DOCUMENTOS DE TRABAJO

BILTOKI

D.T. 97.19

Time-dependent outside option
in an alternating offers bargaining model

Jesús Rubio

Departamentos de
ECONOMETRÍA Y ESTADÍSTICA (E.A. III),
FUNDAMENTOS DEL ANÁLISIS ECONÓMICO,
HACIENDA Y SISTEMAS FISCALES (E.A. II)
e INSTITUTO DE ECONOMÍA PÚBLICA

eman ta zabal zazu

Universidad del País Vasco
Euskal Herriko Unibertsitatea

Facultad de Ciencias Económicas.
Avda. Lehendakari Aguirre, 83
48015 BILBAO.

Documentos de Trabajo BILTOKI

Editado por los Departamentos Economía Aplicada II (Hacienda), Economía Aplicada III (Econometría y Estadística), Fundamentos del Análisis Económico (Teoría Económica) e Instituto de Economía Pública de la Universidad del País Vasco.

Depósito Legal No.: BI-2747-94

ISSN: 1134-8984

Time-dependent outside option in an alternating offers bargaining model *

Jesus Rubio †

Abstract

In this work I consider an alternating offers bargaining model in which a time-dependent outside option is introduced. The purpose of this work is to analyze relationships between S.P.E. utility pairs induced by such a kind of outside option, as well as to compare them to results obtained under alternative assumptions.

JEL: C78

Keywords: Alternating offers bargaining game, outside option.

*I would like to express my thanks to J.M. Aizpurua for helpful guidance while doing this work. I also want to thank M.P. Espinosa for helpfull comments and suggestions, as well as all other participants of a seminar discussion at the Euskal Herriko Unibertsitatea. I also benefited from comments by the participants of: E. A. R. I. E. 21st. Annual Conference, Chania, September 1994; I Encuentro en Teoria de Juegos, Bilbao, September, 1994; X Jornadas de Economía Industrial, Madrid, September, 1994; XIX Simposio de Análisis Económico, Barcelona, December 1994; and Third Summer Meeting on Strategic Equilibria and Applications, Valencia, July 1995. This research has been partially financed by the project EJ-GV:PI9346.

†Ekonomi Analisiaren Oinarriak Departamentua. Euskal Herriko Unibertsitatea. Lehendakari Aguirre Etorbidea 83, Sarriko, 48015 Bilbao. Spain. Telephone: 34(4)4797400; Fax: 34(4)4797474; e-mail: jeprupej@bs.ehu.es

1 Introduction

Rubinstein (1982) introduced the so-called alternating offers bargaining game, and proved that there exists a unique subgame-perfect equilibrium (S.P.E.) partition in it. Binmore, Shaked and Sutton (1989) formulated the Outside Option Principle in an alternating offers bargaining game in wich players have outside options. They proved that there exists a unique S.P.E. partition in this modificated game and that it coincides with that of the game by Rubinstein unless the amount unilaterally obtainable by each player exceeds her respective S.P.E. part of the pie in Rubinstein's game. Outside options are modelled constant over time, that is, the amount that a player could obtain by unilaterally terminating the negotiation is always the same. The posterior literature [e.g., Binmore (1985), Sutton (1986), Osborne and Rubinstein (1990) or more recently Dalmazzo (1992)] maintains this assumption.

I consider a time-dependent outside option, that is, by unilaterally quitting the negotiation different amounts could be obtained at differents times. An outside option function is incorporated specifying what part of the pie could be attained by opting out at a given time. No constraint is imposed on the evolution of the function over time.

When analysing relationships between S.P.E. utility pairs induced by such a kind of outside option we can obtain results such as the following two.

First, it is enough for the outside option function to be effective to exist *one arbitrarily late time* at which the value given by the time-dependent outside option function is greater than the part of the pie obtainable in the absence of the outside option function. That is a sort of restatement of the Outside Option Principle.

Second, only when a time-dependent outside option is considered could Player 2 prefer to threaten to opt out at a time posterior to every one where she maximizes the utility obtainable by opting out.

This work is organized as follows. In Section 2 the game is formulated. In Section 3 the concept of equilibrium used is presented. In Section 4 some results are proposed.

2 The game

The game is a standard bargaining game of alternating offers in which

(i) the two players have time preferences with the same constant discount factor $0 < \delta < 1$,

(ii) Player 2 has an outside option that can be exercised only when responding to an offer, and

(iii) the outside option is time-dependent according to a given function, the outside option function, which can be defined as follows:

Definition 1 (Outside option function) An outside option function f is a function from the set of times at which a player can opt out into the set $\mathcal{R}$, which specifies the amount the player opting out could obtain by opting out.

In this model, this function specifies what amount Player 2 could obtain by opting out in response to an offer by Player 1. That is,

$$f : \{0, 2, 4, \ldots\} \longrightarrow [0, 1)$$

Therefore, Player 2 could, alternatively to saying *yes* or *no*, opt out obtaining an amount $f(t) \in [0, 1)$, whose utility is $f(t)\delta^t$. In this event Player 2 would obtain 0. Alternatively, if some agreement $(x_1, x_2) \in X = \{(x_1, x_2) \in \mathcal{R}_+^{\in}$ so that $x_1 + x_2 = 1\}$ is reached at some time $t \in \{0, 1, 2, \ldots\}$ Players 1 and 2 obtain respectively the following utilities: $U_1 = x_1\delta^t$ and $U_2 = x_2\delta^t$, that is, their valuation at $t = 0$ of their respective shares at t.

This game will be henceforth referred to as a bargaining game of alternating offers with common discount factor δ where Player 2 has an outside option function f, or alternatively as a (δ, f) game.

3 The equilibrium concept

The equilibrium concept to be used is subgame-perfect equilibrium.

4 Results

Proposition 1 (S.P.E. utility pairs) *Given a (δ, f) game such that $f(t) < 1$ forall $t \in \{0, 2, 4, \ldots\}$, the unique S.P.E. utility pair is:*

$$(U_1^*, U_2^*) =$$

$$\left(1 - \max_{t \in \{0,2,4,\ldots\} \cup \{\infty\}} [\sum_{t=0}^{t=t} (-1)^t \delta^t] - f(t)\delta^t, \max_{t \in \{0,2,4,\ldots\} \cup \{\infty\}} [\sum_{t=1}^{t=t} (-1)^{t+1} \delta^t] + f(t)\delta^t\right)$$

Proof: I proceed in three lemmas.

Lemma 1 *Given a (δ, f) game such that $f(t) < 1$ for all $t \in \{0, 2, 4, \ldots\}$, if it is a credible threat at time 0 that Player 2 is going to opt out at a given time $\hat{t} \in \{0, 2, 4, \ldots\}$ unless an agreement is reached at this time or before, then the unique S.P.E. utility pair is*

$$(U_1^{\hat{t}}, U_2^{\hat{t}}) = ([\sum_{t=0}^{\hat{t}} (-1)^t \delta^t] - f(\hat{t})\delta^{\hat{t}}, [\sum_{t=1}^{\hat{t}} (-1)^{t+1} \delta^t] + f(\hat{t})\delta^{\hat{t}}).$$

Proof: Given a (δ, f) game such that $f(t) < 1$ for all $t \in \{0, 2, 4, \ldots\}$, assume that it is a credible threat that Player 2 is going to opt out at time $\hat{t} \in \{0, 2, 4, \ldots\}$ if no agreement is reached at this time or before. In this case, the unique S.P.E. share obtainable by Player 1 at $\hat{t}$, is $x_1 = 1 - f(\hat{t})$, whose utility is $U_1 = \delta^{\hat{t}} - f(\hat{t})\delta^{\hat{t}}$. Following backwards, the unique S.P.E. share obtainable by Player 2 at $\hat{t}-1$, where it is her turn to make an offer, is $x_2 = 1-[1-f(\hat{t})]\delta$, whose utility is $U_2 = \delta^{\hat{t}-1} - \delta^{\hat{t}} + f(\hat{t})\delta^{\hat{t}}$. The unique S.P.E. share obtainable by Player 1 at $\hat{t}-2$, where she offers, is $x_1 = 1 - \{1 - [1 - f(\hat{t})]\delta\}\delta$, whose utility is $U_1 = \delta^{\hat{t}-2} - \delta^{\hat{t}-1} + \delta^{\hat{t}} - f(\hat{t})\delta^{\hat{t}}$. At time $\hat{t}-\hat{t}$, where it is Player 1's turn to make an offer, the unique S.P.E. utilities (as well as shares) obtainable, respectively by Player 1 and Player 2 will be

$$U_1^{\hat{t}} = x_1 = \delta^{\hat{t}-\hat{t}} - \delta^{\hat{t}-\hat{t}+1} + \delta^{\hat{t}-\hat{t}+2} - \delta^{\hat{t}-\hat{t}+3} + \delta^{\hat{t}-\hat{t}+4} - \delta^{\hat{t}-\hat{t}+5} + \ldots + \delta^{\hat{t}} - f(\hat{t})\delta^{\hat{t}} =$$

$$= 1 - \delta + \delta^2 - \delta^3 + \delta^4 - \delta^5 + \ldots + \delta^{\hat{t}} - f(\hat{t})\delta^{\hat{t}} = [\sum_{t=0}^{\hat{t}}(-1)^t\delta^t] - f(\hat{t})\delta^{\hat{t}}$$

$$U_2^{\hat{t}} = x_2 = [\sum_{t=1}^{\hat{t}}(-1)^{t+1}\delta^t] + f(\hat{t})\delta^{\hat{t}}$$

The proof of Lemma 1 is completed.

Remark 1 *The utility Player 2 can obtain by credibly threatening to opt out at some $\hat{t}$ has two sources. On the one hand, the outside option itself, whose discounted value is $f(\hat{t})\delta^{\hat{t}}$. On the other hand, the negotiation process, from which a non-negative and time-increasing gain $[\sum_1^{\hat{t}}(-1)^{t+1}\delta^t]$ can be obtained by credibly threatening to opt out.*

Lemma 2 *Given a (δ, f) game such that $f(t) < 1$ for all $t \in \{0, 2, 4, \ldots\}$, if and only if*

$$\hat{t} \in \arg\max_{t\in\{0,2,4,\ldots\}} [\sum_{t=1}^{t}(-1)^{t+1}\delta^t] + f(t)\delta^t$$

then it is a credible threat at time 0 that Player 2 is going to opt out at time $\hat{t} \in \{0, 2, 4, \ldots\}$ unless an agreement is reached at this time or before.

Proof: I proceed in two steps.

Step 1 (If) Given a (δ, f) such that $f(t) < 1$ for all $t \in \{0, 2, 4, \ldots\}$, if

$$\hat{t} \in \arg\max_{t\in\{0,2,4,\ldots\}} [\sum_{t=1}^{t}(-1)^{t+1}\delta^t] + f(t)\delta^t$$

then it is a credible threat at time 0 *that Player 2 is going to opt out at time* $\hat{t} \in \{0, 2, 4, \ldots\}$ *unless an agreement is reached at this time or before.*

Proof: Given a (δ, f) such that $f(t) < 1 \text{forall} t \in \{0, 2, 4, \ldots\}$, if time $\hat{t}$ is such that

$$[\sum_{1}^{\hat{t}}(-1)^{t+1}\delta^{t}] + f(\hat{t})\delta^{\hat{t}} = x_2$$

and x_2 is not smaller than

$$x_2' = [\sum_{1}^{t}(-1)^{t+1}\delta^{t}] + f(t)\delta^{t}$$

for any other $t \in \{0, 2, 4, \ldots\}$, then at $t = 0$ it holds that

1. no threat of opting out at a time different from $\hat{t}$ could induce a greater S.P.E. utility for Player 2, and

2. since
$$\lim_{t\to\infty}[\sum_{1}^{t}(-1)^{t+1}\delta^{t}] + f(t)\delta^{t} = \frac{\delta}{1+\delta}, \tag{1}$$
the S.P.E. utility obtainable by Player 2 without using her outside option (that is the unique S.P.E. utility obtainable in (δ)) is not greater than that obtainable by threatening at $t = 0$ to opt out at $\hat{t}$ if credible.

So, if the offer by Player 1 at $t = 0$ is less than

$$x_2 = [\sum_{1}^{\hat{t}}(-1)^{t+1}\delta^{t}] + f(\hat{t})\delta^{\hat{t}},$$

Player 2 prefers to say *No* and demand at $t = 1$ her S.P.E. level of utility (which is, $[\sum_1^{\hat{t}}(-1)^{t+1}\delta^{t}] + f(\hat{t})\delta^{\hat{t}}$).

This argument can be used at every $t \in \{0, 2, 4, \ldots\} \leq \hat{t}$, so that, at all these times, Player 2 prefers to go on to the next time rather than to opt out or agree.

Now, at $\hat{t}$ the following unequality is satisfied for all $t \geq \hat{t}$:

$$[\sum_{1}^{\hat{t}}(-1)^{t+1}\delta^{t}] + f(\hat{t})\delta^{\hat{t}} \geq [\sum_{1}^{t}(-1)^{t+1}\delta^{t}] + f(t)\delta^{t}.$$

Hence

$$f(\hat{t}) \geq [\sum_{\hat{t}+1}^{t}(-1)^{t+1}\delta^{t}] + f(t)\delta^{t-\hat{t}} > f(t)\delta^{t-\hat{t}}$$

where

$$\lim_{t\to\infty}[\sum_{\hat{t}+1}^{t}(-1)^{t+1}\delta^{t}] + f(t)\delta^{t-\hat{t}} = \frac{\delta}{1+\delta},$$

so that the share obtainable by opting out at $\hat{t}$ ($f(\hat{t})$) is neither less than

(i) every share inducible at $\hat{t}$ by means of a credible threat with opting out at $t > \hat{t}$ ($[\sum_{\hat{t}+1}^{t}(-1)^{t+1}\delta^{t}] + f(t)\delta^{t-\hat{t}}$ for every $t > \hat{t}$), nor less than

(ii) her utility in the unique S.P.E. in Rubinstein's game with common discounting factor δ ((δ)), which is $\frac{\delta}{1+\delta}$.

Therefore, opting out at $\hat{t}$ is really credible.

The proof of Step 1 is completed.

Remark 2 *Any credible threat pays to Player 2 not less than the unique S.P.E. in the absence of outside function.*

Step 2 (Only if) Given a (δ, f) *such that* $f(t) < 1$ *for all* $t \in \{0, 2, 4, \ldots\}$, *only if*

$$\hat{t} \in \arg\max_{t\in\{0,2,4,\ldots\}}[\sum_{t=1}^{t}(-1)^{t+1}\delta^{t}] + f(t)\delta^{t}$$

then it is a credible threat at time 0 *that Player* 2 *is going to opt out at time* $\hat{t} \in \{0, 2, 4, \ldots\}$ *unless an agreement is reached at this time or before.*

Proof:

Definition 2 (Dominated threat) Any threat of opting out at a time $t \in \{0,2,4,\ldots\}$ such that there exists another time $\{0,2,4,\ldots\} \ni \hat{t} \neq t$ at which

$$[\sum_{1}^{\hat{t}}(-1)^{t+1}\delta^t] + f(\hat{t})\delta^{\hat{t}} > [\sum_{1}^{t}(-1)^{t+1}\delta^t] + f(t)\delta^t$$

is said to be a dominated threat (threat $\hat{t}$ dominates threat t).

Remark 3 *Threat $\hat{t}$ dominates threat t in the sense that if $t < \hat{t}$, Player 2 at t would prefer to threaten to opting out at $\hat{t}$ rather than to opt out at t, and if $t > \hat{t}$, Player 2 at $\hat{t}$ would prefer to opt out at $\hat{t}$ rather than to threaten to opting out at t.*

Therefore, only if

$$[\sum_{1}^{t}(-1)^{t+1}\delta^t] + f(t)\delta^t$$

reach a maximum at some $t \in \{0,2,4,\ldots\}$ can it be said that threatening to opt out at this t is not dominated by another alternative threat.

The proof of Step 2 is completed.

The proof of Lemma 2 is completed.

Lemma 3 *Given a (δ, f) game such that $f(t) < 1$ for all $t \in \{0,2,4,\ldots\}$, if the set*

$$\arg\max_{t\in\{0,2,4,\ldots\}}[\sum_{t=1}^{t}(-1)^{t+1}\delta^t] + f(t)\delta^t$$

is empty, then the unique S.P.E. utility pair is

$$(U_1^*, U_2^*) = ([\sum_{0}^{\infty}(-1)^t\delta^t] - f(t)\delta^t, [\sum_{1}^{\infty}(-1)^{t+1}\delta^t] + f(t)\delta^t)$$

$$(= (\frac{1}{1+\delta}, \frac{\delta}{1+\delta}))$$

Proof: Given a (δ, f) such that $f(t) < 1$ for all $t \in \{0, 2, 4, \ldots\}$, if the set

$$\arg \max_{t \in \{0,2,4,\ldots\}} [\sum_{t=1}^{t} (-1)^{t+1} \delta^t] + f(t)\delta^t$$

is empty, then it is satisfied for all $t \in \{0, 2, 4, \ldots\}$ that

$$[\sum_{1}^{\hat{t}} (-1)^{t+1} \delta^t] + f(\hat{t})\delta^{\hat{t}} \leq [\sum_{1}^{\infty} (-1)^{t+1} \delta^t]$$

That occurs because f has been assumed to be upper bounded (more precisely $f(t)$ has been assumed to be smaller than or equal to one for all $t \in \{0, 2, 4, \ldots\}$). In this case it holds that

$$\lim_{t \longrightarrow \infty} f(t)\delta^t = 0$$

Since we have that

$$[\sum_{1}^{\infty} (-1)^{t+1} \delta^t] = \frac{\delta}{1 + \delta}$$

Player 2 prefers at any time $t \in \{0, 2, 4, \ldots\}$ to demand the S.P.E. utility pair in the absence of outside option rather than to make any outside option based demand.

The proof of Lemma 3 is completed.

The proof of Proposition 1 is completed.

4.0.1 Effective outside option threat

Definition 3 (Effective outside option threat) An effective outside option threat is a time $t \in \{0, 2, 4, \ldots\}$ such that,

(i) at $t = 0$ a threat of opting out at this time is credible, and

(ii) this threat induces an S.P.E. utility different from that in the absence of outside option.

It is clear that by definition every effective outside option threat will be a credible one. The opposite will not be satisfied if a credible outside option threat induces an S.P.E. utility pair equal to that in the corresponding game without outside option.

4.0.2 Outside Option Principle

An important result in literature on outside options in bargaining games is the so-called Outside Option Principle [Binmore, Shaked and Sutton (1988)].

Referring to a (δ, f) game such that $f(t) = b < 1$ for all $t \in \{0, 2, 4, \ldots\}$ as a (δ, b) game, the Outside Option Principle states that in a given (δ, b) game some effective outside option threat exists if and only if

$$b > \frac{\delta}{1+\delta}$$

When a time-dependent outside option is considered, this result can be restated as follows:

Proposition 2 (Time-dependent Outside Option Principle) *Given a (δ, f) game such that $f(t) < 1$ for all $t \in \{0, 2, 4, \ldots\}$, some effective outside option threat exists if an only if $f(t) > \frac{\delta}{1+\delta}$ for some $t \in \{0, 2, 4, \ldots\}$.*

In this manner, it is enough to exist **one arbitrarily late time** at which the outside option value exceeds the S.P.E. portion obtainable at this time by the player in the absence of outside option for the outside option to be effective. Thus, even if the outside option specifies zero at every time except for an arbitrarily late time where the previous condition holds, the outside option function will have some effect on the S.P.E. utility pair inducible without using any outside option.

Proof: Given a (δ, f) game such that $f(t) < 1$ for all $t \in \{0, 2, 4, \ldots\}$, if there exists some time $\hat{t} \in \{0, 2, 4, \ldots\}$ such that $f(\hat{t}) > \frac{\delta}{1+\delta}$, then

$$\delta^{\hat{t}-1} + \delta^{\hat{t}} - f(\hat{t})\delta^{\hat{t}} > \delta^{\hat{t}-1} + \delta^{\hat{t}} - \frac{\delta}{1+\delta}\delta^{\hat{t}}$$

is satisfied, and also

$$[\sum_{1}^{\hat{t}}(-1)^{t+1}\delta^{t}] + f(\hat{t})\delta^{\hat{t}} > [\sum_{1}^{\hat{t}}(-1)^{t+1}\delta^{t}] + \frac{\delta}{1+\delta}\delta^{\hat{t}}$$

which is the same as

$$[\sum_{1}^{\hat{t}}(-1)^{t+1}\delta^{t}] + f(\hat{t})\delta^{\hat{t}} > \frac{\delta}{1+\delta}$$

With $f(t)$ being upper bounded (1) holds and hence some effective outside option threat has to exist.

Corollary 1 *Let $\hat{t}$ be an effective outside option threat. If there exists a time $\bar{t} < \hat{t}$ such that*

$$f(\bar{t})\delta^{t} = f(\hat{t})\delta^{\hat{t}}$$

then Player 2 prefers to threaten to opt out at $\hat{t}$ (which is later) rather than to threaten to opt out at $\bar{t}$.

4.0.3 Optimal outside option

Definition 4 (Optimal outside option) An optimal outside option is a time $t \in \{0, 2, 4, \ldots\}$ at which Player 2 maximizes the utility she can get by means of exercising her outside option.

That is, an optimal option is a time

$$t \in \{arg \max_{t \in \{0,2,4,\ldots\}} f(t)\delta^{t}\}$$

Proposition 3 (Effective threats and optimal options) *(i) No effective threat can be prior to some optimal outside option.*

(ii) Given a (δ, b) game, an effective threat is also an optimal outside option.

(iii) Given a (δ, f) game such that $f(t) < 1$ for all $t \in \{0, 2, 4, \ldots\}$, an effective threat may be subsequent to an optimal outside option.

When constancy over time of outside options is required, then an effective outside option threat is always an optimal outside option. This result might seem to be natural. However, it turns out that when a time-dependent outside option is considered, the above characterization disappears, giving way to the possibility that every effective outside option threat could be, somewhat paradoxically, subsequent to every optimal outside option. In other words, the related player could prefer to threaten to opt out at a time subsequent to every one where she maximizes the utility obtainable by opting out.

Proof of (i): $g(t) = [\sum_1^t (-1)^{t+1} \delta^t]$ is a non-negative function increasing in time t. Therefore, the argument maximizing $f(t)\delta^t$ in $\{0, 2, 4, \ldots\}$ cannot be greater than the argument maximizing $g(t) + f(t)\delta^t$ in the same set.

Proof of (ii): Given a (δ, b) , since f is an upper bounded function, an optimal outside option threat $\hat{t}$ always exists.

The existence of an effective outside option threat $\hat{t} + N$ subsequent to the optimal outside option $\hat{t}$ requires the existence of some odd N such that the strictly positive gain induced by making use of a later threat (specifically N times later, from $\hat{t}$ to $\hat{t} + N$) is greater than the loss suffered by the outside option threat from $\hat{t}$ to $\hat{t} + N$, that is,

$$\sum_1^N (-1)^{n+1} \delta^{\hat{t}+n} > f(\hat{t})\delta^{\hat{t}} - f(\hat{t} + N)\delta^{\hat{t}+N}.$$

With f being constant, if the previous unequation is satisfied for some N, then the following one is also satisfied for this N

$$\delta^{\hat{t}}[\delta(1-\delta) + \delta(1-\delta)\delta^{4-2} + \delta(1-\delta)\delta^{6-2} + \ldots + \delta(1-\delta)\delta^{N-2}] >$$

$$\delta^{\hat{t}}[f(\hat{t})(1-\delta^2)+f(\hat{t})(1-\delta^2)\delta^{4-2}+f(\hat{t})(1-\delta^2)\delta^{6-2}+\ldots+f(\hat{t})(1-\delta^2)\delta^{N-2}],$$

which can be simplified as follows

$$\delta(1-\delta) > f(\hat{t})(1-\delta^2)$$

This occurs if and only if $\frac{\delta}{1+\delta} > f(\hat{t})$ which is contradictory to the existence of an effective outside option threat.

Proof of (iii): Let f be such that $f(t)\delta^t = f(t+2)\delta^{t+2}$ is satisfied for some $t \in \{0, 2, 4, \ldots\}$. It is clear that

$$[\sum_{1}^{t}(-1)^{t+1}\delta^t] + f(t)\delta^t < [\sum_{1}^{t+2}(-1)^{t+1}\delta^t] + f(t+2)\delta^{t+2}.$$

This inequality holds even when $f(t)\delta^t$ is infinitesimally greater than $f(t+2)\delta^{t+2}$.

References

[1] Binmore, K.G., 1985, Bargaining and coalitions, in Roth, A. (ed.), Game-theoretic models of bargaining, Cambridge University Press.

[2] Binmore, K.G., A. Shaked and J. Sutton, 1989, An outside option experiment, Quarterly Jounal of Economics, 104, 753-770.

[3] Dalmazzo, A., 1992, Outside options in a bargaining model with decay in the size of the cake, Economic Letters 40, 417-421.

[4] Osborne, M.J. and Rubinstein, A., 1990, Bargaining and markets, (Academic Press, San Diego, CA).

[5] Rubinstein, A., 1982, Perfect equilibrium in a bargaining model, Econometrica, Vol. 50, 97-109.

[6] Shaked, A. and J. Sutton, 1984, Involuntary unemployment as a perfect equilibrium in a bargaining model, Econometrica, Vol. 52, 1351-1364.

[7] Sutton, J., 1986, Non-cooperative bargaining theory: An introduction, Review of Economic Studies 53, 709-724.