Control Distribution of Partially Decoupled Multi-level Manipulators with 5 DOFs

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Abstract

Multi-level manipulators are those mechanisms in which two or more levels, that define the main chain of the manipulator, are joined in parallel to each other. Besides, each level is linked to the base in parallel by some limbs. Based on the idea of multi-level manipulators and using the concept of plain leg-surfaces, the synthesis of partially decoupled manipulators with five degrees of freedom is presented. Among the different possibilities that exist to design the main chain of the manipulator, one is selected and the different manipulators that can be obtained from this option are analyzed. The concept of control distribution per level is presented and compared with the distribution of degrees of freedom per level. Finally, each of the proposed manipulators is studied and those which decouple the rotations are chosen.
Keywords: synthesis, decoupled control, multiple platforms, parallel manipulators

It is well known that a parallel manipulator can offer advantages with respect to serial manipulators, but their disadvantages include a smaller

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workspace and lower dexterity due to a high motion coupling. Though it is true that 6 DOF manipulators have the advantage to be able to maneuver around singularities and avoid link interference, they may present a more complicated mechanical assembly as well as a more complex actuation system. This is why the concepts of lower mobility [1] or limited DOF [2] parallel manipulators might be introduced.

In the industrial context, there is a wide variety of applications that require only 4 or 5 DOF, such as Shcönflies' mechanisms for automation and pick and place, and other manipulators for riveting, drilling and machining applications. In this sense the 3T2R motion pattern can cover a wide range of applications including, among others, 5-axis machine tools and welding. One of the manipulators more recently proposed in this area is introduced in [3]. Due to its large rotational capability it results in a very suitable mechanism for 5 face-machining and similar applications. Also, in medical applications that require simultaneously mobility, compactness and accuracy around a functional point, a 5-DOF parallel mechanisms can be regarded as a very promising solution [4].

However, for a parallel manipulator to become a machine some requirements have to be fulfilled. One of the main ones is that the solution tothe position problems has to be as simple and decoupled as possible. This will provide high speeds and a quick control and will make the calibration easier. To this purpose, this paper focuses on 3T2R parallel manipulators with partially decoupled motions.

The general methods used for the structural synthesis of parallel mechanisms can be divided into two approaches: the constraint-synthesis method based on the screw theory, [1], [5], [6], [7] and the Lie subgroup synthesis method based on the algebraic properties of a Lie group of the Euclidean displacement set, [8], [9], [10], [11]. Also G. Gogu proposed a new methodology based on the Theory of Linear Transformations achieving numerous contributions on the synthesis of manipulators with different degrees of freedom [12], [13], [14], [15].

Two strong tendencies evolve from the approaches described above. The first main stream is the search for symmetric parallel manipulators. When a parallel manipulator has a fully symmetrical structure, which means identical chains, symmetrical assembly conditions and symmetrical actuators will present isotropy performance on workspace and kinematics. Several works can be mentioned in this area, like Fang and Tsai's 4-5 degrees manipulators [16] or Huang and Li's 3 to 5 degrees symmetric manipulators [1], [17]. Kong and Gosselin followed this approach to obtain spherical manipulators [18] and in [19] 18 new symmetrical 2T3R manipulators are presented. More recently, the type synthesis of 4-DOF nonoverconstrained parallel mechanisms with three translations and one rotation is developed in [20]. Also, Oliver and Pierrot described how to obtain symmetric manipulators using the kinematic chains of the Delta manipulator and an articulated platform. The relative movements of this element allow the rotational degree of freedom of the final element [21], [22]. The idea of designing a manipulator with an articulated traveling plate is proposed again in [23].

The second line of research focuses on finding solutions that lead to a simpler resolution of the kinematic problem. Platforms with decoupled degrees of freedom are especially relevant in this field. G. Gogu uses the

Theory of Linear Transformations in order to develop the structural synthesis of fully isotropic Shcönflies manipulators [24] and almost regular 3T2R platforms [25]. Also, the Multipteron family developed by the University of Laval includes the Tripteron platform [26], [27],with three decoupled translational degrees of freedom, which is used to create the Quadrupteron [28], and Pentapteron [29], by adding one and two rotational degrees of freedom respectively.

In addition to these two main trends, recent works can also be mentioned in which computational methods are developed to solve the kinematic problem of these manipulators. In [33] and [34] a new algorithm is presented to solve the forward kinematic problem of parallel manipulators, with improved accuracy and optimized time when compared with previous methods.

Finally, it is worth nothing that there is still room for research on decoupled and symmetrical manipulators with five degrees of freedom. The symmetrical manipulators presented up to date have complex assembly and control conditions; on the other hand, though it is true that manipulators with simpler control have been achieved, it has been at the cost of losing symmetry. This article focuses on partially decoupled manipulators with almost symmetrical limbs.

However, it should be mentioned that translational parallel manipulators have been widely studied by several authors. Among the existing literature references [30] must be highlighted, where some input-output (I/O) decoupled TPMs are described, providing some basic concepts that are used as preliminary concepts in the present article.

This paper studies the possibilities of joining multiple platforms in order to achieve partially decoupled 5 DOF parallel manipulators with relatively simple equations for the control. The seminal idea has been previously presented in [31] and allows an intuitive approach of the synthesis. Some manipulators with four or five degrees of freedom with a simple control can be obtained as a result. The main contribution of this work, consists on introducing the idea of control distribution per level in as opposed to the conventional concept of degrees of freedom per level. In addition, following the recommendations for the synthesis stated in [32], some 3T2R manipulators with decoupled rotations are presented as a result.

1. Preliminary concepts

1.1. The multi-level morphology

The multi-level morphology concept was first introduced in [31] and in [32] and it is based on the assembly of several platforms in parallel. The manipulator consists of a main chain composed by two or more levels joined together by lower kinematic pairs, until the mobile platform (MP) with the attached tool is reached. On the other hand, the n kinematic chains required to link the manipulator to the frame are distributed in parallel among the different levels, see Fig. 1.

Based on the Theory of Groups of Displacements [8] a procedure to combine the degrees of freedom in order to obtain the final motion pattern in several stages can be stated. The displacement set of the lowest level will be determined by the intersection of the motion generators of the kinematic chains that link it to the frame in parallel. When this level is joined to the next level, the displacement subgroup of the joint is added to the displacement set of the lowest level. Kinematic chains that join this upper level to the frame have to be chosen in order to be compatible motion generators with the new displacement set. This procedure is repeated up to the MP.

1.2. Type synthesis of the main chain for 5 DOF manipulators

The process starts with defining the type of motion required at the MP and finding how the final displacement can be decomposed among the different levels. The synthesis presented in [32] focused on manipulators of 5 degrees of freedom with 3T2R mobility using revolute pairs (R), universal joints (U) and prismatic (P) pairs in the main chain, and it led to a high number of possible structures.

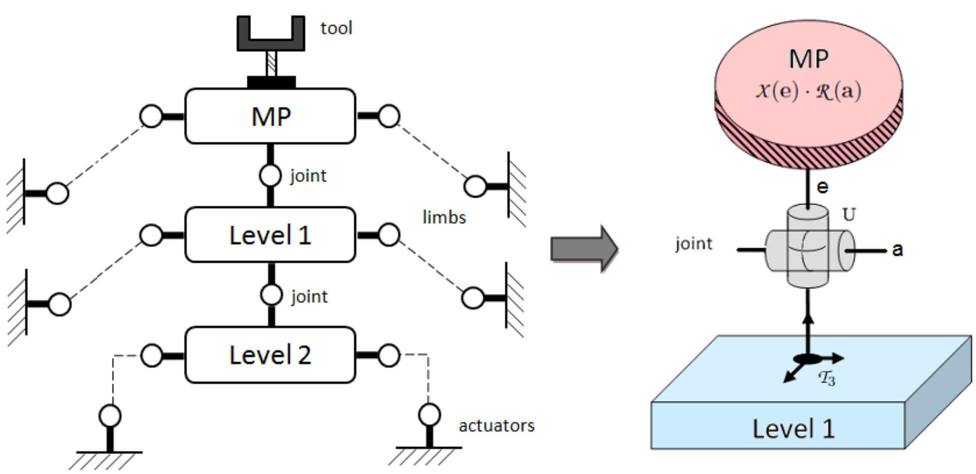


Figure 1: Multi-level main chain studied.

In this paper, taking into account the synthesis of the main chain developed in [32] and in order to obtain decoupled motions, we will focus on assigning the translations to the lowest level. Besides, looking for a minor mechanical complexity we will define only two levels. So the possibilities are restricted to the case shown in Fig. 1.

1.3. Type synthesis for kinematic chains in different levels

Limbs for the Lowest Level: in the lower level as stated in the previous paragraph, kinematic chains whose intersection generate the $\mathcal{T}_3$ subgroup are needed. Besides, we are looking for decoupled translations, so the synthesis is based on using the kinematic chains proposed in [30] and patented in [35] to define the linear input-output parallel translational manipulators.

At this point it is necessary to recall the leg-surface concept, which was first introduced in [30] and later used in [31] and [32]. We are looking for kinematic chains where, if the actuated joint is locked, the end-joint p_i moves on a plain surface π, called the leg-surface of the limb i. Among all kinematic chains that produce a planar leg surface, we are interested in those in which the actuator produces that the plane π moves in the direction of its normal vector $\mathbf{n}_i$.

Limbs for Upper Level: in order to keep the resultant motions as decoupled as possible, we will use kinematic chains with a planar leg-surface compatible with the final movement of the MP, allowing the rotations obtained at the upper level by the U pair of the main chain, see Tab. 1.

R($\mathbf{a}$), R($\mathbf{b}$), R($\mathbf{c}$), R($\mathbf{d}$) and R($\mathbf{e}$) define revolute pairs R with axes $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$, or $\mathbf{e}$ respectively; P($\mathbf{u}$), P($\mathbf{v}$), P($\mathbf{w}$) refer to prismatic pairs along $\mathbf{u}$, $\mathbf{v}$ or $\mathbf{w}$ and S($\mathbf{o}$) is a spherical pair S that defines rotations around point O.

KC for lowest level	KC for upper level	Conditions for Leg-surface	$\mathbf{n_i}$
P(**u**)· P(**v**)· P(**w**)	———	$\mathbf{u} \perp \mathbf{v}$; $\mathbf{v} \perp \mathbf{w}$	**u**
P(**u**)· P(**v**)· R(**c**)· R(**d**)	P(**u**)· P(**v**)· R(**c**)· S(**o**)	$\mathbf{u} \parallel \mathbf{c} \parallel \mathbf{d}$; $\mathbf{v} \perp \mathbf{u}$	**u**
P(**u**)· R(**b**)· P(**w**)· R(**d**)	P(**u**)· R(**b**)· P(**w**)· S(**o**)	$\mathbf{u} \parallel \mathbf{b} \parallel \mathbf{d}$; $\mathbf{w} \perp \mathbf{u}$	**u**
P(**u**)· R(**b**)· R(**c**)· P(**q**)	———	$\mathbf{u} \parallel \mathbf{b} \parallel \mathbf{c}$; $\mathbf{q} \perp \mathbf{u}$	**u**
P(**u**)· R(**b**)· R(**c**)· R(**d**)	P(**u**)· R(**b**)· R(**c**)· S(**o**)	$\mathbf{u} \parallel \mathbf{b} \parallel \mathbf{c} \parallel \mathbf{d}$	**u**
R(**a**)· R(**b**)· P(**w**)· P(**q**)	———	$\mathbf{a} \parallel \mathbf{b} \parallel \mathbf{q}$; $\mathbf{a}, \mathbf{b}, \mathbf{q} \perp \mathbf{w}$	$\mathbf{w} \times \mathbf{q}$
R(**a**)· R(**b**)· P(**w**)· R(**d**)· R(**e**)	R(**a**)· R(**b**)· P(**w**)· R(**d**)· S(**o**)	$\mathbf{a} \parallel \mathbf{b}$; $\mathbf{d} \parallel \mathbf{e} \perp \mathbf{w}$; $\mathbf{a}, \mathbf{b} \perp \mathbf{d}, \mathbf{e}$	**d**
R(**a**)· R(**b**)· R(**c**)· P(**q**)· R(**e**)	R(**a**)· R(**b**)· R(**c**)· P(**q**)· S(**o**)	$\mathbf{a} \parallel \mathbf{b}$; $\mathbf{c} \parallel \mathbf{e} \perp \mathbf{q}$; $\mathbf{a}, \mathbf{b} \perp \mathbf{c}, \mathbf{e}$	**c**
R(**a**)· R(**b**)· R(**b**)· R(**d**)· P(**s**)	———	$\mathbf{a} \parallel \mathbf{b}$; $\mathbf{c} \parallel \mathbf{d} \perp \mathbf{s}$; $\mathbf{a}, \mathbf{b} \perp \mathbf{c}, \mathbf{d}$	**c**
R(**a**)· R(**b**)· R(**c**)· R(**d**)· R(**e**)	R(**a**)· R(**b**)· R(**c**)· R(**d**)· S(**o**)	$\mathbf{a} \parallel \mathbf{b}$; $\mathbf{c} \parallel \mathbf{d} \parallel \mathbf{e}$; $\mathbf{a} \perp \mathbf{c}$	**c**

Table 1: Kinematic chains for the lowest and the upper level.

2. Considerations for distribution of control among the different levels

In the previous section the synthesis of both the main chain and the kinematic chains suitable for each level have been defined. In this section, these concepts are used to define a large number of partially decoupled manipulators.

The conventional parallel manipulators are composed by a single level and therefore all degrees of freedom of the manipulator are controlled there. In our case, as we are working with manipulators containing different levels connected in parallel, it is necessary to distinguish between degrees of freedom and control. This means that the degrees of freedom of the lower level do not necessarily have to be controlled from this level; some of them can remain free and thus be controlled from the upper level.

On the basis of a main chain in which the lower level has three translations, while two rotations are added in the upper level, the different options for control and the manipulators that can be obtained for each case are going to be analyzed, see Fig. 4. Some considerations for a good distribution of control are:

1. An adequate choice of actions to generate the rotation at the MP. It is clear that in order to generate a rotation we need two different forces. One possibility is to use an action and its reaction on the axis (see Fig. 2.up: action on P_i, reaction on P). This approach makes it possible to obtain decoupled equations. Another option from the point of view of the transmission of torque, is using two actions out of the axis (see Fig. 2.down: both actions on points P_i). These will control the rotation, but also one translation. Therefore, the position equations of these displacements will be coupled.
2. Define the control of the remaining translations at the lower level that will be based simply on the perpendicularity of the normals $\mathbf{n_i}$. It is important to highlight that, since we are using the kinematic chains used in the linear translational parallel manipulators, each actuator of the lower level controls the displacement of the associated leg-surface (in all cases a plane π_i) along its normal $\mathbf{n_i}$. Hence, the displacement of the end-joint of that limb P_i is defined in that direction $\mathbf{n_i}$ by the input of its limb, while the other displacements are defined by the inputs of the other limbs when assembled (see Fig. 3). The number of translations controlled from the lower level depends on how many limbs are assembled in this level. If the lower level is defined by less than three limbs, the other translations will be controlled from the upper level. Since the condition has been imposed that it is necessary

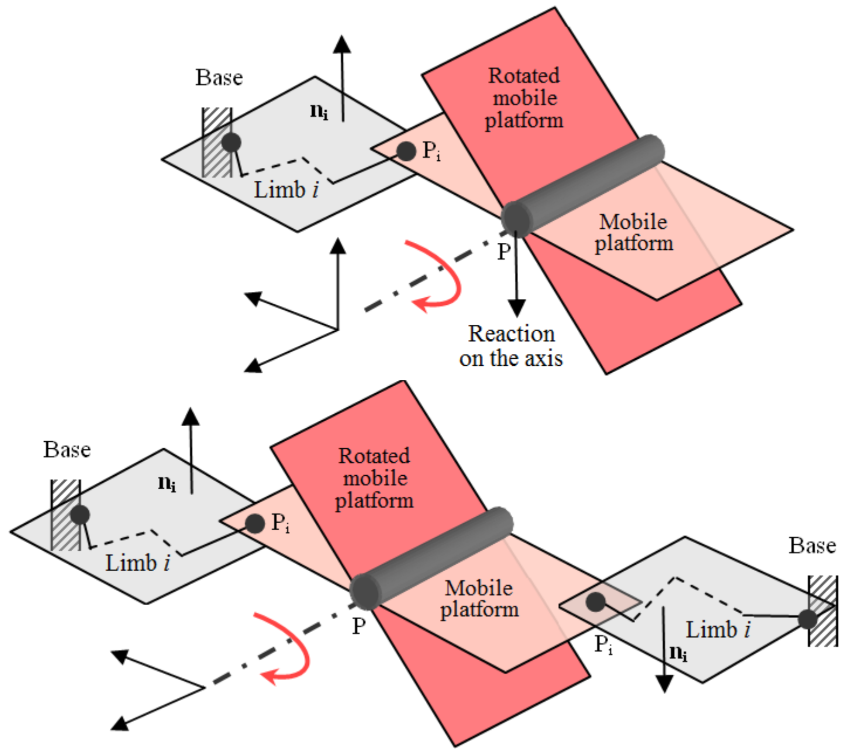


Figure 2: Different options to control the rotations

> to assembly the kinematic chains with their corresponding leg-surfaces orthogonally orientated, it is clear that the translations of the lower level will remain uncoupled. Each of these translations will be directly controlled by its corresponding actuator.

Bearing these two ideas in mind, the options for distributing the control of five degrees of freedom among the two levels can be stated. In Fig. 4 the scheme of the proposed structure and different options for control is shown. The conditions for assembly are described later in tables and the manipulators which were obtained are shown in the figures.

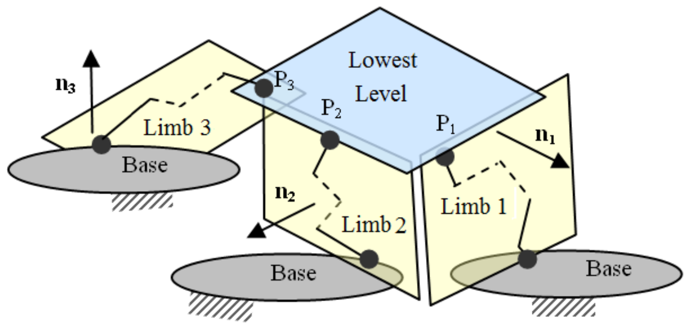


Figure 3: Orthogonal assembly of the kinematic chains in the lower level to control the three translations of point P on the mobile platform

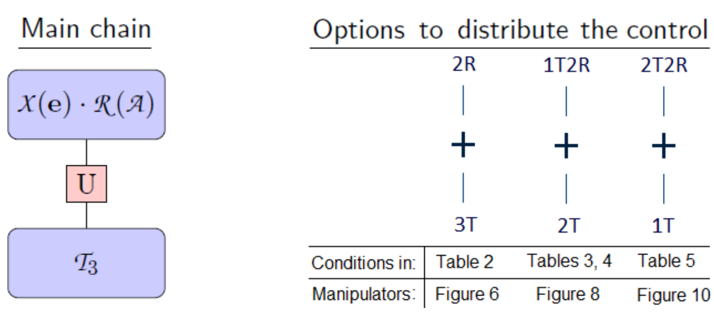


Figure 4: Options to control the five degrees of freedom

3. Synthesis of two-level manipulators (3T+2R, 2T+1T2R, 1T+2T2R) with five degrees of freedom

The synthesis is focused on the fact that the distribution of control starts deciding how many translations are going to be controlled in the lower level. At this point, it is interesting to highlight that since we are working with three orthogonal directions and five degrees of freedom, it is clear that we will never obtain five fully uncoupled movements. In fact, the degrees of freedom controlled by kinematic chains with equally orientated leg-surfaces will be coupled. The orientation of the upper level planes will determine the degree of coupling of the different degrees of freedom.

The double manipulators of five degrees of freedom obtained are described in Tab. 2, Tab. 3, Tab. 4 and Tab. 5.

3.1. First option for the control: 3T+2R

In this option, the lower level has a translational motion controlled by three limbs, with orthogonal leg-surface normals ($\mathbf{n_1}$, $\mathbf{n_2}$ and $\mathbf{n_3}$), which decouple the corresponding position equations. The remaining two legs of the upper level control both rotations. On the mobile platform, the limbs attached have associated leg-surfaces (π_4 and π_5) and they have to remain parallel to one or two of the planes used in the lower platform. The conditions for the end-joints of these limbs, S pairs G_4 and G_5, and the different platforms obtained, are presented in table 2.

3.2. Second option for the control: 2T+1T2R

Another possibility is based on the following reasoning: if only two translations are controlled in the lower level, the limbs needed to control the two rotations, and the remaining translation, will be attached to the upper level. Depending on how these limbs are joined, the first or the second rotation will be controlled by a couple of forces instead of by a single action. The platforms obtained from this morphology appear in Tab. 3 and Tab. 4. In the cases shown in Tab. 3 the first rotation is controlled by only one action and the second rotation will be controlled by a couple of forces. Tab. 4 shows

<table>
<tr><td colspan="3">5 DOF: $\mathcal{X}(\mathbf{e}) \cdot \mathcal{R}(\mathcal{A})$</td></tr>
<tr><td>Distribution of the Control</td><td colspan="2">3T+2R</td></tr>
<tr><td>Low level</td><td colspan="2">Constitution: 3 Limbs from Tab. 1

Conditions: $\mathbf{n}_1 \perp \mathbf{n}_2 \perp \mathbf{n}_3$</td></tr>
<tr><td>Joining pair</td><td colspan="2">Joint for the Main Chain: $U_{\mathbf{e},\mathbf{a}}$

Conditions for the joint: $\mathbf{e} \parallel \mathbf{n}_1$</td></tr>
<tr><td>Mobile Platform</td><td>Constitution:
2 Limbs from Tab. 1

Conditions for the limbs:
$\mathbf{n}_4, \mathbf{n}_5 \perp \mathbf{n}_1$

Conditions for S pairs:
• G_4 on $\mathcal{A}$ axis
• G_5 on a line $\perp$ to the plane defined by $\mathcal{A}$ and $\mathcal{E}$, containing $U_{\mathbf{e},\mathbf{a}}$</td><td>Constitution:
2 Limbs from Tab. 1

Conditions for the limbs:
$\mathbf{n}_4 \perp \mathbf{n}_1$ AND $\mathbf{n}_5 \parallel \mathbf{n}_1$

Conditions for S pairs:
• G_4 on $\mathcal{A}$ axis
• G_5 on a line $\perp$ to the plane defined by $\mathcal{A}$ and $\mathcal{E}$, containing $U_{\mathbf{e},\mathbf{a}}$</td></tr>
</table>

Table 2: Double manipulators with $\mathcal{X}(\mathbf{e}) \cdot \mathcal{R}(\mathbf{a})$ motion: 3T+2R Control

the first rotation which is controlled by a couple of forces and there will be only a remaining limb for the second rotation.

3.3. Third option for the control: 1T+2T2R

In the last option, only one translation is controlled in the lower level and the remaining movements are controlled in the upper level. In this

5 DOF: $\mathcal{X}(\mathbf{e}) \cdot \mathcal{R}(\mathbf{a})$		
Distribution of the Control	2T+1T2R	
Low level	Constitution: 2 Limbs from Tab. 1 Conditions: $\mathbf{n}_1 \perp \mathbf{n}_2$	
Joining pair	Joint for the Main Chain: $\mathrm{U}_{\mathbf{e},\mathbf{a}}$ Conditions for the joint: $\mathbf{e} \parallel \mathbf{n}_1$	
Mobile Platform		
Constitution: 3 Limbs from Tab. 1 Conditions for the limbs: $\mathbf{n}_3, \mathbf{n}_4, \mathbf{n}_5 \perp \mathbf{n}_1$ Conditions for S pairs: • G_3 on $\mathcal{A}$ axis • G_4 and G_5 on a plane $\perp$ to $\mathcal{A}$ symmetrical to U	Constitution: 3 Limbs from Tab. 1 Conditions for the limbs: $\mathbf{n}_3, \mathbf{n}_4 \perp \mathbf{n}_1$ AND $\mathbf{n}_5 \parallel \mathbf{n}_1$ Conditions for S pairs: • G_3 on $\mathcal{A}$ axis • G_4 and G_5 on a plane $\perp$ to $\mathcal{A}$ symmetrical to U	Constitution: 3 Limbs from Tab. 1 Conditions for the limbs: $\mathbf{n}_3 \perp \mathbf{n}_1$ AND $\mathbf{n}_4, \mathbf{n}_5 \parallel \mathbf{n}_1$ Conditions for S pairs: • G_3 on $\mathcal{A}$ axis • G_4 and G_5 on a plane $\perp$ to $\mathcal{A}$ symmetrical to U

Table 3: Double manipulators with $\mathcal{X}(\mathbf{e}) \cdot \mathcal{R}(\mathbf{a})$ motion: 2T+1T2R Control

case and according to the geometric conditions, the two rotations will be controlled either by a unique force or by a couple of forces. Thus, the following manipulators are obtained, see Tab. 5.

It is important to highlight that, while the different limbs are being

<table>
<tr><td colspan="3">5 DOF: $\mathcal{X}(\mathbf{e}) \cdot \mathcal{R}(\mathbf{a})$</td></tr>
<tr><td>Distribution of the Control</td><td colspan="2">2T+1T2R</td></tr>
<tr><td>Low level</td><td colspan="2">Constitution: 2 Limbs from Tab. 1

Conditions: $\mathbf{n}_1 \perp \mathbf{n}_2$</td></tr>
<tr><td>Joining pair</td><td colspan="2">Joint for the Main Chain: $\mathrm{U}_{\mathbf{e},\mathbf{a}}$

Conditions for the joint: $\mathbf{e} \parallel \mathbf{n}_1$</td></tr>
<tr><td>Mobile Platform</td><td>Constitution:
3 Limbs from Tab. 1

Conditions for the limbs:
$\mathbf{n}_3, \mathbf{n}_4, \mathbf{n}_5 \perp \mathbf{n}_1$

Conditions for S pairs:
• G_3 and G_4 on $\mathcal{A}$ axis symmetrical to U
• G_5 on a line $\perp$ to the plane defined by $\mathcal{A}$ and $\mathcal{E}$, containing $\mathrm{U}_{\mathbf{e},\mathbf{a}}$</td><td>Constitution:
3 Limbs from Tab. 1

Conditions for the limbs:
$\mathbf{n}_3, \mathbf{n}_4 \perp \mathbf{n}_1$ AND $\mathbf{n}_5 \parallel \mathbf{n}_1$

Conditions for S pairs:
• G_3 and G_4 on $\mathcal{A}$ axis symmetrical to U
• G_5 on a line $\perp$ to the plane defined by $\mathcal{A}$ and $\mathcal{E}$, containing $\mathrm{U}_{\mathbf{e},\mathbf{a}}$</td></tr>
</table>

Table 4: Double manipulators with $\mathcal{X}(\mathbf{e}) \cdot \mathcal{R}(\mathbf{a})$ motion: 2T+1T2R Control

attached to any of the two levels according to the conditions exposed in Tab. 2, Tab. 3, Tab. 4 and Tab. 5, in order to control all the degrees of freedom, at least one limb must be orientated in each of the three principal directions.

5 DOF: $\mathcal{X}(\mathbf{e}) \cdot \mathcal{R}(\mathbf{a})$		
Distribution of the Control	1T+2T2R	
Low level	Constitution: 1 Limb from Tab. 1 Conditions for the pairs of the low level: G_1 pair on $\mathcal{E}$ axis	
Joining pair	Joint for the Main Chain: $U_{\mathbf{e},\mathbf{a}}$ Conditions for the joint: $\mathbf{e} \perp \mathbf{n}_1$	
Mobile Platform	Constitution: 4 Limbs of the Tab. 1 Conditions for the limbs: $\mathbf{n}_2 \perp \mathbf{n}_1 \perp \mathcal{E}$; $\mathbf{n}_3 \parallel \mathbf{n}_1$ OR $\mathbf{n}_3 \parallel \mathbf{n}_2$; $\mathbf{n}_4 \parallel \mathcal{E}$ AND $\mathbf{n}_5 \perp \mathcal{E}$ Conditions for S pairs: • G_2 and G_3 on $\mathcal{A}$ axis symmetrical to U • G_4 and G_5 on a line $\perp$ to the plane defined by $\mathcal{A}$ and $\mathcal{E}$, symmetrical to U	Constitution: 4 Limbs of the Tab. 1 Conditions for the limbs: $\mathbf{n}_2 \perp \mathbf{n}_1 \perp \mathcal{E}$; $\mathbf{n}_3 \parallel \mathbf{n}_1$ OR $\mathbf{n}_3 \parallel \mathbf{n}_2$; $\mathbf{n}_4 \parallel \mathcal{E}$ AND $\mathbf{n}_5 \parallel \mathcal{E}$ Conditions for S pairs: • G_2 and G_3 on $\mathcal{A}$ axis symmetrical to U • G_4 and G_5 on a line $\perp$ to the plane defined by $\mathcal{A}$ and $\mathcal{E}$, symmetrical to U

Table 5: Double manipulators with $\mathcal{X}(\mathbf{e}) \cdot \mathcal{R}(\mathbf{a})$ motion: 1T+2T2R Control

4. Considerations for an uncoupled control

Following the information of the previous tables and depending on the relative orientation between the different kinematic chains used in each level, it is possible to obtain different manipulators for each of the control options proposed. To analyze which of the proposed manipulators has a higher degree of uncoupled movements, it is necessary to solve the position problem for each case.

4.1. How to get decoupled rotations for the 3T+2R control

This corresponds to the first case shown in Fig. 4 and conditions for assembly are described in Tab. 2 If any of the actuators of the lower level is moved the corresponding translation will be directly controlled. The two rotations that define the orientation of the mobile platform are controlled in the upper level. A fixed frame $(O, \mathbf{i}, \mathbf{j}, \mathbf{k})$ and a moving frame $(P, \mathbf{u}, \mathbf{v}, \mathbf{w})$ attached to the MP are defined, as shown in Fig. 5. In order to obtain the loop-closure equations in a simple way, the mobile frame must be orientated so that the $\mathbf{u}$ and $\mathbf{v}$ axes coincide respectively with the directions of rotation $\mathcal{E}$ and $\mathcal{A}$ of the universal joint $\mathrm{U}_{\mathbf{e},\mathbf{a}}$. The rotation matrix from the moving frame to the fixed frame is defined by a rotation angle α around the fixed axis $\mathbf{i}$ and by a rotation angle β around the mobile axis $\mathbf{v}$:

$$
\begin{aligned}
{}^{\mathbf{0}}_{\mathbf{1}}\mathbf{R} = &\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \cdot \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} = \\
&\begin{bmatrix} \cos\beta & 0 & \sin\beta \\ \sin\alpha\cdot\sin\beta & \cos\alpha & -\sin\alpha\cdot\cos\beta \\ -\cos\alpha\cdot\sin\beta & \sin\alpha & \cos\alpha\cdot\cos\beta \end{bmatrix}
\end{aligned} \tag{1}
$$

The loop-closure equation for kinematic chains 4 and 5 is the following:

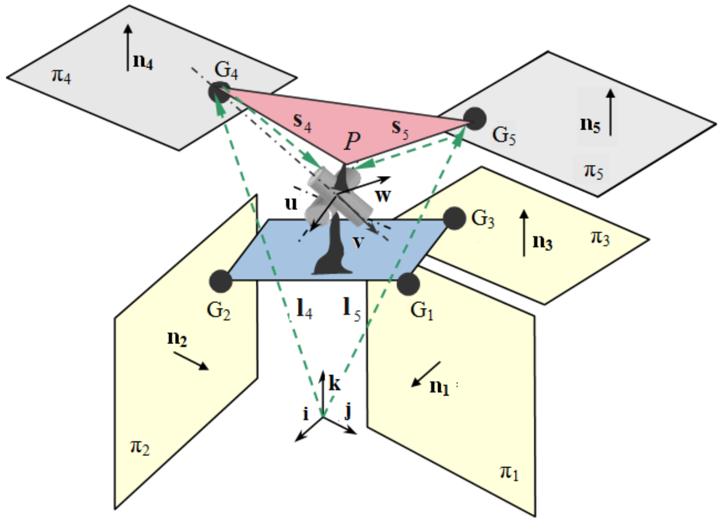


Figure 5: Relative orientation between planes for the 3T+2R configuration

$$\mathbf{p} = \mathbf{l}_i + \mathbf{s}_i \qquad i = 4, 5 \tag{2}$$

where, $\mathbf{p} = \begin{bmatrix} P_x & P_y & P_z \end{bmatrix}^T$ defines the position vector of the reference point P, $\mathbf{l}_i = \begin{bmatrix} l_{ix} & l_{iy} & l_{iz} \end{bmatrix}^T$ defines the position of the last pair of the analyzed limbs G_i which joins the corresponding limb with the MP. Finally $\mathbf{s}_i$ vectors go from G_i pairs to the P point of the MP. According to the conditions exposed in Tab. 2, G_4 and G_5 must be respectively on mobile axes $\mathbf{v}$ and $\mathbf{w}$.

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} l_{ix} \\ l_{iy} \\ l_{iz} \end{bmatrix} + \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ \sin\alpha \cdot \sin\beta & \cos\alpha & -\sin\alpha \cdot \cos\beta \\ -\cos\alpha \cdot \sin\beta & \sin\alpha & \cos\alpha \cdot \cos\beta \end{bmatrix} \cdot \begin{bmatrix} s_{iu} \\ s_{iv} \\ s_{iw} \end{bmatrix} \tag{3}$$

Depending on the different orientations of the chains of the upper level, $\mathbf{n}_4$ and $\mathbf{n}_5$, the following equations are got:

- $P_y = l_{4y} + s_{4v} \cdot cos\alpha$
- $P_z = l_{4z} + s_{4v} \cdot sin\alpha$

- $P_x = l_{5x} + s_{5w} \cdot sin\beta$
- $P_y = l_{5y} - s_{5w} \cdot sin\alpha \cdot cos\beta$
- $P_z = l_{5z} + s_{5w} \cdot cos\alpha \cdot cos\beta$

Combining the different possibilities, three manipulators are obtained, see Fig. 6. It can be observed that in the last one, both rotations remain decoupled.

4.2. How to get decoupled rotations for the 2T+1T2R control

These manipulators have a lower level with three translations T_3; the kinematic chains for these legs are chosen from Tab. 1, and joined with the normals $\mathbf{n}_i$ of their corresponding leg-surfaces orthogonally to each other. Therefore three directions in this level are decoupled. As in the lower level there are only two limbs, two of the three translations will be controlled leaving the last one free. This third translation is controlled from the upper level and therefore it will be coupled with the two rotations. From the different control options presented in Tab. 3 and in Tab. 4, the last ones are chosen because more symmetrical manipulators with more decoupled motions are obtained. As in the previous case, two different frames are defined, the fixed frame $(O, \mathbf{i}, \mathbf{j}, \mathbf{k})$ and the mobile one $(P, \mathbf{u}, \mathbf{v}, \mathbf{w})$, with the same orientation as in the previous case. The rotation matrix that relates both frames is the same one given in (1), see Fig. 7.

Now the equations of the kinematic chains of the upper level are obtained.

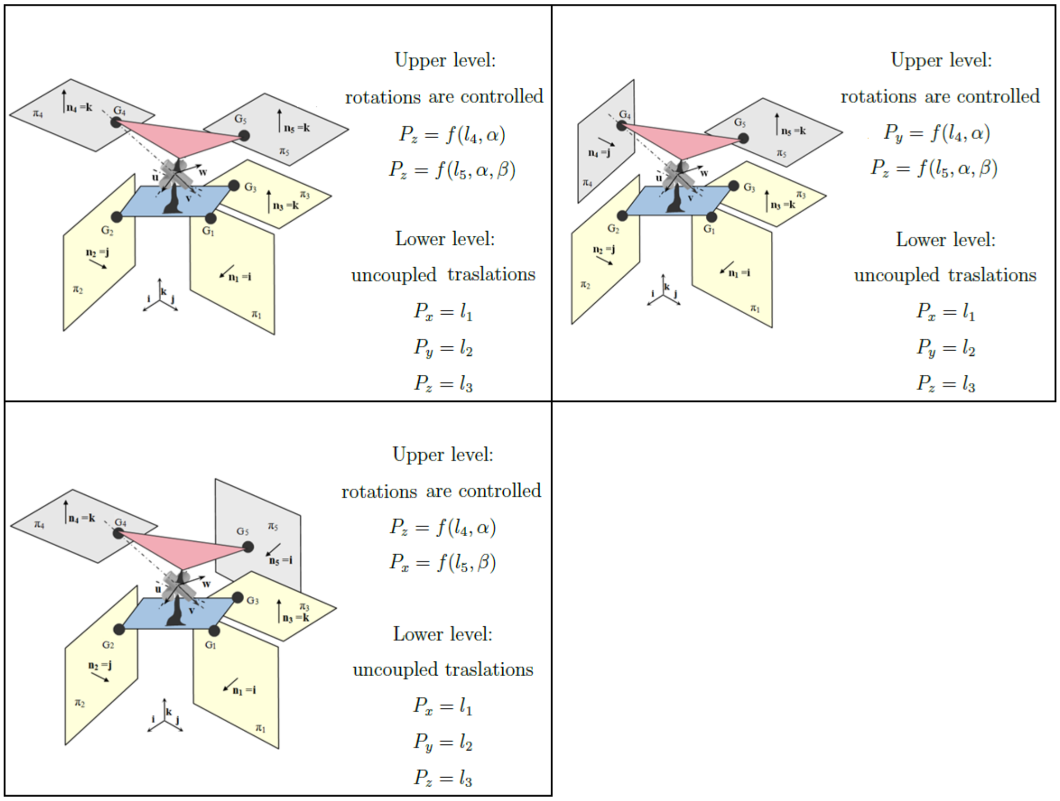


Figure 6: Double manipulators with 3T+2R control

These equations control the coupled degrees of freedom. The loop-closure equation for the kinematic chains of the upper level is,

$$\mathbf{p} = \mathbf{l}_i + \mathbf{s}_i \qquad i = 3, 4, 5 \tag{4}$$

where the vectors $\mathbf{p} = \begin{bmatrix} P_x & P_y & P_z \end{bmatrix}^T$, $\mathbf{l}_i = \begin{bmatrix} l_{ix} & l_{iy} & l_{iz} \end{bmatrix}^T$ and $\mathbf{s}_i$ are defined in the same way as in the previous case. According to the conditions

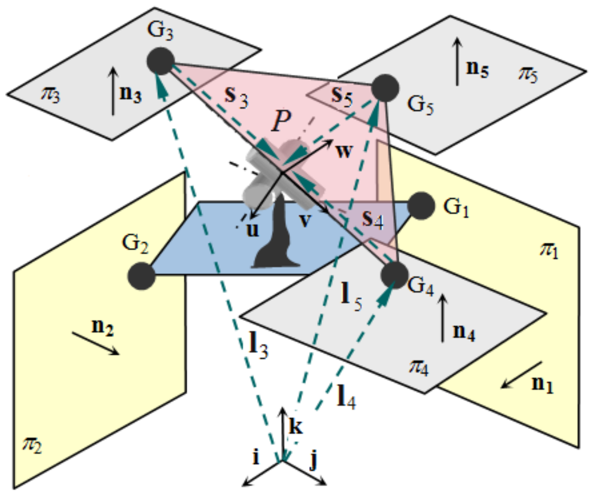


Figure 7: Relative orientation between planes for the 2T+1T2R configuration

exposed in Tab. 4, points G_3 and G_4 should be placed on the mobile axis $\mathbf{v}$ and point G_5 on the axis $\mathbf{w}$.

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} l_{ix} \\ l_{iy} \\ l_{iz} \end{bmatrix} + \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ \sin\alpha\cdot\sin\beta & \cos\alpha & -\sin\alpha\cdot\cos\beta \\ -\cos\alpha\cdot\sin\beta & \sin\alpha & \cos\alpha\cdot\cos\beta \end{bmatrix} \cdot \begin{bmatrix} s_{iu} \\ s_{iv} \\ s_{iw} \end{bmatrix} \tag{5}$$

Depending on the different orientations of the chains of the upper level, $\mathbf{n}_3$, $\mathbf{n}_4$ and $\mathbf{n}_5$, the following loop-closure equations are obtained:

• $P_y = l_{3y} + s_{3v} \cdot cos\alpha$	• $P_y = l_{4y} + s_{4v} \cdot cos\alpha$
• $P_z = l_{3z} + s_{3v} \cdot sin\alpha$	• $P_z = l_{4z} + s_{4v} \cdot sin\alpha$

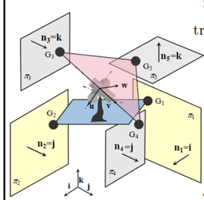
- $P_x = l_{5x} + s_{5w} \cdot sin\beta$
- $P_y = l_{5y} - s_{5w} \cdot sin\alpha \cdot cos\beta$
- $P_z = l_{5z} + s_{5w} \cdot cos\alpha \cdot cos\beta$

The different combinations obtained are shown below, see Fig. 8. In the last two cases, the two rotations are decoupled.

4.3. How to get decoupled rotations for the 1T+2T2R control

In this case, the lower level is composed of only one kinematic chain with T_3 motion and only one of its translations is controlled from this level. In consequence, the remaining translations will be controlled from the upper level. Again, it is necessary to define two frames, the fixed one $(O, \mathbf{i}, \mathbf{j}, \mathbf{k})$ and the mobile one $(P, \mathbf{or}, \mathbf{v}, \mathbf{w})$ and they are oriented along the $\mathbf{w}$ and $\mathbf{v}$ axes respectively, coinciding with the rotation axes $\mathcal{E}$ and $\mathcal{A}$ of the universal joint $\mathrm{U}_{\mathbf{e},\mathbf{a}}$, see Fig. 9.

The rotation matrix and the loop-closure equations of the kinematic chains with coupled degrees of freedom are shown below. In this case, the



Upper level:
rotations and coupled
traslation are controlled

$P_y = f(l_3, \alpha)$

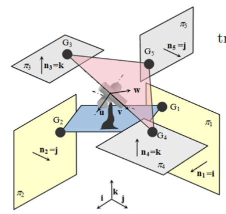
$P_y = f(l_4, \alpha)$

$P_z = f(l_5, \alpha, \beta)$

Lower level:
decoupled traslations

$P_x = l_1$

$P_y = l_2$



Upper level:
rotations and coupled
traslation are controlled

$P_z = f(l_3, \alpha)$

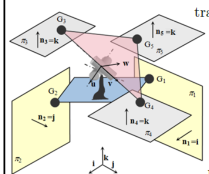
$P_z = f(l_4, \alpha)$

$P_y = f(l_5, \alpha, \beta)$

Lower level:
decoupled traslations

$P_x = l_1$

$P_y = l_2$



Upper level:
rotations and coupled
traslation are controlled

$P_z = f(l_3, \alpha)$

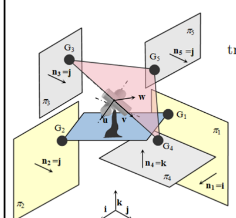
$P_z = f(l_4, \alpha)$

$P_z = f(l_5, \alpha, \beta)$

Lower level:
decoupled traslations

$P_x = l_1$

$P_y = l_2$



Upper level:
rotations and coupled
traslation are controlled

$P_y = f(l_3, \alpha)$

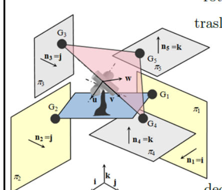
$P_z = f(l_4, \alpha)$

$P_y = f(l_5, \alpha, \beta)$

Lower level:
decoupled traslations

$P_x = l_1$

$P_y = l_2$



Upper level:
rotations and coupled
traslation are controlled

$P_y = f(l_3, \alpha)$

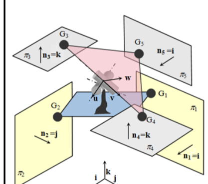
$P_z = f(l_4, \alpha)$

$P_z = f(l_5, \alpha, \beta)$

Lower level:
decoupled traslations

$P_x = l_1$

$P_y = l_2$



Upper level:
rotations and coupled
traslation are controlled

$P_z = f(l_3, \alpha)$

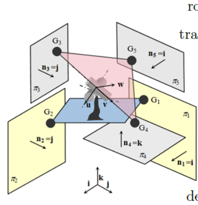
$P_z = f(l_4, \alpha)$

$P_x = f(l_5, \beta)$

Lower level:
decoupled traslations

$P_x = l_1$

$P_y = l_2$



Upper level:
rotations and coupled
traslation are controlled

$P_y = f(l_3, \alpha)$

$P_z = f(l_4, \alpha)$

$P_x = f(l_5, \beta)$

Lower level:
decoupled traslations

$P_x = l_1$

$P_y = l_2$

Figure 8: Double manipulators with 2T+1T2R control

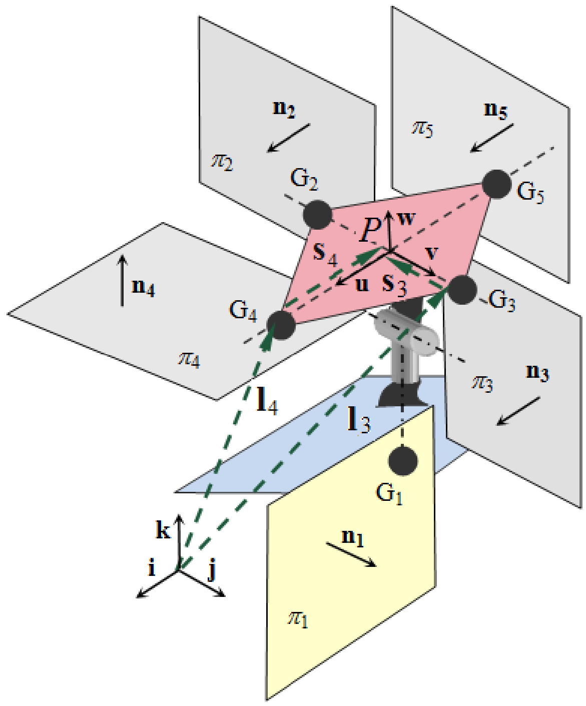


Figure 9: Relative orientation between planes for the 1T+2T2R configuration

rotation matrix is defined by a rotation angle γ around the fixed axis $\mathbf{k}$ and by a rotation angle β around the mobile axis $\mathbf{v}$.

$$
\begin{aligned}
{}^{\mathbf{0}}_{\mathbf{1}}\mathbf{R} = &\begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} = \\
&\begin{bmatrix} \cos\gamma\cdot\cos\beta & -\sin\gamma & \cos\gamma\cdot\sin\beta \\ \sin\gamma\cdot\cos\beta & \cos\gamma & \sin\gamma\cdot\sin\beta \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}
\end{aligned} \tag{6}
$$

$$\mathbf{p} = \mathbf{l}_i + \mathbf{s}_i \quad i = 2, 3, 4, 5 \tag{7}$$

where the vectors $\mathbf{p} = \begin{bmatrix} P_x & P_y & P_z \end{bmatrix}^T$, $\mathbf{l}_i = \begin{bmatrix} l_{ix} & l_{iy} & l_{iz} \end{bmatrix}^T$ and $\mathbf{s}_i$ have the same meaning as in the previous cases.

According to the conditions in Tab. 5, the points G_2 and G_3 will be placed in the mobile axis $\mathbf{v}$ and points G_4 and G_5 must be in the mobile axis $\mathbf{u}$.

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} l_{ix} \\ l_{iy} \\ l_{iz} \end{bmatrix} + \begin{bmatrix} \cos\gamma \cdot \cos\beta & -\sin\gamma & \cos\gamma \cdot \sin\beta \\ \sin\gamma \cdot \cos\beta & \cos\gamma & \sin\gamma \cdot \sin\beta \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \cdot \begin{bmatrix} s_{iu} \\ s_{iv} \\ s_{iw} \end{bmatrix} \tag{8}$$

Depending on the different orientations for the chains of the upper level, $\mathbf{n}_2$, $\mathbf{n}_3$, $\mathbf{n}_4$ and $\mathbf{n}_5$, the following loop-closure equations are obtained:

• $P_x = l_{2x} + s_{2v} \cdot sin\gamma$ • $P_y = l_{2y} + s_{2v} \cdot cos\gamma$	• $P_x = l_{3x} + s_{3v} \cdot sin\gamma$ • $P_y = l_{3y} + s_{3v} \cdot cos\gamma$
• $P_x = l_{4x} + s_{4u} \cdot cos\gamma \cdot cos\beta$ • $P_y = l_{4y} + s_{4u} \cdot sin\gamma \cdot cos\beta$ • $P_z = l_{4z} - s_{4u} \cdot sin\beta$	• $P_x = l_{5x} + s_{5u} \cdot cos\gamma \cdot cos\beta$ • $P_y = l_{5y} + s_{5u} \cdot sin\gamma \cdot cos\beta$ • $P_z = l_{5z} - s_{5u} \cdot sin\beta$

And combining the different possibilities, the obtained manipulators are shown below, see Fig. 10. It can be seen that the last two cases keep decoupled rotations.

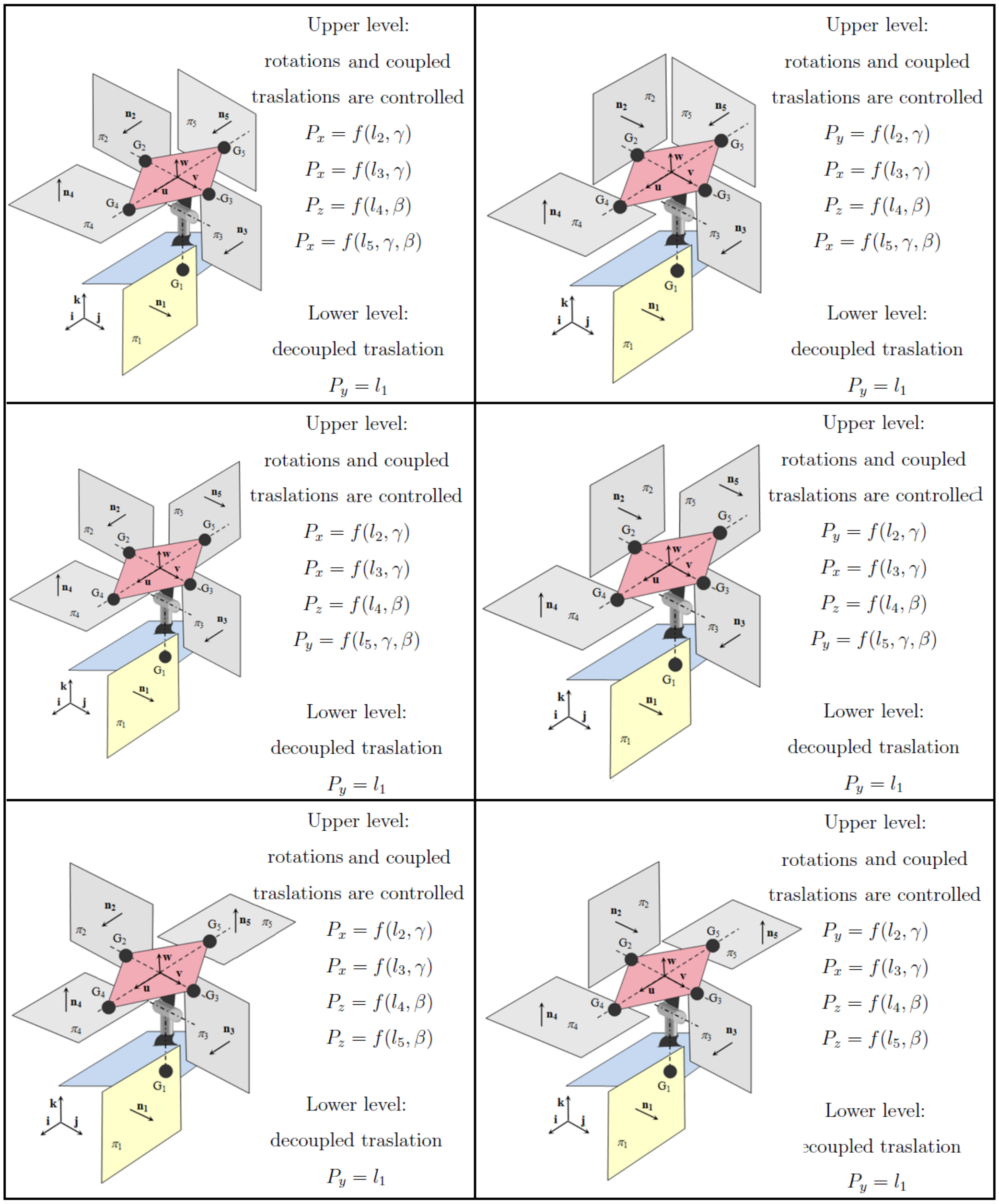


Figure 10: Double manipulators with 1T+2T2R control

As a conclusion, it must be highlighted that for any of the control options proposed, and assuming that points G_i have been placed according to the synthesis presented in the previous section 3, decoupled rotations are obtained if the following conditions are fulfilled,

1. The actuators of the kinematic chains that control the rotation about the fixed axis $\mathbf{e}$ of the $U_{\mathbf{e},\mathbf{a}}$ joint, must be perpendicular to it.
2. The actuators of the kinematic chains that control the rotation about the mobile axis $\mathbf{a}$ of the $U_{\mathbf{e},\mathbf{a}}$ joint, must be parallel to the fixed axis $\mathbf{e}$.

5. Case Study

In this section, one of the morphologies with 3T+2R has been selected from Tab. 2, in order to illustrate the main contributions proposed in the paper. In this case, as two rotations are controlled in the upper level and as the orientations of their actuators are the same as the orientation of two actuators of the lower level, the corresponding translations will get coupled. The third translation is decoupled and controlled from the lower level.

As shown in the model of Fig. 11, the reference point P is on the $U_{\mathbf{e},\mathbf{a}}$ joint between the first level and the mobile platform. The first level is connected to the frame by three CPR limbs that allow to control the three translations The other two CPS limbs (limbs 4 and 5), which are attached to the mobile platform, enable control of the rotations but they will interfere in two of the three translations.

We are looking for decoupled rotations so that kinematic chains must be orientated following the synthesis conditions presented in Tab. 5 and in the last configuration shown in Fig. 6. This means that $\mathbf{n}_4$ must be perpendicular to $\mathbf{n}_1$ and $\mathbf{n}_5$ parallel to $\mathbf{n}_1$. Besides, S pairs (points G_4 and G_5) must be respectively placed in the mobile axis $\mathcal{A}$ and on a line perpendicular to the plane defined by the axis of the $U_{\mathbf{e,a}}$ joint, according to the conditions presented in Tab. 2.

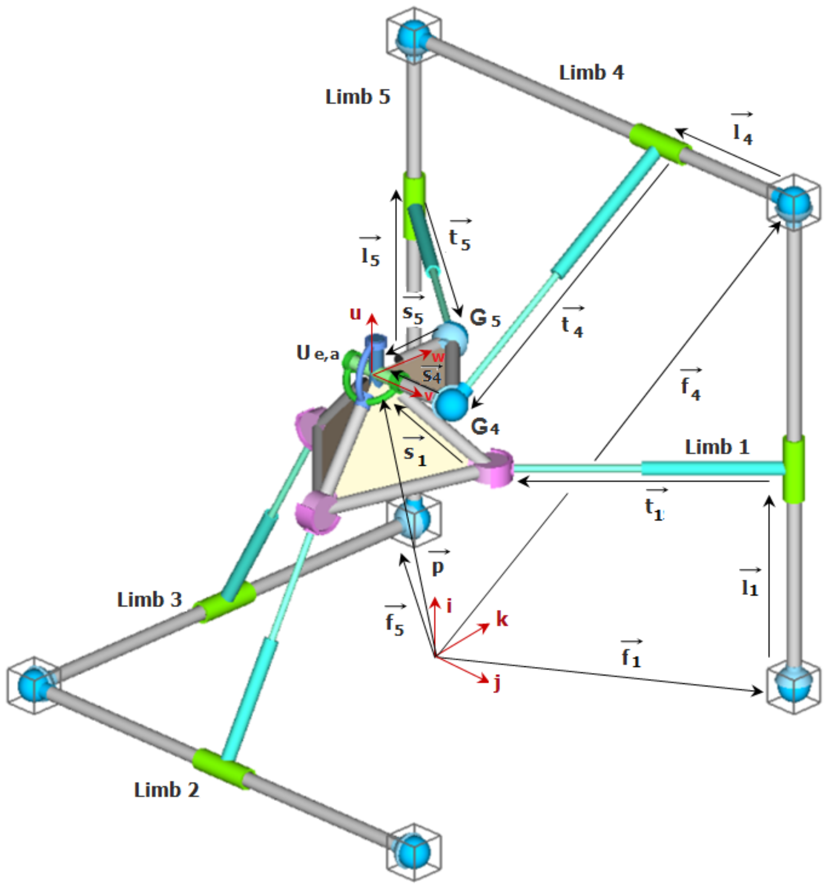


Figure 11: 5 DOF manipulator with 3T+2R control.

In order to write the loop-closure equations of the position analysis, it is necessary to define a fixed frame $(O, \mathbf{i}, \mathbf{j}, \mathbf{k})$ and a moving frame $(P, \mathbf{u}, \mathbf{v}, \mathbf{w})$

attached to the MP, as shown in Fig. 11. The rotation matrix, from the moving frame to the fixed frame is defined by a rotation angle α around the fixed axis $\mathbf{i}$ and by a rotation angle β around the mobile axis $\mathbf{v}$, see equation (1):

$$ {}^{\mathbf{0}}_{\mathbf{1}}\mathbf{R} = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ \sin\alpha\cdot\sin\beta & \cos\alpha & -\sin\alpha\cdot\cos\beta \\ -\cos\alpha\cdot\sin\beta & \sin\alpha & \cos\alpha\cdot\cos\beta \end{bmatrix} \tag{9} $$

The loop-closure position equations for limbs 1, 2 and 3 are:

$$ \mathbf{p} = \mathbf{f}_i + \mathbf{l}_i + \mathbf{t}_i + \mathbf{s}_i \quad i = 1,2,3 \tag{10} $$

where, $\mathbf{p} = \begin{bmatrix} P_x & P_y & P_z \end{bmatrix}^T$ defines the position vector of the reference point P, $\mathbf{f}_i = \begin{bmatrix} 0 & f_{iy} & f_{iz} \end{bmatrix}^T$ is the fixed position of the reference point on the linear table i, input values are defined by vector $\mathbf{l}_i = l_i \cdot \mathbf{d}_i$ where $\mathbf{d}_i$ is the unit vector in the direction of the linear table and l_i the value of the input displacement, $\mathbf{t}_i$ defines the passive stroke of the limb i, and finally, the vectors $\mathbf{s}_1 = \begin{bmatrix} s_{1x} & s_{1y} & s_{1z} \end{bmatrix}^T$, $\mathbf{s}_2 = \begin{bmatrix} s_{2x} & s_{2y} & s_{2z} \end{bmatrix}^T$ and $\mathbf{s}_3 = \begin{bmatrix} s_{3x} & s_{3y} & s_{3z} \end{bmatrix}^T$ define vectors P_iP on the rigid body of the lower level.

For limb 1, eq (10) yields:

$$ P_x = f_{1x} + l_1 + s_{1x} \tag{11} $$

For limb 2:

$$ P_y = f_{2y} + l_2 + s_{2y} \tag{12} $$

and for limb 3:

$$ P_z = f_{3z} + l_3 + s_{3z} \tag{13} $$

It can be seen that as we are working with limbs defined for input-output translational manipulators, decoupled and linear equations for the X, Y and Z positioning of P are obtained.

The loop-closure equations for limbs 4 and 5 are:

$$\mathbf{p} = \mathbf{f}_i + \mathbf{l}_i + \mathbf{t}_i + \mathbf{s}_i = \mathbf{f}_i + \mathbf{l}_i + \mathbf{t}_i + {}^{\mathbf{0}}_{\mathbf{1}}\mathbf{R}^{\mathbf{1}}\mathbf{s}_i \quad i = 4, 5 \tag{14}$$

where the vector ${}^{\mathbf{1}}\mathbf{s}_i = \begin{bmatrix} s_{ix} & s_{iy} & s_{iz} \end{bmatrix}^T$ is best expressed in the moving frame, and to reference it to the fixed frame, it is necessary to premultiply it by the rotation matrix. Then, similar equations to the ones presented in section 4 are obtained.

For limb 4, as G_4 should be placed in the mobile axis $\mathbf{v}$, $\mathbf{s}_4 = \begin{bmatrix} 0 & s_{4v} & 0 \end{bmatrix}^T$ so ${}^{\mathbf{0}}_{\mathbf{1}}\mathbf{R}^{\mathbf{1}}\mathbf{s}_4 = \begin{bmatrix} 0 & s_{4v} \cdot cos\alpha & s_{4v} \cdot sin\alpha \end{bmatrix}^T$ and, eq (14) yields,

$$P_y = f_{4y} + l_{4y} + s_{4v} \cdot cos\alpha \tag{15}$$

and for limb 5, as G_5 should be placed in the $\mathbf{w}$ axis, $\mathbf{s}_5 = \begin{bmatrix} 0 & 0 & s_{5w} \end{bmatrix}^T$ so ${}^{\mathbf{0}}_{\mathbf{1}}\mathbf{R}^{\mathbf{1}}\mathbf{s}_5 = \begin{bmatrix} s_{5w} \cdot sin\beta & -s_{5w} \cdot sin\alpha \cdot cos\beta & s_{5w} \cdot cos\alpha \cdot cos\beta \end{bmatrix}^T$ and, eq (14) yields:

$$P_x = f_{5x} + l_{5x} + s_{5w} \cdot sin\beta \tag{16}$$

To calculate the inputs in terms of the output pose of the MP, we have to obtain the values of the actuators l_i from equations (11), (12), (13), (15) and (16).

To express the output coordinates in terms of the input values, P_x, P_y and P_z are directly defined in equations (11), (12) and (13) and α and β are obtained from equations (15) and (16).

Once both the inverse and direct kinematics are solved, it is important to highlight the form of the equations that have been got. Besides being linear equations, P_x, P_y and P_z coordinates are decoupled and directly defined by the corresponding inputs l_1, l_2 and l_3. The other motions are partially decoupled, α depends on l_2 and l_4 while β depends on l_1 and l_5. As all the motions are decoupled or partially decoupled, the relation between the position of the platforms and the inputs is very intuitive.

6. Conclusions

In this paper, the synthesis of two-level manipulators with five degrees of freedom has been deeply studied. The idea of distributing the control of the degrees of freedom among different levels is proposed in contrast to the traditional approach of controlling all the motions from just one level. In this way, for manipulators with three translations in the lower level and two rotations in the upper level, different options to distribute the control of the motions have been analyzed.

For each of the control distribution options presented, the orientations of different actuators in both levels have been analyzed. The coupling of the motions has been studied for each of the proposed manipulators and several manipulators of five degrees of freedom with decoupled rotations have been obtained.

Any of the proposed manipulators, transmits the inputs to the mobile platform in a robust way, obtaining a homogeneous distribution of efforts that provides good stiffness and accuracy to the mechanism. Besides, these designs have the advantage of an intuitive understanding of the manipulator's operation that makes it easier for the technician to know how the machine behaves.

Acknowledgment

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[1] Huang Z., Li Q.C., C., 2002. "General methodology for type synthesis of symmetrical lower-mobility parallel manipulators and several novel manipulators". *International Journal of Robotics Research,* **21** (2), pp. 145–190.

[2] Joshi S.A., Tsai L. W., C., 2002. "Jacobian analysis of limited-dof parallel manipulators". *Journal of Mechanical Design,* **124**, pp. 254–258.

[3] Shayya S., Krut S., Company O., Baradat C., Pierrot, F., 2014. "A novel (3T-2R) parallel mechanism with large operational workspace and rotational capability". *2014 IEEE International Conference on Robotics and Automation*, pp. 5712–5719.

[4] Piccin O., Bayle B., Maurin B., de Mathelin M., (2009). "Kinematic modeling of a 5-DOF parallel mechanism for semi-spherical workspace". *Mechanism and Machine Theory,* **44** (8), pp. 1485–1496.

[5] Caricato M. , 2005. "Fully-isotropic four degrees-of-freedom parallel mechanisms for Schoenflies motion". *International Journal of Robotics Research,* **24**, pp. 397–.414

[6] Kong X., Gosselin C., 2004. "Type synthesis of 3T1R 4-dof parallel manipulators based on screw theory". *IEEE Transactions on Robotics and Automation,* **20** (2), pp. 181–190.

[7] Zeng Dx., Huang Z., 2011. "Type synthesis of the rotational decoupled parallel mechanism based on screw theory". *Science China-Technological Sciences,* **54** (4), pp. 998–1004.

[8] Hervé J. M., 1978. “Analyse structurelle des mécanismes par groupe des déplacements”. *Mechanism and Machine Theory,* **13** (4), pp. 437–450.

[9] Li Q., Huang Z., Herve J. M. , 2004. “Type synthesis of 3R2T 5-DOF parallel mechanisms using the Lie group of displacements”. *IEEE Transactions on Robotics and Automation,* **20** , pp. 173–180.

[10] Altuzarra O., Sandru B., Pinto C., Petuya V., 2011. “A simmetric parallel Schonflies-motion manipulator for pick-and-place operations”. *Robotica,* **29**, pp. 853–862.

[11] Hernandez A., Ibarreche JI., Petuya, V. Altuzarra, O., 2014. “Structural Synthesis of 3-DoF Spatial Fully Parallel Manipulators”. *International Journal of Advanced Robotic Systems,* **11**.

[12] Gogu G., 2008. “Structural synthesis of parallel robots. Part 1: methodology”. *Springer , Dordrecht.*

[13] Gogu G., 2009. “Structural synthesis of parallel robots. Part 2: translational topologies with two and three degrees of freedom”. *Springer , Dordrecht.*

[14] Gogu G., 2010. “Parallel mechanisms with decoupled rotation of the moving platform in planar motion”. *Proceedings of the institution of mechanical engineers part C- Journal of mechanical engineering science,* **224**, pp. 709–720.

[15] Gogu G., 2011. “Maximally Regular T2R1-Type Parallel Manipulators With Bifurcated Spatial Motion”. *Journal of Mechanisms and Robotics,* **3**.

[16] Fang Y., Tsai L-W., 2002. "Structure synthesis of a class of 4-dof and 5-dof parallel manipulators with identical limb structures". *International Journal of Robotics Research,* **21** (9), pp. 799–810.

[17] Huang Z., Li Q. C. , 2002. "On the type synthesis of lower-mobility parallel manipulators". *Proceedings workshop on fundamental issues and future research directions for parallel mechanisms and manipulators*, Quebec-Canada, pp. 272–283.

[18] Kong X., Gosselin C., 2004. "Type synthesis of 3-dof spherical parallel manipulators based on screw theory". *Journal of Mechanical Design,* **126**, pp. 101–108.

[19] Zhu, S. J., Huang, Z., 2007. "Eighteen fully symmetrical 5-DoF 3R2T parallel manipulators with better actuating modes". *The International Journal of Advanced Manufacturing Technology,* **34** (3-4), pp. 406–412.

[20] Guo S., Fang YF., Qu HB., 2012. "Type synthesis of 4-DOF nonover-constrained parallel mechanisms based on screw theory". *Robotica,* **30**, pp. 31–37.

[21] Company O., Marquet F., Pierrot F., 2003. "A New High-Speed 4-DOF Parallel Robot Synthesis and Modeling Issues". *IEEE Transactions on Robotics and Automation,* **19** (3), pp. 411–420.

[22] Company O., Krut S., Pierrot F., 2006. "Singularity analysis of a class of lower mobility parallel manipulators with articulated traveling plate". *IEEE Transactions on Robotics,* **22** (1), pp. 1–11.

[23] Song YM., Gao H., Sun T, Dong G., Lian BB., Qi Y., 2014. "Kinematic analysis and optimal design of a novel 1T3R parallel manipulator with an articulated travelling plate". *Robotics and Computer-Integrated Manufacturing,* **30** (5), pp. 508–516.

[24] Gogu G., 2007. "Structural synthesis of fully-isotropic parallel robots with Schönflies motion via Theory of Linear Transformations and evolutionary morphology". *European Journal of Mechanics / A-Solids,* **26**, pp. 242–269.

[25] Gogu G., 2009. "Structural synthesis of maximally regular T3R2-type parallel robots via Teory of Linear Transformations and evolutionary morphology". *Robotica,* **27** (1), pp. 79–101.

[26] Kong X., Gosselin C.M., 2002. "Kinematics and Singularity Analysis of a Novel Type of 3-CRR 3-DOF Translational Parallel Manipulator". *International Journal of Robotics Research,* **21** (9), pp. 791–798.

[27] Kong X., Gosselin C.M., Foucault S., Bonev I., 2004. "A Fully Decoupled 3-dof Translational Parallel Mechanism". *Parallel Kinematic Machines International Conference, 2004*, pp. 595–610.

[28] Richard P.-L., Gosselin C.M., Kong X., 2006. "Kinematic Analysis and Prototyping of a Partially Decoupled 4-DOF 3T1R Parallel Manipulator". *Proceedings of the 2006 ASME Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, Philadelphia, USA.

[29] Kong X., Gosselin C.M., 2005. "Type Synthesis of 5-DOF Parallel Manipulators Based on Screw Theory". *Journal of Robotic Systems,* **22** (10), pp. 535–347.

[30] Kong X., Gosselin C., 2002. "Type synthesis of linear translational parallel manipulators". *Advances in Robot Kinematics*, pp. 453–462. Springer Netherlands

[31] Altuzarra O., Loizaga M., Petuya V., 2007. "Partially decoupled parallel manipulators based on multiple platforms". *Proceedings of the IFToMM World Congress*, France.

[32] Altuzarra O., Loizaga M., Pinto C., Petuya V., 2010. "Synthesis of partially decoupled multi-level manipulators with lower mobility". *Mechanism and Machine Theory,* **45**, pp. 106–118.

[33] Rolland L., Chandra R., 2014. "The forward kinematics of the 6-6 parallel manipulator using an evolutionary algorithm based on generalized generation gap with parent-centric crossover". *Robotica,* **32** (6), pp. 1–22.

[34] Chandra R., Rolland L., 2015. "Global-local population memetic algorithm for solving the foward kinematics of parallel manipulators". *Connection Science,* **27** (1), pp. 22–39.

[35] Kong X., Gosselin C., 2004. "Cartesian parallel manipulators". *United States Patent 6729202*