EEG and MEG Evidence of a Predominant Number Code in Bilinguals and its Significance for Developmental Dyscalculia

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Resumen amplio en castellano

Los números son uno de los pilares en nuestra sociedad actual y los encontramos en el día a día, por ejemplo, en canales de televisión, fechas, señales, números de teléfono, etc. Además, usamos los números para cuantificar, ordenar, clasificar y medir prácticamente todo: velocidades, tiempo, estaturas, propinas, etc. Un sistema numérico defectuoso tiene consecuencias tanto en la infancia como en la vida adulta.

La presente tesis se centra en el estudio del funcionamiento numérico en bilingües. Cómo procesan las matemáticas las personas bilingües es una cuestión de interés actual y que tiene muchas incógnitas por resolver, como por ejemplo qué papel desempeña realmente el lenguaje utilizado para la adquisición temprana de las matemáticas.

El sistema cerebral encargado del funcionamiento numérico ha atraído la curiosidad en neurociencia durante mucho tiempo y numerosos estudios han buscado respuestas a un sinfín de cuestiones relacionadas con los números, como las representaciones más simples de magnitud en nuestro cerebro (Carey, 2001; Nieder y Dehaene, 2009) o el procesamiento de la aritmética en bilingües (Spelke y Tsivkin, 2001; Salillas y Wicha, 2012). Las particularidades del sistema numérico en bilingües son objeto de estudio per se, pero, además, dicho estudio puede ayudar a resolver muchas dudas sobre los lazos entre las matemáticas y el lenguaje en general. De hecho, hay estudios que han demostrado que los bilingües ya muestran una clara preferencia por la lengua en la que se han adquirido las matemáticas, no sólo en aritmética (Salillas y Wicha, 2012) sino también en procesamientos más simples y desvinculados del lenguaje, como la simple comparación entre numerosidades (Salillas y Carreiras, 2014).

El objetivo de esta tesis es comprobar los patrones de dominancia de los dos códigos que usan los bilingües para el procesamiento matemático. En otras palabras, queremos estudiar si hay una lengua que es dominante para las matemáticas y qué determina tal dominancia. Nuestra hipótesis es que una de ellas, aquella utilizada durante el aprendizaje numérico temprano, es la que establece el código verbal preferido para las matemáticas en bilingües, incluso si estos bilingües son equilibrados y fluidos en ambas leguas. A esta lengua, o medio verbal preferido, la llamamos *lengua del aprendizaje de las matemáticas* (LL^{math}) en comparación con *la otra lengua* (OL).

Basándonos en estudios previos (Salillas, Barraza y Carreiras, 2014; Salillas y Carreiras, 2015, Salillas y Wicha, 2012) aquí nos interesa estudiar el peso de la representación léxica más básica en cada lengua. Es decir, si es el caso que, por ejemplo, en bilingües perfectamente fluidos en ambas lenguas, la representación de *uno* en castellano, difiera de la representación en euskera *bat*. En principio tal desequilibrio es planteado como independiente de representaciones equilibradas para palabras no numéricas como *mar* vs. *itsaso*.

Tal desequilibrio en la representación de las palabras número es fácilmente observable atendiendo a lo que se conoce como coste de cambio entre lenguas (*switch cost* en inglés). Es sabido que el desequilibrio entre lenguas genera un coste asimétrico cuando cambiamos de una lengua a otra, este desequilibrio depende de la dirección del cambio: es más costoso ir de la lengua menos dominante a la más dominante (Green, 1992). Por lo tanto, si tal asimetría ocurre para palabras número en bilingües fluidos y ese desequilibrio está originado en la LL^{math}, podremos mostrar que es el aprendizaje temprano lo que determina la dominancia lingüística para el material numérico. De manera crucial, ese desequilibrio entre palabras-número, será independiente de un uso fluido de ambas lenguas durante la comunicación de material no numérico. La presente tesis utiliza por tanto poblaciones bilingües equilibradas euskera-castellano en distintas tareas y además gracias al uso de técnicas de Neuroimagen - electroencefalograma (EEG), magnetoencefalograma (MEG) - observaremos la base neuronal de las representaciones numéricas bilingües. Finalmente, se estudian las implicaciones que tales patrones de dominancia numérica tienen cuando el sistema numérico falla en la discalculia de desarrollo, en población bilingüe.

Experimentos

Experimento 1

El **Experimento 1** investiga la dominancia en los códigos para las matemáticas. La hipótesis de este experimento es que en bilingües equilibrados en su primera lengua y en su segunda lengua (L1/L2) el factor determinante para la preferencia de un código será la experiencia de aprendizaje, es decir la LL^{math}. Considerando estudios previos de Potenciales Evocados con una tarea de cambio de lenguas (cambio o *switch*) como Chauncey, Holcomb y Graigner (2008) y en estudios conductuales (Costa y Santesteban, 2004), la predicción es que el cambio entre los códigos numéricos LL^{math} y OL generará un cambio asimétrico en el componente N400, reflejando un coste mayor al pasar desde la OL a la LL^{math}, siendo esta asimetría en el cambio de códigos independiente de la dominancia L1/L2.

Métodos

Doce bilingües equilibrados en euskera-castellano participaron en este estudio. Todos ellos aprendieron ambas lenguas antes de los tres años de edad. De estos 12 participantes, seis aprendieron las matemáticas en euskera y seis en castellano. Todos los participantes fueron evaluados en la dominancia de ambas lenguas con una adaptación al castellano y euskera del Boston Naming Test (Kaplan, 1983), y a través de una entrevista oral.

La tarea consistía en clasificar los números en formato verbal que aparecen en pantalla como mayor o menor de "*6*". El rango de números variaba de 3 a 9 y los estímulos fueron presentados en orden aleatorio, pero siempre siguiendo el orden AABB, dónde "A" y "B" son las lenguas en las que los números se presentaban. Se crearon un total de 480 ensayos de los cuales la mitad eran ensayos en castellano y la otra mitad en euskera, controlando la distancia entre los números presentados y el número referencia (6) y que todos los números aparecían el mismo número de veces en ambas lenguas. Los participantes realizaron la tarea en una cabina insonorizada y los estímulos se presentaban en una pantalla de ordenador con fondo negro. Cada estímulo aparecía en pantalla durante 500 ms y entre estímulos la pantalla estaba en negro durante un tiempo aleatorio entre 1.500 y 2.000 ms. Los datos fueron recogidos usando un sistema de EEG de 27 canales más un electrodo de referencia en el mastoides izquierdo. Otros 6 electrodos se usaron para grabar los pestañeos y movimientos oculares. Estos últimos electrodos se colocaron encima y debajo del ojo izquierdo y en las sienes. Tras el preprocesamiento de la señal, el EEG se segmentó en base a la presentación de las palabras número y promedió para calcular los potenciales evocados para cada condición experimental.

Había dos condiciones de cambio (AB y BA) y dos condiciones de no cambio (AA y BB) que eran predecibles por los participantes. Un total de 480 ensayos se presentaron para cada participante en dos formatos: una mitad (240) en castellano (p.ej. *tres*) y la otra mitad en euskera (p.ej. *hiru*). Del total de los ensayos, 120 eran palabras número en euskera precedidas por palabras número en castellano, 120 eran palabras número en castellano precedidas por palabras número en euskera, 120 palabras número en castellano precedidas por palabras número en castellano y 120 palabras número en euskera precedidas por palabras número en euskera. La distancia entre los números presentados y el número de referencia (6) estaba controlada (había el mismo número de ensayos para las distancias 1, 2 y 3), y las palabras número eran las mismas en ambas lenguas.

Resultados y conclusiones

El objetivo de este estudio era comprobar si las representaciones numéricas dependen de la dominancia del lenguaje en general o por el contrario dependen de la dicotomía LL^{math}/OL. Esto se observó a través del cambio entre los códigos verbales numéricos (palabras número). Los resultados muestran una asimetría en el coste del cambio de códigos cuando se usan los códigos numéricos, produciéndose un coste mayor de cambio de lenguas cuando se cambia en la dirección OL-LL^{math} en comparación con la dirección contraria (LL^{math}-OL). Ello se observó en la amplitud del componente N400, el cual tiene una amplitud mayor para el cambio en la dirección hacia la LL^{math} que en la dirección a la OL. Esta asimetría es independiente de la dominancia L1/L2, ya que cuando se realizaron los análisis agrupando los ítems en base a la dominancia general del lenguaje (L1/L2) los cambios entre una lengua y otra eran similares en ambas direcciones (L1-L2 y L2-L1).

A partir de estos resultados, se concluye que la dominancia relativa de los códigos numéricos es independiente de la dominancia de los códigos generales del lenguaje. Por lo tanto, la distinción a tener en cuenta cuando se construyan modelos de cognición numérica ha de ser la distinción LL^{math}/OL y no L1/L2.

Experimento 2

El **Experimento 2** tiene los mismos objetivos y diseño que el Experimento 1, sin embargo, el procedimiento es diferente: en lugar de emplear una manipulación explícita de la información cuantitativa a través de la comparación numérica se utilizó una tarea de paridad, es decir, clasificar los números que aparecen en la pantalla como par o impar. Además, en lugar de usar una secuencia predecible de presentación del tipo AABB, se usó una presentación de estímulos aleatoria. El procedimiento de presentación era un primado enmascarado en el cual los participantes ven una máscara formada por ocho almohadillas ("########"), que fue posteriormente sustituida por el estímulo de primado durante un período de tiempo casi imperceptible (40 ms) pero capaz de ser procesado inconscientemente, este primado era inmediatamente seguido del estímulo que los participantes tenían que clasificar como par o impar. Los estímulos que forman el primado podían ser en la misma lengua, o diferente, del estímulo que tenían que clasificar. Los estímulos a clasificar podían ser en euskera o castellano, creando así cuatro condiciones diferentes al igual que en el Experimento 1: dos condiciones de cambio (LL^{math} a OL y OL a LL^{math}) y dos condiciones en las que los estímulos del primado y el estímulo a clasificar estaban en la misma lengua, es decir dos condiciones de no cambio (LL^{math}-LL^{math} y OL-OL). Se crearon un total de 504 ensayos, la mitad (252) eran en euskera y la otra mitad en castellano. Estos a su vez fueron divididos en ensayos de cambio (126 ensayos de cambio castellano-euskera y 126 ensayos de cambio euskera-castellano) y ensayos de no-cambio (126 ensayos de no-cambio euskera-euskera y 126 ensayos de no cambio castellano-castellano).

Estos cambios en el diseño se introdujeron con el fin de averiguar si los costes asimétricos de cambio de código encontrados en el primer experimento eran debidos a la tarea, o por el contrario se deben a la dominancia LL^{math}/OL. Las predicciones son las mismas que en el Experimento 1: habrá un cambio asimétrico que dependerá de la dominancia LL^{math}/OL.

Métodos

14 participantes tomaron parte en este estudio, todos ellos bilingües equilibrados en euskera-castellano, siete de ellos aprendieron matemáticas en euskera y siete las aprendieron en castellano. A todos ellos se les aplicaron las mismas medidas de evaluación en ambas lenguas que en el Experimento 1. Los métodos de recogida de datos fueron los mismos que en el Experimento 1.

Resultados y conclusiones

Los resultados del Experimento 2 corroboran los del primer experimento ya que una vez más se encontró un coste asimétrico de cambio entre lenguas, reflejado en el componente N400: el cambio hacia el código más dominante (OL-LL^{math}) muestra una mayor amplitud de onda. Lo más llamativo de este experimento es que los participantes no eran conscientes del cambio y, además, no se manipulaba la información sobre magnitud numérica de una manera explícita (es decir, los juicios de par/impar no implican atención de si un número es mayor o menor). Este cambio inconsciente fue diseñado a propósito para saber si los resultados del Experimento 1 eran producidos a nivel léxico o por el contrario eran debidos a la tarea y viendo los resultados se puede interpretar que los cambios asimétricos ocurren a nivel léxico.

Experimento 3

El **Experimento 3** fue diseñado con el objetivo de averiguar que redes neuronales son las responsables de los cambios entre los dos códigos para las matemáticas. En los Experimentos 1 y 2 se encontraron cambios de código asimétricos similares a los que se encuentran en el lenguaje en bilingües no equilibrados (Chauncey et al., 2008; Jackson, Swainson, Cunnington y Jackson, 2001). Así que el siguiente paso lógico era descubrir si las redes neuronales subyacentes a los cambios de los códigos para las matemáticas y para el lenguaje en general son los mismos. Estudios que examinan este tipo de cambios muestran que las regiones implicadas son la corteza prefrontal dorsolateral, regiones anteriores como las áreas de Brodmann 45 y 9 y el córtex cingulado anterior (Abutalebi et al., 2012; Hernández, Dapretto, Mazziotta, y Bookheimer, 2001; Rodriguez-Fornells, De Diego Balaguer, y Münte, 2006; Wang, Xue, Chen, Xue y Dong, 2007).

Métodos

12 participantes realizaron la tarea utilizando la técnica de magnetoencefalografía (MEG). Al igual que en los Experimentos 1 y 2, los participantes eran bilingües equilibrados euskera-castellano, de los cuales la mitad habían aprendido las matemáticas en euskera y la otra mitad en castellano. La MEG tiene una mejor resolución espacio temporal y un mayor número de sensores que el EEG, gracias a ello permite una mejor estimación de fuentes en combinación con una imagen de resonancia magnética del cerebro de cada participante.

Resultados y conclusiones

Los resultados de los análisis muestran otra vez un coste de cambio asimétrico en los códigos para las matemáticas en la N400, con mayor coste para la dirección OL-LL^{math}. Los campos evocados (ERFs) localizan el coste de cambio en las regiones frontales del cerebro. La estimación de fuentes para esos ERFs determinó la implicación de regiones frontales del cerebro (Área de Brodmann 9, Cingulado Anterior y Corteza Dorsolateral Prefrontal). Estos resultados son similares a los encontrados en otros experimentos que observan el cambio de lengua. Pero además, según diversos estudios éstas regiones activas en el cambio de lenguas también se activan ante un cambio de tarea en general (Abutalebi y Green, 2007; Craik y Bialystok, 2006; Garbin et al., 2010; Luk, Green, Abutalebi, y Grady, 2012). Esto implica que el control cognitivo que se ejerce durante los cambios entre lenguas y los cambios entre palabras número puede formar parte de un mismo sistema ejecutivo general. Sin embargo, el origen de la activación de tal sistema de control común proviene de sistemas léxicos con dominancia independiente, para las matemáticas (palabras número) y para el lenguaje en general.

Experimento 4

El **Experimento 4** trata de profundizar en el conocimiento del papel de la lengua de aprendizaje temprano en el procesamiento matemático cuando el sistema numérico falla. En este experimento se investigó el trastorno de la discalculia en bilingües, un síndrome que afecta a un 6% de la población y que implica más lentitud en el aprendizaje de las matemáticas y su procesamiento con un uso inmaduro del cálculo como puede ser el conteo con los dedos. Hoy en día es muy común el estudio de estos trastornos, sin embargo, a pesar de estar en un mundo multilingüe este trabajo es el único que, hasta el momento, ha estudiado esta condición en poblaciones bilingües y multilingües. El estudio de este síndrome en poblaciones bilingües nos puede proporcionar información muy valiosa de los lazos del lenguaje con las matemáticas. En este estudio se evalúan las posibles interacciones entre los efectos distancia y la lengua con la que se operan estos efectos. Los efectos de distancia implican que las distancias entre números se procesan más rápido y más precisamente cuando la distancia que separa dos números es más pequeña, a medida que la distancia aumenta, el procesamiento se hace más lento y menos preciso. Este efecto distancia se ha encontrado en estudios conductuales y en ciertos componentes de ERPs en torno a los 200 ms y se toma como un índice de acceso a la representación numérica más básica y esencial. Por lo tanto, en este experimento se exploran las posibles interacciones entre los efectos de distancia y el lenguaje, es decir, si los efectos de distancia ocurren en cada lengua y si esos efectos de distancia ocurren en las mismas regiones cerebrales para cada lengua. Además, se evalúa si hay diferencias en estos procesos entre personas con discalculia que son bilingües y sus controles equivalentes, todo ello con análisis de EEG y estimación de fuentes.

Métodos

En este estudio participaron un total de 14 niños bilingües euskera-castellano con edades comprendidas entre los 8 y los 13 años. La mitad de los niños que participaron en este estudio (un total de siete) habían sido diagnosticados previamente con discalculia y se han evaluado usando el programa Dyscalculia Screener (Butterworth, 2003). Los otros siete niños forman el grupo control equivalente en sexo y edad con el primer grupo. Aunque ambos grupos demostraron una mejor eficiencia en castellano en tareas del lenguaje general, la lengua de aprendizaje de las matemáticas para todos ellos era euskera.

Para la tarea se creó un paradigma de adaptación con seis palabras número en castellano y seis en euskera correspondientes con los dígitos 1, 2, 3, 7, 8 y 9. Se crearon ocho listas repetidas ocho veces en cada lengua. Cuatro listas usaban los estímulos 1 y 2 como adaptación y 3 y 7 como "desvíos", y otras cuatro listas usaban los estímulos 8 y 9 como adaptación y 3 y 7 como "desvíos". Cada estímulo se presentaba durante 200 ms y había un intervalo entre estímulos de 1000 ms.

Los datos de EEG fueron grabados y preprocesados usando los mismos procedimientos que en los Experimentos 1 y 2.

Resultados y conclusiones

Los resultados mostraron que los efectos distancia (es decir, el acceso al código nuclear numérico) eran diferentes para ambos grupos, y que tales efectos de distancia dependían de la lengua del estímulo. El grupo control mostró efectos de distancia en ambas lenguas en el componente N2, mientras que el grupo con discalculia solo mostró efectos de distancia para la LL^{math}, también en la N2. En la estimación de fuentes el grupo control mostró una mayor activación de las regiones parietales del hemisferio derecho para la LL^{math} mientras que el grupo discalcúlico las mostró en el izquierdo para ese mismo código, además este último grupo utiliza una red en el hemisferio izquierdo que incluye áreas frontales, perisilvianas e inferior-parietal que no utilizaban los controles. Para la OL solamente el grupo control mostró efectos de distancia en el hemisferio derecho con una red neuronal que consta de áreas frontales en inferioparietales lo que sugiere un procesamiento de la magnitud menos automatizado.

Conclusiones generales de la tesis

El objetivo de esta tesis era explorar los vínculos entre el lenguaje y las matemáticas a través del bilingüismo. Esta investigación ha mostrado efectos convergentes utilizando distintas técnicas de neuroimagen (EEG, MEG) y distintos paradigmas experimentales. Se han observado los efectos de dominancia de la LL^{math} teniendo en cuenta las dos lenguas de los bilingües y se han investigado los patrones de cambio de lenguas usando tareas clásicas con estímulos numéricos en lugar de los estímulos tradicionales. Además, se ha investigado el acceso a la magnitud en bilingües con el síndrome de la discalculia para saber más sobre la preferencia de un código para el acceso a la magnitud, así como para conocer las implicaciones de ser discalcúlico bilingüe.

Teniendo en cuenta todos los resultados de esta tesis, se ha demostrado que en la población estudiada los cambios entre los dos códigos para las matemáticas son asimétricos y que, por lo tanto, un bilingüe equilibrado puede no serlo ante una entrada numérica. En resumen, la dominancia para las matemáticas está modulada por la LL^{math} y no por la L1. Por otro lado, el control cognitivo sobre las palabras número parece asimilarse al aplicado sobre el lenguaje natural y sobre la ejecución de tareas no lingüísticas. Además, se ha demostrado que la LL^{math} es el código dominante para las matemáticas en su representación más básica: en la discalculia bilingüe la LL^{math} es el único código que permite un acceso al código nuclear numérico (efecto de distancia) y dicho acceso difiere en su base neural también en población sana.

Como posibles investigaciones futuras en el campo de cómo los bilingües procesan los dos códigos para las matemáticas podrían enfocarse en las redes neuronales se utilizan específicamente para la LL^{math} y para la OL en el procesamiento de la magnitud en tareas no numéricas, por ejemplo, para comparar tamaños de objetos. Ya que el procesamiento de la magnitud tiene áreas comunes (Sokolowski, Fias, Mousa y Ansari, 2016), sería apropiado entender hasta qué punto la LL^{math} modula el acceso a la magnitud en tareas que no usen números en sus formas verbales en estas regiones comunes del procesamiento de la magnitud.

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1. The Numerical Representation Framework

Numbers are one of the pillars in today's society; they can be found in various aspects in life, apart from the obvious counting and arithmetic, such as TV stations, dates, signals, our favorite player's number, etc. Numbers are also used to label, rank, order, quantify and measure almost everything. We can calculate speed of trains, the most complex equations or even simple everyday tasks such as the percentage of a tip in a restaurant.

Over the last decades research into cognitive number processing has made considerable progress and one of the central questions has been how basic numerical knowledge is represented in our minds (Carey, 2001; Dehaene, 1999b; Dehaene, Piazza, Pinel, and Cohen, 2003; Nieder and Dehaene, 2009; Spelke, 2000; Whalen, Gallistel, and Gelman, 1999). It is believed that humans have an innate system for number representation (Dehaene, 1997) also present in human infants and animals. This system is considered as an abstract, non-verbal representation of magnitude, and mostly, independent of language. Although the representation of exact magnitudes can be shaped by other numerical information acquired during the early school years, it needs to be learned, since it is not part of the preverbal human core quantity representation. Moreover, magnitude information is encoded or mapped in various symbolic notations (e.g. Arabic, number words) allowing a more precise manipulation of quantity and having an effect on individuals' abilities to compare and represent certain magnitudes (Gilmore, McCarthy, and Spelke, 2010; Holloway and Ansari, 2008; Moyer and Landauer, 1967; Temple and Posner, 1998). There is enough evidence proving that children and adults can manipulate numerical information without symbols (Barth, La Mont, Lipton, and Spelke, 2005; Pica, Lemer, Izard, and Dehaene, 2004; Whalen et al., 1999) and the use of these numerical symbols serves to increase the precision of magnitude representation (Holloway and Ansari, 2008). In turn, the general view is that there is a pre-existing intuitive system, a core quantity knowledge, that forms the basis for the development of symbolic representation. In order to have a better understanding of the general framework of mathematical cognition, the most basic concepts and most important models will be described in the numerical formats section.

1.1 An innate system for magnitude

The ability to estimate magnitudes is based on an *Approximate Number System* (ANS). This system is essential for the development of numerical skills and it is present in human and non-human species (Butterworth, 2010; Cantlon, Platt, and Brannon, 2009; Nieder and Dehaene, 2009). Comparative studies have shown that a variety of non-verbal animals and human infants are able to detect the approximate difference in magnitude between two sets and perform elementary calculations (Agrillo, Piffer, Bisazza, and Butterworth, 2012; Dehaene, 1999a; Flombaum, Junge, and Hauser, 2005; Rumbaugh, Savage-Rumbaugh, and Hegel, 1987; Woodruff and Premack, 1981). The ANS plays a crucial role in the human capacity for estimating and comparing approximate numerosities during the course of numerical knowledge development (Feigenson and Halberda, 2004; Gallistel and Gelman, 2000). Nevertheless, this system is also involved in more complex numerical knowledge, including arithmetic (Butterworth, 2010; Gilmore, McCarthy, and Spelke, 2007; Gilmore et al., 2010). The ANS has been assessed in several studies using non-symbolic number comparison tasks (e.g. the identification of the larger amount between two arrays of dots or objects). Studies suggest that infants and humans, and even animals, have a domain-specific representation of number and elementary arithmetic operations (Feigenson and Halberda, 2004; Starkey and Cooper, 1980). Behavioral studies show number perception and discrimination abilities in the non-verbal stages of infants, and in animals that also lack of verbal communication. This type of performance can be found in animals without training in numerical tasks and in inexperienced rhesus macaques (Flombaum et al., 2005). Hauser et al., (2003) further suggested that monkeys detect violations in operations of small sets of items, but only when the ratio between the observed and the expected outcome is favorable. On the other hand, single-cell recording studies have shown number-selective neurons in monkeys' prefrontal and parietal areas, comparable to the prefrontal and parietal number-sensitive areas in the human brain (Nieder and Dehaene, 2009). Sensitivity of infants towards quantity happens at a very early age (Libertus and Brannon, 2009). They can discriminate between groups of objects when the quantities involved are small (1, 2 or 3 items; Antell and Keating, 1983; Starkey and Cooper, 1980; Strauss and Curtis, 1981). Lipton and Spelke (2003) and Xu and Spelke (2000) showed that 6-month-old infants can also discriminate between large sets of objects such as arrays of 8 dots or from an array of 16, but fail to discriminate 8 from 12. At 9 months, infants are able to discriminate 8 from 12 but fail to discriminate 8 from 10 (Lipton and Spelke, 2003). Infants can also engage rudimentary arithmetic; Wynn, (1992) showed five-month-old participants additions and subtractions on small sets of objects. Thus, sensitivity of infants towards quantity happens at a very early age (Libertus and Brannon, 2009).

This sense of approximate magnitude is based on *Webber's law.* It states that the discriminability between two numerosities varies as a function of the ratio between them. In other words, for equal numerical distances, discrimination of two numbers worsens as their numerical size increases. Proof of this is the existence of distance and numerical size effects (Moyer and Landauer, 1967). These two effects are not only present in humans but also in animals and they index the activation of the analogue magnitude system (Dehaene, 1992, 1999b; Gallistel and Gelman, 1992). Moyer and Landauer (1967) were the first to demonstrate the distance effect with a numerical comparison task. They proved that the accuracy of magnitude discrimination was influenced by both the linear distance and absolute magnitude value. For example, we know that discriminating the relative magnitude of 1 versus 8 is faster than for 1 versus 3; or that discriminating the relative magnitude of 1 versus 9 is faster than for 11 versus 19. Thus, the speed of processing also decreases proportionally to the number size being represented (number size effect). In general terms, Weber's law is taken as a key characteristic of core numerical knowledge (Dehaene, 1992, 1999b; Gallistel and Gelman, 1992).

This core numerical system is included in all models of numerical cognition: *The Triple Code model* (Dehaene and Cohen, 1995; Lemer, Dehaene, Spelke, and Cohen, 2003) proposes this core numerical representation to be an analogue magnitude or quantity code accessible from all formats of numbers; also McCloskey´s Abstract Code Model proposes an abstract semantic around which a whole model is built, similar to what Campbell and Clark (1992; 1998) refers as "*Analogue Magnitude Representation*".

1.2. Numerical formats

There are different notations or formats through which magnitudes can be accessed. Number symbols are generally acquired in at least two forms: Arabic numerals (1, 2, 3...) and number words (one, two, three…). Contrary to core magnitude knowledge, symbolic numerical codes are culturally established (Ansari, 2008) and therefore they are learned and retrieved from long-term memory. In turn, the phonological code for the Arabic digits is not specified, mainly because the relation between the visual symbol representing the magnitude and its specific verbal format is arbitrary. The influence that number codes have in magnitude processing has been of special interest over the past years (Cohen Kadosh, Lammertyn, and Izard, 2008; Damian, 2004; Dehaene et al., 1997; Kadosh and Walsh, 2009; Nathan and Algom, 2008; Noël et al., 1997; Seron and Noel, 1995). The general assumption is that number representation can be directly influenced by number codes and that number codes affect the manipulation of number representation (Noël and Seron, 1997; Zhang and Norman, 1995). Hence, a debate has been established in regard to the existence of one unique numerical representation, independent from the surface code (Dehaene and Cohen, 1995) or instead, notation specific representations (Cohen Kadosh, Cohen Kadosh, Kaas, Henik, and Goebel, 2007). In fact, all the proposed models for the architecture of the numerical system try to provide a comprehensive account of the numerical representation system based on different numerical formats and modalities.

The **Abstract-Code Model** (McCloskey, 1992; McCloskey, Caramazza, and Basili, 1985) proposes a modular architecture of the numerical representation system based on three modules (comprehension, production and calculation). Each module is connected to a central magnitude code defined as both abstract and amodal (see Figure 1). The model's conception is that there is an abstract semantic representation level on which arithmetic fact retrieval from long-term memory is built. In addition, arithmetic facts retrieval is independent from any input format (Arabic, verbal) and processed through an abstract semantic representation system (e.g. 2x5 and *two times five* are processed in the same central system). The abstract semantic module constitutes a compulsory filter engaged in all number processing operations and calculation mechanisms. With regard to number codes, this model implies that core numerical processing is achieved independent on the surface code once this code has been transcoded to the abstract semantic representation. Thus, no interactions between numerical formats and core magnitude processing are proposed.

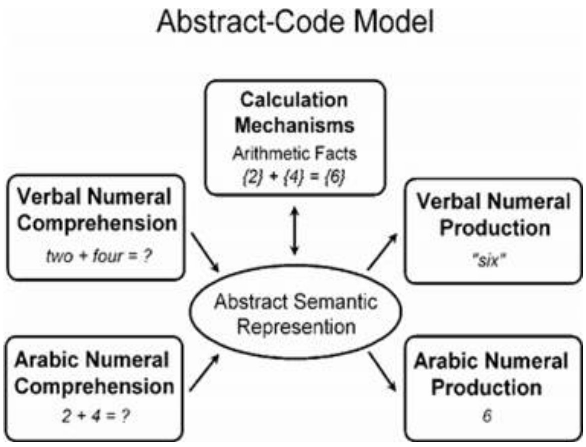

Figure 1. Illustration of the Abstract Code Model (McCloskey, 1992; McCloskey et al., 1985)

In contrast, the **Encoding-Complex Hypothesis** by Campbell and Clark (Campbell and Clark, 1988, 1992) is based on the assumption that multiple formats are connected in a sort of encoding-decoding network of numerical computations. The strengths and weaknesses of those connections will directly depend on both the task and individual peculiarities mediated by learning experiences. The core of the Encoding-Complex Model resides in the number of effective networks available to operate efficiently at three levels: comprehension, production and arithmetic-fact retrieval, which is related to practice and familiarity with the specific formats (Campbell and Epp, 2004; Campbell and Xue, 2001). To exemplify how the model functions, the authors propose that whenever a numeral is presented, different formats of representations are activated at the same time as an "associative network". All the information associated with the format in the specific task, becomes more or less active depending on an inhibitory and excitatory mechanism. For example, during Arabic-to-verbal transcoding the model allows a direct association between the two codes (i.e. reading aloud implies transcoding from the visual- Arabic code to the verbal format) combined with indirect associative connections (e.g. mental number lines, visual-motor procedures) (Campbell and Clark, 1992).

Another model attempting to explain the functioning of the number representation system is the **Triple-Code Model** (Dehaene and Cohen, 1995; Dehaene et al., 2003). This model postulates that numerical information is based on three representational systems: visual-Arabic, verbal-auditory and analogue quantity or magnitude code. These three distinct systems exchange information during numerical operations and, depending on the task, will recruit one specific code (see Figure 2). The model's prediction is that some operations (e.g. arithmetic facts) are learned and stored verbally, when problems are presentenced in an Arabic code (e.g. 3x5) they are converted into a verbal format (e.g. three by five) and therefore retrieved through a verbal route. On the other hand, the analogue magnitude representation is independent from surface codes and accounts for the semantic of numbers. In this representation, access to quantity would not be mediated by any format since the model assumes that only the analogue code has the inherent meaning of magnitude. Thus, as in the Abstract Code Model, any effect of the surface code should be additive.

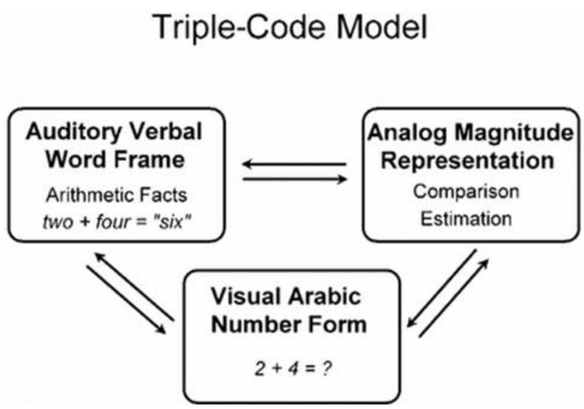

Figure 2. Illustration of the Triple Code Model (Dehaene and Cohen, 1995; Dehaene et al., 2003).

1.3. Neuroimaging Evidence

One of the Triple Code Model strengths is its explicit neuroanatomical proposal (Dehaene, Piazza, Pinel and Cohen, 2003), by which there are three neuronal circuitries in the parietal lobes to account for each of the three proposed numerical systems. Based on meta-analytic neuroimaging data, this model predicts selective neural activation depending on the type of numerical information to be processed. Specifically, these authors (Dehaene, 2009; Dehaene et al., 2003; Nieder and Dehaene, 2009) proposed a neuroanatomical version of their Tripe Code Model with three parietal circuits for number processing: the Intraparietal Sulcus (IPS), the Angular Gyrus and the Posterior Superior Parietal System. The IPS would process core magnitude representations. The left Angular Gyrus would support the manipulation of numbers in verbal form; and the Posterior Superior Parietal System would support attentional orientation of the spatial dimension of numbers (i.e. the Mental Number Line).

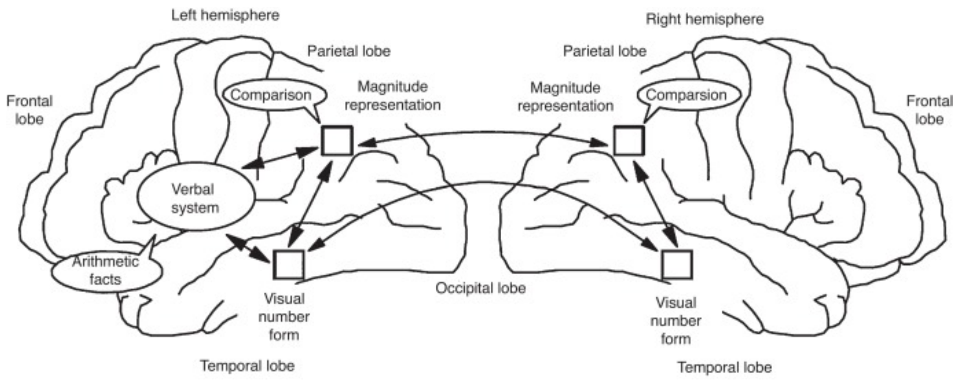

Figure 3. Schematic Functional and Anatomical Architecture of the Triple-code Model (Dehaene and Cohen,1995). The localization of the main areas thought to be involved in the three numerical codes is depicted on a lateral view of the left and right hemispheres. The arrows indicate a functional transmission of information across numerical codes.

Several studies have explored the representation of numbers as a function of the different notations in the IPS using functional Magnetic Resonance Imaging (fMRI). The common finding among these studies was that the IPS was activated during numerical processing independently of numerical notation (Chochon, Cohen, van de Moortele, and Dehaene, 1999; Dehaene, 1996; Dehaene, Spelke, Pinel, Stanescu, and Tsivkin, 1999; Piazza, Mechelli, Butterworth, and Price, 2002; Pinel, Dehaene, Rivière, and LeBihan, 2001). Therefore, the main hypothesis is that the representation of numbers in an abstract fashion engages the IPS, which is considered the neural substrate for core numerical representation (Bhatia and Ritchie, 2012; Cantlon, Libertus, et al., 2009; Dehaene, Dehaene-Lambertz, and Cohen, 1998; Dehaene et al., 1999; Eger, Sterzer, Russ, Giraud, and Kleinschmidt, 2003; Kadosh and Walsh, 2009; Nieder and Dehaene, 2009; Piazza, Pinel, Le Bihan, and Dehaene, 2007; Pinel et al., 2001; Rosenberg-Lee, Tsang, and Menon, 2009; Venkatraman, Ansari, and Chee, 2005). However, the notion that the IPS is the key region for number processing has been challenged. Many neuroimaging studies reported activation in regions of the frontal cortex during number processing (Cohen Kadosh, Cohen Kadosh, Kaas, et al., 2007; Dormal, Dormal, Joassin, and Pesenti, 2012; Eger et al., 2003; Franklin and Jonides, 2009; Hayashi et al., 2013; Kadosh and Walsh, 2009). It is proposed that although there might be an abstract representation system of numerical magnitude in the brain, it is not only limited to the IPS, but includes regions across the parietal cortex that are engaged in number processing and frontal regions which are activated dependently of the format being processed (see Sokolowski, Fias, Mousa, and Ansari, 2016) .

There are other studies that show another active circuitry of the parietal cortex affected during mathematical operations, the left Angular Gyrus. These networks show more activation during the performance of exact arithmetic such as multiplications or additions, those sorts of operations in which the verbal format is involved and needed for arithmetic fact retrieval (Dehaene, 1992; K. M. Lee, 2000). On this subject, those studies comparing different notational effects (e.g. verbal versus Arabic) while performing simple arithmetic show that difficulty increases with the verbal format condition (Campbell and Alberts, 2009; Cohen Kadosh, Henik, and Rubinsten, 2008; Damian, 2004; Noel et al., 1998). Other neuroimaging techniques such as Event-related Potentials (ERP's) have also shown parietal activation during performance of simple arithmetic in different formats and modalities (Dehaene, 1996, 1997).

Apart from the areas mentioned before, there are classical language areas in the left hemisphere that are active during the processing of numbers in their verbal notation, though the quantity code is not affected (Dehaene and Cohen, 1991; Dehaene et al., 1999). In this regard, the main claim is that numerical symbols do not modify the number core representation when they are incorporated into the numerical system, although recent views hypothesize that symbols introduced in the math system may be linked to quantity in an automatic manner (Dehaene, 2009). In this way, format dependencies on the distance effect can be explained (e.g., Cohen Kadosh et al., 2007).

How the process of integration of numerical symbols occurs is still uncertain and unspecified however. Actually, there are recent neuroimaging findings challenging previous views and pointing more directly to the fact that the analogue magnitude representation can be somehow affected by verbal notation systems (Nuerk, Weger, and Willmes, 2005; Salillas, Barraza, and Carreiras, 2015; Salillas and Carreiras, 2014). These findings open a debate about the possible influence of language in magnitude representations, and highlight the importance of studying the impact of the verbal format (Dehaene, 1992, 1996; Salillas and Carreiras, 2014). These studies add upon the notation specificity debate (Cohen Kadosh, Henik, et al., 2008), showing some modulation of neural signatures in core numerical representations by language and verbal numerical symbols.

In conclusion, there is a general consensus that numerical formats activate differently the magnitude representation system in the brain depending on the format. Symbols seem to change the neural basis of number processing and access to magnitude might be modulated by the input format (Campbell and Epp, 2004; Cohen, Ito, and Hatta, 2003; Cohen Kadosh, Cohen Kadosh, Kaas, et al., 2007; Cohen Kadosh, Henik, et al., 2008; Dehaene, 2009; Fias, Reynvoet, and Brysbaert, 2001; Kadosh and Walsh, 2009; Sokolowski et al., 2016).

2. Language and Number Representation

Language constitutes the basis of communication and is the most powerful tool not only to share actions or ideas, but also numerical concepts. Although language is the media for transmitting these abstract concepts, the role of language in the evolution of the innate sense of magnitude is not properly defined by any existing theory. The current view is that some basic aspects of number cognition development (e.g. arithmetic facts, counting) strongly depend on language (Butterworth, 2010; Carey, 2001; Delazer et al., 2005; Gelman and Butterworth, 2005; Nieder and Dehaene, 2009; Piazza, 2011; Salillas and Wicha, 2012; Spelke and Tsivkin, 2001). It is well known that solving simple arithmetic problems involves language processing. Nevertheless, there is also support for the idea that language and numerical ability are independent and thus have different underlying processes (Cappelletti, Butterworth, and Kopelman, 2001; Cipolotti, Butterworth, and Denes, 1991; Varley, Klessinger, Romanowski, and Siegal, 2005). In turn, the questions of whether and how language modifies our numerical system, as well as what are the neural bases of such possible modulations are not completely responded to date. In this section we are going to review the influence language may have in the numerical knowledge and how this connection is reflected in the brain.

2.1. The link between language and numbers

First, the role of language in number development has been the focus of some research (Carey, 2004). During the first stages of development, language is obviously part of the process of the acquisition of a more complex numerical knowledge beyond the approximate quantity sense that humans and non-humans share (Feigenson, Dehaene, and Spelke, 2004; Gordon, 2004; Hodent, Bryant, and Houde, 2005; Pica et al., 2004). For example, the ability to solve complex calculations is based on our capacity to manage numerical procedures that are mediated by verbal reasoning (i.e. language). A high level of mathematical reasoning can only be reached if an exact quantity representational system is present, and this is possible thanks to language (Carey, 2004; Feigenson et al., 2004). The link between language and number appears before any mathematical learning and formal instruction with the acquisition of number words and counting (Dehaene et al., 1999; Feigenson et al., 2004; Gelman and Galistell, 1979; Wynn, 1990).

On the other hand, studies on cultures with a very limited range of number words to refer to numerical concepts provide us with relevant insights about the relation between language and mathematics. Pica et al. (2004) investigated the Munduruku (an Amazonian tribe) who only have five number words. In their study the Munduruku were able to compare the relative magnitude of large numbers of dots, with a similar performance to the French-speaking controls. Similarly, Gordon (2004) asked participants from the Piranha culture in Brazil, which only has words for "one" and "two" and a single term for larger quantities ("many"), to build sets of nuts or batteries that matched an example set in number. Participants were accurate in constructing sets of one, two and sometimes three objects; however, when they had to construct sets of numbers larger than three precision dropped. The conclusion to be drawn from these studies is that development of exact magnitude is based on the acquisition of numerical verbal forms but once they are mastered, language (verbal format) is not compulsory to manipulate exact quantities. Indeed, exact arithmetic, contrary to approximate number processing, is thought to be represented in a specific language-coded format.

In turn, a preverbal system is based first on the "approximate number system" (ANS), which is approximate and becomes more imprecise as numerosity increases (following Weber's law) and another system, perhaps more debatable, restricted to the precise and automatic processing of sets of 1 to 3 objects, often referred to as the "object tracking system" (Feigenson and Halberda, 2004). These systems will be, however, the bases for symbolic mathematics, a cultural achievement that will allow humans to reach mathematical complexity. Hence, a basic number representational system accounts for approximate numerosities and this system is present before language acquisition. A verbal counting system should not be essential for having a core numerical representation; however, learning verbal numbers seems to be fundamental in order to develop the representation of exact quantities (see Carey, 1998, 2004; Dehaene, Izard, Pica, and Spelke, 2006; Feigenson et al., 2004; Spelke and Tsivkin, 2001).

Furthermore, exact arithmetic is thought to be represented specifically in a language-coded format (Butterworth, Reeve, Reynolds, and Lloyd, 2008; Dehaene et al., 1999; Spelke and Tsivkin, 2001). Exact calculations are learned by rote memory, as opposed to approximate estimations or numerosity, which do not rely on language as they are supposed to be held through the quantity code (Dehaene, 2009). Nevertheless, the sole linguistic nature of arithmetic facts has also been challenged (McCloskey, 1992; Noël et al., 1997; Semenza, Salillas, Di Pellegrin, and Della Puppa, 2016)

Understanding the role of language in accessing magnitude is essential to acquire a full picture of how numbers are represented and stored. The connection of language with the basic aspect of magnitude representation is still an open question (Dehaene, Bossini, and Giraux, 1993; von Aster and Shalev, 2007). In this respect, it has been proposed a role of language beyond the context of exact arithmetic. Recent findings suggest a linguistic permeability of quantity code originated during early learning math that would remain in adults' magnitude representation system (Salillas et al., 2015; Salillas and Carreiras, 2014). These recent findings point to the idea that language may have a crucial role in the processing of core numerical magnitude, as there would be established a link between them during early math learning and the acquisition of numerical verbal symbols.

2.2. Neuroimaging Evidence

Within the parietal lobe, the intraparietal sulci are assumed to be essential for the appreciation of quantity (see, for a review, Dehaene et al., 2004). We have already mentioned the role of the IPS in the processing in numerical magnitudes (Ansari, 2008; Cipolotti and Butterworth, 1995; Cohen Kadosh, Henik, et al., 2008; Dehaene and Cohen, 1995; Dehaene et al., 2003). But when the human brain is processing exact calculations we find more active regions apart from the IPS. Neuroimaging studies observed activation within fronto-parietal areas for simple and complex arithmetic problems (e.g. Gruber et al., 2001; Venkatraman et al.,2005; Delazer et al., 2003).

Most of neuroimaging studies gather around the idea that human number cognition is based in the integration of system for the non-verbal representation of approximate quantities and a language-based system for exact arithmetic. Dehaene et al., (2003) argued that the left Angular Gyrus is mainly involved in the verbal coding of numbers because it was strongly activated during small problems of addition and multiplication that require the retrieval of arithmetic facts stored in the verbal memory (Chochon et al., 1999; Dehaene, 1999a; Dehaene et al., 2003; Delazer et al., 2003, 2005; Grabner et al., 2009; Ischebeck et al., 2006; Zago et al., 2001). The left AG seems also to play a major role during the transfer of facts between arithmetic operations (Ischebeck, Schocke, and Delazer, 2009). For example, by comparing problem solving of small versus large problems over different arithmetic operations, a significant difference was found in the left AG (Grabner et al., 2009), which supports its role in arithmetic fact retrieval. Within exact calculation, the left AG shows greater activation for operations that require access to a rote verbal memory of arithmetic facts, such as multiplication, than for operations that are not stored and require some form of quantity manipulation. For instance, the left AG increased activation for multiplication relative to both subtraction and number comparison (Chochon et al., 1999; K. M. Lee, 2000). Additionally, Delazer (2003, 2005) contrasted untrained versus trained conditions in arithmetic facts; this contrast showed a significant focus of activation in the left AG. Untrained problems showed stronger activation in fronto-parietal areas such as the IPS and the left inferior frontal gyrus than previously trained problems. Trained problems, on the other hand, showed stronger activation in the left AG than untrained problems. This shift of activation within the parietal lobe from the IPS to the AG was interpreted to represent a shift from calculation to result retrieval from long-term memory. The left IPS showed significant activations, as well as the inferior parietal lobule. These results also indicate that the AG is closely linked to the retrieval of information stored in long-term memory. Significant activation was found in the left inferior frontal gyrus, which may be accounted for by higher working memory demands in the untrained as compared to the trained condition. These studies ultimately show that the observed relative increase in activation in the left AG was specifically related to result retrieval.

Additional of the dissociation between the exact and approximate systems are neuropsychological studies with patients suffering from brain lesions (Dehaene et al., 2003; Delazer and Benke, 1997; Lemer et al., 2003). These studies show double dissociations between lesions in left inferior parietal areas and left prefrontal damage and lesions in parietal regions: patients with lesions in the left frontal regions and left inferior parietal regions showed deficits in performing exact calculations related to verbal memory such as multiplications, while the ability of performing approximate calculations (e.g. magnitude comparison) was intact; the opposite was found in patients with lesions in parietal regions.

In summary, the reviewed brain imaging evidence suggests firstly, that the role of language in numerical cognition is conditioned to the exact arithmetic system but possibly also on most fundamental numerical knowledge. Secondly, arithmetic facts are integrated within language cortical networks mainly because they are learned verbally and retrieved by rote.

3. Bilingualism

Bilingualism offers a window to the study of the links between math and language. Referring to the same numerosity through to different codes implies specificities that should not be ignored. Given that nowadays bilingualism is more the rule than the exception, and that around half of the world's population is bilingual (Bhatia and Ritchie, 2012; Grosjean, 2010) and two thirds of the world's children are raised in a bilingual environment (Crystal, 1996), these possible peculiarities for math processing in bilinguals require attention. Before getting into a more profound exposition on bilingual math we will first introduce some relevant issues regarding bilingualism, relevant to the present thesis.

The term bilingualism refers to those individuals who have learned more than one linguistic code for oral and written communication (Grosjean, 2010). One must bear in mind that bilingualism should not be simplified as a dichotomy of speaking two languages, inside bilingualism we can find a wide range of categories related to fluency and other linguistic domains (Centeno and Obler, 2001; Hamers and Blanc, 2000; Macnamara, 1967). There is a wide range of dimensions to take into account when categorizing bilingualism: age of acquisition (AoA), percentage of use, context and cultural identity among others (Baker, 2011; De Houver, 2009; Flege, Mackay, and Piske, 2002; Grosjean, 2010; Grosjean and Li, 2012; Hazan and Boulakia, 1993; Hernandez, 2013; Kroll, Dussias, Bogulski, and Valdes Kroff, 2012; Valdes, Brookes, and Chavez, 2003) Some of these aspects are relevant for this work: The AoA and the relative proficiency.

A language can be acquired very differently across and within societies, going from individuals learning two languages with both languages present in extensive contexts from birth (this would be the case of simultaneous bilinguals in regions such as the Basque Country in Spain), to late learners of a second language (L2) who have less contextual presence in the environment (e.g. learning an L2 without natural immersion). The AoA makes the distinction between these two kinds of bilinguals, being the first group called simultaneous bilinguals, and the second group late learners of an L2 or late bilinguals. The influence of the age of acquisition in the level of competence has been challenged in many studies (Bosch and Sebastian-Galles, 2003; Gandour et al., 2007; Kim, Relkin, Lee, and Hirsch, 1997; Perani et al., 1998, 2003); some of them supporting the notion of a critical period (DeKeyser, 2005; DeKeyser and Larson-Hall, 2005; Lenneberg, 1967). It is well known that some aspects of a language such as morphology or phonology do not reach a native level when learned in adulthood (Bialystok and Miller, 1999; Long, 1990; Pinker, 1994) and that acquiring an L2 becomes increasingly difficult after puberty (Flege, Munro, and Mackay, 1995; Johnson and Newport, 1989; Weber-Fox and Neville, 1996). At the neuroanatomical level, separate representation networks of bilinguals´ L1 and L2 have been related to differences between both languages' age of acquisition (Abutalebi, 2008; Chee, Hon, Lee, and Soon, 2001; García-Pentón, Pérez Fernández, Iturria-Medina, Gillon-Dowens, and Carreiras, 2014; Perani et al., 1998).

The level of proficiency defines the bilingual speakers' ability in the different competences of a language (e.g. fluency and comprehension). *Balanced bilinguals* are the speakers who have the same abilities in all the competences of a language, whereas *unbalanced bilinguals* are those speakers whose language abilities are superior in one language (usually their L1) than in the other. It is very important to interpret the term "balanced" appropriately as an exact equal level of proficiency when assessing its degree for research purposes. The importance of this fact will be revisited in following sections. Indeed, the level of proficiency has an impact on the degree of brain activation related with the early or late age of acquisition. In other words, bilinguals with similar levels of proficiency have different level of activation in the L2 compared to L1, having the late bilinguals higher activation whereas early bilinguals had no different values of activation (Kovelman, Shalinsky, Berens, and Petitto, 2008; Perani et al., 2003; Perani and Abutalebi, 2005).

The procedures to assess the level of proficiency, and therefore the level of bilingualism, include self-reporting measures and linguistic competence tests. The former gather information about participants' AoA, percentage of use or speaking contexts in the form of questionnaires and/or interviews (Li, Sepanski, and Zhao, 2006). However, if we want a more objective and standarized measure of the individuals'' proficiency, the most common way is to evaluate it by the use of tests. One of the most frequently used techniques is the naming task, which requires the naming of pictures of objects, graded in difficulty, in both languages. Perhaps the Boston Naming Test (BNT) by Kaplan, Goodgalss and Weintraub (1983) is the most known test and one of the most frequently used to assess bilingual dominance. The BNT offers a standardized measure based on the performance criteria and enjoys widespread use in clinical and experimental research, adapted to at least nine different languages (Moreno and Kutas, 2005; Rosselli et al., 2000; Salillas and Wicha, 2012; Verhoef, Roelofs, and Chwilla, 2009).

3.1. Bilingualism and Math Cognition: the case of arithmetic

Bilingualism could imply a specific case for math. The fact that the bilingual manages two verbal codes for the same numerosity sets the question of whether these codes are equally represented or, on the contrary, imply an imbalance with consequences for math processing. In addition, the aforementioned dissociations between the neurofunctional bases for approximate vs. exact arithmetic imply that perhaps exact arithmetic is not equally represented in the two languages either. Some studies have addressed these questions (Bernardo, 2001; Dehaene et al., 1999; Martinez-Lincoln, Cortinas, and Wicha, 2015; Salillas and Wicha, 2012; Spelke and Tsivkin, 2001).

Initial research on math in bilinguals used bilingualism as a mean for testing the approximate vs. exact dissociation. Given the crucial role of language in exact arithmetic, as defended by the Triple Code Model, initial studies focused on this specific math process. This approach studied the influence of language in bilinguals arithmetic performance, with and without training in their L1 or L2 (Bernardo, 2001; Campbell, Kanz, and Xue, 1999; Dehaene, 1999b; Marsh and Maki, 1976; Salillas and Wicha, 2012; Spelke and Tsivkin, 2001). Particularly, comparisons have been made in bilingual children and adults in terms of stronger and weaker language impact (L1 vs L2) during arithmetic-solving tasks (Campbell and Epp, 2004; Frenck-Mestre and Vaid, 1993; Rusconi, Galfano, and Job, 2007; Secada, 1991).

Thus, bilingualism serves as perfect way for testing the link between exact arithmetic and language (Frenck-Mestre and Vaid, 1993; Marsh and Maki, 1976; Rusconi et al., 2007; Salillas and Carreiras, 2014; Salillas and Wicha, 2012; Spelke and Tsivkin, 2001). It is well known that bilinguals tend to perform arithmetic facts (e.g. multiplications) in one particular language (Spelke and Tsivkin, 2001). In Spelke and Tsivkin (2001) three experiments were conducted in order to sort out the influence of language in bilinguals' numerical representation. There were exact arithmetic problems (e.g. additions) and approximate problems (e.g. estimating approximate cube roots), all of them presented in the numerical verbal format. Participants were Russian-English bilinguals who were trained to solve the problems in one of the two languages. After participants were trained they tended to perform better in the trained language independently of it being L1 or L2, and similar performances in the approximate operations. This suggests the idea that exact arithmetic is represented in a more language-specific form. Thus, language and math interact during children´s learning of arithmetic.

Most of bilingual research on math cognition demonstrates that bilinguals feel more comfortable and perform better on the language in which they have learnt arithmetic in school than in the other language (Marsh and Maki, 1976) and worse performance when numerical problems were posed in bilinguals' weaker language or L2 (Frenck-Mestre and Vaid, 1993; Morales, Shute, and Pellegrino, 1985).

These questions are pivotal to the present thesis, but our approach differs from these early studies. Our main interest is focused on the study of actual specificities in the bilingual numerical system as it is acquired in usual bilingualism. Specificities that come from a possible unbalance between the two languages that are in turn, the medium for math communication and numerical information exchange. Recent evidence suggests that in fact, language plays an important role in math learning and that bilinguals' numerical processing is directly modulated by some elements such as the language of math instruction.

3.2. Bilingual math and the language of learning math

The consequences of bilingualism in the development of numerical skills have attracted interest over the last decades and have taken into account critical factors such as AoA, the language of instruction in early learning of math or language proficiency. These studies have also permitted a deep look in the role played by language in our math system. Based on current evidence, it is assumed that mathematical development in bilinguals normally involves one of the two languages preferentially (Bernardo, 2001; Grabner et al., 2012; Martinez-Lincoln, Cortinas, and Wicha, 2015; Salillas, Barraza, and Carreiras, 2015; Salillas and Carreiras, 2014; Salillas and Wicha, 2012). It is also well-known that bilinguals often translate or switch languages when carrying out simple arithmetic facts or for mathematical thinking in general (Moschkovich, 2007). This preference for a language for number processing could be the language in which math has been studied (Bernardo, 2001; Clarkson and Galbraith, 1992; Frenck-Mestre and Vaid, 1993; Geary, Cormier, Goggin, Estrada, and Lunn, 1993; Kolers, 1968). Nevertheless, the question of the language preference for magnitude representations in bilinguals requires other important considerations related with whether and how language connects also with our core numerical knowledge (Salillas and Carreiras, 2014).

The influence of learning experiences in setting a preferred verbal code for arithmetic was vaguely contemplated in the Encoding-Complex Model (Campbell, 1994, 2005; Campbell and Clark, 1988; Campbell and Epp, 2004). The model claims that the bilingual arithmetic memory system keeps a relatively strong link with the language used for learning and retrieve arithmetic. The connection between one of the languages and the arithmetic memory networks, as well as the analogue magnitude code, will depend on the prior experience in direct retrieval of numerical information and not on the proficiency of the language (see Figure 4).

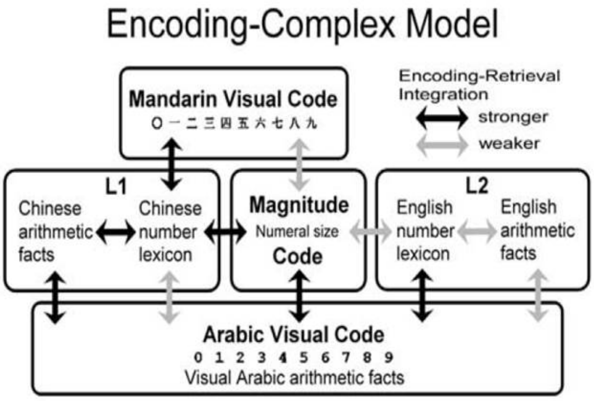

Figure 4. Illustration of the Encoding-Complex hypothesis by Chinese-English bilinguals (Campbell and Epp, 2004). The arrows have two different colors according to strength. Grey arrows illustrate weak integration of the interfacing processing, black arrows represent strong integration.

A more pivotal question highlighted in literature refers to the consequences of early learning in adults´ numerical representations. In an educational bilingual environment, the language of formal instruction has enormous influence in knowledge representation (Clarkson and Galbraith, 1992; Malt and Wolff, 2010) and the influence of math learning experiences with one of the two languages has been tested in bilingual children (Clarkson and Galbraith, 1992; Cummins, 1984; Cummins and Gulutsan, 1974; Kempert, Saalbach, and Hardy, 2011; Moschkovich, 2007; Spelke and Tsivkin, 2001). In the case of math, consequences can affect early numerical knowledge development as well as math competence later in life. The impact of early learning in arithmetic processing networks has been recently investigated also in adult bilinguals. Salillas and

Wicha (2012) using ERPs tested arithmetic memory networks in adult bilinguals who only learned arithmetic in one of their two languages. Independently of the language dominance, bilinguals showed stronger and more accurate arithmetic memory networks in the language of learning exact arithmetic. These results led authors to conclude that math and language connection is maintained in adulthood and language proficiency does not alter the arithmetic networks established during early learning. In a subsequent study, Martinez-Lincoln et al., (2015) tested, however, if explicit experience using the language in which there is no arithmetic experience (i.e. bilingual teachers repeating arithmetic facts and teaching them in the other language to the students) could equate the strength of the language in which arithmetic facts were acquired. The same paradigm and technique as in Salillas and Wicha (2012) was applied to the special case of elementary school teachers, adding a new variable: the language in which the participants had been teaching arithmetic for an average of 9 years. This language could be either the language of learning arithmetic (LA+) or the other language in which they were proficient but not arithmetic facts were acquired (LA-). The authors showed that extensive teaching of arithmetic facts in LA- equated the strength of arithmetic facts to the LA+. Crucially, once more in this study, the pattern of dominance between languages for arithmetic facts was independent from overall proficiency in each language.

These studies show that language plays an important role in those number (and essentially verbal) tasks that require the retrieval of exact arithmetic (Dehaene, 1999b; Rusconi et al., 2007; Salillas and Wicha, 2012; Van Rinsveld, Brunner, Landerl, Schiltz, and Ugen, 2015). To clarify and in line with Dehane ´s Triple Code Model, exact calculations (e.g. multiplication and addition) are thought to be coded in a specific language contrary to approximate number processing. The reason is that they are learned and retrieved by verbal rote. Thus, until very recently the association between language and numbers in bilinguals was conceived in relation to the management of exact arithmetic (Rusconi et al., 2007; Salillas and Wicha, 2012). Very little research has considered the specific role of language beyond the context of arithmetic facts. Thus, one can consider not only the language of learning, encoding and storing arithmetic facts, but a broader concept: The *Language of Learning Math* (LL^{math}) as the language used for the learning of all mathematical concepts. A linguistic context in which more core numerical development is carried out. LL^{math} includes but is not restricted to the LA+, and it will be contrasted to the Other Language (OL) as the language that, albeit being perfectly fluent, is not the language used for early math learning.

The role of language in mathematics comes from the idea that in early learning stages the numerical concepts are manipulated verbally (e.g. multiplication tables, equation rules, additions, substractions, matching between numerical symbols and their meaning, etc.) and therefore several kinds of numerical information is stored in long-term memory preferentially in one language. In that sense, Salillas and Carreiras (2014) proposed that language should play a further role also in the more basic magnitude representation level. In bilinguals, the basic quantity system seems to be shaped by one particular language. Therefore, we need a broader concept to define the language of preference for magnitude representation that needs to be independent of the language proficiency and beyond arithmetic fact retrieval, that is, LL^{math}. This proposal was initially supported by electrophysiological data, exploring the neuroanatomical and functional networks that link the core number representations and language (Salillas et al., 2015; Salillas and Carreiras, 2014). This link is hypothesized to occur during early math learning (Salillas et al., 2015; Salillas and Carreiras, 2014).

3.4 Neuroimaging evidence

Following the previous section, a critical factor should be how the learning process of numerical knowledge is influenced by language and how it determines the brain organization and functioning of the numerical system in bilinguals. The neuronal correlates in the process of learning exact arithmetic have been widely explored (Dehaene and Cohen, 1997; Dehaene et al., 2003; Delazer et al., 2003; Delazer and Benke, 1997; Venkatraman, Siong, Chee, and Ansari, 2006). The general finding is a difference in the language networks (AG, left inferior frontal gyrus) when comparing exact and arithmetic mathematical processing. One study comparing solving exact and approximate calculations by bilinguals in one of their languages was Venkatraman et al. (2006). In their study, a group of English-Chinese bilinguals were trained in solving approximate and exact arithmetic calculations in one of their languages. Afterwards, they were tested and scanned using fMRI, and the main findings showed that there was greater activation for exact problems presented in untrained vs. trained language in the left inferior frontal gyrus and the AG. In contrast, comparison of approximate problems presented in the untrained vs. the trained language modulated regions in bilateral posterior parietal cortex.

Another study by Lin, Imada and Kuhl (2012), observing this time exact calculations, found language differences in the form of greater activation for L2 exact addition in the left inferior frontal area in bilinguals. A negative correlation between brain activation and behavioral performance during mental addition in L2 was observed in the left inferior parietal area. These results provide evidence of the bilinguals' preference for one code in computing mathematical operations.

There are also studies that do not focus on training but on the average role of each of the languages in bilinguals. Specifically, they have studied the impact of the language of learning arithmetic facts in the exact arithmetic system in equally proficient bilinguals, thus setting apart LL^{math} and proficiency, by keeping proficiency constant (Martinez-Lincoln et al., 2015; Salillas and Wicha, 2012). In Salillas and Wicha (2012) measured both the strength of the arithmetic networks and their quality in English-Spanish bilinguals who were presented with arithmetic facts solutions in the LA+ (Language of Learning Arithmetic[1]) as opposed to the LA- (Other Language). Solutions could be correct, incorrect (3x2=7) or incorrect but related to the correct solution (e.g. 3x2=9). The ERP results showed that the N400 effect was larger for incorrect solutions that for correct solutions for the LA+, and that this amplitude in the N400 was modulated by relatedness only in the LA+. The N400 is thought to reflect the automatic spread of activation among representation of arithmetic facts (Niedeggen, Rösler, and Rosler, 1999). Indeed, in the LA- there was not an amplitude difference between related and unrelated solutions. Therefore, the quality and actual form of those networks was different, less precise, for the language in which arithmetic facts was firstly acquired. It is worth to highlight that these effects were independent on the relative overall proficiency between languages as measured by picture naming and fluency tasks. Martinez-Lincoln et al. (2015) did a similar study, but in this case, they tried to solve if explicit experience using LA- in the case of bilingual teachers repeating arithmetic facts and teaching them in the LA- to other students could equate in strength to that of the LA+. The results showed that extensive teaching of arithmetic facts in LA- equated the strength of arithmetic facts activations to the LA+. This was demonstrated by similar N400 amplitude differences between correct and incorrect solutions in both languages. Although the authors did not report the N400 relatedness effects, these results show that it was exposure to arithmetic facts in one

[1] Notice the difference between the broader concept of the LL^{math} and LA+. The former refers to the language in which all the mathematical concepts were acquired whereas the latter may simply refer to arithmetic facts of a given language (e.g. multiplications).

language that determined the imbalance between languages as in Salillas and Wicha (2012).

In addition to the influence of language in exact arithmetic, the linguistic traces in the quantity code as result of early learning processes of numerical verbal symbols has also been studied using ERPs. Salillas and Carreiras (2014) dug into the effect of the language and early learning into the core numerical representation. The task they used was a simple comparison task in which participants did number comparisons on pairs of digits. The most interesting part of this study was that authors tested two different groups of Spanish-Basque balanced bilinguals whose only difference was their LL^{math} since the authors were looking for a possible association between a specific wording system and the core numerical knowledge. Spanish and Basque numerical systems differ in that the Spanish system follows the base 10 system (e.g., the number word for 58 is 50 and 8, “cincuenta y ocho”) whereas in Basque, number words follow the base 20 system (e.g., the number word for 58 is 2x20 and 10 and 8, that is, 40 and 10 and 8, “berrogeita hamasei”). Base 20, however, is restricted to number words in Basque, so any impact on any number effect under study that is related to this vigesimal system should be interpreted as coming from the number word and therefore linguistic. The results showed that a different ERP pattern was associated to the Basque wording system (base 20) as a function of the LL^{math}. The most important finding was that an earlier N1-P2 distance effect appeared for pairs of digits related through the base 20 system and only for those participants whose LL^{math} was Basque. This component involves the transition wave from a negative peak at 100 ms to a positive peak at 200 ms, and usually indexes numerical semantic processes (access to quantity), together with a subsequent P2p[2]. This led the authors to suggest that verbal signatures in the core magnitude representation system are due to early learning math. In a subsequent experiment using EEG oscillatory analysis and the same data as in their previous study, the same authors provided consistent evidence supporting the same hypothesis. They observed that the brain waves whose frequency was close to the band of 40Hz were synchronized (gamma band synchronization) to numbers that were related through the vigesimal system, only for the group whose LL^{math} was Basque. Synchronized electrodes were localized to the left hemisphere in frontoparietal sites and then extended bilaterally. As numerical comparison

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[2] A number-specific ERP component emerging over the parietal areas around 200 ms after stimulus presentation.

of Arabic digits does not imply a verbal code, all these effects would originate in number words. Importantly, and similar to Salillas and Carreiras (2014), very proficient bilinguals whose LL^{math} is Spanish did not show this synchronization in the gamma band. These results provide information about when number words attach to quantity in the balanced bilingual participants, during early math learning. Ultimately, what these studies show is that verbal input during simple numerical tasks is not necessary for obtaining verbal effects, which influence our quantity manipulation.

To sum up, the results showed in the previous studies suggest that language plays an important role in exact arithmetic and it could influence the core numerical representation. Additionally, arithmetic facts are integrated in language cortical networks mainly because they are learned verbally and retrieved by rote, but the link of math and language does not seem to be restricted only to that, and a possible linguistic shaping of the basic quantity representation system can be proposed (Salillas et al., 2015; Salillas and Carreiras, 2014). This latter proposal will be one of the topics of research in the present dissertation, in the special case of Develomental Dyscalculia. Since the reviewed papers in this chapter show that bilinguals have a preference for one code in mathematics (i.e. LL^{math}), this might have consequences for cases in which basic numerical knowledge is altered, such as developmental dyscalculia. This syndrome is of special importance in the understanding the processing of mathematics at a neuronal level and also bilingual dyscalculia can help to understand the links between math and language.

4. Unbalanced Math in Bilingual Minds?

According to the proposed relevance of early learning in setting (1) the strength and quality of arithmetic memory networks and (2) a preferential link between one of the languages and core numerical knowledge, a most essential question arises: Are the two number words systems in a bilingual equally represented? Most of the experimental work in this thesis addresses this question. This is to say that a proficient bilingual may have an equally strong representation for the word '*sea'* and the word '*mar'*. However, the same bilingual might present an unbalanced representation for number words, '*one'* vs. '*uno'*. As we will see, a straightforward way of measuring such an unbalance is through the study of *language switching* and the relative costs implied in the switching between the two number word systems. Language switching has been profusely studied in bilingual language cognition, as we expose below.

4.1. Switch cost as a measure for lexical unbalance for math

One commonality to all bilinguals is the ability to switch between languages. When bilinguals perform this switch, there is always an effort. This effort is known as *switch cost* (Costa and Santesteban, 2004; Jackson, Swainson, Cunnington, and Jackson, 2001; Meuter and Allport, 1999; Palmer, van Hooff, and Havelka, 2010). Balanced bilinguals switch between L1 and L2 indistinctly with the same effort; this is known as *symmetric switch cost*. However, unbalanced bilinguals shown an asymmetry in the switch, that is to say the switch in one direction always takes more effort than in the other; this is known as asymmetric switch cost (Costa and Santesteban, 2004; Duñabeitia, Dimitropoulou, Uribe-Etxebarria, Laka, and Carreiras, 2010; Meuter and Allport, 1999). In the last decades, switch cost has been examined in numerous ways and there are several models that aim to explain this switch of which two are worth mentioning.

Though one may came across different ideas of how bilinguals process and switch languages, there are two main views on the consequences of frequent switching between languages in bilinguals. There are authors that propose that bilinguals have advantages over monolinguals in attentional resources, memory skills and executive control mechanisms. In other words, speaking several languages can lead to benefits that go beyond the realm of language, impacting on global cognitive functioning (Bialystok, Craik, and Luk, 2012). These advantages are somehow boosted by bilingualism due to the constant need of bilinguals to inhibit one of their languages while they use the other. This language inhibition is believed to be a part of a greater general mechanism of inhibition and since language inhibition is used every day it helps to improve the general inhibition mechanism (Abutalebi et al., 2012; Craik and Bialystok, 2006; Garbin et al., 2010). Another view on this matter is that the mechanism of inhibition used in languages belongs solely to the language system and therefore is not part of any general mechanism (Abutalebi et al., 2008; Calabria, Hernandez, Branzi, and Costa, 2012).

The first model to account for an asymmetry in the switch is the Revised Hierarchical Model (RHM) by Kroll and Stewart (1994). In this model L1 and L2 lexical representations are stored in separate lexicons that are connected to each other and to a common semantic system. The strength of the connections proposed in this model is asymmetric since the connections from the lexical representations in the L2 to their L1 translations are stronger than the other way around. Additionally, L1 words activate conceptual representations with more strength than L2 words do. This model assumes that these asymmetries disappear once the bilinguals become more proficient in their L2 since the links between the L2 and the conceptual representations become stronger.

The Inhibition Control (IC) hypothesis (Green, 1998) proposes that bilinguals' expressing and comprehending a communicative intention may be an inherently competitive process. Managing competing systems such as phonology, syntax, prosody and in reading they must manage distinct mappings of orthography to phonology. In other words, syntactically-specified lexical concepts in different languages might compete for selection (Green, 1986, 1993, 1998; Hermans, Bongaerts, De Bot, and Schreuder, 1998; M.-W. Lee and Williams, 2001; Poulisse, 1999). This models' proposal is that bilinguals may become adept specifically at selecting responses in the face of competing cues even in non-verbal tasks (Bialystok, Craik, Klein, and Viswanathan, 2004; Craik and Bialystok, 2006), thus, such competition is resolved by inhibiting any active, non-target language competitor. In the case of unbalanced bilinguals, it can take longer (and therefore more effort) to switch into the more dominant language (L1) compared to the less dominant language (L2). Although the precise circumstances in which this type of asymmetry arises are not established by the IC theory.

Another model, the BIA+ (Dijkstra and van Heuven, 2002), proposes that language information is accessed through the lexical representations of words. This model is an update of the Bilingual Interactive Activation (BIA) model (Dijkstra and Van Heuven, 1998), which is itself a bilingual extension of the Interactive Activation model (McClelland and Rumelhart, 1981). It is a word recognition model and is based on the hypothesis that bilinguals have a shared bilingual lexicon and each word is recognized as belonging to one language or another. It is basically the ability to access the right word in the right context. When a word is presented to a bilingual, both languages would be initially activated. Once the word is recognized, the use of language nodes (i.e. language tags) selectively inhibit the words in the other language producing lexical decision. The original BIA model contained a representational layer with two language nodes (one for each language) and these language nodes had top-down connections to the lexicon. Each language node collected activation from words of the corresponding language and inhibited words from the other language, processing costs following a language switch were the result of inhibition of the inappropriate language. However, the BIA+ model accounts for these switch costs as they are almost exclusively the result of executive control factors and thus external to the language system and related instead to a more general control mechanism (Dijkstra and van Heuven, 2002), similar to Green's IC model. All in all, albeit implying some differences the two models account for an asymmetry in the switch for unbalanced bilinguals.

This aspect is of special relevance for this thesis: first, we will observe the symmetry in switches between number words as an index of the balance or unbalance between the two number word systems, positing that even in overall balanced bilinguals an unbalance should be shown for number words; second, we will study the nature of the control system that operates on number words in bilinguals, positing that, although the relative unbalance for number words mismatch the otherwise balance for general language in proficient bilinguals, very likely the control mechanism that control for relative activations is shared with language and with other cognitive domains.

4.2. Neuroimaging evidence

This section comprises part of the neuroimaging evidence supporting the idea of both *switch costs* based on the models reviewed in the previous section. Bilingual language processing and control has been investigated in numerous studies (Abutalebi et al., 2008; Abutalebi and Green, 2007; Chee, Tan, and Thiel, 1999; De Bleser et al., 2003; Hernandez, Dapretto, Mazziotta, and Bookheimer, 2001b; Hernandez, Martinez, and Kohnert, 2000; Perani et al., 2003; Swainson et al., 2003; Wang, Xue, Chen, Xue, and Dong, 2007; Yetkin, Yetkin, Haughton, and Cox, 1996). A common finding in these studies is a significant difference in activation in the brain regions involved in language switching. For example, Hernandez et al. (2001b) performed an fMRI study with early Spanish–English bilinguals who had to name objects in one language or switch between languages. Increased activity was found in the left dorsolateral prefrontal cortex for the switching condition relative to the non-switching condition. Similar findings were reported by Chee, Soon, and Ling Lee (2003). Additionally, Rodriguez-Fornells et al. (2002) investigated the neural correlates of language selection. They recruited a group of early Catalan–Spanish balanced bilinguals. The main aim of their study was to determine how bilinguals inhibit the non-target language (Catalan in that study). They addressed this question by combining ERPs and fMRI. The results were compared to a control group of Spanish monolinguals selecting visually presented real Spanish words intermixed with pseudo-words. Activation of a left anterior prefrontal region (Brodmann areas 45 and 9) was only observed in the group of bilinguals. Aside from emphasizing this selective effect for bilinguals, we stress that this study confirms that even highly proficient bilinguals need inhibition mechanisms.

Another ERP study by Jackson et al. (2001) examined language switching in a digit-naming task. Participants named digits in the target language cued by the color of the digit. The N2 component, recorded over the left frontocentral region and typically associated with response inhibition, was much more negative when individuals switched from naming in L1 (the habitual language) to naming in L2. Such data are consistent with the notion that the more dominant language requires more active suppression (see also Verhoef et al. 2009 for evidence that attentional engagement as confirmed by the N2 component amplitude is at the root of different switch cost patterns).

Jackson et al. (2001) also examined the effects of switching on activity in the parietal cortices and found that switching induced an increase over the parietal cortices in the late positive complex (LPC) associated with increased demands on response selection as in the Stroop interference. The authors related these results with the idea that language switching increased frontal and parietal activity consistent with the requirement to inhibit ongoing activity and select a relevant response in the face of competition. Complementary to this, Chauncey et al. (2008) studied unbalanced bilinguals and found an asymmetric switch cost in another component, the N400. In their study participants had to perform a semantic categorization task (classify items as animals or non-animals) while before the target words participants were presented with words that were perceived unconsciously. These unconscious words could be in the same or different language (L1 or L2) as the word they had to classify with, being harder to switch from the less dominant language (L2) to the more dominant language (L1). The results showed a larger N400 modulation for the switch cost in the L2 to L1 direction than vice versa. Additionally, Duñabeitia et al. (2010) performed a similar study with early bilinguals who showed a switch effect in the N2 component for the both directions of the switch (i.e. L2 to L1 and L1 to L2) and found no asymmetric switch, concluding that the asymmetry observed by Chauncey et al (2008) was language-dominance related.

In sum, the revised studies in this section demonstrate that switching languages in bilinguals requires an extra effort, and that this effort is asymmetric in unbalanced bilinguals since one of the directions of the switch shows higher activation or bigger amplitude in the ERP components. These switch costs could be due to a general control mechanism.

These facts establish the ground on which most of this thesis relies, namely, that with regard to number words, this unbalance will be set by LL^{math} and not by general proficiency. The final experimental work aims to further explore the impact of LL^{math} in a more profound level of representation, the core numerical knowledge, essentially altered in Developmental Dyscalculia.

5. Math Related Disorders: Dyscalculia.

Developmental Dyscalculia (DD) is a disorder of numerical development and mathematical learning happening in the frame of normal IQ. It has a prevalence of 6%, which makes it as prevalent as dyslexia (a developmental disorder that affects the ability to read in otherwise normally developing individuals) and there is also a high degree of comorbidity between DD and dyslexia. DD is characterized by slow and error-prone learning and retrieval of arithmetic facts from memory (Jordan and Montani, 1997). Additionally, there is use of immature calculation, problem-solving and counting strategies and a delay in the transition from finger counting to verbal counting and fact retrieval (Geary, 2004). Additionally, people with DD present heterogeneous symptomatology, including spatial working memory deficits or symptoms due to a failure in broad executive functions as attention (Henik, Rubinsten, and Ashkenazi, 2011; Rubinsten and Henik, 2009).

Different explanations of its causes suggest possible core numerical deficits (Butterworth, 1999, 2005, 2011; Geary, 2004; Landerl, Bevan, and Butterworth, 2004; Piazza et al., 2010; Rousselle and Noël, 2007; Wilson and Dehaene, 2010). People with DD potentially lack essential numerosity representation due to functional (Price, Holloway, Räsänen, Vesterinen, and Ansari, 2007) and morphological alterations in the numerical neural networks (Molko et al., 2003; Rotzer et al., 2008). The prevalent view for many years has been that DD is caused by an anomaly in the core magnitude representation (Butterworth, 1999, 2005, 2010; Geary, 2004; Wilson and Dehaene, 2010).

There are alternative explanations emphasizing the role of numerosity-to-symbol matching and the subsequent modification of the ANS (Piazza et al, 2010; Noël and Rousselle, 2011). Thus, a core deficit could arise later on during symbol acquisition. In normal circumstances, the acquisition of number words for counting precedes non-verbal Arabic number symbols, which might lead to qualitative modifications in ANS (Piazza, Pica, Izard, Spelke, and Dehaene, 2013) that also allow exact calculation. For DD children, a process of linearization is impaired (Piazza et al., 2010). In fact, there are researchers who have observed specific alteration of the processing of quantity when using symbols in contrast to an apparent spared quantity processing when non-symbolic numerical stimuli (i.e. dot patterns) are used (Noël and Rousselle, 2011; Rousselle and Noël, 2007).

5.1. Dyscalculia in bilingual populations.

Although DD has such a significant prevalence the management of two codes in bilingual DD (bDD) is a circumstance no study has yet contemplated behaviorally or at the neuronal level. Given the mentioned linguistic imbalances for math in bilinguals, one might predict that the input language will matter even more when a core number processing deficit occurs in bDD. The differences between the two languages in bilinguals described above should be based in differences in the brain networks sustaining these effects: either the computation of quantity implies a wider network of interacting systems or core number parietal areas show a better integration with the preferred language in a sort of neural recycling for that specific symbolic system (Dehaene, 2009).

The study of syndromes and lesions is a powerful tool to understand the proper functioning of the neural networks implicated in math. And although there is no research on bDD, this syndrome is the second pillar of this thesis and it will provide unprecedented work on DD.

5.2. Neuroimaging evidence.

This section summarizes the most relevant neuroimaging studies regarding Developmental Dyscalculia trying to find out out which brain regions are affected by this syndrome. At the neural level, Intraparietal Sulcus (IPS) differences during the processing of non-symbolic numerosities have been reported (Price et al., 2007). Kucian et al. (2006), when comparing DD children and controls, found similar activation patterns for both groups for approximate and exact calculations but with some activation differences: DD children presented weaker and more diffuse activation than the controls. Additionally, studies using symbolic notations have found that notation modulates the activity of frontoparietal activations, as meta-analysis studies demonstrate (Kaufmann, Wood, Rubinsten, and Henik, 2011). In turn, DD children differ from controls in not only parietal function but also frontal functions during number processing (Rotzer et al., 2008). In fact, some residual numerical processing in DD has been found in frontal areas, possibly reflecting a greater use of working memory processes than controls (Cappelletti and Price, 2014). Further, there is evidence of disconnection between the areas linked by the superior longitudinal fasciculus in DD. That is, deficient fiber projection between parietal, temporal and frontal areas has recently been reported in DD (Kucian et al., 2014). In addition, transcranial electrical stimulation over left posterior parietal areas when learning symbol-quantity associations ameliorates those associations in DD by facilitating neuroplasticity (Iuculano and Cohen Kadosh, 2014).

In addition, ERP studies that investigated numerical magnitude processing in DD children showed distance effects that did not differ between controls and DD at an early time window during a symbolic number comparison task (Soltész, Szucs, Dékány, Márkus, and Csépe, 2007) . However, at a later time window, the controls showed a significant distance effect over the right parietal areas while the DD group showed no such effect. After further analysis, the authors showed DD participants also presented an effect on the fronto-central electrodes that the controls did not. Therefore, the authors suggested executive control differences between groups, in keeping with working memory deficits contributing to DD (Geary, 2004). All these studies point to the fact that DD is characterized at the neuronal level by atypical properties in parietal and frontal regions and that DD children show atypical behavior and atypical functional activation in the parietal areas during basic numerical processing.

6. The Present Study

With the aim to provide a proper background for the reader, in the previous chapters we have described the main theoretical and empirical insights contemplated in Math Cognition and the main topics of interest that concern the present thesis. Evidences of the connection of language and numbers have been strongly remarked consistent with classical and recent research findings. The impact of language in the accessing to core magnitude representation has been postulated as an ongoing research question in literature, including neuroimaging studies. In order to understand the role of language in the core magnitude system, advances in math cognition research have been followed by research on bilingualism. Bilingualism is a key to the understanding of bilinguals' preferences for one of the two codes they have for math and studies about how bilinguals manipulate and switch languages have been reviewed. The concept of code switching has been introduced as a way to measure the possible unbalance between number word systems in bilinguals. Finally, a specific numerical syndrome (Dyscalculia) has been described and postulated as an ongoing research in the field of bilingualism.

In the present study, we will observe two main points: **1) the unbalance for numerical wording systems in balanced bilinguals** and **2) further exploration of LL^{math} in core numerical systems through dyscalculia** using neuroimaging and source imaging reconstruction techniques**.**

It has been already shown that bilinguals have a preference for one of their two codes in arithmetic representations (Martinez-Lincoln et al., 2015; Salillas and Wicha, 2012; Spelke and Tsivkin, 2001) and more recently it has been shown that LL^{math} might be the language of preference for accessing magnitude (Salillas et al., 2015; Salillas and Carreiras, 2014). This preference is proposed to take place during early math learning. At the same time this preference will show an unbalance in the switch between the codes for math and consequently, balanced bilinguals will be more "proficient" in their LL^{math}. **This effect should manifest as an asymmetric switch cost when switching between their two codes for math.**

These preferences for LL^{math} should also be consistent in the study of bilingual bDD. We will explore **how distance effects occur in each of the languages in**

dyscalculic children as compared to controls and their respective neural source activations.

Experiment 1 investigates dominance in the codes for math. We hypothesize that in L1/L2 balanced bilinguals, the determinant factor for a preferential code for math will be early learning experience (i.e. LL^{math}). In order to sort out this preference we will distinguish between the two codes for math: the language for learning math (LL^{math}) and the Other Language (OL) as the determinant factors of this relative dominance. The novelty of this experiment is that this LL^{math}-OL distinction has not been previously considered in numerical tasks as the dichotomy L1/L2 is always considered. Based on previous ERP switch studies (Chauncey et al., 2008; Jackson et al., 2001) we predict that switching between LL^{math} and OL will generate asymmetric N400 switch costs. A larger switch cost should appear in the OL to LL^{math} transition whereas a lower switch cost should be found in the other direction. This asymmetry should be independent from L1/L2 proficiency.

Experiment 2 aimed the same goal as Experiment 1, but it additionally enquires about the roots of this asymmetric switch cost. In order to find out about this, this experiment was designed to avoid executive functions and avoid explicit magnitude manipulation so as to know whether the imbalance is truly lexically driven (code dependent). In this Experiment participants will be unaware of the switch so as to avoid the aforementioned general mechanisms. Additionally, we aim to find similar asymmetric switches between the codes for math and therefore replicate the results from Experiment 1.

Experiment 3 has the objective of knowing the brain bases of switch cost when number words are manipulated and whether they are the same or different than in language and in general switch mechanisms. This experiment has also a secondary goal: observing if the found unbalance for math found in Experiments 1 and 2 is caused by the numerical tasks or contrary, it can also occur in linguistic tasks.

Experiment 4 has a different goal than previous experiments. This experiment is aimed to tests bilingual developmental dyscalculic children and explore distance effects across languages; additionally, brain source estimation of the found ERP distance effects will be performed with MEG to gain knowledge of possible differences in the brain bases for the distance effects for bDD as compared to controls.

Together, these four experiments provide a measure of the brain's electrophysiological response (ERP/MEG) to numerical information and switch cost when items are presented in LL^{math} and in the OL. In the next section, the methods and results of the four presented experiments will be described together with the fundamental conclusions and implications that this study has in the field of Math Cognition and Bilingualism.

Unbalanced Math in Bilingual Minds: Experiments 1 to 3

Experiment 1

Introduction

Independence between language and math has been challenged by proposals suggesting that the acquisition of number symbols and counting modulates the core numerical magnitude system (Halberda, Mazzocco, and Feigenson, 2008; Piazza et al., 2013). Research on bilingual math representations has provided further empirical support by showing that the bilingual numerical system could include linguistic traces (Salillas et al., 2015; Salillas and Carreiras, 2014). Hence, a preference for one of the languages not only in exact arithmetic but also in the fundamental number representations has been suggested. The present study directly investigates this possible lexical unbalance for math: that is, whether there is an unbalanced dominance for the two bilingual numerical lexicons that runs independently of the relative proficiency in general linguistic representations.

Most of the research about the effects of the linguistic component in the math system has focused on exact arithmetic. Several studies suggest that the encoding of exact arithmetic is verbal, thus arithmetic facts are stored verbally and subsequently verbally retrieved (e.g. Dehaene and Cohen, 1995; Delazer and Benke, 1997; Lemer, Dehaene, Spelke, and Cohen, 2003; Spelke and Tsivkin, 2001) but see (Noël et al., 1997; Noel et al., 1998). When more than two number words are available in bilinguals, it appears that only one of the two languages becomes linked to exact calculation (Campbell and Epp, 2004; Campbell et al., 1999; Frenck-Mestre and Vaid, 1993; Marsh and Maki, 1976; McClain and Huang, 1982). In fact, bilinguals often report the inner switch to one of their languages for counting or for arithmetic fact retrieval.

One crucial question is whether that preferred language is also the dominant language, L1 or whether contrarily, bilingualism implies an independent dominance pattern for mathematical representations. This question has not been directly addressed yet. Some studies have addressed the representation of arithmetic facts in bilinguals (Bernardo, 2001; Salillas and Wicha, 2012; Vaid and Menon, 2000), focusing on the role of early learning in arithmetic representations. More recently, we have extended the question further to the impact of each of the languages in core numerical knowledge: in

Salillas and Carreiras (2014) it was suggested that one of the languages might enter into the core of magnitude. This language would correspond to the language used for learning math during early education (LL^{math}). Thus, early math learning would impact not only arithmetic, but also essential numerical knowledge. However, a direct test of the relative dominance between the multiple number words in bilinguals is missing. A classical way to study such dominance pattern is through the observation of the cost of switching between the two verbal codes.

Switch costs are found when bilinguals switch languages, switch trials elicit longer reaction times and differential ERP effects (Costa and Santesteban, 2004; Macizo, Bajo, and Paolieri, 2012; Meuter and Allport, 1999; Moreno, Rodriguez-Fornells, and Laine, 2008; Palmer et al., 2010). Specifically, behavioural studies have reported asymmetric switch costs in bilinguals with unbalanced proficiency. However, when testing balanced bilinguals, language switches are similar in both directions (L2 to L1 and L1 to L2; Duñabeitia et al., 2010). In Meuter and Alport (1999), participants named items in their L1 or L2. Results showed that bilinguals named items faster in their L1 than in their L2 in non-switch trials. However, in switch trials, subjects named items more slowly in their L1 than in their L2. Based on the Inhibitory Control (IC) model by Green (1998), the authors argued that this additional time is needed mainly because the more dominant language (L1) requires more inhibition during L2 naming trials, since inhibition is carried on to the next trial, a switch from L2 to L1 needs to overcome this inhibition, hence making this switch harder than a L1 to L2 switch. Importantly, a different switch pattern seems to appear for balanced bilinguals. Costa and Santesteban (2004) contrasted language switching performance between balanced and unbalanced bilinguals and only the unbalanced group showed asymmetric switch costs. What determines the asymmetry or symmetry according to these studies is the level of proficiency that ultimately modifies the mechanisms of inhibition or selection in the two lexicons in production tasks (Costa and Santesteban, 2004) or the automaticity of the activations for words in the two languages during comprehension (Duñabeitia et al., 2010).

Asymmetric switch costs have frequently been found in ERP studies in at least two different components: the N2 and the N400 (Chauncey et al., 2008; Jackson et al., 2001; Jackson, Swainson, Mullin, Cunnington, and Jackson, 2004; Verhoef et al., 2009)

Being these studies always based on unbalanced bilingual samples, asymmetric switch costs uncover the cost of switching to the dominant language. In turn, these components are differentially modulated by switch direction, with larger amplitudes for the L2-L1 switch than in the L1-L2 direction, as contrasted with the corresponding non-switch conditions. Dissociation between components is also reported for each switch direction, suggesting also neurofunctional differences: Chauncey et al (2008) found a switching effect in the N400 when switching in the L2-L1 direction, and in the N250 in the L1-L2 direction. Overall, behavioural and electrophysiological studies converge in showing that relative proficiency between languages modulates switch costs, and switch asymmetry appears in unbalanced bilinguals. Therefore, an asymmetry between switch directions can be taken as an index of proficiency unbalance between verbal codes.

To test whether the LL^{math} makes a difference in the proficiency pattern for number words in balanced bilinguals, we will distinguish between LL^{math} and the OL as compared to the L1/L2 dichotomy. Such distinction has not been considered previously, albeit using non-numerical tasks (Meuter and Allport, 1999). Here we will sustain that while our bilinguals will be L1/L2 balanced bilinguals in terms of proficiency and general language use; early learning experience (i.e. LL^{math}) is what will determine the proficiency pattern for number word representations based on previous findings (Salillas and Carreiras, 2014; Salillas and Wicha, 2012). Thus, if what determines the relative representational strength of the two numerical lexicons is just overall language proficiency (L1/L2), balanced bilinguals should show symmetric switch costs even when considering the LL^{math}-OL distinction for analysis, because both languages would have equivalent dominance based on general language functioning. However, if a larger switch cost appears in the OL to LL^{math} transition this would index a different dominance pattern for math, based on LL^{math}. Moreover, only an *apparent* symmetric switch cost should appear when the L1/L2 dichotomy is considered, because when L1/L2 is considered for analysis, a latent LL^{math}, that for some participants coincides with L1 while for others coincides with L2, would favour the appearance of comparable switch costs between L1 and L2. We will present two converging studies addressing this hypothesis.

Methods

Participants

Participants were twelve healthy right-handed Spanish – Basque bilinguals (9 females, 3 males, mean age = 22, range = 22-26 years. All of them were exposed to Basque and Spanish before the age of 3. Of the 12 participants, 6 learned math in Basque and the other 6 learned it in Spanish. From the 6 participants who learned math in Basque, 3 were slightly more proficient in Basque. From the 6 participants who learned math in Spanish, 4 were slightly more proficient in Spanish. Therefore, LL^{math} coincided with the more proficient language in 58% of our sample, whereas LL^{math} coincided with the less proficient language in 42% of our sample.

Language assessment

Language proficiency was assessed in two different ways in both languages (Basque and Spanish) which consisted of (1) the Boston Naming Test (Kaplan et al., 1983) (2) another test, the BEST: The Basque English Spanish Test, was developed locally to measure the proficiency in both languages (range 1-77). The procedure of this test was similar to the Boston Naming test. The latter test also included an oral interview which assessed not only the general vocabulary knowledge, but also general fluency and knowledge of the language, i.e. how participants formulated sentences correctly in both languages, verb conjugations, ability to get their messages through, etc. Scoring in the interview went from 0 to 5, being 0 the lowest score (complete lack of the knowledge being tested) to 5 (complete mastery of the language). Additionally, participants also reported percentage of daily use of each language. A summary of these measures can be found in Table 1. Participants reported what they considered their L1 was. Additionally, participants reported which of the languages was LL^{math} (the first language used for math learning in school, in which language they learnt arithmetic and which language they used for counting and calculation).

	LL^{math}=Spanish		LL^{math}=Basque	
	Spanish	**Basque**	**Spanish**	**Basque**
BNT	53	42	50	48
BEST	76	62	76	71
% Daily use	57	43	50	50
Interview	5	4.8	5	4.8

Table 1. Scores in the different language tests (Experiment 1): BNT: the Boston Naming Test (Kaplan et al., 1983). **BEST:** The Basque English Spanish Test, developed locally to measure the proficiency in the three languages (range 1-77). **% of Daily use:** The approximate percentage of daily use of the language reported by the participants. **Interview**: A personal interview with the participants in which their general language skills were measured and scored from 1 (the lowest) to 5 (the highest).

Stimuli and Procedure

Stimuli consisted of six numbers in their verbal form. Numbers ranged between *3* and *9*, using *6* as the reference. Stimuli were randomized with the trials always following this order: AABBAA where 'A' and 'B' are the languages in which the numbers were presented (Basque and Spanish). Therefore, there were two non-switch conditions ("AA" or "BB"), and two switch conditions ("AB" and "BA") which will be predictable by participants as opposed to Experiment 2 in which the switch will be unpredictable. A total of 480 trials were created which were presented (as depicted in Figure 5) in two formats: one half (240) in Spanish (e.g. *"cinco"- five*), and one half (240) in Basque (e.g. "*bost"- five*). From the total of 480 trials, 120 were number words in Basque preceded by Spanish number words, 120 were Spanish preceded by Basque, 120 Spanish preceded by Spanish, and 120 Basque preceded by Basque. Distance between the presented numbers and the reference number 6 was controlled for and the same numbers appeared in each of the languages.

Participants were in a sound-attenuated chamber. The stimuli were presented using Presentation software (version 14.7) on the center of a monitor located 60 cm from the participant. Each sequence began with a fixation point ('*') appearing for 1300 ms, which was immediately followed by the stimuli, appearing for 500 ms and followed by a blank screen for 1500 to 2000 ms, the maximum time to respond. Participants were asked to perform a number comparison task, responding whether the number word displayed on the screen was *bigger or smaller than* 6 by using one of the two buttons of a Logitech precision gamepad. In the case that a response was not given, the following trial started. The number 6 was used as reference because it is a cognate for Spanish (*seis*) and Basque (*sei*).

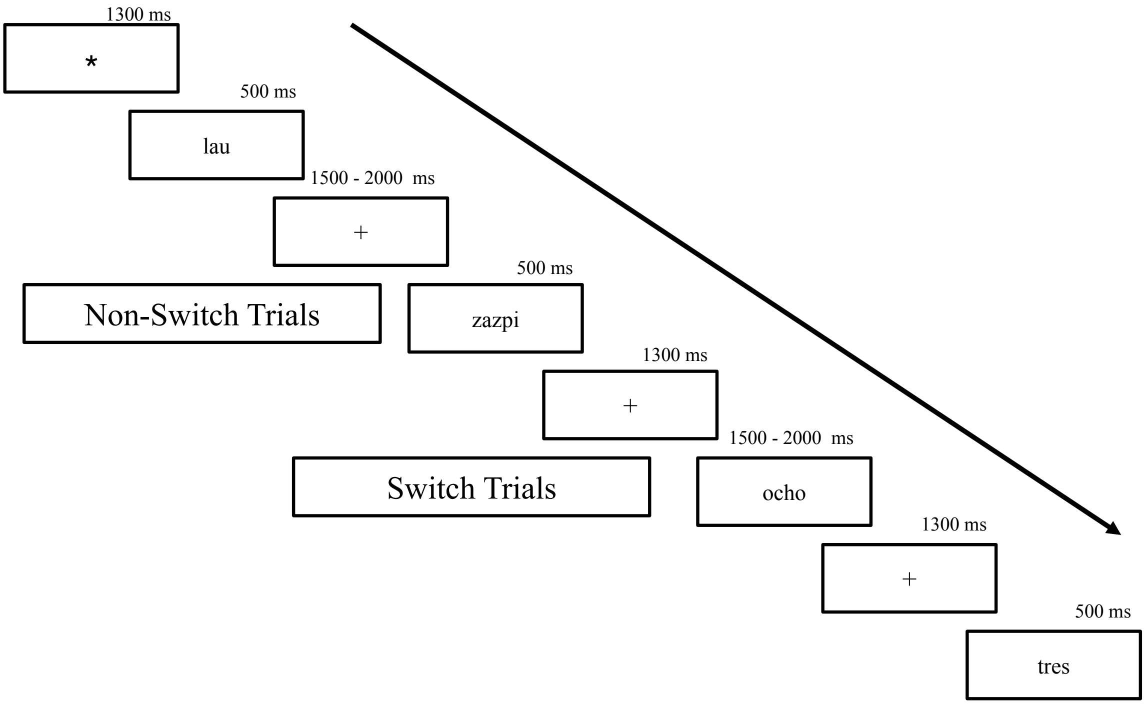

Figure 5. Example of trials (Experiment 1). Participants had to compare numbers as smaller as or bigger than 6. At the beginning of each trial an asterisk (*) was presented for 1300 ms. In each trial the stimulus was presented for 500 ms, then an inter-trial screen containing a cross "+" appeared for an interval between 1500 and 2000 ms, time in which the participant had to give the answer. Stimuli were Basque (lau-*four*, zazpi-*seven*) and Spanish (ocho-*eight*, tres-*three*). Stimuli were divided into 4 conditions: 2 non-switch conditions (LL^{math} items followed by LL^{math} items, OL items followed by OL items) and 2 switch conditions (LL^{math} items followed by OL items, OL items followed by LL^{math} items).

EEG recording and analyses

The EEG was recorded from 27 scalp electrodes embedded in an Easy-Cap in a 10-system array, which was referenced online to the left mastoid. Six free electrodes were used to record blinks (below the eye), horizontal eye movements (outer canthi). Electrode impedances were maintained below 5 kΩ. The EEG was amplified with Brain Amp amplifiers, with the band pass set from 0.01 to 100 Hz, and sampled at a rate of 1000 Hz. The output of the amplifiers was fed into a 32 channel 12-bit analogue-to-digital converter on a PC computer. *Brain Vision Recorder* software was used to deliver event codes to the data acquisition PC synchronously with the onset of EEG activity to the events of interest. Data were re-referenced off-line to the algebraic sum of the left and right mastoids, and subsequently averaged for each experimental condition and time-locked to the onset of the second number. A digital band-pass filter set from 0.1 to 30 Hz was used on all of the data prior to running analyses to reduce high frequency content that was irrelevant to the components of interest. Baseline correction used the 100 ms pre-stimulus. Trials with artifacts due to eye movements, excessive muscle activity, or amplifier blockage were eliminated offline before averaging. Artifact rejection criteria were a minimum to maximum baseline-to-peak allowed voltage of +-70 µV, a maximum voltage gradient of 75 µV per sample point, a maximal difference of 150 µV in intervals of 100 ms and a minimum voltage of 0.5 µV in intervals of 50 ms. All electrodes were assessed for artifacts. Analyses were reported for each critical stimulus relative to a 100 ms pre-stimulus baseline.

Mean amplitudes of relevant latency bands were first analyzed in a 27 (electrode) x 2 (Switch: switch/non-switch) x 2 (Direction of switch) repeated measures ANOVA. When an interaction by electrode was shown, ANOVAS were performed for the electrodes in which this interaction was significant. In a first analysis, the contrast L1/L2 was considered. Proficiency was considered taking into account the measures displayed in Table 1. Items were classified as L1 or L2 items depending on each participant's L1 and L2. In other words, for those participants whose L1 was Basque the conditions L1-L1, L2-L1, L2-L2 and L1-L2 were Basque-Basque, Spanish-Basque, Spanish-Spanish and Basque-Spanish, respectively. In contrast, for participants whose L1 was Spanish the conditions L1-L1, L2-L1, L2-L2 and L1-L2 were Spanish-Spanish, Basque-Spanish, Basque-Basque and Spanish-Basque, respectively. The four conditions were collapsed between languages for analysis. Contrasts for *switch costs* entailed the contrast between the switch condition and the non-switch condition for each of the directions (to L1: L2-L1 vs. L1-L1 and to L2: L1- L2 vs. L2- L2).

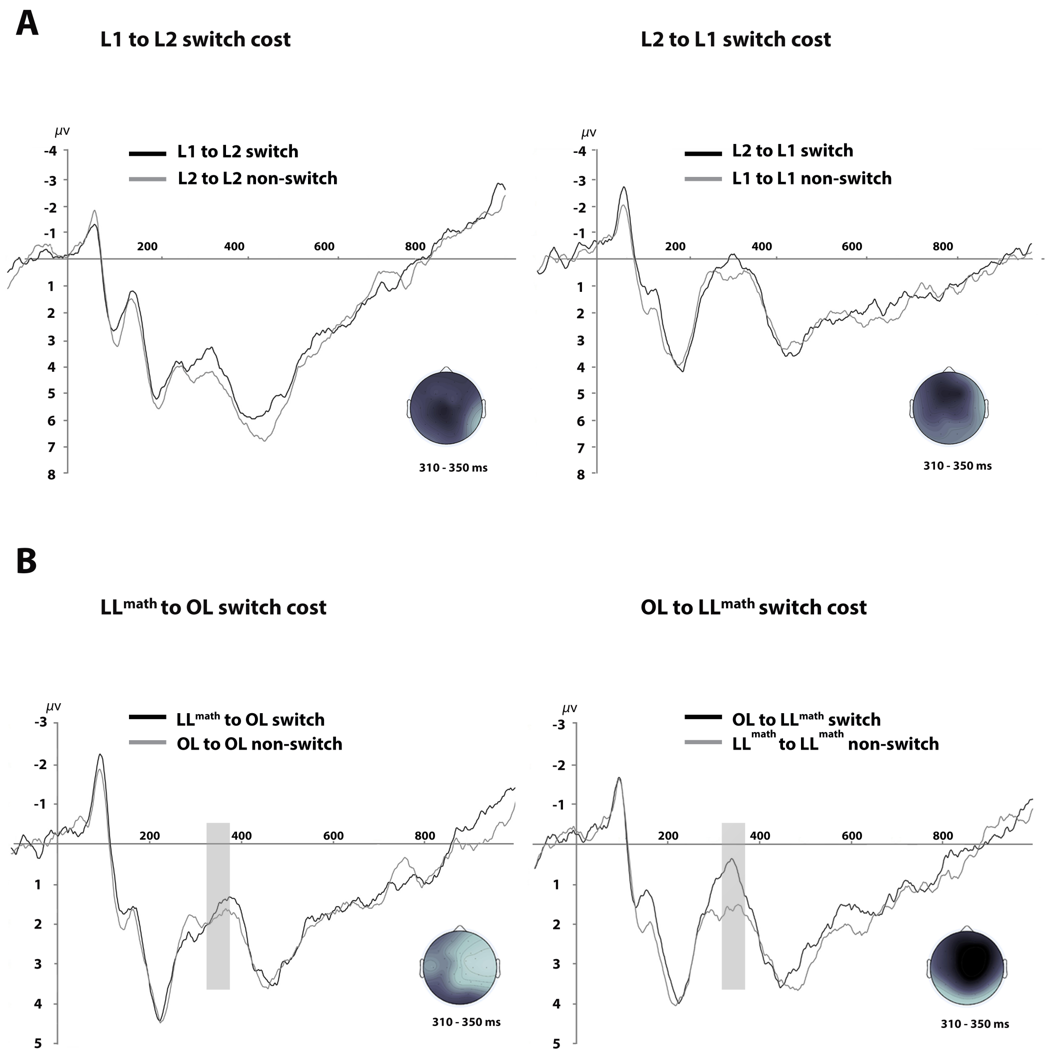

Figure 6. N400 switch cost effect (Experiment 1). a) N400 switch cost effect for L1 to L2 (left) and for L2 to L1 (right). **b)** An asymmetric switch cost is found between 310-350 ms.: the switch cost only appeared for the OL to LL^{math} switch condition versus non-switch (right) compared to the LL^{math}–OL switch condition versus non-switch (left).

In a second analysis, the contrast LL^{math}-OL was considered from participants whose LL^{math} was Basque (conditions LL^{math}-LL^{math}, OL-LL^{math}, OL-OL, LL^{math}-OL, i.e., Basque-Basque, Spanish-Basque, Spanish-Spanish, Basque-Spanish, respectively), and for participants whose LL^{math} was Spanish (conditions LL^{math}-LL^{math}, OL-LL^{math}, OL-OL, LL^{math}-OL, i.e., Spanish-Spanish, Basque-Spanish, Basque-Basque, Spanish-Basque, respectively). The four conditions were collapsed between languages for analysis, providing exactly the same stimuli between conditions. Contrasts for *switch costs* entailed the contrast between the switch condition and the non-switch condition for each of the directions (to LL^{math}: OL-LL^{math} vs. LL^{math}-LL^{math} and to OL: LL^{math}-OL vs. OL-OL).

Results

A visual analysis of the ERP data showed that a component peaking between 300 and 400 ms was overall more negative for the switch conditions than for the non-switch conditions. This difference started at 310 ms and ended at 350 ms as revealed by a consecutive 50 ms time windows analysis contrasting all switch vs. non-switch trials.

Symmetric switch costs between L1 and L2 (N350 310-350 ms): The electrode x switch x direction ANOVA considering the L1/L2 distinction showed a main effect of switch (F (1, 11) = 16.049, p=.002), and no significant direction x switch interaction (F (1,11) =0.07, p>.250), thus showing a symmetric switch cost when the L1–L2 distinction was considered (see Figure 6a).

Asymmetric switch costs between LL^{math} and OL (N350 310-350 ms.): The electrode x switch x direction ANOVA considering the LL^{math}-OL distinction revealed however, asymmetric switch costs (electrode x switch x direction interaction: *F* (1,26) = 3.826, p<0.0001). Two similar analyses taking the 27 electrodes x switch for each direction (one for OL-LL^{math} vs. LL^{math}-LL^{math} and another for LL^{math}-OL vs. OL-OL) were carried out separately: the switch to LL^{math} analysis showed a significant switch cost F (1,11) = 8.381, p=.015. But non-switch cost appeared for the opposite direction, to OL (switch effect *F* (1,11) =.224, p > .250). A significant switch cost for the OL-LL^{math} vs. LL^{math}-LL^{math} direction appeared distributed in most part of the electrodes (Fp1, Fp2, F3, F4, C3, C4, P3, F8, T8, T7, Fz, Cz, Pz, FC1, FC2, CP1, CP2, FC6; F (1,11) = 9.02, p=.012). In turn, switch trials were more negative than the non-switch trials only for the switch to LL^{math} (see Figure 6b).

To further ensure that these switch costs were really dependent on the participants' LL^{math} and not on an actual proficiency in LL^{math} vs. OL, we performed correlation tests to the relative LL^{math}-OL proficiency (difference between BNT in LL^{math} minus BNT in OL). These analyses showed no correlation between the relative proficiency in LL^{math} versus OL and the N400 switch cost difference (r=.310, n=12, p=.326).

Discussion

The switch asymmetry found in the ERPs agrees with previous literature in unbalanced bilinguals when measuring L1-L2 switching (Chauncey et al., 2008; Jackson et al., 2001, 2004; Verhoef et al., 2009). Crucially however, the results point to an LL^{math} dominance over the OL in perfectly L1/L2 balanced bilinguals. In turn, it seems that the found unbalance is due to the LL^{math}-OL dichotomy and not the L1/L2 dichotomy. As predicted, an only apparent symmetric switch cost appeared when considering L1/L2 for analyses, given that L1= LL^{math} for the 58% of the participants, whereas L2=LL^{math} for 42% of the participants.

These results are similar to those of Jackson et al. (2001), both in the location and in the timing of the switch cost effects. Although they consider their results as part of a late N2, we believe that they suit better in an N400 in accordance to the existent literature on code-switching (Christoffels, Firk, and Schiller, 2007; Jackson et al., 2001; Verhoef et al., 2009).

These results suggest that it is not the L1/L2 distinction we need to have into account when building models of mathematical representation but the LL^{math}-OL dichotomy, since perfectly balanced bilinguals who do not show a switch asymmetry when data in collapsed based on their L1 dominance, exhibit an asymmetry when collapsed based on an LL^{math} dominance.

Experiment 2

Introduction

Experiment 2 had the same experimental design and goals as Experiment 1, however, it implied a very different procedure: (1) instead of using an explicit manipulation of quantitative information through numerical comparison, a parity task was used. Judgements about parity imply the classification of numbers as odd of even irrespective of their numerical magnitude. Thus, only implicit activation of magnitude occurs (Dehaene et al., 1993; Gevers et al., 2010; Gevers, Verguts, Reynvoet, Caessens, and Fias, 2006). (2) Instead of using an AABBAA language presentation sequence, in which the switch is predictable, the language presentation sequence was random. According to previous studies (Chauncey et al., 2008) this implies that lexical representations should drive possible switch costs. (3) Finally, the procedure of presentation of the prime in this experiment was masked. Masked priming would uncover switch costs driven by lexical representational strengths guiding cognitive drawbacks during language switching (Altarriba and Basnight-Brown, 2007; Chauncey et al., 2008; Chauncey, Grainger, and Holcomb, 2011; Grossi, 2006). Hence, this experiment was designed with the goal of gaining precision in the uncovering of relative lexical strengths, by unlinking our measures from numerical and non-numerical executive processes. However, our predictions regarding LL^{math}-OL asymmetries were the same than in Experiment 1.

Methods

Participants

Fourteen healthy right-handed Spanish-Basque bilinguals (9 female, 5 males, mean age = 24.26 years, range = 19-34 years) participated in this experiment. All of them were early bilinguals, exposed to Basque and Spanish before the age of 3. Of the 14 participants, 7 learned math in Basque and the other 7 learned it in Spanish. From the 7 participants who learned math in Basque, 3 were slightly more proficient in Basque, from the 7 participants who learned math in Spanish, 6 were slightly more proficient in

Spanish. Therefore in 64% of our sample LL^{math} coincided with the more proficient language and in 36% of our sample, LL^{math} coincided with the less proficient language.

Language assessment

Language proficiency was assessed with the same three different measures in both languages (Basque and Spanish) as in Experiment 1 (see Table 2).

	LL^{math}=Spanish		**LL^{math}=Basque**	
	Spanish	Basque	Spanish	Basque
BNT	54	48	52	50
BEST	77	62	76	74
% Daily use	60	40	49	51
Interview	5	4.8	4.8	5

Table 2. Scores in the different language tests (Experiment 2).

Stimuli and Procedure

Stimuli were similar to Experiment 1, consisting of numbers in their verbal form. The numbers used in the experiment ranged between *1* and *9*, excluding *6 as it is a cognate number*. Stimuli could be presented in two formats: in Spanish (e.g. *"cinco"*-*five*), or in Basque (e.g. "*bost"*- *five*).

Participants were asked to perform a parity task so that they classified the numbers as odd or even. Each sequence began with a forward mask composed of hash-marks (########) displayed during 500ms. The forward mask was replaced at the same location on the screen by a lower case prime item for 40 ms. The prime was immediately replaced by the target in uppercase letters that remained in the screen for 1000 ms. All target words were followed by an asterisk (*) to indicate when the participants should respond (see Figure 7 for a schema of the trials in the task). A total of 504 trials were created, one half (252) were Basque trials, and the other half (252) were Spanish trials. These trials were again divided into switch trials (126 Basque-Spanish switch trials and 126 Spanish-

Basque switch trials); and non-switch trials (126 Spanish-Spanish non-switch trials and 126 Basque-Basque non-switch trials) and distance was equated across all the trials. The design was identical to Experiment 1: 2 (switch: switch / non-switch) x 2 (direction: to LL^{math} to OL) x 27 (electrode).

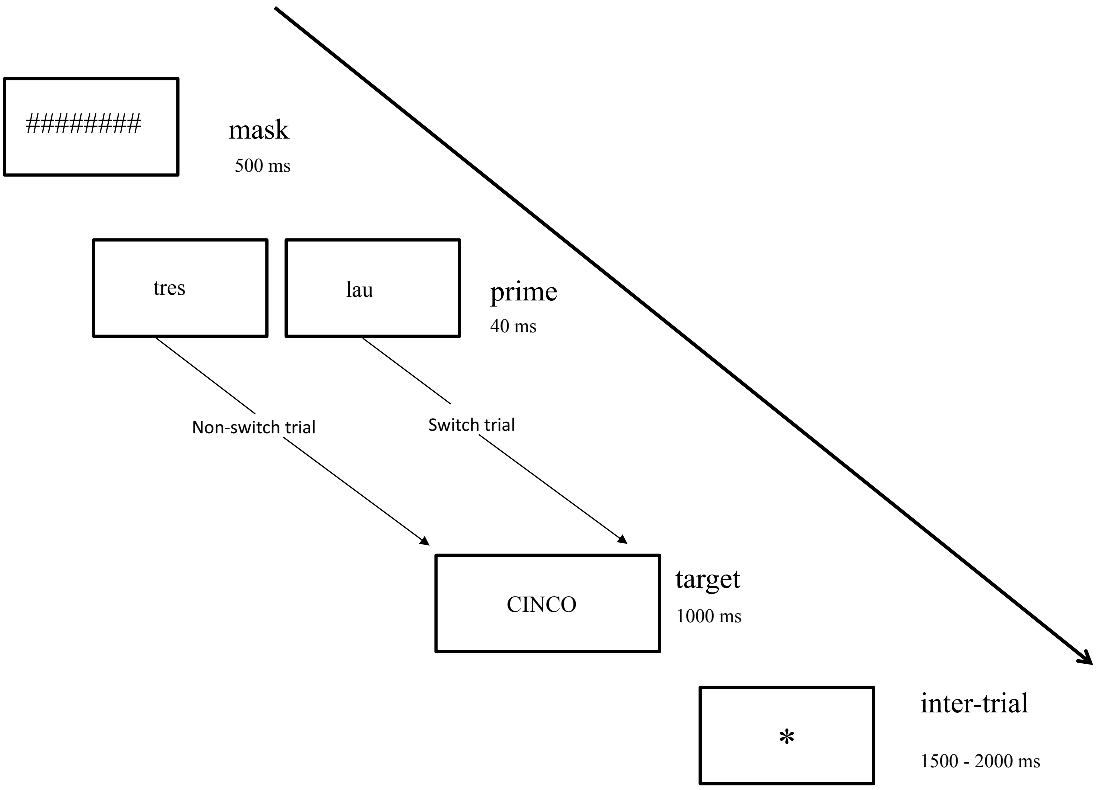

Figure 7. Example of trials (Experiment 2 – masked priming paradigm). Participants had to classify numbers as odd or even. Each trial consisted of a mask (########) appearing for 500 ms. followed by a prime lasting for 40 ms, and finally the target lasted for 100 ms. An inter-trial asterisk appeared for between 1500-2000 ms, when participants had to provide the delayed response. The prime could be in the same or different language as the target, but they were never the same items or translations. As in Experiment 1 Stimuli were Basque (e.g., lau-*four*, zortzi-*eight*) and Spanish (e.g., cinco-*five*, tres-*three*); and there were 4 conditions: 2 non-switch conditions (LL^{math} primes followed by LL^{math} targets; OL primes followed by OL targets) and 2 switch conditions (LL^{math} primes followed by OL targets, OL primes followed by LL^{math} targets). The figure depicts an example of a switch and non-switch trial in Spanish, but in the task targets were both in Spanish and in Basque.

EEG recording and analyses

EEG recording and analyses were the same as in Experiment 1.

Results

As in Experiment 1, visual inspection of the data showed a negativity between 200 and 250 ms, and 300 and 500 ms modulated by switching, with larger negativity for the switch conditions than for the non-switch conditions. Consecutive time windows of 50 ms contrasting all switch vs. non-switch trials determined the exact latency period for analysis (400 to 450 ms) and no significant effects were found for the 200-250 ms time window.

Symmetric switch costs between L1 and L2 (N400: 400-450 ms): The first ANOVA including the 27 electrodes including switch and switch direction as factors revealed a main effect of switch (F (1, 11) = 16.049, p=.002), and no significant direction x switch interaction (F (1,11) =0.07, p>.250), thus showing a symmetric switch cost when the L1 – L2 distinction was considered (see Figure 8).

Asymmetric switch costs between LL^{math} and OL (N400: 400-450 ms): The first ANOVA including the 27 electrodes including switch and switch direction as factors revealed a switch x direction interaction (F (1,13) = 8.82, p=0.05). The switch cost effect was restricted to the OL to LL^{math} switch in the following electrodes: FC2, Cz, CP2, Fz, FC1, Pz, F3, C4, F4, CP1, P4, C3, FC6, CP6, FC5, T8, F8, Fp2 (F (1,13) = 8.82, p= 0.011). The LL^{math} to OL switch showed no significant cost (F (1,13) = 0.17, p>.250). *S*uggesting that the switch trial was also significantly more negative than the non-switch trial only in the switch to LL^{math}. Therefore, an asymmetric switch cost was also found here (see Figure 8).

By repeating the analysis procedure from Experiment 1 we ensured that these switch costs were really dependent on the participants' LL^{math} and not on their L1; the same correlation tests showed no correlation between the relative proficiency in LL^{math} versus OL and the N400 switch cost difference (r=.383, n=14, p=.176).

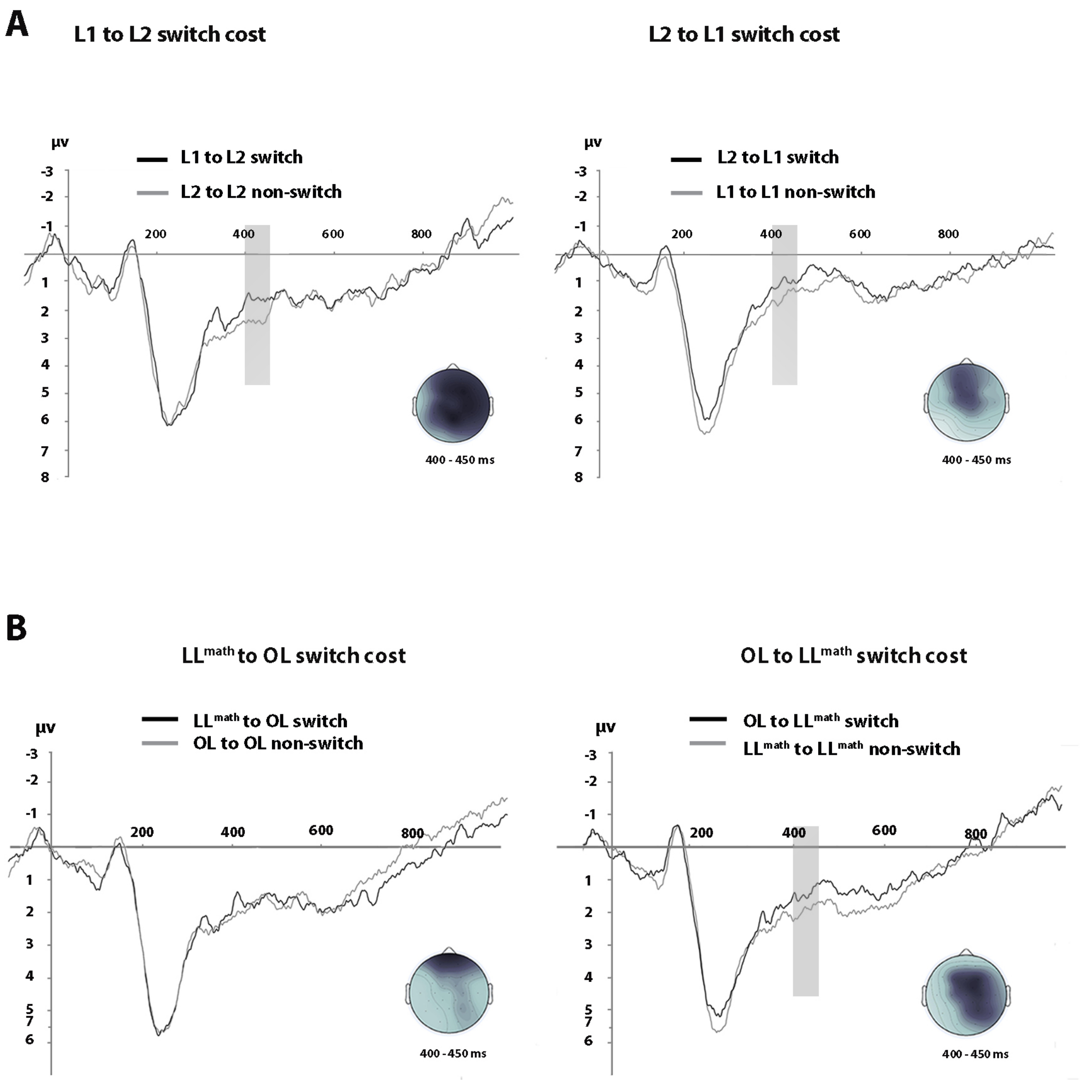

Figure 8. N400 switch cost effect (Experiment 1). a) N400 switch cost effect for L1 to L2 (left) and L2 to L1 (right) vs. their respective non-switch conditions OL-OL (left) LL^{math}-LL^{math} (right). The switch is similar in both directions (i.e. symmetric switch); **b)** N400 switch cost effect for LL^{math} to OL (left) and OL to LL^{math} (right) vs. their respective non-switch conditions OL-OL (left) LLmath-LL^{math} (right) An asymmetric switch cost is found between 400-450 ms in the OL to LL^{math} switch condition versus non-switch (right) compared to the LL^{math} –OL switch condition versus non-switch (left).

Discussion

This second experiment replicates the results of Experiment 1, even though a parity task was used and, thus, participants did not explicitly manipulate quantity. There is again an N400 effect in the OL-LL^{math} direction. The most striking fact is that in this experiment the switch was masked and therefore, unconscious for the participants. This unconscious switch was intentionally designed in the experiment in order to find out whether the results of Experiment 1 were due to a lexically imbalance in the codes for math. All these factors favoured overall, an interpretation of switch costs as lexically driven. Hence, different strengths in the lexical representations for LL^{math} and OL can be suggested. This lexical unbalance dissociates from the L1/L2 dichotomy for general language use.

Our effects were moreover localized in the N400 component, which is a component that is sensitive to lexico-semantic variables (Alvarez, Holcomb, and Grainger, 2003; Hoshino, Midgley, Holcomb, and Grainger, 2010; Moreno, Federmeier, and Kutas, 2002; Van Der Meij, Cuetos, Carreiras, and Barber, 2011). This fact also speaks in favour of lexically driven switch cost effects (Chauncey et al., 2008).

Experiment 2 also replicated the symmetric switch costs during the consideration of the L1/L2 dychotomy. Attending to the asymmetrical switch found contrarily, i.e. when considering the LL^{math} - OL dichotomy, the symmetry must be taken just as an apparent main effect. Similar to Experiment 1, LL^{math} was equal to L1 for the 64% of our sample, whereas LLmath was equal to L2 for the 36% of our sample, leading again to a symmetrical effect when grouping items according to L1/L2 for analyses.

Experiment 3

Introduction

The goal of this experiment was to find the neural networks responsible for the mechanisms underlying the costs of switching between the two codes for math. In Experiments 1 and 2 we found an asymmetric switch cost which is similar to those found in language in unbalanced bilinguals in ERPs (e.g. Chauncey et al., 2008; Jackson et al., 2001; Moreno et al., 2010); this asymmetry was due to the $LL^{math/}OL$ and not to the L1/L2 dichotomy that is reported in these studies. So, the next logical step is to find out if the neural networks for the switch in the mathematical codes are the same as the ones used in general language switching. Studies examining language switching at the neuronal level show that the regions implicated in these switches are usually frontal areas as the dorsolateral prefrontal cortex (DLPFC) (Hernandez, Dapretto, Mazziotta, and Bookheimer, 2001a; Hernandez et al., 2000; Rodriguez-Fornells, De Diego Balaguer, and Münte, 2006; Wang et al., 2007) or left anterior prefrontal regions including pars triangularis (Brodmann areas 45 and 9). Rodriguez-Fornells et al. (2002) reported language-switching involving the left ACC as well (Abutalebi et al., 2008; Crinion et al., 2006; Van Heuven, Schriefers, Dijkstra, and Hagoort, 2008; Wang et al., 2007)

Experiment 3 measured magnetic brain activity during a similar paradigm than Experiment 2. The use of Magnetoencephalography (MEG) allows to estimate where in the cortex the switch-costs are originated, while preserving the same good temporal resolution provided by EEG. Experiment 3 implies a new task however: it is a linguistic task (lexical decision) that will further test the task independency of the observed effects. That is, if such described unbalances were due to the use of numerical tasks (number comparison and parity judgments), no such asymmetry dependent on early learning should appear.

Generally, the description of the neural basis for switch costs between LL^{math} and OL can provide information in two main ways: (1) It can inform us whether the same mechanisms of switching apply to switching between number words and hence; (2) if the mechanisms are the same, then the origin of the predicted asymmetries would in fact be caused by different representations and not to different control mechanisms for the control of number word activations. That is to say, that for number words a different relative dominance between codes would appear in bilinguals but the control mechanisms applied to the lexical systems would not differ.

Methods

Participants

Participants were twelve healthy right-handed Spanish – Basque bilinguals (6 females 6 males), mean age = 26, range = 21-30 years. All of them were exposed to Basque and Spanish before the age of 3. Of the 12 participants, 6 learned math in Basque and the other 6 learned it in Spanish.

Language assessment

Language proficiency was assessed with the same three different measures in both languages (Basque and Spanish) as in Experiment 1 (see Table 3).

	LL^{math}=Spanish		**LL^{math}=Basque**	
	Spanish	Basque	Spanish	Basque
BNT	54	49	54	55
BEST	77	62	71	72
% Daily use	57	43	60	40
Interview	5	5	5	5

Table 3. Scores in the different language tests (Experiment 3).

Stimuli and Procedure

Stimuli were similar to Experiment 1 and 2, consisting of numbers in their verbal form. The numbers ranged as in Experiment 2 (between *1* and *9*, and excluding *6* since it is a cognate number). And again, stimuli could be presented in two formats: in Spanish (e.g. *"cinco"-five*), or in Basque (e.g. "*bost"- five*).

In this experiment participants were asked to perform a lexical decision task so that they classified the items appearing on the screen as words or non-words. The masked priming sequence was equal to that of the Experiment 2: the sequence begun with a forward mask composed of hash-marks (########) displayed during 500ms. The forward mask was replaced at the same location on the screen by a lower case prime item for 40 ms. The prime was immediately replaced by the target in uppercase letters that remained in the screen for 1000 ms. All target words were followed by an interrogation (?) to indicate when the participants should respond (see Figure 9 for a schema of the trials in the task). A total of 384 trials were created, one half (192) were Basque trials, and the other half (192) were Spanish trials. These trials were again divided into switch trials (96 Basque-Spanish switch trials and 96 Spanish-Basque switch trials); and non-switch trials (96 Spanish-Spanish non-switch trials and 96 Basque-Basque non-switch trials) and distance was equated across all the trials Additionally, a total of 640 distractors were included; these distractors were pseudo-words derived from the numbers used in the stimuli changing letters in a range from two to four letters in order to make them pseudo-words (e.g. *tres* - *fres*), but none of these trials were included in the analysis (see Figure 9).

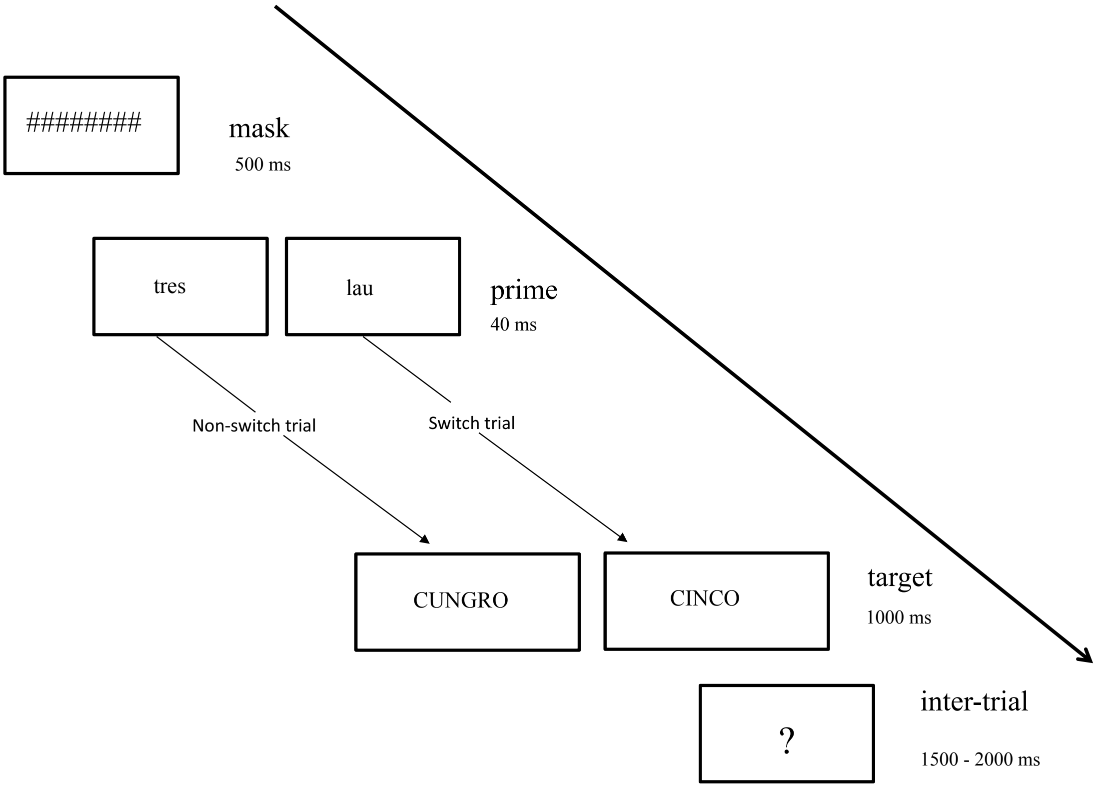

Figure 9. The design of this experiment was the same as in Experiment 2. But in this case the target could be a pseudo-word (*cungro*) or a word (*cinco- five in Spanish*). The figure depicts an example of a trial with Spanish as a target, but in the task targets were both in Spanish and in Basque.

MEG analyses

MEG data were acquired in a magnetically shielded room using the whole-scalp MEG system (Elekta-Neuromag, Helsinki, Finland) installed at the BCBL. The system is equipped with 102 sensor triplets (each comprising a magnetometer and two orthogonal planar gradiometers) uniformly distributed around the head of the participant. Head position inside the helmet was continuously monitored using four Head Position Indicator (HPI) coils. The location of each coil relative to the anatomical fiducials (nasion, left and right preauricular points) was defined with a 3D digitizer (Fastrak Polhemus, Colchester, VA, USA). This procedure is critical for head movement compensation during the data recording session. Digitalization of the fiducials plus ~100 additional points evenly distributed over the scalp of the participant were used during subsequent data analysis to spatially align the MEG sensor coordinates with T1-weighted MPRAGE magnetic resonance brain images acquired on a 3T magnetic resonance imaging (MRI) scan (Siemens Medical System, Erlangen, Germany). MEG recordings were acquired continuously with a bandpass filter at 0.01-330 Hz and a sampling rate of 1 kHz. Eye-movements were monitored with two pairs of electrodes in a bipolar montage placed on the external chanti of each eye (horizontal electrooculography (EOG) and above and below right eye (vertical EOG)). Source reconstruction in the cortical surface and volumetric segmentation was reduced to 15000 vertices in order to simplify analyses, and performed with the Freesurfer image analysis suite, which is documented and freely available for download online (http://surfer.nmr.mgh.harvard.edu/). Briefly, this processing includes motion correction and averaging of multiple volumetric T1 weighted images (when more than one is available), removal of non-brain tissue using a hybrid watershed/surface deformation procedure automated Talairach transformation, segmentation of the subcortical white matter and deep gray matter volumetric structures (including hippocampus, amygdala, caudate, putamen, ventricles). Freesurfer morphometric procedures have been demonstrated to show good test-retest reliability across scanner manufacturers and across field strengths (Han et al., 2006; Reuter et al., 2012).

Data pre-processing

To remove external magnetic noise from the MEG recordings, data were preprocessed off-line using the temporal Signal-Space-Separation method (Taulu and Kajola, 2005) implemented in Maxfilter 2.1 (Elekta-Neuromag). MEG data were also corrected for head movements, and bad channels were substituted using interpolation algorithms implemented in the software. Subsequent analyses and heartbeat and EOG artifacts and data analysis was performed with Brainstorm (Tadel, Baillet, Mosher, Pantazis, and Leahy, 2011), which is documented and freely available for download online under the GNU general public license (http://neuroimage.usc.edu/brainstorm).

Source estimation analyses (MNE):

The method used to estimate the sources distributed in the cortex was MNE (Gramfort et al., 2014) based on all the sensors for each participant/condition average time course. We used the T1-weighted MRI scans for each individual. Analyses were performed individually using the standard weighted minimum norm estimate (wMNE) generating a source model for the 15.000 vertices. Before the wMNE source reconstruction was calculated, the covariance matrix required for computing the wMNE source reconstruction was obtained based on the data during a 100 ms baseline period. An overlapping spheres method was used to estimate the forward model. Estimated source activations were standardized using a Z-score transformation with respect to the average and standard deviation of the source activity during the 100 ms baseline period. The Z-score source space activity was then projected to a template (ICBM152 anatomy) for all subjects/condition and averaged for visualization. Relevant latency bands (300-500 ms) from the Event Related Field (ERF) analyses (see below) were averaged across time in the Z-score source files and exported for each condition/participant into 3D activation maps for statistical analyses at the group level. These analyses consisted of n=12 F-tests (switch ≠ non-switch) using SPM8 with an uncorrected threshold of $p<0.001$.

ERF analyses

Sensor clusters and latency bands were identified from previous studies and from Experiments 1 and 2 (Blanco-Elorrieta and Pylkkänen, 2016; Chauncey et al., 2008; Christoffels et al., 2007; Duñabeitia et al., 2010; Jackson et al., 2001). MEG sensors were regrouped into a total of eight clusters, that is four clusters by hemisphere (temporal left, temporal right, paracentral left, paracentral right, parietal left, parietal right, frontal left, frontal right). Based on Experiment 1 and 2 results, a window between 300 and 500 ms was considered for the analyses. This time window was divided into 50 ms smaller windows to better capture the effects (similar to Experiments 1 and 2). T-tests were performed in each time window comparing mean amplitudes of relevant latency bands in each cluster. Again, the contrast LL^{math}-OL was considered from participants whose LL^{math} was Basque (conditions LL^{math}-LL^{math}, OL-LL^{math}, OL-OL, LL^{math}-OL, i.e., Basque-Basque, Spanish-Basque, Spanish-Spanish, Basque-Spanish, respectively), and for participants whose LL^{math} was Spanish (conditions LL^{math}-LL^{math}, OL-LL^{math}, OL-OL, LL^{math}-OL, i.e., Spanish-Spanish, Basque-Spanish, Basque-Basque, Spanish-Basque, respectively). The four conditions were collapsed between languages for analysis, providing exactly the same stimuli between conditions. Contrasts for *switch costs* entailed the contrast between the switch condition and the non-switch condition for each of the directions (to LL^{math}: OL-LL^{math} vs. LL^{math}-LL^{math} and to OL: LL^{math}-OL vs. OL-OL).

Results

Source space

In order to observe the evolution in the brain sources the original 300-500 ms window was split into four smaller windows (300-350, 350-400, 400-450, 450-500). The first time-window that showed a switch effect was the **400-450** showing activation in the left middle frontal gyrus and anterior cingulate gyrus (BA32). In the **450-500** ms time window similar effects were revealed in the dorsolateral prefrontal cortex (DLPFC) (see statistical parametric maps in Figure 10 and exact peak coordinates in Table 4).

TIME (ms)	Brain region	Peak MNI coordinates (x,y,z)	T-value	Z_0-value	p-value
400-450	*BA 32 (Anterior Cingulate Gyrus)*	-2, 42, 16	3.98	3.42	.000
	Left Middle Frontal Gyrus	-22, 4, 54	3.70	3.29	.000
450-500	*BA 9 (Dorsolateral Prefrontal Cortex)*	-54, 32, 26	3.79	3.29	.000

Table 4. Main effects in the two different time windows (400-450 and 450-500 ms) showing the regions where the effect of switch was located, their coordinates and their respective statistical values (T, Z_0 and p-value)

ERFs

A visual analysis of the ERF data showed that a component peaking between 300 and 500 ms was more negative for the switch conditions than for the non-switch conditions (see Figure 11). This difference started at 400 ms and ended at 500 ms as revealed by a consecutive 50 ms time windows analysis (i.e. 400-450 and 450-500 ms) contrasting all switch vs. non-switch trials. The t-tests in each region showed a switch effect only for the OL-LL^{math} vs. LL^{math}-LL^{math} switch in the 400-450 ms time window in the left frontal area $t=-2.289$, $p= .043$ and in the 450-500 ms the left temporal areas showed an effect of switch t =2.487, p=.030. However, the t-test in the LL^{math} –OL switch direction did not show any significant results in any region.

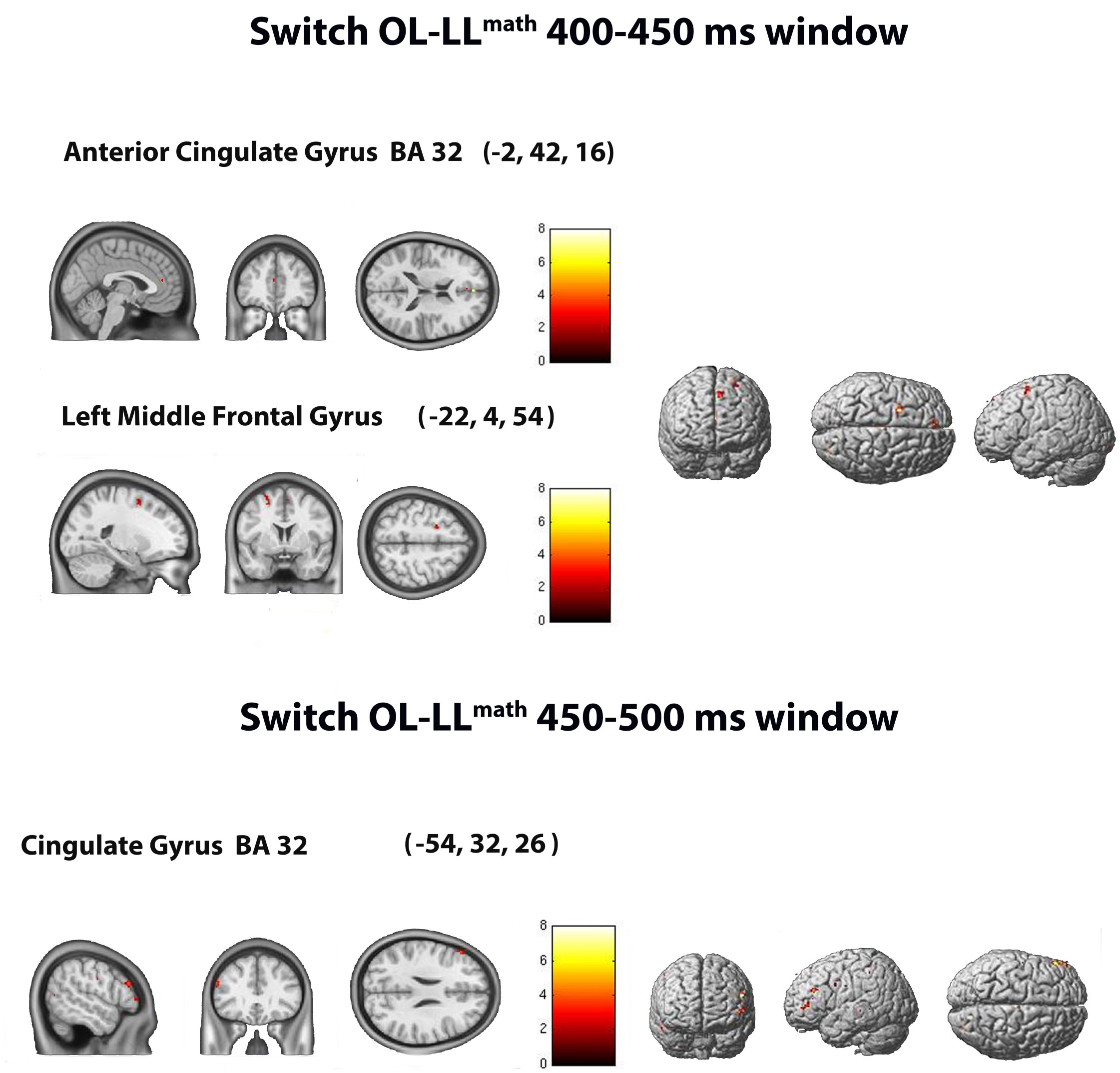

Figure 10. The source analyses revealed main switch effects around 400ms in the left hemisphere. The first time-window (400-450 ms) showed the effect in the anterior cingulate gyrus and left middle frontal gyrus. In the second time-window, the effects were similar to those in the previous time-window in the left inferior frontal gyrus.

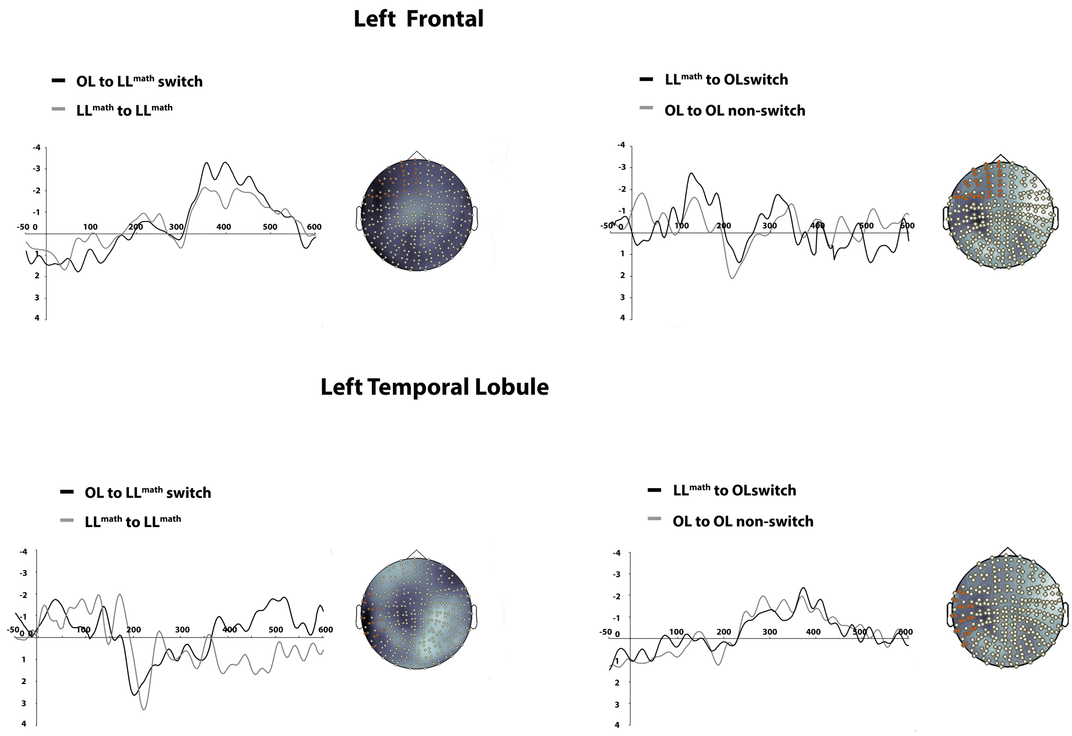

Figure 11. Event related fields (ERF) showing the asymmetry in the switch comparing the OL-LL$^{\mathbf{math}}$ (left part of the figure) versus the LL$^{\mathbf{math}}$-OL switch (right part of the figure). The OL-LL^{math} direction showed the asymmetry in the switch in two different time windows (400-450 and 450-500 ms). In the top of the image the switch effect was found in the left-frontal cluster in the 400-450 ms time window. In the bottom of the image the effect was found in the left temporal cluster for the 450-500ms time window.

Conclusions

This experiment addressed the neurophysiological effects of the switch cost between the two codes for math and investigated the anatomical basis of these costs. We contrasted the switch versus the non-switch conditions in both the LL^{math} and OL in a lexical decision task.

The results in this experiment nicely converge with the results in Experiments 1 and 2. We found an asymmetry in the switch costs being the OL-LL^{math} switch direction the one showing the switch cost. Additionally, the source estimation analyses helped to estimate the brain regions behind this switch and localized it in the left hemisphere, more concretely in frontal and temporal regions. These results are in accordance with previous studies observing the neuroanatomical bases of language switches for language (Abutalebi et al., 2008; Abutalebi and Green, 2007; Blanco-Elorrieta and Pylkkänen, 2016; Hernandez et al., 2001a, 2000; Rodriguez-Fornells et al., 2005; Wang et al., 2007). The switches are located in the anterior cingulate gyrus similar to Blanco-Elorrieta and Pylkkänen (2016), the left middle frontal gyrus and the DLPFC, which coincides with previous studies locating the switch and inhibition processes in these regions (Abutalebi and Green, 2007). These results further suggest that code-switching for math and the general language code-switching share the same neural mechanisms. These could match with the idea that language control mechanism are a subdomain of a general control mechanisms (Craik and Bialystok, 2006; Garbin et al., 2010; Luk, Green, Abutalebi, and Grady, 2012). Thus, also here, the mechanisms controlling for number word activations coincide with those reported for general task switching. Our data also suggest that it is language proficiency what modulates these switches (Christoffels et al., 2007; Costa and Santesteban, 2004; Duñabeitia et al., 2010), albeit for math, the dominant language will always be the one in which numerical knowledge was first acquired. As predicted, the pattern of dominance for math is determined by LL^{math} even though shared control mechanisms are used.

Interim discussion (Experiments 1 to 3)

Experiments 1 to 3 were designed to explore the dominance of the linguistic codes for math (whether they are LL^{math} dependent or not) and to reveal the mechanisms underlying this dominance. Participants were balanced bilinguals who have learned math in one of their two languages (this LL^{math} could be the same as their L1 or not). In all three experiments, the results show an asymmetric switch cost in the codes for math, the switch cost always occurring in switches from the OL to the LL^{math}. These results are replicated across three different tasks implying direct access to magnitude as number comparison (Experiment 1) or indirect access to magnitude as the parity task or the lexical decision task (Experiments 2 and 3) and with two different neuroimaging techniques (EEG and MEG).

Experiment 1 showed the asymmetric switch in the N400. The task was an overt switch in which participants were completely aware of the language switches. This switch was independent of L1/L2 dominance. Experiment 2, was designed to find out more about the general mechanisms underlying this asymmetry. The task was a masked priming design which made participants unaware of the switches. This way we emphasized on possible lexically driven switch costs. The results of this Experiment replicate those of Experiment 1, meaning that the unbalanced lexical representations for the two number word systems were the cause of the asymmetric switch. Moreover, these results suggest that for math, the dominant language is the one in which math was first acquired. The L1/L2 distinction is not valid when talking about the codes for math, it is the LL^{math}-OL the one that must be taken into account. Natural language and math codes seem to be independent Finally, Experiment 3 shows that the mechanisms for task switching, language switching and the control mechanisms triggered by the switch between number words might be the same. In fact, this experiment was designed to study the active neural sources during language switches. The experiment was performed using MEG technique and source estimation analyses. The ERF results replicated those of Experiments 1 and 2 finding the asymmetry again in the N400. Moreover, the active sources during switches for the codes for math were located in the frontal areas, similar to language switching and general switch mechanisms. Although being the same mechanisms for all the switches, the results show that it is the dominance in the codes used in the switches what marks the asymmetric switch.

With these experiments, the importance of LL^{math} to the simple lexical representation of number words was further extended for the first time, although its importance in arithmetic and numerical magnitude processing had already been shown (Salillas et al., 2015; Salillas and Carreiras, 2014; Salillas and Wicha, 2012). Since the asymmetry in the switch demonstrates that balanced bilinguals have a clear preference for the LL^{math} even at the most basic representational level, and independently of the L1/L2 dominance, numerical lexical representations seem to operate separately from the general language lexical representations.

From the Lexical Representations of Number to Core Numerical Knowledge: The Case of Bilingual Developmental Dyscalculia

Experiment 4

Introduction

One step further to find out about the role of early learning in math processing is studying the syndromes affecting mathematics. As we have mentioned before, dyscalculia is a disorder of numerical development and mathematical learning with different explanations (Butterworth, 1999, 2010; Geary, 2004; Gelman and Butterworth, 2005; Noël and Rousselle, 2011; Piazza et al., 2010; Wilson and Dehaene, 2010). The prevalent view has been that DD is ultimately caused by an anomaly in the core magnitude representation (Butterworth, 1999; Geary, 2004; Gelman and Butterworth, 2005; Wilson and Dehaene, 2010). However, no study has yet investigated the management of two codes in bilingual DD (bDD). Since we have demonstrated the linguistic imbalances for math in bilinguals, we predict that the input language will be the key factor when accessing core magnitude representation in bDD.

Therefore, this study will present ERP data and source estimation of bilingual dyscalculic children when doing basic numerical operations with an input in either LL^{math} or the other language (OL) in bDD individuals and matched control participants. Specifically, we used an adaptation paradigm (Grill-Spector, Henson, and Martin, 2006; Henson and Rugg, 2003) allowing the measure of passive computation of numerical distance (Hsu and Szucs, 2012). Numerical distance effect implies that close numerical distances are computed faster and more accurately than far numerical distances. This effect is thought to reflect the manipulation of the core numerical system (Cohen Kadosh, Cohen Kadosh, Kaas, et al., 2007; Piazza et al., 2007) and has been also revealed in certain ERP components. Perhaps the most consistent ERP index of the distance effect occurs between 190 and 210 ms. in the transition between the N1 and the P2P (Libertus, Woldorff, and Brannon, 2007). Effects of numerical distance at this latency have been revealed also using time-frequency analyses (Szũcs, Soltész, Jármi, and Csépe, 2007).

Hence in Experiment 4, by presenting numerical information in the two languages (LLmath/OL) to bDD children and matched control children we were able to explore:

(1) possible interactions between distance effects and the input language (i.e. whether distance effects occurred in each of the languages for the control group and whether those distance effects were localized in the same brain loci for each of the languages).

(2) whether there was any difference between the control and bDD for those distance effects both in the ERPs and in the estimated brain sources of those ERPs. Overall, this experiment allowed for the exploration of specificities in the management of quantity in bDD, that are dependent on the input language and early math learning.

Methods

Participants

A total of 14 Basque - Spanish bilingual children aged between 8 and 13 years old took part in this study. 7 children previously diagnosed with Developmental Dyscalculia and tested using the Dyscalculia Screener (Butterworth, 2003) and 7 age-sex matched controls (see Table 5). Despite the two groups showed better word retrieval efficiency in Spanish, which suggests Spanish dominance (see Table 6), LL^{math} was Basque for all participants.

ID	Age	Gender	WISC-R		DS	
			Similarities	Arithmetic	CAP 1	CAP2
s01	9.00	M	9	3	2	4
s02	13.00	F	14	5	2	3
s03	12.00	F	8	8	2	3
s04	11.00	M	10	7	2	3
s05	12.00	F	11	4	2	2
s06	8.00	M	14	7	2	3
s07	9.00	M	10	5	4	2
			10.9	**5.6**	**2.3**	**2.9**

Table 5 – WISC-R and Dyscalculia Screener (DS) score for the DD group. The two capacity tests from the DS were taken as the main diagnostic criteria for DD: CAP1 – Dot Enumeration and CAP2 – Numerical Stroop.

	Mean age	BNT Basque	BNT Spanish
Control Group	10.3	26.4 (3.6)	43.2 (5.9)
DD Group	10.9	29.8 (6.2)	41.5 (6.4)

Table 6 – Boston Naming Test scores for the two groups. The table displays the average age, and socres in the BNT for both groups in Basque and in Spanish.

Stimuli and Procedure:

An adaptation paradigm (Grill-Spector et al., 2006; Henson and Rugg, 2003; Hsu and Szucs, 2012) was created. 6 number words in Spanish and 6 number words in Basque corresponding to the digits *1*,*2*,*3*,*7*,*8*,*9* ("*uno*, *dos*, *tres*, *siete*, *ocho*, *nueve*" in Spanish and "*bat*, *bi*, *hiru*, *zazpi*, *zortzi*, *bederatzi*" in Basque) were used to create 8 stimuli lists repeated 8 times in each language. 4 stimulus lists were created with the corresponding number words to *1* and *2* in each language as the adaptation stimuli with the corresponding number words to *3* as a close distance deviant and the corresponding number words to *7* as a far distance deviant (as follows): 1-2-1-2-1-2-1-2-1-3; 2-1-2-1-2-1-2-1-2-3; 1-2-1-2-1-2-1-2-1-7; 2-1-2-1-2-1-2-1-2-7. Similarly, the other 4 stimulus lists were also created with the corresponding number words to *8* and *9* in each language as the adaptation stimuli and the corresponding number words to *7* as a close distance deviant and the corresponding number words to *3* as a far distance deviant (as follows): 8-9-8-9-8-9-8-9-8-7; 9-8-9-8-9-8-9-8-9-7, 8-9-8-9-8-9-8-9-8-3; 9-8-9-8-9-8-9-8-9-3. In all the lists the deviants were always the same; however, *3* could serve as a close distance deviant when the adaptation items were *1* and *2* and far when the adaptation items were *8* and *9*. The opposite happened with *7*; it could serve as a close distance deviant when the adaptation items were *8* and *9* and far when the adaptation items were *1* and *2*. Every list was repeated 8 times having a total of 32 close-distance trials and 32 far-distance trials in each language.

Each stimulus was presented for 200 ms with an inter stimulus interval of 1000 ms. In order to avoid confounds related to spatial locations and sizes stimuli were presented on different locations around the center of the screen, and with random sizes. During each trial, a fixation cross was presented on the center of the screen. Participants were required to fixate on the cross throughout trials. Their task was to press a button when the colour of the stimulus was orange which randomly happened 90 times during the experiment. This task was used simply to maintain attention. Each trial was followed by an intertrial interval of 1000 ms. 26 more lists of fillers were included in the experiment to distract participants from the real objective of the experiment.

Data recording and analysis

ERP analyses

The EEG was recorded following the same procedures as in Experiments 1 and 2.

Source estimation analyses (MNE)

The method used to estimate the sources distributed in the cortex was MNE (Gramfort et al., 2014) based on all the electrodes for each participant/condition average. We used an MRI-T1 template constructed for the age range 7.5-13.5 from the MNIC (Fonov et al., 2011). Cortical reconstruction and volumetric segmentation was performed with the Freesurfer image analysis suite (as in Experiment 3). Analyses were performed using the standard weighted minimum norm estimate (wMNE) and mapped to a source model of 15.002 electric dipoles. Before MNE was calculated, the covariance matrix required for computing the wMNE source reconstruction was obtained based on the data during a 100 ms baseline period. An overlapping spheres method was used to estimate the forward model. Estimated source activations were standardized using a Z-score transformation with respect to the average and standard deviation of the source activity during a 100 ms baseline. The Z-score source space activity for all subjects/condition was then averaged for visualization. For each group and language, relevant latency bands from the ERP analyses were averaged across time in the Z-score source files. Then 7 participants from each group were compared through F-tests (close ≠ far) using SPM8 with an uncorrected threshold of $p<0.03$.

Results

ERPs

Visual inspection of the ERP data showed a negative component peaking around 200 ms in the LL^{math} condition for both the control and bDD groups and was more positive for the far conditions than for the close conditions in LL^{math}. Additionally, the component revealed the same pattern in the OL condition for the control group. However, no effects were found in the bDD group in the OL condition.

The time window centered on the negativity peak from 170 to 210 ms was analyzed. We analyzed the effect of distance in electrode clusters based on proximity to sites reported in previous studies (Libertus, Woldorff, and Brannon, 2007; Szucs and Csépe, 2004; Temple and Posner, 1998).

We performed ANOVAs on the mean amplitudes over the test windows with each language (LL^{math} vs. OL), distance (close vs. far), hemisphere (left vs. right), anteriority (anterior vs. posterior sites) and electrode as within-subject factors and group (control vs. bDD) as a between-subject factor. The electrodes included in the analyses were divided into clusters as follows: Fp1, F7, F3 and C3 (frontal left hemisphere); Fp2, F8, F4 and C4 (frontal right hemisphere); CP1, CP5, P3 and P7 (posterior left hemisphere); and CP2, CP6, P4 and P8 (posterior right hemisphere).

The general ANOVA revealed a main effect of distance $F(1,12) = 10.72$, $p = .007$, meaning that both groups (control and bDD) showed a distance effect. Results also showed that both groups shared a distance effect in the same time window, but the locations of the effect and the languages in which it was found varied for each group (language x distance x anteriority: $F(1,12) = 6.74$, $p = .023$; language x distance x hemisphere: $F(1,12) = 7.81$, $p = .016$; and language x distance anteriority x hemisphere x electrode x group: $F(3,36) = 3.38$, $p = .44$).

In order to disentangle these interactions and find out which languages and regions the distance effects were located in for each group, we performed a similar ANOVA that included the same factors in the bDD and control groups separately. The control group showed a close-to-significant main effect of distance $F(1,12) = 5.42$, $p = .059$ meaning that the distance effect may be present in both LL^{math} and OL. However, the interactions of language x distance x anteriority $F(1,12) = 13.01$, $p = .011$ suggested that the location of distance effects might differ between languages. Separate ANOVAs (language x distance x hemisphere x electrode) of the anterior and posterior electrodes were performed to specify the location of the distance effect. The ANOVA of the anterior sites showed two interactions: language x distance x electrode $F(3,18) = 4.030$, $p = .039$ and distance x electrode $F(3,18) = 4.275$, $p = .040$, meaning there were different distance effects for each language for certain frontal electrodes. Then t-tests comparing the two distances for each electrode in each language revealed a distance effect in the LL^{math} in the electrodes Fp1 ($t = -7.218$), F3 ($t = .246$, $p = .049$), F4 ($t = 3.83$, $p = .009$), whereas no effects were found in the OL.

In the posterior sites, there was another marginal main effect of distance $F(1,6) = 4.60$, $p = .076$, but also a language x distance $F(1,6) = 6.302$, $p = .046$ interaction. An ANOVA of each language in the posterior sites revealed a main distance effect for LL^{math} ($F(1,6) = 11.86$, $p = .014$) and a marginal interaction of distance x hemisphere for OL ($F(1,6) = 5.30$, $p = .061$). The left posterior cluster showed a main effect of distance in the OL ($F(1,6) = 7.15$, $p = .037$) but no effects in the right cluster. Given that the distance effect appeared to be restricted to a smaller time window, a new ANOVA was performed using a smaller time window (190 to 210 ms). Main distance effects in the left hemisphere ($F(1,6) = 6.46$, $p = .044$) and right hemisphere ($F(1,6) = 6.3471$, $p = .041$)

appeared in this later, smaller window. Overall, these results point to an overall similarity in the location of the ERP distance effect for the two format inputs.

When looking at the bDD group separately, we found a main effect of distance (F (1,12) = 7.75, p =.03) and an interaction of language x distance x hemisphere (F (1,12) = 6.96, p = .039). These results are similar to the control group results in that both languages appear to show a distance effect. However, in the case of the bDD group, the interactions show a difference of the distance effect between hemispheres and no interaction of anteriority, thus suggesting different locations for the distance effect. The two ANOVAs performed for each hemisphere (language x distance x anteriority x electrode) revealed an interaction of language x distance F (1,6) = 7.18, p = .037 in the left hemisphere and no significant distance effects in the right hemisphere. A separate ANOVA (distance x anteriority x electrode) for each language (LL^{math} and OL) in each hemisphere (left and right) showed a main effect of distance (F (1,6) = 6.60, p = .042) for LL^{math} in the left hemisphere and no other significant effects. These results suggest that there is a difference in the distance effect of the bDD group, with LL^{math} being the only language showing such effect and it being located in the left hemisphere. In contrast, the OL showed no distance effects (see Figure 12).

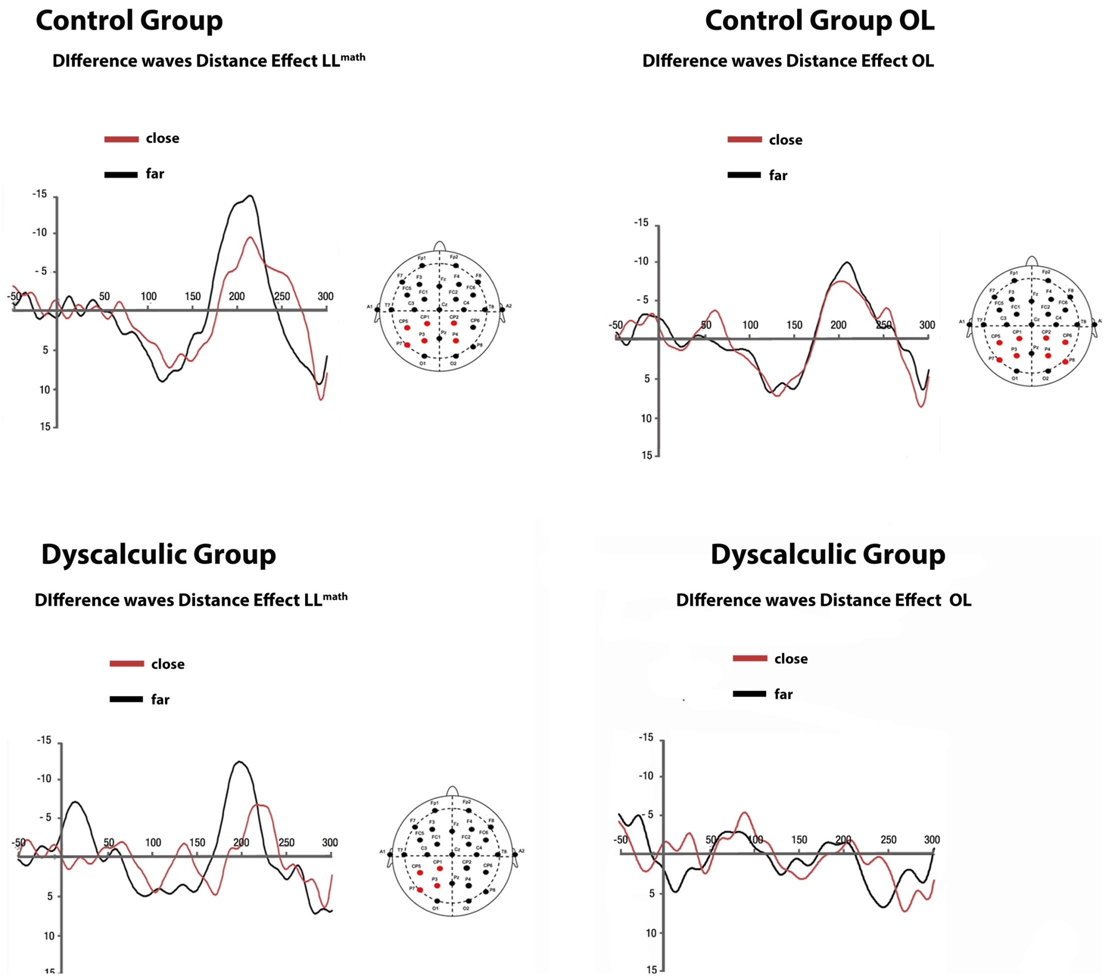

Figure 12. ERPs and scalp voltages for the distance effect in the control and DD group depending on the input language. A. The control group showed a main distance effect in the posterior electrodes around the same time window (170-210 ms) in both LL^{math} and OL. Voltage maps (difference wave close – far) show similar locations for the distance effect in both languages. However, in the OL the effects (**B)** mainly occur in a smaller time window (around 190-210 ms). The DD group showed the effect only in LL^{math} in the same time window as controls although with a left-lateralized distribution. The electrodes selected for each display are shown in red next to each wave.

Source space

In order to observe possible evolution in the brain sources that generated the reported ERP distance effect, the 170 to 210 window was split into two smaller time windows (170-190 and 190-210) (see Figure 13 and Table 7). The control group showed a main brain source for the LL^{math} distance effect during the first 170-190 ms interval in the right supramarginal gyrus (peak coordinates: x = 64, y = -42, z = 42) and the right superior parietal lobule (x = 30, y = -75, z = 47) during the following 190-210 ms interval. The ERP distance effect for the OL during the 170-190 ms interval originated in the right frontal operculum (x = 51, y = 18, z = 7), extended dorsally to the supramarginal gyrus and subsequently, extended more focally at the right supramarginal gyrus (x = 60, y = -31, z = 42) during the 190-210 ms interval.

In contrast, the ERP distance effect for LL^{math} in the bDD group had its source in a broader set of areas in the left hemisphere: along the left perisylvian (x = -54, y = -24, z = 8 and middle frontal gyrus (x = -52, y = 2, z = 46) during the first 170-190 ms interval and at the left angular gyrus (x = -60, y = -36, z = 24), middle frontal gyrus (x = -64,y = 36, z = 24) and inferior frontal gyrus (x = -44, y = 46, z = 28) during the 190-210 ms interval. Hence, though both groups showed distance effects in the ERPs for LL^{math}, the different scalp locations of those effects had indeed different brain origins. For OL, the distance effect shown by the control group also differed from the LL^{math} distance effect in its brain source, involving a frontoparietal executive network.

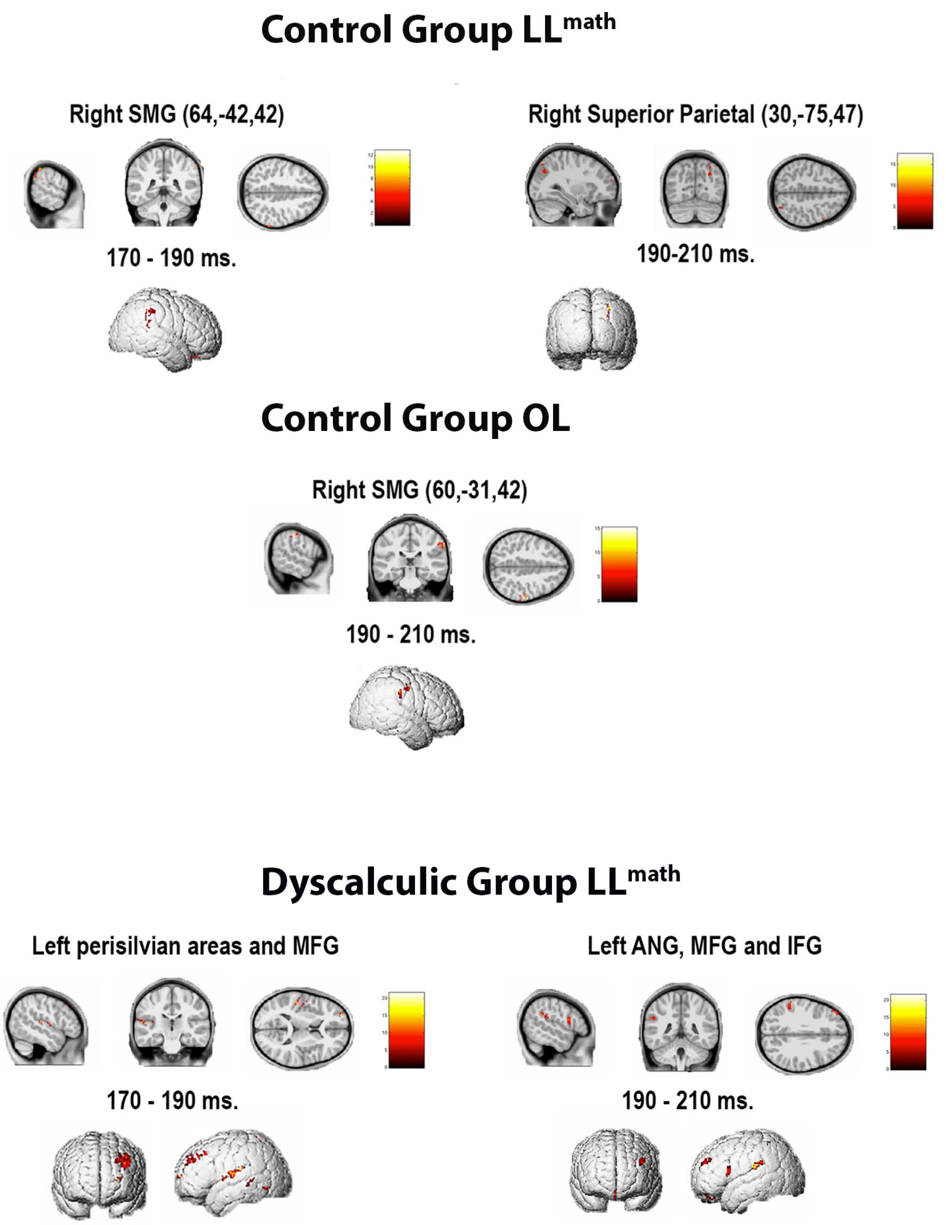

Figure 13. ERPs´ source estimation for the distance effect in each group depending on the input language. For the control group, the distance effect to input in LL^{math} entailed focal parietal sources: supramarginal gyrus (SMG) and then right superior parietal activations, including the intraparietal sulcus (IPS). However, when the input is in OL both the inferior frontal gyrus (operculum) and SMG appeared as the sources generating the distance effect. In both cases sources were located in the right hemisphere. Interestingly, for the bDD group distance effects with LL^{math} input were generated in the left hemisphere including left perisylvian, the left angular gyrus (ANG), and frontal areas (middle frontal gyrus (MFG) and inferior frontal gyrus (IFG)). Thus, despite distance effects appeared in all these conditions the ERP generators were based on very different brain networks. Statistical parametric maps show F values.

		TIME (ms)	Brain region	MNI coordinates (x, y, z)	F-value	Z_0-value	p-value
CONTROL GROUP	**LM**	170-190	*Right Supramarginal Gyrus*	64, -42, 42	7.73	2,13	.017
		190-210	*Right Superior Parietal Lobule*	30,-75, 47	7.77	2.07	0.02
	OL	170-190	*Right Frontal Operculum*	51, 18, 7	7.41	2.09	0.018
		190-210	*Right Supramarginal Gyrus*	60, 31, 42	12.57	2.65	0.004
bDD group	**LM**	170-190	*Left Perisylvian*	-54, -24, 8	17.44	3.02	0.001
		170-190	*Left Middle Frontal Gyrus*	-52, 2, 46	9.66	2.36	0.009
		190-210	*Left Middle Frontal Gyrus*	-64, 36, 24	20.61	3.2	0.001
		190-210	*Left Inferior Frontal Gyrus*	-44, 46, 28	9.85	2.39	0.009
		190-210	*Left Angular Gyrus*	-60, -36, 24	20.61	3.2	0.001

Table 7. Main effects in the two different time windows (170-190 and 190-210 ms) showing the regions where the effect of switch was located, their coordinates and their respective statistical values (T, Z_0 and p) for both groups (control and bDD).

Discussion

Both groups showed a distance effect in the ERPs when using the LL^{math}. However, no distance effect was found in when using the OL as input for the bDD group. Thus, data suggest that both groups have a prevalence of the LL^{math} when manipulating quantity. In addition, there are some differences between groups in the source localization of the distance effect with LL^{math} input. Although both groups showed activation in parietal areas, the bDD group showed activation in areas of the left hemisphere, whereas the control group only showed activation in the right hemisphere. Moreover, the bDD group relied in a left hemispheric network, involving frontal, perisylvian inferioparietal areas, although they may seem close to the Heschl gyrus, the fact is that the highest number of active voxels were located in the left cerebellum which has been shown to be of relevance in magnitude processing (Tang et al., 2006). These sources were not shown by the control group for LL^{math}, for whom the distance effect was more focal in right parietal sites. As per OL in the control group, distance effect involved a right lateralized network comprising frontal and inferioparietal areas, suggesting a less automatic processing of magnitude in the OL. These results are in agreement with (Salillas et al., 2015; Salillas and Carreiras, 2014) which showed that the bilingual core numerical system (as indexed by the distance effect) could include linguistic traces and suggested that one of the languages might enter into the core of magnitude representation.

It is also important to note that all the children in this experiment were more proficient in Spanish, however their LL^{math} was Basque. This entails a nice mismatch between the dominance for language and the dominance for math, where crucially the preference for LLmath is replicated.

This experiment addressed the access to core numerical magnitude representations in bilinguals in the case of bDD. In this unexplored circumstance, the results show that given the aforementioned linguistic imbalances shown in the previous experiments in this thesis, the input language indeed matters even more when a core number processing deficit occurs. The results in this experiment demonstrate that in bDD, in which the differences between languages in the passive computation of basic numerical processes are exacerbated, the accessing to core numerical magnitude depends completely on the LL^{math} since the OL showed no distance effect neither in the ERP analyses or the source estimation analyses. In other words, when everything goes well, very proficient bilinguals should show efficient behavior using both codes, albeit relying on different brain networks.

General discussion

Taken together these results support the predictions formulated in this thesis. First, we have demonstrated that the switch costs between the two codes for math are asymmetrical. Based on the significant differences found on the ERP analyses, we have shown that the LL^{math} and not the L1 modulate these asymmetries; such effects are consistent in three experiments contrasting code switching. Additionally, these switches seem to follow a similar mechanism that language switching and, according to some authors, might trigger general cognitive control mechanisms (Abutalebi et al., 2012; Craik and Bialystok, 2006; Dijkstra and Van Heuven, 1998; Garbin et al., 2010; Green, 1998). Moreover, we have demonstrated that the LL^{math} is indeed the dominant code for math in more nuclear numerical representation, since in bilingual dyscalculia the LL^{math} is the only code showing distance effect and brain source for distance effects also differed between languages for the control group.

Asymmetric switch costs in the codes for math

In the present study, the unexplored relative dominance between the two codes for math has been investigated during the performance of different code-switching tasks. Results show that asymmetric switch costs occur when switching between LL^{math} and OL in perfectly balanced bilinguals. This asymmetry is similar to other switch cost asymmetries reported in general language between the first and second languages (L1 and L2) in unbalanced bilinguals (Alvarez et al., 2003; Chauncey et al., 2008; Costa and Santesteban, 2004; Duñabeitia et al., 2010; Macizo et al., 2012; Meuter and Allport, 1999; Moreno et al., 2002; Palmer et al., 2010; Proverbio, Leoni, and Zani, 2004). The transition from the non-dominant to the dominant code (OL to LL^{math}) implies a larger negativity (a bigger switch cost) at the N400 latency. This switch cost does not depend on the relative proficiency between L1 and L2 (since in this switch there were no asymmetries) but on the LL^{math} and OL relative proficiency.

Dominance of the L1/L2 codes has been extensively explored and several models accounting for the switches and accounting for these asymmetries have also been postulated such as the BIA+ or the IC, which propose that the switch mechanisms in language are part of a general control mechanism. Additionally, only one of the several math cognition models accounts for a bilingual switch: the model for bilingual mathematical cognition developed by Campbell and Epp (2004). This model already proposed an unbalanced representation between the two numerical lexicons. The two lexicons would follow the same pattern of dominance that arithmetic fact representations, with stronger representations for the dominant language, L1. According to the encoding complex hypothesis (Campbell, 1994; Campbell and Clark, 1992), the history of interactive encoding-retrieval processes would explain actual numeric representations. Attending to the present data, it is not the L1/L2 distinction what must be considered when modeling bilingual math representations, but the dichotomy LL^{math}-OL. This is not a trivial question, especially when considering those bilinguals whose LL^{math} mismatches their more proficient language (L1). It means that the pattern of dominance for math runs separately of general language. It appears that the words “bi” and “dos” (two), at least in the context of numerical operations, have not the same relative activation than “itsaso” and “mar” (sea). In terms of the cost of inhibition or competition between languages, the present data suggest that for a person who learned math in Spanish albeit equally proficient in both languages, “itsaso” would be equivalent to “mar”, but the word “bi” would require stronger inhibition of the early learned number word “dos”, at least when performing numerical operations. This agrees with previous observations for arithmetic fact retrieval (Bernardo, 2001; Salillas and Wicha, 2012), where the language used for early learning not only determines the quality and strength of memory networks for arithmetic but also the lexical imbalance between the languages for math.

In sum, arithmetic memory networks depend on early learning (Salillas and Wicha, 2012). Balanced bilinguals have mathematical concepts that are accessed more efficiently in the language in which they learned simple arithmetic. But also, the most basic numerical representation has showed linguistic traces, inherited from early learning (Salillas and Carreiras, 2014). When exposure to number words is associated with quantity during early learning, number representations might be shaped by that particular language. The present data crucially extends these questions and highlights the importance of early learning to lexical representations of number words. The bilingual number word system is unbalanced and can cohabit with more balanced representations for non-numerical words in fluent bilinguals.

Code-switching mechanisms

The existing explanations of language switching point the idea that switch costs are caused by a general domain task-control mechanism. The Bilingual Interactive Activation (BIA+) model (van Heuven and Dijkstra, 2010) and the Inhibitory Control model (IC) (Green, 1998) propose that inhibitory processes for language are just part of executive control factors external to the language system. These models imply what is called "task schema": they are part of the general control system and link the output of lexical processing to a behavioral response. Several studies have given support to this hypothesis giving evidence for the engagement of the anterior cingulate cortex (ACC) in non-language tasks (for review, see Carter and Van Veen, 2007), as well as shared involvement between language and domain-general cognitive control (De Baene et al., 2015). The results in our Experiment 3 yielded similar results having more frontal activations when switching languages.

Due to the fact that in our task, responding to the OL or responding to LL^{math} implied the same output response, as a numerical decision to the very same set of stimuli, and still implied asymmetric switching costs, results would support inhibitory mechanisms acting upon unbalanced lexico-semantic representations. Both task and stimuli for LL^{math} were identical to task and stimuli in OL, and consequently they could not imply different task schemas. In addition, the masked priming paradigm used in Experiments 2 and 3 made the switch from one language to another unconscious, and too brief to activate any task schema (Chauncey et al., 2008). Still, asymmetric switch costs were found. Moreover, the results in Experiment 3 helped to locate the regions of the switch in frontal regions, more concretely in anterior cingulate gyrus (BA 32), left middle frontal gyrus and DLPFC, which agrees with previous studies locating the sources of the switch in frontal regions (Abutelabi et al., 2007; Hernandez et al., 2000, 2001; Rodriguez-Fornells et al., 2005; Wang et al., 2007; Blanco-Elorrieta and Pylkänen, 2016). These locations coincide with the location of general task switching mechanisms in frontal areas of the brain (Craik and Bialystok, 2006; Garbin et al., 2010; Abutalebi et al., 2012). The anterior cingulate cortex has been related to attention and comprehension (Botvinick, Braver, Barch, Carter, and Cohen, 2001; Carter, Botvinick, and Cohen, 1999; Frith, Friston, Liddle, and Frackowiak, 1991; Hikosaka and Isoda, 2010) and the left middle frontal gyrus and DLPFC have been reportedly involved in inhibition (Abutalebi and Green, 2007), which consequently matches the idea of having the same switch mechanism for both LL^{math}/OL codes and L1/L2 codes. In conclusion, the control mechanisms applied to numerical lexicons seem of similar nature to those applied to general language, as shown in paradigms using non-numerical words, in the sense that they are sensitive to different baseline activations as reflected by the asymmetry of the switch costs. Therefore, while two different dominance patterns would be at work for numerical and non-numerical words, similar control mechanisms could be acting in both domains.

LL^{math} and dyscalculia (access to magnitude)

Most of the studies in math cognition in bilingualism have centered around the idea proposed by the triple-code model (Dehaene and Cohen, 1995) that access to magnitude is made through all input forms and that arithmetic is learned by rote via language. The studies in the field of bilingual numerical cognition have shown that indeed bilinguals have a preference for a language in which they learn calculations (Spelke and Tsivkin, 1999; Salillas and Wicha 2012) and even practice in the other language can sometimes equate the strength in the neural networks for both languages (Martinez-Lincoln et al., 2016). However, recently it has been proposed that early learning will facilitate an integration of the LL^{math} also in our core numerical knowledge (Salillas and Carreiras, 2014; Salillas et al., 2015); supporting this proposal is Experiment 4 in this thesis. In Experiment 4 both bilingual DD (bDD) and matched controls showed distance effects in the ERPs when using the LL^{math}. Nevertheless, only the control group showed the distance effect when using the OL. Thus, the data suggest that both groups have a preference for the LL^{math} when manipulating quantity, independently of their L1. Moreover, there are differences between groups in the source localization of the distance effect with LL^{math} as input. Both groups show activation in the parietal areas; however, the bDD group had activation in the left hemisphere whereas the control group showed activation in the right hemisphere. Moreover, the bDD group relied on a left hemisphere network that involved the frontal, perisylvian and inferoparietal areas, differing from the control group, for whom the distance effect under LL^{math} input was more focalized in right parietal sites similar to what Soltész et al., (2007) found. As for OL in the control group, the distance effect involved a right lateralized network comprising frontal and inferoparietal areas, suggesting a less automatic processing of magnitude in the OL. Importantly, the present results must be put in the context of an LL^{math} mismatching L1, hence showing again the independence between a dominant language for math and general language use. This data suggests interactions between the verbal codes and core numerical magnitude marker, the distance effect. What is more, input language and bDD interact as shown by the absence of a distance effect in this group. Right-lateralized distance effects in superior parietal areas (including the occipital part of the IPS and the supramarginal gyrus) for the control group in LL^{math} suggest an integration of this language with classical quantity and calculation areas (Dehaene, 2003; Menon, 2014). The control group processed distance differently when the input was in OL, which again involved a right lateralized network comprising frontal and inferofrontal areas. This suggests that executive processes are involved when input is in OL. In general, this also points to a somehow different management of distance than in LL^{math} input for the control group. Moreover, the bDD group seems to also use executive processing, as well as more explicit linguistic processes when computing distance in LL^{math}. In turn, this implies a worse integration between the preferred code and core quantity knowledge in bDD. All things considered, distance effects that are detectable in the ERPs imply very different neurofunctional bases (and networks) for the two groups when functioning in LL^{math} and for the two verbal codes in the control group.

Final remarks

Balanced bilinguals have mathematical concepts that are accessed more efficiently in the language in which they learned simple arithmetic. Even the most basic numerical representation has showed linguistic traces, inherited from early learning (Salillas and Carreiras, 2014). When exposure to number words is associated with quantity during early learning, number representations might be shaped by that particular language. The present data crucially extend the importance of early learning to lexical representations of number words. The available empirical evidence to date suggests that we depart from abstract and amodal numerical knowledge. However, the acquisition of number words during early education and, likely, repeated exposure to math content in one of the languages may facilitate an integration of that language in the storage of arithmetic facts and also, in our core numerical knowledge. Our studies suggest a window for the study of the math–language relationship. Moreover, they emphasize the relevance of studying bilingual math. Though this peculiar imbalance for math in bilinguals may lead to similar observed behavior for both language inputs, the underlying brain networks appear to differ between the preferred and non-preferred language for math. This makes the study of bilingual math relevant, as more complex math functioning is based on the differential neural bases that we have outlined. As we have shown, this relevance becomes more evident in bDD, in this case, the differences between languages when computing basic numerical processes intensifies. That is, in very proficient bilinguals would nod yield any behavioral differences when using both codes for math; nevertheless, the brain networks being used are different for each code. However, the preference should be evidenced when math complexity increases or when core math functions are essentially altered, as in bDD. Additionally, the control mechanisms applied to numerical lexicons and general language seem to be the same as those general control mechanisms, as shown in paradigms using non-linguistic stimuli and source location analyses, based on the asymmetric switch costs found and their neural locations.

In sum, the present work demonstrates that early math learning shapes core numerical magnitude and it also creates an unbalanced dominance for the two codes which, in turn, makes use of the same control mechanisms as general language and general control tasks.

Future directions

Future research on how bilinguals process the two codes for math from a neurocognitive account may address the specific brain structures that operate for both LL^{math} and OL in magnitude processing for non-numerical tasks. Since magnitude processing has been demonstrated to have common areas for numerical and non-numerical processing (Sokolowski et al., 2016), it would be appropriate to understand to what extent does the LL^{math} modulate access to magnitude in other tasks which do not use numbers in their verbal forms in these common areas for magnitude processing. Additionally, it would also be suitable to dig out more about to what extent the LL^{math} modulates non-numerical tasks including number words. This approach would again require a combination of neuroimaging methods (EEG, MEG and fMRI). By using a multimodal imaging approach, along with source estimation analyses, we could address the specific brain structures that operate for the LL^{math} (and the OL) in magnitude processing including numerical and non-numerical stimuli (such as size or time comparison of items presented in their written forms) and contrasting the activation of the traditional areas found active for magnitude comparison across the different varieties of stimuli. Moreover, it would also be interesting to look at the influence of LL^{math} in non-numerical tasks that include numbers (e.g. memorization tasks). Furthermore, more research into the switch mechanisms employed for language, math and general switch-mechanisms should be made, in order to disentangle the differences among mechanisms (if any) and their commonalities.

In this regard, exploring the ontogenetic and cultural components of the relation between language and numerical cognition remains useful to understand the numerical competence acquisition and mathematical performance in child and adult bilinguals. Such investigation would require the contribution of other cognitive research areas such as Psycholinguistics. Finally, the most practical effects of language upon numerical cognition reside in its pedagogical consequences as bilingualism has become a crucial factor in the educational context. On such issue, the link between math and language requires considerable research effort.

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