

LINE-OF-BALANCE SCHEDULING METHOD

ANALYSIS OF LOB SCHEDULING METHOD

JAIME UBIETA ASTIGARRAGA



TABLE OF CONTENTS

LIST OF TABLES	ii
LIST OF FIGURES	ii
1. INTRODUCTION	1
2. THE LINE OF BALANCE METHOD	3
2.1. Description of the LOB method	4
2.2. LOB diagram.....	7
3. ACCELERATION OF PROJECTS USING THE LOB METHOD	12
3.1. Acceleration routine	12
3.2. Required date of completion.....	24
4. LOB METHOD USAGE SURVEY AND RESULTS.....	28
5. CONCLUSIONS	32
REFERENCES.....	34
APPENDIX I: Acceleration process example	35



LIST OF TABLES

Table 1 Activities in the project	4
Table 2 Calculated values with the LOB	6
Table 3 Starting and finishing time table of the unit.....	10
Table 4 Project data.....	17
Table 5 Project data after first acceleration.....	19
Table 6 Project data after second acceleration.....	21
Table 7 Project data after total acceleration.....	22
Table 8 Initial project data (date completion analysis)	25
Table 9 Data of the survey.....	29

LIST OF FIGURES

Figure 1 Example of the network diagram of a produced unit	4
Figure 2 Buffer time disposition between activities.....	7
Figure 3 Movement between units of a crew in an activity.....	8
Figure 4 Movement between units of two crews in an activity.....	8
Figure 5 Order of time calculation depending of production rates	10
Figure 6 LOB diagram of the example	11
Figure 7 Effect of increasing number of crews on activity B.....	14



Figure 8 Effect of increasing number of crews on activity B 15

Figure 9 Acceleration process flowchart (inspired by Tokdemir, 2002) 16

Figure 10 Initial LOB diagram (example) 18

Figure 11 Initial resource histogram..... 18

Figure 12 LOB diagram after first acceleration of activity F 20

Figure 13 Resource histogram after first acceleration of activity F 20

Figure 14 LOB diagram after second acceleration of activity F 21

Figure 15 Resource histogram after second acceleration of activity F 22

Figure 16 LOB diagram after total acceleration 23

Figure 17 Resource histogram after total acceleration..... 23

Figure 18 Initial LOB diagram (date completion analysis)..... 25

Figure 19 Acceleration of activity A (project duration 20 days)..... 26

Figure 20 Acceleration of activity C (project duration 23 days)..... 27

Figure 21 % of companies that use LOB in their projects 29



1. INTRODUCTION

In the construction industry there are projects such as construction of high-rise buildings or pipelines that consist of repetitive activities. For these projects has proved the Line of Balance (LOB) method to be the most suitable scheduling tool of all existing linear scheduling techniques (Sarraj 1990).

The LOB was first used by the Goodyear Company and was developed by the U.S Navy Department in the early 1950's for programming and controlling repetitive projects. It was later developed by the National Building Agency (in the United Kingdom) to be used in repetitive housing projects.

The LOB is a variation of linear scheduling methods that allows the project activities to stay "in balance" so that the production rate of the operations allows each activity in the project to be continuously performed and the planned project goals be achieved. The LOB schedule can be plotted into an easy to understand graph where the project progression can be followed, because it provides information of the activity rates and durations. LOB allows a better understanding of a project composed of repetitive activities than any other scheduling technique, because it allows the possibility to adjust activities' rates of production. It allows a smooth an efficient flow of resources and requires less time and effort to produce than network schedules (Arditi and Albulak 1986).



In this document the LOB method is presented, as well as an explanation of the algorithms that are used when applying this method in a project with repetitive activities. Later the acceleration routine is explained in detail.



2. THE LINE OF BALANCE METHOD

LOB is a resource driven scheduling system for projects with repetitive activities. The main objective is to find the required resources for each activity. The project is developed according to a schedule with a continuous flow in the activities.

The LOB diagram provides a vivid overview of the project's overall status by quantitatively representing the cumulative completions of activities associated with a level of planned number at given time (Suhail and Neale 1994). It graphically reveals any imbalance that suggest a deviation from the plan due to the actual uneven progress of activities, and promptly enables management to focus on assessing the deviation quantitatively (Khisty 1970)

The LOB method is based on the number of required units to be delivered at the end of the project. This number of units is what sets the production rate of the overall project. The rate of production of every activity should be higher than the rate of delivery, so that the project is finished within the projected time.

The rates of production of the different activities are obtained by analyzing data from previous projects, or by experience of line managers and workers in similar activities. The LOB diagram is developed in the assumption that the production rates are uniform in each activity. But the rates between different activities may be different.

2.1. Description of the LOB method

In this section, the LOB method is explained using a simple example of a project that involves repetitive activities.

Company ABC has been asked to deliver 60 units to a client at a target rate of 3 units per week. A typical working week consists of 5 days of 8-hour shifts. The construction of each unit consists of the activities in series displayed in the following table.

Table 1 Activities in the project

Activity	Worker-hours per unit	Workers per crew	Activity buffer (days)
A	100	3	2
B	300	8	2
C	350	9	1
D	50	2	2
E	210	5	-

The first thing that needs to be done is to develop a network diagram of one of the units to be produced. The network diagram is presented in Figure 1:

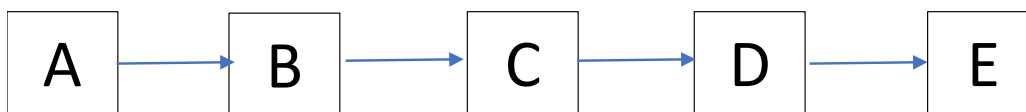


Figure 1 Example of the network diagram of a produced unit

In this example, all activities are in series for the sake of simplicity.

The table displays information about the estimated worker-hours and the optimum crew sizes for each activity. This information yields a natural rhythm for each activity defined as the optimum rate of output that a crew of optimum size will be able to produce (Arditi and



Albudak 1986). This crew size assures that no idle time between unit exists and that the project is developed in the most effective way.

The buffer time is displayed in Table 1. The buffer time is the extra time added to the time estimate of an activity to keep the project on time, and to ensure a smooth flow of resources through the units because some activities may be delayed in some units.

The next step is to calculate the duration of each activity. So that the schedule can be later developed. This is done by developing a table with the available information.

To complete the table, several equations must be used.

Theoretical crew size at the objective production rate:

$$G = \frac{r * \text{Worker hours per unit}}{N^{\circ} \text{ hours per worker/week (40hours)}}$$

This equation calculates the number of workers that should theoretically be used to achieve the objective production rate ($r=3$ units/week).

Actual crew size:

$$G_a = \text{multiple of } G$$

Since the manpower available in Company ABC is organized in teams, and the theoretical gang size is most probably not an integer, it is necessary to round the result to the closest number of teams. In this equation G is the number of workers on each team.

Actual output rate



$$R_a = r * \frac{G_a}{G}$$

The equation calculates the real rate of production of the activity that will be used to draw the LOB graph later. In this case G is the theoretical gang size.

Activity duration for one unit in days:

$$D = \frac{\text{Worker hours per unit}}{\text{Workers per gang} * N^o \text{ hours per worker/day (8 hours)}}$$

This equation calculates the duration for one unit to be completed.

Time from the start of the first unit to the start of the last unit:

$$T = \frac{(Q - 1) * N^o \text{ working days/week}}{R_a}$$

This is done for every activity that takes place in the project. This will allow the scheduler to draw the LOB diagram later.

Table 2 is developed using the equations presented.

Table 2 Calculated values with the LOB

Activity	Worker-hours per unit	Workers per crew	Theoretical crew size at specified output rate	Actual crew size	Actual output rate	Activity duration for one unit (days)	T (days)
A	100	3	7.5	9	3.6	4	82
B	300	8	22.5	24	3.2	5	92
C	350	9	26.25	27	3.09	5	96
D	50	2	3.75	4	3.2	3	92
E	210	5	15.75	15	2.86	5	103

2.2. LOB diagram

With the calculated values and the buffer times between activities the LOB diagram can be drawn. It should be noted that when the following activity has a smaller rate of production than the current, the buffer time between activities must be placed after the first unit. This is, the following activity should start after the buffer time has passed after finishing the first unit in the previous activity. On the other hand, when the following activity's rate of production is higher, the buffer time needs to be placed after the last unit. This is, the following activity on the last unit can only be performed once the buffer time has passed after finishing the last unit in the preceding activity. Figure 2 displays these situations.

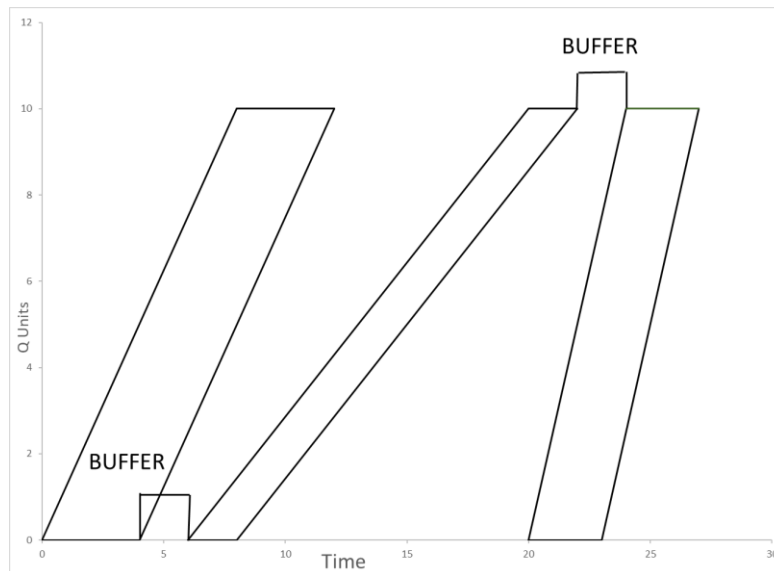


Figure 2 Buffer time disposition between activities

It can be seen in Table 2 that each activity is performed by more than one crew. Each crew can work in only one unit at a time. So, in case there are more than one crew working in an activity, after a crew finishes working on a unit it would move itself to the next available

unit. In Figures 3 and 4 the same activity is plotted, where 6 units are to be produced using crews of a natural rhythm of 0.5 units/day. If only one crew is working in the activity, the movement of this crew between units will be as shown in Figure 3. If two crews are used, the movement of these crews are shown in Figure 4. Where one crew is displayed in black and the other in red.

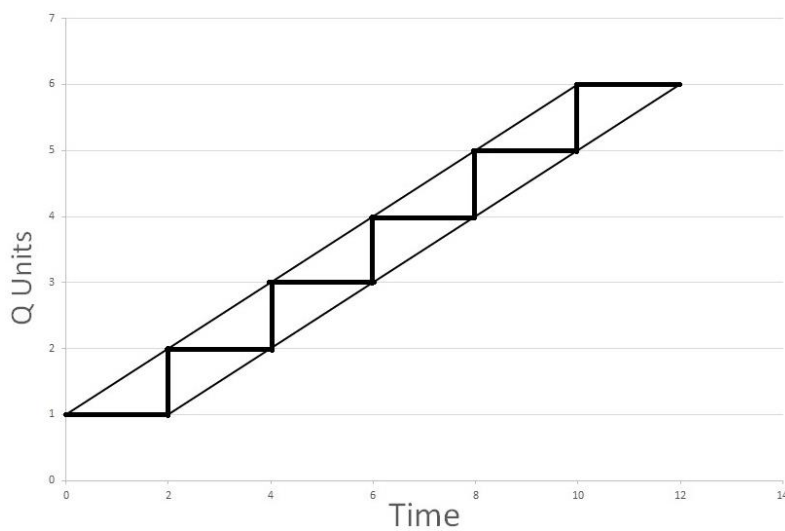


Figure 3 Movement between units of a crew in an activity

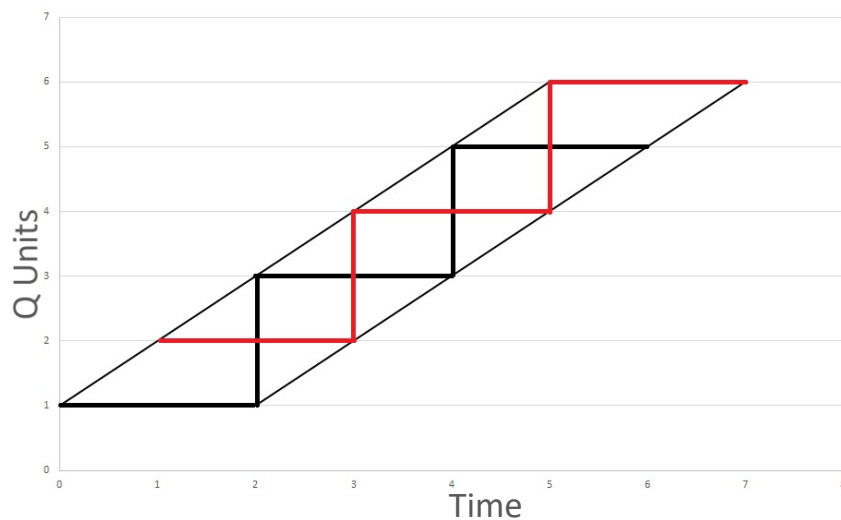


Figure 4 Movement between units of two crews in an activity



The way of developing the LOB diagram is as follows. The first unit starts at time 0, knowing the rate of production of the first activity one can draw the first line, the slope of the line is equal to the rate of production of the activity. By adding the duration of a unit activity at the end of the line, the finish time of the first activity is calculated. Connecting the finish time of first and last units in the activity the second line is drawn. This line must be parallel to the first line.

Then, depending on whether the following activity's rate of production is smaller or higher, the second activity is drawn. As explained before, if the following activity is slower the buffer time must be positioned after the finish of the first unit. Otherwise, the buffer time is placed after the finish time of the last unit.

If the second activity is slower than the preceding activity, the unit starting time line is drawn from unit one to the last unit. The slope of the line is again the rate of production of the activity. Placing the duration of the activity in the end of the line and drawing a parallel line to show the unit finish times, the second activity is drawn.

In case the activity's rate of production is faster than the preceding activity's rate, the unit starting time line is drawn beginning from the last unit to the first. The unit finish time line is drawn like explained before.

Figure 5 depicts the explained procedure of drawing the LOB diagram.

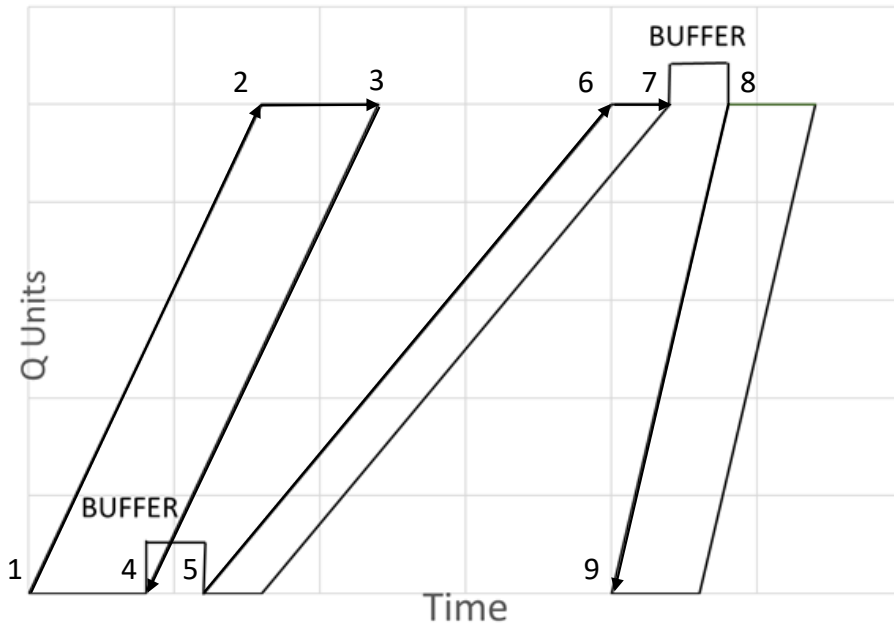


Figure 5 Order of time calculation depending of production rates

Table 3 is completed using the equations and conditions stated before.

Table 3 Starting and finishing time table of the unit

Unit	A		B		C		D		E	
	Start time	Finish time	Start time	Finish time	Start time	Finish time	Start time	Finish time	Start time	Finish time
1	0	4	6	11	13	18	22	25	27	33
2	1	6	8	12	14	19	24	27	29	34
3	3	7	9	14	16	21	25	28	31	36
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
59	81	85	97	101	107	112	113	116	129	134
60	82	86	98	103	108	113	114	117	131	136

From Table 3 it can be deduced that the project will take 136 days to complete. With the data in the table, one can proceed to draw the LOB diagram.

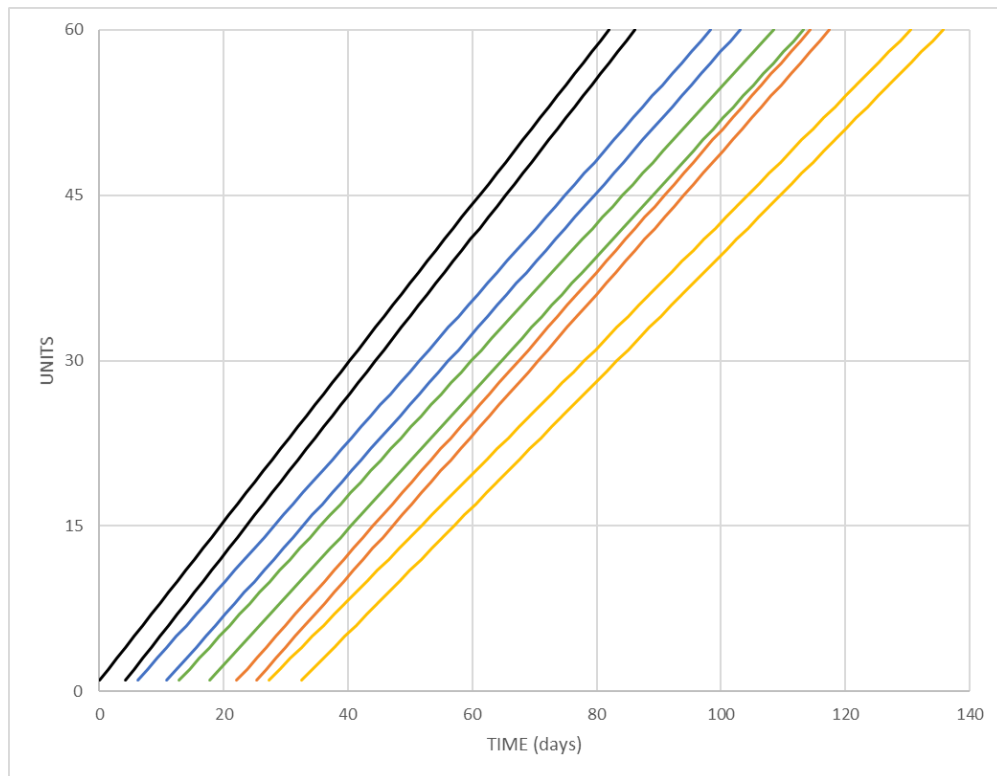


Figure 6 LOB diagram of the example

The LOB diagram drawn is used to keep track of the project in a very easy to visualize manner. It allows to keep track of the units that are finished and at the same time know exactly at every moment where the workers should be and where resources are needed. The use of different colors for each activity makes the understanding of the LOB simple and quick.



3. ACCELERATION OF PROJECTS USING THE LOB METHOD

When working on projects with multiple activities and multiple relations between them, the LOB scheduling is not simple. In this section some of the acceleration of projects procedure is presented.

3.1. Acceleration routine

The project is finished when the last unit of the last activity is finished and ready to be delivered. The time at which this last unit is finished is what determines the project time. In many cases, the project duration will be longer than the contract time. To solve this problem, the activities need to be done at a higher rate of production so that the project duration meets the requirements in the contract.

The only way that a project production rate can be effectively accelerated without incurring in higher costs is to increase the number of crews. Alternatives like overtime, more equipment or extending crew sizes are not realistic. They involve an increase in direct cost because only the optimum crew size can achieve maximum productivity in an activity. Using faster and more efficient equipment or better construction methods might accelerate the project, but since the company would have used those in the first place, acceleration would not be possible.

Cost optimization can be achieved by using multiples of the optimal number of crews. They yield a multiple of the natural rhythm and therefore the productivity is the optimal one. Once the number of crews used in an activity is established, it should remain constant



throughout the completion of the entire project in order to take advantage of the continuity in the labor force.

An important aspect of the acceleration of projects is that the total number of worker hours needed to finish the project remains constant regardless of the number of crews allocated in each activity and the duration in days of the project. This might seem counterintuitive, but it is what happens. The number of units to be delivered and the worker hours needed to complete each one are always constant for each activity, meaning that the same amount of hours will be worked by one crew than by two crews, the duration from the start of the activity to the end of it is what varies.

The acceleration routine needs to be performed following certain priorities. This way a cost-effective compression of the project duration can be achieved. The first priority is the number of available crews for the activities. If there are no available crews, the acceleration of the activity is impossible. The second priority is the duration of the activities. The activity with the longest duration is the one with the slower production rate. Therefore, this activity will be the one with a higher potential for compression. Once the activities have been analyzed the order of compression can be set and started.

It is important that the rules of priority are used because if not the project duration might be increased. In Figures 7 and 8 two different examples are presented where it can be seen how choosing the right activity to compress is important. In Figure 7 it can be observed how increasing the number of crews in the activity with the longest duration, and the slower production rate, results in shortening the project completion date. An additional crew

doubles the rate of production and halves the duration of the activity. On the other hand, in Figure 8 is seen how increasing the rate of production of an activity other than the one with the longest duration leads to a longer project duration. This happens because the second activity has preceding and succeeding activities with longer durations.

Example 1:

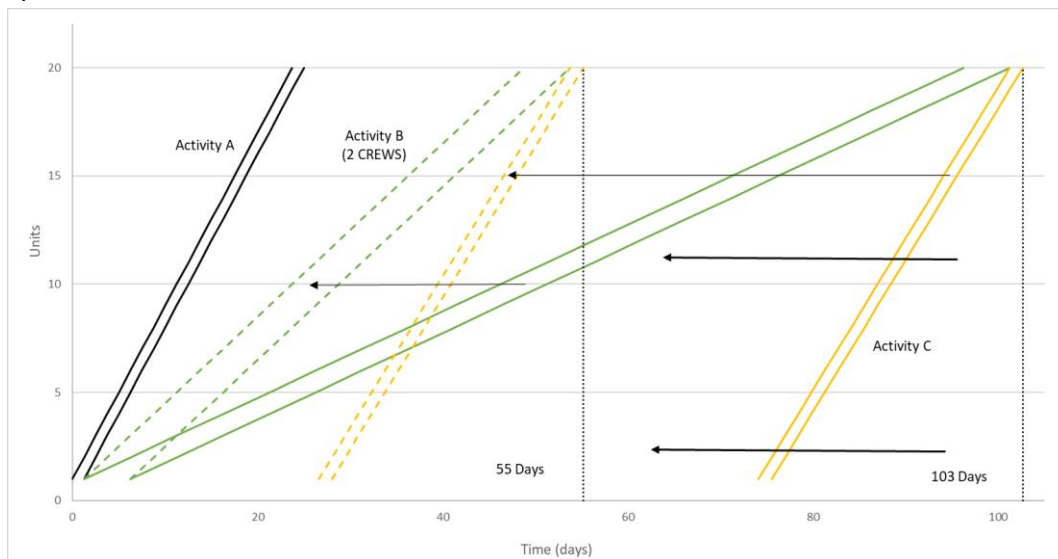


Figure 7 Effect of increasing number of crews on activity B

Example 2:

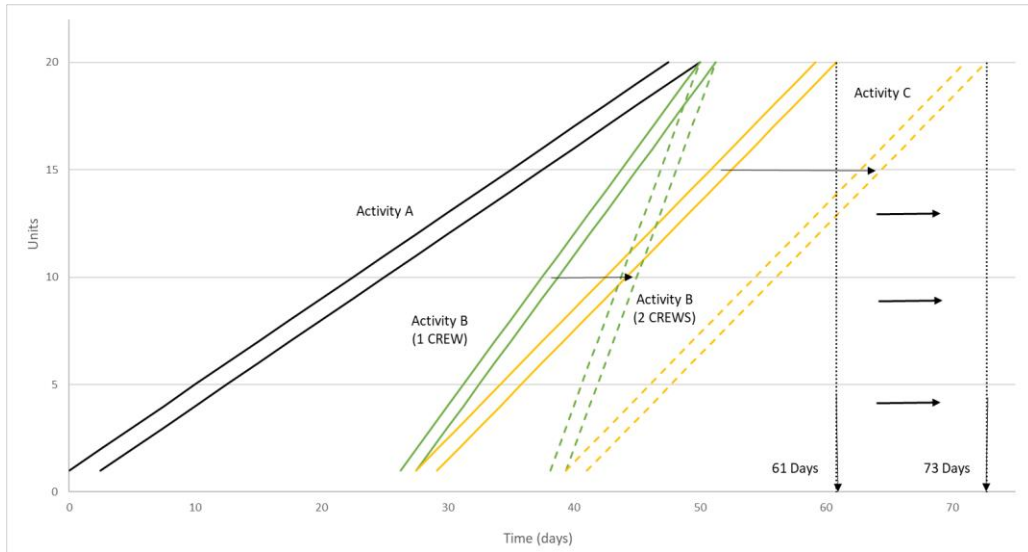


Figure 8 Effect of increasing number of crews on activity B

In Figure 9 a flowchart where the acceleration process is explained is presented. Following the steps presented a project can be accelerated and the completion date can be shortened.

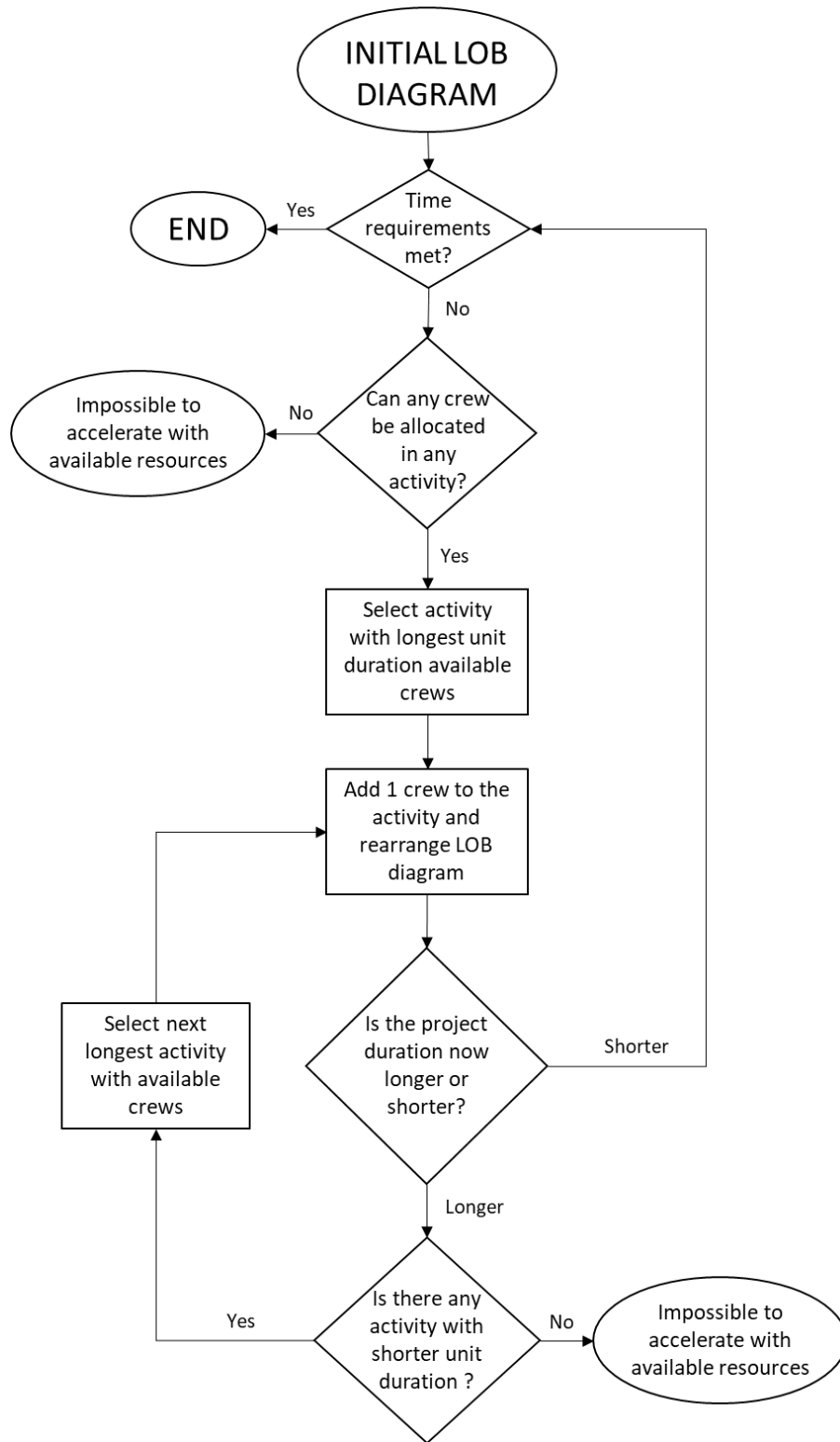


Figure 9 Acceleration process flowchart (inspired by Tokdemir, 2002)



An example is developed to show how the acceleration method is used. A project with six activities (A, B, C, D, E, F) and 4 crews available for each project is to be accelerated as much as possible. The workers work 5 days a week, 8 hours a day. The data of the project is shown in Table 4.

Table 4 Project data

Activity	Natural rhythm (units/week)	Actual rate (units/week)	Optimal size crew	Nº of crews	Available crews	Unit duration (days)	Activity duration (days)
A	2	2	5	1	3	2.5	47.5
B	3	3	3	1	3	1.7	31.7
C	4	4	8	1	3	1.3	23.8
D	1.5	1.5	6	1	3	3.3	63.3
E	2.25	2.25	7	1	3	2.2	42.2
F	1	1	4	1	3	5	95

With the data available and the equations presented on this document the first LOB diagram is built. Figure 10 shows the LOB diagram and Figure 11 the resource histogram. The project duration is 156 days. The total amount of worker hours needed to perform the project is 13,289 hours. The number of worker hours is always the same regardless the number of crews used in each activity, because multiples of the optimum crew size are used each time the activities are accelerated.

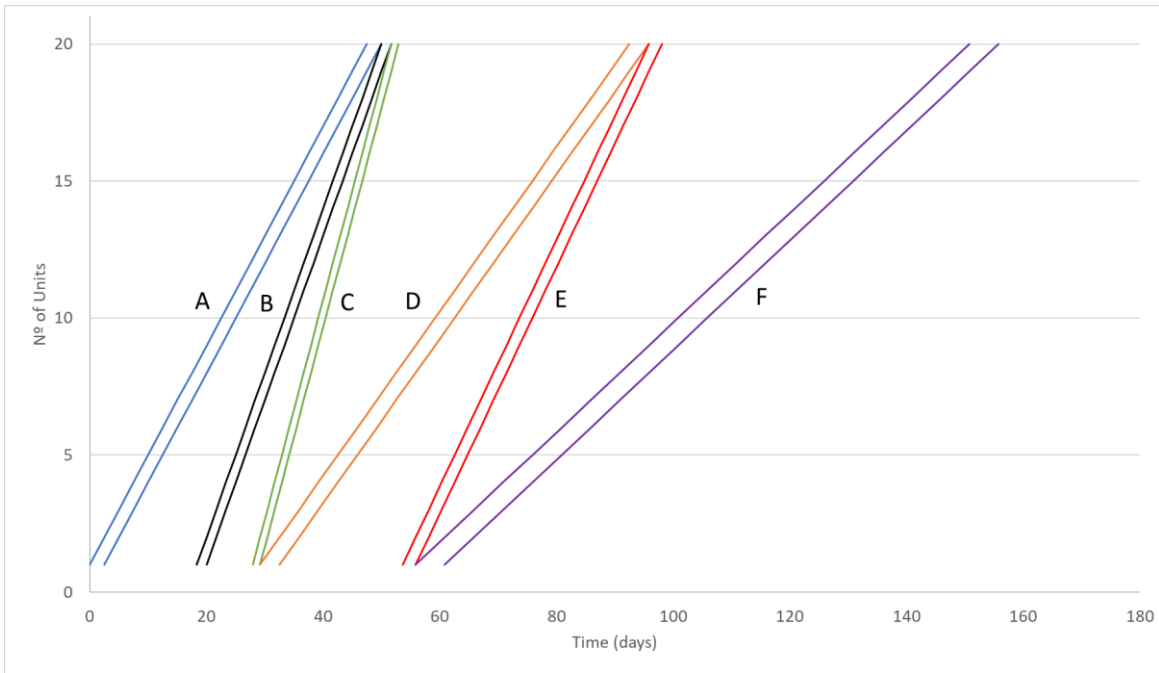


Figure 10 Initial LOB diagram (example)

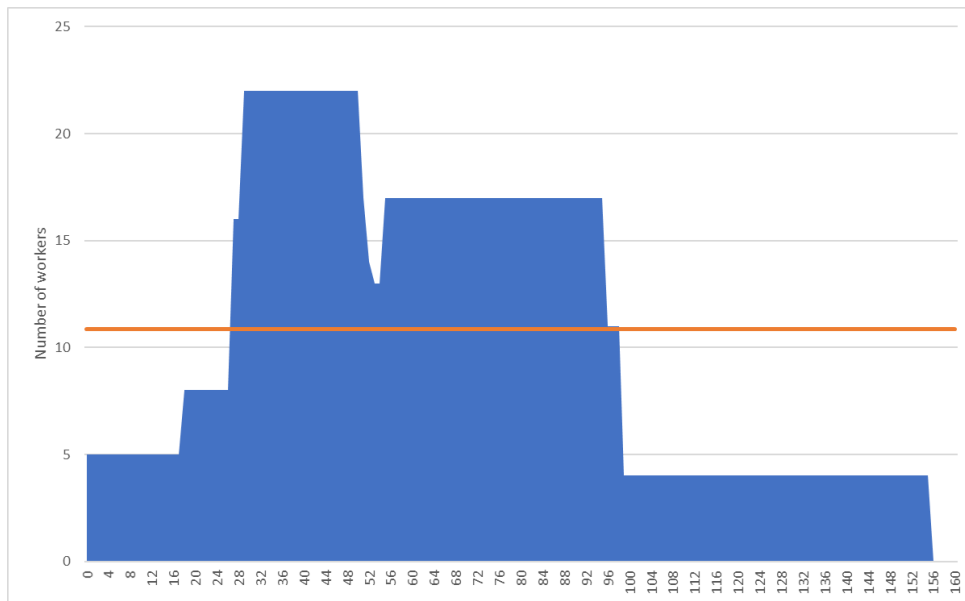


Figure 11 Initial resource histogram

Maximum workers: 22

Average number of workers: 11

Worker hours: 13,289 hours



It can be seen how many activities can be accelerated. In this example every activity has a maximum of 4 crews available to work on each activity. This means that the project duration can be shortened. Following the flow chart, we choose activity F as the first to be accelerated as it has the longest unit duration. The preceding activity is shorter and as it is the last activity of the project, a crew is allocated in the activity and the LOB diagram is built again. Table 5 and Figures 12 and 13 show the new project data, LOB diagram and resource histogram. The project duration has been reduced from 156 days to 108 days.

Table 5 Project data after first acceleration

Activity	Natural rhythm (units/week)	Actual rate (units/week)	Nº of crews	Available crews	Unit duration (days)	Activity duration (days)
A	2	2	1	3	2.5	47.5
B	3	3	1	3	1.7	31.7
C	4	4	1	3	1.3	23.8
D	1.5	1.5	1	3	3.3	63.3
E	2.25	2.25	1	3	2.2	42.2
F	1	2	2	2	5	47.5

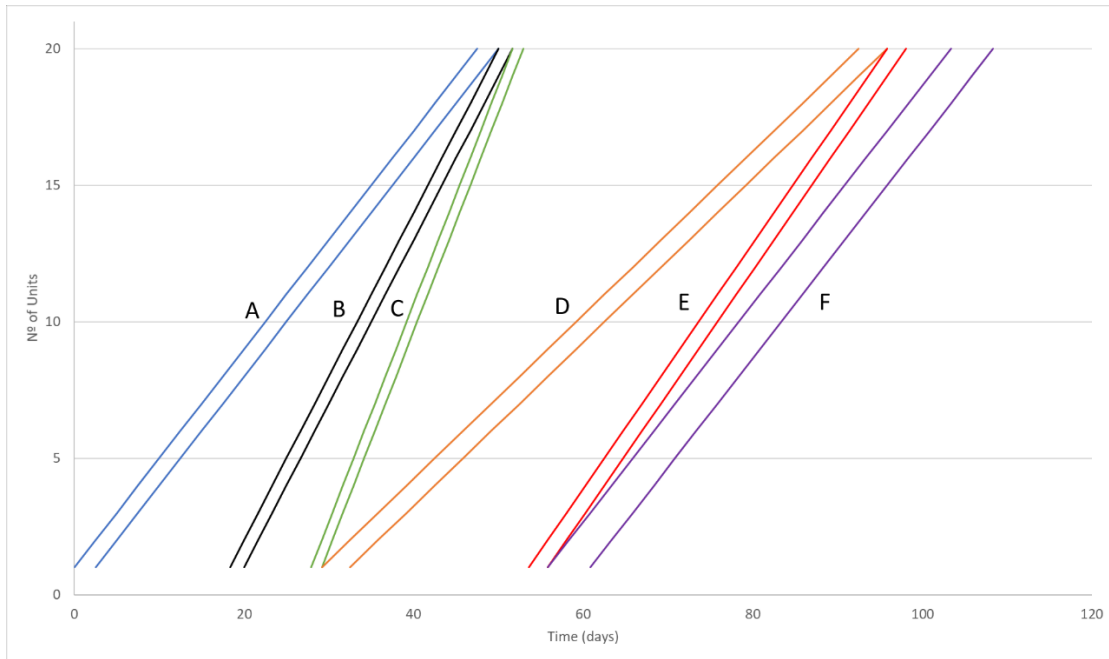


Figure 12 LOB diagram after first acceleration of activity F

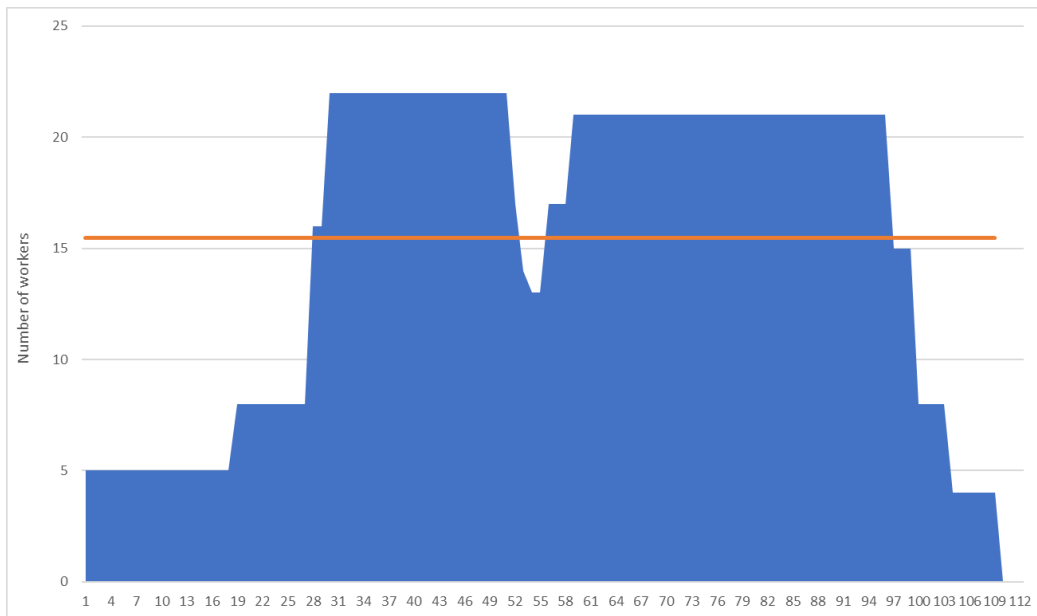


Figure 13 Resource histogram after first acceleration of activity F

Maximum workers: 22

Average number of workers: 15

Worker hours: 13,289 hours

The project can still be accelerated because there are still available crews to be allocated. The longest unit duration is F. Both its preceding and succeeding activities are shorter, so F can be accelerated. A crew is allocated, and the new LOB diagram is built. Table 6 and Figures 14 and 15 show the new data for the project.

Table 6 Project data after second acceleration

Activity	Natural rhythm (units/week)	Actual rate (units/week)	Nº of crews	Available crews	Unit duration (days)	Activity duration (days)
A	2	2	1	3	2.5	47.5
B	3	3	1	3	1.7	31.7
C	4	4	1	3	1.3	23.8
D	1.5	1.5	1	3	3.3	63.3
E	2.25	2.25	1	3	2.2	42.2
F	1	3	3	1	5	31.7

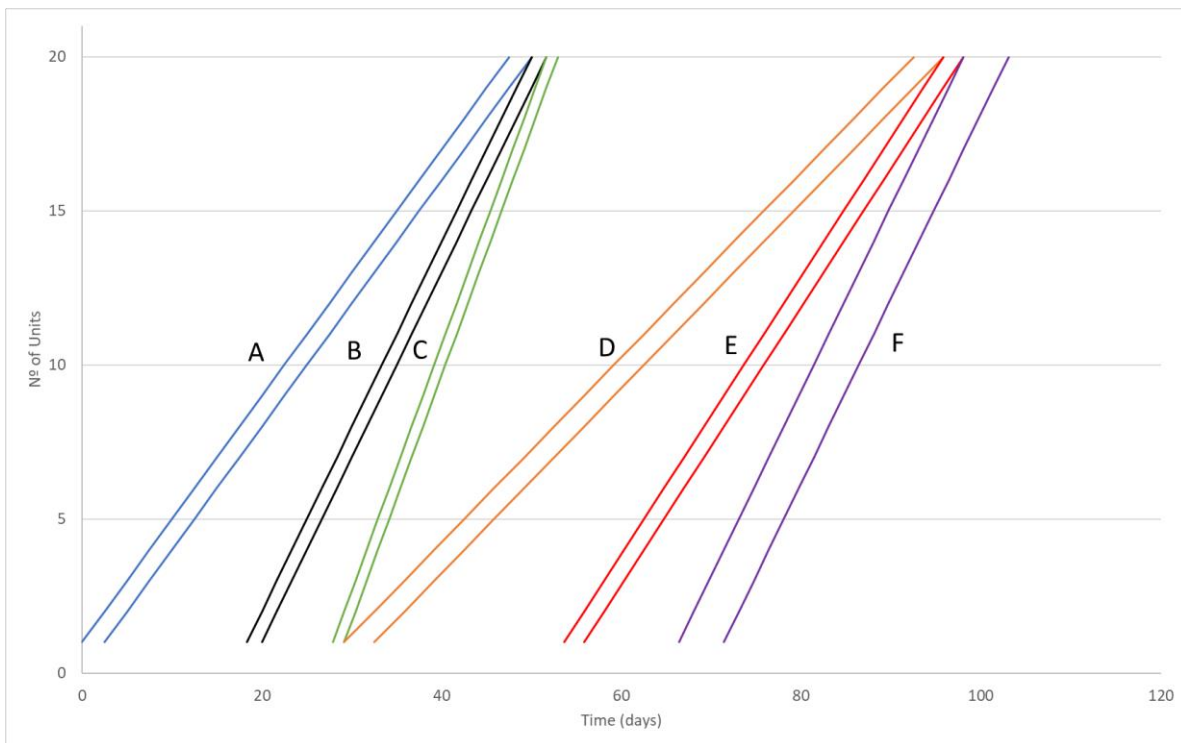


Figure 14 LOB diagram after second acceleration of activity F

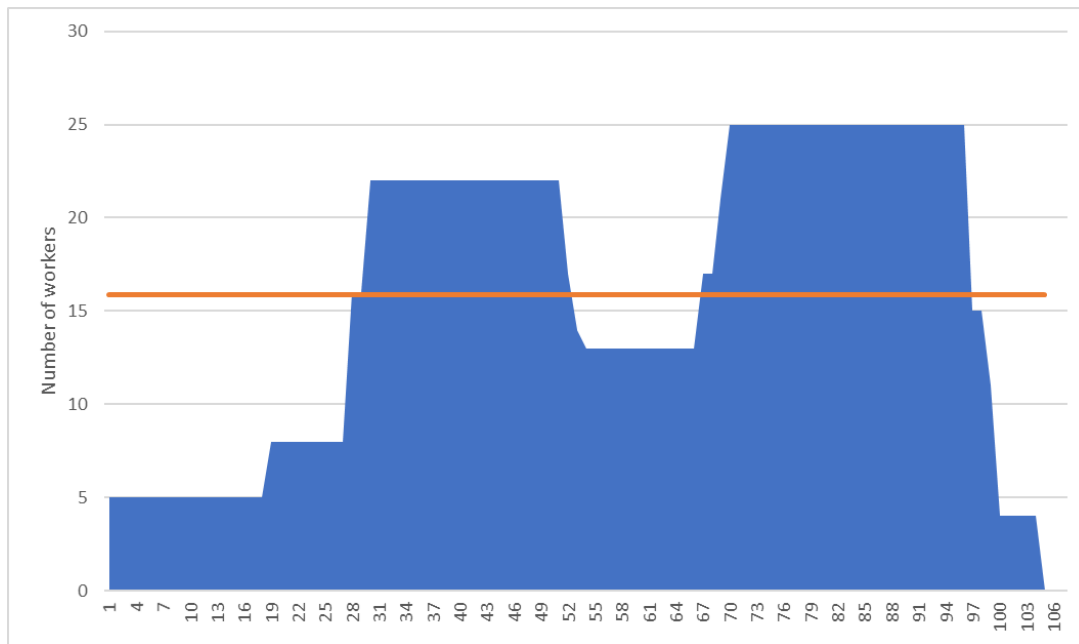


Figure 15 Resource histogram after second acceleration of activity F

Maximum workers: 25

Average number of workers: 16

Worker hours: 13,289 hours

Following with the process, new crews are assigned to each activity until all available crews are assigned or until the project cannot be accelerated. After some iterations (shown in Appendix I) the example project is accelerated to its limit. Table 7 and Figures 16 and 17 show the final data table and the LOB diagram.

Table 7 Project data after total acceleration

Activity	Natural rhythm (units/week)	Actual rate (units/week)	Nº of crews	Available crews	Unit duration (days)	Activity duration (days)
A	2	6	3	1	2.5	15.8
B	3	6	2	2	1.7	15.8
C	4	4	1	3	1.3	23.8
D	1.5	4.5	3	1	3.3	21.1
E	2.25	4.5	2	2	2.2	21.1
F	1	4	4	0	5.0	23.8

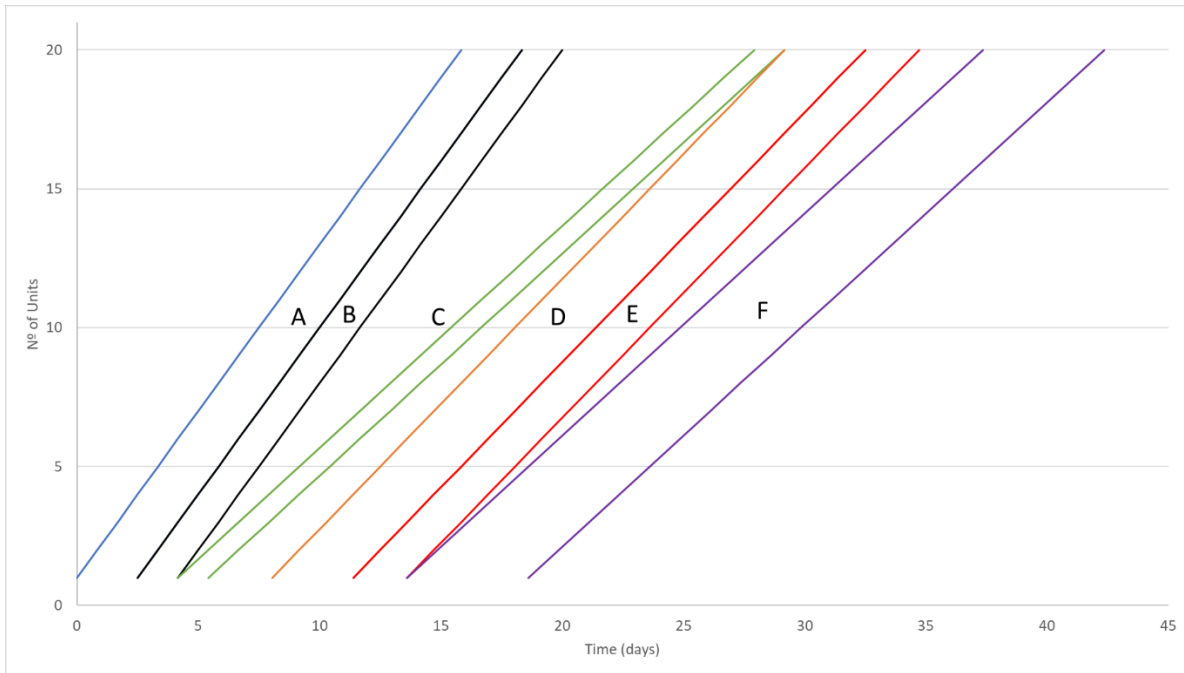


Figure 16 LOB diagram after total acceleration

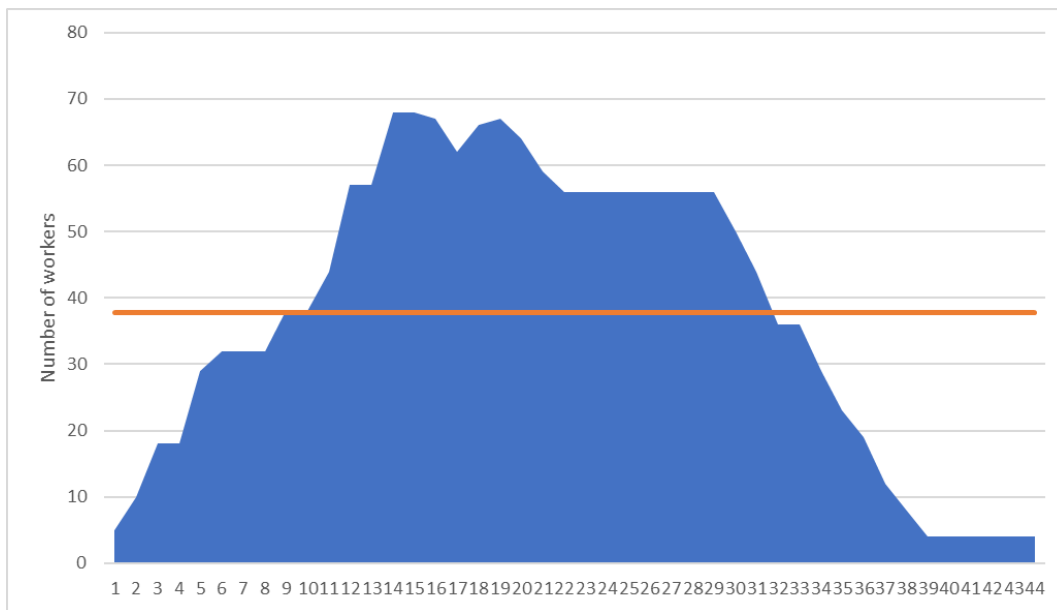


Figure 17 Resource histogram after total acceleration

Maximum workers: 68

Average number of workers: 38

Worker hours: 13,289 hours



Looking at the project data table one can see how the project cannot be accelerated more. Extra crews are still available in some activities, but these activities do not fulfil the requirements to have new crews allocated to them. The only activity that could be accelerated would be F, but there are no crews available for this activity.

The initial duration of the project was 156 days, with the acceleration process this duration has been reduced to 43 days. This shows how valuable the use LOB diagram can be on project planning.

3.2. Required date of completion

In the example presented in the previous section the project is accelerated as much as possible. However, this is not always desirable. Sometimes the date of completion specified in the contract is achieved before all activities are accelerated. In those cases, the project is accelerated so that the required completion date is met. There might exist multiple combinations of resource allocation to meet this requirement, but the combination that generates a project duration that is less than the required duration but is closest to the required duration is selected. This is done by following the instructions presented in the flowchart Figure 9. For example, if there are two activities that can be accelerated to get a slightly shorter completion date than the duration specified in the contract, the activity that generates a completion date that is less than but closest to the contractual date is chosen for acceleration. This is graphically shown in the next example. In this example, the contract specifies that the project has to be finished before 24 days. Developing an initial LOB

diagram, it is seen in Figure 18 that the project duration is 27 days if one crew is assigned per activity. Therefore, the schedule must be accelerated.

Table 8 Initial project data (date completion analysis)

Activity	Natural rhythm (units/week)	Actual rate (units/week)	Nº of crews	Available crews	Unit duration (days)	Activity duration (days)
A	2.5	2.5	1	3	2	18
B	4	4	1	3	1.3	11.3
C	3	3	1	3	1.7	15

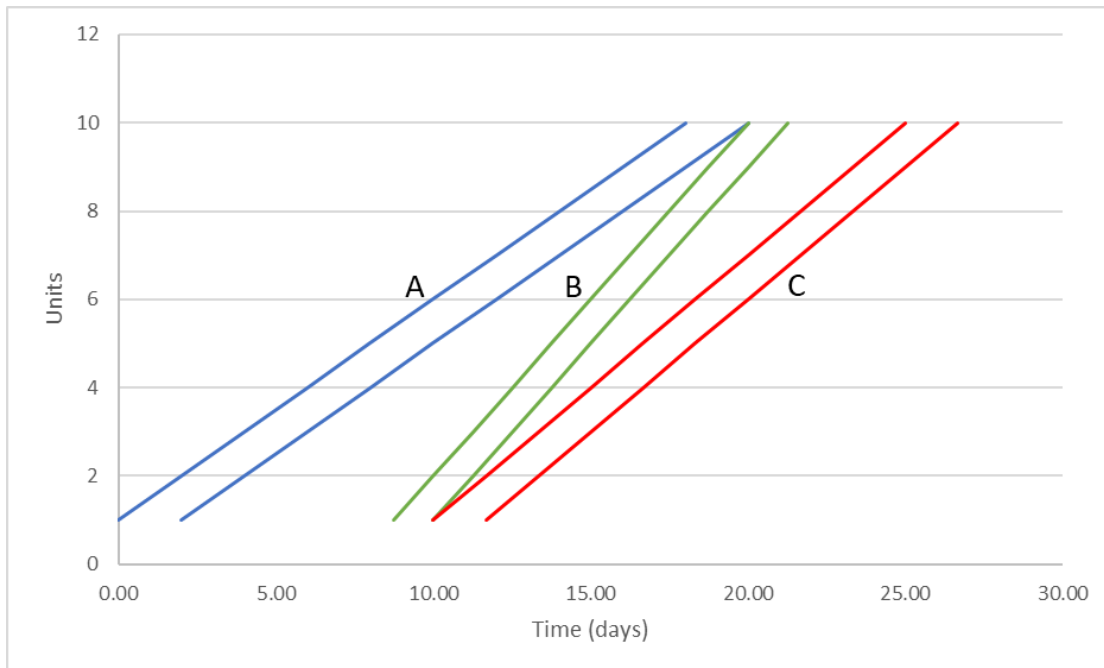


Figure 18 Initial LOB diagram (date completion analysis)

Given the configuration of the LOB diagram, it is clear that accelerating Activity B will only cause an extension of the project duration. Therefore, acceleration of activity B is not desirable. Activities A and C can be accelerated if an additional crew is assigned to them. In Figures 19 and 20 both alternatives are depicted. It is shown that accelerating activity A

reduces the project duration to 20 days, whereas accelerating activity C reduce the project to 23 days. Since the second alternative generates a project duration that is less than but closest to the duration specified in the contract, that alternative is chosen.

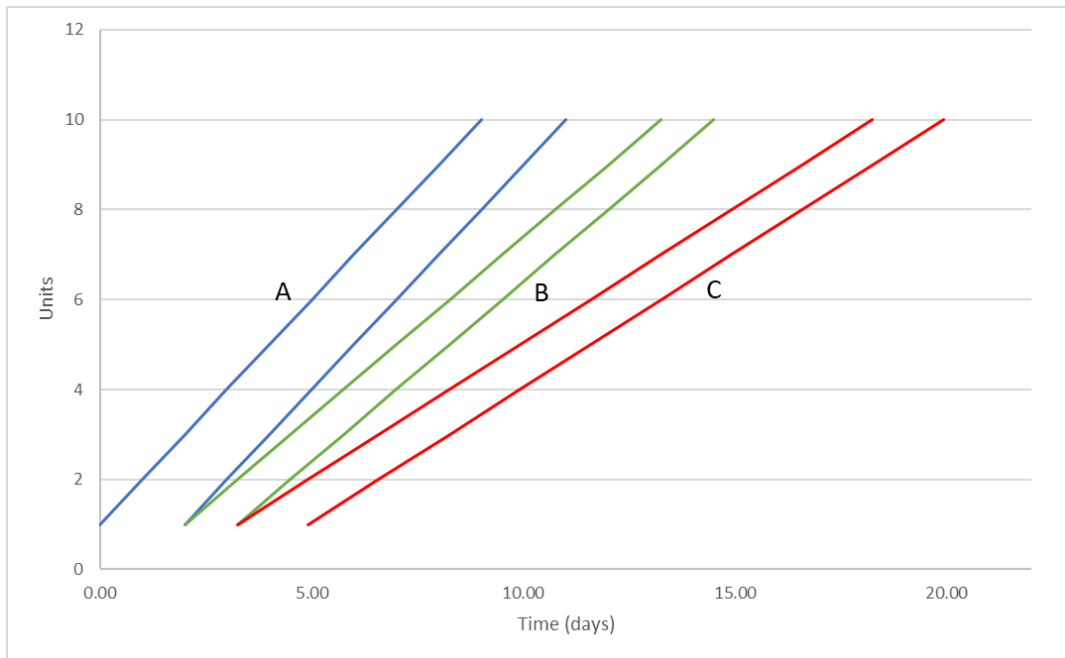


Figure 19 Acceleration of activity A (project duration 20 days)

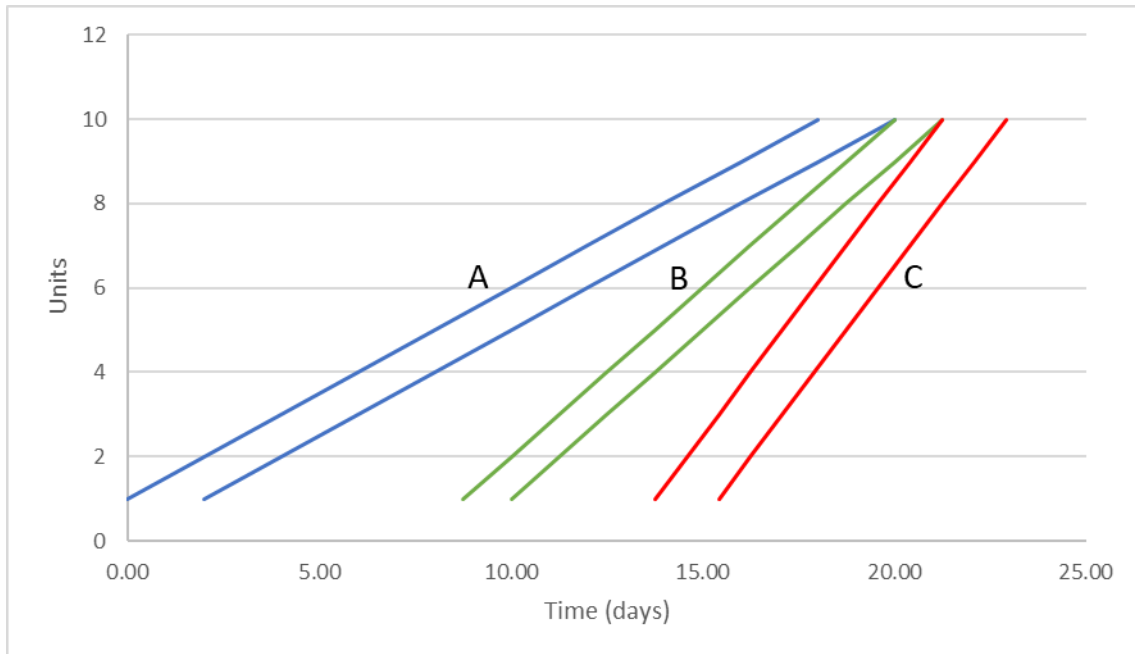


Figure 20 Acceleration of activity C (project duration 23 days)

This example is a rather simple demonstration of the situation. In a real schedule, a multitude of activities may be involved in this process, a multitude of activities may be involved in this process, and generating all combinations may be quite troublesome. This process needs to be computerized in order to create an application that can be implemented without much effort.



4. LOB METHOD USAGE SURVEY AND RESULTS

A survey was conducted to determine the extent to which the LOB scheduling method is used by Spanish companies and what type of project it is used for. The results show that the use of the LOB method is not common and other methods are used for scheduling projects.

A total of 157 randomly selected Spanish companies that operate in the construction, demolition and similar industries were contacted. Only 15 of those companies responded (Rate of reply 9.6%) and only one of the 15 responded positively to the question of whether they use LOB or not. Among the 14 companies that responded that do not use linear scheduling 4 said they use Gantt Diagrams, one said they use the LEAN method and 9 did not specify which method they use.

The selection of the companies was made by identifying different Spanish associations in which these companies are members. The associations are SEOPAN, ANCI, AEDED, ALTAP, ARPHO and AESMIDES.

The size of the sample is not big enough to extrapolate the conclusions to all industries and even if it proves that the LOB method is fairly unknown, the extent of its use might be different. The results are shown in Table 9 and Figure 21.

Table 9 Data of the survey

Question	N. of companies
Use LOB or linear scheduling?	
Yes	1
No	14
Total	15
Which scheduling method is used?	
LOB	1
Gantt Diagram	4
LEAN	1
Not responded	9
Total	15

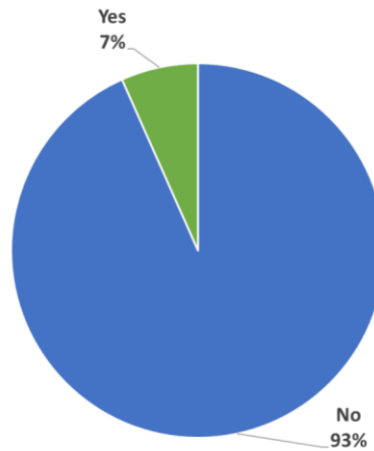


Figure 21 Percentage of companies that use LOB in their projects



The survey sent to the companies was written in Spanish. The original and the translated surveys are presented below.

Survey in Spanish:

- Los Métodos de Planificación Lineal (p.e. Line of Balance) son métodos que se utilizan en la planificación de proyectos de naturaleza repetitiva. ¿Utiliza su empresa métodos de este tipo en la planificación de proyectos?
 - Si
 - No

- En caso afirmativo. ¿En qué tipo de proyectos lo hace?
 - Construcción de edificios/ rascacielos
 - Construcción de carreteras
 - Construcción de oleoductos/ tuberías
 - Otro (Especificar):

- En caso contrario. ¿Qué otro método de planificación utiliza su empresa?
 - Diagrama de Gantt
 - CPM (Critical Path Method - Camino de la Ruta Crítica)
 - PERT (Project Evaluation and Review Techniques)
 - CCPM (Critical Chain Project Management - Método de la Cadena Crítica)
 - LEAN
 - Otro (Especificar):

Survey in English:

- Linear Scheduling Methods, such as Line of Balance method, are methods used to schedule projects of repetitive nature. Does your company use any of these methods in the scheduling of projects?
 - Yes
 - No



- If answered affirmatively. In which type of project do you use it?
 - Construction of high-rise buildings
 - Construction of roads
 - Construction of pipelines
 - Other (Specify)

- If answered negatively. What other type of scheduling method does your company use?
 - Gantt Diagram
 - CPM (Critical Path Method)
 - PERT (Project Evaluation and Review Techniques)
 - CCPM (Critical Chain Project Management)
 - LEAN
 - Other (Specify)



5. CONCLUSIONS

The LOB scheduling method has been consistently proven to be the best method for scheduling repetitive projects, such as pipeline or high-rise buildings construction. It allows to keep track of the development of the project in an easy way. The use of a LOB diagram allows planners to detect any imbalance in the schedule and to act in the source of the problem.

Understanding of the equations involved in the development of the LOB method and knowing how to construct the LOB diagram is vital to be able to benefit from the full potential of the LOB method. One of the main benefits of the LOB diagrams is the ease to use and understand them.

The acceleration routine allows the planner to allocate efficiently the available crews in the project, reducing the total duration of the project, while ensuring that the total worker hours remains constant.

The use of multiples of the natural rhythm is the key to ensure that the LOB method is used correctly. Only this way is the project done in the most effective way and the worker hours are minimum. If a different crew size to the optimal one is used the total worker hours grows because of the existence of idle time between units.

The LOB method remains fairly unused. A possible explanation for this is the fact that LOB is not normally taught in college programs, and that no computer software is currently available for companies to use. The development of appropriate curricula and software



would allow planners of different industries to widely use LOB and to fully benefit from its use of the method and extend its use among them.



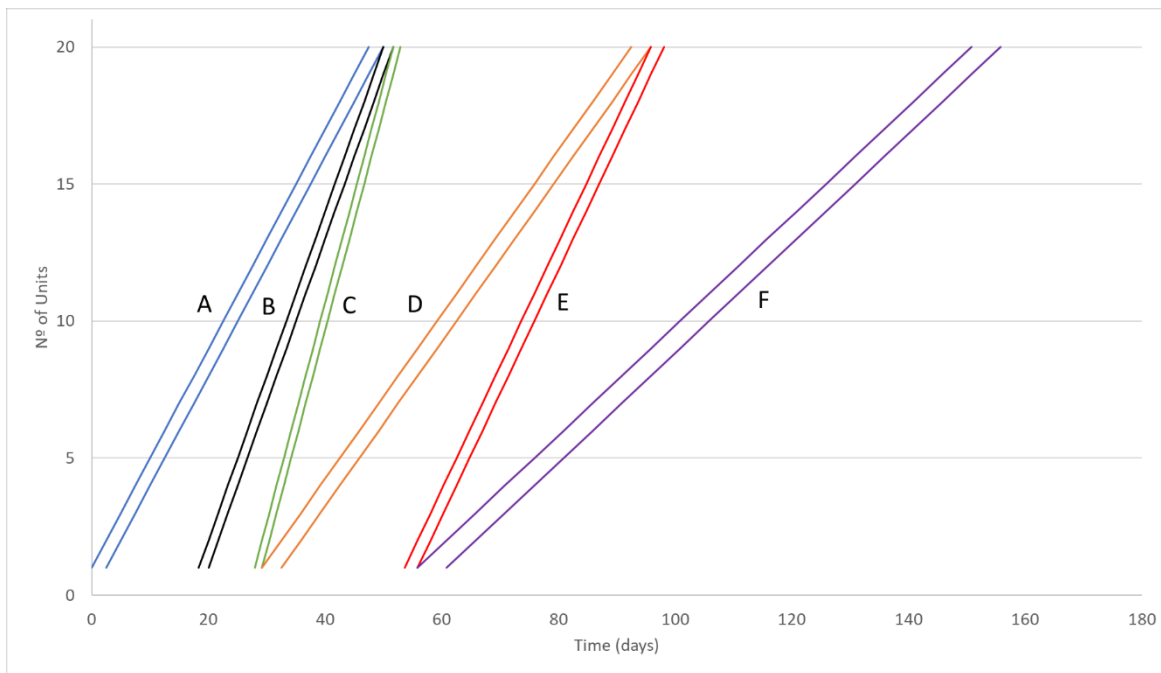
REFERENCES

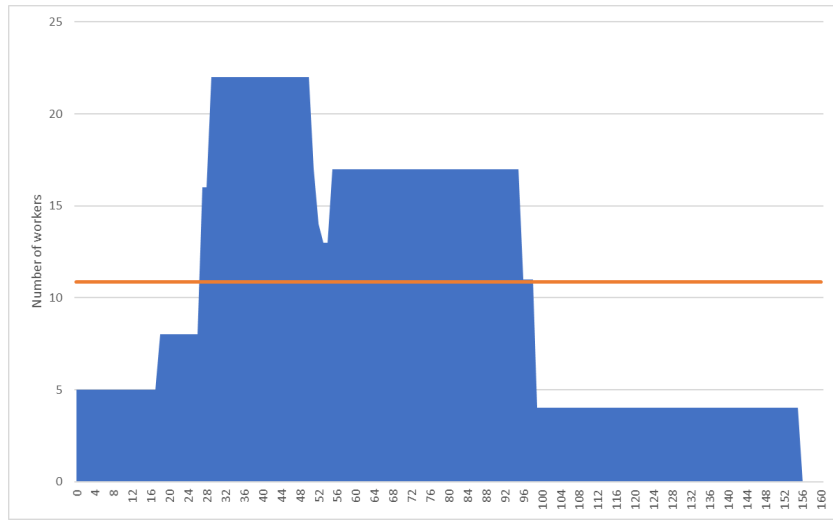
- Arditi, David, and M. Zeki Albudak. 1986. "Line-of-Balance scheduling in pavement construction." *Journal Construction Engineering Management* 411-424.
- Arditi, David, Onur B. Tokdemir, and Kangsuk Suh. 2002. "Challenges in Line-of-Balance Scheduling." *J. Constr. Eng. Manage.* 545-556.
- Khisty, C. J. 1970. "The application of the Line of Balance technique to the construction industry." *Indian Concrete J.* 297-320.
- Odeh, Ibrahim. n.d. *Course: Construction Scheduling; Lesson: Linear Construction Operations and Line of Balance.* <https://www.coursera.org/lecture/construction-scheduling/l>.
- Sarraj, Zohair M. Al. 1990. "Formal development of Line-of-Balance technique." *J. Constr. Eng. Manage.* 689-704.
- Suhail, Saad A., and Richard H. Neale. 1994. "CPM/LOB: New methodology to integrate CPM and Line of Balance." *J. Constr. Eng. Manage.* 667-684.

APPENDIX I: Acceleration process example

Initial project data, LOB diagram and resource histogram:

Activity	Natural rhythm (units/week)	Actual rate (units/week)	Optimal size crew	Nº of crews	Available crews	Unit duration (days)	Activity duration (days)
A	2	2	5	1	3	2.5	47.5
B	3	3	3	1	3	1.7	31.7
C	4	4	8	1	3	1.3	23.8
D	1.5	1.5	6	1	3	3.3	63.3
E	2.25	2.25	7	1	3	2.2	42.2
F	1	1	4	1	3	5	95





Project duration: 156 days

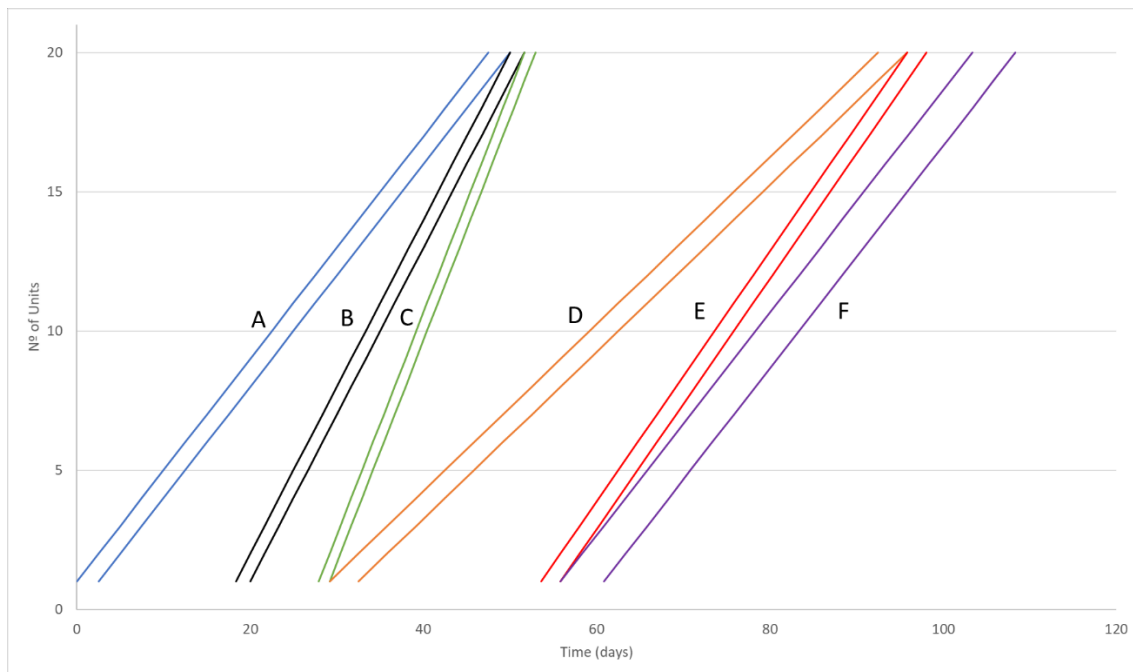
Maximum workers: 22

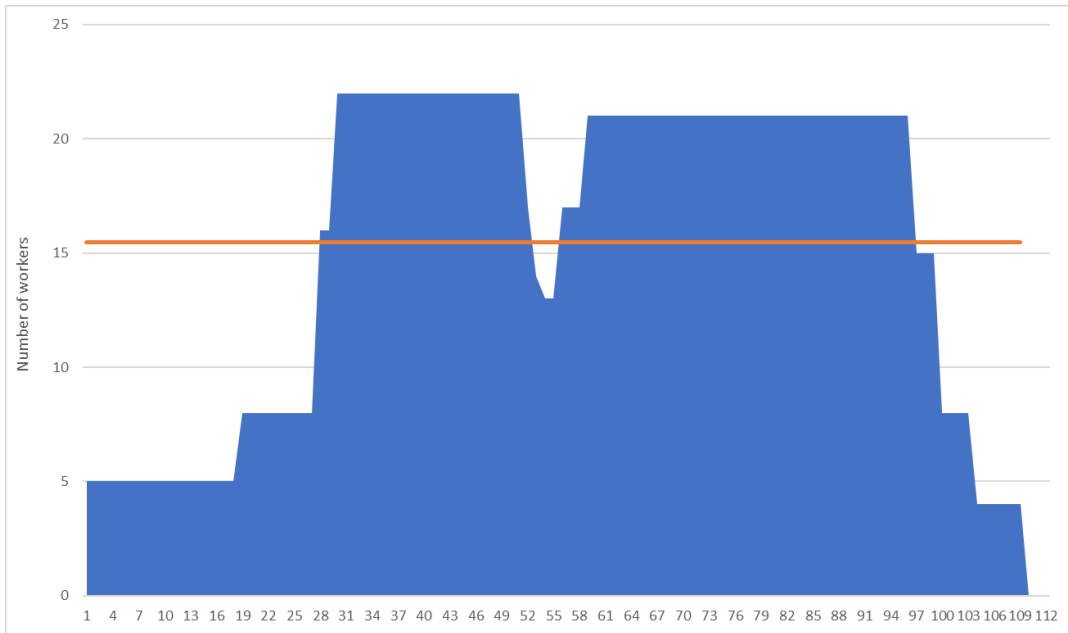
Average number of workers: 11

Worker hours: 13,289 hours

First Iteration: Crew allocated in activity F

Activity	Natural rhythm (units/week)	Actual rate (units/week)	Nº of crews	Available crews	Unit duration (days)	Activity duration (days)
A	2	2	1	3	2.5	47.5
B	3	3	1	3	1.7	31.7
C	4	4	1	3	1.3	23.8
D	1.5	1.5	1	3	3.3	63.3
E	2.25	2.25	1	3	2.2	42.2
F	1	2	2	2	5	47.5





Project duration: 108 days

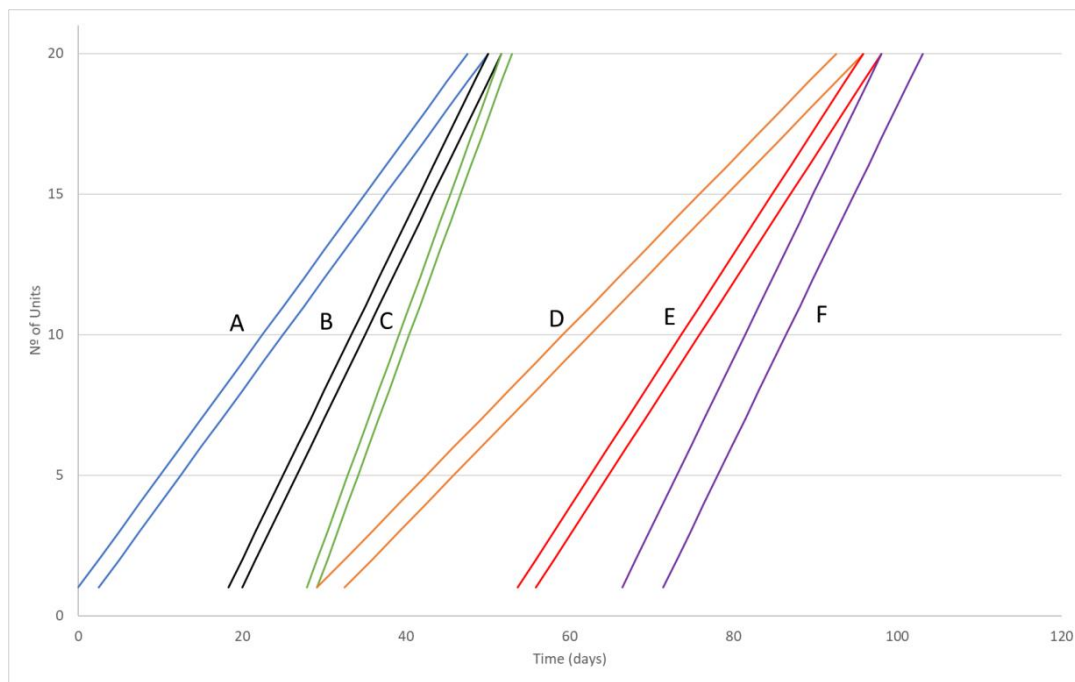
Maximum workers: 22

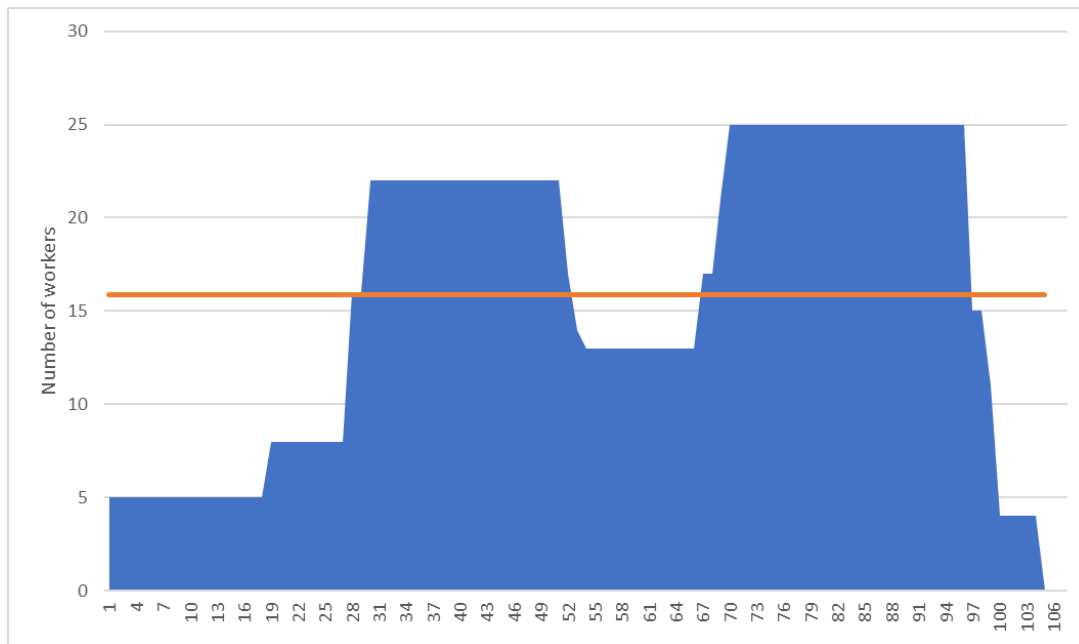
Average number of workers: 15

Worker hours: 13,289 hours

Second Iteration: Crew allocated in activity D

Activity	Natural rhythm (units/week)	Actual rate (units/week)	Nº of crews	Available crews	Unit duration (days)	Activity duration (days)
A	2	2	1	3	2.5	47.5
B	3	3	1	3	1.7	31.7
C	4	4	1	3	1.3	23.8
D	1.5	1.5	1	3	3.3	63.3
E	2.25	2.25	1	3	2.2	42.2
F	1	3	3	1	5	31.7





Project duration: 103 days

Maximum workers: 25

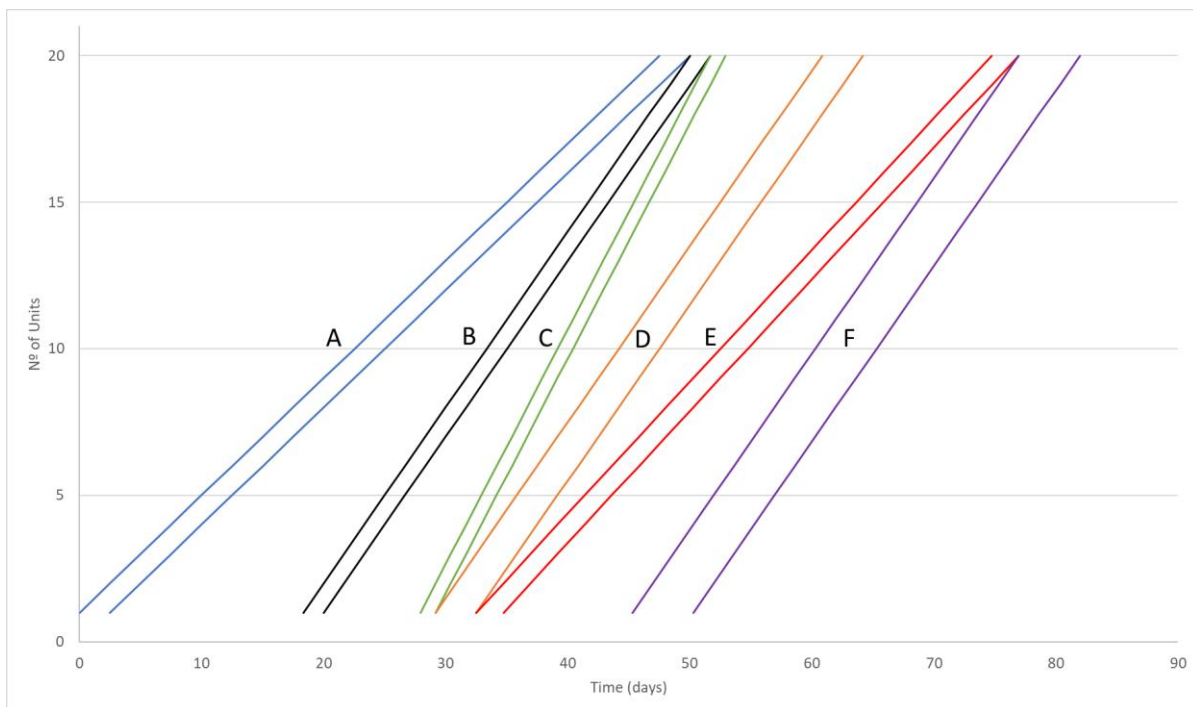
Average number of workers: 16

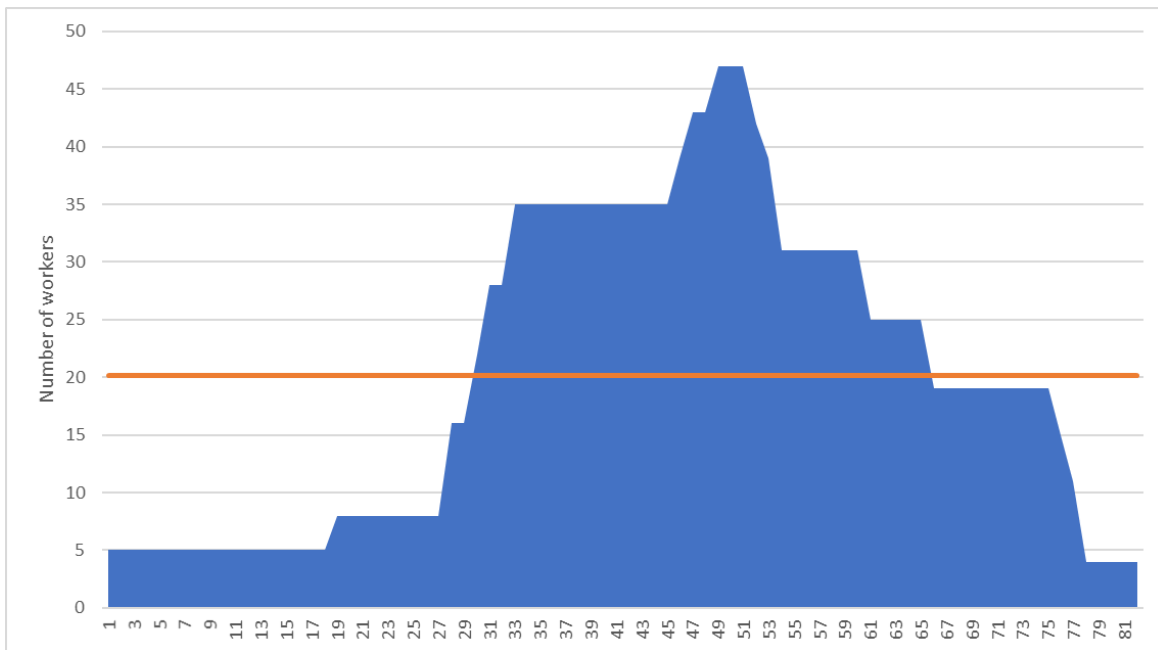
Worker hours: 13,289 hours



Third Iteration: Crew allocated in activity D

Activity	Natural rhythm (units/week)	Actual rate (units/week)	Nº of crews	Available crews	Unit duration (days)	Activity duration (days)
A	2	2	1	3	2.5	47.5
B	3	3	1	3	1.7	31.7
C	4	4	1	3	1.3	23.8
D	1.5	3	2	2	3.3	31.7
E	2.25	2.25	1	3	2.2	42.2
F	1	3	3	1	5.0	31.7





Project duration: 82 days

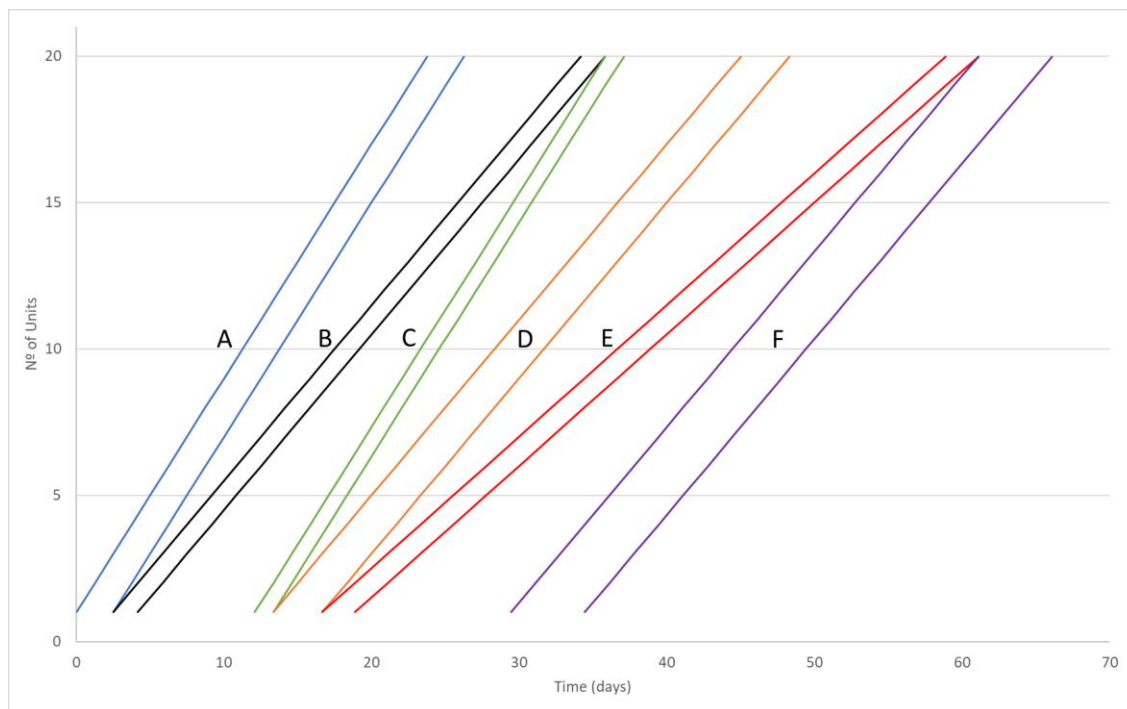
Maximum workers: 47

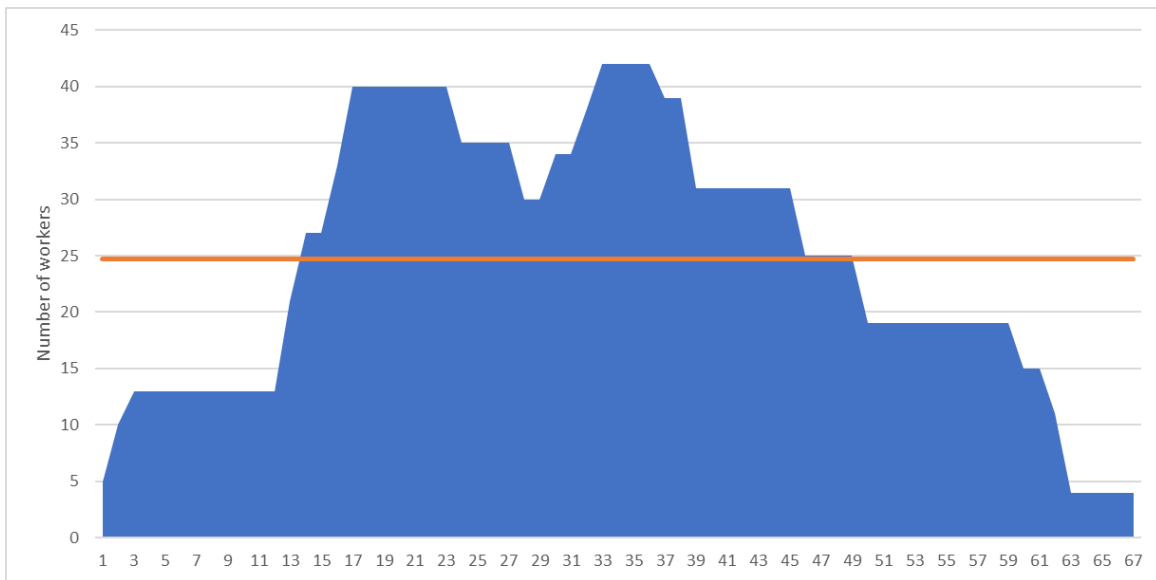
Average number of workers: 20

Worker hours: 13,289 hours

Fourth Iteration: Crew allocated in activity A

Activity	Natural rhythm (units/week)	Actual rate (units/week)	Nº of crews	Available crews	Unit duration (days)	Activity duration (days)
A	2	4	2	2	2.5	23.8
B	3	3	1	3	1.7	31.7
C	4	4	1	3	1.3	23.8
D	1.5	3	2	2	3.3	31.7
E	2.25	2.25	1	3	2.2	42.2
F	1	3	3	1	5.0	31.7





Project duration: 66 days

Maximum workers: 42

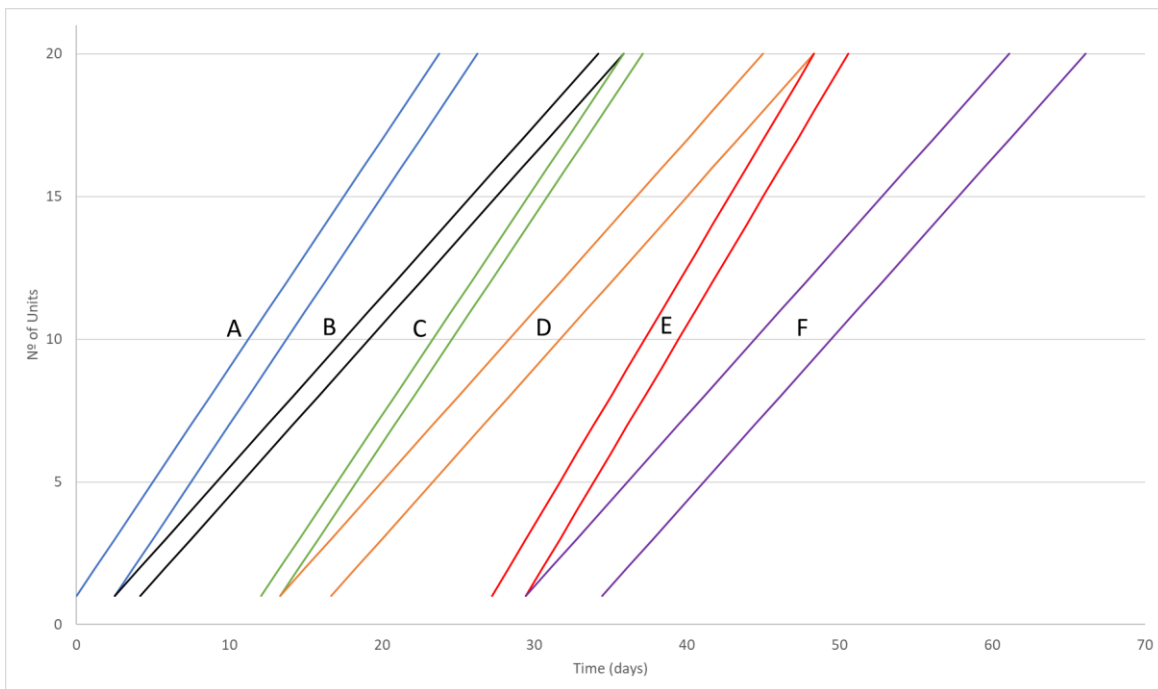
Average number of workers: 25

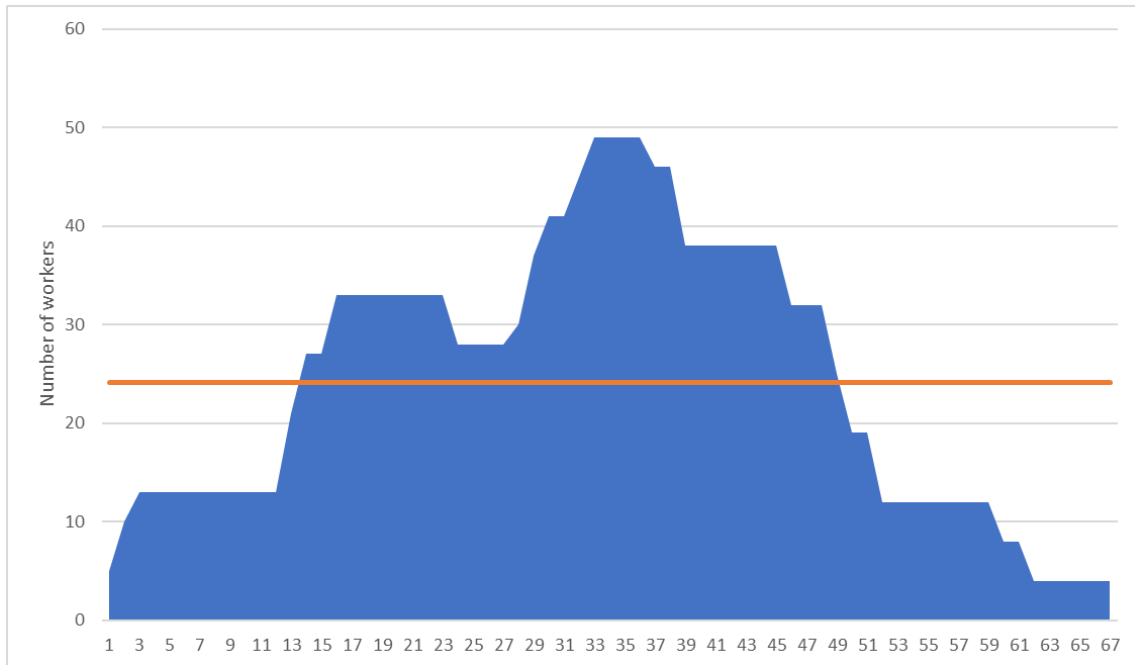
Worker hours: 13,289 hours



Fifth Iteration: Crew allocated in activity B

Activity	Natural rhythm (units/week)	Actual rate (units/week)	Nº of crews	Available crews	Unit duration (days)	Activity duration (days)
A	2	4	2	2	2.5	23.8
B	3	3	1	3	1.7	31.7
C	4	4	1	3	1.3	23.8
D	1.5	3	2	2	3.3	31.7
E	2.25	4.5	2	2	2.2	21.1
F	1	3	3	1	5.0	31.7





Project duration: 66 days

Maximum workers: 49

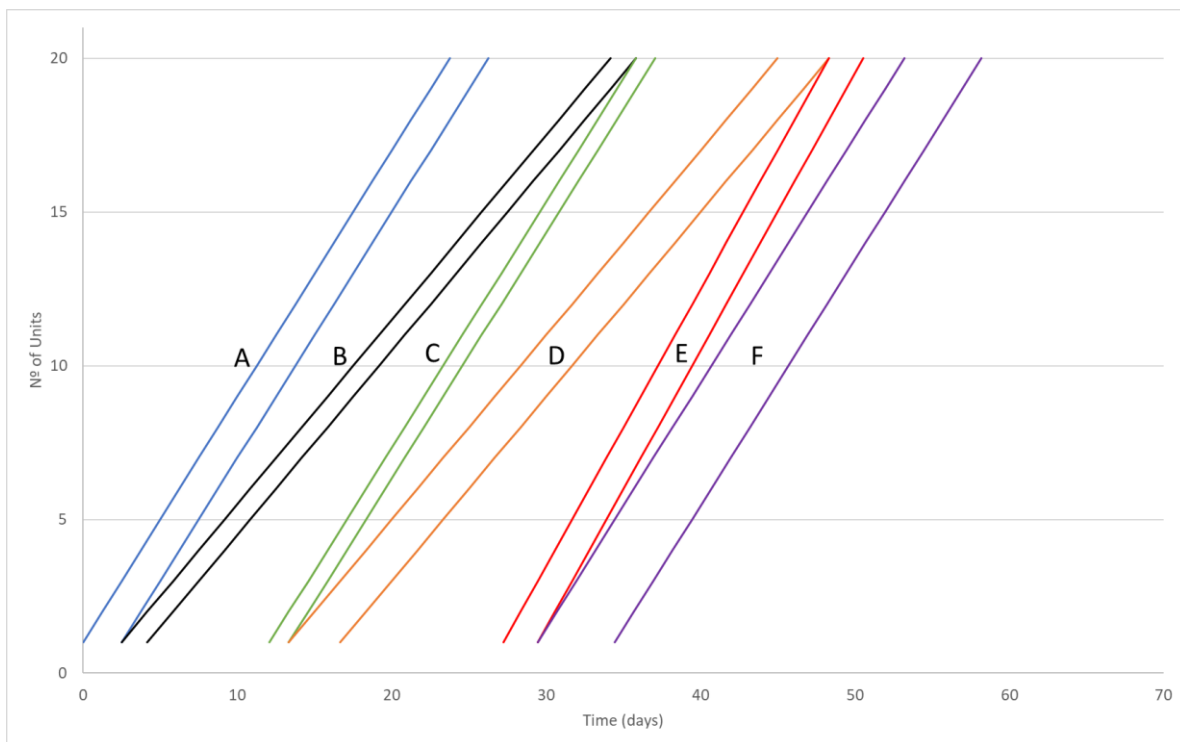
Average number of workers: 24

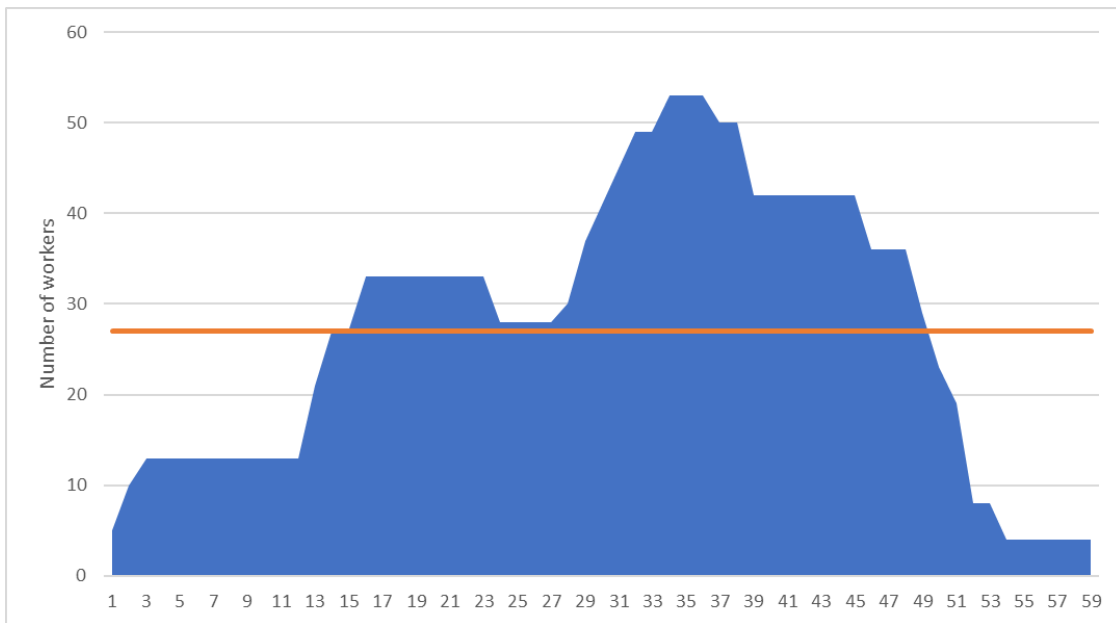
Worker hours: 13,289 hours



Sixth Iteration: Crew allocated in activity F

Activity	Natural rhythm (units/week)	Actual rate (units/week)	Nº of crews	Available crews	Unit duration (days)	Activity duration (days)
A	2	4	2	2	2.5	23.8
B	3	3	1	3	1.7	31.7
C	4	4	1	3	1.3	23.8
D	1.5	3	2	2	3.3	31.7
E	2.25	4.5	2	2	2.2	21.1
F	1	4	4	0	5.0	23.8





Project duration: 58 days

Maximum workers: 49

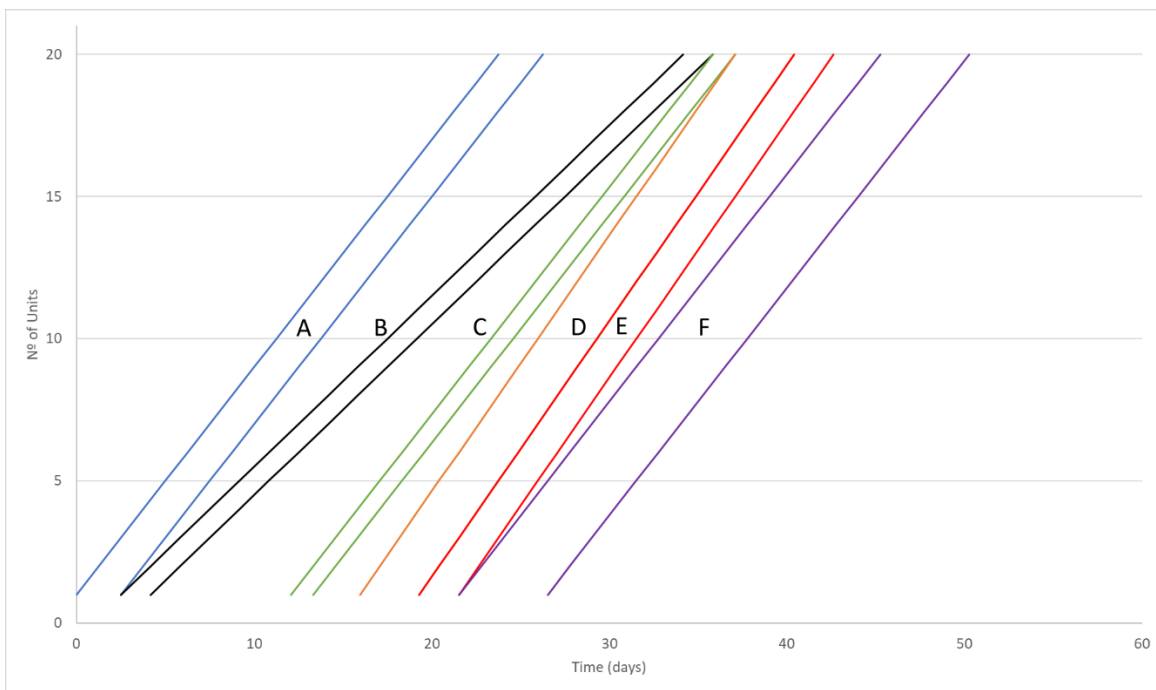
Average number of workers: 24

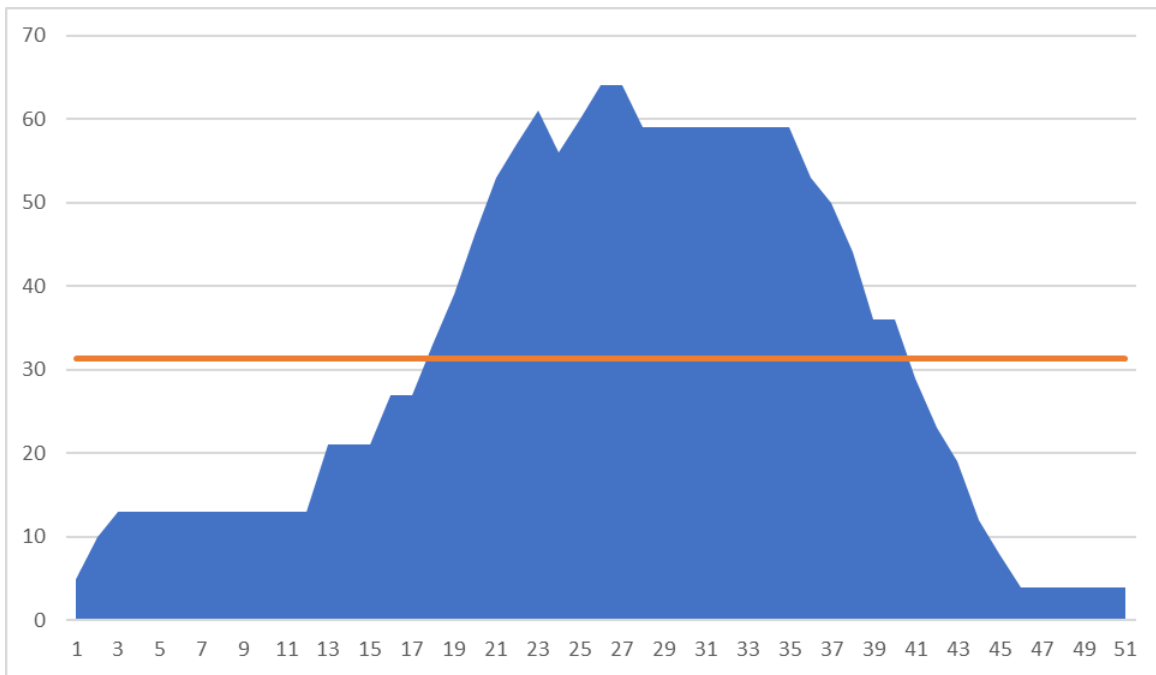
Worker hours: 13,289 hours



Seventh Iteration: Crew allocated in activity D

Activity	Natural rhythm (units/week)	Actual rate (units/week)	Nº of crews	Available crews	Unit duration (days)	Activity duration (days)
A	2	4	2	2	2.5	23.8
B	3	3	1	3	1.7	31.7
C	4	4	1	3	1.3	23.8
D	1.5	4.5	3	1	3.3	21.1
E	2.25	4.5	2	2	2.2	21.1
F	1	4	4	0	5.0	23.8





Project duration: 50 days

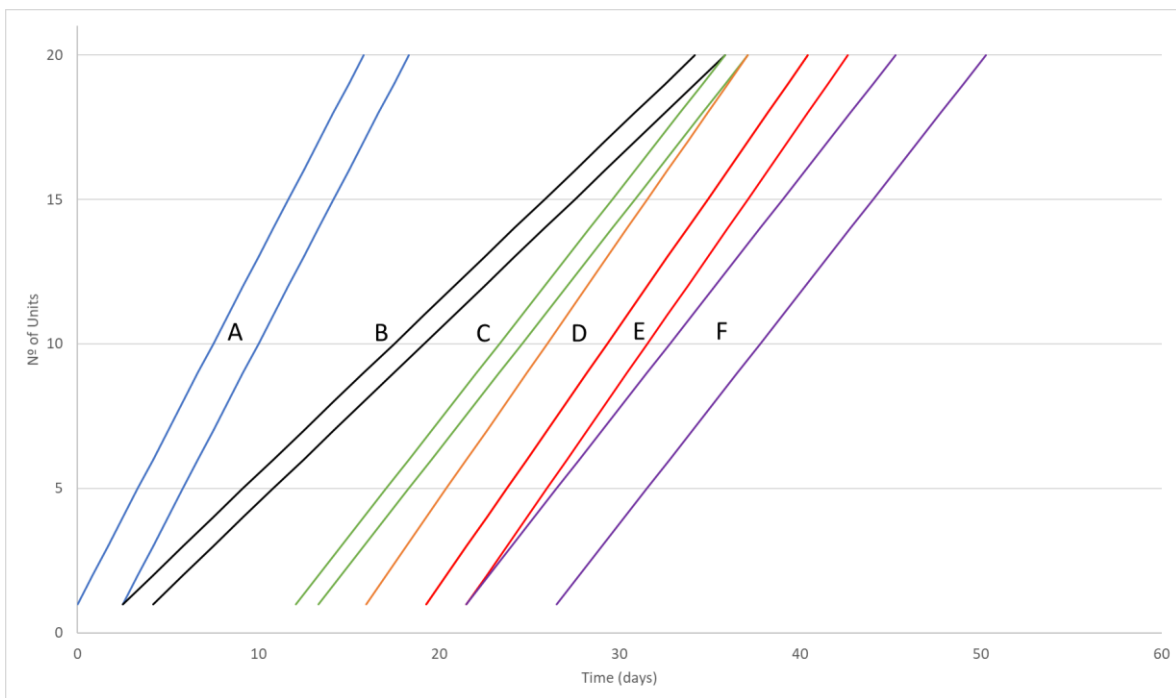
Maximum workers: 64

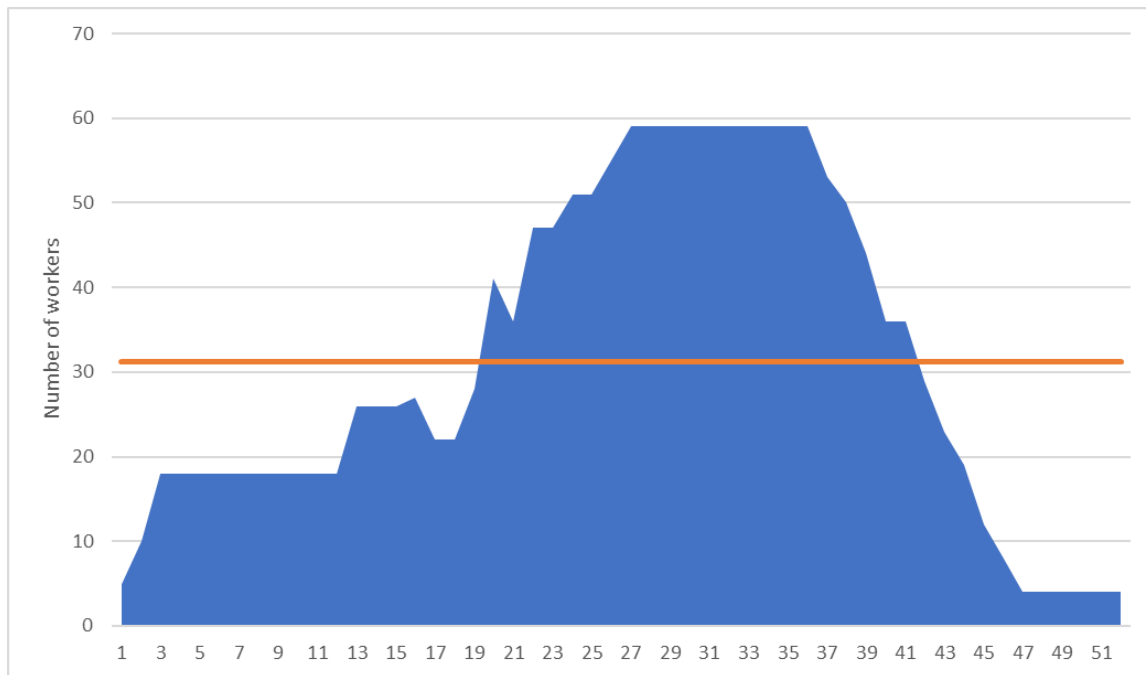
Average number of workers: 31

Worker hours: 13,289 hours

Eighth Iteration: Crew allocated in activity A

Activity	Natural rhythm (units/week)	Actual rate (units/week)	Nº of crews	Available crews	Unit duration (days)	Activity duration (days)
A	2	6	3	1	2.5	15.8
B	3	3	1	3	1.7	31.7
C	4	4	1	3	1.3	23.8
D	1.5	4.5	3	1	3.3	21.1
E	2.25	4.5	2	2	2.2	21.1
F	1	4	4	0	5.0	23.8





Project duration: 50 days

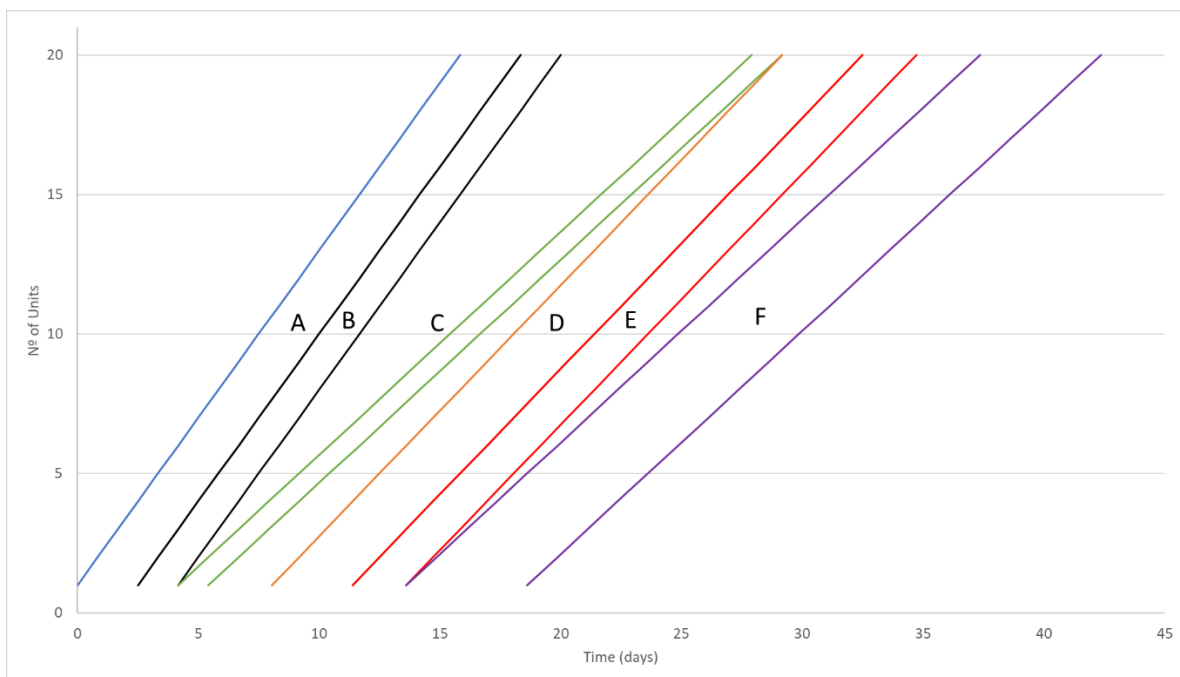
Maximum workers: 59

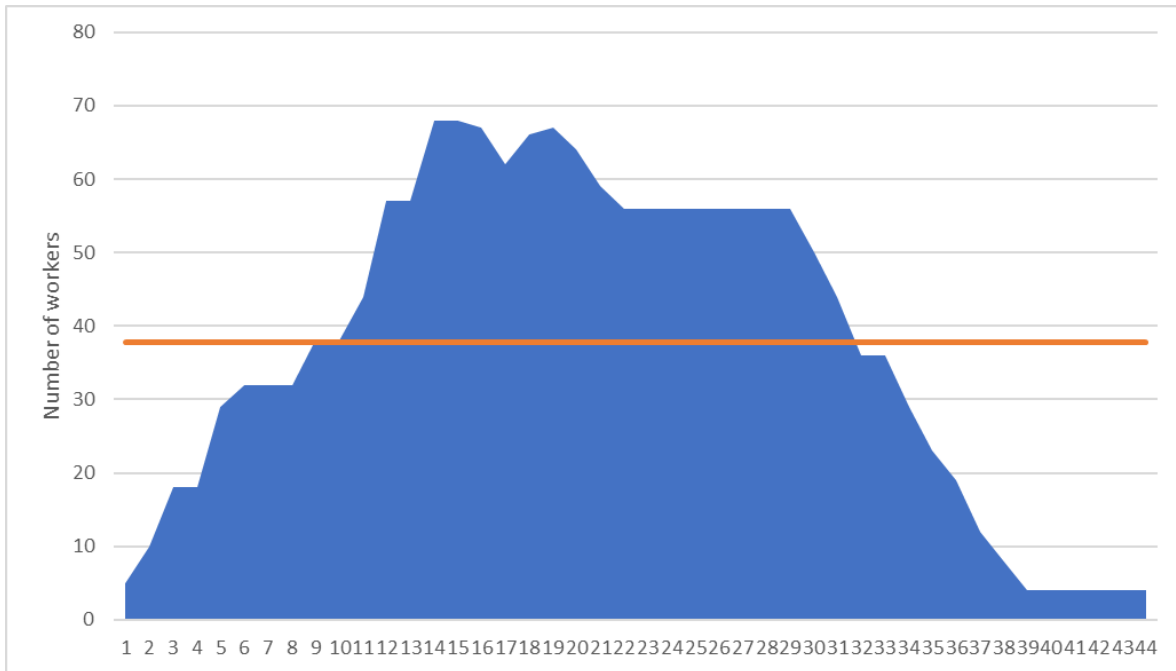
Average number of workers: 31

Worker hours: 13,289 hours

Last Iteration: Crew allocated in activity B

Activity	Natural rhythm (units/week)	Actual rate (units/week)	Nº of crews	Available crews	Unit duration (days)	Activity duration (days)
A	2	6	3	1	2.5	15.8
B	3	6	2	2	1.7	15.8
C	4	4	1	3	1.3	23.8
D	1.5	4.5	3	1	3.3	21.1
E	2.25	4.5	2	2	2.2	21.1
F	1	4	4	0	5.0	23.8





Project duration: 42 days

Maximum workers: 68

Average number of workers: 38

Worker hours: 13,289 hours