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Abstract

We propose and analyze, focusing on non-adiabatic effects, a technique of manipulating quantum spin systems based on local ‘cutting’ and ‘stitching’ of the Heisenberg exchange coupling between the spins. This first operation is cutting of a bond separating a single spin from a linear chain, or of two neighboring bonds for a ring-shaped array of spins. We show that the disconnected spin can be in the ground state with a high-fidelity even after a non-adiabatic process. Next, we consider inverse operation of stitching these bonds to increase the system size. We show that the optimal control algorithm can be found by using common numerical procedures with a simple two-parametric control function able to produce a high-fidelity cutting and stitching. These results can be applied for manipulating ensembles of quantum dots, considered as prospective elements for quantum information technologies, and for design of machines based on quantum thermodynamics.

1. Introduction

Theory of quantum control and its applications [1, 2] attract considerable attention of researchers. The abilities of producing and transforming quantum states on demand become important also in connection with research on quantum computation and information [3]. The problems in this field are usually related to the studies of the system evolution under time-dependent Hamiltonians and the resulting preparation of the desired final states with the maximum possible fidelity.

In general, these problems can be formulated as follows. Assume that we impose an external control on a given quantum system in such a way that its initial Hamiltonian H_i and the final Hamiltonian after time T , H_f are known. We want to achieve the ground state of H_f as a result of the unitary evolution governed by a time-dependent Hamiltonian $H(t)$, with $H(0) = H_i$ and $H(T) = H_f$, whose initial state is the ground state of H_i . The adiabatic passage is a well-known way [4] to realize this process. However, in order to satisfy the conditions of the adiabatic theorem, one has to implement the evolution of slow-varying Hamiltonian for a very long time, where decoherence may ruin the quantumness of the system.

One of the strategies for quantum control is the ‘adiabaticity shortcut’ [5] to allow the design of the Hamiltonian $H(t)$ in such a way that the system terminates at the ground state of H_f at a relatively short T . There are lots of proposals for making such adiabatic-like passages in a finite, short time; for instance, the proposal of transitionless quantum driving [6, 7], pulse/noise control [8, 9], and invariant-based inverse engineering [5]. However, these proposals require the ability to control the *entire* quantum system [10–12]. For example, to cut a spin chain in a short time with ‘adiabatic shortcut’, one has to control and change with time *each* single spin and *each* spin–spin interaction bond [13] in the chain.

In the present paper, we investigate the shortcut via only *local control* of a complex quantum system described by the Heisenberg model for interacting spins in an external magnetic field. First, we focus on the problem of ‘cutting’ or disentangling of a such system into two parts. Second, we consider inverse problem of stitching of two systems into one—the process which can be characterized as a quantum assembly. In general, the current adiabatic quantum computation or quantum annealing [14] is a multipartite stitching process, as implemented, for example, in *D*-wave quantum processors. The initial Hamiltonian H_i may describe a non-interacting independent spin system, and with time evolution the system is stitched as a correlated entity described by the site-bond spin model H_f , such as the Ising model. Our direct numerical simulations show that such kind of control can be achieved with a high-fidelity in the non-adiabatic domain. These quantum processes with a finite time duration can be also interesting as a non-adiabatic counterpart to a local quench dynamics of a suddenly cut and stitched spin chains [15], and for bond impurity chain cutting [16]. Another related field is the quantum thermodynamics [17], or the physics of quantum heat machines [18–21], where one part of the spin chain can be considered as a working medium, connected to and disconnected from the other part, which plays the role of a thermal reservoir.

In general, such a spin system can be realized in an ensemble of quantum dots, proposed as hardware elements for quantum information processing [22] as well as in other spin-based systems with a similar perspective [23]. The single spin manipulation in a quantum dot has become a well-controllable procedure [24, 25] by now. The spin–spin interaction, strongly dependent on the electron tunneling rate between the dots can be controlled electrically by a fast gating of the tunneling channels [26].

2. Separation of a quantum system

2.1. General description: evolution and fidelities

Here we describe the problem in general, without referring to a specific system. Let us assume that some joint quantum system consists of two parts: A and B. Initially, there is an interaction between A and B, such that the ground state $|\psi_0\rangle$ of the joint A+B system generally cannot be presented as a direct product of A and B states: $|\psi_0\rangle \neq |\varphi_{0A}\rangle \otimes |\varphi_{0B}\rangle$, where $|\varphi_{0A(B)}\rangle$ is the ground state of system A(B). Assume that initial Hamiltonian of this joint system has the form:

$$H_i = H_0 + V, \quad (1)$$

where V corresponds to the interaction between A and B parts. We can also write the following relation: $(H_0 + V)|\psi_0\rangle = \lambda_{\min}|\psi_0\rangle$, where λ_{\min} is the minimal eigenvalue of the Hamiltonian (1). Let us assume the ‘switch-off’ of the V interaction while the unitary evolution $U(t)$ is governed by the time-dependent Hamiltonian $H(t) = H_0 + g(t)V$, with initial state $|\psi(t=0)\rangle = |\psi_0\rangle$, $g(t \leq 0) = 1$, and $g(t \geq T) = 0$, where T is the time of cutting. The state at time t can be written as $|\psi(t)\rangle = U(t)|\psi_0\rangle$, where

$$U(t) = \mathcal{T} \exp \left[-i \int_0^t H(s) ds \right], \quad (2)$$

with \mathcal{T} denoting the time-ordering. Here and below we use the units with $\hbar \equiv 1$. To characterize the evolution of the subsystems A and B we introduce their reduced density matrices presented as traces over the other one:

$$\rho_{A(B)}(t) = \text{tr}_{B(A)} |\psi(t)\rangle \langle \psi(t)|. \quad (3)$$

After the interaction V between A and B is switched off, we may consider A and B as two separate systems, which however, can be entangled. Our goal is to make the switch-off process in such a way that the system A is finally in its ground state (see figure 1) regardless of the final state of the system B. Although the sizes of these systems are different, their purities $\text{tr} \rho_{A(B)}^2(t)$ and entropies $S_{A(B)}(t) = -\text{tr} \rho_{A(B)}(t) \ln \rho_{A(B)}(t)$ are equal to each other [27].

The final Hamiltonian can be written as $H(t \geq T) = H_0 = H_{0A} + H_{0B}$, where H_{0A} and H_{0B} have their own ground states $H_{0A(B)}|\varphi_{0A(B)}\rangle = \lambda_{\min A(B)}|\varphi_{0A(B)}\rangle$. To trace the proximity of A system to the ground state of H_{0A} , we introduce time-dependent parameter

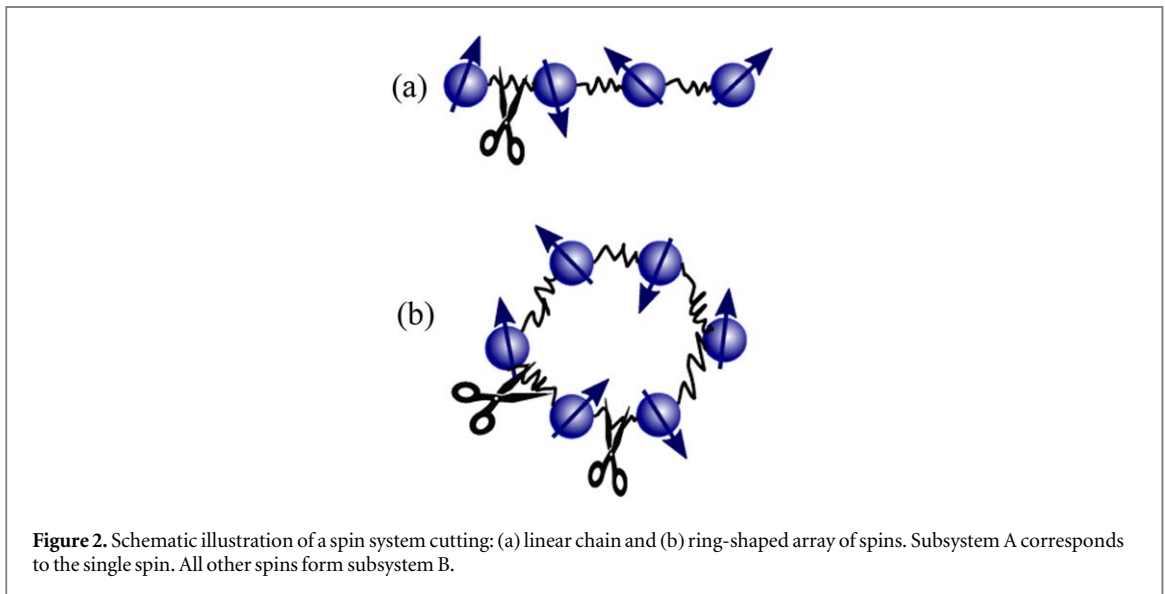
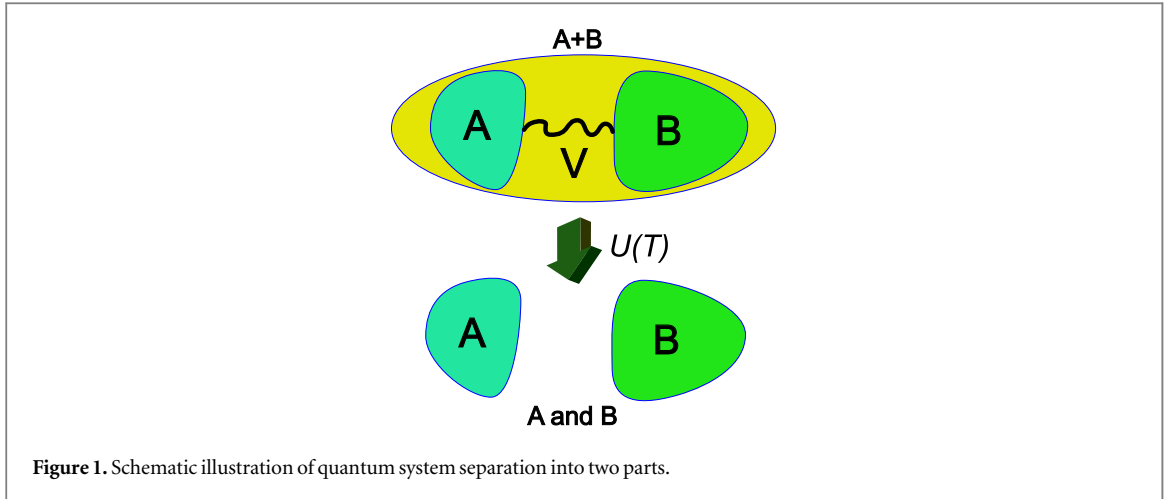
$$f_c(t) = \sqrt{\langle \varphi_{0A} | \rho_A(t) | \varphi_{0A} \rangle}, \quad (4)$$

and the resulting fidelity of the cutting process $f_c \equiv f_c(T)$.

A complementary general parameter, describing the proximity of the state at given t to the ground state of $H(t)$, $|\psi_0(t)\rangle$, is defined as:

$$f_g(t) = |\langle \psi_0(t) | \psi(t) \rangle|, \quad (5)$$

and corresponding fidelity $f_G \equiv f_g(T)$. It is easy to prove the inequality: $f_c \geq f_G$, such that the f_G is the lower bound for f_c . We will use f_G to describe the reverse problem for controllably entangling initially separated A and B parts into the joint A+B system.



Before proceeding to the numerical analysis, we notice the importance of the commutator $[H_0, V]$. It is easy to see from equation (2) by using Magnus and Zassenhaus [28] formulas that if $[H_0, V] = 0$, the fidelities f_C and f_G do not depend on $g(t)$. Therefore, we assume that $[H_0, V] \neq 0$.

Since the propagator (2) for our problem is not presentable in a closed analytical form, one cannot produce an exact equation for $g(t)$ assuring a shortcut to adiabaticity. In general, several numerical algorithms for optimal control of quantum systems have been proposed (see, e.g. [29]) and implemented [30]. Recently, an optimized algorithm has been developed for the control pulses for practical qubits [31].

In this paper, we present numerical results of the successful optimization of the $g(t)$ function by choosing appropriate set of parameters for its representation. The target fidelity f_C or f_G , being a functional of $g(t)$, becomes then a function of these parameters. We will show how this set can be optimized, study the properties of the optimal solutions, and provide a picture in basic physical and mathematical terms explaining main features of the proposed algorithm.

2.2. The model: Heisenberg chain in magnetic field

We consider a chain with N spins (see figure 2) placed in a uniform external magnetic field with the following Hamiltonian

$$H = J \sum_{n=1}^{N-1} \sigma_n \sigma_{n+1} + B \sum_{n=1}^N \sigma_n^z, \quad (6)$$

where $\sigma_n = (\sigma_n^x, \sigma_n^y, \sigma_n^z)$ is the Pauli matrix vector, B is the external magnetic field along the z direction, $J = \pm 1$ for (anti-) ferromagnetic coupling, and n is the spin number. For the ring-shaped array we add $J\sigma_N \sigma_1$ term to equation (6) to assure the system periodicity. We consider the whole chain as the $A + B$ system and the first

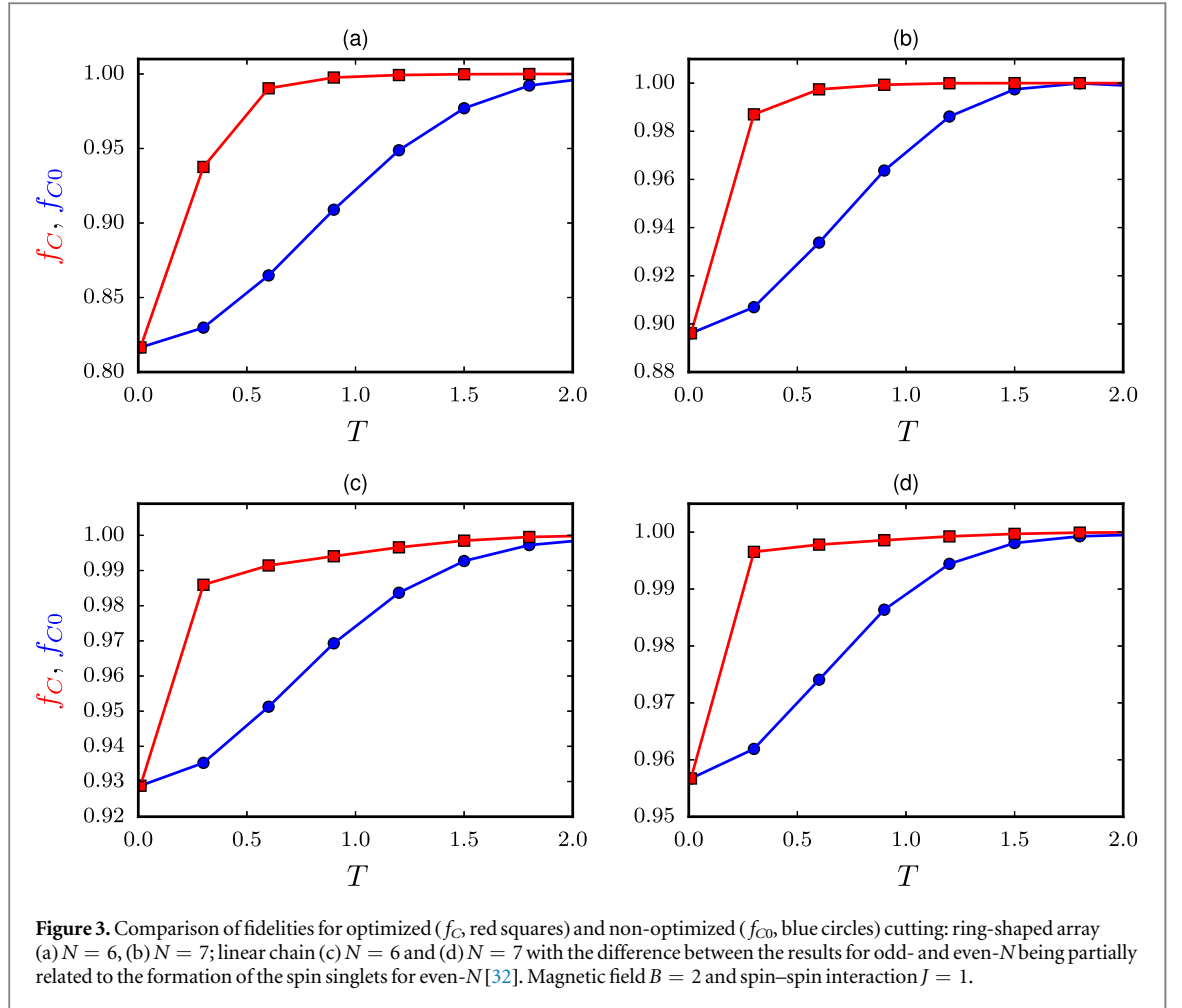


Figure 3. Comparison of fidelities for optimized (f_C , red squares) and non-optimized (f_{C0} , blue circles) cutting: ring-shaped array (a) $N = 6$, (b) $N = 7$; linear chain (c) $N = 6$ and (d) $N = 7$ with the difference between the results for odd- and even- N being partially related to the formation of the spin singlets for even- N [32]. Magnetic field $B = 2$ and spin–spin interaction $J = 1$.

spin as the A system (see figure 2). In such a case, we have $H_0 = J \sum_{n=2}^{N-1} \sigma_n \sigma_{n+1} + B \sum_{n=1}^N \sigma_n^z$ and $V = J \sigma_1 \sigma_2$, while for a ring-shaped array $V = J \sigma_1 \sigma_2 + J \sigma_1 \sigma_N$. After separating this single spin, we obtain $H_{0A} = B \sigma_1^z$ with the eigenvalue $\lambda_{\min A} = -B$ and spin-down eigenstate $|\varphi_{0A}\rangle = |\downarrow\rangle_1$. Thus, we have satisfied $[H_0, V] \neq 0$, where the time dependence of the coupling is due to the time dependence of the exchange interaction in the corresponding bonds. It is obvious that for a ferromagnetic coupling with $J < 0$, the ground state of the A + B system is disentangled [32] with $|\psi_0\rangle = |\varphi_{0A}\rangle \otimes |\varphi_{0B}\rangle$ and therefore the cutting with $f_C = 1$ can be made instantaneously. Thus, we concentrate only on the nontrivial anti-ferromagnetic coupling $J > 0$ and take $J = 1$ as the energy unit.

2.3. Parametrization of $g(t)$ and numerical results

We begin with a polynomial representation of $g(t)$ in the form:

$$g(t) = \begin{cases} 1 + \sum_{n=1}^K a_n \left(\frac{t}{T}\right)^n, & \text{for } 0 \leq t \leq T, \\ 0, & \text{for } t > T, \end{cases} \quad (7)$$

where a_2, \dots, a_K are free parameters and $a_1 = -(1 + a_2 + a_3 + \dots + a_K)$. Thus, the fidelity is a function of a_2, \dots, a_K as $f_C \equiv f_C(a_2, a_3, \dots, a_K, T)$. In the realization with the simplest linear decrease of the V term, where $a_1 = -1$, and $a_{n>1} = 0$, we define

$$f_{C0} \equiv f_C(0, 0, \dots, 0, T); \quad \lim_{T \rightarrow \infty} f_C = \lim_{T \rightarrow \infty} f_{C0} = 1. \quad (8)$$

Our goal is to find a set of parameters $\mathbf{a} = \{a_2, a_3, \dots, a_K\}$ which maximizes the fidelity f_C for a non-adiabatic process with a given finite time T . To find the proper \mathbf{a} , we use Broyden–Fletcher–Goldfarb–Shanno (BFGS) optimization method [33].

In figure 3 we show f_C for optimized cutting and f_{C0} for non-optimized cutting as a function of T for the ring-shaped and linear chains with $N = 6$ and 7 , $J = 1$, and $B = 2$. We use two free parameters a_2 and a_3 , with $K = 3$ in equation (7). Numerical simulations have been performed by using a Python SciPy package with

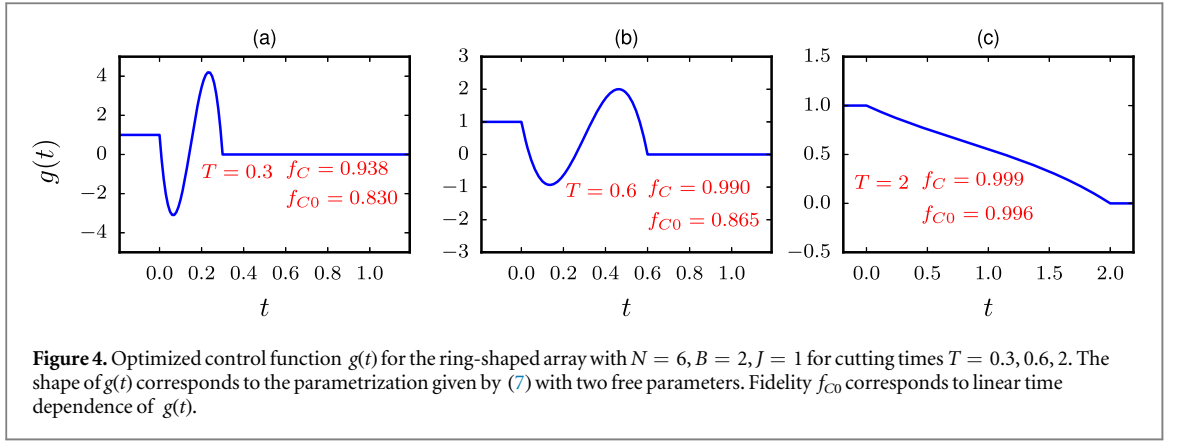


Figure 4. Optimized control function $g(t)$ for the ring-shaped array with $N = 6, B = 2, J = 1$ for cutting times $T = 0.3, 0.6, 2$. The shape of $g(t)$ corresponds to the parametrization given by (7) with two free parameters. Fidelity f_{C0} corresponds to linear time dependence of $g(t)$.

Table 1. Non-optimized and optimized fidelity for ring-shaped array of spins with $N = 6, B = 2, J = 1$.

	$T = 0.3$	$T = 0.6$	$T = 0.9$	$T = 2$
f_{C0}	0.830	0.865	0.910	0.996
f_C	0.938	0.990	0.998	0.999
a_2	122.8	54.3	20.0	0.87
a_3	-82.0	-36.3	-13.5	-0.72

the built-in BFGS optimization method. Unitary operator in equation (2) was approximated with 300 time steps, and the error in fidelities is less than 0.001. Gradient for BFGS was approximated as a finite difference with a step of 0.1 in the parameter space. Note, that the magnetic field B should not be very high, because in such a case the ground state becomes fully spin polarized and disentangled. In figure 4 we show the optimal shape of the control function $g(t)$ for the ring-shaped array of spins. The linear chain has a similar shape of $g(t)$. Obviously, $g(t)$ becomes closer to the simple linear dependence with increasing the cutting time T . Numerical data for this setting is presented in table 1. As can be seen from figure 3 and table 1, the same high value of f_C can be achieved in two different ways: by a slow linear adiabatic turning off the interaction, and via a more complicated but faster switching. Thus this second way for achieving the demanded state can be considered as the adiabaticity shortcut.

In figure 5 we show f_C as a function of the parameter a_2 (a_3) with a fixed at the optimal value a_3 (a_2). Here $T = 0.6$ for the ring-shaped array with $N = 6, J = 1$, and $B = 2$, where the optimal values are $a_2 = 54.31$ and $a_3 = -36.33$ (the optimal values correspond to the red dashed vertical lines in figure 5 and are shown in the second column in table 1). As can be seen in figure 5 when parameters a_2 and a_3 have a small deviation ($\approx 2\%$) from their optimal values, our proposal is still efficient. The parameter a_3 must be tuned with more accuracy than parameter a_2 .

In figure 6, we show the time-dependent $f_g(t)$ in equation (5) and $f_c(t)$ in equation (4) for optimized and non-optimized (linear) cutting protocols. One can see that while in the middle of the optimized process, $f_c(t)$ can be smaller than that for the non-optimized cutting, at $t = T$, it becomes higher. Moreover, as can be seen from the behavior of $f_g(t)$, optimized process is less adiabatic than the non-optimized one during most of the evolution time. However, as mentioned above, we are interested only in the final state.

In addition, we note that even in the case of the energy-levels crossing in the cutting process (i.e. when at some time t' we have $E_0(t') = E_1(t')$, where $E_0(t)$ and $E_1(t)$ is the ground and the first excited state energy of the instantaneous $H(t)$, respectively), which happens, e.g., in a ring-shaped array with $N = 7, B = 2, J = 1$, the behavior of f_C (or f_{C0} corresponding to the linear switch-off) is the same as without the crossing. Fidelities f_C and f_{C0} increase up to 1 with the evolution time T (see figure 3(b)). However, calculated $f_C = 0$ here, meaning that A and B parts become disentangled after cutting, with A being in its ground state and B in a pure state orthogonal to the ground one.

2.4. Robustness of the control

Now we consider the influence of the ‘apparatus’ noise [34, 35] in the control function $g(t)$ on the fidelity of cutting. The noise is simulated as a set of rectangular pulses with a fixed length Δt and the random strength $\Delta g(1/2 - r)$, where Δg is a characteristic strength, and r is a random number in the half-interval $[0, 1)$. This noise is added to the smooth $g(t)$ given by equation (7) for the linear chain. The results of simulations

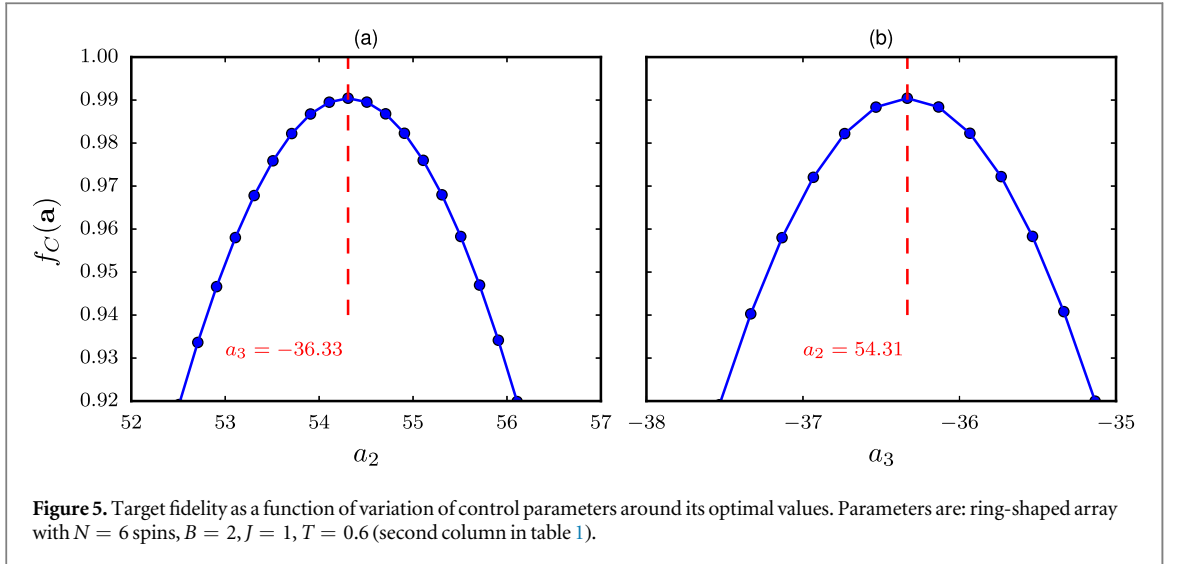


Figure 5. Target fidelity as a function of variation of control parameters around its optimal values. Parameters are: ring-shaped array with $N = 6$ spins, $B = 2$, $J = 1$, $T = 0.6$ (second column in table 1).

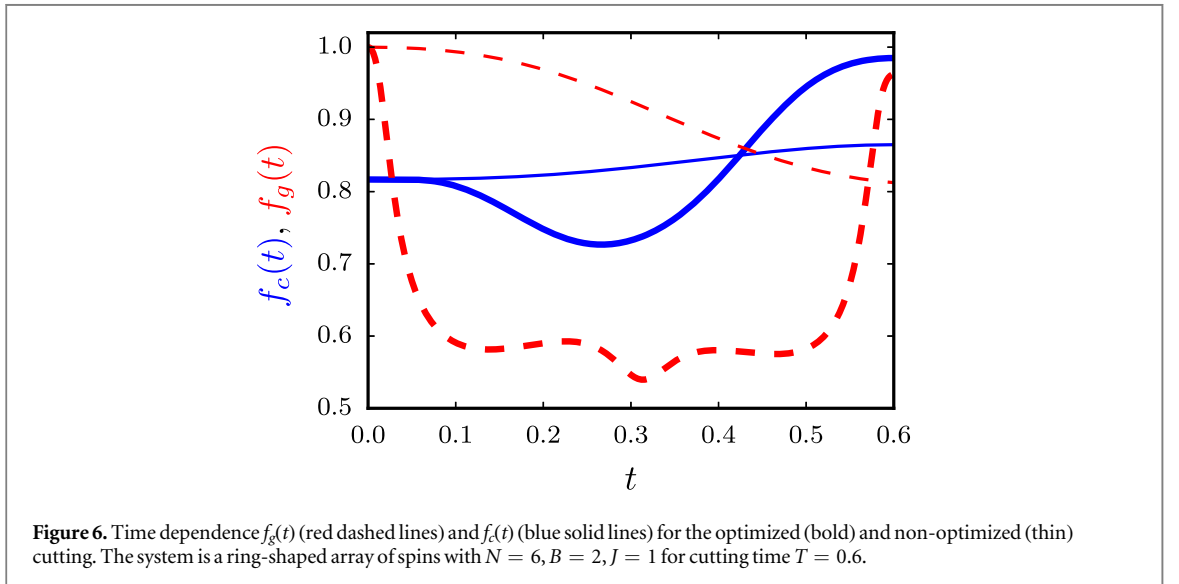


Figure 6. Time dependence $f_g(t)$ (red dashed lines) and $f_C(t)$ (blue solid lines) for the optimized (bold) and non-optimized (thin) cutting. The system is a ring-shaped array of spins with $N = 6$, $B = 2$, $J = 1$ for cutting time $T = 0.6$.

presented in figure 7 lead to conclusion that our proposal is robust against small fluctuations in the control function. Also, as can be seen in figure 7, a noise with a high characteristic frequency influences the fidelity less than that with a low frequency.

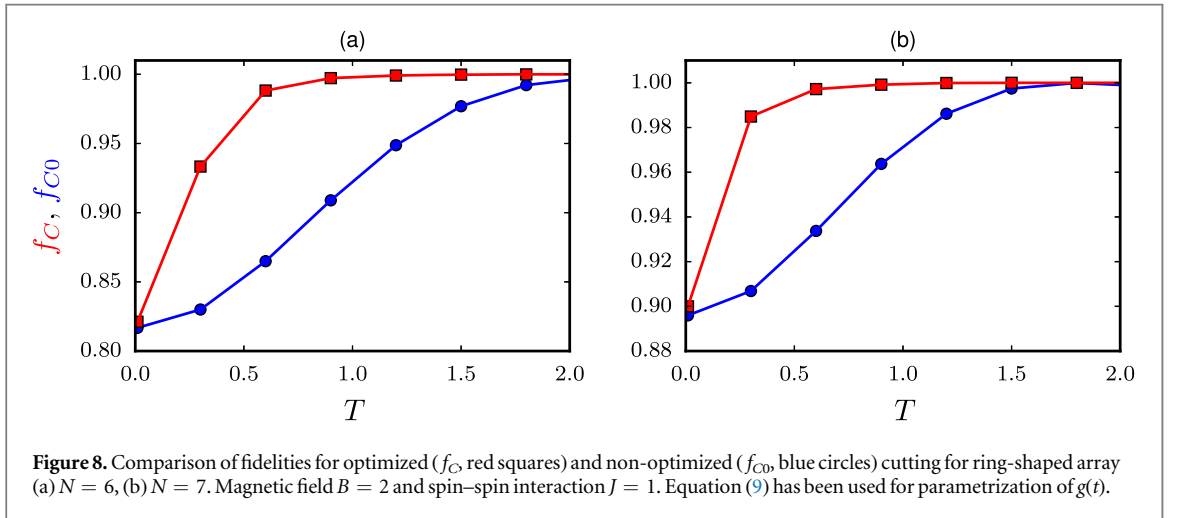
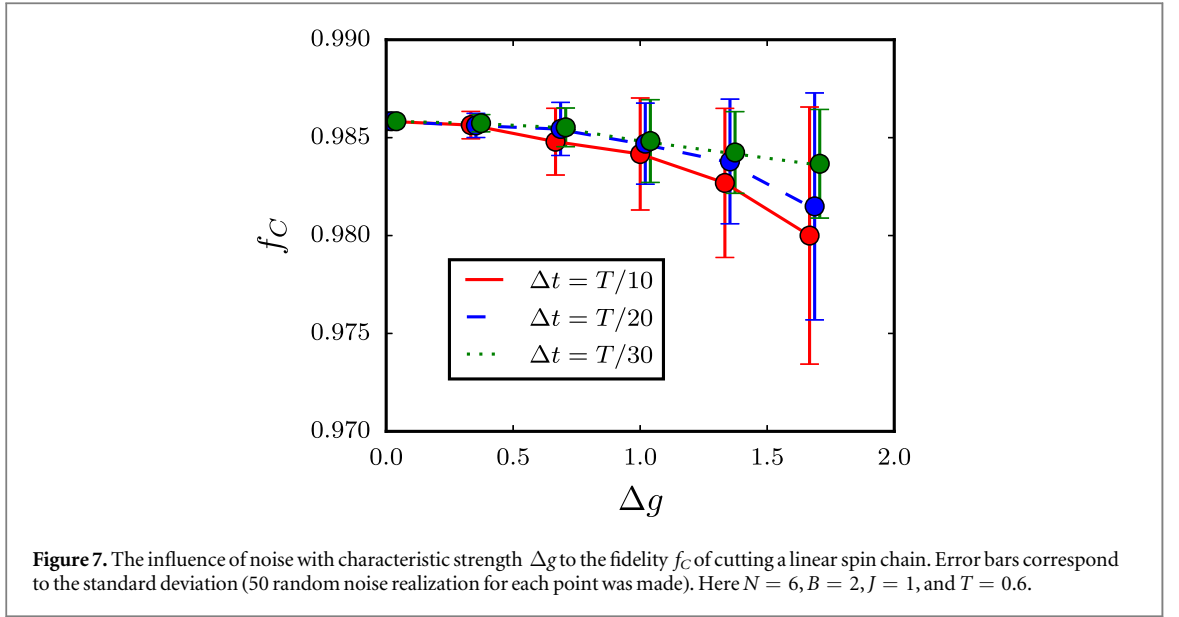
2.5. Other $g(t)$ -realizations

Since we are not restricted in choosing the parametrization of the control function $g(t)$, here we compare the results of different representations. According to the boundary conditions, we can write $g(t)$ as follows:

$$g(t) = \begin{cases} 1 - \frac{t}{T} + \sum_{n=1}^K b_n \sin\left(\frac{n\pi t}{T}\right), & \text{for } 0 \leq t \leq T, \\ 0, & \text{for } t > T, \end{cases} \quad (9)$$

where b_1, \dots, b_K are free parameters. To compare the parametrizations in equations (7) and (9), we chose $K = 2$ in equation (9) and repeat the optimization for the cutting of a ring-shaped array. The results are presented in figure 8. As can be seen in the figure, the effectiveness of the optimization algorithm for different $g(t)$ representations is almost the same.

Since in our simulations we use the gradient-based BFGS optimization method, we do not need to calculate the fidelity f_C for each \mathbf{b} on a specially chosen dense grid. Nevertheless, it is instructive to look at the 'landscape' of f_C as a function of two free parameters and make sure that the optimization algorithm works correctly. Moreover, this data may be of independent interest. In figure 9, we show the landscapes corresponding to different parametrizations of $g(t)$ and the same physical setting (ring-shaped array, $N = 6$, $T = 0.6$, $J = 1$, and



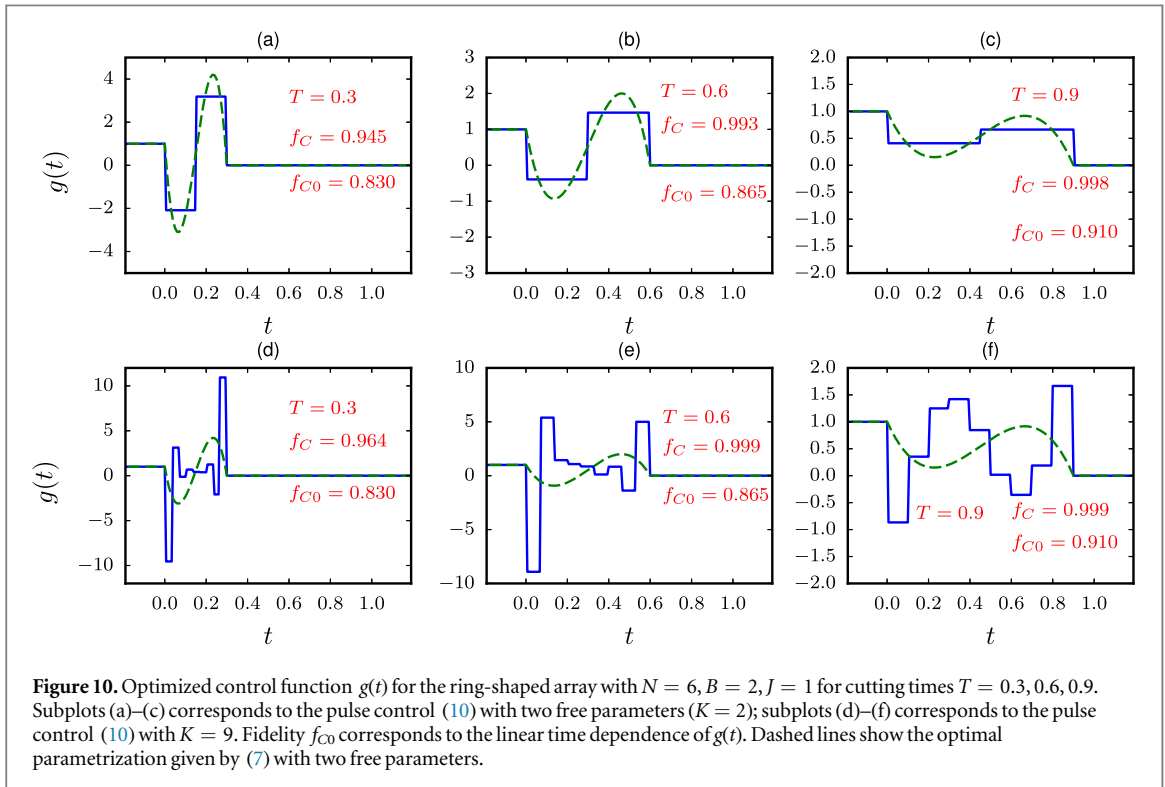
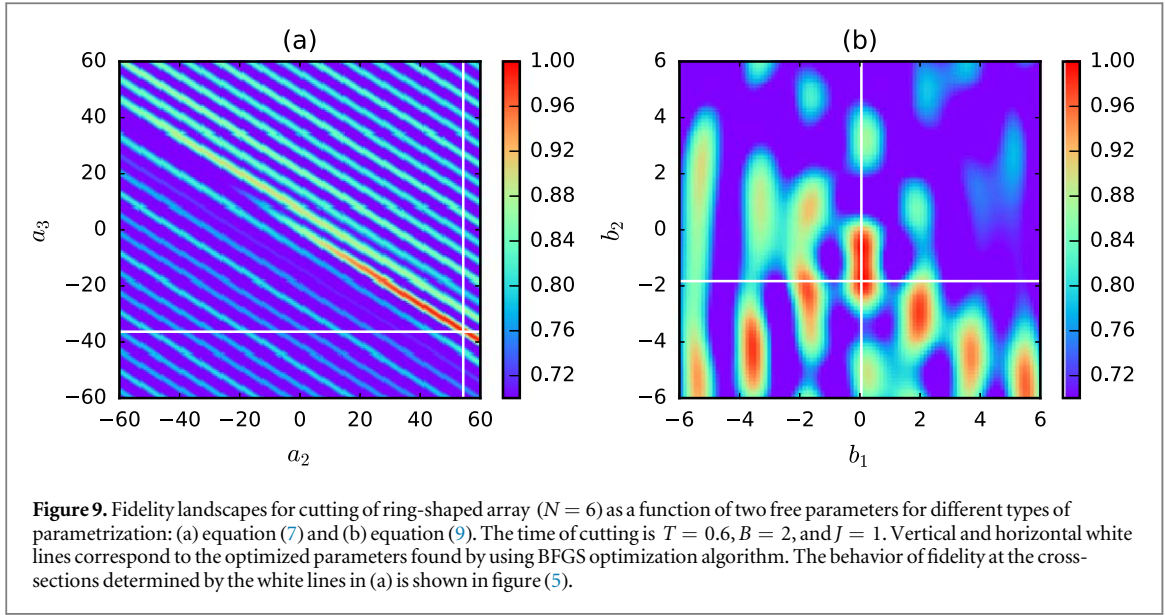
$B = 2$). Intersections of white lines in figures 9(a), (b) are positioned at the optimal sets of parameters corresponding to the results of the BFGS algorithm. For both realizations, the optimal fidelities and control functions are very close to each other. It is interesting to observe that both the landscapes have a feature in common. The point corresponding to the best f_C is located on the same high-fidelity ‘island’ as the $(0, 0)$ -point, making it the right initial state for the global-minimum search algorithm, as it was employed in our simulations.

We can also consider $g(t)$ as a sequence of K rectangular pulses whose amplitudes form the set of free parameters $\mathbf{c} = \{c_1, \dots, c_K\}$, and the duration of each pulse is $\Delta t = T/K$:

$$g(t) = \sum_{n=1}^K c_n [\theta(t - (n-1)\Delta t) - \theta(t - n\Delta t)], \quad (10)$$

where $\theta(t)$ is the Heaviside step function.

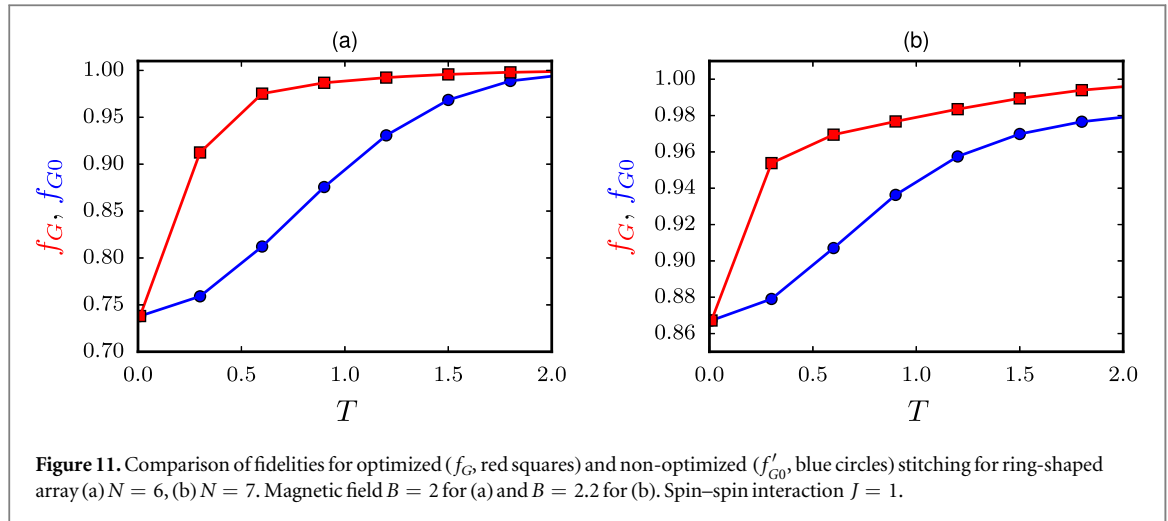
In figures 10(a)–(c), we show the optimal shape of $g(t)$ for $K = 2$, and in figures 10(d)–(f) $K = 9$ is used. Our setting is the same as in figure 4. By comparing the first row in figure 10 with the shapes in figure 4 (dashed lines), we see the correspondence between the shapes of smooth and pulse controls. Moreover, the optimization time for a pulsed shape with $K \approx 10$ is much faster than the parametrized optimization with smooth shape of $g(t)$ even for $K = 2$. This is because for optimization of a smooth $g(t)$, the algorithm divides the $[0, T]$ interval into more than 100 pieces to accurately calculate the propagator (2) while for the pulse control the evolution interval is divided into the given number of K pieces.



3. Stitching of spin chain

Now we consider the opposite process: a stitching of quantum system. In this setting, we have a disentangled initial state of the A+B system, which is the ground state of the Hamiltonian H_0 . Then, we switch on the interaction V between A and B systems, and the desirable final state of the joint A+B system is the ground state of the Hamiltonian $H_0 + V$. This approach, which offers a new class of operations such as increasing the size of a quantum system, is of a practical importance, being a possible way of initializing a spin chain for quantum information processing (see e.g. [36]).

Since both states of the A and B parts are important at the start and at the end of the stitching, we use the fidelity f_C to describe this process. Thus we have to satisfy more strict condition in order to achieve the high-fidelity: the energy gap between the two lowest states of the Hamiltonian $H = H_0 + g(t)V$ cannot be zero for any $g(t) \in (0, 1)$. This requirement originating from the adiabatic theorem is not applicable to the chain cutting, where only the final state of the A system is essential.



Here we consider parametrization of the control function for the stitching process $g(t)$ expressed in the same way as for the cutting (7) by changing $t \rightarrow T - t$:

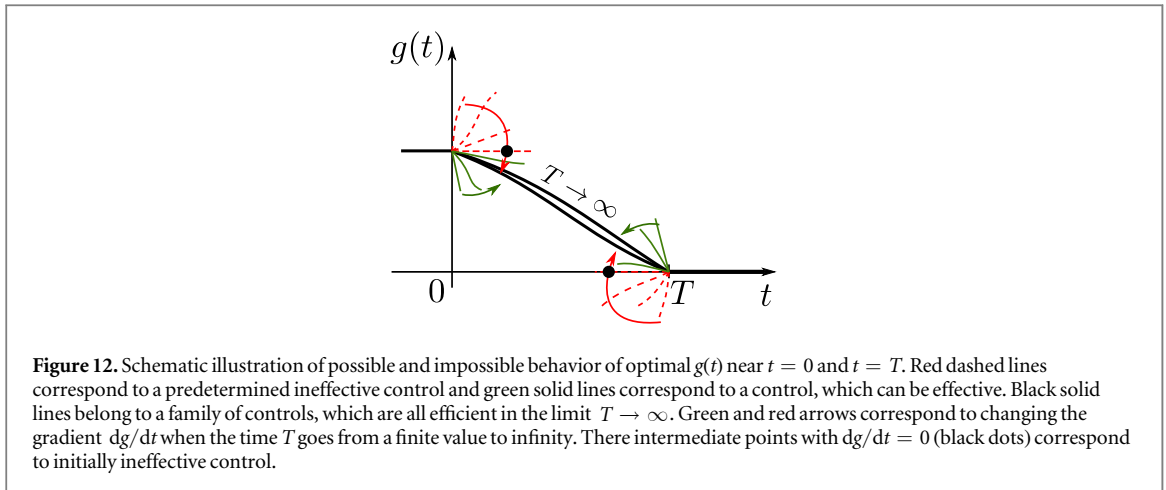
$$g(t) = \begin{cases} 1 + \sum_{n=1}^K d_n \left(\frac{T-t}{T}\right)^n, & \text{for } 0 \leq t \leq T, \\ 1, & \text{for } t > T, \end{cases} \quad (11)$$

where $1 + \sum_{n=1}^K d_n = 0$. In addition, as in the case of cutting, we also use here the fidelity f_{G0} , similar to that defined in equation (8). Note that the polynomial parametrization of $g(t)$ does not imply that the optimal control shape for stitching can be directly reconstructed from the optimal $g(t)$ for the corresponding cutting since the optimal sets of parameters are essentially different for these processes. As an example we show in figure 11 the results for the optimization of the stitching process for a ring-shaped array.

Here we make a general comment related to the cutting and stitching processes. The ground states of the initial and final Hamiltonians can be degenerate for the cutting or stitching process. In such a case we have the ground state subspace, and we have to choose the initial ground state $|\psi_0\rangle$ in such a way that $\langle \psi_0 | \psi'_0 \rangle \rightarrow 1$ for $|\psi'_0\rangle$ being a non-degenerate ground state of the Hamiltonian $H = H_0 + (g(0) + \delta g)V$, where $g(0) = 1$ and $\delta g \rightarrow -0$ ($g(0) = 0$ and $\delta g \rightarrow +0$) for the cutting (stitching) process.

4. Discussion

As can be seen from the shapes of control functions $g(t)$ in figure 10, they have similar patterns for all cases of cutting, clearly seen for the fast processes. In all these patterns, the interaction V enters a ferromagnetic domain with $g < 0$ at the beginning, and jumps back to a strong anti-ferromagnetic coupling before the end of the process (see figures 4(a), (b)). This behavior can be explained by the following heuristic arguments. In the adiabatic limit ($T \rightarrow \infty$) almost linear $g(t)$ -dependence is the sufficient control for high output fidelity, as shown in figure 4(c). First, let us assume, that a non-adiabatic control which we have found for a finite $T = T_0$ has positive derivative $dg(+0)/dt > 0$ in the case of a smooth control (see red dashed lines in figure 12), or the amplitude of first pulse $c_1 > 1$ in the case of a pulse control (equation (10)). With the increase in T , the shape of the control function must transform into a linear $g(t)$ continuously (red arrows in figure 12). This continuity implies that the derivative $dg(+0)/dt$ will become negative $dg(+0)/dt \rightarrow -1/T$ (or first pulse amplitude will go to $1 - 1/K$) with increasing time T . Therefore, for some intermediate $T' > T_0$, the derivative should become zero, $dg(+0)/dt = 0$ (or $c_1 = 1$). However, this vanishing derivative means that for this particular value $T = T'$ the control loses the efficiency, since at the beginning of the interval $[0, T']$ it does not influence the system. On the contrary, if $dg(+0)/dt < 0$, for $T = T_0$ (or $c_1 < 1$) one can make continuous transformation of the control function to $T \rightarrow \infty$ without crossing such 'no-control' point when $dg(+0)/dt = 0$. The same reasoning can be made for the $g(t)$ -behavior near the final $t = T$ point, where one must have $dg(T-0)/dt < 0$ or $c_K > 0$ for the continuous and pulsed control, respectively. After analyzing all these options, we are left with the only possibility to have $dg(+0)/dt < 0$ and $dg(T-0)/dt < 0$, as confirmed by our numerical simulations. This behavior of $g(t)$ for $T < 1$ agrees well with the physical picture of formation of the spin ground state in the Zeeman field. The initial decrease in $g(t)$ to negative values forms a ferromagnetic coupling of the spin that is going to be separated (system A) with the nearest neighbor(s) from the rest of the chain (system B). This coupling



decreases the quantum fluctuations in the corresponding Heisenberg spin–spin bond and the spins forming this bond together become easier oriented by the Zeeman field.

This general behavior of the control function is corroborated by the analysis of figure 9 (b): the peak of the high-fidelity island is located at a negative value of b_2 parameter while b_1 is close to zero, corresponding to a fast change in the sign of the exchange coupling at the corresponding bond. By using parametrization (9) and inequalities $dg(+0)/dt < 0$ and $dg(T-0)/dt < 0$, one can analytically write the restrictions on b_1 and b_2 as: (i) $b_2 < 1/2\pi - b_1/2$ and (ii) $b_2 < 1/2\pi + b_1/2$. Subsequent numerical analysis shows that the high-fidelity islands are located below the lines corresponding to these inequalities, in agreement with our reasoning. Note that the above reasoning is valid only if the requirements of the adiabatic theorem can be satisfied in the $T \rightarrow \infty$ limit, i.e. in the absence of the energy-level crossings for $g \in [0, 1]$.

The amplitude of $g(t)$ increases with decreasing of T , as expected from the time-energy uncertainty relation and quantum speed limit [37]. However, it is easy to see that for effective control this amplitude should not be very large. To prove this, let us consider the pulse-shaped $g(t)$, where the evolution during the pulse can be described by $U(t) \approx \exp[-i(g-1)Vt]$. Thus, due to the local nature of V , the initial state cannot be changed to the demanded one by such evolution. Therefore, in the limit $T \rightarrow 0$ ($\psi(T) \rightarrow \psi_0$), the increase in $g(t)V$ cannot make the fidelity f_C higher than the non-optimized f_{C_0} . As a result, optimized and non-optimized curves merge in the anti-adiabatic limit $T \ll 1$ ($T = 0.01$ in figure 3).

In general, our exact numerical simulations are limited by small number of spins N since even for $N < 10$ the optimization takes a relatively long time. However, the control function does not strongly depend on N for long chains. Taking into account that the spin wave velocity (e.g. [38, 39]) is of the order of J , one can see that during the time T the non-equilibrium spin wave propagate from the perturbed bond the distance (in the units of the spin–spin separation) of the order of JT . Therefore, for $N \gg JT$, the $g(t)$ –function should be only weakly N -dependent. However, the effect of parity, that is the difference between odd and even N can influence the shape of $g(t)$.

Although we presented here cutting protocols for detachment of a single spin from a chain, our results go far beyond preparation of a spin 1/2 in the ground state. First, we demonstrated here the proof of concept of the high-fidelity non-adiabatic separation of a complex system, which is a new type of a quantum operation. Second, our proposal works well when we cut more than a single spin. For example, the non-optimized linear cutting of two spins from $N = 5$ open chain with $B = 2.1$ and $T = 0.6$ yields $f_{C_0} \approx 0.26$. However, our calculations demonstrate that under the pulse-shape control (10) with $K = 2$ and $c_1 = -5.4$, $c_2 = 4.1$ the target fidelity becomes $f_C \approx 0.79$. Third, we note that cutting a single spin with the perfect fidelity $f_C \approx 1$ means that the remaining part of the chain is in a known *pure* (not necessarily the ground) state, which in its turn can be transformed by unitary operations to a desirable pure state of $N - 1$ spins.

5. Conclusion

We have shown the feasibility of the unitary non-adiabatic cutting and stitching of complex quantum systems with a demanded high-fidelity output. The key feature of our proposal is the possibility of using *local* control instead of the global one. The optimal shape of the control function can be found by the gradient numerical optimization. We show that even a simple two-parametric control can be effective for cutting and stitching the complex quantum system with a high-fidelity. Since our approach is numerically exact, this proposal is directly applicable only for relatively small quantum systems. However, the increase in the system size is not expected to

modify our results qualitatively. Even when the time of the process is short, a demanded fidelity can be achieved, and thus shortcut to adiabaticity can be realized. Different parametrizations of the control function can be used with a high-fidelity of the results robust against small variations of parameters and noise in the control. Our results can stimulate further investigations in the adiabaticity shortcut problems, including application in quantum heat transfer and annealing processes. From the experimental point of view, they can be realized by the electrical control of spin states in ensembles of quantum dots, where the modulation of spin–spin interaction, including change in the sign of the exchange coupling J [26] can be done by gating the interdot tunneling and the states inside the dots on the time scale as short as 10^{-2} ns, assuring a highly coherent evolution.

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