

# ON ELECTRONS AND REFERENCE<sup>1</sup>

W. BALZER and G. ZOUBEK

## INTRODUCTION

The reference of individual terms like "electron" provides a stumbling block for any theory of reference in science -not to speak of reference in general. In addition, the particular term "electron" is of special interest because it "lived through" at least four clearly differentiable theories (up to now), and its denotations accordingly show a great deal of variation. The philosophical question of why one and the same term is used with such different denotations, or, more simply, what constitutes the "real" referents of the term, or what "really are electrons", will only serve here as an overall problem in order to direct more specific investigations. The more mundane problem we tackle here is simply to get clear about how to characterize the referents of individual terms in scientific theories. More recently, this problem was discussed mainly by Kripke and Putnam<sup>2</sup> in connection with the causal "theory" of reference, the two main examples being "H<sub>2</sub>O" and "electron". In our project<sup>3</sup> both examples were studied in detail.

In this paper we will first describe the physical meaning of "electron" (Sec. I), then provide some theoretical background (Sec. II) relative to which the term can be defined (Sec. III). After that, we will offer a general account of the reference of definable terms in scientific theories (Sec. IV) which on the one hand is exemplified by our case studies of "electron" and "H<sub>2</sub>O"<sup>3</sup> and on the other hand generalizes previous structuralist attempts at reference<sup>4</sup>. Finally, we will address the question of the impact of determination on reference (Sec. V).

As a background theory to which the reference of "electron" is relativized we take wave mechanics, i.e. Schrödinger's theory<sup>5</sup> which preceded "classical" quantum mechanics in the forms introduced by von Neumann. By not sticking to the most modern theory (which would

be theory of elementary particles in our case) we admit for an explicit relativization, namely to one or more historically given theories. We do not believe, however, that this commits us to take party in the philosophical controversy on realism versus idealism. On the contrary, we think that we provide some (admittedly very small) preliminary step to substantiate his controversy. Our particular choice of a theory is mainly motivated by our aim of studying some substantial predecessor of quantum mechanics so that the transition to the latter may be investigated in the next step.

### I ELECTRONS IN PHYSICS

In physical literature electrons are characterized in various ways, and usually it is the conjunction of all or most of these characterizations which is held to provide a complete "definition". A first feature is that electrons are "parts" of atoms: particle-or wave-like, and with different models of the atom. Second, electrons have certain properties: they are endowed with masses and electrical charges of particular magnitudes, and they also have spin ( $=h/4\pi$ ). Third, electrons are characterized by their "behaviour" in certain well-specified situations like experiments with cathode rays in external fields (J.H. Thomson), scattering on material atoms (P. Lenard, Franck-Hertz), diffraction of waves of electrons (Davidsson, Germer, G.P. Thomson) and Stern-Gerlach-experiments. Often classical experiments (that is, experiments which provide intended applications of a classical theory, like mechanics or classical electrodynamics) like the determination of the ratio "electrical charge/mass" provide prototypes for the situations in which electrons show their distinctive behaviour.

Here is a typical citation<sup>6</sup>: "Electron: An elementary particle which is the negatively charged constituent of ordinary matter. The electron is the lightest known particle which possesses an electric charge. Its rest mass is  $m_e = 9.1 \times 10^{-28}$ g, about 1/1836 of the mass of the proton or neutron, which are, respectively, the positively charged and neutral constituents of ordinary matter. Discovered in 1895 by Sir J.J. Thomson in the form of cathode rays, the electron was the first elementary particle to be identified".

Another ingredient of the meaning of "electron" consists of the historical developments in which electrons were found or had some essential role to play.

## ON ELECTRONS AND REFERENCE

As is well known, electrons were first observed in cathode rays by J. Plücker (1857), but discovered and identified by J.J. Thomson (1897) 40 years later! It was a long run from the first observation of cathode phenomena until the identification of these phenomena as particle like cathode rays, caused by the "smallest" atoms of electricity which possess a certain fixed electrical charge and fixed mass. The name "electron" was introduced for electricity atoms by George J. Stoney in the 1870th. After his discovery of the electron J.J. Thomson postulated his famous model of the atom which consisted of electrons distributed over a homogeneous positively charged sphere such that the whole system was electrically neutral.

Later on Thomson's model was replaced by that of E. Rutherford (1911), stimulated by the  $\alpha$ -ray experiments of H. Geiger and E. Marsden (1913). Instead of a homogeneous sphere one has a small positive nucleus and a hull of pointlike negative electrons.

This model of an atom was the starting point for N. Bohr's model as well as for E. Schrödinger's atom. In order to make Rutherford's atom stable, N. Bohr (1913) introduced his famous quantum postulates (among others, the quantisation of angular momentum and the postulate for frequencies) whereas E. Schrödinger (1926) (stimulated by the work of L. de Broglie (1924/25)) could derive all that in a "natural way" from his wave-view of electrons, as a consequence of his wave equation for the H-electron. Schrödinger's treatment of the H-atom was a paradigm for subsequent quantum-mechanical frame-theories of atoms and electrons<sup>7</sup>.

From these brief remarks it is quite clear that theoretical pictures play an important role in the characterization of electrons: Bohr's and Rutherford's models of the atom, and later on Schrödinger's account of quantum mechanics. Also, electrodynamics and mechanics are essential for they provide the background for the understanding of diffraction and scattering experiments.

We will concentrate on one such theoretical picture, that is, on one theory in the following, and see how far the referents of the term "electron" can be specified with respect to this theory.

## II WAVE MECHANICS

We adopt the structuralist view of theories, according to which an empirical theory has the form of a *net of theory-elements* all of which are *specializations* of one common *basic element*<sup>8</sup>. Roughly, the basic element represents the basic laws valid in all applications while each specialization represents some special law which only holds in some restricted, "special" domain of applications. We will not present the full theory-net of wave mechanics here<sup>9</sup> but concentrate on the basic element which is essentially given by Schrödinger's equation, plus one specialization to cover what we call Coulomb-Lorentz systems.

Wave mechanics (WM) provides a theoretical picture for systems like Rutherford-atoms, free particles, phenomena of electrical resistance and flow in metals and semi-conductors, phenomena in crystals and of capacity of heat. These systems in a first step are conceived of as systems of small moving particles which are endowed with mass and charge. In a second step this picture is refined in the following way: Each state of the Hamiltonian phase-space is replaced by a wave in the configuration space of coordinates, the coordinates of the particles systems by a density function  $\rho$  (in Schrödinger's view a weight function for classical states) while mass and electrical charge are kept in their original form. Intuitively, at each instant  $t$  and for each particle  $a$ , if we integrate the "part" of  $\rho$  in configuration space concerning all particles  $a' \neq a$  we obtain the "part" of  $\rho$  which corresponds to particle  $a$ . The form and the development over time of these waves is theoretically governed by Schrödinger's equation which uses two "theoretical" concepts. One is the so-called *state-vector* ( $\tau$ ) which describes the state of the wave at a certain fixed instant. The second theoretical term is the *wave-operator* ( $W$ ) which has a role analogous to the force function in classical mechanics<sup>10</sup>.

DI a)  $x$  is a *model of wave mechanics* ( $x \in M$ ) iff there exist  $n$ ,  $P$ ,  $T$ ,  $\kappa$ ,  $\rho$ ,  $m$ ,  $K$ ,  $\Psi$ ,  $W$  such that

$$x = \langle P; T, \mathbb{R}, \mathbb{R}^+, \mathbb{N}_n; \kappa, \rho, m, K, \Psi, W \rangle$$

and 1)  $P$  is a finite non-empty set,  $||P|| = n$ ,

2)  $T \subseteq \mathbb{R}$  is a real interval,

3)  $\kappa: \mathbb{N}_n \rightarrow P$  is bijective,

ON ELECTRONS AND REFERENCE

- 4)  $\rho: \mathbb{R}^{3n} \times T \rightarrow \mathbb{R}$
- 5)  $m: P \rightarrow \mathbb{R}^+$ ,
- 6)  $K \in \mathbb{R}^+$ ,
- 7)  $\psi = \mathbb{R}^{3n} \times T \rightarrow \mathbb{C}$  is such that certain requirements of continuity and differentiability are satisfied<sup>11</sup> and for all  $t \in T$ :

$$\psi_t \in L^2(\mathbb{R}^{3n}, \mathbb{C}) \text{ and } \int |\psi_t(q)|^2 dq \neq 0$$

- 8)  $W$  is a function assigning to each  $t \in T$  a symmetric operator  $W(t)$  in the Hilbert space  $L^2(\mathbb{R}^{3n}, \mathbb{C})$ <sup>12</sup>
- 9)  $\rho = |\psi|^2$
- 10) for all  $t \in T$ :  $W(t)$  is defined for  $\psi_t$
- 11) for all  $t \in \dot{T}$

$$\begin{aligned} \sqrt{-1}K(D_{3n+1}\psi)_t &= \\ &= -\sum_{j \in \mathbb{N}} K^2/2m(\kappa(j)) (D_{3j-2}^2 + D_{3j-1}^2 + D_{3j}^2) \psi_t + W(t)\psi_t \end{aligned}$$

- b)  $x$  in a potential model of wave mechanics ( $x \in M_p$ ) iff  $x$  is like in a) and requirements a1) - a8) are satisfied

Here,  $P$  denotes the finite set of "particles" occurring in the system. By the bijective map  $\kappa$  each particle is denoted by a natural number. This yields a natural ordering of the particles needed in expressions of the form  $\psi(q_1, \dots, q_{3n}, t)$ .

The problem of whether elements of  $P$  denote *individual* "particles" (of whatever nature) rather than *types* of particles leads us right into the foundations of quantum mechanics and cannot be decided here. In Sec. IV we will find an argument in favour of the "types of particles"-interpretation. But we do not feel convinced by this single argument<sup>13</sup>. Therefore, we simply leave the issue in its double-faced state-typical for quantum mechanics.  $T$  represents the time interval during which the system is considered. The density  $\rho$  in Schrödinger's *classical view* is a weight function for the intensity of possible classical states. In particular, for  $n=1$ ,  $e\rho$  should be the charge density of the particle, if  $e$  is its electrical charge<sup>14</sup>. But in the final, commonly accepted *statistical interpretation* due to M. Born (cf. e.g. M. Jammer (1966), p. 281 ff.) the normalised density  $\rho^1 := \rho / \int \rho(q, t) dq$  is a probability density related to de particles' positions  $q = (q_1, \dots, q_{3n})$ .  $m$  is the mass function and  $m(\kappa(i))$  the mass of particle  $i$ .  $K$  represents Planck's constant divided by  $2\pi \cdot \tau$  is the state vector. In the wave picture relevant for wave

mechanics, at each instant  $t$ ,  $\tau_t$  roughly represents the spatial distribution of the "particles". A bit more precisely:  $\Psi_t$  carries all possible information which can be obtained about the quantum system by measurement at time  $t$ . The operator  $W$  is difficult to interpret directly: it clearly is a theoretical term. In most cases  $W(t)$  is the operator expressing multiplication by a coordinate dependent potential  $U$  ( $U: \mathbb{R}^{3n} \rightarrow \mathbb{R}$ ). With these interpretations, axioms 7, 9 and 10 of D1-a may be expressed more verbally. The central axiom D1-a11, Schrödinger's equation, roughly expresses the following.  $D_{3n+1}^\tau$  is the derivation of  $\tau$  with respect to time, i.e. the measure of change of  $\tau$ . The expression " $D_{3j-2}^2 + D_{3j-1}^2 + D_{3j}^2$ " designates the Laplace operator with respect to the coordinates  $q^{(j)} = (q_{3j-2}, q_{3j-1}, q_{3j})$  which in a certain "potential way" belong to particle  $\kappa(j)$  and are the "j-projections" out of the whole configuration  $q \in \mathbb{R}^{3n}$  of the particle system. The three components  $3j-2, 3j-1, 3j$  may be regarded just as making up the "position function" of particle  $\kappa(j)$ . This operator multiplied by  $(-\hbar^2 / 2m(\kappa(j)))$  and summed over all particle numbers  $j$  gives the "free part" of the whole wave operator (which is the sum of the free part and the interaction part  $W$ .) Then Schrödinger's equation says:

The application of the whole wave operator to the wave function equals the time derivative of  $\Psi$  multiplied by Planck's constant and the imaginary unit  $\sqrt{-1}$ . For  $n = 1$  in the usual scalar potential situation we have (with " $\frac{\hbar}{2\pi}$ " for " $\hbar$ ")

$$\sqrt{-1} \cdot \frac{\hbar}{2\pi} \cdot \frac{\partial \Psi}{\partial t} = -\left(\frac{\hbar}{2\pi}\right)^2 \left/ 2m \left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} \right) \Psi + U\Psi \right.$$

It is easily seen that the right hand operator comes from a classical Hamiltonian

$H(q_1, q_2, q_3, p_1, p_2, p_3) = (2m)^{-1} (p_1^2 + p_2^2 + p_3^2) + U(q_1, q_2, q_3)$  by the correspondence  $p_i \rightarrow \frac{\hbar}{2\pi\sqrt{-1}} \cdot D_i$ ,  $U \rightarrow M(U)$  (where  $M(U)$  is multiplication by  $U$ , and  $q, p$  are classical Cartesian positions and momenta). For further explanations we refer to (Zoubek, 1986).

D1-b yields a model theoretic statement of the "vocabulary" of WM plus its "grammar" and some further technical stipulations. Intuitively, potential models are those systems for which it is meaningful to ask whether they are models of WM or not.

## ON ELECTRONS AND REFERENCE

We introduce a set  $I$  of intended applications of WM, that is, real systems to which WM is intended to apply by assuming that each intended application consists of a substructure of some potential model which represents all the data known about the system. A substructure of structure  $x$  is a structure all of whose components are subsets of the components of  $x^{15}$ . We might consider the empirical claim of WM as stating that all intended applications of WM "are" (i.e. can be extended to) models of WM. But a claim of this form would neglect important features concerning relations or links between different models. The most convenient form of stating such links is in the form of *constraints*<sup>16</sup>, that is, by characterizing admissible combinations (=sets) of potential models. Four constraints are of special importance in WM.

D2 Let  $X$  be a set of potential models of WM.

- a)  $X$  satisfies the identity constraint<sup>17</sup> for  $K$  and  $m$  ( $X \in C_1$ ) iff for all  $x, y \in X$ :  $K^x = K^y$  and (for all  $a \in P^x \cap P^y$ :  $m^x(a) = m^y(a)$ )
- b)  $X$  is bounded ( $X \in C_2$ ) iff there is some  $l \in \mathbb{N}$  such that  $|| \bigcup \{P^x / x \in X\} || \leq l$
- c)  $X$  satisfies the full mass interpretation constraint ( $X \in C_3$ ) iff  $X \in C_2$  and each mass value occurring in members of  $X$  also is realized in an intended application<sup>18</sup>.
- d)  $C := C_1 \cap \dots \cap C_3$  is called the global constraint of WM
- e)  $WM := \langle M_p, M, C, I \rangle$  is the theory-element of wave mechanics with  $I$  as a set of substructures of potential models representing the intended applications.

An empirical claim to the extent that the set of the intended applications can be extended to a set  $X$  of models of WM which also satisfies the global constraint  $C$  comes much closer to what actually is claimed with WM about reality. Such a claim comprises the additional statements that Planck's constant is the same in all applications (which is to be expected from a universal constant), that the same (type of) particle has identical mass in different systems, that there are only finitely many different (types of) particles, and that no reference to purely theoretical particles is needed. In  $C_3$ , a particle is treated as purely theoretical if it has a mass value different from those of all "real" particles, i.e. those

occurring in intended applications. Note that in  $X \in C_2$  there always will be particles with minimal mass values.

While Schrödinger's equation is intended to hold in all intended applications of WM there are many other special laws obtained from special laws of mechanics by means of the correspondence principle<sup>19</sup>. We need just the most important one of these in the following<sup>20</sup>.

D3 a)  $x$  is a *Coulomb-Lorentz model* ( $x \in M^{CL}$ ) iff there exist  $n, P, T, \kappa, \dots, W, A_1, A_2, A_3, \Phi, e, c$ , such that  $x = \langle P, \dots, W, A_1, A_2, A_3, \Phi, e, c \rangle$  and

1)  $\langle P, \dots, W \rangle$  satisfies D1 - a-1 through 11

2) for  $i \in \{1, 2, 3\}$  :  $\Phi, A_i : \mathbb{R}^3 \times T \rightarrow \mathbb{R}$  are such that all partial derivatives of second order exist

3)  $e : P \rightarrow \mathbb{R}$

4)  $c \in \mathbb{R}_+$

5) for all  $t \in T$  and all  $q$  for which the right-hand side is defined:

$$-\sum_j k^2/2m(\kappa(j)) (D_{3j-2}^2 + D_{3j-1}^2 + D_{3j}^2) g + W(t)g =$$

$$\sum_{1 \leq j \leq n} \left[ 1/2m(\kappa(j)) \sum_{1 \leq i \leq 3} (k/| -1D_{3(j-1)+i} + e(\kappa(j))/c \right.$$

$$\left. \cdot M((A_i^{[j]})) + e(\kappa(j))M((\Phi^{[j]}))_t \right] g +$$

$$1/2 \sum_{1 \leq j, l \leq n, j \neq l} e(\kappa(j))e(\kappa(l)) R(j,l)^{-1} g$$

b)  $x$  is a potential Coulomb-Lorentz system ( $x \in M_P^{CL}$ ) iff  $x$  is as in a) and  $x$  satisfies 2) - 4) of part a) and  $\langle P_x, \dots, W_x \rangle \in M_P$

c) for  $x = \langle P, \dots, W, A_1, \dots, c \rangle \in M_P^{CL}$  we write  $\sigma(x) := \langle P, \dots, W \rangle$  and for  $X \in \text{Po}(M_P^{CL})$ ,  $\sigma^1(X) := \{\sigma(x)/x \in X\}$

d) for  $i = 1, 2, 3$  :  $C_i^{CL}$  is defined by  $X \in C_i^{CL}$  iff  $\sigma^1(X) \in C_i$

e) for  $i = 1, 2, 3$  :  $C_{3+i}^{CL}$  is defined by replacing "mass" by "charge", "m" by "e" and "K" by "c" in the definition of  $C_i$



ON ELECTRONS AND REFERENCE

- f)  $C^{CL} := C_1 \cap C_2 \cap \dots \cap C_6$  is the global constraint of CL
- g)  $CL := \langle M_p^{CL}, M^{CL}, C^{CL}, I^{CL} \rangle$  is the theory element of Coulomb-Lorentz systems with  $I^{CL}$  as a set of substructures of potential Coulomb-Lorentz systems representing intended applications.

The potential models of CL are obtained from those of WM by adding functions  $A_i, \Phi, e$  and a constant  $c$  which represents the vector-potential  $A = \langle A_1, A_2, A_3 \rangle$  the scalar-potential  $\Phi$ , the electrical charge function  $e$  ( $e(a)$  is particle  $a$ 's charge) and the velocity of light. The models satisfy and additional axiom D3 -a-5 which says that the whole wave operator (the sum  $W_0 + W$ ) is constructed out of its constituents  $A_i, \Phi, e, c$  in the very special manner due to the correspondence to classical CL-systems.

This axiom, together with D3 -a-1 and D1 -a-11, guarantees that  $e$  is an *electrical* charge as opposed, say, to a magnetic one. D3-c captures the relation between CL and WM: by cutting off the additional functions we pass over from CL to WM. Constraints  $C_i^{CL} (i=1,2,3)$  are just those of WM, and constraints  $C_i^{CL} (i=4, 5, 6)$  are the strict analoga to those of WM, but now required for charge instead mass and for the velocity of light  $c$  instead of the constant of action  $K$ . Typical intended applications of CL are particles created in cathode rays (electrons) interacting with homogeneous electromagnetic fields or a lightening atomgas in such fields (Stark-1 Zeeman effects). For further reference we note the following theorem.

T1 if  $X \in e(I)$  is a shorthand for "for each  $z \in I$  there is  $x \in X$  such that  $z$  is a substructure of " $x$ " and if for each  $z \in I^{CL}, \sigma(z) \in I$  then, for some suitable subset  $I^+ \subseteq I$ : if  $X \in Po(M^{CL}) \cap C^{CL} \cap e(I^{CL})$  then  $\sigma^1(X) \in Po(M) \cap C \cap e(I^+)$

Proof: Let  $X \in Po(M^{CL}) \cap C^{CL} \cap e(I^{CL})$ . by D3-a-1:  $\sigma^1(X) \in Po(M)$ , and by D3-d,  $\sigma^1(X) \in C$ . Let  $I^+ := \{ \sigma(z) / z \in I^{CL} \}$ . By assumption,  $I^+ \subseteq I$ . If  $w \in I^+$  then there is some  $z \in I^{CL}$  such that  $w = \sigma(z)$ , and from the premiss we obtain some  $x \in X$  such that  $z \subseteq x$ . But  $z \subseteq x$  implies  $\sigma(z) \subseteq \sigma(x)$ . So there is some  $v$ , namely  $v := \sigma(x) \in \sigma^1(X)$  such that  $w \subseteq v$ , that is  $\sigma^1(X) \in e(I^+)$  %

### III A THEORETICAL DEFINITION OF "ELECTRON"

We now can make precise some of the major items of Sec. I and thereby obtain a theoretical definition of "electron". It is clear that such a definition cannot make explicit all the subtleties of the meaning of the term which are also not exhausted by what was said in Sec. I. It is not very clear whether most of these subtleties nevertheless are logically contained in the definition.

Certainly, the property of having spin is not expressible in the vocabulary of WM. This inadequacy is neglected here deliberately for we focus on a given theory (WM) which existed long before spin effects were discovered. For other properties, like that referring to scattering experiments, the situation simply is not clear. We do not want to argue for the physical adequacy of our definition because this is a game without end. Instead, our main purpose is to use this definition for discussing questions of reference.

As a first step, one might try and define electrons relative to a given model of WM as those particles of the models which have minimal mass. However, it may be the case that a given model does not contain any "real" electrons in this sense. So this definition sometimes would characterize the wrong things. It is clear that we have to refer not to single models but to whole arrays of models, that is, to sets of models satisfying certain constraints. It is certainly necessary to require to validity of  $C_1$  (identity of mass values of identical particles in different systems), and also of  $C_2$  because otherwise there might be no minimal mass value. Suppose we define electrons relative to a given set  $X \in \text{Po}(M) \cap C_1 \cap C_2$  as those particles occurring in systems of  $X$  which have minimal mass ("minimal" now with respect to  $X$ ). Still this might produce unintended results for  $X$  might be some abstract set of purely formally (say set theoretically) defined entities such that the minimal "mass" value with respect to  $X$  still is different from the real mass value of electrons. This shows that we cannot remain at the purely formal level of definitions of models and constraints. In order to characterize electrons as those real entities we are after, some reference to real systems, i.e. to intended applications, is needed. It is most convenient to presuppose that the given set  $X$  of systems satisfies constraint  $C_3$ , that is, all, and in particular the minimal mass values occurring in systems of  $X$  are values for real particles which

## ON ELECTRONS AND REFERENCE

occur in some intended application. Note that by assuming  $C_3$  we implicitly refer to  $I$ . Still, a further inadequacy might come about as follows. Eventhought  $X$  satisfies  $C_3$  and the therefore each mass value occurring in  $X$  is associated with a real particle it might be the case that  $X$  consists entirely of (extensions of) intended applications which do *not* contain electrons, and the therefore the minimal mass value in  $X$  would be larger than that for real electrons. In this case  $X$  would be a representation of some subset of intended applications only. It is clear how to exclude this possibility. We assume that  $X$  represents the *whole* array of intended applications in the sense that each intended application  $z \in I$  has an extension in  $X$ . Let us write " $X \in e(I)$ " for "for each  $z \in I$  there is  $x \in X$  such that  $x$  is an extension of  $z$ " (intuitively: " $X$  is a set of extension of  $I$ "), and assume that, for the set  $X$  considered,  $X \in e(I)$  holds.

The appropriate entity to which the definition of "electron" has to be relativized therefore is a set  $X$  of models satisfying constraints  $C_1, C_2$  and  $C_3$  and being an extension of  $I$ . Formally we may write  $X \in \text{Po}(M) \cap C \cap e(I)$  and we define electrons with respect to such an  $X$  as those particles which occur in some systems of  $X$  and have mass values minimal in  $X$ .

D4 if  $X \in \text{Po}(M) \cap C \cap e(I)$  then  $E$  is an electron with respect to  $X$  iff there is  $x \in X$  such that

- 1)  $E \in P^X$
- 2) for all  $y \in X$  and all  $E' \in P^Y$ :  $m^X(E) \leq m^Y(E')$

Obviously, the set  $\mathcal{E}(X)$  of all electrons with respect to  $X$  is a *mass-type* (with respect to  $X$ ), if we define a mass-type  $S$  with respect to  $X$  in general by the following condition. there is some  $\alpha \in \mathbb{R}$  such that for all  $a$ :  $a \in S$  iff there is  $x \in X$  such that  $a \in P^X$  and  $m^X(a) = \alpha$ . Though trivial, the following theorem shows that we have a definition proper. Moreover, the set of electrons with respect to  $X$  does not really depend on "purely theoretical parts" of  $X$  which go beyond  $I$ . That is  $\mathcal{E}(X)$  depends on  $M$  and  $I$  but not on special features of  $X$  (besides those expressed in  $C$ , of course).

T2 In each  $X \in \text{Po}(M) \cap C \cap e(I)$  the set of all electrons with respect to  $X$  is uniquely determined. Moreover, if  $X$  and  $X'$  are elements of  $\text{Po}(M) \cap C \cap e(I)$  then  $\mathcal{E}(X) = \mathcal{E}(X')$

Proof: Let  $E \in \mathcal{E}(X)$ , i.e.  $\exists x \in X (E \in P^X \wedge \forall y \in X \forall a \in P^Y (m^X(E) \leq m^Y(a)))$ . We have to show that  $\exists x' \in X' (E \in P^X \wedge \forall y' \in X' \forall a' \in P^{Y'} (m^{X'}(E) \leq m^{Y'}(a')))$ . By D2-c is  $z \in I$  such that  $E \in P^Z$  and  $m^X(E) = m^Z(E)$ . But  $X' \in e(I)$ , so there is  $x' \in X'$  such that  $z \subseteq x'$  which implies (1)  $m^{X'}(E) = m^Z(E) = m^X(E)$ , and in particular  $E \in P^{X'}$ . Now let  $y' \in X'$  and  $a' \in P^{Y'}$ . Since  $X' \in C_3$  there is  $z' \in I$  such that  $a' \in P^{Z'}$  and  $m^{Y'}(a') = m^{Z'}(a')$ . But  $X \in e(I)$ , so there is  $y \in X$  such that  $z' \subseteq y$  which implies that  $a' \in P^Y$ . By assumption this yields  $m^X(E) \leq m_Y(a')$ , so by (1),  $m^{X'}(E) \leq m^{Y'}(a') = m^{Y'}(a')$ . The converse implication is proved in the same way %

At this stage even the most modest physicist will protest and hold that this definition of "electron" is "false" because no reference at all is made to charge and to electric properties which after all are the essential ones by which electrons were first identified. Well, we can easily produce another definition which refers to minimal electric charge instead minimal mass. The only problem here is to bring the notion of electric charge into play, and we solve it by passing over from WM to CL. In models of CL electrical charge ( $e$ ) is a primitive. The definition of "electron" in CL then proceeds strictly analogous to that in WM. We have to assume analogous constraints  $C_4^{CL}$ ,  $C_5^{CL}$ ,  $C_6^{CL}$  which guarantee that the same particle has the same charge in different systems ( $C_4^{CL}$ ) that there are only finitely many different values for charge ( $C_5^{CL}$ ), and that all charge values are associated with real particles  $C_6^{CL}$ .

D5 If  $X \in \text{Po}(M^{CL}) \cap C^{CL} \cap e(I^{CL})$  then  $E$  is an electron with respect to  $X$  iff there is an  $x \in X$  such that 1)  $E \in P^X$  and  $e^X(E) < 0$  and 2) for all  $y \in X$  and all  $E' \in P^Y$ : if  $e^Y(E') < 0$  then  $e^Y(E') < e^X(E)$

We write  $\mathcal{E}^+(X)$  for the set of all electrons with respect to  $X$ .

T3 In each  $X \in \text{Po}(M^{CL}) \cap C^{CL} \cap e(I^{CL})$   $\mathcal{E}^+(X)$  is uniquely determined, and if  $X'$  is another element of  $\text{Po}(M^{CL}) \cap C^{CL} \cap e(I^{CL})$  then  $\mathcal{E}^+(X) = \mathcal{E}^+(X')$

Proof: Like the proof of T2 %

We now have two rather independent definitions of "electron", and there is some need for comparison. Let us first look at the situation very formally. For each  $X^+ \in \text{Po}(M^{CL}) \cap C^{CL} \cap e(I^{CL})$  the corresponding set  $\sigma^1(X^+) \in \text{Po}(M) \cap C$ , and under the assumption of T1 we proved

## ON ELECTRONS AND REFERENCE

even that  $\sigma^1(X^+) \in e(I^+)$  for some suitable subset  $I^+ \subseteq I$ . We now might formally ask whether, for given  $X^+ \in \text{Po}(M^{\text{CL}}) \cap C^{\text{CL}} \cap e(I^{\text{CL}})$ ,  $\mathcal{E}^+(X^+)$  is identical with  $\mathcal{E}(\sigma^1(X^+))$ . The answer is a clear "no". Without proof it is clear from looking at the places where "m" and "e" occur in the respective axioms that particles with minimal m values (in WM) need not have minimal e values (in CL), and vice versa. That is, both definitions are theoretically independent in the sense just indicated.

So let us look, secondly, at the empirical data. Here we find that in most cases particles with minimal mass are also minimally charged. There is, however, a class of exceptions (not to speak of exceptions transcending the frame of WM and CL): ions. Ions may have the charge of an electron but a much larger mass. Since ions fall in the domain of applications of WM and CL we have to state that both definitions differ also empirically. In the present frame the empirical relation between both definitions may be expressed by saying that, for  $X^+ \in \text{Po}(M^{\text{CL}}) \cap C^{\text{CL}} \cap e(I^{\text{CL}})$ :  $\mathcal{E}(\sigma^1(X^+)) \subsetneq \mathcal{E}^+(X^+)$ . That is, particles with minimal mass (in  $\sigma^1(X^+)$ ) form a proper subset of particles with minimal charge. In fact the case of ions shows that a definition merely in terms of charge would be too broad: it would cover unintended cases.

There are three possible reactions in situations like that before us. The first is to dismiss the whole idea of *defining* empirical concepts: science is so vague and ambiguous that any definition will turn out inadequate. The second alternative is to choose the most narrow definition and take it as *the* correct one; the third amounts to admit for a multiplicity of definitions. Each of these reactions faces difficulties. The first is at odds with the idea of a *theory* about science (for "theory" implies a certain amount of precision, idealization, and therefore definition). The second is exposed to the objections against constructivism and conventionalism. The third is in danger of inconsistency: if we have different non-equivalent definitions we cannot say that they define the *same* thing.

As far as physics is concerned it seems to us that physicists oscillate between alternatives one and three, generally feeling uneasy when pressed for definitions. In our view, alternative three is most fruitful for developing theories about science and for establishing a theory of scientific reference. We believe that it is the task of such a theory of reference to incorporate multiple definitions ("multiple reference", "theoretical overdetermination") into a comprehensive,

consistent picture. Of course, the *fact* that multiple reference occurs or that scientific concepts are overdetermined has been observed by many others.

The situation does not change when we pass over to reference. If we ask to what entities the term "electron" in theory T refers to a natural answer is "to those entities distinguished by the definition available in T". In the case of WM and along the lines of D4 we find that "electron" in WM refers to real particles with minimal mass values. This can be made precise as follows.

D6 E is an electron in WM iff there is some X such that

- 1)  $X \in \text{Po}(M) \cap C \cap e(I)$
- 2) E is an electron with respect to X

In others words, the set of electrons of WM,  $\mathcal{E}(WM)$ , is defined as the union of all electrons with respect to all sets  $X \in \text{Po}(M) \cap C \cap e(I)$ . The operation of taking the union here really is redundant for, by T2 all sets  $\mathcal{E}(X)$ ,  $\mathcal{E}(X')$  with  $X, X' \in \text{Po}(M) \cap C \cap e(I)$  are identical. If we define "E is an electron in CL" ( $E \in \mathcal{E}(CL)$ ) along the same lines then the empirical fact that electrons in WM form a proper subset of those in CL mentioned above means that the referents of the term "electron" in WM as a matter of fact also are referents of the term in CL but not conversely.

We may summarize these findings by stating that the referents of the term "electron" in WM and CL are given by elements of the different sets  $\mathcal{E}(WM)$  and  $\mathcal{E}(CL)$ , respectively. We reserve judgement as to whether this might represent an objection against the general approach towards reference pursued here. Certainly, this example and even the general notions of the following section are still far away from a general theory of reference, and only from the point of view if such a theory the evaluation of situations where common terms have different referents is in order. We believe that a good theory of reference will be able to deal with such problems.

#### IV REFERENCE OF SPECIFIED TERMS IN EMPIRICAL THEORIES

The examples considered above are typical for a whole class of cases, namely those where reference of an individual term (as contrasted to a relational one) is at stake for which there is a theoretical definition relative to a given theory.

From the present examples and the one of "H<sub>2</sub>O" in stoichiometry<sup>3</sup> we may easily abstract a general schema of how to characterize the referents of definable individual terms in empirical theories in general.

Such a general schema has been offered for relational terms<sup>21</sup>, and here is extended to definable terms in general. We start from the most basic and simple structuralist notion a *theory-element*<sup>22</sup>. A theory-element  $T = \langle M_p, M, C, I \rangle$  consists of a class  $M_p$  of *potential models*, a class  $M$  of *models* such that  $M \subseteq M_p$ , a *global constraint*  $C \subseteq \text{Po}(M_p)$  and a set of *intended applications* which is a subset of the class of all substructures of potential models. As before we write " $X \in e(I)$ " for expressing that set  $X (X \subseteq M_p)$  is an extension of  $I$ , i.e. for each  $z \in I$  there is some  $x \in X$  such that  $z$  is a substructure of  $x$  (and  $x$  an extension of  $z$ ). A simple empirical claim can be formulated with such a theory-element  $T$ , namely, that there is an extension  $X$  of the set of intended applications ( $X \in e(I)$ ) which is a set of models ( $X \in \text{Po}(M)$ ) and in addition satisfies the constraints ( $X \in C$ ):

there is  $X$  such that  $X \in \text{Po}(M) \cap C \cap e(I)$ .

The potential models (and therefore also the models) are assumed to be of the following general form:

$$\langle D_1, \dots, D_k; A_1, \dots, A_l; R_1, \dots, R_n \rangle$$

where  $k, l, n, \in \mathbb{N}$ ,  $k, n \geq 1$ ,  $D_1, \dots, D_k$  are sets of (non-mathematical) unspecified objects,  $A_1, \dots, A_l$  are sets of mathematical objects, and  $R_1, \dots, R_n$  are relations of given types  $\tau_1, \dots, \tau_n$  over the sets  $D_1, \dots, D_k, A_1, \dots, A_l$ . The types  $\tau_i$  are the same for all potential models, and may be of higher order. We further assume that all the items  $D_1, \dots, R_n$  can be construed as sets, and that potential models and models can be characterized by means of set theoretic statements.

Next we have to clarify the status of a new, non-primitive term in a given theory  $T$  (like "electron" in WM). The most natural way of looking at such new terms is to treat them as introduced by definition. But the notion of definition has to be used with caution in the context of empirical theories. Usually, such theories do not allow for a natural first-order formulation so that definability becomes a rather vague and controversial concept. Also, the big discussion about "definability" of theoretical terms in logical empiricism should be kept in mind. For these reasons we do not restrict ourselves to strict cases of definable

terms, and take a more general and -with respect to scientific theories- more realistic approach.

Still, the problem remains to say what we mean by a new, non-primitive "term" of  $T$  since we did not use any notions of a syntax or a language of  $T$ . It is most convenient to stick to this usage and introduce the items needed by means of set theoretic definitions (which only in the beginning look a bit unfamiliar). Let's say that for a theory  $T$  with models of the form stated above, and for each  $i \in \mathbb{N}$ :  $\bar{R}_i := \{R_i^X / X \text{ is a potential model of } T\}$  is the  $i$ -th term of  $T$ . If  $\tau$  is a  $(k+1)$ -type, i.e. some set theoretic construction scheme starting with  $k+1$  given sets, and forming successively cartesian products and power sets out of previously constructed sets, we say that  $\bar{t}$  is a new term of type  $\tau$  in  $T$  iff  $\bar{t} = \{y / \exists x(x \in M_p \wedge y \in \tau(D_1^X, \dots, D_k^X; A_1^X, \dots, A_l^X))\}$  where  $\tau(D_1^X, \dots, A_l^X)$  denotes the set constructed out of  $D_1^X, \dots, D_k^X, A_1^X, \dots, A_l^X$  according to the construction scheme given by  $\tau$ . We say that  $t$  is a new term of  $T$  iff there is some  $(k+1)$ -type  $\tau$  such that  $\bar{t}$  is a new term of type  $\tau$  in  $T$ . In the case of electrons in WM, for instance,  $\tau(D_1, \dots, D_k; A_1, \dots, A_l) = \tau(P; T, \mathbb{R}, \mathbb{R}^+, \mathbb{N}_n) = P$ , that is,  $\tau$  is just the first projection, and the term "electron" in the sense of our definition is just the set of all (types of) possible particles.

Now instead of requiring a new term  $\bar{t}$  to be definable we consider the case where  $\bar{t}$  is simply characterized by some set theoretic formula  $B$  which yields some (not necessarily unique) connection between  $\bar{t}$  and "the rest of"  $T$ . The development in Sec. III suggests that formula  $B$  should talk about the new term and, at least, an admissible combination  $X \in \text{Po}(M) \cap C \cap e(I)$ . For reasons not to be seen from the present example we also want to permit that  $B$  talks about some single, fixed model of  $T$ . A new term characterized by such a formula we call a new, specified term of  $T$ . We say that  $\bar{t}$  is a new term of  $T$  specified by  $B$  iff 1)  $\bar{t}$  is a new term of  $T$  and 2)  $B$  is a set theoretic formula containing no free variables other than  $t$ ,  $x$  and  $X$  such that

$$\forall t, x, X (B(t, x, X) \rightarrow (t \in \bar{t} \wedge x \in M_p \wedge X \subseteq M_p)).$$

In (Balzer, 1985) the referents of the  $i$ -th term  $\bar{R}_i$  of  $T$  were characterized as those relations  $R_i^+$  which are obtained as the union of all relations  $R_i$  occurring in the systems of a given set  $X$  of extensions. That is,  $R_i^+$  is a referent of  $\bar{R}_i$  in  $T$  iff there is some  $X$  such that



## ON ELECTRONS AND REFERENCE

1)  $X \in \text{Po}(M) \cap C \cap e(I)$ , 2)  $R_1^+ = \bigcup \{ R_1^x / x \in X \}$ .

We now generalize this account to include new, specified terms of T.

D8 If T is an empirical theory and  $\bar{t}$  a new term of T specified by B then t is a referent of  $\bar{t}$  (in T) iff

$$x \quad X(X \in \text{Po}(M) \cap C \cap e(I) \wedge x \in X \wedge B(t,x,X))$$

In other words, the referents of a new specified term  $\bar{t}$  are those correctly typified entities t which occur in some model x in some admissible set X of models, and which satisfy the characteristic formula B. Note that D8 covers new, specified terms in general and not only individual terms. In the latter case we may replace "entities" in the previous sentence by "objects". Note also, that the referents are characterized by means of "all of" T. They depend on the axioms proper of T which define the models, on the second order axioms which define the constraints, and also on the real systems to which T applies, the intended applications. Thus in concrete cases a whole lot of systems may go into the determination of a referent. In this sense our definition is clearly holistic. In the example of electrons (D6) formula B(t,x,X) would just be the formula expressing that E is an electron with respect to X, i.e. the *definiens* of D4. Note that in the case x is quantified so that B only has the form B(t,X).

Concerning the relation of D8 and the account for relational terms sketched above the situation is this. In the case of relational terms we have the referents built up as unions of relations occurring in an admissible set of models. Such unions do not occur in D8. Still, we may say that D8 generalizes the relational case for this union comes up with relational terms in general i.e. for defined relational terms as well). The union just serves to construct an "overall" referent which in ordinary language is often indicated by definite article. Besides this difference, D8 clearly is more general because it contains the additional characterization B which in general may define terms  $\bar{t}$  of arbitrarily complicated types.

In applying these notions to the example at hand we see at once that "electron", in fact, is an individual term. What is not immediately clear is whether the referents of this term are uniquely determined by D8 and by the formula B given in D4. The reason for this brings us

back to the question of interpretation of the sets  $P$  of "particles". It is easily shown by counterexample that the axioms of WM -as stated in D1 and D2- are not sufficient to guarantee uniqueness of "electron" in WM. Intuitively, in the "individual particles" interpretation a model of WM may contain many different electrons (i.e.  $E$ 's satisfying D6), and the same is true for admissible sets  $X \in \text{Po}(M) \cap C \cap e(I)$ . So D7-4 cannot be satisfied. It is only in the "type of particles" interpretation that intuitively we may have a situation in which type of electrons in models and admissible sets are uniquely determined by the minimal-mass requirement (i.e. by D6). This seems to yield an argument against the "individual particles" interpretation (as mentioned in Sec. II).

These intuitive considerations, however, do not automatically follow from our formulation of WM. In order to enforce uniqueness of "electron", and simultaneously the "type of particles" interpretation we have to add one further axiom to those of WM. We choose a formulation via mass: each mass value belongs to only one particle, not only in single models but also in admissible set of models. So the axiom is about many models and therefore takes the form of a constraint. Note that in an admissible set  $X \in C_1$  we can speak of mass function with respect to  $X$  defined by  $\cup\{m^X/x \in X\}$ .

D9  $X$  satisfies the uniqueness constraint for particles ( $X \in C_4$ ) iff  $\cup\{m_x/x \in X\}$  is injective

If we pass over from WM to  $UWM = \langle M_p^U, M^U, C^U, I^U \rangle = \langle M_p, M, C \cap C_4, I \rangle$  by simply adding  $C_4$  to the axioms of WM we obtain a theory in which the referents of "electron" are unique.

T4 There is at most one referent of the new term "electron" specified by formula B as given in D4 in UWM

Proof: Let  $E$  and  $E'$  be electrons in UWM, that is (1) there is

$X \in \text{Po}(M) \cap C^U \cap e(I)$  and  $x$  such that  $E \in P^X$  and for all  $v \in X$  and  $a \in P^V$ :  $m^X(E) \leq m^V(a)$  and (2) there is

$Y \in \text{Po}(M) \cap C^U \cap e(I)$  and  $y \in Y$  such that  $E' \in P^Y$  and for all  $w \in Y$  and  $a' \in P^W$ :  $m^Y(E') \leq m^W(a')$ . By D2-c and (2) there is  $u \in I$  such that  $E' \in P^u$  and (3)  $m^Y(E') = m^u(E')$ . But by (1):  $X \in e(I)$  and so for  $u \in I$  there is an extension  $x_1$  of  $u$  in  $X$ . By definition of extension this implies (4)  $m^{x_1}(E') = m^u(E')$ . Now take in

## ON ELECTRONS AND REFERENCE

(1)  $v:=x_1$  and  $a:=E'$ . Then, by (1),  $m^X(E) \leq m^{X_1}(E')=m^U(E')=m^Y(E')$  by (4) and (3), i.e. (5)  $m^X(E) \leq m^Y(E')$ . In the same way we infer from (1) and (2): (6)  $m^Y(E') \leq m^X(E)$ . From (5) and (6) we obtain  $m^X(E)=m^Y(E')$ , from (3) and (4)  $m^Y(E')=m^{X_1}(e')$ , so  $m^X(E)=m^{X_1}(E')$ . But  $x, x_1 \in X$  and  $x \in C_4$ , then yield  $E=E'$  %

T4 shows that D6 -at least under the "type of particles" interpretation- may be regarded as a definition of "electron" in WM.

Furthermore, by applying the general definition of referents in D8 we see that the referents of the term "electron" are those objects which occur in models  $x$  of admissible sets  $X \in \text{Po}(M) \cap C \cap e(I)$  of WM and which are electrons in the sense of D6. We spent some effort in Sec. III to produce an adequate definition of "electron" in WM. The specification of the referents of the term according to D8 will be as adequate as the previous definition D6. We note again the holistic features of this specification.

### V DETERMINATION AND REFERENCE

The present account of reference naturally raises a question about the role of measurement and determination in connection with reference. The most natural idea about reference, after all, is that a term's denotation is given ("defined") by the methods of measurement available for that term. So even if one does not subscribe to operationalism one may feel uneasy about our account which does not even mention feature of measurement.

We therefore want to finish by briefly considering where and how measurement is incorporated in our notions. Three points are of particular importance here, and we will discuss them in turn. Before this, however, some general remarks about measurement may be helpful. In structuralist terms measurement may be treated by means of the notions of a measuring model and of a method of measurement. A method of measurement for a fixed term simply may be represented by a class of models, each model having the additional property of being a measuring model. That is, in each model the function (or relation) which interprets the given term is uniquely determined (up to transformation of scale) by the other functions occurring in the model and by the axioms which characterize the whole class of models<sup>23</sup>. In a given theory, methods of measurement may be obtained as subclasses of

the class of models. Usually, the special laws associated with the theory yield methods of measurement if further *and hoc* requirements are added (restricting the numbers of objects, the geometrical or kinematical configurations, or the impact of certain parameters or arguments). Such methods usually serve to determine the theory's "theoretical" terms (if a distinction can be drawn between theoretical and non-theoretical terms in the theory at all). The non-theoretical terms (if there are any) usually are determined via other, presupposed theories, so that corresponding methods of measurement are classes of models of those other theories. Hooke's law which is a special law mechanics, for instance, yields a method for measuring forces which are mechanics-theoretical, whereas the law of cosine (regarded as a physical law about the relations among measuring rods and light rays) which is a special law of physical geometry yields a method for measuring positions which are mechanics-non-theoretical.

#### 1) The Role of Measurement

On closer inspection of D8 above it should become clear now that a term's referent is indeed essentially dependent on measurement. For we may safely assume that among the measuring models for that term there are many which represent (are extensions of) real systems and which therefore are intended applications. In such measuring models the value(s) of the function (which interprets the term under consideration) is uniquely determined by values of other functions of the model. Since the model represents a real system we may say that it serves to determine the "real" value(s) of the function under consideration in terms of other (presupposed or predetermined) "real" values. These determined values then by means of the systems being represented by a proper model enter into the theoretical *Überbau*, namely, first, into the class of models, and from there via constraints are spread to other, connected models. So an admissible set  $X$  of models will be subject to severe restrictions concerning the range of values for the function in question: only those values will be admitted which occur in the measuring models for that function. And by means of our definition these restrictions hold for the referent of the corresponding term, too.

To put it differently, a term's referent is restricted by (real or possible) measurements in the following way. In order to be "part" or "element"

## ON ELECTRONS AND REFERENCE

of referent, an entity has to occur in a system  $x$  which is an extension of some real system, some intended application (because of the part " $X \in e(I)$ " in D8). But  $x$  very likely is linked via constraints with some measuring model  $y$  for the term under consideration which on the one hand is an intended application, and on the other hand serves to determine just the original entity. So if a corresponding measurement was made in systems  $y$  the precise values(s) of the original entity in  $x$  would be obtained. In this way real or possible measurement serve to fix at least essential part of the term's referent.

### 2 Underlying Theories

This does not hold however, for all the theory's terms, for not every theory has measuring models for all of its terms. If a theory  $T$  does not provide measuring models for term  $t$  then  $t$ 's referent will be fixed either in a purely theoretical way or it will be fixed *in practice* (contrary to the scheme underlying D8) with the help of measurements involving theories different from  $T$ , that is, theories underlying  $T$ . This situation is a familiar one in WM and in CL. Most of the classical experiments, like Millikan's, can hardly be construed as models of WM or CL. The most natural move rather is to see them as yielding (measuring) models of theories underlying WM and CL, like electrodynamics, classical mechanics or geometrical optics.

It would be desirable to have a concept of reference covering such situations. We admit that our present account does not do this. But we emphasize that this consequence was to be expected from our relativizing all considerations to one single theory. It is rather obvious how to proceed further: just substitute "theory  $T$ " in the present definitions by "theory net underlying  $T$ " plus some formalism to carry measured values from underlying theories to the one considered. This formalism already is at hand in the form of the concept of intertheoretical links<sup>24</sup>. In particular, it would be desirable to analyze concrete measurements like the one mentioned before, or determinations of the ratio  $m/e$  in such an extended conceptual frame and to work out the precise role they play in fixing a term's referent ("referent" now in an extended sense relative to a theory-net).

### 3) Multiple Determination

A final point in connection with determination is that, *as a rule*,

there is more than one method of measurement for each of a theory's terms. These methods can be applied independently of each other, and it is seen empirically that they yield (approximatively) identical results. Usually, this is what makes us believe in having picked out "real" features of the world to which we refer by the theory's terms.

We certainly are not the first to emphasize that such a multiplicity of possibilities of determinations for the same term is of central importance for saying that a term "really" refers.

### NOTES

- 1 This paper was written under DFG project Ba 678/3-1.
- 2 For instance (Kripke, 1972) and (Putnam, 1975).
- 3 See note 1). The case of "H<sub>2</sub>O" was treated by B. Lauth, see (Lauth, 1987).
- 4 See (Balzer, 1985), Sec. 23 and (Balzer, 1987).
- 5 See (Schrödinger, 1926).
- 6 (Encyclopedia, 1960), p. 501-2.
- 7 See (Anderson, 1964), (Jammer, 1966), (Owen, 1955) and (Teichmann, 1984) for historical accounts, as well as the original papers (Bohr, 1913), (Geiger and Marsden, 1913), Rutherford, 1911) and (Schrödinger, 1926).
- 8 See (Balzer, Moulines, Sneed, 1987), Chap. IV.
- 9 See (Zoubek, 1987).
- 10  $\mathbb{N}$ ,  $\mathbb{N}^+$ ,  $\mathbb{R}$ ,  $\mathbb{R}^+$ ,  $\mathbb{R}^{3n}$ ,  $\mathbb{C}$  denote the sets of natural numbers, of numbers  $\{1, 2, 3, \dots, n\}$ , of real numbers, positive real numbers, vectors of real numbers of length  $3n$ , and of complex numbers, respectively.  $\|X\|$  denotes the cardinality of set  $X$ .
- 11 See (Zoubek, 1987) for a detailed statement of these conditions. We denote by  $\tau_t$  the function defined by  $\tau_t(a) = \tau(a, t)$ .  $D_i \tau$  denotes the  $i$ -th partial derivative of  $\tau$ , and  $D_i^2 \tau$  the second partial derivative.  $\overset{\circ}{\tau}$  is the interior of  $\tau$ .
- 12 This Hilbert space consists of all functions  $\Psi: \mathbb{R}^{3n} \rightarrow \mathbb{C}$  for which  $|\Psi|^2$  is integrable (modulo identity up to the Lebesgue integral). By means of  $\langle \Psi | \tau \rangle := \int \Psi \tau^* d\lambda^{3n}$  a scalar product is defined in  $L^2(\mathbb{R}^{3n}, \mathbb{C})$  where  $\lambda^{3n}$  denotes the Lebesgue measure on  $\mathbb{R}^{3n}$ , and  $\tau^*$  the conjugate of  $\tau$ . These notions as well as the general properties of Hilbert spaces can be looked up, e.g., in (Achieser and Glasmann, 1968). A linear operator  $W$  in the Hilbert space  $H$  is just a linear, partial function  $W: H \rightarrow H$ . Such a  $W$  is called symmetric iff, for all  $\Psi, \tau \in \text{Dom}(W)$ ,  $\langle W\Psi, \tau \rangle = \langle \Psi, W\tau \rangle$ . We use modern mathematical terminology. In physics, such operators are sometimes called hermitian or even self-adjoint. Von Neumann uses the stronger notion of self-adjointness ("hyper-maximality") in the modern sense.
- 13 Note that Pauli's principle is not contained in the axioms of D1.

## ON ELECTRONS AND REFERENCE

- 14 See (Schrödinger, 1926), p. 109.
- 15 See (Balzer, 1983) for a precise definition.
- 16 See (Balzer, Moulines, Sneed, 1987) Chap. II.
- 17 We write  $m^x, K^x, P^x$  etc. for the components  $m, K, P$  etc. occurring in structure  $x$ , provided it is clear from the context which structure  $x$  is meant.
- 18 Formally:  $X \in C_3$  iff  $X \in C_2$  and all  $x \in X$  and  $a \in P^x$  there is  $z \in I$  such that  $a \in P^z$  and  $m^z(a) = m^x(a)$ .
- 19 See (Zoubek, 1986).
- 20 See (Zoubek, 1987) for details.
- 21 See (Balzer, 1985).
- 22 See (Balzer, Moulines, Sneed, 1987) Chap. II for details.
- 23 See (Balzer, 1985) and (Balzer, 1987a) for details.
- 24 Compare (Balzer, Moulines, Sneed, 1987), Chap. II.

### REFERENCES

- ACHIESER, L.I., and GLASMANN, 1968 (5[ ed.): *Theorie der linearen Operatoren im Hilbert-Raum*, Berlin.
- ANDERSON, D.L., 1964: *The Discovery of the Electron*, Princeton.
- BALZER, W., 1983: "Theorie and Measurement", *Erkenntnis* 19, p. 3-25.
- BALZER, W., 1985: *Theorie und Messung*, Berlin-Heidelberg etc.
- BALZER, W., 1987: "Reference and Development of Theories", invited paper, to appear in *Proceedings of the 11.th International Wittgenstein Symposium 1986*.
- BALZER, W. 1987a: "The structuralist View of Measurement: An Extension of Received Measurement Theories", to appear in: *Minnesota Studies in the Philosophy of Science*.
- BALZER, W., MOULINES, C.U., and SNEED, J.D., 1987: *An Architectonic for Science*, Dordrecht.
- BOHR, N., 1913: "On the Constitution of Molecules and Atoms I", *Philosophical Magazine* 26, pp. 476.
- ENCYCLOPEDIA, 1960: *McGraw-Hill Encyclopedia of Science and Technology Vol. 4*, New York-Chicago etc.
- GEIGER, H. and MARSDEN, E. 1913: "The Laws of Deflection of  $\alpha$  Parti-

## ON ELECTRONS AND REFERENCE

- cles Through Large Angles", *Philosophical Magazine* 25, p. 604-623.
- JAMMER, M. 1966: *The Conceptual Development of Quantum Mechanics*, New York etc.
- KRIPKE, S.A., 1972: "Naming and Necessity" in: D. Davidson and G. Harman (eds.), *Semantic and Natural Language*, Dordrecht, pp. 253.
- LAUTH, B., 1987: "Atombegriff, Stöchiometrie und Referenz", un-published manuscript for DFG project Ba 678/3-1
- OWEN, G.E., 1955: "The Discovery of the Electron", *Annals of Science* 11, p. 173-82.
- PUTNAM, H. 1975: "The Meaning of "Meaning", in H. Putnam: *Philosophical Papers Vol II*, Cambridge, Mass., pp. 215.
- RUTHERFORD, E., 1911: "The Scattering of  $\alpha$  and  $\beta$  Particles by Matter and the Structure of the Atom", *Philosophical Magazine* 21, p. 669-688.
- SCHRÖDINGER, E., 1926: "Quantisierung als Eigenwertproblem" (Erste und zweite Mitteilung), *Annalen der Physik* 79, pp. 361 and pp. 489
- TEICHMANN, J., 1984: "Kathodenstrahlen und Elektron", in: F. Fraunberger and J. Teichmann, *Das Experiment in der Physik*, Braunschweig.
- ZOUBEK, G. 1986: "Korrespondenzprinzipien der Quantentheorie, ihre Rekonstruktion und Bewertung im Rahmen des strukturalistischen Theoriekonzepts", to appear in: *Proceedings of the 11.th International Wittgenstein Symposium 1986*.
- ZOUBEK, G., 1987: "Zur Rekonstruktion der nichtrelativistischen Wellenmechanik", unpublished manuscript for DFG project Ba 678/3-1.

Universität München