LEIBNIZ'S LOGICAL SYSTEM OF 1686 - 1690

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ABSTRACT

Logical works of this period, beginning with **Generales Inquisitiones** and ending with the two dated pieces of 1 Aug. 1690 and 2 Aug. 1690, are read as a sustained effort, finally successful, to develop a set of axioms and an appropriate schema for the expression of categorical propositions faithful to traditional syllogistic. This same set of axioms is shown to be comprehensive of the propositional calculus of **Principia Mathematica**, providing that 'Some A is A' is not a **thesis** in an unrestricted sense. There is no indication in the works of this period that Leibniz understood just how significant is this logical system he developed. But it is undeniable that he held tenaciously to this particular set of axioms throughout the period, a set of axioms of great power.

The logical works of this period, in more or less chronological order[1], include:

(i) **Generales Inquisitiones de Analysi Notionum et Veritatum**, 1686. (**Opus.**, 356-99). Henceforth, GI.

(ii) **De Formae Logicae Comprobatione per Linearum Ductus.** (**Opus.**, 292-321). Henceforth, **Comprobatione.**

(iii) A sketch of a calculus, **Opus.**, 259-61.

(iv) A sketch of a calculus, **Opus.**, 261-64.

(v) **Principia Calculi rationalis.** (**Opus.**, 229-31).

(vi) **Primaria Calculi Logici Fundamenta**, 1 Aug. 1690 (**Opus.**, 232-37). Henceforth, **Primaria.**

(vii) **Fundamenta Calculi Logici**, 2 Aug. 1690 (**Opus.**, 421-23). Henceforth, **Fundamenta.**

Couturat notes the symbolism which caracterizes this series of logical essays: they employ upper case letters for **terms** (and, as we shall see, **propositions**), and they use **equality** (or 'equivalence') as their fundamental copula.[2] We note that they also exhibit a movement to an axiomatic system where, from a small set of axioms and rules of inference, the whole of traditional syllogistic, and the standard propositional calculus can be produced. This logical calculus is not without philosophical interest for these logical essays must be read as the formal side and consequence of Leibniz's philosophical thought expressed in the series of sort pieces prior to 1686 such as **Primae Veritatis** culminating in the principal philosophical work of the period, **Discourse on Metaphysics.** Moreover, that Leibniz produces in one calculus the whole of traditional syllogistic and the standar propositional calculus ought to be in some measure surprising, for we have grown accustomed to think of Aristotelian logic and the classical propositional calculus of **Principia Mathematica** as separate traditions. Where formerly, and with Leibniz too, all inference was at root syllogistic and all propositions understood in subject-predicate form, the twentieth century came to grant with Frege and Russell that propositional logic was primary. Thus it is said that those like Leibniz who insisted on construing every statement as subject-predicate in form were bound to fail in developing a logical calculus of any lasting significance.[3] Yet this triumph of the primacy of propositional logic in oue century will increasingly reveal itself as a kind of prejudice, an unquestioned orthodoxy for which grounds are never given, once it is fully acknowledged that propositional logic, predicate logic and even the logic of relations can just as fruithfully and faithfully be generated from "term logic" as all these from propositional logic.[4]

A. GENERALES INQUISITIONES (n. 171) to (n. 200)

I have examined elsewhere the argument of GI, both is philosophical content and its development of a logical calculus.[5] As noted there, Leibniz had in his possession, (n. 1) to (n. 106) of that work, a system faithfull to the syllogism. This further should be noted, that in (n. 1) to (n. 106), Leibniz moves freely back and forth, appealing to earlier sections as justification in proofs, but from (n. 107) to the end (n. 200), there is no proof appealing to principles in (n. 1) to (n. 106). In addition, there are in GI three sets of axioms after (n. 106), at (n.171) (n. 189) and (n. 198), and as we shall see, two of the sets are sufficient and the third nearly so, with an apt schema for categorical propositions and some modest interpolation, to produce the syllogistic. Leibniz struggles in the latter half of GI not to find the axioms for his calculus, but to find that apt schema in which categorical propositions are to be expressed. The schema we would argue must be more than formally correct, 'well-formed' as it were. not permit the deduction of the entire syllogistic; although its character as 'well-formed' is **sine qua none**, it must also preserve distinctions between necessary and contingent propositions, and also permit the extension of the system beyond the categorical syllogism ideally to every species of logical inference. GI does not hit upon the apt schema; instead it leaves us with three contenders to which subsequent works return.

Anticipating the results of the entire movement from the latter half of GI to **Primaria**, (2 Aug. 1690), let us lay down a set of theses all of which occur or can be proven in the earlier parts of GI (from n. 1 to n. 106), and from which, using Scheme III[6] as the expression for categorical propositions, the traditional is able to be derived. Then we shall not of these theses ocuur in the sets of axioms given in (n. 171), (n. 189) and (n. 198). It should be striking to the reader how frecuently Leibniz reproduces the critical elements of this set, why he is wedded to that particular set of axioms, and, therefore, what he is doing in these latter sections of GI.

AXIOMS OF TRADITIONAL SYLLOGISTIC (TS)

1. A = A	(n. 10)
2. AA = A	(n. 18)
3. Non non A = A	(n. 2)
4. (A = B) → (AC = BC)	Lemma 1
5. (A = AB) → (NonB = NonB nonA)	(n. 93, 94)
6. Non B = NonB non(AB)	Lemma 2
7. AB = BA	

Replacement Rule:

B = A or A = B, G(A) I- G(B/A) (n. 1-4)

Substitution Rule:

If G is a **thesis** "about any letters ...(G) is to be undestood of any number of other letters". (n. 26)

Rule of Regress (in two steps):

(1) A = AB nonB ⊢A is **false.** OR (n. 57)

A → B and A → -B ⊢ A is **false.**

(2) (A = B) is **false** ⊢ (A =/= B) (n. 5)

(A =/= B) is **false** ⊢ (A = B) "

SCHEME III:	Aab : A = B	Oab : A =/= AB
	Eab : A = A nonB	Iab: A =/= A nonB

Let us show, first of all, that set of theses is sufficient for the derivation of the traditional syllogism in Scheme III. To do this, we need only prove BARBARA and DARI (CELARENT and FERIO would follow, substituting nonB/B in these two), subalternation (whence follow BARBARI and CELARONT, and by **regress** the entire Second and Third Figures), and conversion rules, to generate the Fourth Figure. For this purpose we shall appeal to two Lemmas, proving Thesis 4 above as Lemma 1, and Thesis 6 as Lemma 2.[7]

LEMMA 1: A = B ⊢ AC = BC

1. A = B/Ergo, AC = BC
2. AC = AC Ax. 1
3. AC = BC 2, B/A by 1

LEMMA 2: ⊢ NonB = NonB non(AB)

1. /Ergo, NonB = NonB non(AB)
2. AB = AB Ax. 1
3. AB = ABB 2, Ax. 2
4. NonB = NonB non(AB) 3, AX. 5

It might be observed that Ax. 1 is redundant[8] but we shall continue to use it for convenience. Commutativity, AB = BA, which appears as an axiom in some sets, might also be thought redundant.[9] But now let us prove the required elements of the traditional syllogism.

CONVERSION **SIMP.** of Iab

(A =/= A nonB) = (B =/= B nonA)

1. A=/=A nonB/Ergo, B=/=B nonA
2. B=B nonA   Neg. conc.
3. A=/=Anon(B nonA)1,B nonA/A by2.
4. A=A non(B nonA)   Lemma 2
5. False: B=B nonA   3, 4
6. B=/=B nonA   5, Regress

And putting B/A; A/B, then

(B =/= B nonA) → (A =/= A nonB)

CONVERSION **SIMP.** of Eab

(A = A nonB) = (B = B nonA)

1. A=A nonB/Ergo, B=B nonA
2. B=/=B nonA   Neg. conc.
3. A=/=A nonB   2, Converse
4. False: B=/=B nonA 1, 3
5. B=B nonA   4, Regress

And, putting B/A, A/B, then

(B = B nonA) → (A = A nonB)

BARBARA

1. M = MP
2. S = SM/Ergo, S = SP
3. S = SMP   2, MP/M by 1.
4. S = SP   3, S/SM by 2.

And with nonP/P above, CELARENT.

DARII

1. M = MP
2. S=/=S nonM/Ergo, S=/=S nonP
3. S = S nonP   Neg. conc.
4. P = P nonS   3, Converse
5. MP = MP nonS   4, Lemma 1
6. M = M nonS   5, M/MP by 1
7. S = S nonM   6, Converse
8. False: S=S nonP 2, 7
9. S =/= S nonP   8, Regress

And with nonP/P above, FERIO.

SUBALTERNATION

1. A = AB/Ergo, A =/= A nonB
2. A = A nonB   Neg. conc.
3. A = AB nonB   2, AB/B by 1
4. False: A = A nonB   3, contrad.
5. A =/= A nonB   4, Regress

And with nonB/B and B/nonB, also **subalternation** of Eab to Oab.

Hence, BARBARI and CELARONT.

At (n. 169), after a change of pen and ink suggesting a return to the manuscript after an absence[10], Leibniz opens a new section of GI, (n. 169 to n. 183), laying down a schema for categorical propositions (Parkinson's Scheme VIII) which had already appeared in another form (n. 129), and explicitily in (n. 151), but without any apparatus of axioms. This expression **secundi adjecti**[11] preserves a relation to possibility and actuality, hence to necessity and contingency, which Leibniz requires if his calculus is to have universal applicability to all logical discourse. At (n. 171) he gives axioms which include all from our set TS except Thesis 5 and 6, the most critical omissions, and Thesis 7 which, while not mentioned explicity in GI, is used throughout. Although he does not show in this section of GI, indeed nowhere in GI, the equivalence of Schemes III and VIII, the one can be reduced to the other in simple transformations.[12] Thus, there is a route, albeit more torturous, to the entire traditional syllogism using Scheme VIII. But is this possible with the axioms at (n. 171)?

Following the axioms at (n. 171), Leibniz proves a number of theorems in (n. 172) to (n. 179). Then at (n. 180) he suggests a theorem which he does not prove:

"(n. 180) [A = non(AC)] → (A = nonC), namely if A is ENS."

This is a theorem which can be proved with the addition of Thesis 6.[13] What Leibniz seems unable to prove solely from the axioms he gives at (n. 171) - he says, *"This must be proved accurately"* - is easily proven with the addition of Thesis 6.

At (n. 189) he presents his second set of axioms, this time in lower-case letters to signify their applicability also to propositional logic, as the sith principle expresses directly:

Principles at (n. 189)

first, aa = a (from which follows a = a, and non a = non a)

second, non non a = a

third, The same term does not contain a and non a

[i.e. (b = a) = non(b = non a)]

fourth, (A contains 1) = (A = xl)

fifth, non a contain non(ab), i.e.

(1 contains a) = (non a contains non 1)

sixth, whatever is said of a term which contains a term can also be said of a proposition from which another proposition follows

seventh, whatever cannot be demonstrated from these principles does not follow formally |**vi formae**|.

Since this set of axioms has all the requisite elements of our TS, including Thesis 5, it is an axiom set adequate to the traditional syllogistic providing there is an appropiate schema (such as Scheme III) for categorical propositions. At (n. 190), there occurs a review of elements which have occurred earlier in GI, and a schema whose significance is not evident in GI - it is hardly examined there - but occurs again in a work which must be thought the logical sucessor to GI, **De Formae Comprobatione per Linearum Ductus.**[14] We identify it, following Parkinson, as Scheme IX: (Aab) A = YB; (Eab) A = Y nonB; (Iab) XA = YB; (Oab) XA = Y nonB. It might be thought regressive for Leibniz to return to a schema employing the 'indefinite' letters (X, Y), something which as early as (n. 87) he had tried to avoid. Assuredly from our knowledge that he already possessed in Scheme III, together with the axioms as they occur in, say, (n. 189) a system and language for categorical propositions adequate to traditional syllogistic, it is regressive. But this leads as to conclude that Leibniz himself did not realize that he has in hand what he had sought. What is wanting, we must assume, is not the set of axioms he requires, for he has repeated the set he laid out in (n. 171), he has now added the one element wanting in that former system, and will, as we shall see, lay out the whole set again in (n. 198).[15] Rather, it is the proper schema which eludes him. There are two matters which perturb him as the movement to various schemata in GI reveals. First, he finds the inequations of Scheme III bothersome; second, he would like to do without expressions involving the indefinite 'Y'. Although he would like to avoid both, the former is no barrier to a quite powerful calculus, something Leibniz himself only comes to recognize in the essays of August, 1690.

At (n. 197), in the final movement of GI, there is another schema useful in conceiving **propositions** as **terms**, hence in the extension of a calculus faithful to traditional syllogistic to other logical discourse.

We call it Scheme X, and draw the reader's attention to the equivalence of that schema and Scheme VIII (which he repeats in n. 199), both being schemata **secundi adjecti** and both easily transformed to the representation of categorical propositions in Boolean algebra:

Scheme X (n. 197) - Scheme VIII (n. 199)			Boolean expression
Aab:	A nonB false	(or nonENS)	A nonB = 0
Eab:	AB false	(or nonENS)	AB = 0
Iab:	AB true	(or ENS)	AB =/= 0
Oab:	A nonB true	(or ENS)	A non B =/= 0

These schemata **secundi adjecti** have the interest and promise of permitting the logical calculus to preserve relevant distinctions of necessity and contingency, and in addition to permit the extension of the system to logical discourse beyond the syllogism. As we shall see, he will return to a schema **secundi adjecti** in subsequent works. Here he tries them in one final formulation of his axiomatic system at (n. 198). As it turns out these axioms together with the expression of categorical propositions in Scheme VIII (or X) is adequate to the traditional syllogistic.

The axioms at (n. 198) state for the first time the Rule of Replacement of TS, and include Theses 2, 3, and implicity 7 (hence also Theses 1 and 4, provable from these). Although neither Thesis 5 nor Thesis 6 is stated explicitly, a form of the equivalence of Aab and its contrapositive, A(non b)(non a) is readily available in the formulae of Scheme VIII.[16] Leibniz mentions that the contradictory relations and conversion **simpliciter** of Eab and Iab are immediately evident. From the transformation effected above of Scheme VIII and Scheme III, it should be abundantly clear that the given axioms at (n. 198) and Scheme VIII, Leibniz has a system adequate to the traditional syllogism. It is therefore not sanguine to close the work as he does with these words, *"In these few propositions, therefore, the fundamentals of logical form are contained."*[17]

B. DE FORMAE LOGICAE COMPROBATIONE PER LINEARUM DUCTUS TO PRINCIPIA CALCULI RATIONALIS

The essays **De Formae Logicae Comprobatione** (ii) and **Principia**

Calculi Rationalis (v), together with the two fragments in **Opus.** 259-61 (iii) and 261-64 (iv) might be conveniently be brought together as clearly subsequent to GI, and earlier than the two essays of August, 1690, which shall be considered in the last part of this paper. These inverting writings explore elements of the system Leibniz developed in GI, sometimes in a kind of blindness to what had been accomplished there, suggesting again that he was not yet fully aware of what he had. So they review and refine, err in places, and, at least in (ii) and (v), explore different schemata for categorical propositions, but without real success. In them one finds confirmation of the movement we have already identified in GI, that, save for Thesis 5 (or 6), Leibniz was securely in possesion of an axiomatic set, and sought to find that apt schemata to complete the logical calculus. We do not attempt to date or order these writings in any rigorous way, but follow the generally accepted view that **De Formae Logicae Comprobatione** (ii) is nearer to GI, and **Principia Calculi Rationalis** (v) nearer to the dated fragmets of August, 1690.[18]

De Formae Logicae Comprobatione is a work in which Leibniz views the syllogism both from the side of extension and from the side of comprehension. For our purposes its chief interest is in its continuity with the argument of GI. After the treatment of syllogisms by means of lines and what are essentially 'Euler circles', he begins a treatment using a modification of Scheme IX, which it might be recalled was virtually the last schema formulated in GI. This central part of **De Formae Logicae Comprobatione** might be considered a complete study of Scheme IX, stretching over ten pages, from which it is supposed he learned something about the requirements of an apt schema. First he modifies Iab and Oab, expressing them as simply the negations of Eab and Aab. Difficulties occur with the moods DARII and FERIO, and after nothing the cause to himself, he dismisses the schema and looks instead to Scheme VIII. Examining immediately the syllogism BARBARA and noting that expressed in Scheme VIII the syllogism is not a form appropiate to a calculus, he writes, "*And also this expression is not apt.*" He returns to the equations of Scheme IX modifying Iab and Oab in another way, examines the whole again and again, modifying always very sistematically so far as is required by difficulties in rendering the schema adequate to all valid syllogisms. As a result, the schema becomes increasingly more complex and farther removed from the axiomatic system of GI. It is not insignificant that the problem underlying the work is the same problem with which GI ended, that is, how within a logic of coincidence to formulate categorical propositions faithful to syllogistic discourse. Here such fidelity to concrete subject matter of syllogisms leads him farther from the logical calculus, and back to the linear methods with which the work began. What ought to be clear is that modifications of Scheme IX, hence representations of categorical propositions using 'indefinites' has not been successful.

The two fragmets at **Opus.** 259-61 (iii) and 261-64 (iv) are similar in structure and content. Both begin with definitions of concepts Leibniz finds necessary to logical discourse and useful in the construction of an axiomatic system. In (iii) and (iv) these elements of an axiomatic set occur:

	(iii)	(iv)
Non nonA = A	(4)	(6)
AA = A	(14)	(7)
(A est B) = (A = AB)	(8)	(8)

From these few elemets, Leibniz proceeds to prove various theorems. The most interesting matters in these derivations of theorems are the unsuccessfulattempts at proving the **contrapositive** relation (Thesis 5 in TS), here in the form (A est B) = (NonB est nonA). At (17) in (iii), and again at (22) in (iv), he gives 'proofs' neither of which is valid. Little wonder! for we have already noted that either Thesis 5 or Thesis 6 must be taken as an axiom, and without one, the other cannot be derived from the remaining axioms of TS. These fragmets emphasize the importance of Thesis 5 and Thesis 6 for an axioms set adequate to traditional syllogistic, for without them conversion **simpliciter** cannot be proved, something Leibniz comes to recognize emphatically in the stidies of August 1690.

The short but virtually complete work **Principia Calculi Rationalis** is similar to the other works of this period which we have considered in as much as it is concerned with producing the syllogism from a system of axioms, indeed the axioms we have come to expect from GI (n. 171), (n. 189) and (n. 198), as well as the two fragmets above. But it differs from these, using as an axiom 'Some A is A' (expressed in a notation utlizing 'indefinites') instead of Thesis 5 or Thesis 6. The work uses this different schema for categorical propositions, differing from Scheme III in Iab and Oab.[19] We call it Scheme XI:

	Aab: A = AB (or 'A est B')
Scheme	Eab: A = A nonB (or 'A est nonB')
XI	Iab: QA = QAB (or 'QA est B')
	Oab: QA nonB (or 'QA est nonB')

In this schema he demonstrates (Ax. 1) the primary moods of the First Figure. Then demonstrating as a **thesis** QA = QAA ('Some A is A'), he proves **subalternation** through DARII and FERIO.[20] By **subalternation**, the secondary moods of the first figure, BARBARI and CELARONT, are demostrated from BARBARA and CELARENT. The 'Rule of Regress' is stated as an axiom (Ax. 5), and through **regress** the Second and Third Figures are derived from the First.

The procedure he follows in the work can be completed as sketched in **De Formis syllogismorum Mathematice definiendis,**[21] to produce the Fourth Figure. To reach that figure, **conversion** rules are proved from selected Second and Third Figure syllogisms, using as premise 'All A is A' or 'Some A is A', i.e.

(Eab.Abb) → Eba (CESARE): conversion **simp.** of Eab.
(Aaa.Aab) → Iba (DARAPTI): conversion **per accidens** of Aab.
(Eab.Ibb) → Oba (FESTINO): conversion **per accidens** of Eab.
(Aaa.Iab) → Iba (DATISI): conversion **simp.** of Iab.

BRAMANTIP is BARBARA with a converted conclusion, from which by **regress** CAMENOP and FELAPTON; CAMENES is CELARENT with a converted conclusion, from which by **regress** follow FRESISON and DIMARIS. Thus the whole syllogism can be proven through the axioms as given in **Principia Calculi Rationalis.**

C. THE COMPLETION OF THE LOGIC OF 1686 - 1690: PRIMARIA AND FUNDAMENTA

We finally arrive at those works in which Leibniz states without hesitancy the axioms of TS, and derives through them the elements of the syllogism using Scheme III. Recall that there must have been some confusion about Thesis 5 in GI, for although it is stated as an axiom in (n. 189), it does not occur in (n. 198), and the attempts to prove it in the sketches at **Opus.**, 259-61, and 261-64 fail. In the Preface to **Primaria**, Leibniz returns to the **contrapositive** relation and somehow grasps that conversion **simpliciter** of Eab (or Iab) is bound up with that relation. Indeed, if any one of these three is given, conversion **simpliciter** of Eab (or Iab), Thesis 5, Thesis 6, the others can be proven in Scheme III from the other theses of TS. It is hardly surprising to find Thesis 5 as one of the axioms of **Primaria**:

	Axioms of **Primaria**	Thesis in TS
(1)	A = B |it is the same as| A = B is true.	
(2)	A =/= B |it is the same as| A = B is false.	
(3)	A = A	(1)
(4)	A =/= B nonA	
(5)	A = non nonA	(3)
(6)	AA = A	(2)
(7)	AB = BA	(7)
(8)	(A = B) = (nonA = nonB) = (A non =/= B)	
(11)	(A = B) → (AC = BC), but not conversely	(4)
(12)	(A = AB) = (nonB = nonB nonA)	(5)

From these axioms and categorical propositions stated in Scheme III we have already shown that the whole syllogism follows. Note that Thesis 7 of TS is stated here (Ax. 7) for the first time, although it is used by Leibniz throughout the works of this period. Moreover, there are none among the axioms of **Primaria** that are not **theses** of TS.[22]

Fundamenta, the short piece written the day after **Primaria** where, he says, *"things are ordered better"*, confirms that Leibniz has grasped the significance of Thesis 5 and Thesis 6. Its axioms in general express the axioms of **Primaria** (hence of TS), with these exceptions: **Fundamenta** supresses **Prim.**(3), which as we know can be proved through the other axioms, and **Prim.**(4), which is prove above. **Fundamenta** adds, at (17) NonB = nonB non(AB), Thesis 6, and from it proves Thesis 5 at (19). All the elements of the axiom set TS are present therefore in **Fundamenta**. But there is more.

We can show that given the axioms of TS, which are wholly present in GI, (n. 189), in **Primaria** and **Fundamenta**, the axioms for the propositional calculus of **Principia Mathematica** can be demonstrated. [23] The axioms of PM, excluding the redundant Ax. 4, together with two appropiate definitions required for our proofs, are these:

AXIOMS OF PM	DEFNS.
1. (p v p) → p	1. (A v B) = non(nonA nonB)
2. q → (p v q)	2. (A → B) = (A = AB)
3. (p v q) → (q v p)	= (nonA v B)
4. [p v (q v r)] → [q v (p v r)]	= non(A nonB)
5. (q → r) → [(p v q) → (p v r)]	

The proofs of these axioms, using the theses of TS as well as proof procedures such as **conditional proof** (CP), are given as follows:

Axiom 1

1. /[non(nonp nonp)] → p		
2. non(nonp nonp)		ACP
3. non(non p)		2, Th.2
4. p		3, Th.3
5. non(nonp nonp)	p	2-4, CP
6. (p v p) → p		5, Defn. 1

Axiom 2

1./q = q non(nonp nonq)	
2.non(nonq)=non(nonq) non(nonp nonq)	nonp/A, nonq/B, Th.6
3.q = q non(nonp nonq)	2, Th.3
4.q → non(nonp nonq)	3, Defn. 2
5.q → (p v q)	4, Defn. 1

Axiom 3		Lemma 3 (for the proof of Ax. 5)	
1./non(nonp nonq) → non(nonq nonp)		1.(A = BC) → (A = AC)	
2.non(nonp nonq)	ACP	2. A = BC	ACP
3.non(nonq nonp)	2, Th.7	3. AA = ABC	2, Th.4
4.non(nonp nonq) → non(nonq nonp)		4. A = ABC	3, Th.2
	2-3, CP	5. A = BCBC	4, BC/A
5.(p v q) → (q v p)	5, Defn.1	6. A = BCC	5, Th.2
		7. A = AC	6, A/BC
		8. Lemma 3	2-7, CP

Axiom 5

1.	/ (q = qr) → [non(nonp nonq) → non(nonp nonr)]	
2.	q = qr/ non(nonp nonq) → non(nonp nonr)	
3.	non(nonp nonq)/ non(nonp nonr)	
4.	nonr = nonr nonq	2, Th.5
5.	nonp = nonp nonp q	3, Defn.2
6.	nonq = nonq p	5, Th.5
7.	nonr = nonr nonq p	4, nonq p/nonq by 6
8.	nonr = nonr p	7, Lemma3,nonr nonq/B, p/C
9.	non(nonp nonr)	8, Defn.2
10.	[3] → [9]	3-9, CP
11.	[2] → [10]	2-10, CP
12.	(q → r) → [(p v q) → (p v r)]	11, Defns. 1, 2

Clearly then the axioms TS, together with an appropiate proof procedure, are comprehensive of the propositional calculus of **Principia Mathematica**, and thus to whatever by extension the axioms of PM can generate. This same set of axioms TS can generate all the twenty-four syllogisms traditionally valid. Yet this produces a **conundrum** of sorts, since whatever can be proven from the axioms of PM is truth-functionally valid, but neither the weakened nor the streghtened syllogisms as stated in Scheme III are truth-functionally valid is their straightforward statement. For example, BARBARI, stated as

[(M = MP) . (S = SM)] → non(S = S nonP)

is not truth-functionally valid. This matter must be given more attention.

CONCLUDING REMARKS

In this paper we have discovered three axiom systems faithful to the syllogism: (1) the system of axioms in GI (n. 189) with Scheme VIII for categorical propositions; (2) the system of axioms in **Principia Calculi Rationalis** which uses a schema with 'indefinites' to express particular propositions (Scheme IX), and in that schema the thesis 'QA = QAA' (interpreted as 'Some A is A'); (3) the system of axioms TS together with Scheme III for categorical propositions, as found in **Primaria** and **Fundamenta.** We have also shown that in the third system the propositional calculus, and surely with it monadic predicate logic, can be derived. It remains for us to discuss the status of the first and second systems, especially regarding their possible extension to the propositional calculus.

The first system does not possess the ease of manipulation of a system of equations. The schema could, of course, be transformed into a Boolean algebraic schema, or alternatively it could be transformed into Scheme III via the transformation rules provided in section A of this paper. But what about Thesis 5 in that case? It is not required as an axiom if we use Scheme VIII, but it (or its derivate, Thesis 6) is required as an axiom for the derivation of the axioms of PM. Thus, because of the difficulty even of expressing Thesis 5 using Scheme VIII, because also the force of the inference is lost,[24] it is hardly worth the trouble of doing the required modifications to produce the whole in that first system. Leibniz exhorts us to abandon Scheme VIII for equations.[25]

The second system uses as an axiom 'QA = QAA'. How shall this be expressed in a system comprehensive of the propositional calculus? If without restrictions 'Some A is A' is allowed as a thesis, then consider how this might be expressed in Scheme III:

A =/= A nonA

In the propositional calculus, if (a =/= a nona) can be derived as a thesis, then since [a = (a =/= a nona)], 'a' itself is a thesis, and the axiomatic system is Post-**inconsistent**, inconsistent with respect to negation (since by similar reasoning 'nona' could also be derived), and absolutely inconsistent given the Rule of Regress, for every WFF of the system would then be derivable. Thus, a system which has an axiom without restriction (as is the case in this second system) 'Some A is A' is inconsistent on the face of it with the axioms of PM.

But what does this say about the third system? Clearly 'All A is A' (A = AA) is a **thesis** in this system and from (A = AA), does not its subaltern, 'Some A is A' (A =/= A nonA) follow? Is it not the case that (A =/= A nonA) is a **thesis** of the system? Consider the proof: Denying (A =/= A nonA) produces the formula (A = A nonA), which is surely only a contradiction if A is ENS. Thus, 'Some A is A' is a **thesis** only under certain conditions. Leibniz allows the condition 'A is ENS' to mean 'A is **possible**', 'A is **non-contradictory**',[26] 'A is a **thing** (actual or possible)',[27] sometimes simply 'A is **true**'.[28] It cannot be said that he is very clear about the matter. Ideed, it is a concomitant of one of his chief problems in this period of his thought, how to speak of truth and falsity, possibility and impossibility, contingency and necessity while adhering to the fundamental principle, *"In every true affirmative proposition, the predicate is in the subject."*

If this third system is to be comprehensive of syllogistic logic and the propositional calculus, and at the same time avoid inconsistency, 'Some A is A' cannot be a **thesis** in an unrestricted sense. It is for reasons similar to these that Sommers in his system TFL does not allow **subalternation** as an immediate inference, but requires that the derivation of Isp from Asp be through the syllogism DARII, using Iss as the minor premise. Thus, Iss is not for him a **thesis**, but can be given as a premise whenever the logical situation warrants it, specifically, he says, wherever Asp *"has a truth value."*[29] It will be sufficient here to recognize that some restriction must be put on the use of Iss in Leibniz's system, as also on the manner in which **subalternation** is derived and the use of the Rule of Regress where a contradiction occurs **if A is ENS.** It is significant that Leibniz has the phrases 'A is ENS', 'AB is ENS', etc., for it can flag the required restriction when the logical system is employed in syllogistic logic. To lay out such restrictions in a rigorous manner is not a major task.

LEIBNIZ'S LOGICAL SYSTEM OF 1686 - 1690

Leibniz has created in the axioms of GI, repeated in **Primaria** and **Fundamenta** an axiomatic system of great power and interest. There is no indication in the works we have discussed in this paper that he understood just how significant his logical system was. But it is almost inconceivable that he should cling to that particular set of axioms as tenaciously as he obviously did and be wholly ignorant of the importance of his discovery.

NOTES

1 Some are dated, and for those the dates are given. For others the order is established from other evidence as shall appear in its proper place.

2 In **Generales Inquisitiones** and **De comprobatione**, the sign for 'equality' is '='; in the others of the period, the equality sign is '00', as in Descartes. Cf. Louis Couturat, **La Logique de Leibniz**, Paris, 1901, 345, n.2.

3 This is the well-known criticism of Bertrand Russell in **A Critical Exposition of the Philosophy of Leibniz**, London, 1900, especially pp. 12-15.

4 W. O. Quine showed the way as early as 1937 in "Logic based on inclusion and abstraction", **Selected Logical Papers** (New York, 1966), 100-09. See John Bacon, "Sommers and Modern Logic", in **THe New Syllogistic**, ed. George Englebretsen, New York, 1987, 212-22, for the history of Quine's attachment to this early position in the several editions of his **Methods of Logic**. Since Quine there have been those few questioned the common orthodoxy, as Henry Veatch in **Intentional Logic**, New Haven, 1952, and in **Two Logics**, Evanston, 1969, but none more effectively in recent times than Fred Sommers in "The calculus of Terms", **Mind**, 1970, 1-39, and several articles after that, culminating in his **The Logic of Natural Language**, Oxford, 1982.

5 F. E. Andrews (maiden name), "Leibniz's Logic within his Philosophical System", **Dyonisus**, Vol. VII (1983), 73-127.

6 I number the Schemes following G. H. R. Parkinson in his edition with translation of Leibniz, **Logical Papers**, Oxford, 1966. Hereafter, Parkinson.

7 It is important to note that Thesis 5, the assertion of the contrapositive of Aab, can be proven from Thesis 6, as well as the way we are proceeding, proving Thesis 6 from Thesis 5. Thus, either could serve as the axiom in our set. One of the two is required, as we shall see, to prove conversion **simpliciter** of Eab and Iab.

8 It can be proven from Ax.2: AA = A (Ax.2). Therefore, substituting A/AA, A = A.

9 Given this proof: ABBA = ABBA (Ax.1), therefore, BA = AB (Ax.2). But this 'proof' might be thought circular. In any case, commutativity is implicit throughout GI, even though it is not stated explicitly until the essays of August, 1690.

10 **Opus.**, 394, n.3.

11 Instead of thr three elements of subject, copula and predicate usually stated, one can differentiate the four categorical propositions with two elements, where subject and predicate collapse into one term with the copula affirming or denying.

12 The rule might be stated: drop the leftmost A is Sch.III (it is entirely undiscriminating); let 'is nonENS' replace '=' and 'is ENS' replace '=/='; B/nonB and nonB/B throughout.

13 The conclusion (A = nonC) is expressible as **both** (A = A nonC) **and** (nonC = A nonC). The denial of (nonC = A nonC), i.e. (nonC =/= A nonC), yields (nonC =/= nonC non(AC)), substituting non(AC) for A by the premise. This is a denial of Thesis 6.

14 Franz Schupp in his edition and comentary on **Generales Inquisitiones de Analysi Notium et Veritatum** (Hamburg, 1982), 180, assigns **De formae** to the same period as **Generales Inquisitiones**, and Raili Kauppi, **Uber die Leibnizsche Logik**, (Helsinki, 1960), 184-85, argues that it is surely earlier that the fragmets os 1690, employing as it does the sign '=' for equality (as in **Generales Inquisitiones**), whereas in those later works the sign '00' is used;

and further, it cannot be earlier than **Generales Inquisitiones** since it presumes the calculus of thar work without stating a calculus of its own.

15 It is no clear that Leibniz recognizes fully the requirement of Thesis 5 (or 6) at this juncture. He will manifestly be aware of its significance in the logical works of August, 1690.

16 Aab: 'A nonB is nonENS' is clearly equivalent to A(non b)(non a): '(non b)(non non a) is nonENS'.

17 Parkinson, **Logical Papers**, xlix, thinks Leibniz exaggerates; but this is only because he has not seen the whole syllogism given in the principles and schemas that Leibniz has developed there.

18 Wolfgang Lenzen, " 'Non est' non est 'est non' ", **Studia Leibnitiana**, Band XVIII (1986), 19, regards (iii), (iv) and (v) as all written about the same time, and later than GI from both the external evidence that they all employ capital letters for terms and '00' for identity, whereas GI and (ii) employ '=', and the internal evidence that they employ a calculus using '00' rather than 'est'. He also, somewhat incoherently, describes (v) as '(eine) Fragment von August 1690', but still dates it earlier than the two dated fragmets from 1 August 1690 and 2 August 1690, **Primaria** (vi) and **Fundamenta** (vii), which he rightly regards as more sophisticated than (iii) to (v). Cf. Kauppi, 184.

19 Scheme III has appeared to Leibniz to be difficult to manipulate because of the inequations in Iab and Oab. Thus, here he tries to avoid them, but at the price of having to use the 'indefinite' letter which might be thought to take the place of 'some'. AS will finally appear to him, the inequations offer no real obstacle when once he has possesion of the axiom which will permit proof of conversion **simpliciter**. That is, of course, Thesis 5 (or Thesis 6).

20 B is C, QB is B (by Ax.2). Therefore, QB is C. Putting nonC/C, in FERIO then Ebc → Obc.

21 **Opus.**, 410-16.

22 **Primaria** (4) is easily proved by Regress, providing A is ENS; **Primara** (8) also by Regress.

23 Héctor-Neri Castañeda "Leibniz's Syllogistico-Propositional Calculus", **Notre Dame Journal of Formal Logic**, Vol. XVII (1976), 481-500, has previously shown that from a small number of axioms of **Fundamenta**, essentially those we have given in TS, a complete version of the propositional calculus can be derived. But he also says *"... we conclude that Leibniz did not have an adequate incipient grasp of the monadic predicate calculus."* If monadic predicate logic admits of the decision procedure of the truth-table (and it does), and if theoremhood in PM is equivalent to validity in truth-tables, it ought to follow, contrary to one of Castañeda's conclusions, that Leibniz's axioms can be comprehensive of monadic predicate logic.

24 This is the criticism Leibniz has of it in the preface to **Primaria**, **Opus.**, 233: *"Quoniam igitur hac resolutione non facile apparet vis consequentiae, not est habenda pro optima resolutione."*

25 He continues, *"Sic ergo melius reducendo omnia ad aequipollentiam seu quasi aequationem."* **Ibid.**

26 See **Opus.**, 259, 261 for example.

27 GI, (n. 145)

28 GI, (n. 198)

29 **The Logic of Natural Language**, 201.

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