

NEW DIFFICULTIES WITH 'IF...THEN'. THE PARADOX OF THE BUSINESSMAN†

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ABSTRACT: A new problem about 'if...then...' is posed which is related to Curry's paradox much as the barber's paradox parallels Russell's paradox. However, it is not obvious how to solve it.

Keywords: conditional, Curry's paradox, knowledge, material conditional, paradox, relevant conditional.

The so-called 'paradoxes of implication' (either material or strict implication in whatever form) are probably the most famous problems by which the relation between logic and natural language has been called into question. The importance of these paradoxes is attenuated, however, if we choose to consider the formal systems on which they are based as having little connection with the 'if...then' connective of natural language. This is insufficient, however, if we want to take all the problems that arise into account. Smullyan, and later Hazen, among others, have explicitly shown how it is possible to demonstrate anything by using a statement of the form 'if this statement is true, then p', where instead of 'p' we substitute any statement whatsoever, without drawing on implicational properties which are counter-intuitive. It is possible to solve the difficulty by saying that the above-mentioned statement is self-referential in a way which is not permitted in a discourse which aims to be coherent or meaningful. Analogously, the statement 'if I am not mistaken, p' is similar in form to the Hazen-Smullyan paradox and it permits us to prove that 'p', no matter what value 'p' takes, if 'I am not mistaken' is understood in the same way as 'I am not mistaken in saying that if I am not mistaken, p', instead of 'I am not mistaken in saying that p'. This is not what one does, however, when one says for example 'if I am not mistaken, this article will be rejected by The Editor', in such a way that self-referential expressions similar to those found in the Hazen-Smullyan statements are eliminated. With this in mind it is important to see if there exist grammatically acceptable statements (such as the paradoxes of

implication) which lead to absurd conclusions following reasoning patterns which are above suspicion (as in the Hazen-Smullyan paradox). Such statements do in fact exist, as the following example shows.

Consider the case of an ambitious businessman H whose sole interest lies in investing his money in different projects with a view to increasing his fortune as much as possible. H has a team of advisors, experts who have always known exactly what his policy is each time he materializes a decision about a possible investment: invest if and only if it will pay ample profits. Now suppose that H is presented with ten investment proposals in ten different companies. In order to decide what he should do, H orders a detailed study of each case from his team of advisors and he receives from them the following information: projects E1, E3, E6 and E10 will pay ample profits but the investment in the rest will in all certainty be a failure. So he decides to invest in E1, E3, E6 and E10, since in keeping with his policy he invests in one of these firms if and only if the investment will pay ample profits. His decision is correct, but what about the reasoning that has led to it? Suddenly he realizes not when he reflects as follows:

Given my investment policy

- (1) For each E_i , I invest in E_i if and only if I invest in E_i will obtain ample profits.

Suppose that

- (2) I invest in E_i

From (1) and (2) it follows that

- (3) If I invest in E_i then I will obtain ample profits

From (2) and (3) by modus ponens

- (4) I will obtain ample profits

It follows from (1) that

- (5) If I invest in E_i , then I will obtain ample profits

From (1) and (5) then

- (6) I invest in E_i

From (5) and (6) again by modus ponens

(7) I will obtain ample profits

And so, from (6) and (7)

(8) I invest in E_i and I will obtain ample profits

Finally, from (8) it follows that

(9) For each E_i , I invest in E_i and I will obtain ample profits.

It follows from all this that H invests in all ten companies and in each one he will obtain ample profits, which is impossible if the experts (for once) are right. Where is the error in H's reasoning? Observe that his investment policy is quite commonsensical and would be considered so by the majority of businessmen, and that the steps in the argument outlined above do not contain anything which might be considered strange in terms of natural language.

It is worth it looking for a while into the question of how the conditional involved in (1) should be understood. Obviously, if this conditional were a material conditional, then every step in the above derivation would be correct, but (1) would then be clearly unacceptable as an investment policy: in effect, in (1) H would be imposing a requirement on investing to the effect that there exist a connection between investing and obtaining profits. Therefore, it seems clear that, in imposing the truth of 'if I invest in E I will obtain ample profits' as a necessary and sufficient condition on investing in E_i , what he is demanding is that the antecedent of this conditional be relevant to the consequent. This is in keeping with the intuitive idea that guides H's investment policy: in order to invest in E_i what must happen is that investment in E_i leads to the obtaining of ample profits. Thus, the conditional involved in (1) is a sort of relevant conditional, in which case all the steps in the argument leading to the paradoxical conclusion above seem justified (in particular, (5) is justified because there is a deduction of (4) which is relevant with respect to (2)).

Note that, significantly, H would be led to the same problematic result even if his investing policy was more 'realistic', in the following sense: it could, in principle, be the case that at least a statement of the sort 'if I invest in E_i I will obtain ample profits' is true (or false) without the experts and, consequently, H ever knowing it. In those circumstances, the fulfilment of condition (1) could not be guaranteed, so H would be obliged to use a criterion that does not presuppose the particular sort of omniscience implicit in (1). A plausible candidate would be:

- (1)* For each E_i , I invest in E_i if and only if I know that if I invest in E_i I will obtain ample profits.

It is now easy for H to build an parallel argument to the one described above and which leads to the same conclusion. One would just have to replace (1) with (1)*, insert

- (2)* I know that if I invest in E_i I will obtain ample profits

between (2) and (3), and finally insert

- (5)* I know that if I invest in E_i I will obtain ample profits

between (5) and (6). The step from (2)* to (3) would be justified by the classical definition of knowledge. The step from (5) to (5)* would be justified by the fact that H, who we suppose is building the argument, knows, having reached (5), that what is said in (5) is true (since he knows that (1)*, on which (5) is based, is true): this is just what is said in (5)*. Thus, it is clear that this step from (5) to (5)* need not be justified by a deductive rule whereby 'I know that p' can be inferred from p (or from the fact that p has been proved). In fact, this kind of rule would be incorrect. The situation here is more like that of somebody who says that he knows that p after having found a proof of p.

One could even object that (1)* is little realistic as an investment policy because, given the complexities of the market, the experts will hardly ever base their predictions on knowledge but rather on greater or lesser plausibility. But this would be a minor objection: we can assume that, in the case we are concerned with here, the experts have safely concluded that, given their particular nature, investment in projects E_1 , E_3 , E_6 and E_{10} will yield ample profits, while their judgement about the rest of the projects is that it is hardly plausible that investment in them will yield any profit. In these conditions, any reasonable businessman would follow the investment policy expressed in (1)*, and one can easily imagine other situations in which even (1) would be strictly appropriate. But note that at least in all cases in which investment in at least one of the E_i is disastrous, the use of a weakened version of (1)* such as

- (1)** For each E_i , if I know that if I invest in E_i then I will obtain ample profits, then I invest in E_i

would be inadequate. Obviously, from (1)** one cannot derive the paradoxical results in this paper. However, (1)** only gives sufficient conditions for investment, so a businessman who followed this policy could perfectly well

invest in a disastrous project: it is actually compatible with (1)** to invest in every E_j , whether it is disastrous or not.

Naturally the central problem outlined in this article could be formulated in a great variety of ways which have nothing to do with businessmen, but all of them have in common that the principle which leads to absurdity contains a statement of the form $p \leftrightarrow (p \rightarrow q)$ (where ' \rightarrow ' should not be understood in a truth-functional sense). Thus they may be considered as natural language versions of the set theoretical paradox of Curry in the same way as the barber's paradox and others represent an intuitive form of Russell's paradox. Indeed, remember that Curry's class C leads directly to the problematic equivalence $C \in C \leftrightarrow (C \in C \rightarrow q)$ and that, in one of its versions (see, for example, Myhill 1984), the paradoxical derivation which results in the deduction of q is formally identical to the one leading to (7) above. Nevertheless, the analogue seems to me to be limited. Can we get rid of the problem pointed out in this article by quietly saying that businessmen such as H do not exist?

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