grooves left by abrasive grains, they assert that chip width is 8-15 times larger than the chip thickness. Equation 2.6) can also be applied for other cross-sections such as the rectangular chip [Mal08]. Equation 2.7 presents this approach, where  $r_{cu}$  equals the chip width  $b_{cu}$ . In the same way, Equation 2.8 estimates  $h_{cu,max}$  assuming a circular cutting edge [CR96]. In this work, the grain diameter is substituted by the cutting edge radius  $\rho_s$ , which is thoroughly described in Section 2.1.3. Figure 2.4 shows the uncut chip geometry according to the grain shape (rectangular, triangular and circular).

$$h_{cu} = L \frac{v_w}{v_s} \sin \theta \quad (2.4)$$

$$h_{cu,max} = 2L \frac{v_w}{v_s} \sqrt{\frac{a_e}{d_s}} \quad (2.5)$$

$$h_{cu,max} = \sqrt{\frac{6v_w}{Grv_s}} \sqrt{\frac{a_e}{d_s}} \quad (2.6)$$

$$h_{cu,max} = \frac{2v_w}{Gr_{cu}v_s} \sqrt{\frac{a_e}{d_s}} \quad (2.7)$$

$$h_{cu,max} = \left[ \frac{15v_w}{8Gv_s} \sqrt{\frac{a_e}{2\rho_s d_s}} \right]^{\frac{2}{3}} \quad (2.8)$$

In this expressions,  $v_w$  and  $v_s$  are the work and cutting speeds,  $\theta$  the position angle,  $a_e$  the depth of cut and  $d_s$  the grinding wheel diameter. The uncut chip thickness increases linearly from zero to the maximum in the three approaches. For the triangular grain shape the  $h_{cu}$  values are slightly larger than for the circular case assumption.

Rowe [Row10] corrects these  $h_{cu}$  expressions, taking into account wheel deflection. He adds the actual to geometric contact length ratio ( $l_c/l_g$ ) to compensate the wheel deflection, which reduces  $h_{cu}$ .

### 2.1.2 The dressing

The grinding wheel wears during the operation. In addition, the machined material loads the surface voids hindering the engagement to grits. Truing and dressing are the wheel conditioning operations: truing recovers the profile and dressing restores the cutting ability of the surface layer. Since the same process is usually employed in conventional vitrified wheels for both purposes, the term dressing is commonly used to designate conditioning operations. The tool usually is a diamond, but cubic boron nitrate (CBN), silicon carbide or even cast iron are also used. The diamonds can be natural or synthetic, and mono- or poly-crystalline.

Dressing methods are classified according to the kinematics of the operation, as stationary or rotating dressing. The rotating disc is more efficient than the stationary dresser due to the largely increased number of impacts and, hence, higher feed speeds. Stationary dressing uses a single-point or multiple diamonds and, for the same reason, multiple diamond dressers allow the use of higher feeds. However, the profile of the dresser and the impact geometry are unknown with rotating disks and multiple diamond dressing. In turn, the kinematic parameters and the geometry of the dresser are known using the single-point dresser. This facilitates the analysis and monitoring of the basic dressing mechanisms, attracting the interest of researchers. In addition, the use of single-point dresser is widespread in industrial workshops. That is the reason to limit the scope of this thesis to single-point dressers. The developed model and conclusions can be further adapted to multi-point and disk dressers.

Figure 2.5 shows the kinematics of dressing with a stationary single-point dresser. Dressing movements are similar to the ones of the turning process. The main parameters are the dressing depth of cut  $a_d$  and the dressing feed  $f_d$  [MRDI04]. Equation 2.9 defines the dressing feed and Equation 2.10 the overlap-ratio  $u_d$ , where  $d_s$  is the diameter of the grinding wheel,  $v_d$  the dressing speed and  $b_d$  the contact width between the wheel and the dresser at the depth  $a_d$ . The combination of  $u_d$  and  $a_d$  determines the aggressiveness of the operation and the achieved openness of grinding wheel surface: small values ( $u_d < 2 - 3$ ) create open surfaces and large values ( $u_d > 7 - 10$ ) close surfaces. However, these values also depend on the dresser radius and the grit size.

$$f_d = \frac{\pi d_s v_d}{v_s} \quad (2.9)$$

$$u_d = \frac{b_d}{f_d} \quad (2.10)$$

#### Dressing mechanisms

The dynamic friction and the fragile abrasive-bonding material fracture increase the complexity of dressing. In the literature most of the authors agree on the description of the dressing mechanisms [PA54, Vic70, MC71, Bha74, Che95, Hes03, MRDI04, KL08]: abrasive breakage due to induced cracks and bonding

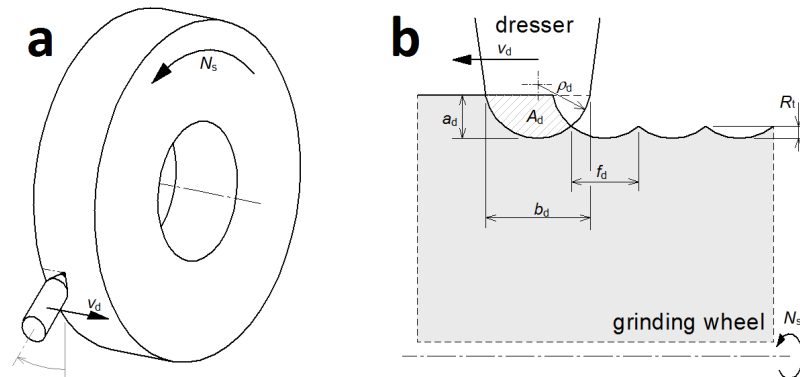


Figure 2.5: Kinematics of stationary dressing: (a) the wheel rotates and the dresser (or the wheel) moves in the transversal direction; (b) detail of the cutting section [MRDI04]

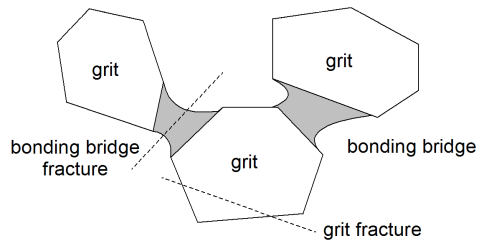


Figure 2.6: Abrasive grit breakage and bonding bridge fracture [Mal08]

bridge fracture releasing an abrasive grit. Figure 2.6 presents the two main mechanisms of material removal in dressing. Klocke and Linke [KL08] divide the second mechanism in two: failure of the grit-bonding material interface and breakage of bonding bridge. However the effect is the same: the grain is released. The depth of the engagement of the diamond grits, the collision number and their intensity determine the shape and the condition of the cutting edges on the surface layer of the wheel [BC95].

Pahlitzsch and Brunswick [PA54] simplify the dressing operation, assuming ductile cutting of abrasive grits by the dresser. The resulting helix of the dresser cannot be observed on the wheel surface, but Bhateja [Bha74] finds out traces of the dresser on the surface of the workpiece, relating the roughness and the dressing feed  $f_d$ . Figure 2.7 shows the resultant wheel surface, the combination of levelling effects and large fractures. Oliveira *et al.* [OBF10] take advantage of these phenomena, proposing an electro-mechanical exciter connected to the dressing tool receives synchronised signal from a control software engraving patterns on the grinding wheel. In this way, the ground surface gets an engineered texture, which may improve its performance.

Malkin and Cook, in the context of an extensive work about grinding wheel



Figure 2.7: Surface created by the dresser on consecutive grains [CR96]

wear, analyse the size distribution of abrasive particles collected during dressing [MC71]. The size of most of abrasive material removed exceeds the dressing depth of cut  $a_d$ . Thus, they conclude that dressing is performed in two stages: abrasive grits are broken until they get a critical size, when are released from the bonding material. They notice that a higher grade of the wheel holds tighter the abrasive, promoting the grit breakage and reducing the critical size. In turn, it is easier to disengage grains of small size due to their higher toughness.

Taking into account these conclusions, they develop a model that estimates the probability of releasing a grit from wheel surface, based on the fraction of abrasive released during dressing, that is, the openness of the surface. To do that, they propose two new parameters: the fraction of abrasive in weight removed by bond fracture  $B$  and the average dressing particle dimension removed by bond fracture  $d_b$ .  $d_b$  reflects the aggressiveness of the dressing. Both concepts are related by Equation 2.11, where  $d_g$  is the average grain diameter. The cubic root arises from the three dimensional (3D) point of view. Equation 2.12 predicts the theoretical grain density  $G_o$  when a plane cuts the body of the wheel. It is estimated according to the volumetric abrasive fraction  $V_g$  and average grain diameter  $d_g$ . Assuming that the grains not available when dressing are released by bonding breakage, Equation 2.13 relates the surface grain density  $G_{dyn}$  and  $G_o$ . It describes the relationship between the size of an available grit for dressing in one dimension with the average grain diameter  $d_g$  and the fraction of abrasive removed in weight  $B$ .

$$d_b = B^{1/3} d_g \quad (2.11)$$

$$G_o = \frac{6V_g}{\pi d_g^2} \quad (2.12)$$

$$\frac{G_{dyn}}{G_o} = \frac{d_g - d_b}{d_g} = 1 - B^{1/3} \quad (2.13)$$

Malkin and Cook do not specify how to define  $B$  from the analysis of the size distribution of dressing particles. Nevertheless, the definition of  $d_b$  is easy measuring  $G_{dyn}$ , counting the reflections of wear flats during grinding. In this

way, the most protuberant elements smaller than  $d_b$  are removed from wheel surface.

Chen, in his PhD thesis [Che95], studies the grain breakout regarding the dressing force. To release a grit, the dressing force will pull out a grain if the force surpasses the grain retention strength. He concludes by means of experimental tests, that the dressing force  $F_d$  has two components: the fracture force  $F_f$  and the friction force  $F_r$ . Equation 2.14 introduces this model, where  $\sigma_s$  is the strength of the wheel,  $A_d$  the dressing section,  $a_d$  the dressing depth and  $f_d$  the dressing feed. The value of  $\sigma_s$  is defined measuring the dressing force  $F_d$ . A grit survives if  $F_d$  does not surpass the grit retention force  $F_g$  defined by Equation 2.15. As  $A_g$  is the section of the remaining grain, the model obtains small ratios of grit breakout. Even though  $\sigma_s$  is easy to determine through dressing forces measurements, the model is not directly related to the obtained topography.

$$F_d = F_f + F_r = \sigma_s A_d + F_r = \sigma_s a_d f_d + F_r \quad (2.14)$$

$$F_g = \sigma_s A_g \quad (2.15)$$

Torrance and Badger [TB00] develop also a model that predicts the surviving fraction of abrasive  $f$  after a dressing pass. The model foresees the grain density  $G_{\text{dyn}}$  after dressing (Eq. 2.16). The parameters of the model are the dressing parameters (dressing depth  $a_d$ , feed  $f_d$  and diamond tip radius  $\rho_d$ ) and grinding wheel characteristics (characteristic strength of the bond  $F_o$  and crushing stress of the abrasive  $\sigma_c$ ).

$$f = \frac{42\sigma_c a_d f_d}{F_o \sqrt{\rho_d}} \quad (2.16)$$

Torrance's model is complete and describes well the effect of dressing aggressiveness. However, it is difficult to define the mechanical characteristics of the bonding material and abrasive. Moreover, these properties are strain-rate dependent parameters in brittle materials. This model does not take into account the topography either.

### 2.1.3 Topography of the grinding wheel surface

In grinding, the surface topography describes the geometry and features of the wheel surface. The topography is the result of the interactions between the dresser and the abrasive surface layer. The number of strikes and their aggressiveness determines the resultant surface.

Both, macro- and micro-topography are important from different points of view. The basic geometry of the grinding wheel is related to the macro-topography and it is the responsible of the deviations on the workpiece surface. The micro-topography of a grinding wheel defines the efficiency of the material removal, due to the influence on the workpiece surface roughness, grinding energy, wheel life.

The topography reproduces the cutting edges sharpness and distribution. It depends on the following variables: grain size, wheel structure (abrasive fraction), dressing conditions and the wear of cutting edges during grinding. Section 2.3.2 reviews the main topography models proposed up till now.

### Topography measurement techniques

The measurements methods are closely related to the measured parameters and the complexity of the surface characterization. Techniques for measurement of grinding wheel topography have been periodically reviewed by several authors [Ver77, Sha96, MRDI04, DWB13]. Surface measurement techniques are classified in two types: direct and indirect methods [MRDI04]. Direct methods are the ones that observe directly the surface topography physically: profilometry, and optic or electronic techniques. Indirect methods deduce the topography studying the variations on grinding forces, temperature or acoustic emissions. In the same way, measurement techniques are classified in static and dynamic methods. In static methods, the wheel stands still during the measurement (stylus, optical or electronic microscope, carbon paper). In turn, dynamic methods obtain information of the grains that take part in the grinding operation. The measurement is done when the wheel is just rotating (acoustic emissions), grinding (razor blade, thermocouple, applied power source, collecting chips) or stopped after grinding (wear flat reflection).

Topography is characterised by one or more parameters, that coincide with the dimension of the model (1D, 2D and 3D). A single characteristic, like grain density, grain sharpness or wear flat fraction, offers valuable information about the surface in a shoot. Indirect and dynamic methods mainly assess the surface openness, defining the grain density. 2D and 3D measurements offer a more tangible description of the topography: 2D captures the transverse cut of the surface and 3D reproduces the topography as a surface function. Both lead the definition of 1D parameters, like grain density, cutting edge height or sharpness. Grinding wheel surfaces can be described by the fractal dimension too [Lia95]. However, the fractal dimension demonstrates a weak relationship with the abrasive potential of particles and, therefore, it is not a suitable measure of the abrasiveness of a wheel [HS95].

Verkerk, in a collaborative work of CIRP [Ver77], compiles and compares the topography measurement methods used in that time. He reports that the cutting edges that belong to the same grain or to adjacent grains might be considered as a single cutting edge, because of the lack of clearance needed for chip formation. In this way, abrasive grits are classified as static and dynamic (or kinematic or active) regarding their participation on material removal according to their depth and distance between them (Fig. 2.8). Stylus 2D profile methods prevail to measure the average static distance between grains  $L_{st}$ , which is used to estimate the static grain density  $G_{st}$  applying empirical models. The carbon paper also measures  $G_{st}$  counting the contacts created rolling a freshly dressed wheel. However, it can not capture the variation of  $G_{st}$  thought depth accurately. On the other hand, wear flats counting and razor blade dynamic methods

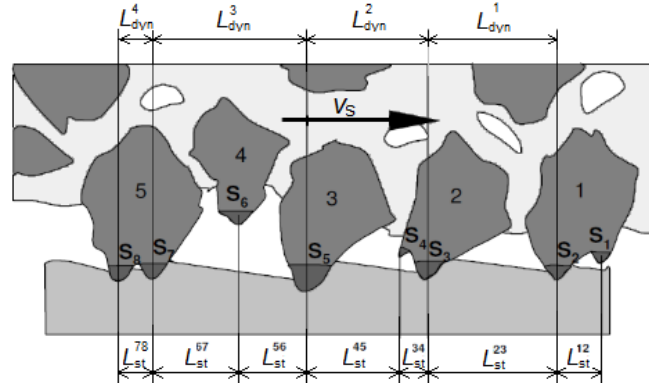


Figure 2.8: Static and dynamic distances between cutting edges ( $L_{st}$ ,  $L_{dyn}$ ) regarding the speed ratio ( $v_s/v_w$ ) [Klo09]

are the main techniques to measure the dynamic parameters. Particularly, the razor blade method counts the force peaks  $N_c$  created when a thin sheet is ground. This allows the definition of the average distance between grains  $L_{dyn}$  (Eq. 2.17) and the grain density  $G_{dyn}$  (Eq. 2.18) [BS74]. In these Equations,  $t_c$  is the contact time,  $v_s$  the cutting speed and  $b_s$  the width of the razor blade (or workpiece). This method can be substituted by anyone that emits a signal when a contact is detected, as the applied source power. The counting of chips removed during grinding could also estimate  $G_{dyn}$ , but not all grits that contact the workpiece remove material.

$$L_{dyn} = \frac{t_c v_s}{N_c} \quad (2.17)$$

$$G_{dyn} = \frac{N_c}{t_c v_s b_s} \quad (2.18)$$

The shape of the cutting edge defines the ability of remove material (sharpness) and contributes to the severity of wear. How these peaks and their spatial arrangements contribute to abrasion is an interesting but complicate problem due to the substantial variations of shape exhibited by dressed wheel surfaces. Numerous techniques have been presented for characterising the sharpness of cutting edges (Fig. 2.9): the degree of roundness or sharpness radius, the defined cutting angles or angularity methods, and the cone-fit analysis or projection ratio.

The first two are cited in the literature to define the sharpness of conventional dressed wheels. Figure 2.9(a) shows the circumscribed circle of radius  $\rho_s$  that represents the cutting edge geometry [Mas52]. The degree of roundness regarding the particle size, the scale of the protrusion and the scratching depth, can be related to erosion too. The technique combines the radius of the particle protrusion with the particle radius. For the dressed wheels, the selection of the protrusion is evident, the most prominent one; but the determination of the



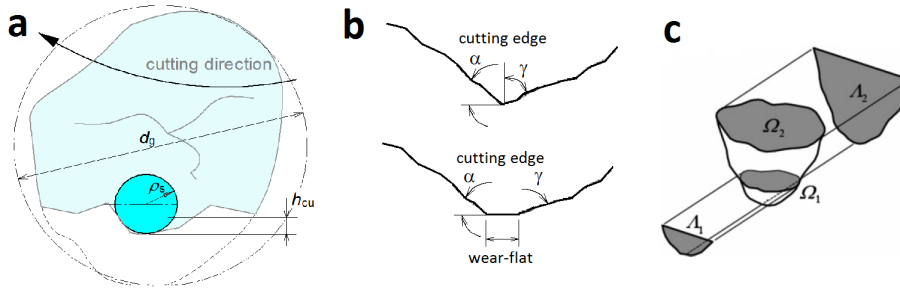


Figure 2.9: Characterisation of the cutting edge sharpness: (a) approximation of the cutting edge with a circumscribed circle of radius  $\rho_s$  [Mas52], (b) analogy of a defined cutting edge geometry [Klo09], and (c) relationship between horizontal and vertical projections [DPS02]

protrusion radius is rather subjective, because depends on the depth. Sharpness is defined as the uncut chip thickness-sharpness radius ratio ( $h_{cu}/\rho_s$ ) [Klo09] or grit-cutting edge radius ratio ( $d_g/(2\rho_s)$ ) [BT93].

The sharpness of a cutting edge can also be defined assuming that has a defined geometry, like turning or milling (Fig. 2.9(b)). For grinding wheels, the average value of the rake-angle  $\alpha$  [Klo09] or slope [TB00] is enough to characterise the sharpness. The effect of the wear can also be implemented to the model. Very negative rake angles are typical in abrasives ( $> 60^\circ$ ). Angularity methods have the same subjective formulation of the parameter, i.e., the definition of the reference depth.

The cone-fit analysis defines the sharpness as the horizontal to vertical projections ratio ( $\Lambda/\Omega$ ), which varies according to the orientation and depth of the measurement (Fig. 2.9(c)). This technique is applied successfully on the characterisation of the sharpness of entire abrasive grits, such as CBN and diamond grits, but it has not described a dressed alumina wheel yet [DPS02]. As the dressed surface follows a pattern, it would be an interesting research line to analyse the implementation of this method on vitreous or resin bonded alumina wheels.

Even though sharpness is a key parameter in grinding performance, its experimental measurement has been vaguely described in the literature. Lal and Shaw [LS75] measure  $\rho_s$  regarding the transverse width-to-depth ratio of the trace created by the single-grit test. They assert that only the penetration has influence on  $\rho_s$ . In turn, the feed and cutting speeds, workpiece material and cutting fluids have no effect on  $\rho_s$ . They obtain high  $\rho_s$  values (75 - 500  $\mu\text{m}$ ), probably due to the single-grit testing conditions do not reproduce the actual grinding conditions of the dressed wheel surface. They do not describe the geometry of the cutting edge of the tested grit.

$\rho_s$  and the cutting edge slope are also measured analysing 2D profiles of wheel surfaces [KL75, BT93]. König and Lortz [KL75] measure the fifth length of the perimeter of the grinding wheel with a stylus, obtaining a representative

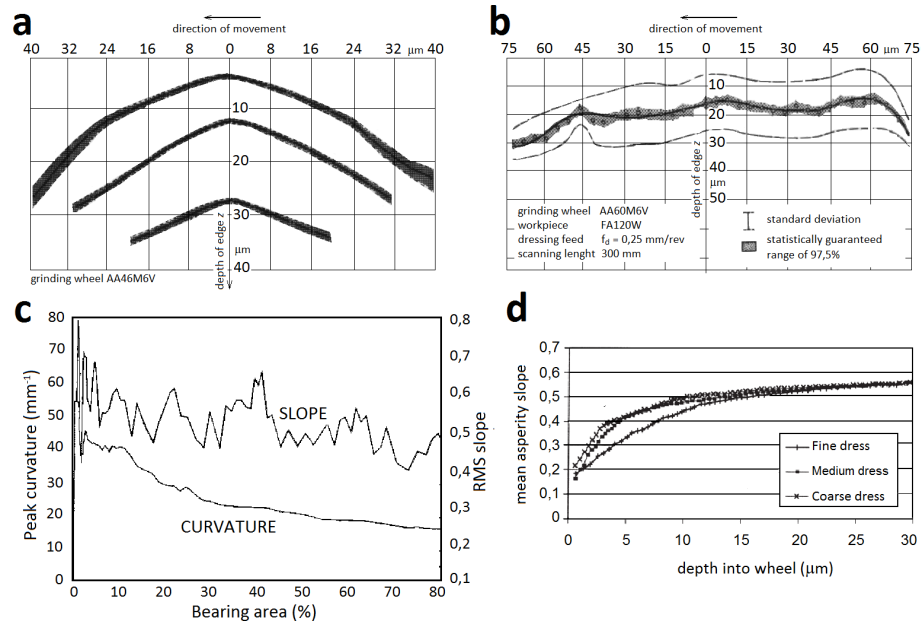


Figure 2.10: (a) Average profiles of the cutting edge and (b) grain according to the depth [KL75]; (c) peak slope and curvature regarding the bearing area after grinding [BT93]; and (d) peak slope regarding dressing conditions [BT00]

the 2D profile. They do not define directly the sharpness, but they present the average profile observed regarding the depth at cutting edge and grain scale (Fig. 2.10(a,b)). The cutting edge profiles allow to estimate  $\rho_s$  and  $\alpha$ , whereas the grain profiles seem noticeably flat due to the action of the dresser (Section 2.1.2). Brenner and Torrance [BT93] analyse several 2D profiles to get the slopes and curvature ( $1/\rho_s$ ) according to the depth. The results are used to study the influence of the sharpness on grinding forces. After each force test, the wheel (wet in acetone) is passed on acetate foils to replicate the surface topography and, then, the profile is measured on the acetate replica by means of the stylus method. The value of the slope oscillates regarding the bearing ratio, while the maximum curvature is observed on the peaks, decreasing slightly through the depth (Fig. 2.10(c)). Other works relate the slope regarding the depth with direct 2D profile measurements with stylus [Sha96, BT00, TB00]. Figure 2.10(d) shows the variation of the slope regarding the depth and dressing condition. The three dressing conditions show a similar blunt slope at the beginning. The slope suffers a steep growth, becoming constant as the depth increases. The three of them converge within the first 12  $\mu\text{m}$ , but coarse dressing is steeper than fine in shallow depths. In combination with the grain density, this facilitates the material removal. All these measurements start out from a random 2D profile measurements, that are not necessarily aligned with the peaks.

Nobody has compared the  $\rho_s$  measurements made with single-grit test [LS75]

and the 2D profile [BT93]. The first measures the consequence of the grit action based on the trace width, and the second directly the 2D profile in the cutting direction.

3D topography measurement methods (optical microscope, 3D stylus and scanning electron microscope (SEM)) get almost all the details of the surface [DWB13]. Their raw information is used mainly to obtain 1D parameters and 2D profiles of the wheel surface. In addition, they provide bearing ratio curves of the surface, which are directly related to the contact stiffness. The raw surface is also used as input in kinematic models that foresee the surface roughness in the workpiece [WZ98]. The steep slopes of grits and voids on the wheel surface pose a problem on 3D measurements. In addition, optical confocal microscope shows difficulties with the reflection of light, detecting points out of the target surface by mistake. Data treatment software solves these drawbacks, creating a 3D surface by interpolation. The required small specimen size and micro-scale measuring surface make SEM not adequate for grinding wheel characterisation.

Optical non-contact measurement approaches include auto-focusing systems, white light interferometers and confocal microscopes. Inasaki [Ina96] uses an auto-focusing system that rely on the fact that the intensity of light reflected from an object. Thus, the in-focus distance can be used to determine the height of a cutting edge. Yan *et al.* and Darafon *et al.* [YRJZ11, DWB13] use a white light interferometer to measure a sample of a grinding wheel. The height of an object is determined examining the interference patterns produced when white light from reflected object of interest and a mobile reference object. Zhang *et al.* [ZXF06] use a confocal microscope, taking two images of the same area from different locations. The height of the cutting edges are determined by triangulation. The surface needs a reflectivity higher than 15 %, but high reflectivity also brings problems, because points out of the surface are detected. Its main features are the wide depth range and the resolution of the measurements (up to 0,1 nm).

The non-contact techniques use relatively small measurement volumes and can only measure replicas or require a destructive preparation of wheels samples. Darafon *et al.* [DWB13] measure the whole diameter of a wheel with a tailor-made instrument. As the sample is large, the captured data amount is significant. Bearing surface, cutting edge density and average distance between cutting edges. The technique seems not to be sensitive on shallow depths, even if a low filter is applied. However, the results show the actual nature of a grinding wheel topography.

The use of replicas avoids reflection problems. But replicas are mainly used because of size issues and the inability to perform *in situ* measurements in the grinding machine to monitor evolution of the wheel surface [MRDI04]. A plastic compound or a soft metallic alloy is used as a replica material. The replica material is applied on the target surface. Once cured, a negative of the surface is obtained. It can be measured directly or used as a mould to get the positive with a similar compound. The replica is measured by interferometer or stylus means, avoiding the problem of specimen size. George [Geo79] uses epoxy resins, Eckert [Eck96] water-based polymers and Cai [CR04] a double

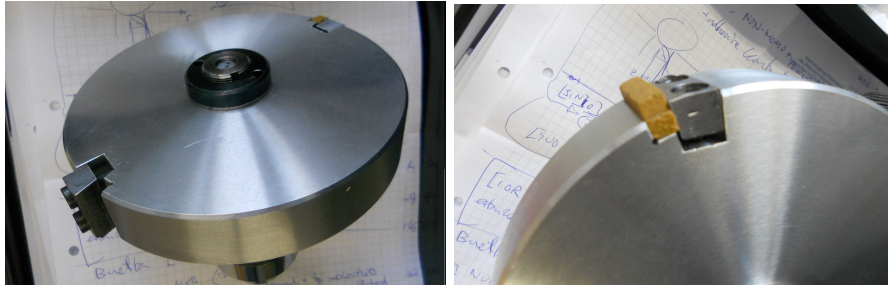


Figure 2.11: The dummy wheel and a CBN wheel specimen

density rubber. The three of them have demonstrated that the correspondents materials are capable of capturing the details of the complex wheel surface. If the replica material is soft, it can not be used to measure by means of stylus method. If the grain size is bigger than 80, surfaces voids are large too, and the replica material breaks out when is taken out from the surface. Brough *et al.* [BBR80] press soft lead on the surface. This technique requires an accurate control on the application force and time.

The RWTH of Aachen designs a dummy wheel to prepare and measure dressed CBN wheel specimens directly (Fig. 2.11). The size of the specimen is adequate for any measurement method, even for SEM. However, the specimens are just freshly dressed, they have not ground the workpiece material. That is, cutting edge wear and chip morphology cannot be studied. However, there is no replica technique able to capture the grit wear.

## 2.2 Grinding contact length

As explained in Chapter 1, the contact area between the grinding wheel and workpiece in surface grinding is determined by the width of the pass  $b_s$  and the contact length  $l_c$  (Fig. 2.12(a)). Usually the contact area is directly characterised by  $l_c$ , because  $b_s$  remains constant in the regular grinding operations. Studying the kinematics of the process and assuming that both, wheel and workpiece are rigid, the geometrical contact length  $l_g$  is deduced (see Eq. 2.3). However, experimental measurements show that the contact length  $l_c$  is 1,5 - 3 times larger than  $l_g$ , mainly due to deflections. The development of a method to estimate the contact length has been a matter of research interest for those groups concerned in the modelling of grinding to enhance the optimisation of the operation.

Figure 2.12(b) shows the phenomena that are combined in the contact area [BSS71]. On the one hand, the machine spindle deforms due to grinding forces and, thus, the effective depth of cut  $a_e$  is smaller than the set depth of cut  $a$ . That reduces directly the contact length. On the other hand, the wheel deforms under contact forces too, enlarging the contact length. The recoverable elastic and permanent plastic deformations in the workpiece are taken into account,

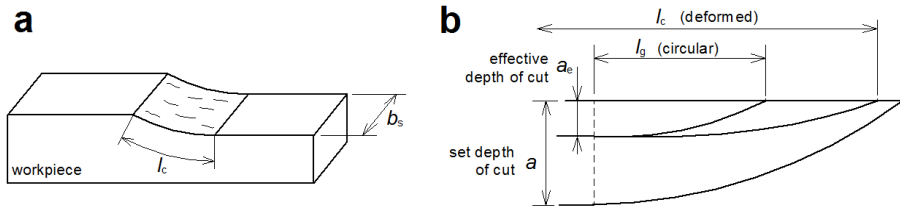


Figure 2.12: (a) Definition of contact length in grinding [MRDI04]; (b) deformations at the wheel-workpiece interface [Sha96]

since workpiece and wheel stand the same force distribution. The combination of the three effects enlarges the contact length.

The contact in grinding is non-conform, the contact is given only in the engaged grits that remove material. Thus, the contact length defines the apparent contact area defined in Figure 2.12(a). In the literature, some authors refer to *real contact length* to designate the *apparent contact length*  $l_c$ . In this work,  $l_c$  is simply named *contact length*.

The following list compiles the influence of grinding parameters in  $l_c$  experimental measurements observed by some authors [GW88, ZV92, QRM97b]:

- the contact length is larger when cooling fluid is used than in dry grinding, due to the reduced friction and the apparition of hydrostatic pressure,
- higher work speeds increase contact lengths as well, due to the increase of normal forces,
- higher cutting speeds leads slightly shorter contact lengths,
- the larger grinding wheel diameters enlarges the contact length,
- the enlargement of the depth of cut, grinding forces and temperatures on the cutting zone (the three of them are related) enlarges the contact length,
- the smoother wheel surface (fine dressing condition, small grit size, wear, loading) the contact length decrease, even though the normal forces increase, and,
- the larger depth of cut decreases the ratio  $l_c/l_g$ .

### 2.2.1 Contact length models

The problem of estimation of  $l_c$  has been tackled from empirical and semi-analytical approaches. The empirical models are the most simple [Mar77, STH83, ZV92]. Lindsay [Lin71] develops a spring model. Semi-analytical models [SS74, ZSNW93a, KS81] stand partially on Hertzian mechanics: smooth surfaces and elastic contact. [BSS71, RMQZ93, HWCC11] take a step ahead

studying the contact in two scales: the individual grain workpiece contact and grinding wheel deflection. Table 2.1 compiles the main contact length models in grinding.

Maris [Mar77] proposes the first empirical contact model, adjusting the expression according to the depth of cut  $a_e$  and speed ratio ( $v_s/v_w$ ). This model has a limited application and does not take into account aspects such as topography. Salje *et al.* [STH83] use the depth of cut as well as the workpiece surface roughness, which has implicitly characteristics of wheel topography. Zhou and Van Lutterwelt [ZV92] propose the simplest empirical model: the sum of an empirical constant and the geometric contact length. The model fits well the experimental results, and it is interesting for a given wheel-workpiece-machine combination. They add an interesting discussion about the uncertainty in contact length measurements, due to dispersion in individual grit contacts.

Lindsay [Lin71] develops a spring model, where each grain behaves as a spring when contacts the workpiece. The stiffness of each spring depends on the hardness grade  $HL$  and structure  $SL$  of the grinding wheel. The model takes into account the grit size and the normal force, but disregards the depth of cut. This model estimates shorter  $l_c$  values than the measured ones.

Some semi-analytical contact models are also found in literature. Zhang *et al.* [ZSNW93a] assign to the wheel elastic deformation the main role in the contact enlargement. Assuming that the deformation curvature is elliptic, the function of the deflection curve is differentiated twice to obtain the expression of the contact length. A single experimental parameter fits the expression to take into account the disregarded factors as surface topography. This parameter can be defined by only one measurement.

Shaw, with Sauer [SS74] and Kumar [KS81], develops two approaches based on Hertzian mechanics, considering the grinding wheel as a continuous homogeneous disc of smooth surface and assuming elastic contact. The two of them need empirical coefficients to fit the results. Both include the elasticity of the grinding wheel, the normal force and the depth of cut. It is remarkable that [SS74] use the average chip thickness and indentation, although the model needs three constants to estimate them. [KS81] estimates 15 % larger contact lengths than the geometrical contact length, too short comparing with experimental measurements.

Brown *et al.* [BSS71] and Rowe and Qi [RMQZ93] study the contact in two scales and combine the effects: the individual grain workpiece contact and grinding wheel deflection on the workpiece. In that way, wheel workpiece stiffness as well as wheel topography are taken into account. Brown *et al.* develop a complex iterative expression that takes into account, beside the Young's modulus of the wheel, the grit size, the depth of cut and the surface grain density. It does not need an empirical coefficient to estimate the contact length, but its results are smaller than those experimentally observed. This high stiffness is probably due to the elastic contact assumption. Rowe and Qi [RMQZ93] modify Brown's model, using the Greenwood-Tripp (GT) rough contact model [GT67] combined with the geometrical study of contact arcs. Wheel topography is taken into account by the  $R_r$  roughness factor, which is defined experimentally.

Table 2.1: Summary of the main contact length models published in grinding

Model	$l_c/l_g$	Contact length $l_c$ expression	Notes
Maris [Mar77]	2	$\left(\frac{u_k}{v_w}\right)^{C_1} e^{C_2 \ln a_e} \sqrt{a_e d_s}$	Empirical
Zhou, Van Luttervelt [ZV92]	2 - 3	$C_1 + \sqrt{a_e d_s}$	Empirical
Sajje <i>et al.</i> [STH83]	1,1 - 1,4	$\sqrt{1 + \frac{R_a}{a_e}}$	Surface roughness
Lindsay [Lin71]	1,33	$10^{-6} \sqrt[3]{\frac{d^2 F_n}{d_g^2 (1.33HL+2.2SL-8)}}$	Elastic spring model
Sauer, Shaw [SS74]	1,3 - 1,7	$\sqrt{1 + 4 \cdot \frac{E^* F_n + (\delta - h_{cu})}{a_e}} = \sqrt{a + C_1 \frac{F_n^{C_2}}{a_e^{C_3}}}$	Hertz, $h_{cu}$ chip thickness and $\delta$ penetration
Kumar, Shaw [KS81]	1,1 - 1,2	$\left(1 + \frac{C_1(1+\nu_s)d_e F_n}{E_s l_g a_e}\right)^{1/2} \left(1 + \frac{2F_n \ln(l_g/2)}{\pi E_s a_e}\right)^{-1/2}$	Hertz mechanics
Zhang <i>et al.</i> [ZSNW93a]	1,5 - 2,7	$R_d \arccos\left(1 - \frac{a_e}{R_d}\right)$ where $R_d = d_e \left(1 + C_1 \cdot \left(\frac{1-\nu_s^2 F_n}{E_s a_e}\right)\right)^{1/2}$	Hertz mechanics, iterative solution
Brown <i>et al.</i> [BSS71]	2 - 2,9	$2 \left(\frac{\left(\frac{9(E^*)^2}{8d_g}\right)^{1/3}}{a_e}\right)^{1/2} \left(\frac{F_n}{C_{st} l_c}\right)^{1/3} + 1,6 \left(\frac{E^* F_n}{a_e}\right)^{1/2}$	Hertz mechanics, contact in two scales
Rowe <i>et al.</i> [RMQZ93]	1,5 - 3	$\left(a_e d_s + \frac{8F_n R_d^2 d_s}{\pi E^*}\right)^{1/2}$	Hertz mechanics and contact in two scales with GT rough elastic contact model
Horn <i>et al.</i> [HWCC11]	1,4 - 2,2	$\left(a_e d_s + \frac{8E_s d_s}{\pi C_1 E^*}\right)^{1/2}$	Rowe-Qi model with elastoplastic contact

Based on Qi and Rowe's formulation, Horn *et al.* [HWCC11] apply a micro-contact model developed by them. Their approach assumes elastic-plastic contact between the spherical asperities and the flat surface. Even though this model is the most complete up-to-date, it also requires to define previously the roughness factor. It has not been validated experimentally. Hou and Komanduri [HK03] and Stepień [Stę09] models study the probability of contact along the contact length, although they do not estimate the contact length.

Summarising, the following characteristics are shared on these models, pointing out their importance. The main are the topography parameters [BSS71, RMQZ93, HWCC11], the depth of cut [BSS71, SS74, Mar77, KS81, STH83, RMQZ93, ZSNW93a, HWCC11], the normal force [Lin71, BSS71, SS74, KS81, ZSNW93a, RMQZ93, HWCC11] and the elasticity of the grinding wheel and the workpiece [Lin71, BSS71, SS74, KS81, ZSNW93a, RMQZ93, HWCC11]. Other parameters are the speed ratio [Mar77], the workpiece roughness [STH83] and the uncut chip thickness [SS74]. Factors not taken into account are packed into empirical coefficients in all models presented up to date, but [BSS71].

### Rowe and Qi (RQ) contact length model

Probably, the model developed by Rowe and Qi (RQ) is the most popular contact model in grinding. They present the RQ semi-empirical model in several papers [RMQZ93, QRM97b]. The starting point is Brown's model [BSS71]: the contact length is the quadratic sum of the geometrical contact length and wheel-workpiece contact according to Hertzian contact conditions at the grit level. But they replace the static contact term by the Greenwood and Tripp (GT) rough contact model [GT67]. The model treats the contacts between spherical asperities statistically, assuming elastic contact. The effect of depth of cut is included with a geometrical study of contact arcs. In this way, the contact length is the combination of three effects (Fig. 2.13(a)): the geometrical contact length assuming a rigid smooth grinding wheel and a depth of cut of  $a_e$ , the elastic static contact between a smooth wheel and a flat surface, and the elastic micro-contacts between the abrasive grits and the workpiece.

Figure 2.13(b) introduces the geometrical analysis. Assuming elastic contact, smooth surfaces and unloaded,  $d_s$  and  $d_2$  are the correspondent diameters of the grinding wheel and the workpiece. Loaded, the contact arc gets the diameter  $d_3$  and coincides with  $ABC$ . If the grinding force is suddenly removed, the workpiece recovers the previous diameter  $d_2$ . Then, the point  $A$  is tangent to the curve  $ADC$  and  $a_e$  is the effective depth of cut. On the one hand, the contact length between cylinders  $l_c$  according to Hertz mechanics [Her1882] is given by Equation 2.19, where  $F'_n$  is the specific normal force and  $E^*$  the equivalent modulus of elasticity. Equation 2.20 offers the expression to calculate  $E^*$  regarding the Poisson ratio ( $\nu_s, \nu_w$ ) and modulus of elasticity ( $E_s, E_w$ ) of the grinding wheel and the workpiece respectively.

$$l_c = \sqrt{\frac{8F'_n d_s}{E^*}} \quad (2.19)$$



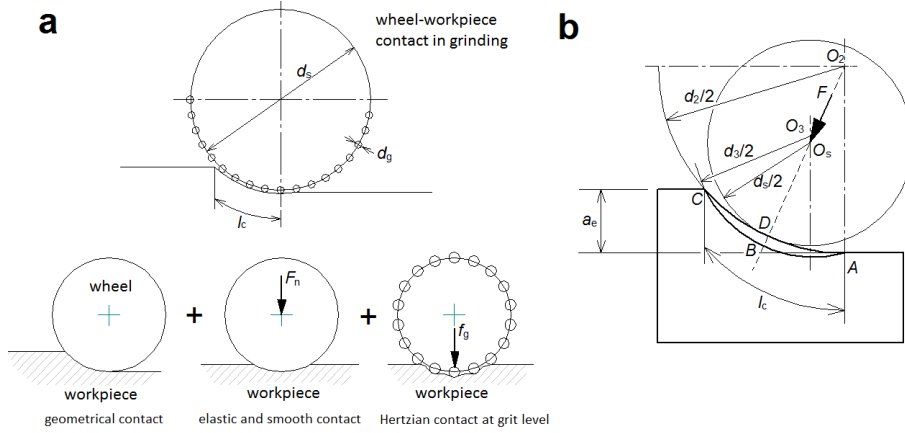


Figure 2.13: Analysis of the elastic deflection between the wheel and workpiece: (a) the combination of three effects, and (b) geometrical study of the contact arcs [RMQZ93]

$$\frac{1}{E^*} = \frac{1 - \nu_s}{E_s} + \frac{1 - \nu_w}{E_w} \quad (2.20)$$

The grinding force increases the wheel diameter from  $d_s$  to  $d_3$ . Equation 2.21 relates the wheel-workpiece pair diameters, regarding Hertz analysis of concave cylinders. And studying Figure 2.13(b), Equation 2.22 relates the workpiece diameter  $d_2$  and the contact length  $l_c$ .

$$d_3 = \frac{d_2 d_s}{d_2 - d_s} \quad (2.21)$$

$$d_2 = \frac{l_c^2}{a_e} \quad (2.22)$$

Combining Equations 2.19, 2.21 and 2.22, the expression of the contact length between wheel and workpiece for smooth surfaces and elastic contact is given by Equation 2.23,

$$l_c = \left( a_e d_s + \frac{8 F_n' d_s}{\pi E^*} \right)^{1/2} \quad (2.23)$$

Equation 2.24 stems from Equation 2.23, where  $l_g$  is the geometrical contact length and  $l_f$  is the static contact term due to the normal force.

$$l_c^2 = l_g^2 + l_f^2 \quad (2.24)$$

The effect of the tangential force is disregarded based on the conclusions of Johnson [Joh87]. This expression coincides with the one developed by Brown [BSS71], but differs on the treatment of the term  $l_f$ . The meaning of  $l_f$  is the

elastic contact between two smooth cylinders based on Hertz mechanics. Rowe and Qi replace the  $l_f$  term by the elastic contact between rough surfaces based on the Greenwood and Tripp (GT) model. That model treats the contact in two scales: on the one hand, the elastic contact between both bodies assuming smooth surfaces macro-scale and, on the other hand, the elastic contact between asperities. Hertz mechanics are applied on both. The GT model assumes that the height of spherical asperities follow the normal distribution and treats them statistically. The development of the GT model multiplies the  $R_r$  roughness factor to the  $l_f$  term.  $R_r$  gathers the characteristics of the surface roughness. Thus, the RQ model obtains the following compact expression (Eq. 2.25),

$$l_c^2 = l_g^2 + (R_r l_f)^2 = a_e d_s + \frac{8R_r^2 F_n' d_s}{\pi E^*} \quad (2.25)$$

In practice, the  $R_r$  roughness factor is defined experimentally for a given grinding wheel, dressing condition and workpiece combination. And the point is that, even though the use of the formula is easy, its accuracy depends on the  $R_r$  factor, which requires complex experimental work. According to the authors,  $R_r$  varies from 7 to 23 [QRM97b]. The RQ model applies the GT model out of application range. In addition, RQ considers the contact in the elastic regime, when the contact pressure intensity surpasses easily the yielding conditions [MAR04]. However, the RQ model obtains good correlation with experimental measurements, probably due to the contribution of bulk deformation is proportional in either, elastic or plastic regime and the empirical  $R_r$  roughness parameter absorbs all the uncertainties of wheel topography.

From the beginning, one of the research lines of this thesis has been to model the  $l_f$  rough contact length numerically regarding the approach proposed by Rowe and Qi.

### 2.2.2 Contact length measurement methods

Even though researchers have developed methodologies for a direct and indirect experimental measurement of the contact length  $l_c$ , there is not a dominant and clearly defined experimental method. Five methods evaluate the contact length: the thermocouple [Pek57], the applied power source (APS) [ZFG85], the quick-stop [SS74, BWW77], the patch grinding [BSS71, KS81] and the acoustic emissions (AE) [BKW13]. A certain level of uncertainty is inherent to all of them due to the dispersion (up to 20 %), beside difficulties in the interpretation of the measurements [GW88, ZV92, QRM97b, QRM01].

Conventional thermocouples measure the temperature studying the variation of electric voltage at the juncture of two conductors. The calibration of the relationship between the temperature and the voltage variation brings the value of the aimed temperature in an indirect way. Conventional thermocouples cannot be ground, so they are not able to measure directly the temperature on the surface. Furthermore, due to the thermal inertia, their response is not fast enough, so they can be used only in slow feed speeds.

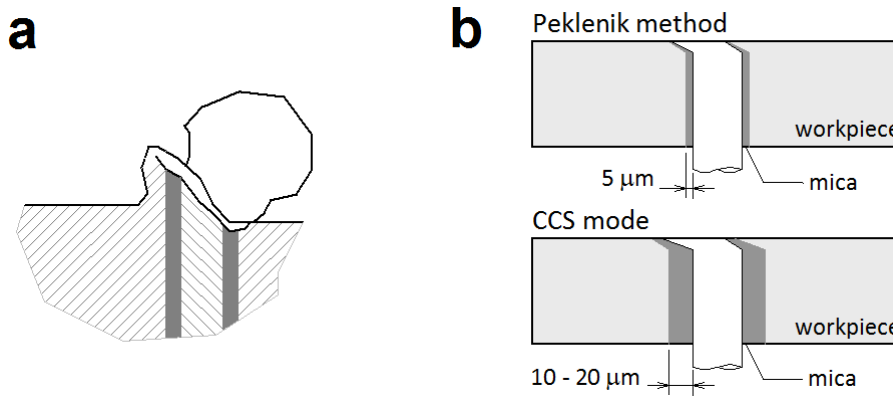


Figure 2.14: (a) Single-pole thermocouple: platinum wire embedded in the workpiece and when a grit passes through a hot junction is created with the workpiece [Pek57]; (b) the thickness of the isolation affects the measurements (Peklenik method and critical contact state (CCS) mode) [GW88]

Peklenik [Pek57] solves these setbacks, developing the single-pole grindable thermocouple. An isolated titanium wire is introduced in the workpiece and the other electrode is linked to the workpiece. Both electrodes are connected to an amplifier, which is plugged in a voltmeter. The circuit is open until an abrasive grain creates a hot junction when passes over workpiece surface (Fig. 2.14(a)). Each single-grit rubbing or cutting on the workpiece surface is observed as a peak in the induced electromagnetic force (EMF) signal, as shown in Figure 2.17. Studying the signal over time the contact length is estimated. An isolated platinum sheet can also be used instead of a wire. The thickness of the isolation and the width of the platinum electrode have direct influence on the sensitivity of the observed temperature, and the calibration is complex. The method even may become unstable too. When the contact area of the platinum wire and the workpiece material extends too deeply under the workpiece surface, the deep hot junction loses the evidence of individual grains attenuating temperature peaks. Thus, a thinner isolation can reach to form and maintain a hot junction spoiling the measurement.

On the one side, Verkerk and Pikelharing [VP75] and, on the other side, Gu and Wager [GW88], study the influence of thickness of the isolation on the EMF signal. To overcome the continuous hot junction and, thus, a continuous EMF signal, [GW88] defines two new states regarding the insulation thickness: the critical contact state (CCS) and the no contact state (NCS) (Fig. 2.14(b)). The hot junction is adjusted to a limiting contact or CCS by varying the thickness of the mica sheet, allowing grains to create their own thermal signal, and giving a clearer picture of the contact zone. Increasing further the thickness, the NCS mode obtains an EMF signal without hot junction related to the contact length, even though it is not valid to measure temperatures. The results obtained by the NCS and CCS modes are similar.

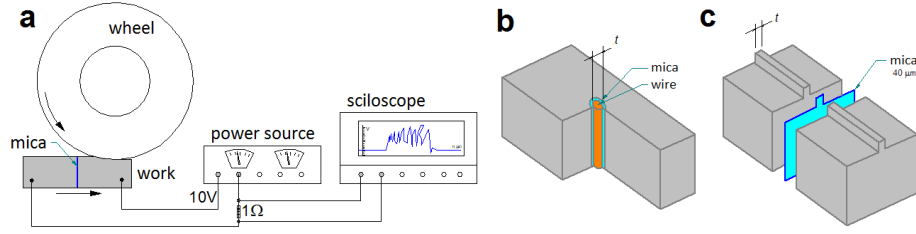


Figure 2.15: (a) Electric circuit of the APS method, where the power source is connected to both parts of the workpiece; (b) an isolated wire and the workpiece; and (c) the workpiece cut in two isolated by a mica sheet [ZV92]

Inspired on the hot junction phenomenon, Zhou *et al.* [ZFG85] develop an interesting variant of the grindable single-pole thermocouple, which measures, not the temperature, but just the contact length. The applied power source (APS) method, instead of the thermoelectric effect, connects the circuit directly to an electric potential source. During grinding, the successive abrasive peaks close the circuit when passing on the isolated wire or sheet. As the temperature measurement is not the aim of the test, the material of electrodes can be anyone, as long as it is conductor. The voltage signal is studied with an oscilloscope. The main advantage of the APS is the intensity of the signal, as well as the high response speed of the oscilloscope. The contact lengths measured by this method are 1,4 - 3 times larger than the geometrical contact length. Analysing the voltage-time APS signals the contact time  $t_c$  is defined. With  $t_c$  and the work speed  $v_w$ , Equation 2.26 calculates the contact length. In the same way, the counting of the number of pulses  $N_c$  allows the estimation of the dynamic grain density  $G_{dyn}$  (Eq. 2.18) and the average distance between grits  $L_{dyn}$  (Eq. 2.17). [GW88, ZV92] offer two variants of the method: the first one inserts an isolated wire in the workpiece, whereas the second cuts the workpiece in two separated by a mica sheet. Figure 2.15 shows the electric circuit of the APS and both variants.

$$l_c = v_w \cdot t_c \quad (2.26)$$

Table 2.2 compiles the APS test conditions and the observed  $l_c$  presented in the literature. It introduces the electrode width  $b$ , the insulation thickness  $s$ , the work and cutting speeds  $v_s$ ,  $v_w$ , the tension  $V$ , and the measured  $l_c/l_g$  ratio range.

The second main family of methods share the measuring principle: they analyse the traces of the wheel on the workpiece surface after a sudden stop of the process. On the one hand, Sauer and Shaw [SS74] by mechanical means and, on the other hand, Brown *et al.* [BWW77] by explosion, present the two variants of the quick-stop method. The workpiece retains the profile of the wheel, so the traces of the wheel can be studied directly on it (Fig. 2.16(a)). Both take into account the elastic recovery of the workpiece. Even though the

Table 2.2: APS test conditions in the literature

	$b$ ( $\mu\text{m}$ )	$s$ ( $\mu\text{m}$ )	$v_s$ (m/s)	$v_w$ (m/s)	$V$ (V)	$l_c/l_g$
Gu, Wager [GW88]	100	10 - 40	10	0,17 - 0,25	6	2 - 2,5
Zhou, Van Lutterwelt [ZV92]	-	(wire, sheet)	22	0,25	-	1,4 - 2,2
Qi <i>et al.</i> [QRM97a, QRM97b]	200	10 - 40	30	0,17 - 0,25	1,2	1,7 - 3 (5,5)

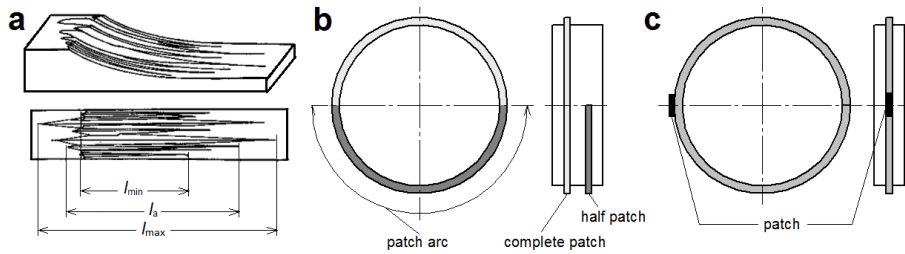


Figure 2.16: (a) Traces left by the wheel on the workpiece surface obtained with the quick-stop technique; definition of the maximum  $l_{max}$  and average  $l_a$  contact lengths [ZV92]; (b) grinding wheel used by Brown, Saito and Shaw for the two slots patch grinding method [BSS71]; (c) grinding wheel used by Kumar and Shaw for the patch grinding method [KS81]

methods are relatively simple, the interpretation of measurements is uncertain, because of the difficulties identifying the starting point of the contact.

The third family analyses the traces on the workpiece surface created by the patch generated on the wheel surface. Brown, with Shaw and Saito [BSS71], propose the patch grinding method. Two slots are made on the wheel surface: one covers the whole perimeter, while the second just half (Fig. 2.16(b)). After grinding with that wheel, the traces of both slots created on the workpiece are analysed regarding the correspondent trochoidal movement, comparing the entrance and exit curvatures. The higher curvature radius proves the elastic deformation of the wheel. The contact length is inferred from Hertz mechanics. Contact lengths measured with this method are 1,9 - 3 times larger than the geometrical contact length. Beside the difficulties of the preparation of the slots, the entrance and exit conditions are tougher than those in the real process, and, self-excited vibrations appear.

Again Shaw, this time with Kumar [KS81], modify the patch grinding method. Figure 2.16(c) shows the patch created on wheel surface after dressing. Then the trace created by the patch on the workpiece surface when grinding is analysed. The contact lengths observed by this method are 25 % larger than the ones obtained with the quick stop method. As the previous method, this method is not reliable, due to the harder conditions at the entrance and exit of the patch, which are just the areas of interest.

Babel *et al.* propose the newest method by means of acoustic emissions (AE) [BKW13]. The noise peaks on the entrance and exit of the wheel on

the workpiece surface are compared with and without wedge. Analysing the length of the peaks, the contact length is inferred. Although it seems simple, its complexity relies on the noise-signal filtering and interpretation.

Time has tipped the balance in favour of the single-pole thermocouple and APS methods, being extensively applied for the contact length measurement. The quick-stop and patch grinding techniques have not had further appearance in the literature, surely due to the complex set-up and interpretation of the measurement. The recent proposal of the AE method has not quoted yet for  $l_c$  estimation, but it may have a promising application in process monitoring.

### 2.2.3 Analysis of the contact length measurement signal

All authors discuss about the uncertainties in the measurement of the contact length and define a confidence interval in the definition of the contact length, regardless the measurement method used. Even under controlled testing conditions, the random distribution of the surface grains cause the variation of the contact length up to 20 %.

Zhou and Van Lutterwelt [ZV92] discuss about the uncertainties in contact length measurements due to the random nature of grinding. Thus, they define the average contact length  $l_a$  and the maximum contact length  $l_{\max}$  (Fig. 2.16(a)). The differences on the APS signals measured with wire-workpiece and workpiece-workpiece variants (Fig. 2.15), directly related to the width of the contacting electrodes, are associated with  $l_a$  and  $l_{\max}$ . The reason is that more grits are exposed and they close the circuit. Figure 2.17(a) superposes measurements made under same conditions varying the width of the contact: the downwards signal corresponds to  $l_a$  (isolated wire, Fig. 2.15(b)) and the upwards signal to  $l_{\max}$  (mica sheet, Fig. 2.15(c)). Therefore, punctual contact length measurements are shorter than using wide exposures. The difference between  $l_{\max}$  and  $l_a$  lies between 22 % - 55 %. This phenomenon can explain the dispersion of contact length values among authors, due to different interpretations in the lecture. In turn, Qi *et al.* [QRM97b] define  $l_a$  as the middle point of the rubbing area, between the end of peak concentration zone and the furthest pulse. With this criterion,  $l_{\max}$  is about 20 % longer than  $l_a$ .

Some authors identify the scratching, ploughing and cutting regions at measurement signals and grinding traces (Section 2.4.2). Gu and Wager [GW88] observe three regions on the single-pole thermocouple measurements in CCS mode, which coincide with the three zones identified by Kops and Shaw [KS82]. Figure 2.17(b) shows these three zones. According to the authors, in the sliding region the first reaction of the grit when it enters in contact is to deflect, rotate and slide on the workpiece surface. When the normal force of the grain is large enough and the metal is soft enough, the grain penetrates the workpiece and starts displacing material. In the cutting zone, the magnitude and the density of the peaks is considerable, which indicates that the hardness of the workpiece surface has dropped, many grains penetrate deep and material is removed. In the second sliding zone, peaks tend to disappear in the tail of the thermosignal. Grits are exiting and only the most protuberant create a hot junction.

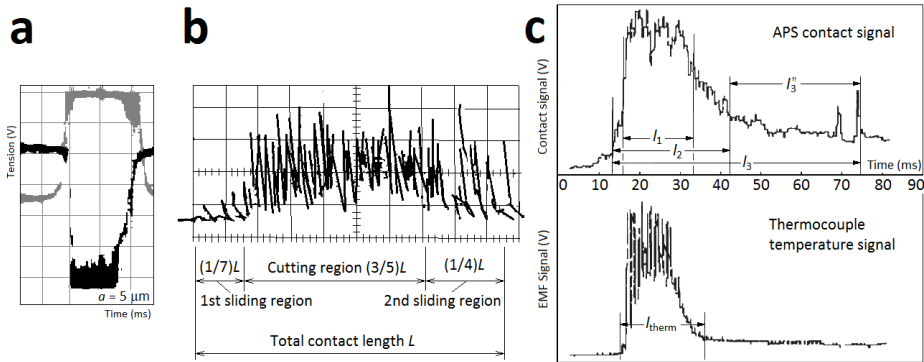


Figure 2.17: (a) APS signals of maximum  $l_{max}$  (upwards wave) and average  $l_a$  (downwards wave) contact lengths [ZV92] (see Fig. 2.16(a)); (b) regions on the contact length (CCS mode) [GW88]; (c) comparison of the signal obtained by the single-pole thermocouple and APS methods [QRM97a]

Qi *et al.* [QRM97a, QRM97b] use other point of view when analysing the APS signal. The three stages are identified but overlapped, as it happens in the actual process. Figure 2.17(c) shows their approach, where the cutting zone ( $l_1$ ) is inside the ploughing zone ( $l_2$ ), which is in turn within the rubbing zone ( $l_3$ ). They argue that the decrease of the signal intensity within zones is due to a smaller chip thickness or grain penetration, as well as a decrease of grain density.

Zhou *et al.* [ZFG85], Gu and Wager [GW88] and Qi *et al.* [QRM97a] compare the single-pole thermocouple and the APS method. Both methods share the no contact state, but differ from the signal source: the grit action induces the EMF signal in the single-pole thermocouple, whereas an applied voltage source creates the signal when a grit closes the APS circuit. The signals obtained by CCS and NCS methods are faint, being near that of the external interference. On the other hand, the APS method obtains a clearer signal. According to [GW88], APS and CCS results are a similar; [ZFG85] notices up to 10 % larger lengths with APS. In turn, [QRM97b] finds the single-pole thermocouple signal blur and unable to capture the rubbing contact ( $l_3$ ). In that way, the thermocouple signal yields a three times shorter contact length than the APS signal (see Fig. 2.17(c)).

## 2.3 Numerical models in Grinding

Up to date, no numerical model has been proposed to simulate the contact length in grinding. However, several numerical models have been presented for different purposes. Among them, thermal models predict the thermal damage on workpiece surface, wheel structure models study the stiffness and surface topography models, combined with wheel and workpiece relative movements,

estimate the surface roughness on the workpiece and grinding forces.

Grinding uses extensively the finite element method (FEM) and finite differences method in thermal models to predict the thermal damage on the workpiece [BAG<sup>+</sup>06, DWB09]. A few papers also introduce modal analysis for grinding by FEM. FEM also allows the simulation of the mechanical behaviour of a heterogeneous body in an extensive domain. Its main drawback is that it would be very difficult to mesh properly the porous structure of the wheel. In addition, the adjustment of the constitutive relationships of the material and contact conditions would be demanding. The use of multi-scale modelling would decrease the size of the model [KBCS09]. However, no FEM model of the wheel structure has been presented up to date.

The RWTH of Aachen [BMKW11] and Zhang *et al.* [ZGS<sup>+</sup>11] have developed similar mathematical models of the grinding wheel structure and topography. Both create a 3D grinding wheel model realistically, which can be treated by computer aided design. Grit geometry and dimensions are parametrised and placed in a random orientation and position by shaking. The bonding material covers the grits with a curvy smooth layer regarding the bonding material fraction, creating an accurate virtual morphology of the grinding wheel. As they model CBN wheels, surface topography is simpler since dressing mechanisms differ. Even both models assert that FEM will be implemented in the future, they have not been further developed. The required element size to describe the rounded geometry of the bonding material and pores, and difficulties in properties homogenisation are the main drawbacks for the use of them for FEM analysis. However, these models reproduce realistically the morphology and topography of the grinding wheel. The mathematical model of Zhang lead the development of the discrete element model of the grinding wheel presented by Li *et al.* [LYZW15a, LYZW15b]. Zhang *et al.* [ZYF<sup>+</sup>15] model a non-porous electroplated diamond wheel by FEM, to simulate the grinding of tungsten carbide with the software DEFORM. The abrasive grits and bonding material are modelled as one, assuming the elastic properties of the diamond. In turn, the workpiece material behaviour is considered viscoplastic.

This Section is focused on the structure models of the grinding wheel and topography models. The aim is to combine both in the contact model, wheel structure and surface topography.

### 2.3.1 Structure models of the grinding wheel

The performance of a grinding wheel depends significantly on its structure and morphology. Grinding wheel structure models overcome the grinding wheel's granular nature. The abrasive grits, the bonding bridges and the pores conform a heterogeneous no-continuous body. Material mechanical properties arise in the macroscopic scale regarding the micro-scale structure, the mechanical properties of each component as well as of their interactions (grains, binder and voids) [CJR13]. The structure models of the grinding wheel reproduce the 3D network that form the abrasive grits and the bonding bridges.

So far, models with noticeable simplifications have been proposed. They



describe wheel elasticity with uniformly distributed spherical grains bonded by elastic cylindrical springs. Moser [Mos80] develops a cubic uniform structure of springs and elements to model ceramic conglomerates. Hessel [Hes03] proposes a similar approach to model the truing of vitrified diamond wheels. The model calculates the maximum stress on bonds to decide if the grit breaks out. Yamada *et al.* [YMLM11] use a similar model to study the stiffness of resin bonded wheels (Fig. 2.18(a)). The model has a big mistake: it matches both, the distance between inner elements and surface elements, according to the surface grain density.

The models based on Moser's approach share the anisotropic behaviour, resulted from the uniform configuration of the elements. 3D micro-tomography analyses show that uniform models are far from the real granular random morphology [BMKW11]. Furthermore, the use of springs does not match the real behaviour of the bonding material, which does not allow the rotation at the interface.

Li *et al.* [LYZW15a, LYZW15b] present the first DEM model of the grinding wheel in two papers. The model studies the stiffness and resistance of the grinding wheel, the force chain in the bonding material and predicts the surface roughness of the workpiece as a kinematic model. It models the mechanical behaviour of a vitrified silicon carbide grinding wheel. Octahedron discrete elements (DE) substitute abrasive grains. The model is created by shaking uniformly distributed elements in the position and orientation like [CR96, KJL97, Dar13]. In order to reproduce bonding bridges geometrically and the binder volume fraction, several elastic beams placed in parallel play the role of a single bonding bridge, creating a complex redundant beam network. In this way, there are several binder spherical DEs between the octahedron abrasive DE. Regarding the elements that connects, there are abrasive-binder beams and binder-binder beams. Beam radius is set arbitrarily regarding a fraction of element diameter plus a normal dispersion component. Figure 2.18(b) shows the parallel beam structure.

The calibration of beam properties is made by numerical and experimental bending, brazilian and compression tests according to the macro-scale maximum load and stiffness. The beam fracture parameters and the initial maximum distance between DE to create a beam (that controls the coordination number (CN) or beams per DE) are the main calibration variables. The model is used to simulate the fragile fracture of the grinding wheel material under compression. Recording the AE signal in the test, the number of noise peaks is compared with the number of beam fractures, to define the maximum distance between DE to create a beam. This comparison is dangerous, because a noise peak does not mean *a priori* a breakage of a beam, that forms a bonding bridge together with many others. In addition, the resistance under compression is highly dependent on strain rate in brittle materials as the grinding wheel: the higher strain rate, the larger resistance. The stiffness (or elasticity) is adjusted regarding the load-time slopes in the experimental compression tests. Time is representative of the resistance but not of the stiffness, which is related to the deformation. Grinding wheels work at high cutting speeds, which enlarges remarkably the resistance of

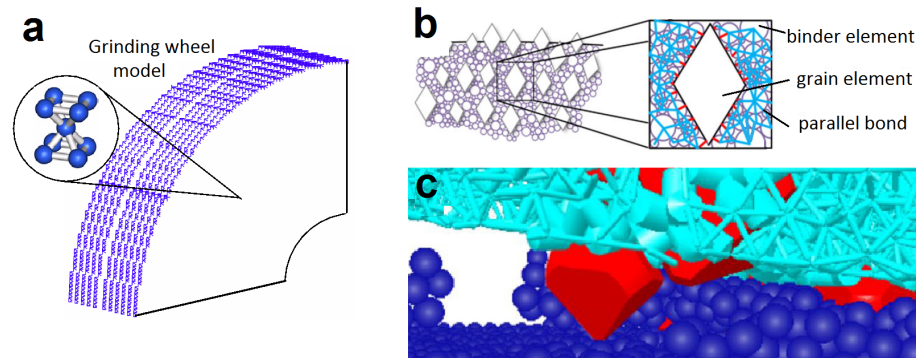


Figure 2.18: (a) Uniform spring model of the grinding wheel [YMLM11]; (b) redundant parallel beam network that models geometrically the bonding material [LYZW15a]; (c) numerical simulation of grinding with the DEM structure model composed of octahedron elements, bonding beams and workpiece elements [LYZW15a]

the conglomerate grit-binder. The model is used to analyse the force chains in the redundant bonding network. The results remark the role of the stresses in the tangential direction.

Beside beams, the model implements the Hertz-Mindlin contact detection algorithm, which is usually applied when modelling loose granular dynamics or compact solids. Due to the shacking method used in the creation of the model, the separation between DEs is large and the algorithm barely works between octahedron DEs, but binder-DEs. As the elements do not contact each other the geometry of the octahedron is also useless. Anyway, the contact algorithm would be interesting to apply on the modelling of wheel packing when manufacturing. They propose element clusters to model the random geometry of grits. This approach allows the modelling of the aspect ratio of grits, influential factor in the packing process of wheels and subsequent wheel structure; but at expenses of computation time. That is the reason for the use of octahedron geometry, because they only need a single element. Anyway, the interactions between elements get complexer as well.

The kinematic model simulates the workpiece roughness, but disregards the effect of the dressing. That is the reason for the differences between observed and obtained numerical surfaces. Figure 2.18(c) shows a snapshot of the simulation, where an abrasive DE surrounded by beams goes through the workpiece surface, composed of spherical DEs. The cutting force of each grain is estimated analysing the peaks of experimental force measurements. In this way, the relative position of the surface grit regarding the workpiece (or chip thickness) is neglected, as well as the effect of overlapped peaks. The relationship between workpiece elements is not explained. The authors claim that the model reproduces the material ploughing during grinding, but they do not clarify if the material is removed.

Nakayama [NBS71] takes the tough work of studying the stiffness of surface grains. Based on Hertz mechanics, develops the expression of the deflection of a surface grain. Chen and Rowe [CR96] and Hecker *et al.* [HRL03] apply Nakayama's model on their respective topography models. Thus, these models take into account surface stiffness, but not the stiffness of the wheel body. However, the approach is an interesting simplification.

Zhang *et al.* [ZSNW93b] find difficult to apply single-grit analytical models to a real abrasive process (see Section 2.4.2), since an extremely detailed description of the grain geometry is needed in order to determine how much the grain rubs, ploughs and cuts. Hence, they propose an iterative model composed by elastic springs that finds the balance regarding the cutting forces the removed material and the elastic deformations. The model output is the distribution of cutting forces along the contact length. But the effect of surface topography is disregarded, a key factor for the contact stiffness.

### 2.3.2 Topography models

The surface topography created by dressing has a direct impact on the grinding performance. Topography models are constructed regarding one or some features described in Section 2.1.3. They are mainly used on Kinematic models to estimate the workpiece surface roughness, uncut chip thickness or grinding forces, applying the relative movements to the surfaces created. Most of the topography models are rigid, so they are not able to model the deformation of the grinding wheel. As seen in Section 2.1.2, grain and bond fracture are the main dressing mechanisms. Dressing conditions modify the aggressiveness of dressing mechanisms, creating an open softer wheel or a close harder one. The topography model should be able to reproduce the surface topography regarding dressing conditions. The dressing, combined with the grit arrangement in the space (average grain size, grain size distribution and position), determine the main features of the surface: peak position and cutting edge sharpness.

Doman *et al.* [DWB06] classify topography models into three types: one, two and three dimension models (1D, 2D, 3D). 1D models characterise wheel surface by a single parameter, such as surface roughness or number of grains exposed. 2D models describe the profile of the grains geometrically. Thus, the grain size distribution, location, protrusion height and cutting edge shapes are studied. In 3D models, grains are spread on the surface with variable position and shape.

1D models are mainly focused on the estimation of the surface static and dynamic grain densities. As explained in Section 2.1.3, abrasive grits are classified as static and dynamic (or kinematic or active) regarding their participation on material removal. Tönshoff *et al.* [TPIP92] notice that 1D empirical models lay on the same four parameters: cutting edge shape, speed ratio, depth of cut and grain size. This remarks the importance of these parameters on the grinding performance, as they also appear on expressions that estimate the uncut chip thickness, grinding forces or ground surface roughness.

Topography models have become more and more sophisticate, reproducing

the randomness of the wheel surface. 2D and 3D models describe geometrically the grinding wheel surface, and this is the reason to focus the review in this kind of models. These models are classified as mechanistic (based on physical grinding features) [CR96, KJL97, WZ98, WZ98, Heg00, CL00, GWW02, NB05, CP08, BMKW11, ZGS<sup>+</sup>11, AK12, CGL<sup>+</sup>13, Dar13, LWBG13, ZYF<sup>+</sup>15], empirical (model coefficients are fit to experimental data) [Ina96] and probabilistic (the probability of occurrence of the factors is evaluated) [YA84, TB00, HK03, NB05, AR05, JGB<sup>+</sup>13] models. Mechanistic and empirical models create a virtual surface regarding some features of wheel surface. They can treat geometrical parameters like grit size and position as random or stochastic. In turn, probabilistic approaches reject this deterministic point of view and evaluate the probability of a grit-workpiece interaction from a stochastic characterisation of the wheel and its surface. Uncut chip thickness models apply normal or Rayleigh's probability density functions to forecast grinding forces and workpiece surface roughness. Rayleigh's function is defined by a single parameter  $\beta$ , which is difficult to determine and has no clear physical meaning [YA84, HL03, AR10].

Not all topography models include the effect of dressing. In some cases, it is justified because they aim to model super-abrasive wheels. Their grain density arise from wheel morphology [BMKW11, ZGS<sup>+</sup>11], or they introduce topography parameters, as the grain density [KJL97, ZYF<sup>+</sup>15]. Others simply implement topography parameters obtained in measurements [Ina96, WZ98]. Some of them disregards the dressing, creating a surface topography that depends only on the wheel morphology, which is a remarkable mistake [AR05, LYZW15b].

Koshy *et al.* [KJL97] introduce the statistical definition of surface parameters, creating a 2D model from 1D parameters. They use the shaking technique for the first time. A virtual environment of grains is created, displacing randomly a group of spheres uniformly arranged. Zhou and Xi [ZX02] and Cao *et al.* [CGL<sup>+</sup>13] propose simple 3D models setting a normally distributed height to a uniform plane mesh, obtaining a rough surface composed by pyramids without traces of dressing. These three models disregard dressing effects. Nguyen and Butler [NB05] generate a numerical surface based on the transformation of a random field, which is adjusted by means of the Johnson transform and the point-wise cumulative distribution function of the measured samples, without the need of physical parameters like protrusion height or grain density.

The probabilistic model of Jiang *et al.* [JGB<sup>+</sup>13] applies the effect of single-point dresser movements and wear in their kinematic model in a simple manner, disregarding the grit random fracture and pull-out. As the uniform dressing is applied on each grain regarding to its position (not regarding the nominal wheel diameter), the model obtains good results. Aurich and Kirsch [AK12] simply cut the protrusions at the reference diameter to model the dressing and wear of a galvanic bonded single-layer CBN wheel.

The models proposed by Chen and Rowe [CR96] and Torrance and Badger [TB00] are dressing models indeed according to the dressing conditions. Chen applies the effect of dressing using a sinusoidal random function that cuts the grit that interfere. The continuous sinusoidal function  $y_p$  has two components

(Eq. 2.27): the uniform helix of the diamond  $y_d$  and a random component  $y_f$  that reflects the fragile fracture of grits.  $h_f$  is the amplitude of the fracture term,  $\omega$  the random frequency and  $\alpha$  the random initial angle. The amplitude  $h_f$  is proportional to diamond-grain intersection area  $A_d$  and the dressing overlap ratio  $u_d$ , and inversely proportional to the dressing feed  $f_d$  (Eq. 2.28). Hege-man [Heg00] uses a similar random periodic function to adjust the ellipsoidal grain parameters and Darafon [Dar13] and Liu *et al.* [LWBG13] expand Chen's model to the 3D domain. Gong *et al.* [GWW02] modify Chen's dressing model, using a continuous random function composed with lines (like [TB00]) within an arbitrarily defined range. However, dressing is applied on every grit regarding its position, not regarding a dressing nominal diameter like [JGB<sup>+</sup>13].

$$y_p = y_d + y_f = f(x) + h_f[\sin(\omega x + \alpha) + 1] \quad (2.27)$$

$$h_f = \frac{A_d \cdot u_d}{4f_d} \quad (2.28)$$

Torrance and Badger [TB00] dress the surface according to the probability of grains that survive a dressing pass. Grain and bond fracture on the wheel surface are represented by a series of angled line segments. Their slopes are stochastically distributed between a maximum and minimum value, creating a rough surface composed of triangles.

Grain protrusion height has been treated as Gaussian [WZ98, ZX02, HK03, CP08, AK12, CGL<sup>+</sup>13], Weibull [TB00] and random [CR96, KJL97, GWW02, NB05, Dar13, LWBG13, ZYF<sup>+</sup>15] variable. Zhou, Hou and Cao use directly the grain diameter  $d_g$  standard deviation to define the peak height, a non-related characteristic. Warneke, Aurich and Zhang do the same but for single-layer CBN and diamond wheels. Koshy and Liu set arbitrarily the peak height within  $0, 1d_g$ ; Gong directly sets a value. Nguyen asserts that the height distribution of wheel topography is non-Gaussian after experimental topography measurements.

As explained in Section 2.1.2, dressing conditions determine the intensity of the operation and, hence, the fraction of grit breakout. Chen [CR96] and Torrance [TB00] take into account bond breakage too, according to a maximum stress defined experimentally (see Section 2.1.2). Darafon [Dar13] applies Chen's model on his 3D topography model. Nobody has used Malkin's model for grit breakout (Eq. 2.11, 2.12 and 2.13), but it can be perfectly implemented on a topography model.

The analysis of cutting edges sharpness starts from the study of abrasive grits morphology [JD11]. Figure 2.19(a) shows the basic grit geometries: cubic, octahedron, amorphous and the combination of them. Diamond and CBN take a structure between octahedron, tetrahedron and cube, facilitating the parametrization [WZ98, BMKW11, ZGS<sup>+</sup>11, AK12, LYZW15b, ZYF<sup>+</sup>15]. Alumina can be mono- or poly-crystalline, regarding the baking parameters and additives used in the manufacturing [MRDI04]. The crushing process and the crystalline structure determines the shape and the damage produced on the grains.

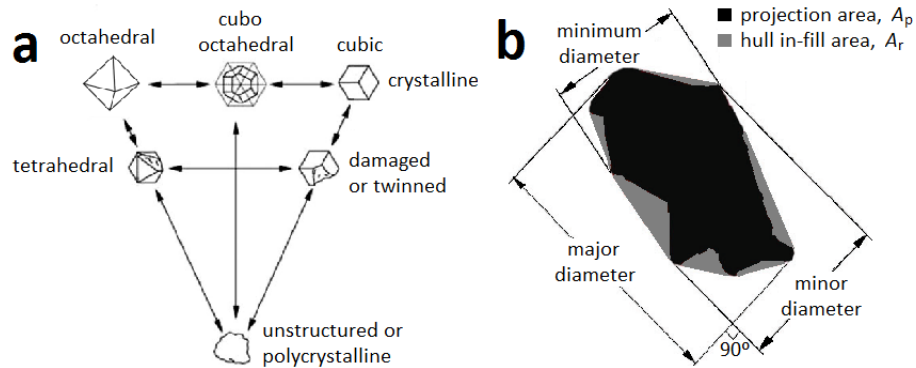


Figure 2.19: (a) Types of abrasive grit geometry [JD11]; (b) projections and shape factor of a grit to study its geometry and cutting ability [DPS02]

Mono-crystalline alumina forms faceted or plate-shaped crystals, whereas polycrystalline alumina is supplied as cubic (blocky), angular (tetrahedral like) and pointed (elongated) grains. Even though they share composition and structure, the chosen grain geometry modifies grain properties (friability, sharpness and wear). Individual grit projections, concavity-convexity analysis and aspect ratio (Fig. 2.19(b)) are used to evaluate the sharpness of abrasive grits. However, these parameters lose their meaning for vitrified alumina wheels after dressing. Thus, they are specially addressed to super-abrasive wheels and loose abrasive processes [DPS02, JD11]. The grain shape has also influence on the compacting process or structure openness of the wheel: the higher aspect ratio enlarges the technological limits of structure, beside the self-sharpening properties during.

Three model types are identified regarding the representation of abrasive grits and cutting edges: (1) abrasive grit models [KJL97, WZ98, HK03, CP08, BMKW11, ZGS<sup>+</sup>11, AK12, LYZW15b, ZYF<sup>+</sup>15], (2) cutting edge models [Ina96, Heg00, CL00, ZX02, CGL<sup>+</sup>13] and (3) abrasive grit-cutting edge combined models [CR96, TB00, GWW02, AR05, JGB<sup>+</sup>13, Dar13, LWBG13]. (1) The abrasive grits models are formed by uncut elements and they act directly as cutting edge. In this way, they disregard the effect of dressing. These abrasive grains are assumed spherical [KJL97, HK03], pyramidal [CP08], tetrahedron [BMKW11, AK12], hexahedron [ZYF<sup>+</sup>15] and octahedron [BMKW11, ZGS<sup>+</sup>11, AK12, LYZW15b] in shape. All of them take into account the grain orientation, but the spherical shape. (2) Cutting edges models only reproduce the cutting edges disregarding the grits. The chosen geometry for the cutting edges are conical or frustum [Ina96, CL00], pyramidal [ZX02, CGL<sup>+</sup>13] or ellipsoidal [Heg00]. Finally, (3) some models create first the granular structure with spherical grains and, then, prepare the surface placing a cutting edge on each surface grain regarding the dressing conditions. They take into account the grain pull-out too. [CR96, LWBG13, Dar13] assume spherical cutting edge geometry, [TB00, GWW02, JGB<sup>+</sup>13] pyramidal and [LWBG13] conical. Cutting edge and grit-cutting edge combined models reproduce the sharpness of the

surface. In grit models, the assertion is relative, depending on the assumed grit geometry and the need of dressing. Grit-cutting edge combined models, beside the wheel topography, model the granular structure of the wheel.

As explained in Section 2.1.3, sharpness is defined regarding the slope of the cutting edge and sharpness radius  $\rho_s$  (Fig. 2.9). Badger and Torrance [BT00, TB00] use the rake angle to define the slope of the pyramids. In turn, Inasaki and Cooper use the rake angle to define the slope of the cones [Ina96, CL00]. Wear is modelled either cutting the peak [AK12, JGB<sup>+</sup>13] or increasing the slope.

On the other hand, the cutting edge geometry is also defined by a circumscribed circle of radius  $\rho_s$ . As the rake angle of an abrasive grit is largely negative, the assumption of a sphere to represent the cutting edge is acceptable. In addition, the shape of the grit cutting edges continuously alters due to wear and fracture suffered during the grinding process, and  $\rho_s$  can be adapted to express the wear condition of surface grits. Among others, [CR96, Dar13] use  $\rho_s$  to define the sharpness. Some authors [KJL97, HK03, HRL03] use directly the average grit diameter  $d_g$  to define the cutting edge sharpness  $\rho_s$ , a remarkable mistake. The simplification of matching the uncut chip thickness and the sharpness radius of the protrusion is questionable too [AR05].

Topography models are rigid, except for [CR96, HRL03], which use springs on surface elements regarding the stiffness of surface grits studied by Nakayama [NBS71]. Thus, they model surface stiffness, but not the actual body stiffness. [AK12] develops a FEM-kinematic coupled model for a single-layer CBN wheel, that takes into account the deflection and temperature of the wheel.

### Framework of a 3D topography model

Doman *et al.* [DWB06], in their review of topography models, propose a general framework for a 3D grinding topography models. Figure 2.20 describes the structure of a physical topography model. The framework is divided in two parts: the first part creates an undressed grinding wheel, while the second part applies the dressing process to produce the final grinding wheel topography model.

The framework faces initially the topology of an undressed grinding wheel, according to the abrasive grit geometry, size, position and volumetric fraction. Usually numerical models represent grits with the simplest geometry, the sphere. However, other geometries, as ellipsoids or parametrised polyhedrons can be used too. Once grits are uniformly sorted, the aleatory position variations of them obtain the desired granular morphology.

The second part of the framework deals with the generation of the surface topography. The dressing mechanisms determine the outcome: grit fracture and bonding bridge fracture. The helix trajectory of the dresser is reflected subtly on the surface, cutting the grits that intercepts. In addition, abrasive grits suffer fragile fracture, even releasing from the binder regarding the dressing aggressiveness. The combination of both, wheel structure and dressing, creates the topography of the wheel regarding the dressing conditions.

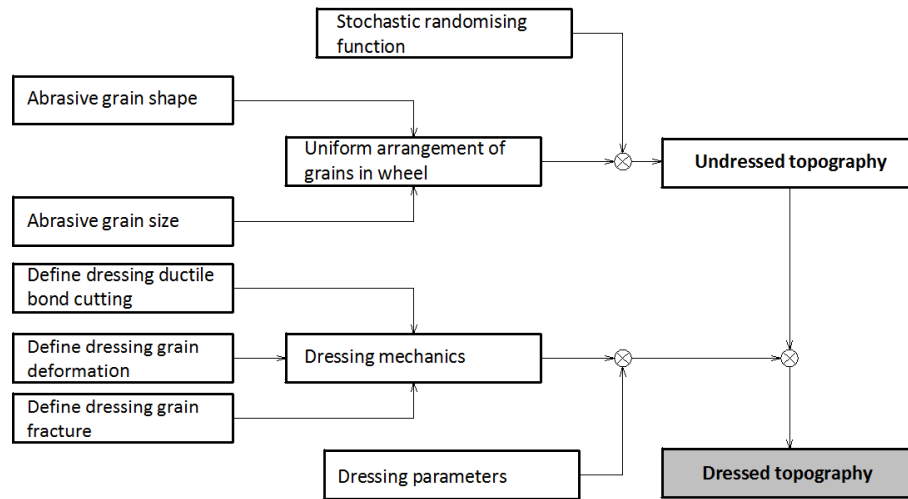


Figure 2.20: Framework of a general 3D topography model [DWB06]

## 2.4 Abrasive grit-workpiece interaction

When modelling the contact between the grinding wheel and the workpiece, the definition of the contact conditions plays a key role. Due to the grinding forces the grinding wheel deforms and penetrates the workpiece. Individual grains plough and cut the workpiece surface, inducing a reaction force on the grits and the workpiece. The sum of all individual reaction forces equals the applied force on the model.

This Section reviews the main concepts of contact mechanics and material removal theory in order to define the interaction between a surface DE and the workpiece. The review includes the variation of material properties at cutting conditions, where material is removed at high strain-rates and temperatures. In these conditions, the metallic workpiece materials show viscoplastic behaviour. The scope of this review is focused on the grinding of steels.

### 2.4.1 Contact mechanics

The contact mechanics branch was born thanks to a young Heinrich Rudolf Hertz. In 1881 during winter holidays in the Alps, due to a snow storm, he was forced to stay at the refuge. That weekend he laid the foundations of contact mechanics writing the paper '*On the contact of elastic solids*' [Her1882]. In that work, he makes an analogy between the electro-magnetic fields and the stress fields in the elastic contact. Until then, contact analysis was tackled from the empirical point of view, relating material mechanical properties to hardness behaviour. Since then, contact mechanics has faced more complex cases (such as adhesion, absorption or friction) with empirical, graphical, analytical and numerical models. Nowadays contact mechanics belongs to the wide knowledge



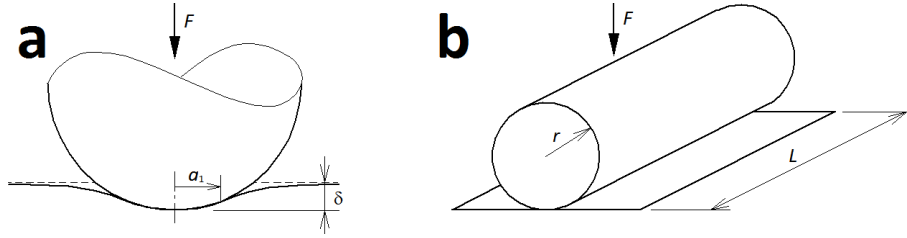


Figure 2.21: Contact with an elastic half-space: (a) rigid sphere and (b) rigid cylinder [Joh87]

branch of tribology.

The generation and correct modelling of the surfaces in contact is a key factor when facing adhesion, friction and wear analyses. For example, the adhesion and friction forces depend on the closeness of the atoms in the contact area. The contact area and geometry define the thermal and electrical conductivity and stiffness at the interface, because they depend on elastoplastic deformations spread at the sub-surface [PHMR05].

In the static contact between two non-conform surfaces, the first contact takes place on a point or a line [Joh87]. The contact zone is small comparing with the bodies dimensions. Under those conditions, contact stresses are confined in a tiny volume and those stresses are independent from the ones created by the external forces. Hertz describes these phenomena and he states the following: *'We can confine our attention to that part of each body which is very close to the point of contact, since here the stresses are extremely great compared with those occurring elsewhere, and consequently depend only to the smallest extent on the forces applied to other parts of the bodies'*. At the opposite end, the contact area on conform surfaces is similar to the dimensions of the bodies, and contact stresses regard the ones stood by the bodies.

The static contact between two smooth surfaces without adhesion nor friction, varies according to the normal force. The problem can be analysed assuming rigid indenter or rigid plane. The first case is the most studied due to the resemblance with the hardness test. Hertz, to analyse the polishing of lenses, proposes the analogy of the magnetic fields of Maxwell to formulate the analytical expressions of the static contact between basic rigid geometries and an elastic half-space [Her1882]. Among others, he defines the force-penetration ( $F - \delta$ ) relationship for the contact between a rigid sphere (Eq. 2.29) and a cylinder (Eq. 2.30) with an elastic half-space. Figure 2.21 shows both cases.

$$F = \frac{4}{3} E^* R^{1/2} \delta^{3/2} \quad (2.29)$$

$$F = \frac{\pi}{4} E^* L \delta \quad (2.30)$$

Actually the indenter is elastic too and this effect is corrected using the equivalent modulus of elasticity  $E^*$  (see Eq. 2.20). In both cases  $R$  is the radius

and  $L$  is the length of the cylinder. In the cylinder-half-space case, the  $F - \delta$  relation is linear for any  $R$ .

As the normal force increases, one of the bodies starts deforming plastically [Joh87]. The situation may be simplified considering either rigid the plane or the half-space, and confining the plastic deformation in the opposite body. Similar results are obtained in both cases. For example, considering a rigid sphere and regarding Hertz, the maximum shear stress in the half-space is at a distance of  $0,47a_1$  from the surface, where  $a_1$  is the radius of the contact area. Plastic flow first occurs at this point when the yield criterion is satisfied. Regarding the Tresca criterion, this will be at a maximum shear stress of  $\sigma_y/2$ , where  $\sigma_y$  is the yield stress of the material. At that point, the average pressure at the interface is  $1,1\sigma_y$ . It is experimentally demonstrated that the volume remains constant under plastic deformation [PW72]. In practice, the deformation at the interface is a combination of elastic and plastic deformation.

As the normal force is further increased, the zone of plastic deformation extends from beneath the indenter until it eventually reaches the surface. At this point, the contact area is still extremely small. The mean pressure over the contact area has risen at this stage to about  $3\sigma_y$  and remains at nearly the same value for subsequent increases in load. The foundations of the indentation hardness testing lay on this constant pressure. Similar results are obtained with indenters of different geometry. Hence, the contact area is directly proportional to the load. This contact condition is called rigid-perfectly-plastic contact and neglects the elastic deformation. In grinding, the contact pressures at grit level surpass largely the yield stress, and the contact can be considered rigid-perfectly-plastic. All the elastic approaches described in Section 2.2.1 neglect the material plastic deformation.

These conclusions can be applied on the analysis of the contact between rough surfaces at the asperity scale, either in the elastic or plastic domain [Joh87]. However, asperities are not uniformly distributed on real surfaces: their radius, height and position follow a given statistic distribution. In real surfaces, when the normal force is increasing, the contact area on the asperities enlarges and new asperities enter in contact.

Archard [Arc57] and the couple Greenwood and Williamson (GW) [GW66] are the pioneers of rough surfaces contact models. Archard, in his analysis of friction on rough surfaces, proves that the real contact area remains constant regarding the normal force, either in elastic or plastic regimes. The famous GW model considers spherical asperities, which heights follow a normal distribution. The Hertz theory is applied independently on each asperity in contact. Greenwood and Tripp (GT) [GT67] modify the GW model to analyse the contact between curved rough surfaces. The GT model also considers that the spherical asperities height follows the normal distribution. Precisely the GT model is the one chosen by Rowe and Qi for their grinding contact length model [RMQZ93] (Section 2.2.1). Bush *et al.* [BGT75] complete the GW model, applying also a normal distribution on the asperity radius and ellipsoids.

Newer models based on GW with elastoplastic character have been proposed. Chan *et al.* [CEB87] propose an elastoplastic model, which assumes spherical

asperities and the conservation of volume. In turn, Zhao *et al.* [ZMC00] model more realistic surfaces, creating variable roughness surfaces with directional properties. In addition, this model smooths out the transition from the elastic to the plastic deformation. His asperities are also spherical. Horng's model [Hor98] considers elliptic asperities and elasto-plastic contact. In this way, the model creates anisotropic rough surface with ellipses. All these models disregard the interaction between asperities, arguing that asperities are far enough from each other. This assumption can be wrong, specially if asperities join together due to large plastic deformations.

Certainly, the easiest model to analyse the contact between surfaces of any geometry in 3D is the method of dimensionality reduction (MDR) or the elastic foundation model [Joh87]. This technique treats one of the surface in a discrete way, placing on it elastic or elastoplastic springs. The other surface is considered rigid. Using the position function of both surfaces, the displacement of each spring is defined and, thus, with the displacement the contact reaction on that point. The simplicity of the method hides the difficulties in the definition of the stiffness of the springs. The use of the MDR technique for the estimation of the grinding contact length is one of the future research lines of this thesis. The stiffness of the springs could be estimated with the proposed wheel DEM model, and the topography with the surface bearing curve.

The finite element method (FEM) can overcome the cited disadvantage. Francis [Fra82] proposes the first FEM contact model for rough and elastic flat surfaces. Kucharski *et al.* [KKPK94] take a step ahead considering elastoplastic contact. Kogut and Etsion [KE02] modify the model for non-conform surfaces, a contact between a sphere and a plane. In addition, FEM simulates successfully the abrasion phenomena of a single-grit (see Section 2.4.2).

Mandelbrot [Man83] changes the point of view of the topography analysis, when demonstrates experimentally the fractal nature of surfaces. The roughness pattern is repeated in different scales of observation. Hybrid models have been proposed to apply fractal surfaces in contact models. Komvopoulos [KY01] proposes a FEM model to simulate the elastic contact between surfaces with compatible fractal scales. Jerier [JM12] models the contact between a rigid fractal surface and an elastic flat body, both composed of discrete elements.

It is interesting that all the theories and models cited in this review, comply the assertion of Archard, that sets the linear relationship between the applied normal force and the contact area.

## 2.4.2 Interactions at grit level

The complex micro-structure of the grinding wheel makes difficult the definition of the cutting edges. Grinding is the result of the sum of the material removed from the workpiece by all individual abrasive grits. Several researches have studied the interactions at grit level experimentally, analytically and numerically. This Section starts compiling the conclusions obtained from experimental tests. Then, the analytical and numerical models that reproduce the single-grit test are introduced.

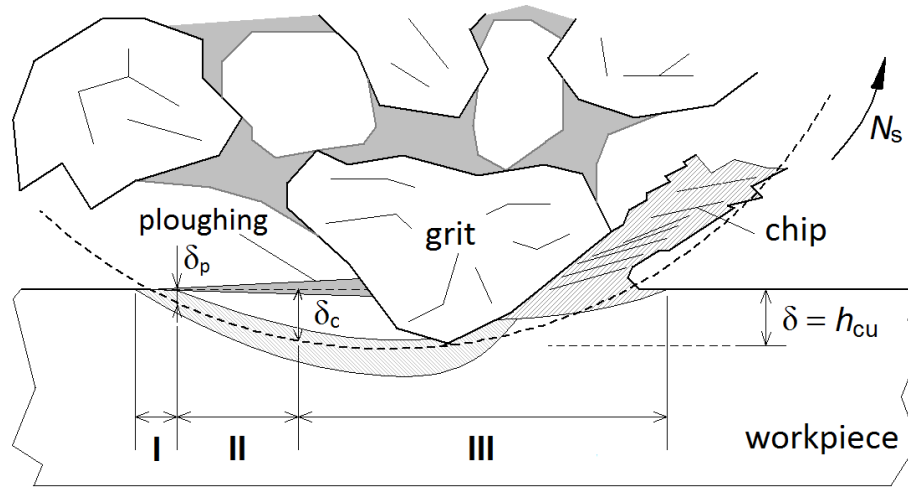


Figure 2.22: Action of an abrasive grit: (I) rubbing: elastic deformation; (II) ploughing: elastic and plastic deformations; (III) ploughing and cutting, elastic and plastic deformations and chip generation (out of scale) [Klo09]

### Cutting edge engagement: the single-grit test

The area of study *material removal micro-mechanics* is borne out of classical metal-cutting research. Early studies already realise that the grinding process is different from metal cutting [BMS52], as grinding produces smaller chips under higher cutting velocities than conventional machining processes with defined cutting edge geometries. Also, the great size disparity between the cutting tip radius and the chip thickness in grinding yields a very different apportionment of energy.

The action of a grain has been described profusely by several authors by means of single-grit tests. They have drawn similar conclusions of the cutting edge engagement on the workpiece surface. Cutting edge engagement is conditionally track-bound because of the relative movement between the grinding wheel and the workpiece. Because of the form of the cutting edge, chips are hardly formed initially. The penetration of the cutting edge  $\delta$  (or uncut chip thickness  $h_{cu}$ ) changes with the position of the grain, varying the contact conditions. The variation of  $\delta$  inspires Hahn's model of the three-phase material removal of single abrasive grain cutting [Hah62], agreed by later authors like [MRDI04, Mal08, Klo09]. The three fundamental phases, namely rubbing, ploughing, and cutting, are shown in Figure 2.22. Jiang *et al.* [JGB<sup>+</sup>13] add a fourth one, the initial non-contact phase, created due to the uneven protrusion height and position.

Hahn asserts that the increase in the specific energy during grinding is the result of the increased rubbing between the grains and the workpiece. Rubbing occurs when the penetration  $\delta$  is smaller than the ploughing depth threshold  $\delta_p$

(Fig. 2.22(I)). Rubbing consists of elastic deformation and heat generated by friction; it is considered wasted energy as no material removal occurs. Kops and Shaw [KS82], using the infra-red technique, analyse the reaction of the grit when gets in contact with the silica glass surface in the single-grit test. They report a sliding and rocking phase before the single grain penetrates the glass. They expect a similar behaviour in grinding, because glass is as hard as the steel. This implies that when a grain on the wheel circumference comes into contact with the workpiece, its first reaction is deflection, rotation and sliding on the workpiece surface, and, when the normal force of the grain is large enough and the metal is soft enough, the grain penetrates into the workpiece.

Once the grit penetrates the workpiece surface, ploughing occurs in the  $\delta$  range of  $\delta_p < \delta \leq \delta_c$ . This phase consists of a combination of elastic and plastic deformations (Fig. 2.22(II)). During this phase, the workpiece material tends to bulge ahead of the abrasive grain and forms a ploughed lip, followed by side flow. The elastic recovery is negligible, but the material displaced plastically generates a groove. However, ploughing is also inefficient, as no material is removed from the workpiece.

Only when the cutting edge has penetrated deep enough into the workpiece, when  $\delta$  gets  $\delta_c$ , the actual chip formation begins [Hah62]. In this phase, beside ploughing, the conditions are sufficient to shear the workpiece material into a chip, removing material from the workpiece (Fig. 2.22(III)). From this point on, as the sequence continues, deformation processes and chip formation appear simultaneously.

Klocke [Klo09] remarks the role of  $\delta$  and  $\delta_c$  on the performance of material removal. He proposes that actually a fraction of  $\delta$  produces chips and defines the effective chip thickness  $h_{cu,eff}$ . This assertion contrasts with kinematic model approaches [DWB06] and experimental single-grit tests [LS75, TFX<sup>+</sup>15], that assume the grit clears all the material that crosses its path: a fraction of the intersected material is removed by the chip, whereas the remaining is displaced and pile-up on both sides of the track.

Chip formation and, thus,  $\delta_c$  are influenced largely by the friction conditions, cutting edge sharpness, cutting speed and shearing conditions (temperature and strain rate) at the material. A better lubrication increases the contact length, enlarging the contact time and promoting the plastic deformation, increasing  $\delta_c$ . So an improved lubrication reduces the efficiency of material removal. The cutting edge sharpness modifies  $\delta_c$  too. Blunt cutting edges promote deformation of the material, leading to larger  $\delta_c$ . Looking at the grain shape [MTO089], an increasing negative rake angle leads to higher plastic material deformation. In addition, the orientation of the abrasive grain according to the feed direction has influence on chip formation. The rising of machining temperatures increases  $\delta_c$ , since the material becomes more ductile. In turn, the higher material malleability decreases  $\delta_c$ , since strain hardening enlarges its effect. The increasing cutting speeds inhibits gradually the plastic deformation.

König and Lortz [KL75], analysing 2D profile measurements of the wheel surface, quantify  $\delta_c$  between 1 and 1,6  $\mu\text{m}$  regarding kinematic parameters (grit size 54). They also define the minimum distance between cutting grains  $l_c$

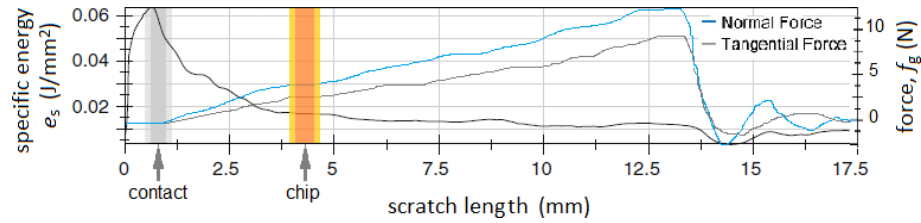


Figure 2.23: Single-grit test: instantaneous specific energy  $e_s$  and grit force  $f_g$  through the contact length [RMK15, RKM15]

to create a chip within 400 - 500  $\mu\text{m}$ , larger than the average grain diameter ( $d_g = 280 \mu\text{m}$ ). So most of the cutting edges that contact the workpiece surface do not remove material. They observe values of  $h_{\text{cu,max}}$  up to 10  $\mu\text{m}$ , almost 30 times larger than the theoretical  $h_{\text{cu}}$  (Eq. 2.5). Tian *et al.* [TFX<sup>+</sup>15] develop a tailored single-grit test to study the removal mechanisms regarding the cutting speed  $v_s$  and  $h_{\text{cu}}$ . The results show that the removal mechanism changes at 100 m/s:  $\delta_c$  varies according to  $v_s$ , from 1,4  $\mu\text{m}$  at 20 m/s, a minimum of 0,6  $\mu\text{m}$  at 100 m/s, to 1,6  $\mu\text{m}$  at 165 m/s. That is, below 100 m/s and increasing speed, the strain rate increases more rapidly than the temperature so that the material plasticity is reduced and the chip formation becomes easier. On the other hand, when the grinding speed increases from 100 m/s to 165 m/s the thermal softening assumes the key role in chip formation. Similar turnaround points are observed on force and pile up ratios measurements. Cai *et al.* [CFJG02] study the influence of  $v_s$  on the force ratio  $\mu$  with the single-grit test:  $\mu$  decreases as  $v_s$  increases. Additionally,  $\mu$  varies using a different abrasive material.

Rasim *et al.* [RMK15, RKM15] present the instantaneous specific energy  $e_s$  and force  $f_g$  measurements (tangential and normal) regarding the contact length position on the single-grit test. The chip formation threshold is determined analysing the videos taken by a high speed camera. As Figure 2.23 shows,  $f_g$  increases almost linearly, whereas  $e_s$  starts with a high value, that falls down rapidly to a sixth of the peak value. The force ratio remains almost constant along the contact length. The peak source is the inefficient rubbing mechanism. After the short rubbing stage, the ploughing starts, decreasing the slope of the fall.  $e_s$  becomes constant once the chip comes up.

As commented in Section 2.2.2, temperature measurements with thermocouple also identify the cutting regions [Pek57, KS82, GW88, ZV92, QRM97b]. First, a few peaks are noticed, a zone that coincides with the sliding region (see Fig. 2.17(c)). The ploughing and cutting zones show up together, with a sudden increase of temperature and peak density. The workpiece material is softened rapidly at this point and much metal is removed. The peaks in the tail of the thermosignal tend to disappear, they corresponds to the most prominent grits and only a few of them are cutting on a surface still at high temperature and low hardness. The cutting conditions of the exiting grits cannot be reproduced

in the single-grit test, because it does not include the effect of the surrounding grits on the temperature.

Kannapan and Malkin [KM72] analyse the proportion of energy consumed by rubbing, ploughing and cutting: the sum of these three terms is the energy consumed in grinding. Rubbing is related to wear, and wear with the tangential force. On the other hand, cutting energy can be approached with the energy needed to melt the chips. The remaining energy is assigned to the ploughing energy. They assert that the energy partition follows the principle of the minimum energy: the mechanism that needs the least energy prevails. In this way, cutting requires less energy than ploughing and rubbing, so when the depth exceeds the  $\delta_c$  threshold, the chip appears. If the depth of cut is smaller than  $\delta_c$ , the energy needed for create a chip is infinite. The size effect of the specific energy in grinding is closely related to the ploughing mechanism.

Malkin and Cook [MC71], in their extensive work about wear, assert that the tangential force is composed of cutting and rubbing forces. The rubbing force term depends on friction and wear flats. To a certain extent, the bonding material can also be in contact with the workpiece, consuming energy and inhibiting the chip formation. Actually, the friction-related part of the total grinding force can be even greater than the part related to chip formation. These two types of forces cannot be clearly distinguished, because they appear coupled in the chip formation process.

### Analytical models

Abrasion analytical models have been developed regarding the three basic mechanisms: rubbing, ploughing (or scratching) and cutting [MRDI04]. These three mechanisms coincide with the abrasive grit-workpiece interaction in grinding. Analytical models evaluate the effect of friction and cutting geometry edge on grit forces. Each abrasion basic model applies different criteria, as the Von Mises yielding criterion, the maximum shear slip lines or the orthogonal cutting theory. Each basic approach helps to estimate the contribution of each mechanisms. In addition, the action of an abrasive grit is approached by analogies of the hardness test and micro-milling.

The analytical approaches of rubbing contact are based on the classic works of Tabor and Bowden [BT39, Tab59]. They study the shallow horizontal movement of a worn grit (Fig. 2.24(a)). Plasticity conditions are defined with the Mohr circle and the shear stress at the tip is lower than the shear flow stress. Tabor modifies the model to include the junction growth observed at the tip (Fig. 2.24(b)). The material flow in rubbing has the effect of increasing the area of the junction without needing to increase the normal force, so the normal stress on the junction is less compressive. The second approach enlarges the friction coefficient  $\mu$ , closer to the actual rubbing conditions.

Regarding the scratching contact (ploughing), Rabinowicz [Rab65] analyses the forces on the cone shown in Figure 2.25. The problem is studied from two points of view: the hardness of the workpiece and the shear stresses. The friction coefficient on both depends on the cone slope. Large angles reduce the friction,

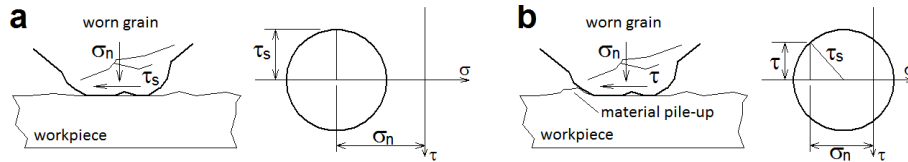


Figure 2.24: Rubbing contact model for a worn grit (a) without [BT39] and (b) with material piled up junction growth [Tab59]

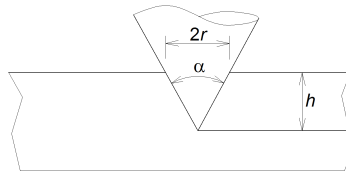


Figure 2.25: Groove created by a cone due to ploughing [Rab65]

as it happens on worn grits. So except for sharp grits, ploughing prevails as leading mechanism.

Slip-lines offer a clarifying graphical representation of the yielding of the material, although they are only applicable in plane stress. They are also based on Mohr's circle. Slip-lines link points of same shear stress. In plastic yielding, they get an angle of  $45^\circ$  with the free surface and the friction angle  $\gamma$  with the grit surface. The indentation solution of Tomlenov [Joh87] foresees the *D-E-F* dead zone under the indenter (Fig. 2.26(a)).

Lortz [Lor79] analyses the scratching of a sphere (ploughing). The material piles up ahead and deforms beneath the sphere (Fig. 2.26(b)). Challen and Oxley [CO79] provide three slip-line models applicable in grinding. The first represents the scratching wave model without wear (Fig. 2.27(a)), the second the scratching wave model with wear (Fig. 2.27(b)) and the third the chip formation (Fig. 2.27(c)). The first one has proved its ability to estimate the friction coefficient. In the second, the material piles up endlessly in front of the indenter, ignoring continuity conditions. The chip formation model follows the

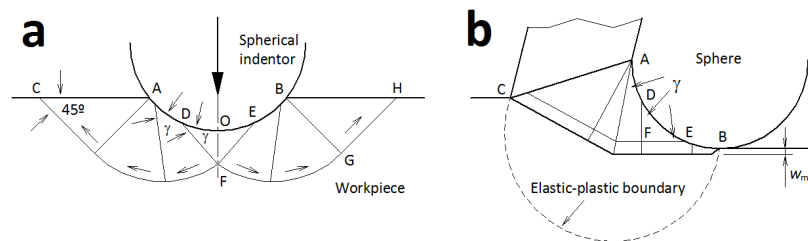


Figure 2.26: (a) Analysis of the indentation with maximum shear slip-lines [Joh87], (b) simplified slip-line field proposed for a grit in grinding [Lor79]



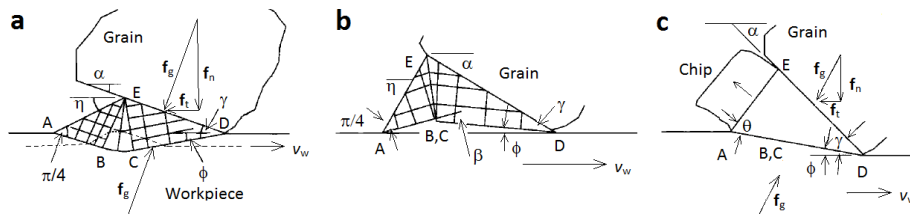


Figure 2.27: Slip-line models of Challen and Oxley: (a) wave model without wear, (b) wave model with wear and (c) chip formation [CO79]

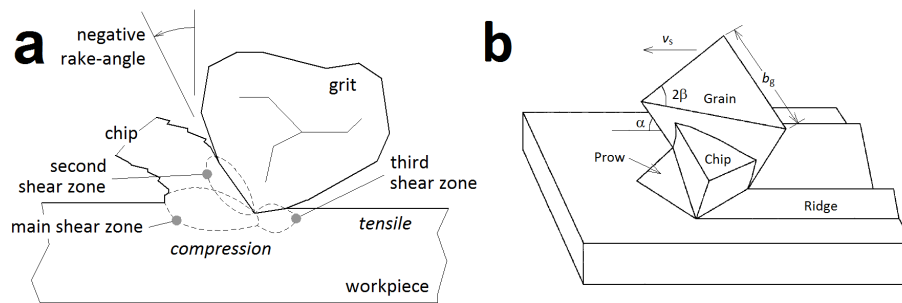


Figure 2.28: (a) Main, secondary and tertiary cutting zones [MRDI04]; (b) chip formation in 3D [WX92]

principle of the minimum required energy. The chip formation always will get over ploughing if the minimum conditions are given. A high friction coefficient leads to compressive stresses under the tip  $D$ . An impact-wave goes through the slip-line  $AD$  pulling out the chip. In scratching, an improved lubrication reduces the tangential force, the opposite of the chip formation model.

The material removal performed by a single grit can also be tackled from the metal cutting theory. Grinding forces depend directly on the shear stresses created on the workpiece by the grit. There are three main zones in metal cutting: the main, the secondary and the tertiary zones (Fig. 2.28(a)). The main cutting zone corresponds to the shear zone. Ideally, this zone is considered a plane, which is straightforward defined by the shear angle. The main cutting plane is between the tip of the grain and the workpiece surface. The secondary zone is located on the interface of the chip and the rake surface of the grit. Friction dominates in that area. The third zone appears under the grit, in the flank surface and ploughing prevails there. The strain rate is higher on the rake surface than on the shear plane, producing high local temperatures and increasing the wear rate. The rake angles are very negative, creating a compressive plastic zone in front of the grain. Once the grain is passed, there is a sudden transition to tensile stresses.

Williams and Xie [WX92] analyse the traverse cutting of a pyramidal grain. Figure 2.28(b) shows the pile up of material in front of the grit, as well as lateral

displacement and chip formation. The grain leaves behind a groove with grown walls. The model foresees that the transition between ploughing and cutting occurs among  $\alpha = 6^\circ - 12^\circ$ .

Torrance, together with Buckley and Badger [TB96, BT00], compare Challen and Oxley's slip line 2D model and the pyramid shape 3D model of Williams and Xie with experimental grinding tests. Roughness measurements are used to define the slope and grain-densities input-data. Both models show their capability to predict cutting forces, being slightly more accurate the 3D model.

Beside the basic analytical models, the action of the grit has been compared with the hardness test and micro-milling. For one grit, the chip removal mechanism involves a micro-extrusion like action: a relatively large volume of material is brought to a fully plastic state for a relatively small amount of material to escape as a chip. Inspired on this fact, Shaw suggests that the analysis of grinding force on a grit can be simplified as an indentation force [Sha71, Sha96]. Figure 2.29(a) shows the Brinell test, where the load  $\mathbf{f}_{\text{HB}}^i$  is supported by the workpiece over an area  $\pi a^2$ , being  $a_1$  the radius of the contact area. In turn, Figure 2.29(b) presents Shaw's hardness model of chip formation. The cutting edge of an abrasive grit is assumed to be a sphere of radius  $\rho_s$ , as explained in Section 2.3.2. While the plastic zone beneath the indenter in the hardness test is completely confined, the grit has an unconfined plastic zone due to the inclined load. As the grit advances horizontally, the material flows upward across the unconfined zone. If the depth that penetrates the grain  $\delta_i$  is enough, the chip arises.

Equation 2.31 estimates the contact force  $\mathbf{f}^i$  in the Brinell hardness test (Fig. 2.29(a)).  $H$  is the Brinell hardness of the workpiece and  $C'$  is a constraint factor, which is defined as the ratio of the average pressure in the contact area to the uniaxial flow stress. In most cases,  $C'$  is about 3. Shaw substitutes the width  $a_1$  with the contact width of the grit  $b_{\text{cu}}^i$  regarding  $\delta_i$ , and the  $H$  Brinell hardness by the contact pressure at the rigid-perfectly-plastic condition, i.e. three times the yield stress ( $3\sigma_y$ ). Thus, this formula internalises the rigid-perfectly-plastic contact hypothesis seen in Section 2.4.1.

$$\mathbf{f}^i = \frac{1}{4} \pi a_1^2 H \frac{C'}{3} \quad (2.31)$$

Chen and Rowe [CR96] and Hecker *et al.* [HRL03] use Shaw's hardness model on their kinematic models. Thus, beside modelling the workpiece roughness, these models estimate grinding forces. The main advantage of the hardness model leads on its simplicity: Equation 2.31 depends only on the sharpness radius, penetration and the yield stress.

The uncut chip thickness  $h_{\text{cu}}$  in fine grinding is of the same order of magnitude as the cutting edge sharpness radius  $\rho_s$  and, hence, the effective rake angle has a large negative value. In turn, Shaw admits that the specific energy in fine grinding is extremely high due to a combination of material displacement and shearing. The plastic strain, strain rate and temperature conditions on both cases differ completely. Hecker adds a correction factor  $C_1$  to take into account the effect of strain and temperature.  $C_1$  is defined experimentally. Another

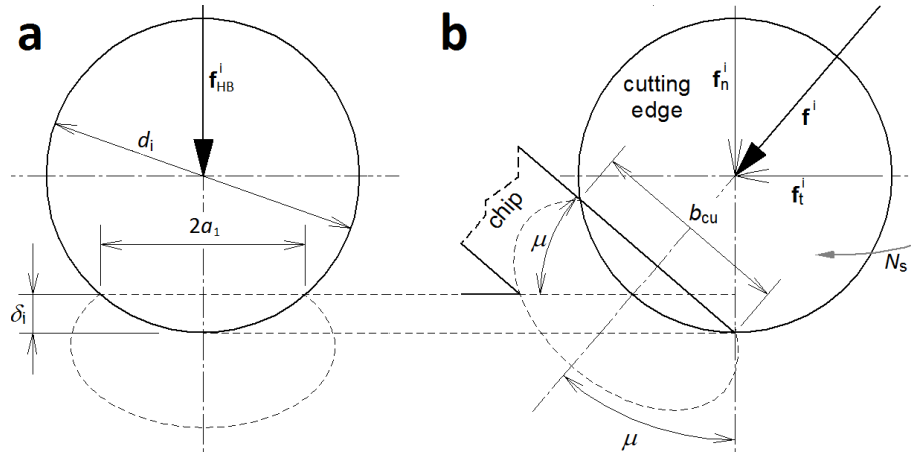


Figure 2.29: Analogy of the hardness model of chip formation: (a) hardness indentation with a spherical indenter; (b) indentation at the cutting edge of an abrasive grit with a sharpness radius  $\rho_s$  and the resultant force  $\mathbf{f}^i$  [Sha71, Sha96]

solution is to take into account the viscoplastic behaviour of the workpiece material, like several single-grit numerical models do. In addition, Shaw's hardness model does not distinguish the rubbing, ploughing and cutting mechanisms, so it does not take into account the remarkable importance of the size effect in grinding.

There have been also attempts to compare grinding and milling. Inspired in the kinematic study of Alden [Ald1914], some authors have developed further the micro-milling analogy to analyse forces at grit level [Ono61, Rub72]. These approaches consider the action of each grit as a cutting edge. In turn, Werner [Wer78] simplifies the turning cutting forces empirical model developed by Kienzle and Victor for turning [KV54]. Aurich and Kirsch [AK12] implement Werner's simplified model on their kinematic model of grinding. Chang and Wang [CW08] apply the analytical model developed by Wang and Zheng [WZ02] on each grit regarding the consumed energy.

In these approaches, the position of an engaging grit and its relative position to the workpiece are defined with the position angle  $\theta$  on the wheel and work coordinates  $(x, y, z)$ , as shown in Figure 2.3. The tangential force on a single-grit can be expressed as the product of the specific grinding force  $k_s$  and the chip section  $A_c$ , which varies continuously with the cutting edge position. The cutting edge can have a basic shape (rectangular, triangular or circular (see Fig. 2.4)) or an irregular geometry [AK12].

Equation 2.32 shows the expression proposed by Ono to estimate the instantaneous single-grit tangential force  $f_t^i$ , where  $k_s$  is the specific force,  $A_c$  the instantaneous cutting section and  $\theta$  the position angle of the grain.  $A_c$  varies according to the instantaneous depth of cut and cutting edge geometry. Ono

defines  $k_s$  empirically as a power function with Equation 2.33, where  $C_1$  and  $C_2$  ( $= 0,25 - 0,5$ ) are constants. In this way,  $k_s$  fits the specific grinding energy  $e_s$ .

$$\mathbf{f}_t^i(\theta) = k_s A_c(\theta) \quad (2.32)$$

$$k_s = C_1 A_c^{-C_2} \quad (2.33)$$

In turn, the Kienzle-Werner approach proposes a similar expression (Eq. 2.34), but applying the exponent  $C_3$  on  $A_c$ . Both authors expand the expressions, achieving relationships of the cutting force in function of the cutting and work speeds, depth of cut and wheel diameter, beside  $A_c$  and the empirical constants. In fact, both models are similar. Chen [Che95] combines both models to study the effect of the exponents. On the one hand, the force depends on the grain density and contact length and, on the other hand, it depends on the penetration. Thus, he concludes that the grinding force is a combination of friction and cutting forces, which agrees with the consequences of the size effect.

$$\mathbf{f}_t^i(\theta) = k_s \cdot A_c(\theta)^{C_3} \quad (2.34)$$

Chang and Wang [CW08] relate linearly the cutting tangential force and the intersected area to the specific grinding energy  $e_s$  ( $k_s = e_s$  in Equation 2.32). This approach disregards completely the abrasion mechanisms and the size effect.

In brief, the grit interacts with the workpiece sliding, ploughing and cutting according to the contact conditions. Rubbing, ploughing (or scratching) and cutting basic analytical models have been compared with the single-grit action in grinding. They show relative resemblance with the actual phenomenon. Although they are not suitable to model the three basic mechanisms at the same time, they offer an interesting information about each one. Two of them have been applied to model the single-grit action in grinding: the maximum shear stress slip-lines and the oblique cutting. Both show good results regarding experimental tests, but the drawbacks come from parameter definition and implementation. Other approaches propose analogies with the hardness test and the micro-milling. Both are easy to implement on force models. The hardness model disregards the effect of the strain-rate and temperature, so it needs to correct the effective yield stress. The micro-milling model is simple, but it requires previous experimental single-grit force measurements.

## Numerical models

In order to describe the chip formation not only qualitatively but quantitatively numerical models have been developed. The analytical models lack the ability to investigate important aspects, such as workpiece strain rate, temperature distributions, wear behaviour and chip formation, which can be analysed with numerical tools. Several FEM approaches exist, mostly 2D, that use idealised grain

shapes [DWB09]. In the same way, the use of atomistic models like molecular dynamics (MD), has gained a deeper understanding about the microscopic material behaviour and material removal mechanisms [MRDI04, DWB06, BAG<sup>+</sup>06]. Most of the single-grit numerical models are based on the concepts of conventional orthogonal metal cutting.

Doman *et al.* [DWB09] classify the single-grit FEM models by the type of material constitutive model used for the workpiece. These constitutive types include elastic, elastoplastic, thermoplastic and ceramic formulations. An elastic constitutive model treats the workpiece as a Hookean material, whereas elastoplastic models take into account the plasticity beside the elastic response. A thermoplastic material adds thermal effects and a ceramic material models includes the initiation, coalescence, and propagation of micro-cracks, as well as damage accumulation in the ceramic which degrades the nominal strength of the material.

To model the chip formation is challenging. Klocke *et al.* [KBH<sup>+</sup>02] create a 2D FEM model with the commercial software of explicit formulation *DEFORM*. The model simulates the creation of the chip under actual grinding parameters, such as depth of cut, cutting speed, and grain shape. The maximum temperatures reach 1700 °C, but the model does not take into account the hardening of the workpiece material due to the high strain rates. Later models include the viscoplastic behaviour of the material due to high strain rates and temperatures. Ohbuchi and Obikawa [OO05] improve the model, simulating continuous and saw-toothed chip formations. The model provides gradients the plastic strain, effective yield stress, temperature and strain rate fields in the cutting zones regarding the depth of cut. It also estimates the threshold penetration  $\delta_c$  in which starts the chip formation according to the yielding conditions (depth of cut, stress and temperature). Park and Liang [PL09] model the single-grit test assuming a viscoplastic material behaviour using the Johnson-Cook model (see Section 2.4.3). This 2D model combines the deflection and heat source of the whole wheel.

Doman *et al.* [DBW09] develop the first 3D FEM model of the single-grit test. A sphere of 2 mm diameter represents the abrasive grain. The model uses an erosion technique to isolate the transition from rubbing to ploughing, removing elements severely distorted that accumulate massive plastic strains. The elastoplastic material behaviour is described by the JC model. A segment-based contact algorithm improves the stability of the simulation comparing with the usual node-to-node penalty algorithm. The model simulates the rubbing and ploughing phases and quantifies the rubbing-ploughing transition depth  $\delta_c$  in 3  $\mu\text{m}$ . This depth makes sense due to the large diameter of the grit. The model is validated experimentally obtaining good correlations. Anderson *et al.* [AWB11] continue Doman's work, modelling and testing a spherical single-grit of 0,5 mm diameter. The model mixes the Lagrangian and Eulerian formulations in the same 3D model, beside the JC constitutive model. The model defines, among others, the ploughing  $\delta_p$  and cutting  $\delta_c$  depths and the specific energies at each stage.  $\delta_p$  values vary between 0,06 - 1  $\mu\text{m}$  and  $\delta_c$  are between 3 - 5,3  $\mu\text{m}$ . These values are also high due to the large grit diameter.

Rasim *et al.* [RKM15] take a step ahead modelling the single-grit test with an amorphous grain geometry. Good correlations are obtained with experimental tests regarding the material pile-up and removed fractions, but the estimated cutting forces are the half of the measured ones. They measure the cutting temperature with the single pole thermocouple, which offers an average overview of the temperature during the single-grit test. They observe an increasing temperature according to the exposed grit cross-area. Akbari *et al.* [ABLW16] model the single-grit test for a diamond grain and titanium alloy. They also validate the results with experimental tests. The testing speed is slow (0,8 m/s), probably due to the because the measurements of temperatures with infra-red camera.

On the other hand, the MD models related to grinding, simulate the material removal at nano-metric scales ( $< 4$  nm), far from the micro-scale depths of cut in grinding. MD shows serious difficulties in the development suitable models in terms of the micro-topography, the definition of potential functions for the material and interactions of interest, and the requirement of a large computation cost for rather small 3D models in absolute size. An alternative approach to a large MD model, where most of the regions only store elastic energy, is to combine the discontinuous atomistic description of MD with an elastic continuous mechanics approach like FEM. Here the atomistic core covers the plastic deformation, while the surrounding continuous mechanics region stores only the excess of elastic deformation. The main difficulty in those hybrid models is the description of suitable interface properties. Further interesting alternative is the use of mesoscopic particles or groups of atoms [KD08, JDCI16].

### 2.4.3 Viscoplasticity

In grinding, the cutting edge interacts with the workpiece at almost-ballistic speeds (20 - 40 - 60 m/s and more). At those cutting speeds the plastic strain and strain rate harden the material, whereas substantial heat is generated at the shear zone [Mal08]. These conditions change completely the mechanical behaviour of the workpiece material, increasing its resistance and facilitating the yielding simultaneously. Hence, the workpiece material shows up a viscoplastic behaviour when cutting.

The theory of viscolasticity is part of the continuous media mechanics branch, that describes the rate dependent inelastic behaviour of solids. Rate dependence means that the deformation of the material depends on the load application speed [LC94]. If the stress surpasses the strain rate dependent stress threshold, the material undergoes unrecoverable deformations. The viscoplasticity in metals is the consequence of dislocations observed at grain scale.

For a qualitative analysis, several characteristic tests are performed to describe the behaviour of viscoplastic materials. These tests are hardening tests at constant stress or strain rate, creep tests at constant force, and stress relaxation at constant elongation. The strain or work hardening is the increase in stress required to produce additional strain after yielding. The stress-strain diagrams remind the ones obtained for strain non-dependent materials, but the stresses

are higher for the same strain when testing at a higher strain rate. Creep tests measure the strain response due to a constant stress at a constant temperature. In turn, the relaxation test is the stress response due to a constant strain for a period of time. These tests characterise the viscosity and determine the relation between the stress and the rate of viscoplastic strain.

There are two approaches to model the viscoplasticity [LC94]. In over-stress or strain rate plasticity models, the stress can surpass a strain independent yield stress and it relaxes according to the time. In those models, the yielding is assumed independent from the strain rate. They can model strain rate variations. The formulation developed by Perzyna [Per66] is the most known.

The second model family takes into account the strain rate in the definition of the yield stress. Their aim, as Tresca and Von Mises failure criteria, is to define an effective yield stress  $\sigma_y$ . Several empirical and semi-analytical yield stress models have been proposed. The semi-analytical models are based on dislocations dynamics and are applied in impact analysis. The Johnson-Cook constitutive model presented in the next Section belongs to this family.

### Johnson-Cook constitutive model

Johnson and Cook [JC83] presented the model developed for the US air force in 1983 in a conference in Holland. Since then, it has become a reference model, due to the clearness in the experimental definition of parameters and the suitability to be implemented in numerical models. The Johnson-Cook (JC) model is completely empirical and Equation 2.35 estimates the effective yield stress  $\sigma_y$ ,

$$\sigma_y(\varepsilon_p, \dot{\varepsilon}_p, T) = [A + B\varepsilon_p^n] \left[ 1 + C \ln \frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_o} \right] \left[ 1 - \left( \frac{T - T_o}{T_m - T_o} \right)^m \right] \quad (2.35)$$

where  $\varepsilon_p$  is the plastic strain,  $\dot{\varepsilon}_p$  and  $\dot{\varepsilon}_o$  actual and reference strain rates, and  $T$ ,  $T_o$  and  $T_m$  actual, reference and melting temperatures.  $A$ ,  $B$ ,  $n$ ,  $C$  and  $m$  are material constants, which are defined experimentally.

The first term in brackets represents the yield stress regarding the strain.  $A$  is the yield stress under quasi-static conditions and  $B$  and  $n$  gathers the effects of the strain hardening. The values of those constants are obtained from either tensile or torque testing. The second term in brackets expresses the effect of the strain rate.  $C$  is defined with tensile and torque tests under several strain rates under the reference temperature. Finally, the third bracket corresponds to the effect of the temperature. The exponent  $m$  is defined testing the material under several temperatures at the reference strain rate.

Its implementation on numerical tools, such as FEM, is easy, because internally work with the variables  $\varepsilon_p$ ,  $\dot{\varepsilon}_p$  and  $T$ . Many authors have assumed viscoplastic material behaviour in grinding, mainly when modelling the single-grit test numerically [KBH<sup>+</sup>02, AWB11, RMK15, ABLW16]. Ding *et al.* [DLL15] apply the JC model together with a phase transformations model on their thermal FEM model of maraging steels grinding. This model simulates the

effect of the strain rate and temperature regarding the whole grinding wheel for the first time. Ohbuchi and Obikawa [OO05] develop a tailored constitutive law, similar to the JC model. In turn, the application of JC model on a kinematic model is not so straight. The strain and temperature conditions vary along the contact arc due to the variable depth of cut. The following Section discuss about the definition of the viscoplastic variables at grit level.

### Discussion about strains and temperatures in grinding

As commented,  $\varepsilon_p$ ,  $\dot{\varepsilon}_p$  and  $T$  are *a priori* unknown in a kinematic model. The determination of these parameters can be tackled from the numerical, analytical and experimental points of view. All of them face the problem of combining the simultaneous action of several neighbour cutting edges, which has influence on the temperature.

Section 2.4.2 compiles the main numerical models that simulate the single-grit test. They calculate the distribution of  $\varepsilon_p$ ,  $\dot{\varepsilon}_p$  and  $T$  regarding cutting conditions. The fundamental results obtained by these works give an excellent idea of the influence and importance of the parameters involved. However, it is difficult to confirm the numerical results by experimental methods. In addition, they do not take into account the effect of surrounding grits on the temperature.

The experimental determination of strain and temperatures is very challenging. In orthogonal cutting, the quick-stop method allows the observation of the plastic strain  $\varepsilon_p$ , disengaging suddenly the tool and freezing the cutting zone. However, it would be difficult to adapt the method to the single-grit test, due to the random geometry of cutting edges and the lateral displacement of material due to ploughing. In turn, the shear plane position  $\phi$  (see Fig. 2.30) and the strain rate  $\dot{\varepsilon}_p$  cannot be measured experimentally.

In analytical models, the material removal theory idealises the shear zone, assuming that the chip is formed instantaneously by shearing in a plane. Figure 2.30 shows the geometry of the orthogonal cutting. The  $\alpha$  rake angle of an abrasive grit is variable and very negative. The cutting occurs in the shear plane, defined by the shear angle  $\phi$ . Eugene Merchant [Mer45] set one of the most important relationships in metal cutting in 1945, derived from the force analysis in orthogonal cutting. His expression relates  $\phi$  to the rake angle  $\alpha$  and the friction angle  $\beta$  (Eq. 2.36), and the obtained  $\phi$  corresponds to the position that minimises the shear energy. The expression assumes that the shear strength of the work material is constant along the plane, unaffected by the strain rate and temperature. Therefore it is considered an approximate relationship rather than an accurate equation. Several modified expressions have been proposed varying the initial assumptions or adjusting with empirical constants [Sch00], but their results are approximate too. Equation 2.37 estimates the friction angle  $\beta$ , regarding the force ratio  $\mu$  and  $\alpha$ .

$$\phi = 45^\circ + \frac{\alpha}{2} - \frac{\beta}{2} \quad (2.36)$$



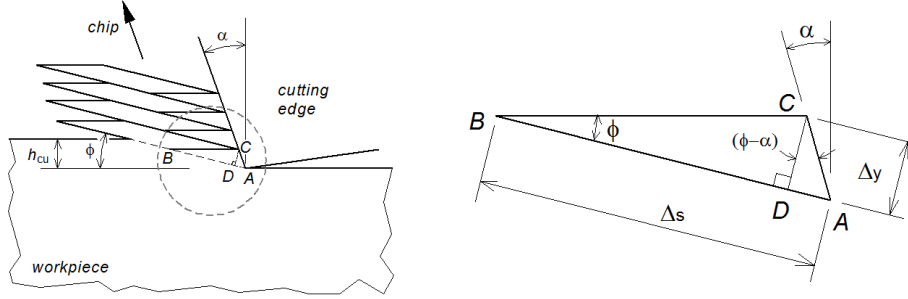


Figure 2.30: Orthogonal cutting with negative rake angles [Sch00]

$$\beta = \arctan(1/\mu) + \alpha \quad (2.37)$$

Assuming pure shear in the cutting plain, Equation 2.38 defines the unit strain  $\varepsilon$ . In turn, the definition of strain rate  $\dot{\varepsilon}$  is not so straight due to the indetermination of  $\Delta y$  (Eq. 2.39, Fig. 2.30). Oxley [OY89], based on experimental observations, proposes the empirical relation shown by Equation 2.40, where  $C_1$  is the material constant,  $v_s$  is the cutting speed and  $h_{cu}$  the uncut chip thickness (or penetration). The strain rate values in grinding are very high, almost comparable with the ones observed in ballistic impacts ( $10^5 - 10^7 \text{ s}^{-1}$ ) [RKM15].

$$\varepsilon = \cot \phi + \tan(\phi - \alpha) \quad (2.38)$$

$$\dot{\varepsilon} = \left( \frac{\Delta s}{\Delta t} \right) \cdot \frac{1}{\Delta y} = \frac{\cos \alpha}{\cos(\phi - \alpha)} \cdot \frac{v_s}{\Delta y} \quad (2.39)$$

$$\dot{\varepsilon} = C_1 \frac{v_s \cos \alpha}{\cos(\phi - \alpha)} \frac{\sin \phi}{h_{cu}} \quad (2.40)$$

Regarding grinding temperatures, the damage created by abusive machining lead to an intense interest of researchers, reflected in the extensive literature. Marinescu *et al.* [MRDI04] review the analytical and numerical prediction methods and measurement techniques. Grinding energies greatly exceed the specific energy required to melt the material removed, so the grain and chip approximately reach the melting temperature of the work material [Mal08]. This is confirmed by numerical simulations of the single grit action, with temperatures up to 1700 K [KBH<sup>+</sup>02]. The surplus energy is dissipated within the remaining workpiece material, the abrasive grains and the fluid. The maximum temperatures are not at the shear plane, but at the rake face due to friction. Direct measurements of temperatures in grinding are difficult by the large number of short random discrete events taking place as individual grains pass through the contact zone. The cooling of the grit is found extremely rapid in the single-grit

test: the temperature drops to one quarter within 1 ms when a grit disengages from the workpiece [USN96]. The single-pole thermocouple, the radiation pyrometer, the infra-red (IR) camera and the metallography analysis are the main temperature  $T$  measurement techniques. Except for the metallography variation, these methods allow the measurement of  $T$  with the whole grinding wheel and a single-grit [USN96, RKM15, ABLW16]. Even though measuring both, it would be difficult to define a certain temperature for a given penetration and position whatever method used.

The single-pole thermocouple test with the whole wheel collects the temperature spikes created by passing grits and the influence of the surrounding grits (or background temperature), whereas the same test with a single-grit captures a single spike. In the first, the depth of cut of each grit is unknown and, in the second, the penetration is known. The observed  $T$  corresponds to an average  $T$  in the main, second and third shear zones (Fig. 2.28(a)). The thinner thermocouple ( $\leq 25\mu\text{m}$ ) has greater discrimination of the local temperature and a faster response time. If the abrasive material is conductive, as CBN, the hot junction could be created between the workpiece and the single-grit. This combination is found in the literature for turning, but it could be adapted to a CBN single-grit test. The pyrometer measures the temperatures on the exposed surfaces: workpiece, chip and wheel or single-grit. The complex set-up and the minimum temperature detectable are the limiting factors of the pyrometers. In turn, IR sensors response quick and do not have a minimum temperature. The temperature distribution in the workpiece sub-surface can be deduced analysing the changes in the hardness and micro-structure through depth. A calibration of the workpiece hardness is needed regarding the temperature and time of heating, creating samples of the structural changes at the corresponding temperatures. Thus, this method needs an extensive start-up work.

$T$  can also be estimated analytically. Only a fraction of the generated heat flux is conducted into the workpiece, designed as workpiece partition ratio  $R_w$ .  $R_w$  varies according to the abrasive type, workpiece material, specific energy, cutting fluid and the contact length. Carslaw and Jaeger [CJ59] set the foundations of the heat conduction theory of moving sources. The action of the grinding wheel can be considered a moving line heat source. Applying Bessel functions, the expression is integrated regarding the time and depth over the length of the grinding contact. The usual heat flux distributions applied in grinding are the uniform and triangular ones. Temperatures may be read from charts and tables, or solved by mathematical software. Restricting the boundary values of the integral, the variation of the temperature along the contact zone can be calculated. Equation 2.41 presents the triangular source solution as a function of time, from  $t = 0$  to the end of the contact arc  $t = t_c (= l_c/v_w)$ ,

$$T = \frac{4q_o}{\beta\sqrt{\pi}}t^{1/2} \left(1 - \frac{2t}{3t_c}\right) \quad \text{where } \beta = \sqrt{(k\rho c)_w} \quad (2.41)$$

$$q_o = \frac{F_t v_s}{l_c b_w} \quad (2.42)$$

where  $l_c$  is the contact length,  $v_w$  the work speed and  $q_o$  the heat flow-rate per unit area (Eq. 2.42).  $\beta$  represents the thermal properties of the workpiece material, where  $k$  is the conductivity,  $\rho$  the density and  $c$  the specific heat.  $T_{\max}$  does not significantly differ regarding the flux distribution, but the position of  $T_{\max}$  varies. The interpretation of measured temperature signals is in correspondence to the assumed heat flux distribution. The flux distribution also reflects the contact pressure in the contact zone. Equation 2.43 estimates the  $T_{\max}$  in grinding, where  $C_1$  is a constant between 0,76 and 1,06, that depends on the Peclet number (Eq. 2.44). The thermal properties of the workpiece material (thermal conductivity  $k$ , density  $\rho$ , specific heat capacity  $c$ ) define the diffusivity  $\alpha$ . The described analytical approach offers approximate temperature values, which accuracy lie on the correct estimation of  $q_o$ ,  $R_w$  and  $l_c$ .

$$T_{\max} = C_1 \cdot R_w \cdot \frac{q_o}{\beta} \cdot \sqrt{\frac{l_c}{v_w}} \quad (2.43)$$

$$Pe = \frac{v_w l_c}{4\alpha} = \frac{v_w l_c}{4} \frac{\rho c}{k} \quad (2.44)$$

Temperature estimation can also be tackled from the empirical point of view. The models that estimate steady-state cutting temperatures, share a basic schema presented by Equation 2.45 [WG06], where  $T$  is the workpiece temperature,  $C_1$ ,  $C_2$ ,  $C_3$  are empirically determined constants,  $v_s$  is the cutting speed and  $a$  is the depth of cut. These models are qualitatively correct and it would be easy to implement in a model where the protrusion geometry and penetration are known. But they show poor correlation when estimating cutting temperatures in operations with discontinuous chip formation as grinding.

$$T = C_1 v_s^{C_2} A_c^{C_3} \quad (2.45)$$

In brief,  $\varepsilon$ ,  $\dot{\varepsilon}$  and  $T$  vary along the trajectory of a grit in contact with the workpiece (rubbing, ploughing and cutting). Ideally, it is desirable to determine  $\varepsilon$ ,  $\dot{\varepsilon}$  and  $T$  regarding the grit penetration in order to define a tailored effective yield stress  $\sigma_y$  with the JC model. Thus, the individual force acting on each grit can be estimated. The experimental determination along the contact arc of them is complex. Only single-grit numerical models offer a complete overview of this parameters, but the validation of their results lays on the same complex experimental work. On the other hand, these parameters can be estimated analytically, assuming rough simplifying hypotheses. The use of analytical models to estimate the cutting on each grit in real time, would increase the complexity and computation requirements when modelling the contact of the whole grinding wheel. In addition,  $T$  would be estimated regarding the position through the contact length, not according to the penetration. In turn, there is almost none empirical model applicable at grit level, surely due to their lack of practical application.

## 2.5 Summary

This Chapter reviews the main scientific knowledge and technological advances in grinding to accomplish the goal of the dissertation, which is to develop a numerical model of the wheel-workpiece contact in surface grinding.

The review has highlighted the main aspects of the contact in grinding. The contact models for grinding proposed to date (Section 2.2.1) remark the elasticity of the grinding wheel and the workpiece, the surface topography and the grinding conditions (depth of cut  $a_e$ , normal force  $F'_n$ ) as main factors. The elasticity of the grinding wheel is found to be an important factor. However, the consideration of elastic behaviour of the workpiece disregards the ploughing and cutting action of grits. The wheel surface topography is created by dressing (Section 2.1.2), which should have a priority treatment in the contact analysis. Finally,  $a_e$  and  $F'_n$  depend on grinding speeds ( $v_s$ ,  $v_w$ ), surface topography, workpiece material and machine stiffness. The RQ model [RMQZ93] stands out from the proposed contact models (Section 2.2.1). The two-scale contact treatment (grit-workpiece and wheel-workpiece) and the geometrical analysis deduction of the composition of the contact arcs are, in any case, original and tricky. Summarising, the wheel model should reproduce the wheel stiffness and granular morphology, the surface topography must be taken into account regarding dressing conditions, and a correct modelling of the grit-workpiece engagement conditions is indispensable. There are several methods to measure the contact length: the single-pole thermocouple, the quick-stop, the patch grinding and AE. A variant of the thermocouple method, the APS, has proven to be an accurate and robust technique (Section 2.2.2). In addition, it can be used to measure the dynamic grain density  $G_{\text{dyn}}$ .

To model the grinding wheel (Section 2.3.1), numerical tools such as the finite element method present noticeable complications when facing the bi-material porous body of the wheel. In turn, uniform spring models create anisotropic bodies, allowing the relative rotation between grains. The kinematic models presented up to date are rigid, because most of them intend to foresee the obtained roughness on the workpiece surface. However, they offer useful solutions to model the surface topography of the wheel. The discrete element method has demonstrated noticeable skills to reproduce the random and granular structure of the wheel. Therefore, it has been chosen as numerical tool to model the grinding wheel.

The surface topography of the wheel (Sections 2.1.3 and 2.3.2) is defined by the surface grain density, peak height and cutting edge sharpness. The wheel surface is generated by dressing and, thus, the topography has to reflect its influence (Section 2.1.2). Rotating and multipoint stationary dressers increase the dressing efficiency, but the single-point dresser allows a tighter control of the dressing parameters. Grit and bonding bridge fractures are the main dressing mechanisms. The dressing aggressiveness determines the proportion of each mechanism, and creates an opener or closer surface altering the surface grain density. The model proposed by Malkin and Cook [MC71] stands out among the models that estimate the fraction of grains pulled out by bonding bridge

fracture. This model defines the minimum size of surface grits before pulling out regarding the surface grain density. Only two authors model the effect of dressing on the topography: Chen and Rowe [CR96] and Torrance and Badger [TB00]. In addition, Chen's model implements the dressing kinematics. The cutting edge sharpness is defined mainly by the rake angle or slope (suitable for conical or pyramidal cutting edges) and the sharpness radius (appropriate for spherical cutting edges). Both are measured in a similar way, by means of wheel topography or single-grit tracks 2D profile measurements. The 3D topography measurements methods are classified as optical (confocal microscope and SEM) and stylus methods (Section 2.1.3).

Beside the model of the grinding wheel and generation of the surface topography, it is indispensable to take into account the grit-workpiece interaction to model properly the wheel-workpiece contact. In a grinding contact model, the correspondent individual contact force is applied on a surface grain if a contact is detected between that grit and the workpiece. Section 2.4.2 reviews the action of an abrasive grit along the contact with the workpiece and compiles the experimental, analytical and numerical studies presented so far. As the aim of grinding is the material removal, the elastic assumption should be left aside. The basic analytical models (slip-lines and orthogonal cutting) are difficult to treat, because some parameters are unknown (friction), undefined (cutting edge geometry) or instantaneous material properties. Shaw's hardness analogy simplifies considerably the instantaneous grit force estimation. It considers implicitly rigid-perfectly-plastic contact. The cutting edges displace and remove material at high strain rates and temperatures on the work surface, which is described by the viscoplastic behaviour of materials (Section 2.4.3).

The model described in the following Chapter lays on the concepts reviewed in this Chapter.



## Chapter 3

# The numerical contact model

*This Chapter introduces the developed DEM model that simulates the contact between the wheel and the workpiece in surface grinding.*

*The procedure to build the model starts defining the tribological system of the contact. The aim of a system is to identify systematically the parameters that have influence on the process, studying the interactions between them and the weight of each one on the output. In grinding, the interaction of several parameters on multiple cutting edges determine the micro-cutting conditions. The review highlights the importance of surface topography and elasticity of wheel and workpiece in the contact analysis of grinding. In turn, the actual grit-workpiece interaction has been largely disregarded in contact models.*

*Then, the adaptation of the DEM formulation to the modelling of the grinding wheel-workpiece contact is described. As seen in the review, DEM is revealed as the most appropriate to model granular environments. The contact detection between discrete elements and interaction algorithms are defined. The DEM program and the calculation loop are also introduced.*

*The Chapter finishes describing the methodology of the construction of the DEM model of the grinding wheel, creation of the surface topography of the wheel and application of the boundary conditions. The model of the grinding wheel, beside the granular morphology, has to describe the mechanical behaviour of the wheel. The topography of the wheel surface is created regarding the dressing conditions. Once created the model, the boundary conditions are applied. The contact detection algorithm and the contact law are defined between the wheel surface elements and the workpiece. Two contact approaches are applied on the DEM model: the first one reproduces the static contact term of the Rowe-Qi model, while the second set applies directly grinding forces on the wheel according to the uncut chip thickness.*





## 3.1 System design

A system can be taken as a black box with several inputs  $\{X\}$  and outputs  $\{Y\}$ . The relationships between both groups are defined by the transference function (Eq. 3.1).

$$X \rightarrow Y \quad (3.1)$$

Grinding is an open system, non-protected and transitory. The inputs and outputs can be broken down into motions, materials, forces and deflections. In addition, there are disturbances to the process such as vibrations which may, in a few cases, be controllable, but not always avoidable. There are also outputs from the process which may be considered as losses, as noise, wear and heat generation.

Regarding the contact in grinding, the contact pressure seems to be applied on the whole apparent area, even though the actual contact area is much smaller [MRDI04]. This is the source of stress concentrations and plastic strains. In addition, the freshly generated surface suffers adhesion, absorption, chemisorption and corrosion due to friction and the removed material. The late effects do not affect directly on the contact length, but they do indirectly on forces, wear and temperatures.

The main factors in contact mechanics are,

- the mechanical properties of both bodies (yield stress, Young's modulus and Poisson's ratio),
- the roughness of both surfaces, and,
- the forces.

In addition, grinding includes the wheel-workpiece relative movement and material removal under high strain rates in grain scale. Each factor must be studied, either individually or together. The grinding wheel is a heterogeneous porous conglomerate. Its surface is very rough, which is created by dressing. In turn, the workpiece is a continuous homogeneous solid. Its surface can be considered smooth comparing with the wheel surface. The machine provokes the relative movements and the subsequent forces. Figure 3.1 lists the variables of each factor and the outputs [Klo09].

### 3.1.1 Numerical model of the contact in grinding

The numerical contact model is developed taking into account the factors listed in Figure 3.1 and considers implicitly the inter-relationships between them. The model has to be able to reproduce the heterogeneous nature of the wheel, implement the surface generation regarding the dressing conditions and apply the grit-workpiece contact conditions. The model follows the flow chart introduced in Figure 3.2, which makes an analogy between real and numerical environments. In the reality, the grinding wheel is a conglomerate of abrasive grains

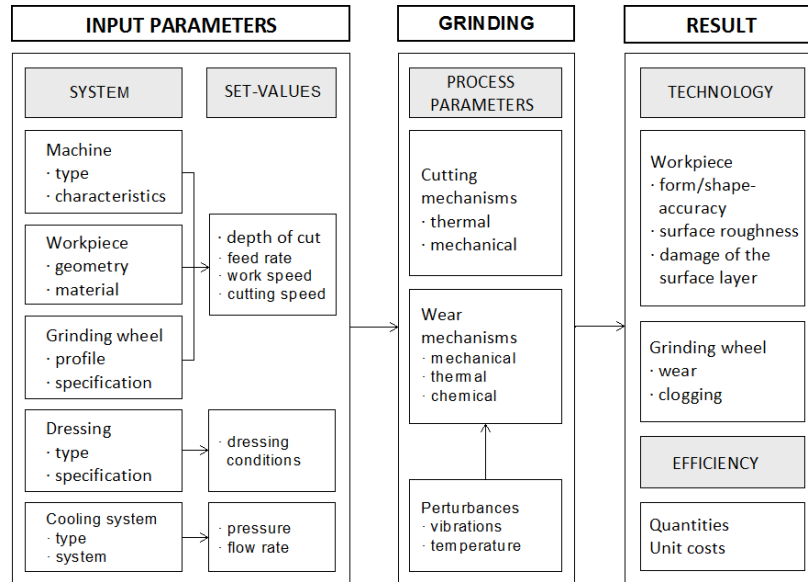


Figure 3.1: Input and outputs of the grinding system [Klo09]

and binder, with certain mechanical properties. The wheel surface is dressed, the working parameters are set and the grinding operation is performed. The numerical environment follows an analogue flow chart: first the model of the grinding wheel is built according to its features, the topography created by dressing is set on the model surface, the boundary and contact conditions are applied and, finally, the simulation is executed.

The numerical model complies the general framework of a general 3D topography model presented by Doman *et al.* [DWB06] to build the wheel and model the topography (see Section 2.3.2). The proposed model improves the framework adding elasticity to the grinding wheel body and considering the interactions between grains. Doman's framework presented in Figure 2.20, corresponds to the highlighted grey area in Figure 3.2.

From now on, the thesis also follows the structure of the flow chart.

## 3.2 The discrete element method

The numerical methods used in material mechanics are classified in discrete and continuous methods. Discrete methods are based on Newtonian mechanics and analyse the interaction between elements with an explicit formulation. The physical phenomenon modelled can be loose particles or solid bodies. Continuous methods consider the volume domains as a continuous media, an adequate hypothesis for a wide range of problems, but difficult to apply on heterogeneous granular bodies like the grinding wheel.

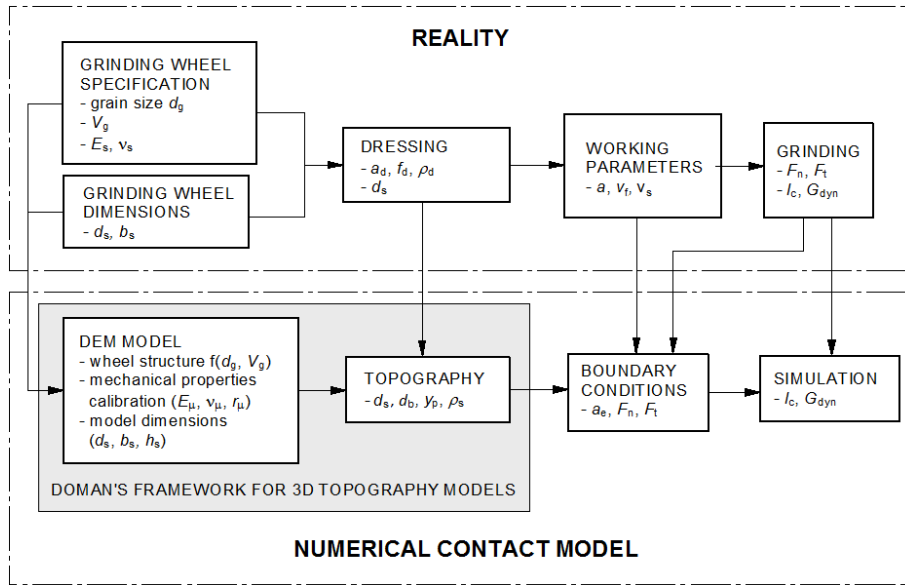


Figure 3.2: Flow chart of the grinding wheel-workpiece contact in the actual and numerical environments

A discrete method is any family of numerical methods for computing the motion of a large number of particles of nano-scale size and above [CJR13]. Discrete methods describe interactions between elements by regular (continuous and sufficiently differentiable) functions involving gaps, relative velocities and reaction forces. The molecular dynamics (MD) and the discrete element method (DEM) stand out among discrete methods. MD models the trajectories of atoms, including the interactions between them. Each atom is defined by the mass and the radius. MD tracks the position vector of the elements, regarding the physical formulation that defines the interaction forces. There is a huge number of atoms and, thus, an efficient algorithm for detecting contacts is required. The discretisation of time leads the numerical simulation: first, the interacting atoms are identified; then, the sum of interaction forces is computed; and, once the acceleration is defined, it is integrated numerically twice, to calculate the speed and the position of each atom on the next iteration. The velocity Verlet, the positions Verlet, the leap frog and the predictor-corrector explicit numerical integration schemes are the most used. MD are applied successfully on the simulation of a single grit action in the nano-scale with very shallow depths of cut ( $< 4$  nm) [MRDI04, BAG<sup>+</sup>06, OC10, MSKM15]. Hence, they cannot be applied on conventional magnitudes of depth of cut used in grinding (1 - 10  $\mu\text{m}$ ).

DEM is inspired in MD, but at micro- and meso-scales. DEM differs from MD in the scale of the domain, the orientation of the elements and the use of different element geometries. Cundall and Strack wrote the reference seed

paper [CS79], applying DEM on a geotechnical study of sedimentary rocks. Its application range has been widened for the study of wear and abrasion [FIB07], the rough contact between surfaces [JM12] and damage on heterogeneous solids, such as concrete or rocks [HDD04], the polishing of silica glass [IBNC08] or carbon fibre composites [IGI<sup>+</sup>10]. A discrete element (DE) can represent either a real grain of granular material (powder, sand, rock, corn...), or a representative volume element (RVE), that models continuous behaviour regarding the scale of the physical phenomena. The random nature of DEM is one of its strong points. DEM tracks the trajectory of several granular elements, studying the interactions between them. The force networks formed in a granular media can be easily visualised using DEM. Such measurements are nearly impossible in experiments with several small particles. Anyhow, DEM are computationally intensive, which limits either the length of the simulation or the size of the model (number of DEs).

The consideration of the contacts between elements has brought the development of two discrete methods families. Smooth contact dynamics are modelled as elastic interactions between rigid spheres. This formulation brings explicit time integration schemes (like MD), appropriate for fast dynamics mechanical problems. On the other hand, non-smooth contact dynamics leads to implicit time integration schemes. This formulation is more suitable for the modelling of static phenomena [MP88].

Based on the highlighted aspects in the state of the art (Chapter 2) and the system design (Section 3.1), DEM has been identified as the most suitable numerical tool to model the structure of the wheel and the contact in surface grinding. DEM has been already used in the modelling of the mechanical behaviour of the grinding wheel [LYZW15a, LYZW15b]. The open formulation of DEM allows to define one or various relationships within the elements of a model, that opens the application of DEM in grinding in a natural way. These Section describes the features of the tailor-made DEM model in order to model the contact in grinding.

### 3.2.1 Adaptation of DEM to the contact model

As commented before, the contact model has to reproduce the mechanical behaviour and granular morphology of the grinding wheel, and use that model to simulate the contact conditions between the surface grits and the workpiece. The morphology description of the wheel requires to model the abrasive grit, the bonding material and the structure that form both of them. In turn, the contact conditions cover the abrasive-abrasive interactions within the wheel and the contact between the surface DEs of the wheel and the workpiece. In addition, the surface topography of the wheel model should be able to be treated to reproduce the dressing features. The flexibility of the formulation is one of the strong points of DEM and it allows to combine these features in a single tailor-made model. The starting point is to assume that a DE represents an abrasive grit. Thus, the wheel is composed of several DEs. In turn, the workpiece is modelled by a single rectangular DE.

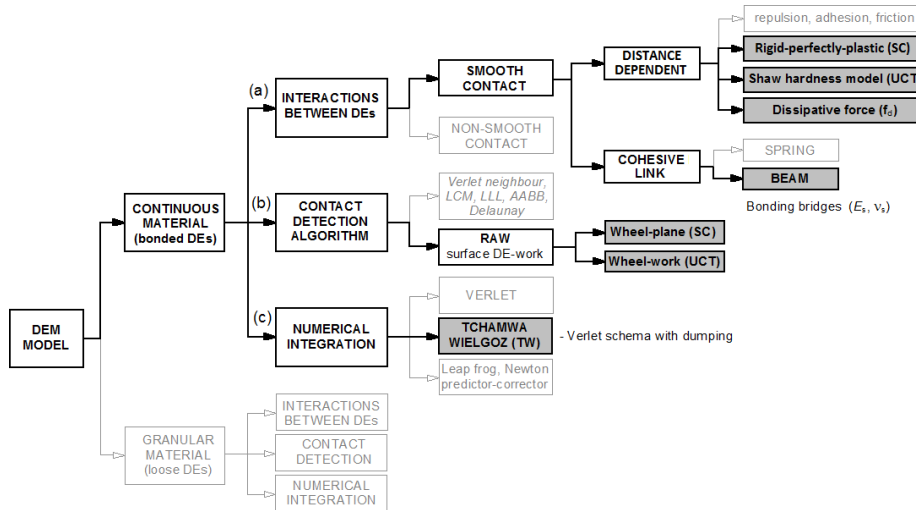


Figure 3.3: Flow chart of the formulation of the proposed DEM contact model

Figure 3.3 shows the flow chart of the chosen tailor-made solution to model the grinding wheel and simulate the contact with the workpiece. Regarding the physical phenomenon, the DEM open formulation can model either a continuous material (e.g. the mechanical behaviour of concrete) or loose granular materials (e.g. the emptying of a corn hopper). The grinding wheel, as a consistent solid, should be modelled as a continuous material. Even is a heterogeneous material, the continuous behaviour of the solid arises due to the combined response of abrasive grains and bonding material.

The formulation of DEM has three grounds in both formulations: (a) the relationship between elements, (b) the contact detection algorithm and (c) the numerical integration method (Fig. 3.3). The combination of different solutions leads several options, adaptable to a wide range of physical phenomena.

The interactions between elements (a) are classified in two main groups: the smooth and non-smooth contact. The smooth contact approach considers that the elements are rigid, whereas the DEs used in non-smooth contact are deformable. The abrasive grits that form the wheel can be considered rigid due to their high hardness. In turn, the workpiece is deformable and the contact conditions with the abrasive grains surpass largely the elastic regime. The flexibility of the smooth contact formulation allows the adaptation of the contact conditions to the modelled interaction. Thus, the interaction between two DEs can be considered a contact (Hertzian elastic, adhesion, friction, ...), cohesive link (spring, beam) and distance dependent (electromagnetic fields, heat conduction, ...). In addition, the explicit formulation of the smooth contact approach offers advantages in the modelling of dynamic phenomena.

Regarding the DEM contact model for surface grinding, there are two types of interactions between elements: the interactions between abrasive grits within

the grinding wheel body, and the contact between surface grits and workpiece. For the first, wheel body elements, elastic beams are chosen. The bonding material creates physical bonds between grains, which stand, beside axial and shear loads, bending moments and torque. Beams reproduce faithfully the behaviour of the bonding material. Unlike beams, contact conditions (Hertz, adhesion, ...) and springs do not prevent the relative rotation between DEs as the bonding material does. And for the second, the interactions between wheel surface DEs and the workpiece, a tailor-made contact law is developed to calculate the individual reaction on each grit. This contact law includes a dissipative force to absorb energy during the simulation and, thus, converge faster.

Regarding the contact detection algorithm (b), the demanding computation requirements make critical the election of the most adequate algorithm. Thus, several algorithms have been proposed to avoid the raw analysis of contacts, i.e. the analysis of the relative position of each DE according to the rest of DEs. Some of the most popular are the Verlet-neighbour list, the linked cell method (LCM) and the Delaunay triangulation. However, the use of beams avoids the contact detection between elements that belong to the grinding wheel. This is specially advantageous, because all DEs but one (the workpiece) belong to the grinding wheel. Regarding surface DE-workpiece contact detection, two tailored algorithms are developed regarding the boundary conditions, which are described in detail in Section 3.5. The use of a single rectangular prism DE to model the workpiece simplifies greatly the contact detection.

The third foundation of DEM is the numerical integration (c) of the new position and orientation of DEs at each iteration. Several integration methods have been adapted to the DEM formulation, as the velocity Verlet, positions Verlet, leap frog or predictor corrector integration schemes. The velocity Verlet integration scheme is most used in DEM. To improve the convergence in quasi-static simulations, Tchamwa and Wielgosz [MGR09] add a numerical damping factor in the Verlet scheme.

The following Sections explain in a deeper way these concepts concerning the grinding contact model.

### The discrete element

DEM usually uses spherical elements, but other geometries can be implemented, such as multi-sphere, ellipsoidal [MI10] and polyhedral elements [HWKES11]. The definition of the spherical DE is noticeably the simplest and its use simplifies greatly the creation of the model and the detection of the contacts between elements. Li *et al.* [LYZW15b], when modelling the grinding wheel, propose a multi-sphere approach to model amorphous abrasive grits. However, they do not develop further the idea. They implement the truncated octahedron element in their model, a Boolean intersection between an octahedron and a spheroid. The shape of the grains has direct influence on the compaction and, thus, on the openness of the wheel structure. As seen in the state of the art (Section 2.3.2), the alumina abrasive grits have cubic, angular and pointed shape. Cubic and angular shapes have an aspect ratio close to one. In turn, pointed (elongated)

grits have one dimension larger than the other two. The contacts between DEs within the wheel are negligible, because the stiffness of the model is brought by the beam network between DEs. Thus, the use of the octahedron or ellipsoidal elements is not justified for the modelling of the wheel structure. Their use would be reasonable just for a very precise modelling of the packing process of grits of definite shapes, as CBN or diamond. On the other hand, the surface topography of the model is numerically modified, so the DE shape does not affect the surface of the model.

As the geometry of the elements only affects on the creation of the model, the spherical geometry assumption proves acceptable and convenient for cubic and angular shapes. In this thesis, the spherical geometry is chosen to model the abrasive grits. If the grain shape is elongated, then the model should be upgraded to a double spherical DE or multi-sphere strategy. The study of the packing process of grinding wheels regarding grain geometry and aspect ratio by means of DEM is one of the future promising research lines that opens this thesis.

A spherical DE in the 3D space is defined by its diameter  $d_{DE}$ , density  $\rho_{DE}$ , position coordinates  $(x, y, z)$  and, depending on the physical event modelled, the orientation too. In this case, the use of beams makes necessary to take into account the orientation of each DE, so it is included in the formulation of the contact model.

The orientation can be expressed by Euler angles, rotation matrices, quaternions or a combination of them. In DEM, the use of unit quaternions  $\mathbf{q}$  offers a precise and powerful representation of the angular velocity and orientation of DEs [ACI15]. In addition, the Verlet integration can also be used for the numerical integration of quaternions. Quaternions define the 3D orientation with four scalar variables. They are an extension of complex numbers with two additional imaginary dimensions, so they contain one real dimension and three imaginary dimensions (square roots of -1). In this way, the easiness to define the orientation and relative rotations has spread its use, from 3D computer graphics to crystallographic planes definition.

Combining both, the frame of a DE is defined by the Cartesian position coordinates and the unit quaternion.

Regarding the workpiece, it is modelled by a single rectangular prismatic DE. This DE works as a reference plane to calculate the penetration of the wheel surface DEs in contact. An individual tailored cutting force is applied on this element, which transmits through the contacting DEs to the grinding wheel. The use of a single DE simplifies greatly the contact detection. The prismatic DE behaves as viscoplastic material when a grain penetrates and applies a reaction force according to the depth.

### Interactions between discrete elements

The interactions between elements are modelled by forces, moments and, sometimes, potentials energies. These are classified as contact, cohesive links and potentials. Figure 3.4 introduces the contact and cohesive (spring and beam)

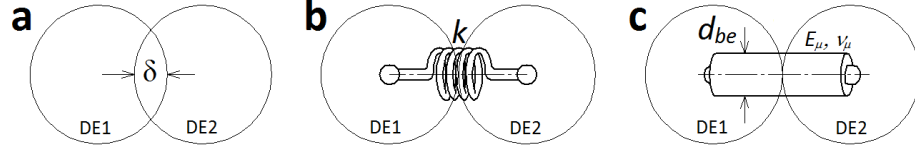


Figure 3.4: Common contact conditions between DEs: (a) contact and potential link ( $f(\delta)$ ), (b) spring and (c) beam [ACI15]

link types.

The interactions between surface elements and workpiece could be treated with the usual contact link approach: Hertz mechanics and Coulomb's friction, together with the dissipation force. However, due to the particular conditions between the abrasive grits and the workpiece surface, an *ad hoc* contact formulation has been developed, adding the usual dissipation force. Two contact definitions are developed. The first one models the rough contact term of the RQ contact model [RMQZ93] and assumes rigid-perfectly-plastic contact [Joh87]. It models the static contact between the grinding wheel and a plane. The variation of material properties due to the strain rate and temperature are upgraded with the JC constitutive model. The second approach implements the hardness analogy of chip formation [Sha71]. In this case, the model estimates the corresponding uncut chip thickness of each surface DE in contact, to calculate the reaction on each DE. The formulations of both contact conditions are explained in detail in Section 3.5. In this way, the DEM model is adapted to these special contact conditions between the surface grits and the workpiece.

As a dynamic model, the dissipative or dumping force appears in the contact reaction between two DEs. The damping force  $\mathbf{f}_d$  ( $\mathbf{f}_{d1}, \mathbf{f}_{d2}$ ) is proportional to the relative velocity of the two DEs  $\dot{x}$  and normal to the contact plane (Eq. 3.2).  $c$  is the damping coefficient, which is calculated with Equation 3.3.  $\alpha$  is the damping factor  $[0, 1]$  and  $\alpha_c$  is the critical dumping of the mass-spring system (Eq. 3.4); so  $c$  varies between  $[0, \alpha_c]$  (Eq. 3.3). Equation 3.4 calculates  $\alpha_c$ , where  $K_c$  is the largest stiffness of the element that compose the couple and  $M_{12}$  is the equivalent mass of both DEs (Eq. 3.5).  $m_{DE1}$  and  $m_{DE2}$  are the correspondent masses of DE1 and DE2. Equation 3.6 calculates  $\dot{x}$ , where  $\mathbf{v}_1$  and  $\mathbf{v}_2$  are the respective speeds of the DEs and  $\mathbf{n}_{21}$  is the unit vector normal to the contact plane. The dumping force  $\mathbf{f}_d$  is added to the contact reaction force of both DEs.

$$\mathbf{f}_d = \mathbf{f}_{d1} = -\mathbf{f}_{d2} = c \dot{x} \mathbf{n}_{21} \quad (3.2)$$

$$c = \alpha \cdot \alpha_c \quad (3.3)$$

$$\alpha_c = 2\sqrt{K_c \cdot M_{12}} \quad (3.4)$$



$$M_{12} = \frac{m_{\text{DE1}} \cdot m_{\text{DE2}}}{m_{\text{DE1}} + m_{\text{DE2}}} \quad (3.5)$$

$$\dot{x} = (\mathbf{v}_2 - \mathbf{v}_1) \cdot \mathbf{n}_{21} \quad (3.6)$$

Regarding the interaction between the elements of the wheel, the contact between them is negligible. The bonding material creates a 3D rigid structure that behaves as a continuous material regarding the representative volume element (RVE) [CJR13]. The scale is clearly defined by the abrasive grit size and the abrasive volumetric fraction (or density). These interactions can be reproduced by cohesive links (Fig. 3.4): normal springs, dual springs (a pair of normal and tangential springs) or Euler-Bernoulli beams. To model the bonding bridges, beams are more adequate than springs, because they prevent the rotation at both ends, supporting, beside axial forces, bending and torque moments. In addition, they allow the crack analysis regarding a given fracture stress. Although this aspect is out of the scope of this thesis, the study of the pull-out of surface grits due to bonding bridge fracture during grinding or dressing, would be possible considering the fracture stress of the beams. These are two of the promising future lines that opens this thesis.

The formulation of the Euler-Bernoulli beam theory, unlike the Timoshenko beam theory, disregards the shear term from the deflection analysis [Tim53]. This simplification becomes noticeable as decreases the aspect ratio (length-to-height) of the beam. The aspect ratio of beams in the DEM model is higher than 3, which creates an error smaller than 2,4 %. The use of Bernoulli beams makes them stiffer. In any case, this effect takes also place during calibration, where the mechanical properties are determined. Therefore, the consequences are diminished. It is an usual practice the application of Euler-Bernoulli beams in the DEM formulation. This work uses the ones proposed by [CWKH08, AICN12, ACI15].

The following Section reviews the formulation of the beams and explains the calibration procedure of their mechanical properties.

### The cohesive beams

The elastic beams are defined by two geometrical parameters (initial length  $L_\mu$  and diameter) and two mechanical properties (Young's modulus  $E_\mu$  and Poisson's ratio  $\nu_\mu$ ). The beams are cylinders of  $d_{\text{be}}$  diameter, which are defined as a fraction  $r_\mu$  of the average diameter of both the DE that connects ( $d_{\text{DE1}}$ ,  $d_{\text{DE2}}$ ) (Eq. 3.7), so it is straightforward defined by the beam diameter ratio  $r_\mu$ . Beams are massless since mass properties are assigned to DEs.  $L_\mu$  is defined by the initial distance between elements. The length variation  $\Delta L_\mu$ , together with orientation variation, is updated every iteration. The variation of position and orientation of DE centres sets the interaction forces between grains: axial and shear forces, torque and bending moments [AICN12, ACI15].

$$d_{\text{be}} = r_\mu \cdot \left( \frac{d_1 + d_2}{2} \right) \quad (3.7)$$

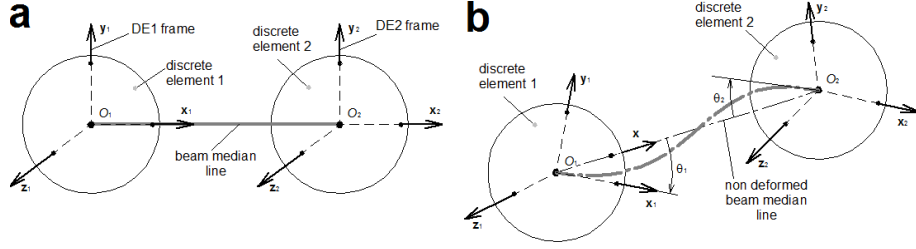


Figure 3.5: Elastic beam (a) relaxed and (b) loaded [AICN12, ACI15]

At the beginning, beams are relaxed (Fig. 3.5(a)) and this position is taken as reference. The DE frames  $F_1(O_1, \mathbf{x}_1, \mathbf{y}_1, \mathbf{z}_1)$  and  $F_2(O_2, \mathbf{x}_2, \mathbf{y}_2, \mathbf{z}_2)$  are placed aligning  $\mathbf{x}_1$  and  $\mathbf{x}_2$  with the beam axis  $(\overline{O_1O_2})$ . The ends of the beam are fixed to the DE centres  $O_1$  and  $O_2$ . The beam is represented by the median line  $(\overline{O_1O_2})$ . Figure 3.5(b) shows the beam in a loaded state induced by the relative displacement of the elements regarding the initial configuration. The reaction force, torque and bending moments are expressed in the beam local frame  $F(0, \mathbf{x}, \mathbf{y}, \mathbf{z})$ . Equation 3.8 defines the unit vectors of the local frame  $(\mathbf{x}, \mathbf{y}, \mathbf{z})$ .

$$\mathbf{x} = \frac{\overrightarrow{O_1O_2}}{|\overrightarrow{O_1O_2}|}, \quad \mathbf{y} = \mathbf{x} \times \mathbf{x}_1 \quad \text{and} \quad \mathbf{z} = \mathbf{x} \times \mathbf{y} \quad (3.8)$$

The orientation variation is defined regarding the axial and tangential angles  $(\theta_1(\theta_{1x}, \theta_{1y}, \theta_{1z}), \theta_2(\theta_{2x}, \theta_{2y}, \theta_{2z}))$  regarding the initial frames at  $O_1$  and  $O_2$ . Consequently, the force  $(\mathbf{f}_{\text{DE1}}, \mathbf{f}_{\text{DE2}})$  and moment  $(\mathbf{m}_{\text{DE1}}, \mathbf{m}_{\text{DE2}})$  reactions on DE1 and DE2 are defined by Equations 3.9, 3.10, 3.11 and 3.12 respectively, where  $L_\mu$  and  $\Delta L_\mu$  are the initial beam length and beam length variation,  $S_\mu$  beam cross section,  $I_\mu$  the moment of inertia,  $I_{p\mu}$  the polar moment of inertia, and  $E_\mu, G_\mu$  are the elasticity and shear modulus of the beam.

$$\mathbf{f}_{\text{DE1}} = E_\mu S_\mu \frac{\Delta L_\mu}{L_\mu} \mathbf{x} - \frac{6E_\mu I_\mu}{L_\mu^2} ((\theta_{2z} + \theta_{1z})\mathbf{y} + (\theta_{2y} + \theta_{1y})\mathbf{z}) \quad (3.9)$$

$$\mathbf{f}_{\text{DE2}} = E_\mu S_\mu \frac{\Delta L_\mu}{L_\mu} \mathbf{x} + \frac{6E_\mu I_\mu}{L_\mu^2} ((\theta_{2z} + \theta_{1z})\mathbf{y} - (\theta_{2y} + \theta_{1y})\mathbf{z}) \quad (3.10)$$

$$\mathbf{m}_{\text{DE1}} = \frac{G_\mu I_{p\mu}}{L_\mu} (\theta_{2x} - \theta_{1x})\mathbf{x} - \frac{2E_\mu I_\mu}{L_\mu} ((\theta_{2y} + 2\theta_{1y})\mathbf{y} - (\theta_{2z} + 2\theta_{1z})\mathbf{z}) \quad (3.11)$$

$$\mathbf{m}_{\text{DE2}} = -\frac{G_\mu I_{p\mu}}{L_\mu} (\theta_{2x} - \theta_{1x})\mathbf{x} - \frac{2E_\mu I_\mu}{L_\mu} ((2\theta_{2y} + \theta_{1y})\mathbf{y} - (2\theta_{2z} + \theta_{1z})\mathbf{z}) \quad (3.12)$$

Unlike finite element method, continuous mechanical behaviour laws cannot be directly introduced into DEM formulation. Material properties, as  $E_s$  and

Poisson's ratio  $\nu_s$ , are emergent properties at macroscopic scale in the DEM model. The mechanical properties of the beams at microscopic scale (Young's modulus  $E_\mu$ , beam radius  $r_\mu$  and Poisson's ratio  $\nu_\mu$ ) arise the macro-mechanical behaviour of the modelled material. This phenomenon is called homogenisation [CJR13]. Section 3.3.1 explains how the wheel hardness or grade letter is related to the Young's modulus  $E_s$  of the wheel. Unlike Li *et al.* [LYZW15b], the volume fraction of binder does not match necessarily with the beams volume, because their role is to provide stiffness to the model, not to represent realistically the geometry of bonding bridges. Thus the model is simpler and requires less computation effort.

The micro-scale mechanical properties are calibrated carrying out several tensile numerical tests on cylindrical specimens, following the procedure set by André *et al.* [AICN12]. There are three micro-scale parameters to calibrate ( $\nu_\mu$ ,  $r_\mu$ ,  $E_\mu$ ) regarding the macro-scale properties ( $E_s$ ,  $\nu_s$ ). Six model sizes are used in the calibration (500, 1000, 2000, 5000, 10000 and 20000 DEs) and four models per size. The reason is that same input-data when constructing the DEM model, creates a different models, due to the randomness of the creation procedure. So the result of each model size is the average of four simulations. Numerical tensile tests measure the axial and transversal strain of the specimens, which allows the definition of the macro-scale properties. Analysing the results graphically, the micro-scale properties are fitted regarding the macro-scale properties.

Figure 3.6 shows the flow chart of the calibration of beam properties. The micro-scale  $\nu_\mu$ ,  $r_\mu$  and  $E_\mu$  parameters affect in a different way in the macro-scale properties. André analyses those interactions by a design of experiments (DOE) [AICN12]. For any specimen size,  $\nu_\mu$  has scarce influence on macro-scale properties. With that in mind, the calibration is performed in two steps. First,  $\nu_\mu$  is set as  $\nu_s$ , sparing a variable. Maintaining fixed  $E_\mu$ , the influence of  $r_\mu$  on  $\nu_s$  is studied. Once  $r_\mu$  is set, the second step analyses the influence of  $E_\mu$  on  $E_s$ . In that way, the three micro-scale variables are set. The defined micro-scale properties are independent to the model size, DE size and abrasive fraction. The mechanical properties are applied after the creation of the DEM model of the wheel, before running the simulation.

### The Tchamwa-Wielgoz (TW) numerical integration

The Tchmwa-Wielgoz (TW) schema modifies the Verlet integration, computing the new position and orientation of all DEs adding some numerical dumping. The TW schema is an off-centre explicit integration scheme, that allows the dissipation of high frequencies. In quasi-static simulations, the system vibrates around the static solution. The dissipation is controlled by the numerical dumping factor  $\beta$ . The second time derivative equality is modified. Hence, the simulation convergence is improved.

To compute the new position and orientation of each DE, the velocity Verlet schema performs a numerical integration of the acceleration and the angular acceleration (using quaternions). The addition of  $\beta$  produces some numerical damping when the time increment  $\Delta t$  is close to the critical value. Mahéo *et*

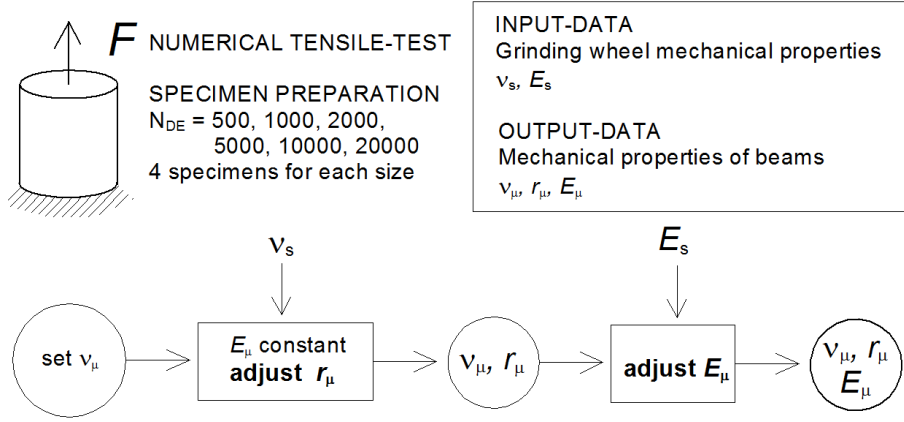


Figure 3.6: Flow chart of the calibration of the micro-scale properties of the beams [AICN12]

*al.* [MGR09] assert that a numerical damping factor  $\beta = 1, 3$  improves the convergence rate to the static solution.

The new acceleration  $\ddot{\mathbf{x}}_{n+1}$  is computed according to the fundamental principles of dynamics. The resultant of the forces applied at the DE gravity centre  $\mathbf{f}_{\text{DE}}$ , depends only on the new computed position  $\mathbf{x}(t)$ . The acceleration of a DE  $\ddot{\mathbf{x}}_{n+1}$  is estimated by Equation 3.13, where  $m_{\text{DE}}$  is the mass of the DE.

$$\ddot{\mathbf{x}}_{n+1} = \frac{\mathbf{f}_{\text{DE}}}{m_{\text{DE}}} \quad (3.13)$$

Given the position, velocity and acceleration components ( $\mathbf{x}_n, \dot{\mathbf{x}}_n, \ddot{\mathbf{x}}_n$ ) of a DE at the step ( $n$ ) and the acceleration  $\ddot{\mathbf{x}}_{n+1}$  at the step ( $n+1$ ), Equations 3.14 and 3.15 estimate the velocity and position at the step ( $n+1$ ). In the same way, given the position, velocity and acceleration quaternions ( $\mathbf{q}_n, \dot{\mathbf{q}}_n, \ddot{\mathbf{q}}_n$ ) of a DE at the step ( $n$ ) and the acceleration quaternion  $\ddot{\mathbf{q}}_{n+1}$  at the step ( $n+1$ ), Equations 3.16 and 3.17 estimate the velocity and position quaternions at the step ( $n+1$ ).

$$\dot{\mathbf{x}}_{n+1} = \dot{\mathbf{x}}_n + \frac{\Delta t}{2} (\ddot{\mathbf{x}}_n + \ddot{\mathbf{x}}_{n+1}) \quad (3.14)$$

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \Delta t \dot{\mathbf{x}}_{n+1} + \beta \frac{\Delta t^2}{2} \ddot{\mathbf{x}}_n \quad (3.15)$$

$$\dot{\mathbf{q}}_{n+1} = \dot{\mathbf{q}}_n + \frac{\Delta t}{2} (\ddot{\mathbf{q}}_n + \ddot{\mathbf{q}}_{n+1}) \quad (3.16)$$

$$\mathbf{q}_{n+1} = \mathbf{q}_n + \Delta t \dot{\mathbf{q}}_{n+1} + \beta \frac{\Delta t^2}{2} \ddot{\mathbf{q}}_n \quad (3.17)$$

Once the new position quaternion  $\mathbf{q}$  and velocity quaternion  $\dot{\mathbf{q}}$  are computed, the Euler relation formulated for quaternions gives the angular acceleration quaternion  $\ddot{\mathbf{q}}$  at the next time step [AICN12].

### 3.2.2 Creation of the DEM model

Beside the DEM formulation, the DEM model is constructed according to targeted physical phenomenon. The modelling of loose particles usually starts from a uniform distribution of DEs. Sometimes, a previous phase of positioning is required (i.e. simulation of free falling of DEs due to gravity) to start up the simulation from a totally random arrangement of DEs.

When modelling continuous bodies, sometimes the uniform arrangement of grains is adequate to the modelled phenomena. However, it creates anisotropic solids, and the modelling of the mechanical behaviour requires randomness and homogeneity of DEs, and isotropy of mechanical properties. The problem is faced shacking a uniformly placed DEs or by numerical sintering. The shacking technique starts with an uniform arrangement of DEs within the model domain. The density can be adjusted using the simple cubic or body-centred cubic unit cell structure. A randomising algorithm displaces each DE, avoiding the interpenetration between them. The shacking technique has been used profusely by topography models in grinding [DWB06].

In turn, the numerical sintering fills randomly the closed model domain of DEs. Andre *et al.* [AICN12, ACI15] propose a procedure to create the DEM model in three stages. A closed volume by elastic walls form the boundaries of the discrete domain. In the first stage, the discrete domain is filled placing DEs randomly. Each DE has a random diameter, that follows a given statistical distribution (uniform or normal). When it is impossible to find a random location without interpenetration within the existing elements, the second stage starts. An iterative process is run to place randomly new elements one by one, re-organising the elements already placed in the discrete domain. The kinetic energy of the discrete domain lowers, ending when the set minimum value is reached. A small interpenetration is allowed, which is corrected in the third stage relaxing the domain walls.

The shacking technique is more simple and faster than the numerical sintering method. However, a model created by shacking shows traces of the initial uniform arrangement in the mechanical properties and behaviour. The numerical sintering has not such problems, because its construction is random from the beginning. By the time writing the thesis, the discrete domains used in numerical sintering have basic geometries, such as a sphere, cylinder or box. In turn, the shacking technique is easily applied on complex geometries created by computer aided design.

As the grinding wheel has a simple geometry, a cylinder, and the isotropic properties are relevant on the modelling of mechanical behaviour, the numerical sintering is chosen to construct the DEM model. To reduce the size of the model, a slide of a cylinder is cut.

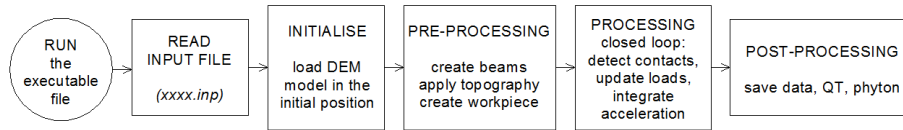


Figure 3.7: Flow chart of a simulation with GranOO [And12]

### 3.2.3 Structure of the DEM model

The contact DEM model is carried out using the granular object oriented (GranOO) workbench developed by the *Institut de Mécanique et d'Ingénierie* (I2M), at the ENSAM of Talence (Bordeaux). GranOO is written in C++, and it allows to design and create DEM models. GranOO is not an executable software. It takes advantage of open libraries (numerical and graphical). It is structured on three libraries: geometrical tools (Cartesian frame, quaternion, vector, matrix, shape, ...), DEM objects (discrete element, cohesive link, contact, set of, ...) and computing architecture (*plugIns*, XML reader, standard template library (STL), ...). The workbench is compatible with multi-physics (mechanics, thermal, electrical) and also supports hybrid FEM-DEM models.

Before starting the DEM simulation, the DEM model is constructed following the numerical sintering procedure. Section 3.3.2 defines the input parameters and the procedure followed in the construction of the DEM model of the wheel. The average DE diameter and DE size distribution are adjusted to fit the average grain size and abrasive volumetric fraction. The DEM model is completed creating the beams between the DEs in contact and generating the surface topography regarding the dressing conditions.

The simulation steps are defined in an input file written in XML. This file calls in the given order the *plugIns*, programs that perform a certain function written in C++. The workbench allows to compile the tailored *plugIns* developed by the user, increasing largely the adaptability to the physical phenomenon.

DEM simulation shares the structure of usual numerical tools: pre-processing, processing and post-processing. Figure 3.7 shows the flow chart of a simulation. First, the input file (*.inp*) is read: the program executes the lines in order one by one, calling the correspondent *plugIn*. In the pre-processing, the DE domain previously created is loaded and the conditions of the simulation are defined (initial speed, external forces, displacement restrictions...). In the processing, the simulation enters in a closed loop. On each iteration, the forces acting on each grain are calculated regarding the given physical laws. With the resultant force and mass on each element, the acceleration, speed and position are calculated, which are used in the next iteration. The time-step between iteration is set as the highest that assures the stability of the simulation.

The simulation ends when the defined maximum iteration number is reached. Finally, the post-processing collects and saves the results data.

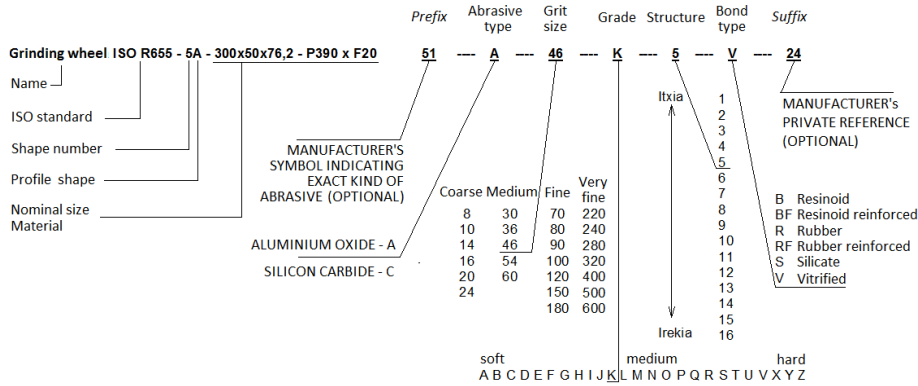


Figure 3.8: Designation of conventional grinding wheels [Mal08]

### 3.3 Grinding wheel DEM model

This Section describes step by step the construction of the DEM model of the grinding wheel. First, the composition and structure of the wheel are described. And then, the procedure to generate the DEM model of the wheel body is explained. The model is built in two steps: first, the granular structure is created and, then, the mechanical properties are transferred to the DEM model.

#### 3.3.1 Characteristics of the grinding wheel

The manufacturing of the wheel (mixing, packing and backing) determines the fragile structure of the grinding wheel. A deep understanding of the creation of the grinding wheel is needed in order to construct the model. However, the definition of the grinding wheel composition is diffuse: even the same manufacturer offers wheels of different characteristics under the same designation, keeping in secret the *know-how*, components and additives.

The grinding wheel dimensions and designation are the required information in order to create the grinding wheel DEM model. Different proportions of abrasive, binder and pores lead several possible combinations of grinding wheels, adaptable to the target application. Then, the main measurement techniques to verify the grinding wheel properties are described, which have been developed *ad hoc* and are continuously improved.

#### Designation of grinding wheels

The designation of grinding wheels is regulated by national and international standards, as ISO R565:1999. The designation has two parts: the first part defines the wheel geometry and dimensions and the second part the composition and mechanical properties. The composition and mechanical properties are defined according to abrasive material, grain size, hardness grade, structure and bond type. Figure 3.8 shows an example.

In the first part of the designation, the shape of the wheel is designated by a number and the profile by a capital letter. The nominal dimensions follow a given order: the outer diameter  $d_s$ , the width  $b_s$  and the inner diameter.

The second part of the designation starts with an optional number defined by the manufacturer, to define the precise abrasive type. The family of abrasive is defined by the following letter. The conventional abrasives are corundum or alumina (A) and silicon carbide (C), whereas the super-abrasives are the diamond (D) and CBN (B).

The following number indicates the grain size. ISO 8486-1:1996 explains how to define abrasive grain size distribution and average grain size. Grinding wheels are formed by a mix of three or four adjacent grain sizes to control grain density and hardness of the wheel. Grit size is defined by the sieve number  $M$  and the average grain size  $d_g$  can be estimated with Equation 3.18 [Mal08]. [KJL97] consider that the grain size follows the normal distribution, hypothesis followed by later authors [HK03, Dar13, JGB<sup>+</sup>13, LWBG13, LYZW15a]. This assumption considers a single mesh size, i.e., neglects the mix of adjacent grit sizes. However, experimental measurements of grain size distribution seem closer to a uniform distribution in actual wheels abrasive mixes, enlarging the size range [MC71].

DEM accepts both types of distribution, normal and uniform. A given abrasive size number fits the uniform distribution, which is composed of abrasive grits sieved between two mesh sizes. On the other hand, the abrasive grit content in a grinding wheel is a mix of two, three or four adjacent grit sizes. In this way, the mix of uniformly distributed populations follows the normal distribution. However, grinding wheel manufacturers are reluctant to publish the technological data of the abrasive grit mix. The uniform distribution has been chosen for the DEM model. As the grit sizes of the mix are adjacent, the standard deviation of the mix coincides more or less with the range of the uniform distribution. So the resulting DEM models would be similar and the error of the assumption of uniform distribution is minimum.

Equation 3.19 defines the range  $\mu_g$ , being  $d_g^1$  and  $d_g^2$  the biggest and smallest grain diameter of the mix regarding sieves used.

$$d_g = \frac{15,2}{M} \quad (3.18)$$

$$\mu_g = \frac{d_g^1 - d_g^2}{2} \quad (3.19)$$

The following letter indicates the grade of the wheel or hardness, being  $A$  the softest and  $Z$  the hardest. The hardness indicates the resistance to pull out a grit from the wheel, a measurement of the adhesion and resistance of bonding bridges.

The structure number states the openness of the wheel and it is defined by the volumetric fraction of abrasive  $V_g$ : from 0 (close structure) to 14 (open structure). Malkin [Mal08] also offers an expression (Eq. 3.20) to estimate  $V_g$  according to the structure number  $S$ .



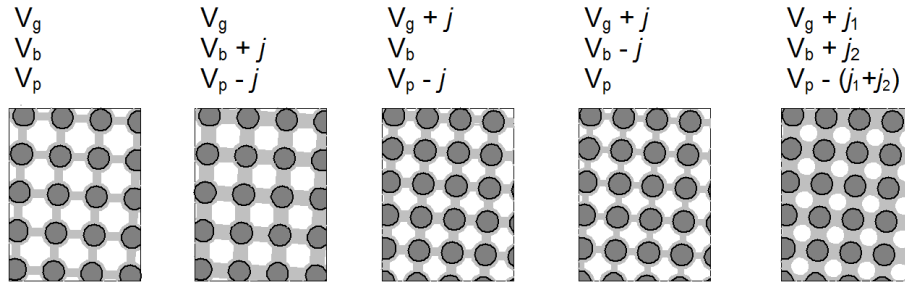


Figure 3.9: Effect of the variation of abrasive, binder and pore proportions on wheel structure [Klo09]

$$V_g = 2(32 - S) \quad (3.20)$$

The following letter defines the type of binder: vitreous (V), resinous (R) and metallic (M). Finally, the last number indicates the maximum cutting speed, which is also indicated by a coloured band on the wheel label.

### Composition of grinding wheels

As a whole, the content of abrasive, binder and pores determines the characteristics and behaviour of the wheel. The volumetric fraction of abrasive grits  $V_g$  define the structure number of a wheel and, together with the bonding material  $V_b$  and pores  $V_p$  fractions, the wheel grade. The sum of  $V_g$ ,  $V_b$  and  $V_p$  is one. Grinding wheel manufacturers carry out a precise control of this proportions in weight when mixing. Figure 3.9 shows the effect of the volumetric content variation on the structure [Klo09].

In turn, Figure 3.10 shows the technological limits of variation on components related to the grade and the structure [Klo09]. The trigonal graph offers an overview of the possible choices. It is specially interesting to follow the lines of same proportion. The size and shape of the pores depend on abrasive grit size and shape. The abrasive fraction can be increased mixing smaller grits (up to 60 %) and/or shaking, and decrease controlling the shape of the grits (up to 30 %). The minimum packing is determined by the minimum contacts per grit. Once defined the abrasive fraction, the remaining volume is shared by the binder and the pores. High porosity helps to distribute the cutting fluid and take out the chip. As the binder fraction increases, the resistance of bonding bridges also increases, obtaining a higher hardness.

The vitrified bonds are obtained sintering the mixed between abrasives and binder powder (silica, calcium and chemical flux). Water or dextrin is added to the mix. The mass can maintain the shape in *green*. The high temperatures inside the furnace promote the diffusion at the interface of the abrasive grits and the binder, creating a chemical and physical bond. The silica forms bonding bridges between abrasive grits, whereas the flux assures the adequate surface

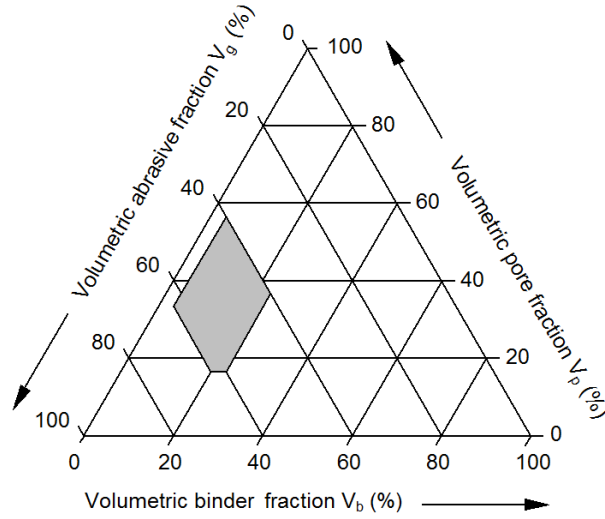


Figure 3.10: Trigonal graph of the composition of the grinding wheels; the range of industrial application is highlighted in grey [Klo09]

stress. Figure 3.11 shows the drop test specimens to verify the consistence and viscosity of the vitreous binder.

### Verification of properties

Grinding wheel manufacturers verify one by one all grinding wheels at 1,2 - 1,5 times the nominal speed. The centrifugal force causes overstress on the wheel, dilating and increasing the stiffness [Smi86]. The increase of stiffness affects on the contact length too. This effect is noticeable specially on resin bonded wheels and high speed grinding. In addition, the fragile structure of the wheel is very dependent of the strain rate and, thus, the resistance is increased, i.e. the grade. Equation 3.21 estimates the overstress at the bore diameter  $\sigma_{\max}$  due to the centrifugal force, where  $\rho$  is the mass density,  $\nu_s$  the Poisson's ratio,  $v_s$  the peripheral speed and  $d_o$ ,  $d_s$  are the bore and outer diameters of the wheel [Mal08].  $\sigma_{\max}$  is sensitive to  $v_s$  through a squared relationship.

$$\sigma_{\max} = \rho \left( \frac{3 + \nu_s}{4} \right) \left[ 1 + \left( \frac{1 - \nu_s}{3 + \nu_s} \right) \left( \frac{d_o}{d_s} \right)^2 \right] v_s^2 \quad (3.21)$$

Regarding the measurement of mechanical properties, the abrasive grits and the grinding wheel pass several verification tests. The ability to remove material lays on abrasive grits. The body of the wheel transmits the movement and holds the grits under grinding forces in a controlled way. In addition, the generated porosity absorbs the cutting fluid. These features have to be measurable and quantifiable with specific tests. The grinding wheel by itself, as a designed



Figure 3.11: Drop test of the binder: the viscosity is verified measuring the spread distance in a given slope (source: Unesa)

man-made and tailored material, receives the stiffness, porosity and cutting properties from the combination of abrasive and bonding material.

Hardness (Knoop and Vickers) and friability tests are the main tests made to the abrasive. Regarding wheel grade or hardness, before it was measured according to its definition, measuring the strength that the bond holds a grit. Decneut [DPS70] lists more than twenty techniques to evaluate the hardness, as the scratching test, conical drilling, bending test or sand blasting. Nowadays it is widely accepted the use of the wheel's Young's modulus  $E_s$  to determine wheel hardness [KF87]: if wheel grade is related to the resistance to pull-out a grit, then hardness and elasticity are also related to it. The elasticity of the wheel, beside the definition of the grade, plays a remarkable role in the contact analysis.

Usually, the elasticity is measured by tensile and compression tests. But the grinding wheel, as a fragile heterogeneous solid, shows an appreciable dependence on the strain rate. The resonance and ultrasound testing allow a non destructive, fast and accurate measurement of the Young's modulus. The resonance analysis measures the natural frequency of the wheel, whereas the ultrasounds measure the speed of a wave that passes through a solid. Although one is the inverse of the other, the resonance test is widely used in the grinding industry. The reason is that the ultrasound measurement requires a submerged environment [Smi86]. Equation 3.22 calculates the Young's modulus regarding the bi-nodal bending mode, where  $f$  is the measured natural frequency,  $d_s$  and  $d$  are the outer and inner diameters of the wheel,  $b_s$  is the width of the wheel,  $\nu_s$  is the Poisson's ratio and  $\rho$  is the density.  $\nu_s$  is also measured by means of

resonance test, regarding tensile and torsion vibration modes.  $\nu_s$  is constant for a given binder type in a wide range of grit size and structure number [DPS70].

$$E_s = 1,074 \frac{(1 - \nu_s^2) f^2 d_s^4 \rho}{(1 - (d/d_s)^2)^2 b_s^2} \cdot 10^{-9} \quad (3.22)$$

### 3.3.2 The DEM model of the Grinding wheel

Figure 3.12 presents the DEM model of the grinding wheel, the workpiece, and the boundary conditions applied on them. The DEs that form the wheel body are connected by a network of beams, which take the role of the bonding bridges.

Now the challenge is to build the morphological and mechanical structure of the grinding wheel from the characteristics of the wheel. The flow chart of Figure 3.13 explains the procedure to obtain the DEM model. The numerical sintering procedure described in Section 3.2.2 has been slightly modified to adapt the created model to the special features of the grinding wheel. Once created the model of the wheel body, the mechanical properties obtained by calibration are applied on the beams. Finally, the workpiece is added on the ensemble.

The characteristics of the grinding wheel are listed in Section 3.3.1. The main parameters to build the grinding wheel DEM model are the average grain diameter  $d_g$ , structure ( $V_g$ , abrasive volumetric fraction) and hardness grade (the correspondent Young's modulus,  $E_s$ ). The influence of the abrasive and bonding material types is internalised in the wheel grade, so they are not an input-data in the DEM model. In the same way, the bonding material and porosity fractions are not described explicitly by the DEM model as Li *et al.* do.

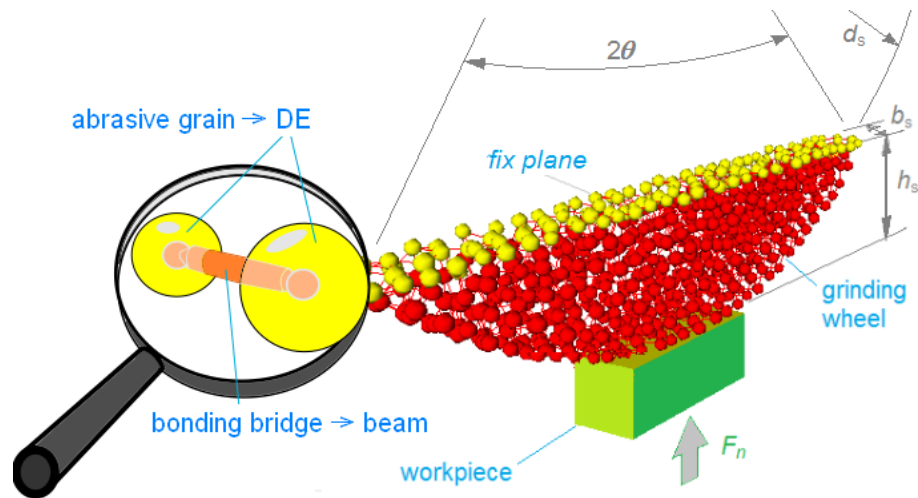


Figure 3.12: Grinding wheel and workpiece DEM model (out of scale)

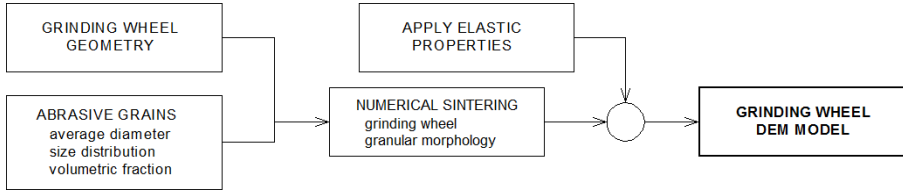


Figure 3.13: Flow chart of the construction of the grinding wheel DEM model

The reason is that the aim is not to model the morphology of the abrasive-binder-porosity combination, but the mechanical behaviour of the wheel material *per se*. In addition, the outer diameter of the wheel  $d_s$  is also needed to reproduce the wheel portion.

From them, the input-data for the numerical sintering to create the DEM model of the grinding wheel are the (a) dimensions of the model, the (b) average grain diameter, the (c) grain size distribution and, the (d) volumetric abrasive fraction.

The dimensions of the model depend on the geometry of the grinding wheel. In surface grinding, grinding wheels are cylindrical with  $d_s$  wheel diameter and  $b_s$  width. The computational cost in DEM has a squared relationship with the element number. In order to reduce the size of the model, just a thin slice of the wheel-workpiece set is modelled. The aspect ratio of a disk is high, due to the thin thickness. The effect arises from an incomplete RVE homogenisation, which is reflected on the calibration of beams with different model sizes. Therefore, the model losses the isotropy in the axial direction, which concerns the Poisson's ratio. However, the model is focused on the deformations and stresses on the contact area, that are unaffected. Thus, the thin thickness of the model is admissible.

In addition, contact stresses and deformations are concentrated in the contact area in the contact between non-conform surfaces [Her1882]. So the wheel model is also cut at the height  $h_s$ , without alter the contact conditions.  $h_s$  is defined by FEM, as the height above the stress is less than a hundredth of the maximum stress. Thus, the dimensions of the DEM model are defined by  $d_s$ ,  $b_s$  and  $h_s$  (Fig. 3.12).

In the DEM model, the average DE diameter  $d_{DE}$  matches the average grain diameter  $d_g$ . The sieve number  $M$ , indicated in the wheel designation, defines  $d_g$ . Thus, Equations 3.18 and 3.19 provide the reference DE diameter  $d_{DE}$  and DE size dispersion  $\mu_g$ . In addition, the DE density of the wheel model has to coincide with the volumetric fraction of abrasive  $V_g$ . The structure number indicates  $V_g$ , which is estimated with Equation 3.20.

On the other hand, bonding bridges bring stiffness to grinding wheels, whereas cylindrical beams provide stiffness to DEM model (Section 3.2.1). The mechanical properties of the beams are calibrated beforehand to match the mechanical properties of the grinding wheel (Young's modulus  $E_s$  and Poisson ratio  $\nu_s$ ). Section 3.2.1 describes the procedure to calibrate beam properties

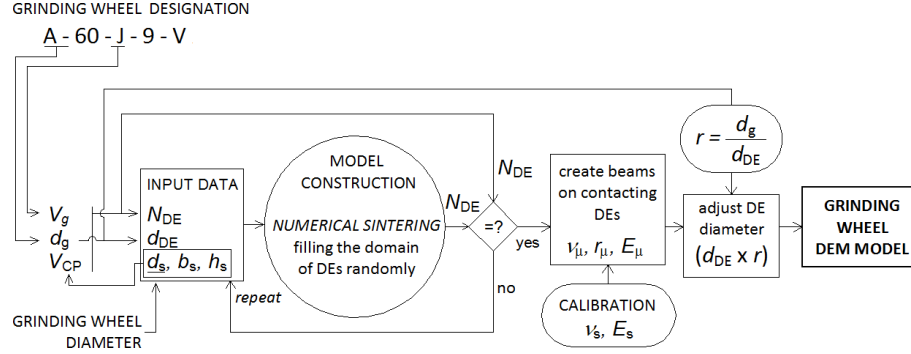


Figure 3.14: Flow chart of the construction of the grinding wheel DEM model

and Section 3.3.1 explains how the hardness grade letter of the wheel is related to the Young's modulus  $E_s$ .

The procedure to construct the DEM model is introduced by Figure 3.14. First, (1) the dimensions of the model ( $d_s$ ,  $b_s$ ,  $h_s$ ) are set (Fig. 3.12). With the model dimensions, the volume of the DEM domain  $V_{DE}$  is defined. Both, the average grain diameter  $d_g$  and the abrasive volumetric fraction  $V_g$  must be adjusted at the same time. (2) The number of DE required  $N_{DE}$  is determined by the volume of the DEM domain  $V_{DE}$ , the average grain diameter  $d_g$  and abrasive fraction  $V_g$ . Equation 3.23 estimates  $N_{DE}$ , where  $d_s$ ,  $b_s$  and  $h_s$  are the wheel diameter, width and height, and  $\theta$  is the semi-angle of the arc of the DEM model (Eq. 3.24). (3) The model is constructed with  $N_{DE}$  as reference value. Once the desired  $N_{DE}$  is achieved, (4) beams are created between the elements in contact. The mechanical properties of the beams are applied regarding calibration results. The numerical sintering procedure saturates with elements the DEM domain, so the average DE diameter  $d_{DE}$  is bigger than the desired  $d_g$ . (5) This is corrected reducing the diameter of all elements by the reduction factor  $r$  (Eq. 3.25). In this way, the DEM model fits both,  $d_g$  and  $V_g$ .

$$N_{DE} = V_{DE} V_g \frac{1}{v_g} = \left[ \frac{d_s^2 b_s}{4} \left( \frac{\pi \theta}{360} - \frac{\sin \theta}{2} \right) \right] V_g \left( \frac{6}{\pi d_g^3} \right) \quad (3.23)$$

$$\theta = 2 \arccos \left( 1 - \frac{h_s}{d_s} \right) \quad (3.24)$$

$$r = \frac{d_g}{d_{DE}} \quad (3.25)$$

Finally, the workpiece is added in the DEM model. As explained in Section 3.2.1, the workpiece is modelled by a single DE with a rectangular prism shape (Fig. 3.12). This special DE models a plain surface. The movement is restricted to a translation in the vertical direction ( $y$  axis), radially regarding the grinding wheel. The rotations are blocked too. As regular DE, the rectangular DE is

perfect rigid, but allows the interpenetration of elements. If an interpenetration is detected between a surface DE and the workpiece surface, a reaction force is applied according to the penetration on both elements, surface DE and workpiece DE (Section 3.5).

## 3.4 Numerical dressing

The surface topography is the direct outcome of dressing. This section applies the topography on the DEM model surface built in Section 3.3 taking into account the effect of the dressing conditions.

The review of the topography models (see Section 2.3.2) has identified the main parameters that define the topography. Among them, there are the grain size, the surface grain density, the peak height and the cutting edge sharpness. These characteristics are common for stationary and rotary dressers. However, as explained in Section 2.1.2, the use of single-point stationary dressers allows the analysis of the kinematic parameters and geometrical features regarding the obtained topography. Therefore, the scope of this thesis is limited to the modelling of the topography created by stationary single-point dressers. The extension of the application range of the model to the rotary and multi-point stationary dressers is one of the pending tasks of this thesis. It can be done defining the previously cited characteristics (surface grain density, peak height and cutting edge sharpness) in a random way on surface grits.

Dressing conditions modify the aggressiveness of dressing mechanisms, creating an opener softer wheel or a closer harder one. Only the most prominent grains make contact with the workpiece, ploughing and removing material. The single-point dresser also leaves a helix trace on the wheel surface. Assuming that a surface grit has a single cutting edge and that the cutting edge is aligned with the DE centre, the topography is implemented in four steps on the DEM model: (1) cutting out the surface, (2) remove the fraction of grains pull-out due to bond fracture, (3) peak depth distribution and (4) average cutting edge radius. Figure 3.15 shows the flow chart of the implementation of topography on the DEM model surface.

The first step prepares the model surface before applying the topography. The assumption that grits are randomly distributed within the wheel implies that grit concentration will rise linearly with depth into the wheel [McA64]. During the creation of the model, even though the numerical sintering assures a homogeneous and random distribution of DEs, they pile up on the elastic walls of the domain. Figure 3.16 shows this effect: the cumulative grain density is not uniform on the surface layer of the freshly created DEM model. This effect is corrected removing the elements of the surface layer about three times the average grain diameter  $d_g$  inward. Hence, the DEM model is constructed with a slightly larger diameter ( $d_s + 3d_g$ ). Thus, the surface is cut to solve the pile-up of grits due to the numerical sintering, and the model surface is left at the wheel diameter  $d_s$ , ready for the application of the topography.

The second step removes the surface elements pulled out due to bond break-

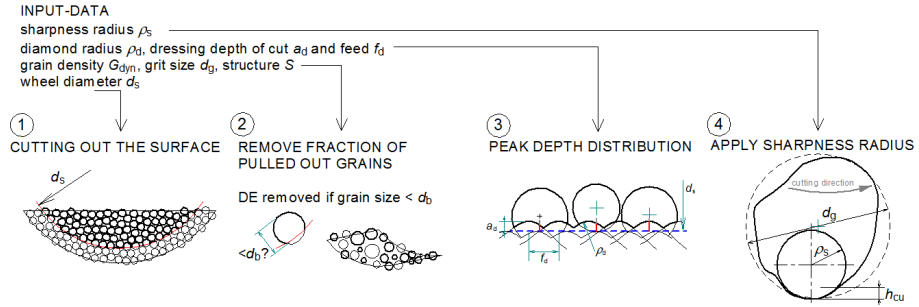


Figure 3.15: Flow chart of the implementation of surface topography on the model surface

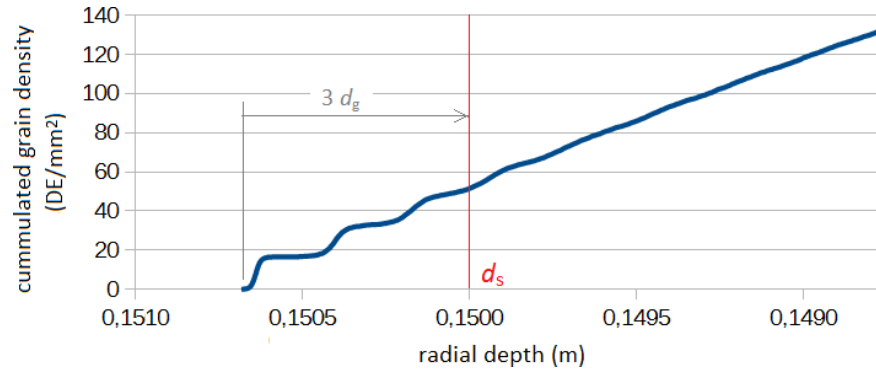


Figure 3.16: Cumulative grain density through radial depth: pile up discrete elements on DEM model boundary and definition of the reference wheel diameter  $d_s$



age, taking into account that the most protuberant grains are removed first. Malkin's model is applied to define the fraction of grains removed by bond fracture [MC71, Mal08]. With the surface grain density measurements  $G_{\text{dyn}}$  and the theoretical grain density  $G_o$  (Eq. 2.12), Equation 2.13 estimates the average dressing particle dimension removed by bond fracture  $d_b$ . In this way, the intercepted elements by the cylindrical surface  $d_s$ , which diameter is smaller than  $d_b$ , are removed from the model surface. These elements are, obviously, the most prominent on the surface.

Now the DEM model surface is ready to be treated. First, the peak position in radial depth  $y_p$  is defined for each surface DE and, then, the cutting edge sharpness is applied on the peak. Regarding Verkerk [Ver77], a surface grit has a single useful cutting edge or peak. In the DEM model, Chen's sinusoidal function [CR96] is applied as a discrete function on surface DE centres to define the peak radial depth  $y_p$  (Fig. 3.17(a)). Chen's assumptions together with Malkin's model, cover the basic dressing mechanisms (bond and grit fracture). Chen's model is the most complete model for single-point dressers so far.

Equation 3.26 reminds Equation 2.27, combining the effect of helix ductile cut  $y_d$  and abrasive fracture  $y_f$ . The circumference function with a moving centre in discrete steps of dressing feed  $f_d$  defines the term  $y_d$  (Eq. 3.27). In Equation 3.27,  $\rho_d$  is the radius of the single-point dresser,  $x_c$  and  $y_c$  are the instantaneous dresser position,  $x$  is the axial position of the peak (or DE centre) and  $a_d$  is the dressing depth of cut.  $x_c$  is a discrete number proportional to the feed  $f_d$  and its value is defined rounding to the nearest value regarding the DE centre axial position  $x$ . And Equation 3.28 defines  $y_f$  as a random fraction  $s_n$  of the fracture term amplitude  $h_f$ , which is defined with Equation 2.28. In this way, the grain density arises according to radial depth  $y_p$ . The mass of the cut DE is reduced according to the new peak position.

$$y_p(x) = y_d(x) + y_f \quad (3.26)$$

$$y_d(x) = \sqrt{\rho_d^2 - (x - x_c)^2} - y_c \text{ where } x_c = \mathbb{Z} \left( \frac{x}{f_d} \right) \text{ and } y_c = a_d - \rho_d \quad (3.27)$$

$$y_f = s_n h_f \quad (3.28)$$

Regarding peak geometry, it reflects the sharpness of the cutting edge. As others authors did, the sphere (cutting edge radius  $\rho_s$  [Mas52, Klo09]) and the cone or pyramid (peak slope or rake angle [TB00]) have been considered (Section 2.3.2). Both are measured in a similar way and both can be implemented on the DEM model. Due to the increment of the slope through the depth [TB00] and very negative rake angles of abrasive grits [Sha96], the sphere geometry with a sharpness radius of  $\rho_s$  has been chosen (Fig. 3.17(b)). In addition,  $\rho_s$  can be adapted to reflect the wear condition. The modelling of the wheel wear, analysing the cutting edge sharpness together with the grit pull-out, is one of the proposed future research lines that opens this thesis. The procedure set

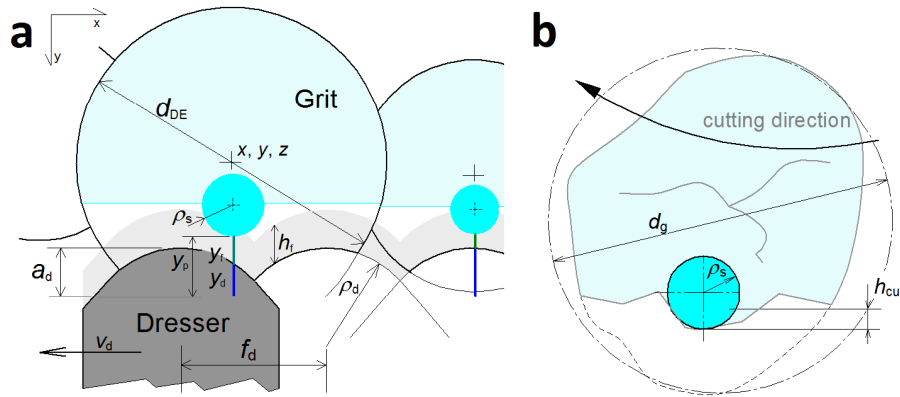


Figure 3.17: (a) The peak of a grit is located at the radial depth  $y_p$  from the outer grinding wheel diameter  $d_s$  and aligned with the DE centre;  $y_p$  is the sum of the ductile cutting of the dresser  $y_d$  and abrasive fracture term  $y_f$  (after [CR96]); (b) grit fractured during dressing and definition of the cutting edge with the sharpness radius (after [Mas52, Klo09])

by König and Lortz [KL75] to measure  $\rho_s$  has been updated to take advantage of 3D topography measurement technologies. The most prominent peaks are identified in the 3D measurements. 2D profiles caught in the cutting direction on those peaks. As grit-workpiece contact is superficial,  $\rho_s$  is defined as the circumference circumscribed within a few microns depth from the peak ( $5 \mu\text{m}$ ). Experimental measurements show that  $\rho_s$  follows a normal distribution with a  $\sigma_\rho$  standard deviation [HK03].

In brief, following the steps described in this Section, the surface of the DEM model is able to reproduce the effects of dressing, keeping the randomness of grinding. In this way, the bearing ratio increases and the grain density arises according to the depth, as it happens in actual wheels.

### 3.5 Boundary conditions

Once the grinding wheel DEM model and the surface topography are created, it is the moment to apply the boundary conditions and run the simulation. In addition to the conventional boundary conditions (movement restrictions and external forces), the DEM model includes a contact detection algorithm between surface elements and the workpiece DE, and applies a reaction force on both if a contact is identified. Figure 3.12 shows the movement restrictions and external forces: the experimentally measured normal force is applied on the workpiece, whereas the cutting plane of the grinding wheel is fixed. In addition, the DEs of one side of the wheel have the axial displacement restricted to avoid buckling. The positions of surface elements and the workpiece are compared on each iteration, applying a reaction force if an interpenetration is detected.

During simulation, the actual maximum penetration, resultant contact force and contact length are monitored. As a damped dynamic system, the result oscillates around the reference force, converging as iterations (or time) advance. The simulation gets balance as the sum of individual grit reactions equals the applied force on the workpiece. To assure dynamic stability, the normal force is applied first as a ramp, becoming constant once the nominal normal force is reached.

As commented in the introduction, two boundary condition sets are applied to the same wheel DEM model: the static contact (SC) and uncut chip thickness (UCT) boundary conditions. Each set has its own contact detection algorithm and contact law. In both, the simulation starts from a non-contact almost-touching position ( $< 10^{-16}$  m) to optimise the computation time. The wheel model does not rotate. However, the centrifugal force that acts on each abrasive grit regarding the distance to the rotation axle, straightens the wheel and induces tensile stresses modifying the behaviour of the wheel. Therefore, the centrifugal force is applied on SC and UCT models (Section 3.5.3).

The SC model simulates the static contact between the grinding wheel. In this way, the rough contact length  $l_f$  term of the RQ model is obtained (Section 2.2.1) and the roughness parameter  $R_r$  is evaluated. On the other hand, the UCT model applies the cutting force exerted by a single-grit when a contact is identified, taking into account the depth of cut  $a_e$  in the contact detection algorithm. Shaw's hardness model of chip formation (Section 2.4.2) is chosen to estimate the cutting force at each cutting edge in contact.

Both models consider the grit-workpiece contact as plastic-perfect rigid and viscoplastic behaviour of the workpiece material. Contacts are limited to grain-workpiece interactions. The contact pressure surpasses largely the elastic regime and, thus, the contact can be considered rigid-perfectly-plastic [Joh87] as seen in Section 2.4.1. Inferred from the maximum shear stress slip lines, contact pressure in plastic regime is about three times the yield stress of the material  $\sigma_y$ .

The plastic strain and the high strain-rate increase material resistance, while high temperatures weaken the material. The viscoplasticity describes the inelastic behaviour of materials regarding the strain-rate and temperature. The Johnson-Cook (JC) model is applied to estimate the effective yield stress  $\sigma_y$  [JC83] on the rigid-perfectly-plastic contact expression. Equation 2.35 estimates  $\sigma_y$  according to the plastic strain  $\varepsilon_p$ , the strain rate  $\dot{\varepsilon}_p$  and temperature  $T$ . These three parameters vary along the trajectory of a grit in contact with the workpiece (rubbing, ploughing and cutting mechanisms). As discussed in Section 2.4.3, the shear plane angle  $\phi$ ,  $\varepsilon_p$ ,  $\dot{\varepsilon}_p$  and  $T$  can be estimated by numerical models. The experimental determination of them regarding the cutting edge penetration would require an extensive single-grit cutting research with a doubtful feasibility. In turn, the analytical approach offers a rough estimation of  $\varepsilon_p$  and  $\dot{\varepsilon}_p$  regarding the depth, but  $T$  regarding the contact position.

As the SC approach models the static contact between the wheel and a plane, the estimated contact area is not related to the actual grinding contact area, so the estimation of  $\varepsilon_p$ ,  $\dot{\varepsilon}_p$  and  $T$  has not sense. And the application on the UCT

model would increase the computation requirements, due to the necessity to define a tailored  $\sigma_y$  for each grit in contact at every iteration. In this thesis,  $\varepsilon_p$ ,  $\dot{\varepsilon}_p$  and  $T$  are considered constant along the contact arc on both approaches, SC and UCT. Anyhow, the error that supposes this rough simplification is smaller than neglecting the viscoplastic behaviour of the workpiece material.

The following Sections describe the boundary and contact conditions of both, SC and UCT models. The contact detection algorithm and contact-law (definition of the reaction force if a contact is detected) are the main differences on both approaches.

### 3.5.1 Static contact (SC) model

As seen in Section 2.2.1, Rowe and Qi [RMQZ93] define the contact length  $l_c$  as the quadratic sum of the geometrical contact length  $l_g$  and rough contact length  $l_f$  (Eq. 2.25). The second term  $l_f$  describes the rough elastic contact between the grinding wheel and a flat plane. The SC model simulates the static contact between the grinding wheel and the workpiece to obtain the  $l_f$  term and the roughness parameter  $R_r$ . The model is upgraded, from elastic contact to a rigid-perfectly-plastic contact and, in addition, it assumes a viscoplastic behaviour on the workpiece material. By itself, the SC model is an innovative and versatile approach for the modelling of the contact between rough surfaces assuming elastic, elastoplastic or rigid-perfectly-plastic contact.

#### Contact detection

At each iteration, the position of each surface DE is compared with the workpiece surface position. Figure 3.18 shows the interpenetration between a surface DE and the workpiece. Equation 3.29 shows the contact detection expression. The penetration  $\delta_i$  is the difference between the grit peak ( $y_i + r_p^i$ ) and workpiece surface ( $y_w - h_w/2$ ) positions.  $r_p^i$  is the distance between peak and DE centre,  $(x_i, y_i, z_i)$  and  $(x_w, y_w, z_w)$  are DE centre and workpiece position coordinates respectively and  $h_w$  is the workpiece height. A positive penetration ( $\delta_i > 0$ ) confirms the contact. The distance  $r_p^i$  is defined in the pre-processing, when applying the topography.  $r_p^i$  is set regarding the DE initial position,  $d_s$  the wheel diameter and  $y_p^i$  the peak radial depth.

$$\delta_i = (y_i + r_p^i) - \left( y_w - \frac{h_w}{2} \right) \quad (3.29)$$

#### Contact law

Once detected the contact, the same and opposite contact normal force  $\mathbf{f}_n^i$  is applied on the DE and the workpiece. The contact force is used along with neighbour DE interactions to calculate the speed and position of the discrete elements in the following iteration.  $\mathbf{f}_n^i$  depends on the penetration  $\delta_i$  and the

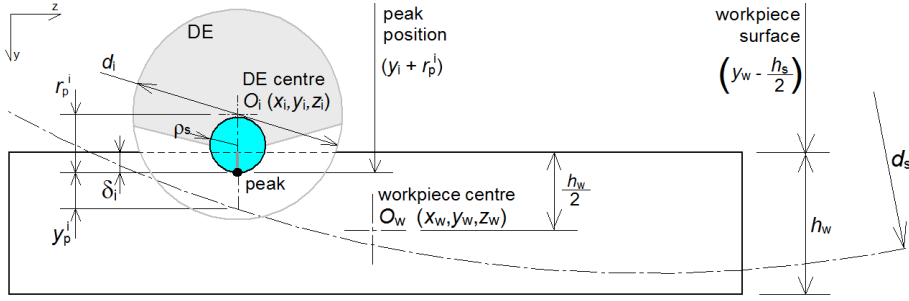


Figure 3.18: Contact detection: interpenetration between a surface DE and the workpiece surface

relative speed of the DE in contact. Its direction coincides with the vertical axis  $y$ . Equation 3.30 defines  $\mathbf{f}_n^i$  for an individual grit-workpiece contact:

$$\mathbf{f}_n^i = \mathbf{f}_c + \mathbf{f}_d \quad (3.30)$$

The first term of Equation 3.30,  $\mathbf{f}_c$ , corresponds to the contact reaction force (Eq. 3.31), which is aligned with  $\mathbf{n}$ , the vertical unit vector, and  $\delta_i$  is the penetration of the DE on the workpiece (Eq. 3.29). Even though DE are spheres of  $d_g$  average grain diameter, a sphere of  $\rho_s$  radius represents grit cutting edge sharpness. The grinding wheel is supposed still, so the trace of the grits get the inverse sphere shape on the workpiece surface.

$$\mathbf{f}_c = 3\sigma_y\pi \left[ \rho_s \sin \left( \arccos \left( \frac{\rho_s - \delta_i}{\rho_s} \right) \right) \right]^2 \cdot \mathbf{n} \quad (3.31)$$

Equation 3.31 considers a rigid-perfectly-plastic contact, since the contact pressure is three times the yield stress  $\sigma_y$  (Section 2.4.1). Even though the model simulates the static contact,  $\sigma_y$  is estimated with the JC model (Section 2.4.3, Eq. 2.35), assuming viscoplastic material behaviour.

The second term of Equation 3.30,  $\mathbf{f}_d$ , reflects the damping in the contact and it is related to the penetration speed. The damping force is calculated by Equation 3.2 (Section 3.2.1), where the equivalent mass  $M_{iw}$  is set with the surface DE mass  $m_i$  and the workpiece DE mass  $m_w$  (Eq. 3.5).

### 3.5.2 Uncut chip thickness (UCT) contact model

The UCT model simulates an instant of the of the grinding wheel when it is removing material from the workpiece. The SC and UCT models share the dressed grinding wheel DEM model and the initial boundary conditions shown in Figure 3.12. But in the UCT model, both, normal and tangential forces are applied on the workpiece. The wheel model does not rotate: the sum of the reactions of the grits in contact equals the forces applied on the workpiece. The contact reaction force at grit level is set regarding the uncut chip thickness  $h_{cu}$  or

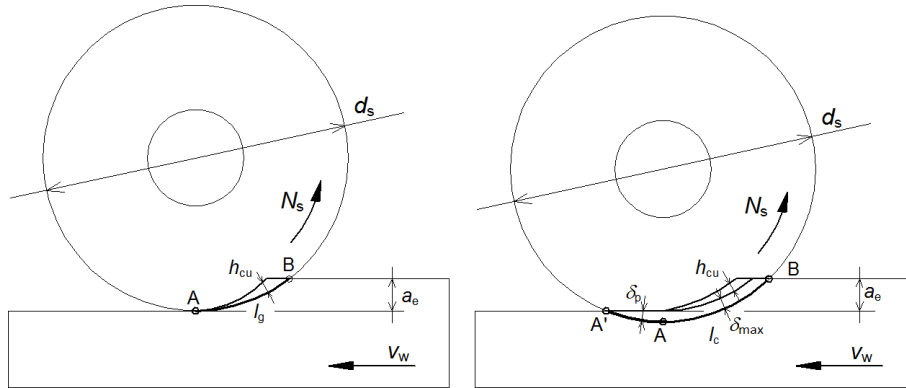


Figure 3.19: Foundations of the UTC model: (a) geometrical contact length  $l_g$  and uncut chip thickness  $h_{cu}$ ; (b) combination of maximum penetration of surface grits and uncut chip thickness ( $\delta_{max} + h_{cu}^i$ )

the penetration in the workpiece. The hardness test analogy of chip formation [Sha71, Sha96] is used to estimate the contact force, assuming a viscoplastic behaviour of the workpiece material. The UCT model takes into account the depth of cut  $a_e$  in the contact detection, so the result is directly the grinding contact length  $l_c$ .

The geometric contact length  $l_g$  considers rigid both, wheel and workpiece. Studying Figure 3.19(a),  $l_g$  is defined by the arc  $A-B$ ; the starting point  $A$  coincides with the bottom quadrant of the wheel. In turn, due to wheel deformation, the contact area enlarges to  $A'$  point in Figure 3.19(b). This model supposes that, on the  $A'-A$  quadrant, the grains ploughs plastically the workpiece surface due to the elastic deformation of the wheel and, on the  $A-B$  quadrant, the grit maintains previous penetration reached in  $A'-A$ , increased by the uncut chip thickness  $h_{cu}$ . On both sides, the penetration of the grits coincides with the actual  $h_{cu}$ , which is used to estimate the force on each grit. The effect of the tracks left by previous grits is disregarded, as well as the material pile up. The implementation of these tracks in the model is one of the pending tasks in the further development of the model.

### Contact detection

The contact detection differs on both sides. Regarding Figure 3.20, on the  $A'-A$  segment  $\delta_i$  is defined and detected as an indentation (Fig. 3.18, Eq. 3.29). On the  $A-B$  segment (Fig. 3.20), the elements that take part are the ones placed on the vertical range of the depth of cut  $a_e$ . Due to the real curvature of the contact zone is unknown, the indentation on the  $A-B$  segment takes the peak depth  $y_p$  as a floating reference. Thus, Equation 3.32 shows that  $\delta_k$  is the sum of the maximum indentation on the  $A'-A$  side  $\delta_{max}$  plus the correspondent uncut chip thickness  $h_{cu}^k$ , taking into account peak height differences  $y_p$ . A positive  $\delta_k$

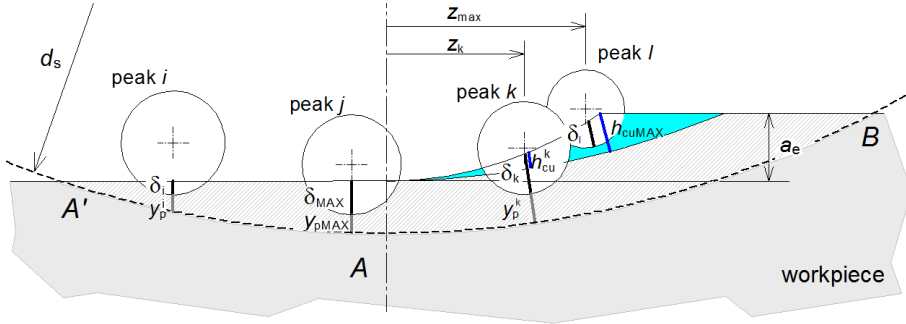


Figure 3.20: Contact detection on  $A'-A$  and  $A-B$  segments, the circles represent grain peaks

confirms the contact. Assuming that  $h_{cu}$  varies linearly through the contact arc, Equation 3.33 estimates its value regarding the maximum uncut chip thickness  $h_{cu,max}$  (Eq. 2.5), the element relative position  $z_k$  and the maximum horizontal contact detected  $z_{max}$ . In this way, taking into account the whole arc  $A'-A-B$ , the contact force takes an irregular triangular profile.

$$\delta_k = \left( \left( \delta_{max}^{A'-A} + y_p^p \right) - y_p^k \right) + h_{cu}^k \quad (3.32)$$

$$h_{cu}^k = h_{cu,max} \cdot \frac{z_k}{z_{max}} \quad (3.33)$$

### Contact law

The normal contact force ( $\mathbf{f}_n^i$ ) in both models, SC and UCT, is the sum of the reaction and dumping forces (Eq. 3.30), sharing the dumping  $\mathbf{f}_d$  term (Eq. 3.2). The analogy of the hardness model of chip formation in grinding proposed by Shaw [Sha71, Sha96] is used to determine the contact force  $\mathbf{f}_c$  (Fig. 2.29). As hardness model, Shaw deduces the reaction as rigid-perfectly-plastic contact. However, plastic strain and the high strain-rate increase material resistance, while high temperatures weaken the material. Then, Equation 2.31 has been adapted to estimate the normal contact force term  $\mathbf{f}_c$ , regarding the force ratio  $\mu$ , the penetration of the grain ( $\delta_i, \delta_k$ ) (Eq. 3.29, Eq. 3.32) and the effective yield stress  $\sigma_y$  obtained with the JC model. In this way, Equation 3.34 defines the normal force  $\mathbf{f}_n^i$  and Equation 3.35 the tangential force  $\mathbf{f}_t^i$ , both regarding the measured force ratio  $\mu$ .

$$\mathbf{f}_c = 3\sigma_y\pi \left[ \rho_s \sin \left( \arccos \left( \frac{\rho_s - \delta_i}{\rho_s} \right) \right) \right]^2 \cdot \sin(\arctan \mu) \cdot \mathbf{n} \quad (3.34)$$

$$\mathbf{f}_t^i = \mu \cdot \mathbf{f}_n^i \quad (3.35)$$

### 3.5.3 Centrifugal force

The centrifugal force is the apparent force that appears on rotating bodies. The centrifugal force expands the wheel radially, increasing the stiffness. In addition, the vitreous binder, as fragile strain rate dependent material, improves its resistance. Newton defined the centrifugal force that acts on a particle  $\mathbf{f}_z^i$  with Equation 3.36, where  $m_i$  is the mass of the particle,  $\omega$  is the angular speed and  $\mathbf{r}$  is the distance to the centre of rotation.  $\omega$  and  $\mathbf{r}$  are vectors. Equation 3.37 shows the module of the centrifugal force  $f_z^i$ . The expression remarks the importance of the rotation speed, because it is squared. The centrifugal force expands the wheel radially, increasing the stiffness.

$$\mathbf{f}_z^i = m_i \omega \times (\omega \times \mathbf{r}) \quad (3.36)$$

$$f_z^i = m_i \omega^2 r \quad (3.37)$$

The centrifugal force  $\mathbf{f}_z$  is applied on each DE of the model, regarding its mass  $m_i$ , rotation speed of the wheel  $N_s$  and distance from wheel centre, which is calculated with  $y_i$  and  $z_i$  position coordinates.  $\mathbf{f}_z$  is applied on the radial direction and Equation 3.38 defines the value of the module  $f_z^i$ . The centrifugal force is applied on both models, SC and UCT.

$$f_z^i = m_i \cdot \left( \frac{\pi N_s}{30} \right)^2 \sqrt{y_i^2 + z_i^2} \quad (3.38)$$

## 3.6 Summary

The discrete element method (DEM) has been chosen to model the grinding wheel morphology and structure. DEM offers a large flexibility to adapt the formulation to the target physical phenomena. The objective is to create the numerical model of the wheel which includes the surface topography. The model aims to simulate the contact in surface grinding.

First, the tribological system of the grinding contact is defined regarding the factors and variables identified in the literature review. Grinding wheel morphology and stiffness, surface topography and grit-workpiece contact conditions are the main aspects that the contact model has to face.

Then, the adaptation of DEM formulation to the physical phenomena is described. The heterogeneous body of the wheel is modelled considering a DE as an abrasive grit, and DEs are bonded by elastic beams. In this way, the wheel is modelled as heterogeneous, granular and porous material. On the other hand, a single rectangular DE represents the workpiece. The formulation of DEM lays on three foundations: the contact detection algorithm, the interactions between DEs and the numerical integration. The use of beams makes unnecessary a contact algorithm between the wheel DEs. The contact detection between surface elements and the workpiece is simplified due to the use of a single DE for the workpiece. The contact algorithm is adapted to the wheel topography features



and the depth of cut. The interaction between wheel elements is governed by beams properties and their relative position. Beam properties are calibrated regarding the wheel material properties. In turn, the interaction between a surface DE and the workpiece is tailored to the physical phenomena according to the approach. The first one models the static contact term of the RQ model. But instead elastic, the contact is considered rigid-perfectly-plastic and the material behaviour is assumed viscoplastic. The second approach simulates directly the grinding contact, taking into account the depth of cut and estimating the reaction force regarding the hardness test analogy of chip formation. The Tchamwa-Wielgoz integration schema is chosen, which provides some numerical dumping to the Verlet schema. And the model is constructed using the numerical sintering technique, which assures the randomness, homogeneity and isotropy of the model.

The methodology of the construction of the contact model has three main parts: the grinding wheel body model, the surface topography and the boundary conditions.

The body of the grinding wheel is created adjusting the DE size and volumetric fraction. To reduce the size of the model, just a thin slice of the wheel is modelled. In order to reduce further this disc is also cut, because the stresses are concentrated in the contact area. Beams bring the stiffness to the body. The mechanical properties of the beams are calibrated to match the properties of the wheel.

Dressing generates the surface, cutting and pulling out surface grains in a brittle way. The freshly created topography depends closely on dressing conditions. The numerical surface is generated in four steps. First, surface DEs are deleted regarding the wheel diameter. Then, the grits released from the surface according to the dressing intensity applying Malkin's model. Regarding Verkerk, a surface grit has a single effective cutting edge. In this way, Chen's topography model is applied in a discrete manner regarding a ductile cutting of the dresser and a random component of the fragile fracture. Cutting edges are characterised as spheres with the sharpness radius.

The boundary conditions are applied in the pre-processing. The model is loaded, the topography applied, the cut of the wheel is fixed and the grinding force is applied on the workpiece. During the simulation the contact detection and reaction force algorithms compare surface DE-workpiece relative position and apply accordingly the reaction force at each iteration. The simulation ends when the sum of reaction forces and applied grinding forces get balanced. Two approaches are proposed. The first one models the static contact term between wheel and workpiece regarding the RQ semi-analytical contact model. The contact is assumed to behave as rigid-perfectly-plastic instead of elastic, as well as viscoplastic due to the severe contact conditions in grinding. The second one models directly the grinding conditions (normal and tangential forces, depth of cut) applying the hardness test analogy of chip formation.



## Chapter 4

# Simulation of the contact in grinding

*This Chapter applies the methodology described in Chapter 3 to analyse the behaviour of the grinding wheel in the contact. Several experimental tests have been made together with the correspondent numerical simulations. The experimental tests provide the input data for the simulations, as well as the aimed contact length. The contact length measurements are compared with the numerical results.*

*Before run the simulations, the mechanical properties of the beams are calibrated according to the grinding wheel grade. To overcome the randomness of DEM models, four models are created with the same parameters for each grinding wheel type. After modifying their surface to reproduce the topography obtained by dressing, the models are ready for the simulations. In the pre-processing, the experimental forces and constraints are applied on the model. In the processing, the new position of each DE is calculated on each iteration regarding contact conditions and surrounding DEs. Finally, the monitored data are analysed. Both models estimate the contact length; the SC gets the roughness factor  $R_r$ ; and, the UCT obtains also the grain density, surface grit displacements and the force per grain. The experimental measurements fit better the UCT results than the ones obtained with the SC model.*

*The Chapter includes a factorial design to analyse the significance of the input parameters on variation of the model output regarding the input parameters. A orthogonal array of two-levels has been used in the design of experiments (DOE). These experiments allow the study of the effect of each factor on the response variable, as well as the effects of interactions between factors on the system response. The variation of stiffness and topography parameters have shown significant influence on the behaviour of the wheel when grinding.*



## 4.1 Experimental work

This Section defines the test conditions and introduces the experimental results. Two vitrified alumina grinding wheels (A46I8V and A60K7V) are tested under three dressing conditions (fine (F), medium (M) and coarse (C)) and two set depths of cut  $a$  (10  $\mu\text{m}$  and 20  $\mu\text{m}$ ). All measurements have been performed without coolant with the surface grinder GER SC 60/40.

Three types of experimental test are used to determine the input data of the model: (a) force measurements (forces and depth of cut), (b) topography measurements (sharpness radius) and (c) APS measurements (grain density and average distance between grits). The APS also measures the contact lengths, which are compared with numerical results. Table 4.1 compiles the testing conditions and measuring instruments.

Table 4.1: Testing equipment and conditions

<b>Grinding Wheels</b>			
Dimensions 300x76,2x30 mm	A46I8V	A60K7V	
Abrasive volume fraction, $V_g$	48 %	50 %	
Average grain diameter, $d_g$ (mm)	0,330	0,253	
Young's modulus, $E_s$ (GPa)	32	43	
Poisson ratio, $\nu_s$	0,2	0,2	
Density ( $\text{kg}/\text{m}^3$ )	1854	2096	
<b>Dressing conditions</b>			
Coarse (C)	$a_d = 20 \mu\text{m}$	$f_d = 0,8 \text{ mm/rev}$	$u_d = 1$
Medium (M)	$a_d = 20 \mu\text{m}$	$f_d = 0,4 \text{ mm/rev}$	$u_d = 2$
Fine (F)	$a_d = 20 \mu\text{m}$	$f_d = 0,2 \text{ mm/rev}$	$u_d = 4$
Needle type single-point dresser		$b_d = 0,8 \text{ mm}$	$\rho_d = 4,06 \text{ mm}$
<b>(a) Force measurements</b>			
Dynamometer plate: Kistler 9257B		500 Hz, force range $\pm 1000 \text{ N}$	
Four runs per test condition	$a = 10 - 20 \mu\text{m}$	$v_s = 24 \text{ m/s}$	$v_w = 0,5 \text{ m/s}$
<b>(b) Topography</b>			
	Confocal microscope	Leica DCM3D	
<b>(c) Grain density, distance between grits and contact length</b>			
Applied power source (APS)	Oscilloscope Lecroy waveRunner 104Xi		
12 runs per test condition	Mica sheet thickness 40 $\mu\text{m}$		
Depth of cut, $a = 5 - 10 - 20 \mu\text{m}$			

The surface grinding machine works at constant rotation speed ( $N_s = 1500$  rpm). Grinding and dressing movements are controlled by numerical control (CNC). A hydraulic drive moves the table. In all tests, the cutting and work speeds ( $v_s$ ,  $v_w$ ) are constants, 24 m/s and 0,5 m/s respectively.

Two conventional vitrified alumina grinding wheels are tested: A46I8V and A60K7V. The first one is made of white alumina, while the second one is of pink alumina. This couple of wheels allows to study the influence of the grain size and the wheel hardness. The workpiece is a rectangular block of 34CrMo4 tool steel tempered and quenched, of 55 HRc hardness.

Regarding dressing, both grinding wheels have been dressed with three dressing conditions. Following the recommendations of the dresser manufacturer (*Winter*), the dressing conditions are set varying the dressing feed  $f_d$  and maintaining constant the dressing depth of cut ( $a_d = 20 \mu\text{m}$ ). The chosen dresser is a

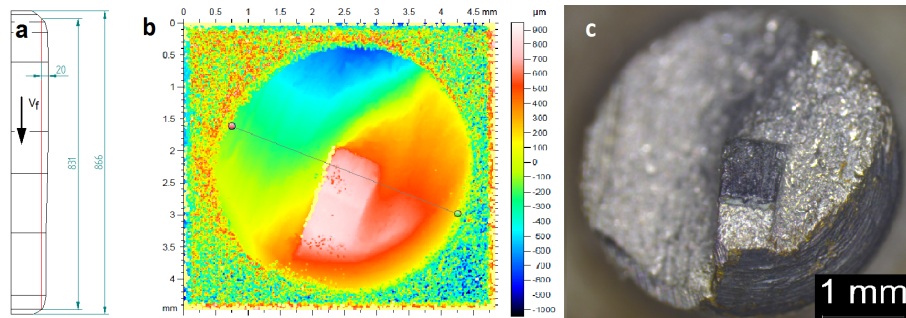


Figure 4.1: (a) Measurement of the dresser profile; (b) confocal microscope contour measurement and (c) top-view photography of the dresser

single-point dresser with a needle like synthetic polycrystalline diamond (PCD). It has the advantage that, once reached a stationary state, maintains the profile through the working life. In turn, it has almost a flat profile. The profile of the dresser has been measured four times during the experiments. The average radius of the dresser tip is  $\rho_d = 4,06$  mm, two orders bigger than  $a_d$ . The average width of the dresser is  $b_d = 0,83$  mm. Figure 4.1(a) shows the profile measurement of the diamond, obtained from 3D topography measurements with the confocal microscope (Fig. 4.1(b)), and, Figure 4.1(c) is a top-view picture of the dresser.

Regarding grinding forces, the experimental grinding forces are directly introduced in the model as input. During simulation, the force is the reference value, as the resultant of individual grit reaction forces converges to the applied force on the workpiece. The effective depth of cut  $a_e$  has been estimated regarding the machine-tool-workpiece system stiffness  $k_m$  (Eq. 2.1) [MRDI04]. The combined stiffness of  $k_m = 0,24$  N/ $\mu\text{m}$  has been defined beforehand as the average of several measurements within the range of cutting widths tested. Even though the DEM models have different widths,  $a_e$  is set according to the value correspondent to the force test conditions ( $b_w = 10$  mm).  $a_e$  is introduced directly in the UCT model, whereas the SC model does not use it; the combination of SC-RQ models takes into account  $a_e$  indirectly with the geometrical contact length  $l_g$ . Forces are measured under two set depths of cut  $a = 10 - 20$   $\mu\text{m}$ . After dressing at the given conditions, four grinding passes are given to avoid the initial transitory condition of the wheel surface. Once the force measurement is done, six spark-out passes assure that the next measurement starts from a known position. Four measurements are made per wheel and dressing condition. Figure 4.2 shows the set-up of the grinding force measurements.

Wheel surface topography is measured with a confocal optical microscope. To prepare easy-to-handle dressed wheel specimens, the dummy wheel technique is chosen (Section 2.1.3, Fig. 2.11). Specimens of 20x20x30 mm dimensions are dressed with the dummy wheel at the given dressing conditions (Fig. 4.3). The direct measurement of surface topography without replica techniques ensures the

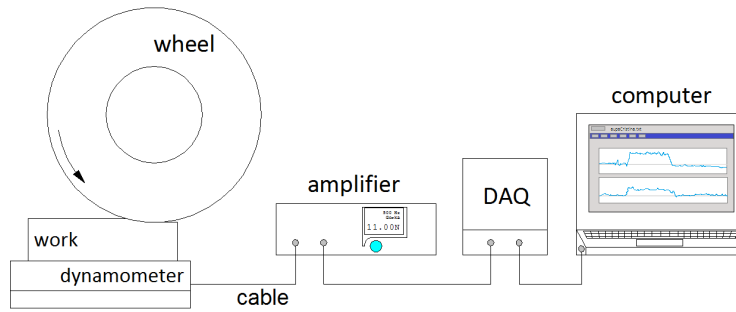


Figure 4.2: Set-up of the grinding forces measurement

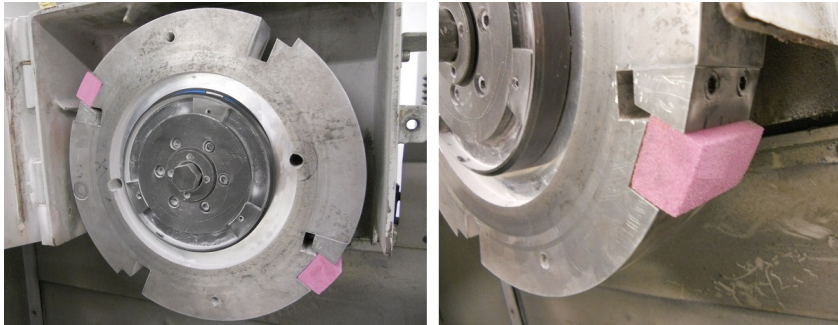


Figure 4.3: The tailored dummy wheel and detail of the specimen fastening

reliability of the data. However, the transitory state of the surface in the first passes of grinding is not taken into account. This transitory stage is produced by the cracks induced by dressing in abrasive grits and bonding bridges. This effect can not be avoided with the dummy wheel due to safety issues.

Figure 4.4(a) shows a 3D topography measurement. The heights are within  $30\ \mu\text{m}$ , which proves the cutting action of dressing. The diamond cuts off the abrasive material that intercepts, and grits break up in a brittle manner, even releasing of the bonding material. The deep gaps correspond to grains removed by bond fracture. The grain density arise from the combination of cutting and release of abrasive grits. These effects are modelled by numerical dressing, cutting off the DEM model surface in a given outer diameter  $d_s$  and deleting surface DEs regarding the average dressing particle size removed by bond fracture  $d_b$  (Section 3.4). In turn, the helix left by the dresser usually is not noticeable in 3D topography measurements on the wheel surface. Figure 4.4(b) shows a SEM stunning image of the transverse cut of the wheel surface taken to a specimen prepared with the dummy wheel. Grits seem clearly cut by the dresser.

The sharpness has been characterised by the sharpness radius  $\rho_s$ .  $\rho_s$  is determined modifying the measuring procedure set by Brenner and Torrance [BT93]. 3D topography measurements (Fig. 4.4(a)) allow to identify the most

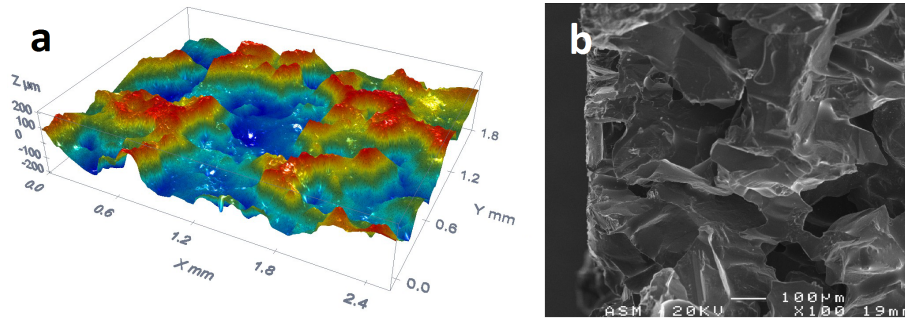


Figure 4.4: (a) Example of a 3D topography measurement with a confocal microscope; (b) a transverse view of the wheel surface taken by SEM (both A60K8V, coarse dressing (C))

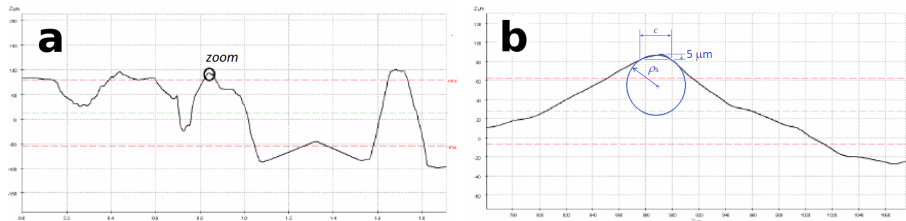


Figure 4.5: Example of  $\rho_s$  measurement: (a) 2D profile of 2 mm in the cutting direction that passes through a peak; (b) zoom of a peak and definition of  $\rho_s$

protuberant peaks and obtain 2D profiles of them in the cutting direction. These 2D profiles, as the one shown as example in Figure 4.5(a), are used to measure the sharpness radius of the cutting edge  $\rho_s$ . Just the tip of the peak removes material in a shallow manner. Hence,  $\rho_s$  is defined as the radius of the circumference that fits in the first 5  $\mu\text{m}$  in depth within the peak. A zoom of the peak allows a comfortable measurement of  $\rho_s$  (Fig. 4.5(b)). At least 12 measurements are made per wheel type and dressing condition. Similar  $\rho_s$  values have been observed for different dressing conditions and wheels (29  $\mu\text{m}$  for A46I8V and 27  $\mu\text{m}$  for A60K7V). The dressing conditions determine the aggressiveness of the operation, which is related to the grit pull-out. In turn, the abrasive fracture occurs in a shallow depth with the blunt dresser under similar conditions. The later one defines the cutting edge sharpness, so it makes sense to obtain similar sharpness characteristics regardless dressing conditions. These values agree with the ones obtained by [KL75, BT93] (16 - 35  $\mu\text{m}$ ), which are measured directly from the freshly dressed wheel in a similar way. [LS75] obtain values of  $\rho_s = 75 - 500 \mu\text{m}$  but measuring non dressed single-grit transverse traces.

The dynamic grain density  $G_{\text{dyn}}$ , the average distance between consecutive grits  $L_{\text{dyn}}$  and the contact length  $l_c$  have been measured by means of the applied power source method, due to its simplicity and accuracy (see Section 2.2.2). The chosen variant is the one that cuts the workpiece in two and a ledge is ground on



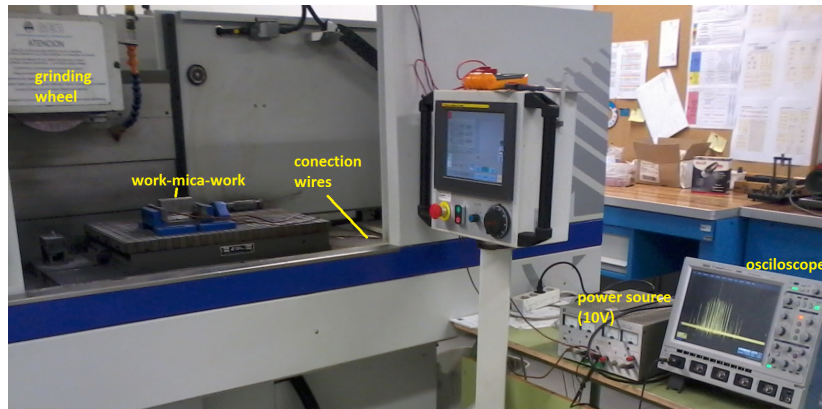


Figure 4.6: The APS set-up

the surface (Fig. 2.15(c)) [ZV92]. Figure 4.6 introduces the test set-up. Both parts are isolated by a mica sheet and connected to an electric potential source. When a grit passes over the protuberance, the circuit closes and a voltage pulse is caught in the oscilloscope. The duration of the signal depends on the kinematic parameters and surface topography. Studying the resultant voltage signal over time, the contact time  $t_c$  is defined.  $l_c$  is estimated with Equation 2.26. In the same way,  $G_{\text{dyn}}$  is defined as the amount of voltage pulses  $N_c$  (contacts) over the wheel contact area (Eq. 2.18). In turn,  $L_{\text{dyn}}$  relates the contacted length on the wheel to  $N_c$  (Eq. 2.17), modifying the razor blade method [BS74].

The measuring width  $b$ , the thickness of the mica sheet  $s$  and the tension  $V$  are the main parameters that govern the APS measurement.  $b$  has influence on the availability of contacts,  $s$  discriminates contacts regarding the intensity and  $V$  concerns the signal intensity. Table 2.2 compiles the parameters used by other authors. The CCS method adjusts the thickness  $s$ , to obtain a clearer and stable signal, avoiding the continuous hot-junction [GW88]. Beside that, when the potential source is not applied, the measurement of the resistance between the isolated parts after grinding shows that circuit is closed. After grinding at the same conditions applying the power-source, the circuit is open. That means that when the current passes thorough the micro-contacts, produces heat that destroys the physical link just after a grit leaves the contact zone. Therefore, the combination of a thick isolation ( $s = 40 \mu\text{m}$ ) and high voltage ( $V = 10 \text{ V}$ ) obtains the clearest signals. When  $b > 0,6 \text{ mm}$ , the high number of contacts increases the risk of a continuous signal, even losing the balance of the measurement (permanent short-circuit). The technique requires further analysis that concerns the influence of the factors on the signal and the comparison with a different technique.

Tests are made with two protuberance widths, measuring the average and maximum contact lengths. Figure 4.7 shows an example of voltage-time (V-ms) signals with both widths measured for the same wheel type, depth of cut

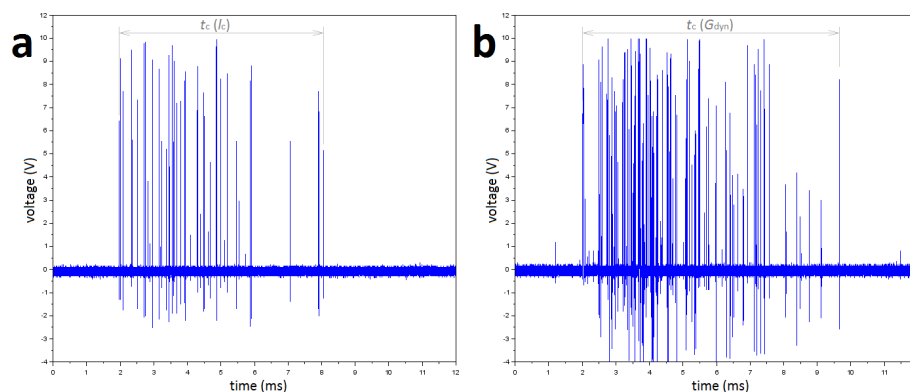


Figure 4.7: APS signal examples regarding protuberance width: (a)  $b = 0,12$  mm and (b)  $b = 0,5$  mm (A46I8V grinding wheel,  $a = 5 \mu\text{m}$ , fine dressing (F))

and dressing condition. Comparing with the APS signals shown in Figure 2.17 [QRM97a], the observed signals are clearer, the pulses are clearly identified. The differences lay on the use of a thicker mica sheet isolation ( $40 \mu\text{m}$  instead of  $20 \mu\text{m}$ ) and the higher voltage applied ( $10 \text{ V}$  instead of  $1,2 \text{ V}$ ).

The DEM model simulates an instant of the contact, not a contact through  $t_c$ , as APS does. In addition, the narrow width of the model also reduces the availability of DEs. Therefore, the simulation result corresponds to the average contact length. In this way, the APS measurements with narrow protuberance ( $b = 0,12$  mm) are used to validate the contact model (Fig. 4.7(a)). In addition, the same signal is used to estimate the average distance between grits  $L_{\text{dyn}}$ . [BS74] discuss about the requirement of a narrower razor blade (or in this case  $b$ ) than cutting edge scratch width to capture a clean signal. As the grain diameter is two or three times  $b$ , there is no overlapping problem. On the other hand, the wide protuberance ( $b = 0,5$  mm) covers a larger area, which is more suitable to determine the dynamic grain density (Fig. 4.7(b)). Three set depths of cut ( $a = 5 - 10 - 20 \mu\text{m}$ ) and 12 runs per test condition are used in the measurements to obtain the regression curves regarding the set depth of cut,  $l_c - a$  and  $G_{\text{dyn}} - a$ . In the APS measurements, the effect of the spindle deflection on the depth of cut has been neglected, due to the small forces generated due to the narrow width of cut.

## 4.2 DEM model set-up

Before run the simulations, the mechanical properties of the beams of the DEM model are calibrated, to adjust the elastic behaviour of the wheel. Simulations are repeated with four different DEM models. As the model construction and surface topography application are completely random processes, the results of simulations are the average values of the obtained contact lengths, as in actual grinding process. Hence, four DEM models are constructed for each grinding

wheel type following the methodology described in Sections 3.3 and 3.4. The model is constructed regarding the dimensions and designation of the wheel. Then, the surface topography is created on the model surface regarding the dressing conditions, grain density measurements and sharpness radius. The boundary conditions (Section 3.5) are set prior simulation, either static contact (SC) or uncut chip thickness (UCT). At the beginning, the force is applied on the workpiece with a ramp, which slope is 1/1000 times the specific normal force. Once reached the nominal force, it remains constant. This assures the stability of the simulation at the early stages. The main parameters are saved during the simulation to track and treat then afterwards.

### 4.2.1 Beam properties calibration

The calibration has been made for the whole range of wheel grades following the procedure described in Section 3.2.1. Calibration results are valid for any grit size and structure number, because the procedure assures the homogenisation of the mechanical properties of the beams as a whole. Several cylindrical DEM specimens of different size are prepared for the tensile numerical tests. Setting the micro-scale Poisson ratio at  $\nu_{\mu} = 0,2$ , a diameter ratio of  $r_{\mu} = 0,538$  is obtained for a macro-scale  $\nu_s$  of 0,2 [KF87]. The second step of the calibration leads a regression curve shown in Figure 4.8, that relates the macro- and micro-scale Young modulus ( $E_s, E_{\mu}$ ). The correlation  $E_s$ -hardness grade has been provided by the grinding wheel manufacturer *Unesa*. The curve fit to a line with a  $R^2$  of 0,9995, so the relationship between  $E_s$  and  $E_{\mu}$  can be considered linear.

Regarding the grinding wheels used in the tests, the wheel A46I8V with an  $E_s$  of 34 GPa, corresponds to  $E_{\mu}$  of 185,5 GPa on the beams. The wheel A60K7V has an  $E_s = 43$  GPa and its beams  $E_{\mu} = 234,3$  GPa. Both share the same micro-scale Poisson's ratio ( $\nu_{\mu} = 0,2$ ) and diameter ratio ( $r_{\mu} = 0,538$ ).

### 4.2.2 Input data

Table 4.2 compiles the input data to create the DEM model and run the simulation. Some of them have been obtained in the experimental work (i.e. grinding forces, grain density), others by simulation (i.e calibration) and, also, applying analytical expressions (i.e. uncut chip thickness).

The construction of the DEM model requires to define the dimensions, the granular morphology and mechanical properties. DEM model dimensions are the wheel diameter  $d_s$ , the width  $b_s$  and the height  $h_s$  (Fig. 3.12).  $d_s$  coincides with the actual wheel diameter. However, in the numerical sintering process, DEs are piled up on the elastic domain walls as shown by Figure 3.16. Therefore when constructing the DEM model,  $d_s$  is enlarged about three times the average grain diameter  $d_g$  outward ( $d_s + 3 \cdot d_g$ ). The nominal  $d_s$  is achieved when applying the topography.

Regarding the model width  $b_s$ , it is a compromise between the model size and available DE in the contact. At least four rows of grains are modelled.

Table 4.2: DEM model input data

<b>Grinding Wheels</b>	A46I8V			A60K7V		
Model dimensions, $d_s, b_s, h_s$ (mm)	300 x 2,24 x 5,1			300 x 1,7 x 5,1		
Abrasive volume fraction, $V_g$	48%			50%		
Average DE diameter, $d_{DE}$ (mm)	0,330			0,253		
Diameter reduction ratio, $r$	0,921			0,944		
Poisson ratio, $\nu_s, \nu_\mu$	0,2			0,2		
Beam diameter ratio, $r_\mu$	0,538			0,538		
Beam Young's modulus, $E_\mu$ (GPa)	185,5			234,3		
Density (kg/m <sup>3</sup> )	1854			2096		
Approx. number of DEs, $N_{DE}$	11000			19100		
<b>Dressing conditions</b>	$a_d = 20 \mu\text{m}$	$b_d = 0,8 \text{ mm}$	$\rho_d = 4,06 \text{ mm}$			
Coarse (C)	$h_f = 20 \mu\text{m}$	$f_d = 0,8 \frac{\text{mm}}{\text{rev}}$	$u_d = 1$			
Medium (M)	$h_f = 10 \mu\text{m}$	$f_d = 0,4 \frac{\text{mm}}{\text{rev}}$	$u_d = 2$			
Fine (F)	$h_f = 5 \mu\text{m}$	$f_d = 0,2 \frac{\text{mm}}{\text{rev}}$	$u_d = 4$			
<b>Cutting edge sharpness</b>	A46I8V			A60K7V		
Cutting edge radius, $\rho_s$ ( $\mu\text{m}$ )	29			27		
Standard deviation, $\sigma_\rho$ ( $\mu\text{m}$ )	4,6			3,8		
	<b>F</b>	<b>M</b>	<b>C</b>	<b>F</b>	<b>M</b>	<b>C</b>
Bond fracture particle size, $d_b$ ( $\mu\text{m}$ )	233	243	259	178	188	199
<b>Max. chip thickness, <math>h_{cu,max}</math> (<math>\mu\text{m}</math>)</b>						
$a = 10 \mu\text{m}$	0,34	0,39	0,51	0,31	0,34	0,4
$a = 20 \mu\text{m}$	0,52	0,58	0,7	0,48	0,48	0,54
<b>Forces</b>						
$a = 10 \mu\text{m}$						
Force ratio, $\mu$	0,53	0,55	0,49	0,49	0,5	0,49
Specific normal force, $F'_n$ (N/mm)	11,2	9,3	4,2	19,3	16,4	10,5
$a = 20 \mu\text{m}$						
Force ratio, $\mu$	0,49	0,45	0,44	0,49	0,45	0,47
Specific normal force, $F'_n$ (N/mm)	7,8	6,9	5,0	15,8	14,4	13,3
<b>Workpiece material viscoplastic properties</b>						
34CrMo4 tool steel	Hardness 55 HRc			$\sigma_y = 1815 \text{ MPa}$		
	$E_w = 210 \text{ GPa}$			$\nu_w = 0,3$		
JC model coefficients [JC83]	$A = 1815 \text{ MPa}$	$B = 477 \text{ MPa}$		$C = 0,012$		
	$n = 0,18$		$m = 1,0$			
Strain and	$\varepsilon = 3,1$		$\dot{\varepsilon} = 3,610^6 \text{ s}^{-1}$			
temperature parameters	$T_o = 300 \text{ K}$	$T_m = 1764 \text{ K}$		$T = 950 \text{ K}$		

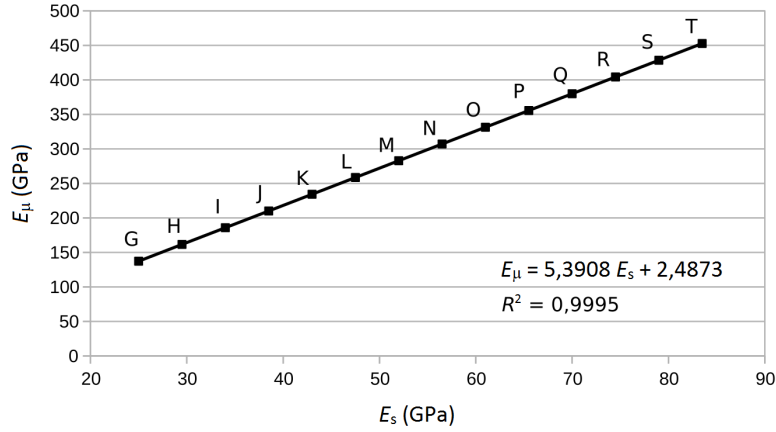


Figure 4.8: Correspondence between grinding wheel hardness grade, elastic modulus  $E_s$ , and micro-scale beam's elastic modulus  $E_\mu$

But due to the distance between grits is greater,  $b_s$  is defined as 6 - 7 times the average grain diameter  $d_g$ . As shown in Table 4.2,  $b_s$  is 2,24 mm for A46I8V and 1,7 mm for A60K7V. Both share the relationship width/DE diameter ( $b_s/d_g$ ) in 6,8. The maximum admissible DE number determines this relation: the lower relationship, the smaller model size and, thus, shorter computation time; on the other hand, the wider relationship covers a larger surface and captures better the insights of the wheel topography. The grain size and density makes the width of the models different, but the equal ( $b_s/d_g$ ) ratio makes comparable the results of them.

Finally, the model height  $h_s$  is defined with FEM as described in Section 3.3.2. Applying the maximum experimental force, the Von Mises stress is less than a hundredth from a  $h_s = 5,1$  mm distance on. Figure 4.9 shows the definition of  $h_s$  by means of FEM. The colour scale has been modified to enhance the simulation results. As the FEM model assumes Hertzian contact (elastic and smooth), the contact area is small and, hence, the stress concentration is large at the contact area. This effect enhances the safety margin of the  $h_s$  calculation.

The granular morphology is defined by the average DE diameter  $d_{DE}$  ( $= d_g$ ) and the abrasive volumetric fraction  $V_g$ . Equations 3.18, 3.20 and 3.23 define  $d_g$ ,  $V_g$  and the number of DEs  $N_{DE}$  (Table 4.2). Thus, with model dimensions and morphology parameters the DEM models are built.

Then, beams are created between the DEs in contact, with their the correspondent mechanical properties ( $E_\mu$ ,  $\nu_\mu$ ,  $r_\mu$ ). A beam is placed on every pair of DEs in contact, before the DE diameter reduction. The created beam has a slightly larger diameter ( $r_\mu/r$ ), in order to compensate the following DE diameter reduction.

Once the DEM model is constructed, the numerical dressing can be applied on the surface. The topography is applied in the pre-processing on each sim-

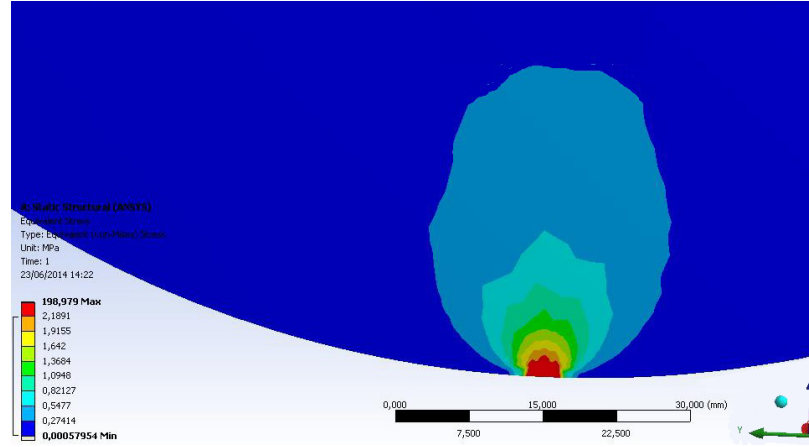


Figure 4.9: FEM analysis of the contact stresses between the grinding wheel and the workpiece to define the model height  $h_s$

ulation regarding the procedure described in Section 3.4. The ductile cutting of the dresser depends on the dressing feed  $f_d$  and the diamond tip radius  $\rho_d$ . The dressing depth of cut is maintained constant on all dressing conditions  $a_d = 20 \mu\text{m}$ . The average particle size released by bond fracture  $d_b$  is calculated regarding the measured grain density at  $a = 20 \mu\text{m}$  depth (see Table 4.2, Eq. 2.12 and 2.13). The fracture amplitude  $h_f$  is calculated with Equation 2.28. In addition, the correspondent sharpness radius  $\rho_s$  is applied regarding a normal distribution with  $\sigma_\rho$  standard deviation. Similar  $\rho_s$  values are observed for different dressing conditions (Table 4.2).

The maximum uncut chip thickness  $h_{cu,max}$  is calculated with Eq. 2.5. Equation 2.17 estimates the required average distance between two consecutive grits  $L_{dyn}$  analysing APS measurements. The use of the  $L_{dyn}$  based approach underestimates the  $h_{cu,max}$  values.

The specific normal forces  $F_n^i$  and force ratios  $\mu$  are also summarised in Table 4.2, and, Figure 4.10 shows the experimental measurements of  $F_n^i$  regarding the dressing feed  $f_d$ . Fine dressing (F) conditions work under higher grinding forces, because of the closer and denser surface topography. In turn, better surface roughnesses are achieved with fine dressing. Regarding the force ratio  $\mu$ , the values are slightly higher for the A46I8V wheel (0,49 - 0,55) than for A60K7V (0,44 - 0,5). When simulating,  $F_n$  is applied on the workpiece.

The centrifugal forces  $f_z^i$  are applied as well on each DE regarding the radial position and the rotation speed of the wheel  $N_s$  (Eq. 3.38). For an outer DE at  $v_s = 24 \text{ m/s}$ ,  $f_z$  is 0,0021 N for a 46 grit size, and 0,0011 N for 60 size. Combining the force of the huge amount of grains  $\Sigma f_z$ , arises the effect of  $f_z$  on the wheel behaviour. If  $v_s$  is double,  $f_z$  is four times bigger.

Regarding the mechanical properties of the workpiece, the SC and UCT models assume viscoplastic material behaviour (Section 2.4.3). As explained

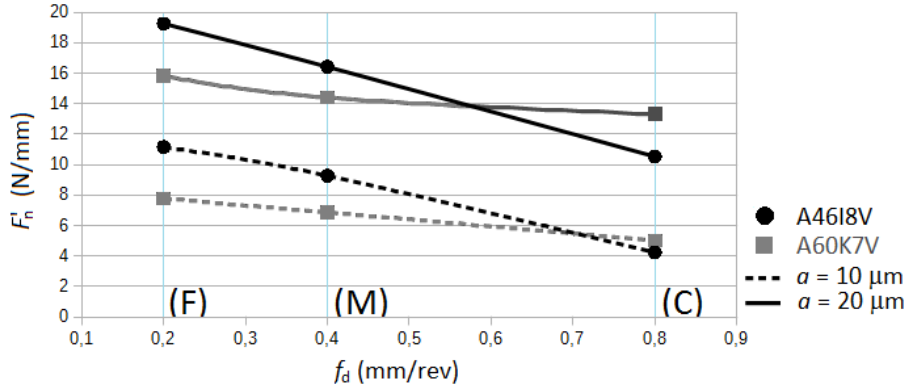


Figure 4.10: Specific grinding forces regarding the wheel type, set depth of cut and dressing feed; (F), (M) and (C) correspond to the dressing conditions

in Section 3.5, their values are assumed constant through the contact arc. A rough estimation is made to define the effective yield stress of the material at cutting conditions with the Johnson-Cook (JC) model. The workpiece is a 34CrMo4 tool steel tempered and quenched. Equation 2.35 introduces the constitutive the JC model for viscoplastic behaviour of metals. It estimates the effective yield stress  $\sigma_y$  regarding  $\varepsilon_p$  the plastic strain,  $\dot{\varepsilon}_p$  and  $\dot{\varepsilon}_o$  the actual and reference strain-rates, and  $T$ ,  $T_o$  and  $T_m$  the actual, ambient and melting temperatures. Johnson and Cook [JC83] list several material constants for a wide sort of materials. The S5 tool steel is among them. As S5 and 34CrMo4 are similar in composition and characteristics, these constants are also applicable for 34CrMo4. The correspondent material constants are listed in Table 4.2.

The review of the material removal theory is focused on the strains and strain-rates in material cutting (Section 2.4.3). Assuming that the cutting edge of an abrasive grit cuts with an average uncut chip thickness of  $h_{cu} = 4 \mu\text{m}$  [KL75] at  $v_s = 24 \text{ m/s}$  speed, with a  $\mu = 0,5$  force ratio and  $\alpha = -60^\circ$  negative rake angle [Sha96], the Ernts-Merchant expression (Eq. 2.36) estimates a shear angle of  $\theta = 13,3^\circ$ . If  $C_1 = 10,6$  [OY89] and applying Equations 2.38 and 2.40, a plastic strain of  $\varepsilon_p = 3,1$  and a strain-rate of  $\dot{\varepsilon}_p = 3,6 \cdot 10^6 \text{ s}^{-1}$  are obtained. These values correspond to almost-ballistic strain ranges [AWB11].

The melting temperature of 34CrMo4 steel is  $T_m = 1764 \text{ K}$  and an ambient temperature of  $T_o = 300 \text{ K}$  as reference temperature. A constant temperature of  $T = 950 \text{ K}$  is chosen. Numerical simulations estimate temperatures around 800 K and 900 K in the shear plane [OO05], where the phase change of the steels takes place becoming weaker [TFX<sup>+</sup>15]. These range is confirmed in the several experimental measurements published up to date with the single-pole thermocouple [LVLL06, DKD<sup>+</sup>10, PSO<sup>+</sup>12, RKM15]. Equations 2.43, 2.44 and 2.42 estimate analytically the maximum temperature in the contact area assuming a triangular moving heat source. Given the following variables ( $q_o = 90 \cdot 10^6 \text{ W/m}^2$ ,  $R_w = 0,9$  (dry grinding),  $k = 23,5 \text{ W/mK}$ ,  $\rho = 7860 \text{ kg/m}^3$ ,

$c = 515 \text{ J/kgK}$ ,  $C_1 = 1,06$ ,  $l_c = 0,004 \text{ m}$ ), Equation 2.43 estimates a maximum temperature of 1060 K, higher than the assumed constant temperature (950 K).

Finally, substituting the constants and parameters on JC constitutive model (Eq. 2.35), an effective yield stress of  $\sigma_y = 810 \text{ MPa}$  is obtained. This  $\sigma_y$  value agree with the values presented by [OO05], who simulate the generation of a serrated chip regarding the depth of cut  $a_e$  by means of FEM. The model obtains a  $\sigma_y$  of 1000 MPa at a depth of cut  $a_e = 1 \mu\text{m}$ , 700 - 900 MPa at  $a_e = 4,6 \mu\text{m}$  and 450 - 1000 MPa at  $a_e = 100 \mu\text{m}$  (rake angle  $\alpha = -45^\circ$ , cutting speed  $v_s = 30 \text{ m/s}$ ). The assumption of constant  $\sigma_y$  creates slightly smaller forces at shallow depths, an assumable error in overall.

### 4.3 Experimental and simulation results

This Section compiles and compares the experimental and simulation results related to the contact in grinding.

The quality of the topography generation is a key factor in the modelling of the contact in grinding. The surface topography of the DEM model is created on the pre-processing. Fig. 4.11 compares the bearing curves measured experimentally (continuous lines) and the ones applied on the DEM model regarding Chen's model (discontinuous lines). The DEM model surface bearing ratios are plotted regarding only the surface cutting edges. The superposition of both coincides up to  $10 \mu\text{m}$  depth, where the experimental bearing ratios increase rapidly. This is due the application of Chen's model in a discrete manner only on grit peaks. However, as the penetration of the cutting edges lays within the range of  $10 \mu\text{m}$ , the discrete application of Chen's model is valid to model the contact. Coarse (C) dressing creates lower bearing ratios than fine (F) dressing, which is reflected in the dressing forces intensities and in the force distributions during grinding.

A simulation lasts between 2 and 6 days with an Intel Xeon E5-2680 CPU at 2,80 GHz regarding the model size. At every iteration, a sensor records the number of DEs in contact with the workpiece  $N_c$ , the maximum penetration  $\delta_{\text{max}}$ , the contact length  $l_c$ , and the resultant reaction normal force in the workpiece. During the simulation, the graphs are often plotted to check out the grade of convergence. When the simulation gets balanced, it is stopped. Taking the resultant normal force as reference, the precise iteration of the solution is determined.  $N_c$ ,  $l_c$  and  $\delta_{\text{max}}$  are defined in that iteration.

As example, Figure 4.12 shows the output sensor of a (a) SC and a (b) UCT simulation. The top-left graphs correspond to  $N_c$ , which is used to estimate the grain density with Eq. 2.18. The top-right graphs corresponds to  $\delta_{\text{max}}$ , that gives a glimpse of the intensity of the contact regarding the available surface cutting edges. The bottom-left graphs indicate  $l_c$ : the upper and lower lines represent the position of the furthest grains in contact on both sides of the contact arc. Thus, the contact length is the distance between both (see also Fig. 4.14). This distance is symmetric regarding the wheel centre in the SC model (Fig. 4.12(a)), and asymmetric in the UCT model (Fig. 4.12(b)). As the



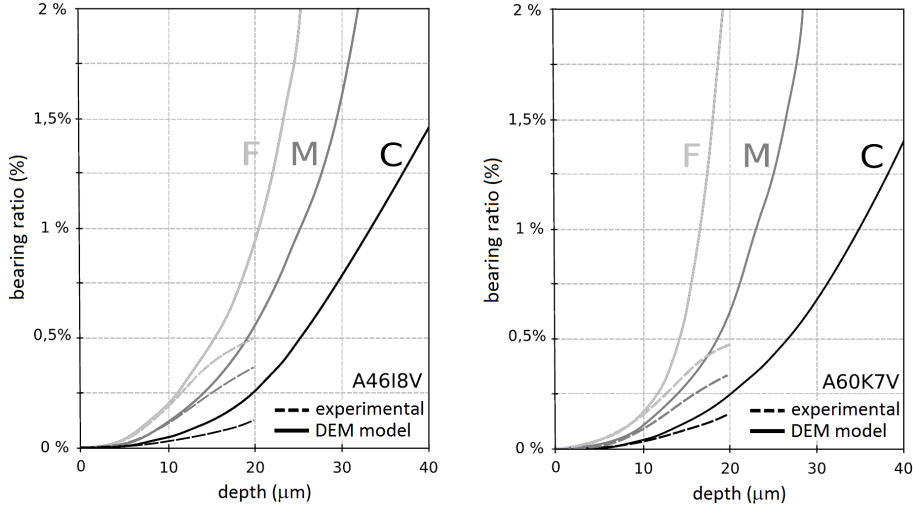


Figure 4.11: Experimental and numerical bearing ratio curves regarding the grinding wheel (A46I8V, A60K7V) and the dressing condition (F, M, C)

UCT model takes into account depth of cut  $a_e$ , the contact on that side is larger than on the plane side. The output for all variables oscillates and converges to the result as iteration number increases. The resultant normal force  $\Sigma f_n$  of the contacting DEs is the reference value in the simulation (bottom-right graph). The simulation is considered in balance when  $\Sigma f_n$  equals the normal force applied on the workpiece  $F_n$ . Generally, the SC model converges a bit faster and oscillates less than the UCT model. This is because every time that a DE enters in contact in the  $A$ - $B$  side of the wheel (see Fig. 3.20) starts with an initial penetration bigger than zero. This initial penetration coincides with the corresponding the uncut chip thickness on the DE location.

Fig. 4.13 shows the standard deviations of the experimental (in grey) and numerical (data point) contact length results for each grinding wheel and dressing condition. First of all, it is remarkable the dispersion that governs both, experimental measurements and simulation results. The DEM model has grasped the randomness of grinding, so all the results are defined by the average and the standard deviation. The dispersion in the numerical results is generally narrower than the observed in the tests. The A60K7V wheel shows a narrower standard deviation, because of the smaller grain size.

The experimental-geometrical contact length ratio ( $l_c/l_g$ ) lay between 1,85 and 2,52 for A46I8V wheel and between 1,92 and 2,2 for A60K7V, as described in the literature [MRDI04]. Table 4.3 compiles the same results, adding the maximum penetration  $\delta_{\max}$ , the roughness factor  $R_r$  for the SC model and the grain density  $G_{\text{dyn}}$  for the UCT model.

From the  $l_f$  results obtained with the SC model,  $l_c$  are calculated using the RQ approach. Results show that the RQ model obtains higher  $l_c$  values than

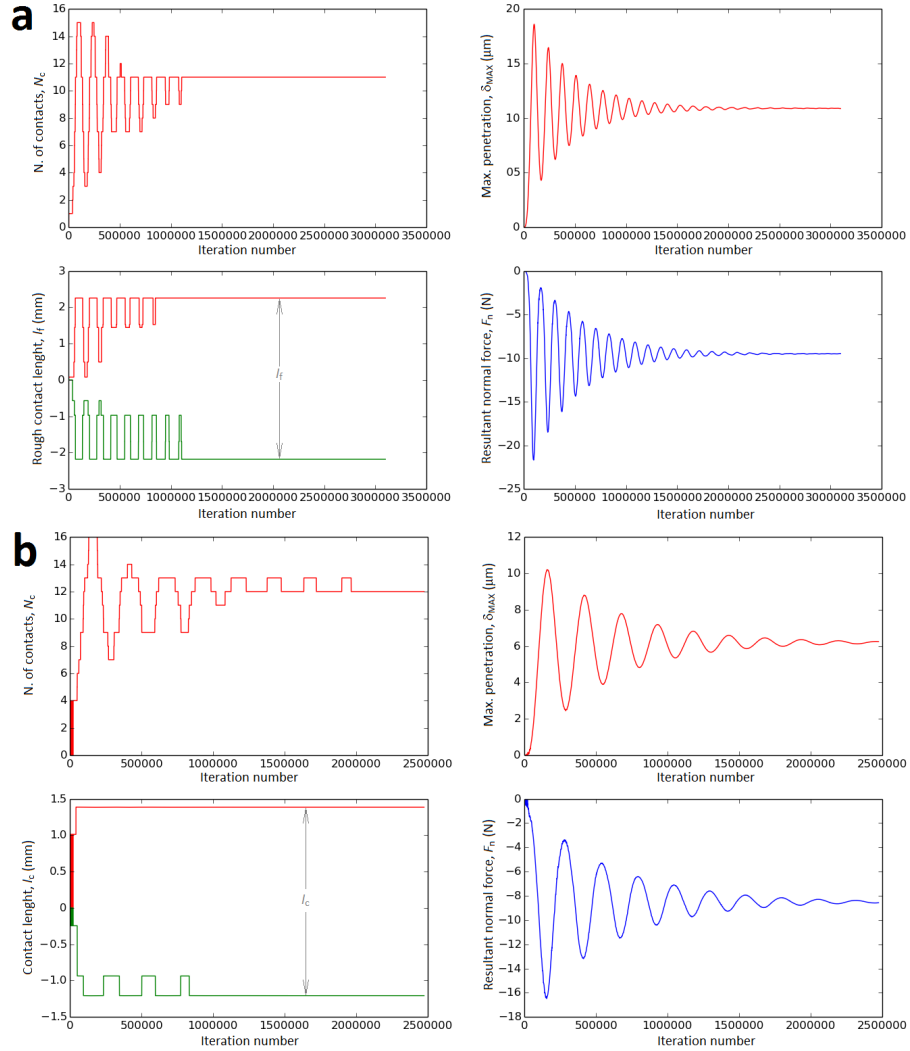


Figure 4.12: Examples of the output of the contact simulation (a) SC model and (b) UCT model (A46I8V,  $a = 10 \mu\text{m}$ ,  $C$  dressing). On both, the top-left graph shows the number of DEs in contact  $N_c$ , the top-right graph the maximum penetration  $\delta_{\max}$  ( $\mu\text{m}$ ); the bottom-left graph the contact length  $l_c$  (mm) as the sum of distance the furthest DEs in contact on both sides; and the bottom-right right the sum of the individual reaction normal forces  $\Sigma f_n$  of the DEs in contact; the horizontal axis indicates the iteration number (or time)

Table 4.3: Experimental and numerical results of contact length, maximum penetration, roughness factor and dynamic grain density

A4618V	Experimental results										SC - RQ					UCT				
	$F_n^i$ ( $\frac{N}{mm}$ )	$\mu$	$l_c$ (mm)	$l_g$ (mm)	$R_r$	$G_{dyn}$ ( $\frac{grits}{mm^2}$ )	$l_f$ (mm)	$l_c$ (mm)	$e\%$	$\delta_{max}$ ( $\mu m$ )	$R_r$	$e\%$	$l_c$ (mm)	$e\%$	$\delta_{max}$ ( $\mu m$ )	$N_c$	$G_{dyn}$ ( $\frac{grits}{mm^2}$ )	$e\%$		
dressing-a	4,2	0,49	2,88	1,56	4,6	1,28	3,77	4,08	42%	8,88	7,2	56%	2,75	-4,6%	6,35	9,8	1,35	5,2%		
C-10	10,5	0,49	4,14	2,14	4,3	1,58	4,44	4,93	19%	16,44	5,4	25%	4,09	-1,1%	7,01	18,2	1,68	6,2%		
M-10	9,3	0,55	2,96	1,32	3,4	2,05	3,96	4,17	41%	11,39	5,1	49%	3,13	5,8%	4,78	18,4	2,22	8,6%		
M-20	16,4	0,5	4,45	1,94	3,9	2,37	5,24	5,59	26%	16,23	5,1	31%	4,24	-4,7%	6,87	23,4	2,1	-11%		
F-10	11,2	0,53	3,08	1,22	3,3	2,62	4,04	4,22	37%	9,15	4,8	43%	2,98	-3,2%	4,28	22,6	2,87	9,5%		
F-20	19,3	0,49	4,12	1,84	3,3	2,93	4,89	5,22	27%	11,54	4,4	33%	4,06	-1,4%	6,01	30,4	2,83	-3,5%		

A60K7V	Experimental results										SC - RQ					UCT				
	$F_n^i$ ( $\frac{N}{mm}$ )	$\mu$	$l_c$ (mm)	$l_g$ (mm)	$R_r$	$G_{dyn}$ ( $\frac{grits}{mm^2}$ )	$l_f$ (mm)	$l_c$ (mm)	$e\%$	$\delta_{max}$ ( $\mu m$ )	$R_r$	$e\%$	$l_c$ (mm)	$e\%$	$\delta_{max}$ ( $\mu m$ )	$N_c$	$G_{dyn}$ ( $\frac{grits}{mm^2}$ )	$e\%$		
C-10	5,0	0,44	2,99	1,52	5	1,83	2,91	3,39	13%	10,3	5,7	13%	2,81	-6,1%	6,5	11,5	1,86	1,5%		
C-20	13,3	0,47	4,24	2,05	4,4	2,38	4,2	4,66	10%	15,8	5	13%	4,08	-3,7%	12,7	20,2	2,26	-5,6%		
M-10	6,9	0,45	3,1	1,44	4,6	2,17	3,17	3,6	16%	8,3	5,3	15%	3,27	5,4%	5,6	15,7	2,19	1,1%		
M-20	14,4	0,5	4,43	2,01	4,5	3	4,27	4,72	6%	13,9	4,9	8%	4,22	-4,7%	7,2	25,7	2,77	-7,6%		
F-10	7,8	0,49	2,71	1,39	3,6	2,83	3,23	3,64	34%	7,1	5,1	38%	2,84	4,6%	3,6	19,5	3,13	10%		
F-20	15,8	0,49	3,84	1,96	3,6	3,23	3,93	4,4	14%	11,5	4,3	19%	4,07	5,9%	6,5	28,5	3,19	-1,4%		

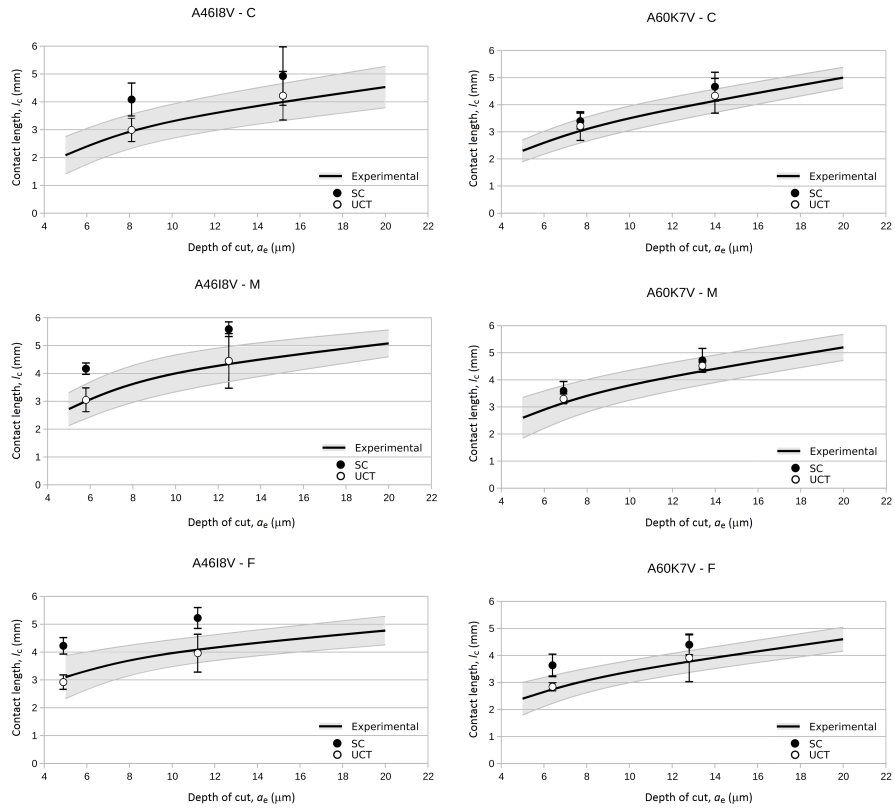


Figure 4.13: Contact length experimental measurements and numerical results, according to the grinding wheel type (A46I8V on the left, A60K7V on the right) and dressing condition (C, M, F)

the observed experimentally. However, RQ results follow the tendency of  $l_c$  regressions. Low roughness factor  $R_r$  values have been obtained experimentally and numerically. However, numerical  $R_r$  values are higher than experimental ones, specially for A46I8V (up to 56 %). The  $R_r$  factors differ even with the same topography. The results do not agree, probably due to the different contact conditions considered in the SC model (rigid-perfectly-plastic instead of elastic contact, and the viscoplastic behaviour of the workpiece material). However, initial numerical simulations assuming elastic contact and quasi-static material properties lead short rough contact lengths ( $l_f < 1$  mm). Maybe, the SC-RQ combination could be a valuable numerical tool to estimate  $l_c$  and  $R_r$ , if the effective yield stress  $\sigma_y$  value would be adjusted regarding experimental measurements.

The UCT  $l_c$  results are closer to the experimental ones (up to 6 %). On both wheels,  $l_c$  seems not to increase according to the dressing condition within the tested range. Wheel grade, centrifugal forces and cutting edge sharpness parameters are similar in both wheels, regardless the dressing condition. Then, the compelling parameters are the specific normal force  $F'_n$ , the depth of cut  $a_e$  and the surface grain density  $G_{dyn}$ . In turn,  $G_{dyn}$  depends on the grinding wheel characteristics (average grain diameter  $d_g$  and volumetric fraction of abrasive  $V_g$ ) and dressing conditions (average particle dimension removed by bond fracture  $d_b$ , dresser radius  $\rho_d$  and overlap ratio  $u_d$ ). Grain density and peak depth determine the bearing ratio of the surface or the supporting surface through radial depth.

Under fine dressing (F) conditions, grinding forces are higher and, thus,  $a_e$  decreases. In turn, the medium dressing (M) obtains larger  $l_c$  than the fine (F) one within the range of variables tested. [MRDI04] asserts that dressing determines the contact stiffness, which is directly related to the bearing ratio. Coarse dressing (C) gets the lowest grain density and maximum penetration  $\delta_{max}$  with the highest  $a_e$ . The single-grit force is related exponentially with the penetration, so less grains stand higher loads penetrating deeper. In turn, the higher  $G_{dyn}$  and, thus, bearing ratio of the fine dressing reduces the penetration in the workpiece. The medium dressing condition meets the combination of  $a_e$  and grain density that obtains the largest contact lengths on both wheels. This confirms the relationship between contact stiffness and bearing ratio, hence dressing.

The DEM model also offers an interesting insight about wheel deformation. The SC model obtains higher penetration  $\delta_{max}$  values than the UCT, due to the symmetrical distribution of grit forces. The UCT model distributes the forces asymmetrically according to the chip thickness, so the force partition resembles the actual. The average  $\delta_{max}$  in the UCT model is 6  $\mu\text{m}$ , which agrees with the results of [KL75]. There is a singular peak of 13  $\mu\text{m}$  in the A46K7V with coarse dressing and  $a = 20$   $\mu\text{m}$ , due to the concentration of load on a low density area.

The UCT model asserts the grain displacement magnitudes due to wheel deflection stated in the literature, about 5  $\mu\text{m}$  [NBS71, Sai90, Sha96]. The maximum grain displacement is 3,7 - 9,1  $\mu\text{m}$  for A46I8V and 2,9 - 6,6  $\mu\text{m}$  for A60K7V. Figure 4.14 shows an example of deflection (A46I8V,  $a = 10$   $\mu\text{m}$ ,  $C$

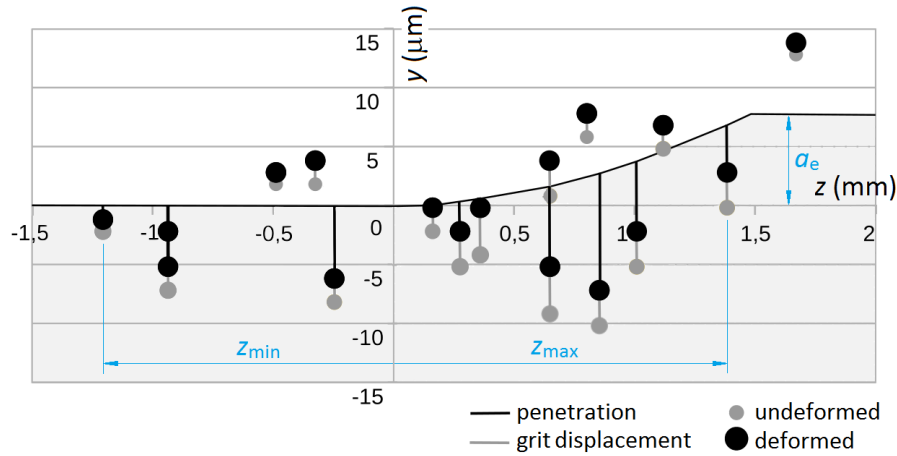


Figure 4.14: Example of wheel deformation: position of surface peaks unloaded and loaded (A60K7V,  $a = 10 \mu\text{m}$ , coarse dressing (C))

dressing). The force intensity on each cutting edge is defined by  $F'_n - G_{\text{dyn}}$  relationship, but grit displacement depends also on the number of beams and the relative position with adjacent grits. Figure 4.14 also shows the geometrical definition of the contact length, as the largest distance between grits in contact in the cutting direction, the sum of  $z_{\text{max}}$  and  $z_{\text{min}}$ . Some surface grits that do not get in touch with the workpiece are also displaced due to the beam network. The black line represents the workpiece surface in the loaded state, and, its relative position of the cutting edges (black circles) determines the penetration. The elastic deformation of the wheel (displacement of surface grits) contributes to the enlargement of the contact length.

The UCT approach foresees the grain density too, because it models directly the contact length. As the UCT approach models an instant of the grinding,  $G_{\text{dyn}}$  is calculated directly as the number of DE in contacts  $N_c$  distributed in the contact area ( $l_c b_s$ ). Numerical  $G_{\text{dyn}}$  results differ from the measured ones (up to 10%). The differences are not proportional under the same dressing conditions. However, the considerable dispersion of  $G_{\text{dyn}}$  experimental measurements (up to  $0,41 \text{ grits/mm}^2$ ) remarks the uncertainties in the APS measurements.

The UCT model also evaluates interesting aspects, such as the average and maximum force per grit, or the average contact pressure. The average force per grit is 1 - 1,5 N for the A46I8V and 0,7 - 1,1 N for the A60K7V. These highlights the role of the grain density in the material removal. The maximum force per grit reaches up to 2,8 N for A46I8V and 4,6 N for A60K7V. These values are interesting to analyse the stresses in the bonding material. Regarding the average contact pressure, it is 1,5 - 4,7 MPa for the A46I8V and 1,8 - 3,9 MPa for the A60K7V. The average pressure hides the ploughing and cutting processes under the apparent contact area, so these values have to be treated carefully.

The assumption of constant effective yield stress  $\sigma_y$  has direct influence on the numerical results. The reaction force per grit is proportional to  $\sigma_y$ . The force and cutting edge position (due to dressing conditions) together with the grain density, determines the penetration of a grit in the workpiece. Therefore, a good approach of  $\sigma_y$  is necessary in order to obtain reliable results with the DEM contact model. As discussed in Section 4.2.2, the range of variation of  $\sigma_y$  regarding the penetration or uncut chip thickness is not very steep, and the estimated value of  $\sigma_y = 810$  MPa seems enough good for the given experimental conditions. However, there is no discussion asserting that a viscoplastic behaviour of the workpiece material should be considered when analysing the grit-workpiece interaction when grinding. To overcome the constant  $\sigma_y$  assumption is one of the improvements pending left by this thesis. The issue can be tackled implementing the analytical strain and temperature expressions according the grit position in the contact arc (see Section 2.4.3). Another option is to substitute Shaw's hardness model by a specific energy model at grit level (micro-milling analogy), thus, avoiding the use of JC model and the viscoplastic behaviour assumption (see Section 2.4.2). The second option would require single-grit force measurements.

Wheel grade, centrifugal forces and cutting edge sharpness are parameters than are not studied in the simulations. Wheels of higher grade are stiffer. Higher centrifugal forces increase the wheel stiffness, due to the induced stresses and improved behaviour of the fragile bonding material at higher cutting speeds. Sharper cutting edges lead more efficient material removal, increasing the depth and, thus, the contact length. The influence of these factors are evaluated in the following Section with a numerical design of experiments.

## 4.4 Factorial experiments with the DEM model

The confidence on a model of almost thirty parameters relies on the good definition of their magnitudes. The numerical contact model for surface grinding, beside the estimation of the contact length and grain density, it is a valuable tool to evaluate the influence of each factor on the contact behaviour. The aim of this Section is to evaluate by means of design of experiments (DOE), the effect of the variation of a factor on the system response and on others factors.

The full factorial design analyses each level of each factor associated with each level of other factors. It is the most efficient method, but it requires the highest effort. Orthogonal arrays reduce largely the effort, assuring a balanced comparison of levels of all the analysed factors. In addition, it identifies the possible interactions between factors and the error made assessing the significance of the effects, defining the confidence limit of the study [Mon08]. As the study is focused on the contact numerical model, the environmental noise is mainly the randomness of the DEM model. There is no optimal or target contact length *a priori*, even though  $l_c$  can be indirectly related to thermal damage and workpiece surface roughness. Although the interest of the SC model as a rough contact model, it is not directly related to the contact in grinding and,

thus, it has been discarded from the DOE.

The contact model has several input factors. The literature review and the development of the contact DEM model (Chapters 2 and 3) have identified the main factors that affect the contact in grinding: the wheel stiffness (the Young's modulus of the grinding wheel  $E_s$  and the cutting speed  $v_s$ ), the surface topography (sharpness radius  $\rho_s$ , dressing conditions, the average particle size of released abrasive grits  $d_b$ , grain size  $d_g$ ), the grinding force ( $F_n$ ,  $F_t$ ), the depth of cut  $a_e$  and the workpiece effective yield stress  $\sigma_y$ . So, nine factors are identified as candidates for the factorial design.

A factorial design of nine factors increases largely the complexity of the design.  $F_n$  and  $a_e$  are closely related by the machine-spindle-wheel stiffness, so they can be considered as one factor. The force is distributed on the surface grits in contact: the larger force, the larger apparent contact area and, thus, the more grits in contact. To focus on the force distribution and contact conditions, the force (and consequently the depth of cut) has been considered constant in the DOE. On the other hand,  $\sigma_y$  is constant and proportional to the contact reaction at grit level. So it has been considered constant too. Thus, the influence of the rest of parameters is estimated with improved sensitivity and accuracy.

All the authors remark the influence of the Young's modulus of the wheel  $E_s$  on the contact length. An increase of  $v_s$  stretches the wheel and modifies the stiffness due to the action of centrifugal forces  $f_z$  on the grits. In addition, its effect has not been studied in the experimental validation of the model. Regarding the surface topography, it is closely interrelated to the dressing conditions. The dressing feed  $f_d$  is the variable taken into account in the experimental set-up to define the dressing conditions, being the dressing depth of cut  $a_d$  constant (see Table 4.1). The dressing conditions define the peak depth ( $h_f$ ,  $y_p$ ) applying Chen's model. On the other hand, the released grit size  $d_b$  determines the openness of the surface, together with the grit size. So the combination of  $f_d$  and  $d_b$  define the availability of cutting edges on the surface. Even though both are interrelated and dependent on dressing, both have been included in the DOE due to the interest of knowing the influence of both independently on the contact. The cutting edge sharpness radius  $\rho_s$  is a topography parameter too, but does not vary significantly regarding the dressing conditions within the range of variables tested. Therefore  $\rho_s$  has been also taken as an independent variable in the DOE. The grain size  $d_g$  has influence on the packing density and, consequently, on the surface grain density.

In this way, the chosen factors in the factorial design are listed and represented by a capital letter. Six factors are still a lot, but acceptable for a two level orthogonal array  $L_8(2^7)$ . Hence, eight experiments (7 degrees of freedom (DOF) in overall) are made to study six sources of variation, analysing each factor under two levels (1 DOF). Thus, the remaining DOF is assigned to the error (variability within the groups). Four DEM models are simulated for each condition to overcome the randomness of the model.



Table 4.4: Allocation of levels and simulation results

Test n.	A	B	e	C	D	E	F	Combination	$l_c$ (mm)	$\delta_{\max}$ ( $\mu\text{m}$ )	$G_{\text{dyn}}$ ( $\frac{\text{grit}}{\text{mm}^2}$ )
1	0	0	0	0	0	0	0	$A_0B_0C_0D_0E_0F_0$	4,39	13,59	2,46
2	0	0	0	1	1	1	1	$A_0B_0C_1D_1E_1F_1$	4,66	14,48	2,64
3	0	1	1	0	0	1	1	$A_0B_1C_0D_0E_1F_1$	3,97	13,16	2,71
4	0	1	1	1	1	0	0	$A_0B_1C_1D_1E_0F_0$	4,69	16,67	2,51
5	1	0	1	0	1	0	1	$A_1B_0C_0D_1E_0F_1$	4,82	19,15	3,54
6	1	0	1	1	0	1	0	$A_1B_0C_1D_0E_1F_0$	4,63	15,93	1,59
7	1	1	0	0	1	1	0	$A_1B_1C_0D_1E_1F_0$	4,97	19,8	1,7
8	1	1	0	1	0	0	1	$A_1B_1C_1D_0E_0F_1$	4,01	11,57	3,2
mark	a	b	ab	c	ac	bc	abc				

A - Young's modulus, $E_s$	25 GPa (G)	83,5 GPa (T)
beam Young's modulus, $E_\mu$	$A_0 = 137$ GPa	$A_1 = 452,7$ GPa
B - cutting speed, $v_s$	$B_0 = 20$ m/s	$B_1 = 40$ m/s
C - sharpness radius, $\rho_s$	$C_0 = 10$ $\mu\text{m}$	$C_1 = 30$ $\mu\text{m}$
D - dressing feed, $f_d$	$D_0 = 0,2$ $\frac{\text{mm}}{\text{rev}}$ (F)	$D_1 = 0,8$ $\frac{\text{mm}}{\text{rev}}$ (C)
E - released particle size, $d_b$ (%)	$E_0 = 0,67$	$E_1 = 0,78$
F - grain size, $d_g$	$F_0 = 253$ $\mu\text{m}$	$F_1 = 330$ $\mu\text{m}$

Constant factors:

normal force, $F'_n = 15,8$ N/mm	force ratio, $\mu = 0,5$
depth of cut, $a_e = 12,8$ $\mu\text{m}$	max. chip thickness, $h_{\text{cu}} = 0,43$ $\mu\text{m}$
effective yield stress, $\sigma_y = 810$ MPa	

Levels are defined (when possible) with the limits of the range of use. The range of the hardness grade in commercial grinding wheels is between G and T. The influence of  $f_z$  is studied at 20 m/s and 40 m/s cutting speeds  $v_s$ . The sharpness radius value varies in the literature from 5  $\mu\text{m}$  to 90  $\mu\text{m}$  (and more), because a lack of definition in the measurement procedure. Assuming that 30  $\mu\text{m}$  is already a quite blunt grain, 10  $\mu\text{m}$  has been chosen as sharp cutting edge [Klo09]. The dressing conditions and grain sizes coincide with the ones used in the experimental tests (see Table 4.1). Finally, the average size of released grits  $d_b$  levels are chosen regarding the minimum and maximum  $d_b$  observed in the tests. Defining  $d_b$  as a fraction of  $d_g$  (%),  $d_b$  is applied easily on both grit sizes. Force, depth of cut, uncut chip thickness and effective yield stress are constant as listed previously.

Table 4.4 introduces the assignment of the level and the results for each factor combination. Three variables are studied in the DOE: the contact length  $l_c$ , the maximum penetration  $\delta_{\max}$  and the grain density  $G_{\text{dyn}}$ . Table 4.5 shows the analysis of variance (ANOVA) of those results regarding the contact length  $l_c$ , the maximum penetration  $\delta_{\max}$  and the grain density  $G_{\text{dyn}}$ .

The ANOVA provides valuable information about the influence of the sources on the behaviour of the model. As commented in the previous Section, the DEM

Table 4.5: ANOVA tables for (a) contact length  $l_c$ , (b) maximum penetration  $\delta_{\max}$  and (c) grain density  $G_{\text{dyn}}$ 

<b>(a) Contact length <math>l_c</math></b>					
Source of variation	Sum of squares	DOF	Mean Square	$F_o$	$P(F(1, 1) \geq F_o) < \alpha$
A ( $E_s$ )	0,06590	1	0,11872	112,09	$P(39, 86 \geq F_o) < 0, 1$
B ( $v_s$ )	0,09162	1	0,05971	56,33	$P(39, 86 \geq F_o) < 0, 1$
$e$ (error)	0,00106	1	0,00106		
C ( $\rho_s$ )	0,00369	1	0,00369	3,48	$P(39, 86 < F_o) > 0, 1$
D ( $f_d$ )	0,57155	1	0,57155	538,57	$P(161, 45 \geq F_o) < 0, 05$
E ( $d_b$ )	0,01360	1	0,01360	12,81	$P(39, 86 < F_o) > 0, 1$
F ( $d_g$ )	0,18681	1	0,18681	176,03	$P(161, 45 \geq F_o) < 0, 05$
Total	0,93	7			

<b>(b) Maximum penetration <math>\delta_{\max}</math></b>					
Source of variation	Sum of squares	DOF	Mean Square	$F_o$	$P(F(1, 1) \geq F_o) < \alpha$
A ( $E_s$ )	9,15	1	9,15	8618	$P(4052, 2 \geq F_o) < 0, 001$
B ( $v_s$ )	0,47	1	0,47	446	$P(161, 45 \geq F_o) < 0, 05$
$e$ (error)	3,72	1	3,72		
C ( $\rho_s$ )	6,22	1	6,22	5856	$P(4052, 2 \geq F_o) < 0, 001$
D ( $f_d$ )	31,41	1	31,41	29597	$P(4052, 2 \geq F_o) < 0, 001$
E ( $d_b$ )	0,71	1	0,71	667	$P(647, 79 \geq F_o) < 0, 025$
F ( $d_g$ )	7,30	1	7,30	6880	$P(4052, 2 \geq F_o) < 0, 001$
Total	58,97	7			

<b>(c) Grain density <math>G_{\text{dyn}}</math></b>					
Source of variation	Sum of squares	DOF	Mean Square	$F_o$	$P(F(1, 1) \geq F_o) < \alpha$
A ( $E_s$ )	0,0113	1	0,0113	11	$P(39, 86 < F_o) > 0, 1$
B ( $v_s$ )	0,0015	1	0,0015	1	$P(39, 86 < F_o) > 0, 1$
$e$ (error)	0,0151	1	0,0151		
C ( $\rho_s$ )	0,0265	1	0,0265	25	$P(39, 86 < F_o) > 0, 1$
D ( $f_d$ )	0,0231	1	0,0231	22	$P(39, 86 < F_o) > 0, 1$
E ( $d_b$ )	1,1754	1	1,1754	1108	$P(647, 79 \geq F_o) < 0, 025$
F ( $d_g$ )	1,8422	1	1,8422	1736	$P(647, 79 \geq F_o) < 0, 025$
Total	3,1	7			

model reproduces the random nature of the grinding wheel. This randomness is also reflected on the factorial design. For example, the standard deviation of the contact lengths in the DOE tests is between 0,12 - 0,57 mm, a remarkable range having in mind that the force and depth of cut are constants in all the experiments.

Table 4.5(a) introduces the ANOVA analysis of the contact length  $l_c$ . The topography plays a remarkable role, specially the dressing feed  $f_d$  (or peak depth) and grain size  $d_g$  at 5 % of probability. Surprisingly, the cutting edge sharpness  $\rho_s$  does not affect significantly  $l_c$ . The elasticity of the wheel  $E_s$  and the cutting speed  $v_s$  are significant at a 10 % level of probability.

Regarding Table 4.5(b), all the sources of variation modify significantly the maximum penetration  $\delta_{max}$ , specially the topography sources ( $f_d$ ,  $\rho_s$ ,  $d_g$ ) and the wheel stiffness  $E_s$ . These can explain the dispersion observed in the  $G_{dyn}$  numerical results. The effective yield stress  $\sigma_y$  is an evident source of variation of  $\delta_{max}$ , and, consequently,  $G_{dyn}$  and  $l_c$ .

Finally, Table 4.5(c) shows that the topography factors dominate the grain density  $G_{dyn}$  output, specially  $d_g$  and  $d_b$ .  $G_{dyn}$  is closely related to the grain availability and that explains the importance of  $d_g$  and  $d_b$ .

The results of the factorial design agree those obtained experimentally. Evidently, they cannot be compared directly. On the one hand, parameters as  $F'_n$  and  $h_{cu,max}$  are considered constant in the factorial design and, on the other hand,  $v_s$  is constant and  $\rho_s$  is almost constant in the experimental work.  $E_s$  do not vary greatly on the tested wheels (I and K grades), so its influence is not noticeable on the observed contact lengths. In turn, topography parameters as  $f_d$  and  $d_g$  do have influence on  $l_c$  as the factorial design predicts. Regarding  $G_{dyn}$ , the factors  $d_b$  and  $d_g$ , also related to the dressing conditions, are dominant in the DOE and the tests.

## 4.5 Summary

In this Chapter, the DEM contact model is applied regarding the experimental work performed in the workshop. The experimental work provides both, the input data required for the simulations and, also, the target contact length  $l_c$ . The DEM models are constructed according the designation and the surface topography is created on them regarding the dressing conditions and topography parameters. The SC and UCT formulations are simulated under same grinding conditions. Finally, the numerical results are compared with the ones observed.

The results show that the SC model together with the RQ model do not fit the experimental results: the predicted contact lengths are longer than the observed ones. The model may require to adjust the contact conditions regarding the experimental contact lengths, specially the effective yield stress. Or it could be that the contact arc analysis do not reproduce the asymmetric forces distribution on the apparent contact zone.

In turn, the UCT approach matches the experimental results within 6 %, reproducing the trend of the contact regarding the characteristics of the wheel

and the dressing conditions. In addition, the DEM model provides valuable information about the deflection of surface grains (within 9  $\mu\text{m}$ ) and contact stiffness. Most of all, the DEM model has reproduced the random nature of the complex grinding wheel structure.

Regarding the DOE,  $L_8$  the fractional factorial design highlights the importance of the wheel stiffness and topography parameters on the contact conditions.

## Chapter 5

# Conclusions

A methodology based on DEM has been presented to analyse the contact in surface grinding. Established fundamental grinding models (i.e. surface topography, contact interface) are integrated into a synthesised DEM model. The 3D structure of the grinding wheel is reproduced representing an abrasive grit by a discrete element and a bonding bridge by an elastic beam. The surface topography is implemented combining Chen's and Malkin's models: Malkin's model describes the openness of the wheel surface due to the grit breakout, and Chen's model is applied on a discrete way, combining the dresser's helix with a fracture term and assigning a spherical cutting edge per DE. Two boundary conditions are used when defining the contact conditions between grits and workpiece: the static contact model, which models the rough contact term of the RQ model, and the uncut chip thickness model, which applies Shaw's hardness test analogy to estimate the single-grit forces. The workpiece is assumed to show viscoplastic behaviour in the contact with the grits. The model provides new insights in the grinding contact evolution.

The discrete element method has proved to be a valuable tool for the modelling of grinding wheels. The DEM model of the grinding wheel reproduces the wheel's granular nature and randomness, beside its stiffness. Chen's topography combined with Malkin's grain breakout model reproduce the peak distribution through depth regarding dressing conditions. The combination has shown the sensitivity to capture dressing hints on grinding forces.

Simulation results show that the RQ model obtains larger contact lengths under the set conditions. The effective yield stress would require a calibration regarding the experimental results. However, the static contact model opens new lines in the contact modelling of rough surfaces. On the other hand, the uncut chip thickness model results fit well the experimental measurements. The model captures the influence of the surface topography and it highlights the combined effect of the normal force, depth of cut and grain density. Shaw's indentation analogy of single-grit force has shown its potential for the estimation of grinding forces in a simple way. But the key of the success lays on the viscoplastic material assumption, which surprisingly has been disregarded so far by contact models.

Among all, the combination of DEM wheel body and surface topography stands out because of the description of the randomness of the grinding wheel, reflected on the dispersion observed in the results.

The versatility of the model formulation allows the adaptation for other purposes, like the workpiece surface roughness, grinding forces, effect of the centrifugal force or thermal analysis including the wheel. Finally, the author is pleasantly surprised that such a complex process as grinding can be modelled numerically, certainly with a not less simple model.

## 5.1 Future work

The following areas need further development,

- to analyse the effect of the cutting speed  $v_s$  and, consequently, the centrifugal force  $f_z$  on the contact length, as well as on the grinding wheel stiffness, expansion and bonding material behaviour,
- to include and evaluate the effect of the process fluids on the contact area,
- to overcome the assumption of constant effective yield stress  $\sigma_y$ :
  - estimating analytically the strains and temperatures regarding the grit position along the contact arc in every iteration, or,
  - applying the micro-milling analogy instead of Shaw's hardness model on the force definition at grit level by the specific energy measured by the single-grit test and, thus, avoiding the viscoplastic behaviour assumption,
- to extend the application range to rotary and stationary multi-point dressers, adapting the numerical dressing procedure, and,
- to take into account the tracks left by previous grits, including the ridges due to the material pile-up on the sides.

The versatility of the model formulation allows the adaptation for other purposes, like,

- to widen the scope, adapting the model to cylindrical, face and profile grinding,
- making the grinding wheel turn, to develop a complete model of the grinding process, which includes the stiffness of the wheel, grinding forces and estimation of the surface roughness on the workpiece,
- to model the wheel wear, modifying the cutting edge sharpness  $\rho_s$  together with the grit pull-out, modifying the formulation of the cohesive beams,
- following previous proposal, to analyse the dressing mechanisms focusing on bond breakage,

- to develop an analogue deterministic and explicit contact model, modelling the topography based on the bearing ratio and probability density function, which would calculate directly the contact length,
- to analyse the packing process of abrasive grits in the manufacturing of grinding wheels to estimate the structure number or volumetric fraction of abrasives, implementing multi-sphere discrete elements, and,
- to model the grinding energy partition between workpiece, chips, process fluid and, specially, grinding wheel, adding a thermal formulation in the interactions between DEs.





# Nomenklatura.

## *Nomenclature*

$A_c$ (mm <sup>2</sup> )	Aldiuneko ebaketa-sekzioa	<i>Instantaneous cutting section</i>
$A_d$ (mm <sup>2</sup> )	Diamantearen eta alearen arteko intersekzio azalera	<i>Area of intersection of the diamond and the grain</i>
$a, a_e$ (μm)	Ezarritako ebaketa-sakonera eta ebaketa-sakonera eraginkorra	<i>Set and effective depth of cut</i>
$a_1$ (mm)	Kontaktu-azaleraren erradiao	<i>Radius of the contact area</i>
$a_d$ (μm)	Diamantaketa-sakonera	<i>Dressing depth of cut</i>
$B$	Askatutako ale frakzioa pisan	<i>Abrasive fraction in weight removed by bond fracture</i>
$b_d, b_s, b_w$ (mm)	Diamante-, harri- eta piezazabalera	<i>Width of the dresser, grinding wheel and workpiece</i>
$C'$	Eremu plastikoaren murrizketa faktorea	<i>Constraint factor of the plastic zone</i>
$C_1, C_2, C_3$	Konstante esperimentalak	<i>Empirical constants</i>
$c$	Moteltze-koefizientea	<i>Dumping coefficient</i>
$d_b$ (μm)	Bataz besteko askatze ale-neurria	<i>Average grit dimension removed by bond fracture</i>
$d_{DE}, d_g$ (mm)	Bataz besteko DEaren diametroa eta ale-neurria	<i>Average discrete element and grain diameter</i>
$d_s$ (mm)	Harriaren diametroa	<i>Grinding wheel diameter</i>
$E_s, E_\mu$ (GPa)	Harriaren eta habeen elastikotasun modulua	<i>Grinding wheel and beam Young's modulus</i>
$e_s$ (J/mm <sup>3</sup> )	Artezketa energia espezifiko	<i>Specific grinding energy</i>
$F'_n, F'_t$ ( $\frac{N}{mm}$ )	Indar normal eta tangenzial espezifikoak	<i>Specific normal and tangential force</i>
$f_{DE}$ (N)	Indar axiala eta ebakitzaila DEan	<i>Axial and shear forces on a DE</i>

$\mathbf{f}_c, \mathbf{f}_d$ (N)	Ale-bakarrean kontaktu-eta moteltze-indarra	<i>Contact and dumping force on a single-grit</i>
$\mathbf{f}_n, \mathbf{f}_t$ (N)	Ale-bakarrean indar normala eta tangenziala	<i>Normal and tangential force on a single-grit</i>
$f_{ce}$ (N)	Ale-bakarrean indar zentrifugoa	<i>Centrifugal force on a single-grit</i>
$f_d$ (mm/rev)	Diamantaketa-aitzinamendua	<i>Dressing feed</i>
$G_{dyn}^*, G_o \left( \frac{\text{grits}}{\text{mm}^2} \right)$	ale-dentsitate teorikoa eta dinamikoa	<i>Dynamic and theoretical grain density</i>
$G_\mu$ (GPa)	Habearen elastikotasun modulua ebakitzailan	<i>Shear elasticity modulus of the beam</i>
$H$ (kg/cm <sup>2</sup> )	Brinell gogortasuna	<i>Brinell hardness</i>
$h_{cu}$ ( $\mu\text{m}$ )	Ebaki-gabeko txirbil-lodiera	<i>Uncut chip thickness</i>
$h_f$ ( $\mu\text{m}$ )	Ale-hausturaren anplitudea	<i>Grit fracture amplitude</i>
$h_s, h_w$ (mm)	Harriaren eta piezaren altuera DEM ereduan	<i>Grinding wheel and workpiece height on the DEM model</i>
$I_\mu, I_{p\mu}$ (m <sup>4</sup> )	Habearen inertzia-momentua eta inertzia-momentu polarra	<i>Moment of inertia and the polar moment of inertia of the beam</i>
$k$ (W/m K)	Konduktibitate termikoa	<i>Thermal conductivity</i>
$k_m^*$ (N/ $\mu\text{m}$ )	Makina-harria-pieza zuruntasuna	<i>Machine-wheel-workpiece stiffness</i>
$k_s$ (N/mm <sup>3</sup> )	Indar espezifikoa ale mailan	<i>Specific force at grit level</i>
$L_\mu$ (m)	Habearen hasierako luzera	<i>Initial length of the beam</i>
$l_c^*, l_g, l_f$ (mm)	Kontaktu-luzera, kontaktu-luzera geometrikoa eta kontaktu-luzera zimurra	<i>Contact length, geometrical contact length and rough contact length</i>
$L^*$ (mm)	Segidako bi aleen arteko batz besteko distantzia	<i>Average distance between two consecutive grits</i>
$M$	Galbahearen sare-neurria	<i>Mesh size</i>
$M_{iw}$ (kg)	$i$ elementuaren eta $w$ piezaren masa baliokidea	<i>Equivalent mass of <math>i</math> element and <math>w</math> workpiece</i>
$\mathbf{m}_{DE}$ (Nm)	Momentuak DEan	<i>Moments on a DE</i>
$N_{DE}$	Elementu diskretu kopurua	<i>Number of discrete elements</i>
$N_P^*$	Behatutako tontor-kopurua APS neurketetan	<i>Number of peaks in the APS signal</i>
$N_c$	Piezarekin kontaktuan da goen DE kopurua	<i>Number of discrete elements in contact with the workpiece</i>
$N_s$ (rpm)	Harriaren biraketa-abiadura	<i>Rotation speed of the grinding wheel</i>

$O_i$	DEaren koordinatu-sistema	<i>Coordinate system of a DE</i>
$\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i$	DEaren orientazio-, abiadura- eta azelerazio-koaternioia	<i>Orientation, angular velocity and angular acceleration quaternions of a DE</i>
$q_o$ (W/m <sup>2</sup> )	Bero-fluxua	<i>Heat flux</i>
$R$ (mm)	Erradioa	<i>Radius</i>
$R_r$	Zimurtasun faktorea (RQ)	<i>Roughness ratio (RQ)</i>
$R_w$	Piezak jasotako bero frakzioa	<i>Heat fraction to the workpiece</i>
$r, r_\mu$	DE eta habe diametroaren murrizpen-faktorea	<i>Discrete element and beam diameter reduction factor</i>
$r_{cu}$	Ebaki-gabeko txirbilaren zabalera-lodiera erlazioa	<i>Width-thickness ratio of the uncut chip</i>
$r_p$ ( $\mu\text{m}$ )	Sorbatzaren (tontorra) eta DE-aren zentroaren arteko distantzia	<i>Distance between the peak and the discrete element centre</i>
$S$	Egitura zenbakia	<i>Structure number</i>
$S_\mu$ (m <sup>2</sup> )	Habearen zeharkako azalera	<i>Cross section of the beam</i>
$s$ ( $\mu\text{m}$ )	Isolatzaille-lodiera	<i>Insulation thickness</i>
$s_n$ [0, 1]	Ausazko zenbakia	<i>Random number</i>
$t_c^*$ (s)	Kontaktu-denbora	<i>Contact time</i>
$T$ (K)	Temperatura	<i>Temperature</i>
$u_d$	Diamantaketa teilakapen-ratioa	<i>Dressing overlap-ratio</i>
$V_{DE}, v_g$ (mm <sup>3</sup> )	Ereduaren domeinuaren eta ale urratzailearen bolumena	<i>Model domain and abrasive grit volume</i>
$V_g$	Urratzaile frakzio bolumetrikoa harrian	<i>Volume fraction of abrasive in the grinding wheel</i>
$v_s, v_w$ (m/s)	Ebaketa- eta aitzinamendu-abiadura	<i>Cutting and work speed</i>
$\mathbf{v}_i, \mathbf{v}_w$ (m/s)	DEaren eta piezaren abiadura	<i>Speed of a discrete element and the workpiece</i>
$x, y, z$ (m)	Posizio koordinatuak	<i>Position coordinates</i>
$y_p$ ( $\mu\text{m}$ )	Sorbatz-sakonera	<i>Grit peak depth</i>
$\alpha, \alpha_c$	Jaulkitze-angelua; moteltze-faktorea eta moteltze kritikoa	<i>Rake-angle; dumping factor and critical dumping factor</i>
$\beta$	Moteltze-faktorea (TW)	<i>Dumping factor (TW)</i>
$\Delta L_\mu$ (m)	Habearen luzera-aldaketa	<i>Length variation of a beam</i>
$\delta_i$ ( $\mu\text{m}$ )	Alearen sarketa piezan	<i>Grit penetration on the workpiece</i>
$\varepsilon_p$	Deformazio plastikoa	<i>Plastic strain</i>
$\dot{\varepsilon}_p$ (s <sup>-1</sup> )	Deformazio-abiadura	<i>Plastic strain rate</i>
$\phi$ (rad)	Ebakidura-angelua	<i>Shear angle</i>
$\gamma$ (rad)	Marruskadura-angelua	<i>Friction angle</i>

$\mu^*$	Indar-arrazoia; marruskadura-koefizientea	<i>Force ratio; friction coefficient</i>
$\mu_g$ (mm)	Ale-neurri tartea	<i>Grit diameter variation range</i>
$\nu_s, \nu_\mu$	Harriaren eta habearen Poissonen modulua	<i>Poisson's ratio of the grinding wheel and the beams</i>
$\theta$ (rad)	Kontaktu-arkuaren posizio-angelua	<i>Position angle in the contact arc</i>
$\theta_i$ ( $\theta_x, \theta_y, \theta_z$ )	DEaren orientazio aldaketa	<i>Variation of DE orientation</i>
$\rho$ ( $\text{kg}/\text{m}^3$ )	Dentsitatea	<i>Density</i>
$\rho_d^*$ (mm)	Diamantearen erradioa	<i>Single-point dresser radius</i>
$\rho_s^*$ ( $\mu\text{m}$ )	Sorbatzaren zorrotasuna edo sorbatz-erradioa	<i>Cutting edge sharpness or peak radius</i>
$\sigma_y$ (MPa)	Isurpen-tentsio eraginkorra	<i>Effective yield stress</i>

## Laburdurak. *Acronyms*

AE	Emisio akustikoak	<i>Acoustic emissions</i>
AG	Askatasun-gradua	-
ANOVA	Bariantza-analisia	<i>Analysis of variance</i>
APS	Aplikatutako potentzial-iturria	<i>Applied power source</i>
CBN	Boro nitruro kubikoa	<i>Cubic boron nitride</i>
CCS	Kontaktu egoera kritikoa	<i>Critical contact state</i>
CN	Koordinazio-zenbaki	<i>Coordination number</i>
DE	Elementu diskretua	<i>Discrete element</i>
DEM	Elementu diskretuen metodoa	<i>Discrete element method</i>
DOE	Saiakuntzen diseinua	<i>Design of experiments</i>
DOF	-	<i>Degree of freedom</i>
EMF	Eragindako indar elektromagnetikoa	<i>Induced electromagnetic force</i>
ETL	Ebaki-gabeko txirbil lodiera	-
FDM	Diferentzia finituen metodoa	<i>Finite differences method</i>
FEM	Elementu finituen metodoa	<i>Finite element method</i>
GranOO	Ale objektuetara orientatua	<i>Granular object oriented</i>
GT	Greenwood-Tripp	<i>Greenwood-Tripp</i>
GW	Greenwood-Williamson	<i>Greenwood-Williamson</i>
IR	Infra-gorriak	<i>Infra-red</i>
JC	Johnson-Cook	<i>Johnson-Cook</i>
KE	Kontaktu estatikoa	-
MD	Dinamika molekularra	<i>Molecular dynamics</i>
MDR	Murrizpen dimentsionalaren metodoa (edo oinarri elastikoen metodoa)	<i>method of dimensionality reduction</i> <i>(aka. elastic foundation model)</i>
NCS	Ez-kontaktu egoera	<i>No contact state</i>
RQ	Rowe-Qi	<i>Rowe-Qi</i>
SC	-	<i>Static contact</i>
SEM	Ekortze mikroskopio elektronikoa	<i>Scanning electron microscope</i>
TW	Tchamwa-Wielgoz	<i>Tchamwa-Wielgoz</i>
UCT	-	<i>Uncut chip thickness</i>



# Bibliography

- [ABLW16] M. Akbari, S. Buhl, C. Leinenbach, and K. Wegener. A new value for johnson cook damage limit criterion in machining with large negative rake angle as basis for understanding of grinding. *Journal of Materials Processing Technology*, 234:58–71, 2016.
- [ACI15] D. André, J.L. Charles, and I. Iordanoff. *3D Discrete Element Workbench for Highly Dynamic Thermo-mechanical Analysis: Gran00*. John Wiley & Sons, 2015.
- [AICN12] D. André, I. Iordanoff, J.L. Charles, and J. Néauport. Discrete element method to simulate continuous material by using the cohesive beam model. *Computer Methods in Applied Mechanics and Engineering*, 213:113–125, 2012.
- [AK12] J.C. Aurich and B. Kirsch. Kinematic simulation of high-performance grinding for analysis of chip parameters of single grains. *CIRP Journal of Manufacturing Science and Technology*, 5(3):164–174, 2012.
- [Ald14] G.I. Alden. Operation of grinding wheels in machine grinding. *Trans. ASM*, 36:451–460, 1914.
- [And12] Damien André. *Modélisation par éléments discrets des phases d'ébauchage et de doucissage de la silice*. PhD thesis, Université Bordeaux 1, 2012.
- [AR05] S. Agarwal and P.V. Rao. A probabilistic approach to predict surface roughness in ceramic grinding. *International Journal of Machine tools and manufacture*, 45(6):609–616, 2005.
- [AR10] S. Agarwal and P.V. Rao. Modeling and prediction of surface roughness in ceramic grinding. *International Journal of Machine Tools and Manufacture*, 50(12):1065–1076, 2010.
- [Arc53] J.F. Archard. Contact and rubbing of flat surfaces. *Journal of applied physics*, 24:981–988, 1953.

- [Arc57] J.F. Archard. Elastic deformation and the laws of friction. In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, volume 243, pages 190–205. The Royal Society, 1957.
- [AWB11] D. Anderson, A. Warkentin, and R. Bauer. Experimental and numerical investigations of single abrasive-grain cutting. *International Journal of Machine Tools and Manufacture*, 51(12):898–910, 2011.
- [BAG<sup>+</sup>06] E. Brinksmeier, J.C. Aurich, E. Govekar, C. Heinzl, H.W. Hoffmeister, F. Klocke, J. Peters, R. Rentsch, D.J. Stephenson, E. Uhlmann, et al. Advances in modeling and simulation of grinding processes. *CIRP Annals-Manufacturing Technology*, 55(2):667–696, 2006.
- [BBR80] D. Brough, W.F. Bell, and W.B. Rowe. Achieving and Monitoring High Rate Centerless Grinding. In *Proceedings of the 21st Machine Tool Design and Research Conference*. Swansea University, 1980.
- [BC95] E. Brinksmeier and M. Cinar. Characterization of dressing processes by determination of the collision number of the abrasive grits. *CIRP Annals-Manufacturing Technology*, 44(1):299–304, 1995.
- [BGT75] A.W. Bush, R.D. Gibson, and T.R. Thomas. The elastic contact of a rough surface. *Wear*, 35(1):87–111, 1975.
- [Bha74] C.P. Bhateja. On the mechanism of the diamond dressing of grinding wheels. In *Proc. Int. Conf. Production Engineering*, pages 733–739, 1974.
- [BKW13] R. Babel, P. Koshy, and M. Weiss. Acoustic emission spikes at workpiece edges in grinding. Origin and applications. *International journal of machine tools and manufacture*, 64:96–101, 2013.
- [BMKW11] K.H. Brakhage, M. Makowski, F. Klocke, and M. Weiss. Grinding Wheel Modeling. Development of a mathematical Model. In *MASCOT11-IMACS/ISGG Workshop*, Rome, Italy, 2011. IAC-CNR.
- [BMS52] W.R. Backer, E.R. Marshall, and M.C. Shaw. The size effect in metal cutting. *Trans. Asme*, 74(1):61, 1952.
- [BS74] J.N. Brecker and M.C. Shaw. Measurement of the effective number of cutting points in the surface of a grinding wheel. In *Proceedings of the International Conference on Production Engineering, Japan Society of Precision Engineers, Tokyo, Japan*, pages 740–745, 1974.



- [BSS71] R.H. Brown, K. Saito, and M.C. Shaw. Local elastic deflections in grinding. *Annals of the CIRP*, 19(1):105–113, 1971.
- [BT39] F.P. Bowden and D. Tabor. The area of contact between stationary and between moving surfaces. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, pages 391–413, 1939.
- [BT93] N. Brenner and A.A. Torrance. Wheel sharpness measurement for force prediction in grinding. *Wear*, 160(2):317–323, 1993.
- [BT00] J.A. Badger and A.A. Torrance. A comparison of two models to predict grinding forces from wheel surface topography. *International Journal of Machine Tools and Manufacture*, 40(8):1099–1120, 2000.
- [BWW77] R.H. Brown, J.G. Wager, and J.D. Watson. An examination of the wheel-work interface using an explosive device to suddenly interrupt the surface grinding process. *Annals of the CIRP*, 26(1):143–146, 1977.
- [CEB87] W.R. Chang, I. Etsion, and D.B. Bogy. An elastic-plastic model for the contact of rough surfaces. *Journal of tribology*, 109(2):257–263, 1987.
- [CFJG02] G.Q. Cai, B.F. Feng, T. Jin, and Y.D. Gong. Study on the friction coefficient in grinding. *Journal of materials processing technology*, 129(1):25–29, 2002.
- [CGL<sup>+</sup>13] Y. Cao, J. Guan, B. Li, X. Chen, J. Yang, and C. Gan. Modeling and simulation of grinding surface topography considering wheel vibration. *The International Journal of Advanced Manufacturing Technology*, 66(5-8):937–945, 2013.
- [Che95] Xun Chen. *Strategy for the selection of grinding wheel dressing conditions*. PhD thesis, Liverpool John Moores University, 1995.
- [CJ59] H.S. Carslaw and J.C. Jaeger. Conduction of heat in solids. *Oxford: Clarendon Press, 1959, 2nd ed.*, 1959.
- [CJR13] B. Cambou, M. Jean, and F. Radjai. *Micromechanics of granular materials*. John Wiley & Sons, 2013.
- [CL00] W.L. Cooper and A.S. Lavine. Grinding process size effect and kinematics numerical analysis. *Journal of manufacturing science and engineering*, 122(1):59–69, 2000.
- [CO79] J.M. Challen and P.L.B. Oxley. An explanation of the different regimes of friction and wear using asperity deformation models. *Wear*, 53(2):229–243, 1979.

- [CP08] S. Chakrabarti and S. Paul. Numerical modelling of surface topography in superabrasive grinding. *The International Journal of Advanced Manufacturing Technology*, 39(1-2):29–38, 2008.
- [CR96] X. Chen and W.B. Rowe. Analysis and simulation of the grinding process. Part I: generation of the grinding wheel surface. *International Journal of Machine Tools and Manufacture*, 36(8):871–882, 1996.
- [CR04] R. Cai and W.B. Rowe. Assessment of vitrified CBN wheels for precision grinding. *International Journal of Machine Tools and Manufacture*, 44(12):1391–1402, 2004.
- [CS79] P.A. Cundall and O.D.L. Strack. A discrete numerical model for granular assemblies. *Geotechnique*, 29(1):47–65, 1979.
- [CW08] H.C. Chang and J.J. Wang. A stochastic grinding force model considering random grit distribution. *International Journal of Machine Tools and Manufacture*, 48(12):1335–1344, 2008.
- [CWKH08] H.A. Carmona, F.K. Wittel, F. Kun, and H.J. Herrmann. Fragmentation processes in impact of spheres. *Physical Review E*, 77(5):051302, 2008.
- [Dar13] Abdalslam Darafon. *Measuring and Modeling of Grinding Wheel Topography*. PhD thesis, Dalhousie University, Halifax, Nova Scotia, April 2013.
- [DBW09] D.A. Doman, R. Bauer, and A. Warkentin. Experimentally validated finite element model of the rubbing and ploughing phases in scratch tests. *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*, 223(12):1519–1527, 2009.
- [DKD<sup>+</sup>10] M. Duscha, F. Klocke, A. D’Entremont, B. Linke, and H. Wegner. Investigation of temperatures and residual stresses in speed stroke grinding via fea simulation and practical tests. *Proceedings in Manufacturing Systems*, 5(3):143–148, 2010.
- [DLL15] Z. Ding, B. Li, and S.Y. Liang. Maraging steel phase transformation in high strain rate grinding. *The International Journal of Advanced Manufacturing Technology*, pages 1–8, 2015.
- [DPS70] A. Decneut, Ja. Peters, and R. Snoeys. *Sonic testing of grinding wheels*. Centre de Recherches Scientifiques et Techniques de l’Industrie des Fabrications Métalliques, 1970.
- [DPS02] D.V. De Pellegrin and G.W. Stachowiak. Assessing the role of particle shape and scale in abrasion using sharpness analysis: Part i. technique development. *Wear*, 253(9):1016–1025, 2002.

- [DWB06] D.A. Doman, A. Warkentin, and R. Bauer. A survey of recent grinding wheel topography models. *International Journal of Machine Tools and Manufacture*, 46(3):343–352, 2006.
- [DWB09] D.A. Doman, A. Warkentin, and R. Bauer. Finite element modeling approaches in grinding. *International journal of machine tools and manufacture*, 49(2):109–116, 2009.
- [DWB13] A. Darafon, A. Warkentin, and R. Bauer. Characterization of grinding wheel topography using a white chromatic sensor. *International Journal of Machine Tools and Manufacture*, 70:22–31, 2013.
- [Eck96] J.D. Eckert. Replica techniques for the study of fracture surfaces and topography study in general. *Praktische Metallographie*, 33(7):369–372, 1996.
- [FIB07] N. Fillot, I. Iordanoff, and Y. Berthier. Wear modeling and the third body concept. *Wear*, 262(7):949–957, 2007.
- [Fra82] H.A. Francis. A finite surface element model for plane-strain elastic contact. *Wear*, 76(2):221–245, 1982.
- [Geo79] A.F. George. A comparative study of surface replicas. *Wear*, 57(1):51–61, 1979.
- [GT67] J.A. Greenwood and J.H.L. Tripp. The elastic contact of rough spheres. *Journal of Applied Mechanics*, 34(1):153–159, 1967.
- [GW66] J.A. Greenwood and J.B.P. Williamson. Contact of nominally flat surfaces. In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, volume 295, pages 300–319. The Royal Society, 1966.
- [GW88] D.Y. Gu and J.G. Wager. New evidence on the contact zone in grinding contact length, sliding and cutting regions. *CIRP Annals-Manufacturing Technology*, 37(1):335–338, 1988.
- [GWW02] Y.D. Gong, B. Wang, and W.S. Wang. The simulation of grinding wheels and ground surface roughness based on virtual reality technology. *Journal of Materials Processing Technology*, 129(1):123–126, 2002.
- [Hah62] R.S. Hahn. On the nature of the grinding process. In *Proceedings of the 3rd machine tool design and research conference*, pages 129–154, 1962.
- [Han06] Ihab M. Hanna. *Thermal modeling of grinding for process optimization and durability improvements*. PhD thesis, Purdue University Indiana, 2006.

- [HDD04] S. Hentz, L. Daudeville, and F.V. Donzé. Identification and validation of a discrete element model for concrete. *Journal of engineering mechanics*, 130(6):709–719, 2004.
- [Heg00] Johannes Bernardus Jan-Willem Hegeman. *Fundamentals of grinding: surface conditions of ground materials*. PhD thesis, University of Groningen, 2000.
- [Her82] Heinrich Hertz. Über die berührung fester elastischer körper. *Journal für die reine und angewandte Mathematik*, 92:156–171, 1882.
- [Hes03] D. Hessel. *Punkterushieren keramisch gebundener Diamantschleifscheibe*. PhD thesis, University of Hannover, 2003.
- [HK03] Z.B. Hou and R. Komanduri. On the mechanics of the grinding process - Part I. Stochastic nature of the grinding process. *International journal of machine tools and manufacture*, 43(15):1579–1593, 2003.
- [HL03] R.L. Hecker and S.Y. Liang. Predictive modeling of surface roughness in grinding. *International Journal of Machine Tools and Manufacture*, 43(8):755–761, 2003.
- [Hor98] J.H. Horng. An elliptic elastic-plastic asperity microcontact model for rough surfaces. *Journal of tribology*, 120(1):82–88, 1998.
- [HRL03] R.L. Hecker, I.M. Ramoneda, and S.Y. Liang. Analysis of wheel topography and grit force for grinding process modeling. *Journal of Manufacturing Processes*, 5(1):13–23, 2003.
- [HS95] M.G. Hamblin and G.W. Stachowiak. A multi-scale measure of particle abrasivity, and its relation to two-body abrasive wear. *Wear*, 190(2):190–196, 1995.
- [HWCC11] J.H. Horng, C.C. Wei, Y.Y. Chen, and Yuh P. Chang. The Micro-contact Analysis of Contact Length between Wheel and Workpiece in Precise Grinding. In *Advanced Materials Research*, volume 146, pages 1930–1936. Trans Tech Publ, 2011.
- [HWKES11] D. Höhner, S. Wirtz, H. Kruggel-Emden, and V. Scherer. Comparison of the multi-sphere and polyhedral approach to simulate non-spherical particles within the discrete element method: Influence on temporal force evolution for multiple contacts. *Powder Technology*, 208(3):643–656, 2011.
- [IBNC08] I. Iordanoff, A. Battentier, J. Néauport, and J.L. Charles. A discrete element model to investigate sub-surface damage due to surface polishing. *Tribology International*, 41(11):957–964, 2008.

- [IGI<sup>+</sup>10] D. Iliescu, D. Gehin, I. Iordanoff, F. Girot, and M.E. Gutiérrez. A discrete element method for the simulation of CFRP cutting. *Composites Science and Technology*, 70(1):73–80, 2010.
- [Ina96] I. Inasaki. Grinding process simulation based on the wheel topography measurement. *CIRP Annals-Manufacturing Technology*, 45(1):347–350, 1996.
- [JC83] G.R. Johnson and W.H. Cook. A constitutive model and data for metals subjected to large strains, high strain rates and high temperatures. In *Proceedings of the 7th International Symposium on Ballistics*, volume 21, pages 541–547. The Hague, The Netherlands, 1983.
- [JD11] M.J. Jackson and J.P. Davim. *Machining with Abrasives*. Springer-Verlag, 2011.
- [JDCI16] M. Jebahi, F. Dau, J.L. Charles, and I. Iordanoff. Multiscale modeling of complex dynamic problems: an overview and recent developments. *Archives of Computational Methods in Engineering*, 23(1):101–138, 2016.
- [JGB<sup>+</sup>13] J.L. Jiang, P.Q. Ge, W.B. Bi, L. Zhang, D.X. Wang, and Y. Zhang. 2d/3d ground surface topography modeling considering dressing and wear effects in grinding process. *International Journal of Machine Tools and Manufacture*, 74:29–40, 2013.
- [JM12] J.F. Jerier and J.F. Molinari. Normal contact between rough surfaces by the discrete element method. *Tribology International*, 47:1–8, 2012.
- [Joh87] Kenneth Langstreth Johnson. *Contact mechanics*. Cambridge university press, 1987.
- [KBCS09] P. Kanouté, D.P. Boso, J.L. Chaboche, and B.A. Schrefler. Multiscale methods for composites: a review. *Archives of Computational Methods in Engineering*, 16(1):31–75, 2009.
- [KBH<sup>+</sup>02] F. Klocke, T. Beck, S. Hoppe, T. Krieg, N. Müller, T. Noethe, H.W. Raedt, and K. Sweeney. Examples of FEM application in manufacturing technology. *Journal of Materials Processing Technology*, 120(1):450–457, 2002.
- [KD08] M. Kalweit and D. Drikakis. Multiscale methods for micro/nano flows and materials. *Journal of Computational and Theoretical Nanoscience*, 5(9):1923–1938, 2008.
- [KE02] L. Kogut and I. Etsion. Elastic-plastic contact analysis of a sphere and a rigid flat. *Journal of applied Mechanics*, 69(5):657–662, 2002.

- [KF87] W. König and H. Follinger. Elastic modulus of grinding wheels and its impact on their in-process behavior II. In *CFI. Ceramic forum international*, volume 64, pages 296–300. Göller, 1987.
- [KJL97] P. Koshy, V.K. Jain, and G.K. Lal. Stochastic simulation approach to modelling diamond wheel topography. *International Journal of Machine Tools and Manufacture*, 37(6):751–761, 1997.
- [KKPK94] S. Kucharski, T. Klimczak, A. Polijaniuk, and J. Kaczmarek. Finite-elements model for the contact of rough surfaces. *Wear*, 177(1):1–13, 1994.
- [KL75] W. König and W. Lortz. Properties of cutting edges related to chip formation in grinding. *Annals of the CIRP*, 24(1):231–235, 1975.
- [KL08] F. Klocke and B. Linke. Mechanisms in the generation of grinding wheel topography by dressing. *Production Engineering*, 2(2):157–163, 2008.
- [Klo09] Fritz Klocke. *Manufacturing Processes 2. Grinding, Honing, Lapping*. Springer-Verlag RWTH edition, 2009.
- [KM72] S. Kannappan and S. Malkin. Effects of grain size and operating parameters on the mechanics of grinding. *Journal of Manufacturing Science and Engineering*, 94(3):833–842, 1972.
- [KS81] K.V. Kumar and M.C. Shaw. The role of wheel-work deflection in grinding operations. *Journal of Manufacturing Science and Engineering*, 103(1):73–78, 1981.
- [KS82] L. Kops and M.C. Shaw. Thermal radiation in surface grinding. *CIRP Annals-Manufacturing Technology*, 31(1):211–214, 1982.
- [KV54] O. Kienzle and H. Victor. Einfluss der wärmebehandlung von stählen auf die hauptschnittkraft beim drehen. *Stahl und Eisen*, 74(9):530–551, 1954.
- [KY01] K. Komvopoulos and N. Ye. Three-dimensional contact analysis of elastic-plastic layered media with fractal surface topographies. *Journal of tribology*, 123(3):632–640, 2001.
- [LC94] J. Lemaitre and J.L. Chaboche. *Mechanics of solid materials*. Cambridge university press, 1994.
- [Lia95] T.W. Liao. Fractal and dds characterization of diamond wheel profiles. *Journal of materials processing technology*, 53(3):567–581, 1995.

- [Lin71] Richard Paul Lindsay. *On the metal removal and wheel removal parameters, surface finish, geometry and thermal damage in precision grinding*. PhD thesis, Worcester Polytechnic Institute, 1971.
- [Lor79] W. Lortz. A model of the cutting mechanism in grinding. *Wear*, 53(1):115–128, 1979.
- [LS75] G.K. Lal and M.C. Shaw. The role of grain tip radius in fine grinding. *Journal of Engineering for Industry*, 97(3):1119–1125, 1975.
- [LVLL06] A. Lefebvre, P. Vieville, P. Lipinski, and C. Lescalier. Numerical analysis of grinding temperature measurement by the foil/workpiece thermocouple method. *International Journal of Machine Tools and Manufacture*, 46(14):1716–1726, 2006.
- [LWBG13] Y. Liu, A. Warkentin, R. Bauer, and Y. Gong. Investigation of different grain shapes and dressing to predict surface roughness in grinding using kinematic simulations. *Precision Engineering*, 37(3):758–764, 2013.
- [LYZW15a] H. Li, T. Yu, L. Zhu, and W. Wang. Analysis of loads on grinding wheel binder in grinding process: insights from discontinuum-hypothesis-based grinding simulation. *The International Journal of Advanced Manufacturing Technology*, 78(9-12):1943–1960, 2015.
- [LYZW15b] H. Li, T. Yu, L. Zhu, and W. Wang. Modeling and simulation of grinding wheel by discrete element method and experimental validation. *The International Journal of Advanced Manufacturing Technology*, pages 1–18, 2015.
- [Mal08] Stephen Malkin. *Grinding technology*. Industrial Press, 2008.
- [Man83] Benoit B. Mandelbrot. *The fractal geometry of nature*, volume 173. Macmillan, 1983.
- [Mar77] M. Maris. *Thermische aspecten van de oppervlakteintegriteit bij het slijpen*. PhD thesis, Katholieke Universiteit te Leuven, Belgien, 1977.
- [Mas52] E.N. Masslow. *Grundlagen der Theorie des Metallschleifens*. Verlag Technik, Berlin, 1952.
- [MC71] S. Malkin and N.H. Cook. The Wear of Grinding Wheels. Part 2. Fracture Wear. *Journal of Engineering for Industry, Transactions of the ASME*, pages 1129–1133, 1971.
- [McA64] H.T. McAdams. A statistical polyhedron model of abrasive grains. *Journal of Engineering for Industry*, pages 75–81, February 1964.

- [Mer45] M. Eugene Merchant. Mechanics of the metal cutting process. ii. plasticity conditions in orthogonal cutting. *Journal of applied physics*, 16(6):318–324, 1945.
- [MGR09] L. Mahéo, V. Grolleau, and G. Rio. Damping efficiency of the tchamwa–wielgosz explicit dissipative scheme under instantaneous loading conditions. *Comptes Rendus Mécanique*, 337(11):722–732, 2009.
- [MI10] Z. Mahmood and K. Iwashita. Influence of inherent anisotropy on mechanical behavior of granular materials based on DEM simulations. *International journal for numerical and analytical methods in geomechanics*, 34(8):795–819, 2010.
- [Mon08] Douglas C. Montgomery. *Design and analysis of experiments*. John Wiley & Sons, 2008.
- [Mos80] Miklós Moser. *Microstructures of ceramics: structure and properties of grinding tools*. Akadémiai Kiadó, 1980.
- [MP88] J.J. Moreau and P.D. Panagiotopoulos. *Nonsmooth Mechanics and Its Applications*, volume 302. Springer Verlag, 1988.
- [MRDI04] I.D. Marinescu, W.B. Rowe, B. Dimitrov, and I. Inasaki. *Tribology of abrasive machining processes*. Elsevier, 2004.
- [MSKM15] A.P. Markopoulos, I.K. Savvopoulos, N.E. Karkalos, and D.E. Manolakos. Molecular dynamics modeling of a single diamond abrasive grain in grinding. *Frontiers of Mechanical Engineering*, 10(2):168–175, 2015.
- [MTOO89] T. Matsuo, S. Toyoura, E. Oshima, and Y. Ohbuchi. Effect of grain shape on cutting force in superabrasive single-grit tests. *CIRP Annals-Manufacturing Technology*, 38(1):323–326, 1989.
- [NB05] T.A. Nguyen and D.L. Butler. Simulation of precision grinding process, part 1: generation of the grinding wheel surface. *International Journal of Machine Tools and Manufacture*, 45(11):1321–1328, 2005.
- [NBS71] K. Nakayama, J. Brecker, and M.C. Shaw. Grinding wheel elasticity. *Journal of Manufacturing Science and Engineering*, 93(2):609–613, 1971.
- [OBF10] J.F.G. Oliveira, A.C. Bottene, and T.V. Franca. A novel dressing technique for texturing of ground surfaces. *CIRP Annals-Manufacturing Technology*, 59(1):361–364, 2010.
- [OC10] A. O Oluwajobi and X. Chen. The fundamentals of modelling abrasive machining using molecular dynamics. *International Journal of Abrasive Technology*, 3(4):354–381, 2010.



- [ÖC12] T.T. Öpöz and X. Chen. Experimental investigation of material removal mechanism in single grit grinding. *International Journal of Machine Tools and Manufacture*, 63:32–40, 2012.
- [Ono61] K. Ono. Analysis of the grinding force. *Bulletin of the Japan Society of Grinding Engineers*, 1:19–22, 1961.
- [OO05] Y. Ohbuchi and T. Obikawa. Adiabatic shear in chip formation with negative rake angle. *International journal of mechanical sciences*, 47(9):1377–1392, 2005.
- [OY89] P.L.B. Oxley and H.T. Young. The mechanics of machining: an analytical approach to assessing machinability. *Ellis Horwood Publisher*, pages 136–182, 1989.
- [PA54] G. Pahlitzsch and J. Appun. Effect of truing conditions on circular grinding. *Ind. Diamond Rev.*, 14:185–189, 1954.
- [Pek57] J. Peklenik. *Ermittlung von geometrischen und physikalischen kenngrößen für die grundlagenforschung des schleifens*. PhD thesis, TH Aachen, 1957.
- [Per66] P. Perzyna. Fundamental Problems in Viscoplasticity. *Advances in applied mechanics*, 9:243, 1966.
- [PHMR05] L. Pei, S. Hyun, J.F. Molinari, and M.O. Robbins. Finite element modeling of elasto-plastic contact between rough surfaces. *Journal of the Mechanics and Physics of Solids*, 53(11):2385–2409, 2005.
- [PL09] H.W. Park and S.Y. Liang. Force modeling of microscale grinding process incorporating thermal effects. *The International Journal of Advanced Manufacturing Technology*, 44(5-6):476–486, 2009.
- [Pom10] Iñigo Pombo. *Estudio teórico-experimental de una nueva tecnología de rectificado ecológico*. PhD thesis, University of the Basque Country UPV/EHU, 2010.
- [PSO<sup>+</sup>12] I. Pombo, J.A. Sánchez, N. Ortega, J.I. Marquinez, B. Izquierdo, and S. Plaza. Contact length estimation in grinding using thermocouple measurement and numerical simulation. *The International Journal of Advanced Manufacturing Technology*, 59(1-4):83–91, 2012.
- [PW72] J. Pullen and J.B.P. Williamson. On the plastic contact of rough surfaces. In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, volume 327, pages 159–173. The Royal Society, 1972.

- [QRM97a] H.S. Qi, W.B. Rowe, and B. Mills. Contact length in grinding: Part 1: Contact length measurement. *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, 211(1):67–76, 1997.
- [QRM97b] H.S. Qi, W.B. Rowe, and B. Mills. Experimental investigation of contact behaviour in grinding. *Tribology international*, 30(4):283–294, 1997.
- [QRM01] H.S. Qi, W.B. Rowe, and B. Mills. Fuzzy contact and its effect on thermal damage. *Key Engineering Materials*, 202:15–24, 2001.
- [Rab65] Ernest Rabinowicz. *Friction and wear of materials*. Wiley, New York, 1965.
- [RKM15] M. Rasim, F. Klocke, and P. Mattfeld. Energy model for grinding processes. In *Thermo-energetic Design of Machine Tools*, pages 35–47. Springer, 2015.
- [RMK15] M. Rasim, P. Mattfeld, and F. Klocke. Analysis of the grain shape influence on the chip formation in grinding. *Journal of Materials Processing Technology*, 226:60–68, 2015.
- [RMQZ93] W.B. Rowe, M.N. Morgan, H.S. Qi, and H.W. Zheng. The effect of deformation on the contact area in grinding. *CIRP Annals-Manufacturing Technology*, 42(1):409–412, 1993.
- [Row10] W. Brian Rowe. *Modern Grinding Techniques*. Wiley Online Library, 2010.
- [Rub72] C. Rubenstein. The mechanics of grinding. *International Journal of Machine Tool Design and Research*, 12(2):127–139, 1972.
- [Sai90] D.P. Saini. Wheel hardness and local elastic deflections in grinding. *International Journal of Machine Tools and Manufacture*, 30(4):637–649, 1990.
- [Sch00] John A. Schey. *Introduction to Manufacturing Processes*. McGraw Hill, 2000.
- [Sha71] M.C. Shaw. A new theory of grinding. In *Proceedings of International Conference on Science in Australia*, pages 1–16, Monash University, Australia, 1971.
- [Sha96] Milton Clayton Shaw. *Principles of abrasive processing*. Oxford University Press, 1996.
- [Smi86] R.L. Smith. The evaluation of NDT techniques for abrasive wheels. *British Journal of Non-Destructive Testing*, 28(2):73–79, 1986.

- [SS74] W.J. Sauer and M.C. Shaw. The role of elastic deflections of the wheel-work interface in surface grinding. In *proceedings of the International Conference on Production Engineering*, pages 645–649, 1974.
- [Stę09] P. Stępień. A probabilistic model of the grinding process. *Applied Mathematical Modelling*, 33(10):3863–3884, 2009.
- [STH83] E. Salje, H. Teiwes, and H. Heidenfelder. Important Results on External Cylindrical Plunge Grinding with Unusual Workpiece Peripheral Speeds and Speed Ratios  $q$  in the Range of 0.2 to 20 000. *CIRP Annals-Manufacturing Technology*, 32(1):241–246, 1983.
- [Tab59] D. Tabor. Junction growth in metallic friction: the role of combined stresses and surface contamination. In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, volume 251, pages 378–393. The Royal Society, 1959.
- [TB96] A.A. Torrance and T.R. Buckley. A slip-line field model of abrasive wear. *Wear*, 196(1):35–45, 1996.
- [TB00] A.A. Torrance and J.A. Badger. The relation between the traverse dressing of vitrified grinding wheels and their performance. *International Journal of Machine Tools and Manufacture*, 40(12):1787–1811, 2000.
- [TFX+15] L. Tian, Y. Fu, J. Xu, H. Li, and W. Ding. The influence of speed on material removal mechanism in high speed grinding with single grit. *International Journal of Machine Tools and Manufacture*, 89:192–201, 2015.
- [Tim53] Stephen Timoshenko. *History of strength of materials: with a brief account of the history of theory of elasticity and theory of structures*. Courier Corporation, 1953.
- [TPIP92] H.K. Tönshoff, J. Peters, I. Inasaki, and T. Paul. Modelling and simulation of grinding processes. *CIRP Annals-Manufacturing Technology*, 41(2):677–688, 1992.
- [USN96] T. Ueda, M. Sato, and K. Nakayama. Cooling characteristics of cutting grain in grinding. *CIRP Annals-Manufacturing Technology*, 45(1):293–298, 1996.
- [Ver77] J. Verkerk. Final report concerning CIRP cooperative work on the characterization of grinding wheel topography. *Annals of the CIRP*, 26(2):385–395, 1977.
- [Vic70] T.J. Vickerstaff. Diamond Dressing. Its Effect on Work Surface Roughness. *Industrial Diamond Review*, 30:260–267, 1970.

- [VP75] J. Verkerk and A.J. Pekelharing. The real contact length in cylindrical plunge grinding. *Annals of the CIRP*, 24(1):259–264, 1975.
- [Wer78] G Werner. Influence of work material on grinding forces. *Annals of the CIRP*, 27(1):243–248, 1978.
- [WG06] L. Wang and R.X. Gao. *Condition monitoring and control for intelligent manufacturing*. Springer Science & Business Media, 2006.
- [WX92] J.A. Williams and Y. Xie. The generation of wear surfaces by the interaction of parallel grooves. *Wear*, 155(2):363–379, 1992.
- [WZ98] G. Warnecke and U. Zitt. Kinematic simulation for analyzing and predicting high-performance grinding processes. *CIRP Annals-Manufacturing Technology*, 47(1):265–270, 1998.
- [WZ02] J.J. Junz Wang and C.M. Zheng. An analytical force model with shearing and ploughing mechanisms for end milling. *International Journal of Machine Tools and Manufacture*, 42(7):761–771, 2002.
- [YA84] M.A. Younis and H. Alawi. Probabilistic analysis of the surface grinding process. *Transactions of the Canadian Society for Mechanical Engineering*, 8(4):208–213, 1984.
- [YMLM11] T. Yamada, M.N. Morgan, H.S. Lee, and K. Miura. Calculation of the contact stiffness of grinding wheel. In *Advanced materials research*, volume 325, pages 54–59. Trans Tech Publ, 2011.
- [YRJZ11] L. Yan, Y.M. Rong, F. Jiang, and Z.X. Zhou. Three-dimension surface characterization of grinding wheel using white light interferometer. *The International Journal of Advanced Manufacturing Technology*, 55(1-4):133–141, 2011.
- [ZFG85] Z. Zhou, X. Fu, and D. Gu. Study on Maximum and Orbitry Contact Length in Surface Grinding. *Abrasive, Abrasive tool and Grinding (Chinese)*, 2, 1985.
- [ZGS<sup>+</sup>11] J.Q. Zhang, P. Guan, C. Su, T.B. Yu, and W.S. Wang. Simulation of grinding wheel with random three-dimensional abrasive and microporous bond. In *Key Engineering Materials*, volume 487, pages 209–214. Trans Tech Publ, 2011.
- [ZMC00] Y. Zhao, D.M. Maietta, and L. Chang. An asperity microcontact model incorporating the transition from elastic deformation to fully plastic flow. *Journal of Tribology*, 122(1):86–93, 2000.
- [ZSNW93a] L.C. Zhang, T. Suto, H. Noguchi, and T. Waida. Applied mechanics in grinding III. A new formula for contact length prediction and a comparison of available models. *International journal of machine tools and manufacture*, 33(4):587–597, 1993.

- [ZSNW93b] L.C. Zhang, T. Suto, H. Noguchi, and T. Waida. Applied mechanics in grinding part ii: modelling of elastic modulus of wheels and interface forces. *International Journal of Machine Tools and Manufacture*, 33(2):245–255, 1993.
- [ZV92] Z.X. Zhou and C.A. Van Lutterwelt. The real contact length between grinding wheel and workpiece - a new concept and a new measuring method. *CIRP Annals-Manufacturing Technology*, 41(1):387–391, 1992.
- [ZX02] X. Zhou and F. Xi. Modeling and predicting surface roughness of the grinding process. *International Journal of Machine Tools and Manufacture*, 42(8):969–977, 2002.
- [ZXF06] X.F. Zhang, H.J. Xu, and Y.C. Fu. Measurement technique of grinding wheel topography based on binocular stereo vision. In *Materials Science Forum*, volume 532, pages 1132–1135. Trans Tech Publ, 2006.
- [ZYF<sup>+</sup>15] X.L. Zhang, B. Yao, W. Feng, Z.H. Shen, and M.M. Wang. Modeling of a virtual grinding wheel based on random distribution of multi-grains and simulation of machine-process interaction. *Journal of Zhejiang University Science*, 1, 2015.