ARISTOTLE'S EXTENSIONAL MODALITY: HINTIKKA'S INTUITIONS, LUKASIEWICZ'S LOGIC AND MIGNUCCI'S VERDICT

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ABSTRACT: The paper discusses interpretations of Aristotle's modal notions by modern commentators (J. Hintikka, J. Lukasiewicz, M. Mignucci). It is shown that the semantics of modal notions which the above mentioned authors attribute to Aristotle is based on the algebraic idea of multiplier.

Keywords: Aristotle, modality, extensionality, multiplier, derivative set.

CONTENTS[1]

1. Preliminaries

Aristotle's modal notions pose the problem to modern commentators (Bekker 1933, Hintikka 1973, Seel 1982). It is enough to mention difficulties in giving an uniform interpretation of Aristotle's notion "*endekhomenon*" which sometimes means in Aristotle "possible" being consistent with "necessary" and sometimes "contingent" being inconsistent with "necessary" (*De int.* 22 a 23-30). In its turn, Aristotle's notion "*anagkê*" (necessary) also manifests an irregular usage. Sometimes what is actual is qualified also as necessary. This is the case when Aristotle derives the apodeitic conclusion from the apodeitic major premise and the assertoric minor premise in the first figure (Barbara) considering at the same time the similar derivation from the assertoric major premise and the apodeictic minor premise as invalid (*An. pr.* 30 a 15-30-b 5). Another case is represented by Aristotle's well-known thesis "what is, necessarily is, when it is" (*De int.* 19 a 23, J. Ackrill's translation (Ackrill 1963)).

In my paper I am going to discuss interpretations of Aristotle's modal notions by J. Hintikka, J. Lukasiewicz and M. Mignucci. I shall not touch upon the question whether these interpretations adequately account all available evidence of Aristotle's texts. My aim is more modest: I will try to show that the semantics of modal notions which the mentioned authors attribute to Aristotle is based on the algebraic idea of multiplier.

2. *Hintikka's "statistical interpretation" of Aristotle's modal notions*

In series of articles which are collected in (Hintikka 1973) J. Hintikka argues for "statistical interpretation" of Aristotle's modal notions. According to Hintikka, Aristotle uses token-reflexive sentences as bearers of truth and falsity, that is, sentences which contain an explicit or implicit reference to the moment of time at which they are uttered. Normally, Aristotle uses a sentence "*p*" as temporally indefinite which tacitly refers to the sentence "*p* now", so that such sentence can change its truth-value in a course of time. In Hintikka's interpretation, Aristotle

> explicitly or tacitly equates possibility with sometime truth and necessity with omnitemporal truth (Hintikka 1964, p. 465).

But this tacit presupposition bounds Aristotle into trouble, when he considers a temporally definite sentence "*p* at t_0" the content of which is specified independently of the moment of utterance. Future singular sentences which Aristotle discusses in the famous chapter IX of *De Interpretatione* give an example of such kind of sentences being tied to a particular event of time.

Why a temporally definite sentence "*p* at t_0" troubles Aristotle when he tries to defend indeterminist position, while a temporary indefinite sentence like "*p* (now)" does not? Because if a sentence "*p* at t_0" is ones true, it is always true. A temporally definite sentence "*p* at t_0" does not change its truth-value with a course of time, as a temporally indefinite sentence "*p* now" does. With the temporal interpretation of modal notions, one comes immediately to a conclusion that "all statements about events that are individual in the sense of being tied to a particular event of time will be either necessary true or necessary false" (*op. cit.*, p. 466). It means that everything that happens happens necessarily: whatever happens at a moment could not fail to happen at it, so that "possibly p at t_0" implies "p at t_0".

However, this conclusion, according to Hintikka, does not challenge Aristotelian indeterminist position, because Aristotle tends to discuss a happening in a moment in terms of temporally unqualified sentences of the type "p now" or "p *simpliciter*". So, he distinguishes between something which is necessary with temporal qualification, "necessarily (p at t_0)", and without it, "necessarily p (now)". Attribution of the last kind of necessity to things is compatible with indeterminism.

According to Hintikka, what really indicates to Aristotle that something is possible or not necessary at a given moment of time is whatever happens in similar circumstances at other (future) moments of time, as discussed in terms of temporally unqualified sentences. The shift of attention from temporally definite sentences to temporally indefinite ones allows Aristotle to discuss a whole range of similar cases in one formulation.

A peculiarity of Hintikka's interpretation can be seen in his analysis of the fragment *De int.* 19 a 12-18, where Aristotle gives an example of situation which is undetermined with respect to the future: this coat may wear out and also it may be cut before it wears out. According to Hintikka, if this is a case, then in a sense one of these possibilities will never be realized. Therefore Aristotle can equate possibility with "sometime true" only by supposing that he deals with statements of the form "a coat will wear out" and "a coat will be cut" or maybe "such and such a coat will wear out or will be cut", but not with statements "*this* coat will wear out" and "*this* coat will be cut" (Hintikka's italic). Thus, according to Hintikka, Aristotle should take a statement as to what is possible in a given moment to a given individual as "an elliptical statement which really says something about all the similar individuals at all the different times" (*op. cit.*, p. 487).

Such an interpretation of modal notions is qualified by Hintikka as "statistical", because statements of possibility were taken by Aristotle to be primarily statements of frequency, wherefore they involve a range of cases.

> Saying that an individual event is possible is for him normally an elliptical way of saying that the relative frequency of similar events on similar occasions is different from zero (*op. cit.*, p. 477).

Statistical account of modal notions is classified by Hintikka as "extensionalistic":

> for us, the extensionalistic account of possibility to which Aristotle resorts scarcely serves to clear up any question as to what can or cannot happen at some particular moment of time (*ibid.*).

3. Topological account on "statistical interpretation"

As far as I understand Hintikka's idea, "statistical interpretation" of modal notions was provoked in the course of Aristotle's defense of the indeterminist position by Aristotle's tacit presupposition that possibility coincides with "sometime true" and necessity coincides with "omnitemporally true".

There are several striking moments in the description which Hintikka gives to his "statistical interpretation":

1. This model appeals to the notion of probability interpreted statistically, that is as relative frequency of occurrence of some particular event. Usually, the statistical interpretation of probability refers to the set of particular events of definite *type* (Kyburg 1970) while Hintikka's "statistical interpretation" refers to the set of *similar* events. In principle, one could define corresponding type of events proceeding from the relation of similarity between particular events, but this step is not a part of conventional statistical interpretation of probability. Thus, it seems to me, Hintikka uses the notion of similarity to say that the "statistical model" deals with a definite type of particular events.
2. Hintikka classifies his "statistical model" as *extensionalistic*. By this he means, it seems to me, that in the "statistical model" different event-tokens occur in different moments of time. A convenient example is the experiment of throwing up a coin. In each moment of time *t* only one event-token occurs (tail-at-*t* or head-at-*t*). If one event-token occurs, another cannot occur. But this is the property of event-tokens which distinguishes them from event-types: the event-type "tail" can be unrealized at *t* while event-type "head" realized, nevertheless, one can say that at the moment *t* two event-types are possible. Thus, it appears that Hintikka uses the logical notion of extensionality to say that the "statistical model" deals with particular event-tokens.

3. The clarification of the notion of possibility in the "statistical model" differs from its conventional clarification. Sentence that utters that something is possible in a given moment refers to *all* similar circumstances at other moments of time. It means that the logical form of such sentence is represented by *universal* sentence. In the case of temporal interpretation of modal notions where the notion of possibility refers to "sometime true" the logical form of the sentence is represented by existential sentence.

Now I try to show that for Hintikka's intuitions about interpretation of Aristotle's modal notions which are manifested in (1), (2), (3) a coherent topological account can be given.

Let us consider the ontology of particular events with a similarity relation $\sim$ (that is, a relation which is reflexive and symmetric). Let S be a set of particular events, and $\sim$ be a similarity relation on S. We shall say that $(S, \sim)$ is a *similarity structure*. There is a standard construction that defines a topological structure on a given similarity structure (Mormann 1996, p. 81).

Definitions. Let $(S, \sim)$ be a similarity structure and $x \in S$. The similarity neighborhood of x is the set $/x/$ of all elements similar to x:

$$/x/ = \{y: y \sim x\}.$$

Define an operator cl on S by

$$cl(X) = \{y: \text{there is an } x \in X \text{ such that } /x/ \subseteq /y/\}.$$

Lemma. cl is a topological closure operator.

The proof that cl is a topological closure operator proceeds from the fact that the inclusion of similarity neighborhoods renders S a partial order. The topology which is defined in this manner is also called order topology.

It is natural to define the event-type which corresponds to the event-token x as its similarity neighborhood $/x/$. Such event-types can be called *simples. Complex* event-types are constructed from simples by means of union and intersection. Let us say that an event-type E (simple or complex) is realized in a set of particular events G iff $G \cap E \neq \emptyset$. An event-type E is realized as event-token x or an event-token x realizes an event-type E iff $x \in E$.

With these preliminary explications, Hintikka's intuition about interpretation of Aristotle's modal notions which was articulated by him as "statistical interpretation" can be grasped as follows:

Definition. It is possible for an event-type E to be realized as event-token x iff E is realized in each neighborhood of x determined by the order topology:

$$x \in Pos(E) \text{ iff for each } G \in N(x)\text{: } G_x \cap E \neq \emptyset$$

where $N(x)$ is the system of neighborhoods of x in the order topology, and G_x is a set G without the element x.

In other words, an event-token x is a possible realization of an event-type E iff x is a limiting point of E. "An event-token x is possible" is an elliptical expression for "an event-token x is possible as realization of an event-type E".

Example. Let us consider a sequence S of event-tokens determined by the coin-experiment. Each event-token has the form "tail-at-t_i" or "head-at-t_i". Similarity relation is defined in a manner that renders similar all event-tokens "tail-at-t_i" and also all event-tokens "head-at-t_i". Thus, we have two event-types: *T(ail)* and *H(ead)*. System of open sets in the correspondent order topology is $\{\emptyset, T, H, S\}$. According to our definition, it is possible for the event-type T to be realized as event-token tail-at-t_i, in other words, the event of occurrence of tail at t_i is possible iff tail-at-t_k (for some k such that $i \neq k$) occurs in each neighborhood of event-token tail-at-t_i. It is easy to verify that this is the case.

Bearing in mind that the set of limiting points of a set is by definition its derivative set, we can conclude that Hintikka's "statistical interpretation" is based on the topological intuition of *derivative set*, where Aristolte's modal notion of possibility is clarified by reference to a derivative operation d which takes a set X to its derivative set dX. In other words, we can suppose that Hintikka's tacit intuitions were topological, while the idea of time which dominated philosophical thinking of the first half of our century transformed them into "statistical interpretation".

In the history of logic there is another example when the derivative operator was introduced by means of temporal notions. Let us consider the notion of *pseudo-topological space* which was defined by P. Simons (1975) in his analysis of provability logic. It was for the first time when a derivative operator explicitly appeared in modal logic. Pseudo-topological space is a Boolean algebra which is endowed with unary operator d which is interpreted as derivative operator, so that it satisfies the following identities:

1. $d0 = 0$,
2. $d(a \vee b) = da \vee db$,
3. $dda \leq da$

(derivative operation on a topological space normally satisfies two first conditions, to satisfy the third one this topological space must be T_D-space, i.e., each point must be isolated in its closure).

But this structure as "logic of futurity" had been introduced by A. Prior (1955, 1957) 20 years before in the context of *Master argument* of Diodorus:

1. $F(p \vee q) \to Fp \vee Fq$
2. $FFp \to Fp$
3. $\neg F \neg T$

Rules of inference are *MP* and $p \to q \,/\, Fp \to Fq$ (mon.) (where "F" means "it will be that" and "T" is the logical constant "true").

4. Some systems of modal logic

Our topological definition explains two of three peculiar moments of Hintikka's "statistical interpretation", namely, the usage of similarity relation and the clarification of logical form of sentence which identifies the possibility of an event as an universal sentence. The third moment, namely the extensionalistic character of the model, can be explained by the analysis of logical system L of J. Lukasiewicz.

J. Lukasiewicz (1953, 1957, 1961), in search of an adequate reconstruction of Aristotle's modal syllogistic, formulated the modal system L. This system has the following axiom schemes:

1. Classical propositional tautologies (PC).
2. $p \to \Diamond p$ (refl.)
3. $p \to q. \to .\Diamond p \to \Diamond q$ (ext.)

The rule of inference is MP. (J. Lukasiewicz interprets $\Diamond$-operator as an operator of possibility).

S. Kripke (1965) formulated this system using necessity-type operator:

1. Classical propositional tautologies.
2. $\Box p \to p$ (refl.)
3. $p \to q. \to .\Box p \to \Box q$ (ext.)

The rule of inference is MP. (Let L_K be Kripke's formulation of L-system).

The peculiarity of L-system is that its modal operators are *extensional* as it is clear from the axiom scheme (3). Thus, analyzing Aristotle's modal notions J. Lukasiewicz for the first time introduced the idea of *extensional modality.*

Logicians had different and opposite attitudes toward this system. According to Lukasiewisz "this system merits future investigation" (Lukasiewicz 1961, p. 296). But according to S. Kripke, "it is not acceptable in general as a system of modal logic" (Kripke 1962, p. 115). From the point of view of G. Hughes and M. Cresswell

> if by a "modal logic" we mean a logic of possibility and necessity, this system takes us to the limit of what we should regard as a modal logic at all (Hughes & Cresswell 1968, p. 310).

L-system has the following properties. It is complete with respect to Kripke-frame which fits the exotic condition: $wRv \to w = v$ (R is an accessibility relation between possible worlds w and v). L-system is Halldén-incomplete (Prior 1957, Kripke 1965) being intersection of two modal systems, namely *Trivial* which is an extension of the classical propositional logic PC by the axiom scheme $\Box p \leftrightarrow p$ and *Falsum* which is an extension of PC by the axiom scheme $\Box p \leftrightarrow \bot$ where $\bot$ is the logical constant "false". (A logical system is Haldén-incomplete, if it has a theorem $p \vee q$ where neither p, nor q are theorems).

From this fact Prior concluded that the operator of modality in L-system is not an unary connective, but a *variable* functor (schematic letter) which takes as its values identity truth-function $I(p) = p$ and constant truth-function $\bot(p) = \bot$. Rejecting Prior's "nominalistic" position, L. Humberstone (1986) subsumed the modality of L-system under the class of *pseudo-truth-functional* connectives. The specificity of this connectives is that being extensional they, nevertheless, cannot be represented as a superposition of truth-functions.

In fact, this discussion was resolved already at the time when L-system was formulated although not in logic, but in algebra. Let us consider the system HI which is an extension of classical logic by axiom schemes (*-schemes are not independent):

1. $\Box (p \vee q) \leftrightarrow \Box p \vee \Box q$ ($\vee$ - distr.)
2. $\Box p \rightarrow p$ (ref.)
*3. $\Box (p \wedge q) \leftrightarrow \Box p \wedge \Box q$ ($\wedge$ - distr.)
*4. $\Box p \rightarrow \Box\Box p$ (trans.)

Rules of inference: MP, $p \rightarrow q \,/\, \Box p \rightarrow \Box q$ (mon.)

According to (1), (3), $\Box$-modality is a representation of $\wedge$, $\vee$-homomorphism on the corresponding Boolean lattice. According to (2), (4), (mon), $\Box$-modality is a representation of topological interior operator.

Let us consider the system MU which is an extension of classical logic by the axiom scheme:

1. $\Box(p \wedge q) \leftrightarrow p \wedge \Box q$ (mult.)

The rule of inference is MP.

The $\Box$-modal operator of the system MU represents the notion of *multiplier.* Multiplier is by definition an algebraic operator which is a $\wedge$, $\vee$-lattice homomorphism and a topological interior operator. This notion was introduced at the end of 50's in (semi)lattice theory. Analogues of multipliers have been studied in many other branches of algebra. This concept also arises in harmonic analysis and in the theory of Banach algebra (Larsen 1965).

Proposition 1. L_K = MU = HI.

The system HC is an extension of classical logic by axiom schemes (*-schemes are not independent):

1. $\Box (p \vee q) \leftrightarrow \Box p \vee \Box q$ ($\vee$ - distr.)
2. $p \rightarrow \Box p$ (ref.)

*3. $\Box (p \wedge q) \leftrightarrow \Box p \wedge \Box q$ ($\wedge$ - distr.)
*4. $\Box\Box p \rightarrow \Box p$ (trans.)

Rules of inference: MP, $p \rightarrow q$ / $\Box p \rightarrow \Box q$ (mon.)

According to (1), (3), $\Box$-modality is a representation of $\wedge$, $\vee$-homomorphism operator. According to (2), (4), (mon), $\Box$-modality is a representation of a closure operator.

The system T* is an extension of classical logic PC by axiom schemes:

1. $\Box (p \rightarrow q) \rightarrow .\Box p \rightarrow \Box q$ (K)
2. $p \rightarrow \Box p$ (ref*).

Rules of inference are MP and p / $\Box p$.

In fact, T* is the extension of the minimal normal modal logic K by the axiom scheme ref*.

Proposition 2. L = HC = T*.

Thus, L-system in Kripke's formulation embodies the notion of the multiplier and in Lukasiewicz's formulation it embodies its dual. What is remarkable is that the idea of multiplier appeared independently and at the same time in logic and in algebra. From this fact we can conclude that there is a real algebraic operator, but not a syntactic variable functor, which corresponds to the modality of L-system.

From the point of view of the possible world semantics, *Trivial* and *Falsum* systems are trivial and have nothing in common. But from algebraic perspective they are *multiplier extensions* of classical logic PC. Thus, there is a well defined object, namely the lattice of multiplier extensions of classical logic (classical logic PC is embedded into this lattice being equivalent to *Trivial*-system).

In this setting one can define operations of sum and product of multipliers (Humberstone 1986), so that the modality of L-system will be the product of the modalities of *Trivial* and *Falsum* systems. What is said above justifies the conclusion that the modality of L-system is a real connective which corresponds to a real algebraic operator, but not a schematic letter.

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                    Inconsistent
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PC ≡ Trivial          o   o Falsum
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5. *Topological semantics*

The possible world model of L-system is trivial as we saw. Nevertheless, L-system has interesting topological models.

Let S = (S,t) be a topological space with a topology t which is defined by means of filters of point neighborhoods (Kuratowski 1966): $s \in S$ $t(s)$ is a filter of point neighborhoods on S iff

1. $\varnothing \notin t(s)$;
2. $Q \subseteq P$ and $Q \in t(s)$, then $P \in t(s)$;
3. $s \in \cap t(s)$;
4. $Q, P \in t(s)$, than $Q \cap P \in t(s)$;
5. for each $Q \in t(s)$ there is $P \in t(s)$: $Q \in t(r)$ for each $r \in P$.

Let us define the valuation function $|\ |_u$ for $\Box$-connective (with standard definition for Boolean connectives):

$|\Box p|_u = 1$ iff $\exists Q \in t(u)\ Q \subseteq |p|$.
$(|p| = \{u\colon |p|_u = 1\})$.

It means that some neighborhood of u is in $|p|$, so that u is an inner point of $|p|$. Let us consider the set CLP of subsets of S which is closed with respect to Boolean operations and each element of which fits the following condition:

$u \in Q \rightarrow \exists G \in t(u)\ G \subseteq Q$.

It means that every element of CLP is closed and open set (*clopen*). Now let us evaluate all formulas of our modal language in the set CLP. With this evaluation the axiom scheme (ref*) $p \rightarrow \Box p$ becomes valid. One can prove that the system T* is complete with respect to this topological model.

Modal operator ◊ which is the dual to □-operator of the system T* is a multiplier (what follows from *Proposition* 1). Its semantics is given by the following condition:

$|\Diamond p|_u = 1$ iff u is a limiting point of $|p|$ where $|p|$ is a clopen.

It means that a derivative operation is a multiplier on the set of clopens of a given topological space. With this conclusion, we can explain the last peculiar moment of Hintikka's "statistical interpretation", namely its extensionalistic character. It is extensionalistic in a sense that it is a model for the *extensional modality*. Thus, Hintikka's intuition was right in that Aristolelian modalities are extensional having the topological interpretation. In this manner the modal system L of J. Lukasiewicz received a topological semantics ten years after its formulation.

Another topological model for L-system proceeds from the fundamental mathematical idea of *localization* (Kuratowski 1966). Let us say that a set X has a property **P** in a point p iff there is a neighborhood of this point E such that $X \cap E \in \mathbf{P}$ (we identify a property **P** with the family of sets that have this property). Let X^* be the set of points where X does not have a property **P** and **P** be an ideal, that is

(i) $X \in \mathbf{P}$ and $Y \subseteq X$ than $Y \in \mathbf{P}$,
(ii) $X \in \mathbf{P}$ and $Y \in \mathbf{P}$ then $X \cap Y \in \mathbf{P}$.

Proposition 3. $G \cap ©X = ©(G \cap X)$, where G is an open set and $©X = X \cap X^*$.

Thus, on the lattice of open sets of a topological space the operation © is a multiplier. Let **P** be the ideal of finite sets. Then X^* is the derivative set for X and © is Kantor's coherence operation.

6. Mignucci's verdict

T*-formulation of L-system can explain partially a strange character of the modality of L-system. According to the axiom scheme (K):

$$\Box(p \rightarrow q) \rightarrow .\Box p \rightarrow \Box q,$$

□-modality is necessity-type modality. But according to the axiom scheme (ref*) $p \to \Box p$, it is possibility-like modality. Thus, we can say that Aristotle's modal notions of necessity and possibility as they represented by the L-system are a "mixture" of conventional notions of necessity and possibility. (Conventional modal notions follow "quantifier analogy", so that operator of necessity behaves like universal quantifier and operator of possibility like existential).

Analysis of Aristotle's modal notions led M. Mignucci to state the following implication as a general law which can be attributed to Aristotle (Mignucci 1990, p. 330):

(R) $p \wedge \mathrm{M}q \to \mathrm{M}(p \wedge q)$,

where "M" signifies "it is possible that".

Mignucci's verdict on (R) was hard:

> the whole Aristotle's proof depends on (R). Unfortunately, (R) is invalid, as is easy to see (*ibid.*).

After such verdict there is no other thing to think about Aristotle than:

> even as great a logician as he was can be trapped in the snares of modalities (*op. cit.*, p. 334).

But as our analysis shows, we can be more optimistic in understanding Aristotle, considering that (R) is a theorem of L-system (as is easy to see from HC-formulation of L-system *modulo* notation).

Notes

1 Section headings introduced by the editors.

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ON THE TRIPLET FRAME FOR CONCEPT ANALYSIS

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ABSTRACT: The paper has two objectives: to introduce the fundamentals of a triplet model of a concept, and to show that the main concept models may be structurally treated as its partial cases. The triplet model considers a concept as a mental representation and characterizes it from three interrelated perspectives. The first deals with objects (and their attributes of various orders) subsumed under a concept. The second focuses on representing structures that depict objects and their attributes in some intelligent system. The third concentrates on the ways of establishing correspondences between objects with their attributes and appropriate representing structures.

Keywords: concepts, models of concepts, triplet description of concept models.

CONTENTS

1. Introduction

For the sake of simplicity, we consider only models of so-called object concepts like *HORSE, STAR* and *ROBIN*. However, with minor reservations our approach may be applied to "abstract" concepts like attribute concepts (*RED, BLACK, SIZE*) and relational concepts (*LOVE, FORCE, DISTANCE*).