Who bears the risk? Analyzing the strategic interaction between regulators and investors when setting incentives for renewable electricity

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Energy policies for promoting investment in renewable energy sources have become crucial for deploying different green energy technologies. Depending on their design, the conventional incentive systems assign the risk to either the policymaker or the investor, affecting the strategic interaction between them when setting a price for the subsidy. Moreover, Feed-in Tariffs, which were the principal subsidy scheme used in Spain, were removed in 2013, mainly because their design led to an unbearable deficit. Farrell et al. (2017), combining option pricing theory and game theory, propose an incentive system for Irish Feed-in Tariffs in which both parties would share the risk. Building on this approach, we develop a methodology to evaluate different optimal incentive schemes for Spain and present an application for 2013 and 2019. We perform an extensive numerical analysis to determine how the different proposals would work for Spain.

Keywords: Renewable Energy; Feed-in Tariff; Efficient Policy; Green Energy; Energy Policy; Energy Economics; Option Pricing.
1 Introduction

Nowadays, one of the biggest concerns worldwide is to achieve a sustainable and clean energy regime. To a greater or lesser extent, most developed countries worldwide have set the goal of fighting global pollution and stopped relying on fossil fuel reserves, increasingly limited and responsible for the emissions of polluting gases into the atmosphere. Although significant advances have been made in this regard, there is still a long way to reach a fully green and sustainable energy scenario. Since the deployment of Renewable Energy Sources (RES) is still more costly than that of other conventional sources, a public subsidy is often required to create a feasible and appetizing investment environment.

There exist many support schemes to incentivize green energy penetration. These schemes include, for example, Renewable Portfolio Standards (i.e., a mechanism that sets an obligation on electricity suppliers to produce a specified fraction of their electricity from renewable energy sources). These Renewable Portfolio Standards might be accompanied by Tradable Green Certificates, which are tradable assets proving that electricity has been generated by renewable energy sources. Green certificates are issued to RES producers, who can trade them to conventional energy suppliers so they can fulfill the quota established in the Renewable Portfolio Standard. In exchange, renewable suppliers receive extra revenue, and the market finds the most efficient way to meet these goals. Another support scheme can be partial, or even full exemption from some taxes and levies to green energy producers. However, Feed-in Tariff (FiT) schemes have become the preferred renewable energy support mechanism in many markets, as they provide greater certainty of remuneration for investors. Nevertheless, their main drawback is the huge costs they usually involve for the regulator, especially if they are not properly designed. That is the main reason why they were abandoned in Spain in 2013 [1].

There are two variants of Feed-in Tariff policy schemes that have been widely used, particularly in Spain: a fixed FiT, and a Feed-in Premium (FiP). A fixed FiT is a mechanism allowing the RES producers to sell the electricity they supply at a fixed price for a specific period. In a FiP scheme, the payment that RES suppliers receive is based on a constant premium offered above the market-clearing price. Until 2013, Spanish RES producers had the option to choose between fixed FiT and FiP subsidy schemes. Indeed, the fixed tariff has been the most widely used FiT design. However, the FiP has been increasingly utilized in Spain, mainly for onshore wind. Since July 2013, the Spanish government suppressed both fixed FiT and FiP subsidies, being renewable energy auctions the current mechanism used for green energy promotion [2]. The benefits of FiT subsidies for renewable energy penetration have been of significant importance, and they may still play a role as long as they are carefully redesigned. Indeed, approximately 64% of today’s global wind power has been promoted through this type of mechanism [3].

Figures 1 and 2 show how both schemes work. The yellow line represents the evolution of the electricity market price, and the green line shows the evolution of the revenue per MWh that a renewable energy investor receives under each policy. Therefore, the green area represents the total cost of the subsidy for the policymaker, whereas the total area (green+yellow) corresponds to the total profits that the investors will receive. As we can see in Figure 1, a fixed tariff regime removes investor exposure to low market prices, being the policymaker the one who bears the risk of market price variability. On the contrary, constant premium policies remove the policymaker’s risk. As shown in Figure 2, under a FiP scheme, the green area is independent of the market price [4]. Since the premium (denoted as X in the figure) is independent of the stochastic market prices, the policymaker has certainty about the cost per MWh that the public subsidy will entail when designing the policy. However, under a FiP scheme, investors are exposed to the full impact of market price fluctuations. It is a well-documented fact that the effectiveness of the FiTs has been attributed to the reduced risk they entail for investors [5]; therefore, transferring all the risk to them might result counterproductive, leading to the conclusion that adequate management of these risks is of capital importance when designing public subsidy policies to promote RES penetration.
In 2011 there were approximately 21,000 MW of wind capacity deployed in Spain. That same year, the Spanish government set the target to achieve a total installed capacity of 35,750 MW by the year 2020 (Plan de Energías Renovables 2011-2020) [6]. As Figure 3 shows, until FiT subsidies were eliminated in 2013, the level of new installed capacity was on the right track, but from then on, it stopped abruptly.

The research question that we want to answer in this thesis is whether incentive schemes that share market price exposure between regulators and investors can improve upon the usual FiT and FiP. We provide an analytical specification of incentive structures that share market price exposure between regulators and investors and present an application for the Spanish market. Indeed, our results indicate that there are risk-sharing incentive schemes that dominate FiT and FiP: they allow reaching the same investment level at a lower cost. Our results have interesting policy implications for future renewable energy regulation: incentive schemes have to be carefully designed, taking into account risk sharing.

Following the approach proposed by Farrell et al. [8], we design subsidy schemes that share out the risks associated with the stochastic nature of electricity price fluctuations among both investors and policymakers. In this context, we analyze schemes based on FiT but incorporating adjustable degrees of market price exposure for investors and regulators. Our analysis will depart from the assumption that the stochastic evolution of the annual electricity price follows a random walk that can be satisfactorily described as a Geometric Brownian Motion (GBM) process. Consequently, the evolution of the payoff that a renewable energy investor receives will follow the Black-Scholes equation but with a different terminal condition, which will depend on the particular design of the FiT in place. Even though FiTs are not financial assets, nor can they be bought or sold, the Black-Scholes equation’s appearance is due to the fact that the payoffs of the corresponding subsidy will depend on the stochastic electricity market price. This fact allows us to obtain analytical solutions for the model, which provides a path of efficient combinations.
of the FiT parameters. By choosing different efficient combinations of these parameters, we can manipu-
late both policymaker’s and investor’s risk exposure levels. Accurately quantifying and distributing these
risks can be an essential stimulus to incentivize green energy penetration. This is the main contribu-
tion of Farrell et al. (2017), who proposed their methodology for a feasible Irish RES deployment scenario.

In a nutshell, our methodology can be summarized as follows:

• First, for every FiT design under consideration, we solve the stochastic model to find analytical
  solutions for the evolution of each policy scheme’s expected payoffs and costs.

• Then, following Farrel et al. (2017), we model the risk-sharing FiT design problem as a strategic
  leader game, where the policymaker (leader) takes into account the strategic response of the in-
  vestors (followers). Considering the expected evolution of payoffs and costs previously calculated,
  the regulator chooses the optimal risk-sharing FiT parameters that incentivize the desired quantity
  of RES deployment.

• Next, we perform a numerical analysis to determine how these proposals would work in Spain for
  wind energy deployment. We calibrate the model to the Spanish electricity market in 2013 (the
  last year with FiTs), and 2019 (the last year with available data).

• Since some of the needed parameters for the Spanish market do not appear in the literature, we
  have to estimate them on our own. In particular, the percentage drift and volatility of the annual
  volume-weighted average price (VWAP) for wind electricity, which depends on the amount of wind
  capacity installed. For that purpose, we build a dataset with the hourly energy production by tech-
  nology and the hourly matching price of electricity, for every hour from 01/01/2014 to 31/12/2019.

• With the estimated parameters, we simulate potential electricity prices evolution according to our
  model, obtaining how the different proposals would work for Spain.

• Finally, we perform a sensitivity analysis, changing several parameters and measuring the impact
  of those variations on the relevant predictions.
2 Mathematical Model

In this section, we present a model for designing optimal pricing rules for the different subsidy schemes we studied. We draw together the separate fields of FiT policy design, game theory, and stochastic financial calculus.

2.1 Preliminaries

One of the cornerstones in the study of financial markets is the so-called Efficient Market Hypothesis. Although there exist several different formulations of this hypothesis, all of them share the following basic features [9]:

1) Markets respond immediately to any new information about an asset price. Corrections of prices are instantaneous, leaving the participants no room for arbitrage.

2) Asset prices reflect all available information. Consequently, it is impossible to “beat the market” consistently on a risk-adjusted basis, since market prices should only react to new information.

One of the initial purposes of this hypothesis was to provide arguments in favor of the feasibility of the Random Walk Conjecture, which states that market prices evolve according to a random walk (price changes cannot be predicted). It is easy to see that if the Random Walk Conjecture holds, then, as a direct consequence, the Efficient Market Hypothesis must hold.

Suppose that at time \( t \), the asset price is denoted by \( S_t \). After an infinitesimal time interval \( dt \), the asset price will change by an amount \( dS_t \). The most common way to characterize the corresponding change on the asset price is to decompose it into two different contributions:

\[
dS_t = a(S_t, t)dt + b(S_t, t)dW_t
\]  

On the one hand, there is a deterministic contribution given by a function \( a(S_t, t) \). On the other hand, we have a random contribution to the asset price change in response to unexpected external effects, given by a deterministic function \( b(S_t, t) \), and the differential \( dW_t \). The term \( dW_t \) contains the randomness, and it is modeled as a Wiener process. A random process \( \{W_t\}_{t=0}^T = \{W_0, W_1, ..., W_T\} \) is a Wiener process when the following properties are met [10]:

- \( W_t \sim N(0, t) \Rightarrow W_0 = 0 \)
- \( \{W_t\}_{t=0}^T \) has independent increments: \( P(W_t - W_s | W_s) = P(W_t - W_s) \)
- \( \text{For } 0 \leq s < t: \quad (W_t - W_s) \sim N(0, t - s) \)

where \( N(x, y) \) represents the normal distribution with mean \( x \) and variance \( y \). From these properties, it follows that a Wiener process is also a Markov process. Since we are usually more interested in the relative change of an asset price than in its absolute change, it may be sometimes more convenient to express Eq.(1) as

\[
1 \text{In practice, it is evident that the time intervals we handle when analyzing a stochastic process, are always discrete. However, the continuous formulation is usually made for analytical convenience. Once we have solved the model for continuous time, the discrete approximation can always be made by taking appropriate time intervals.}

\[
2 \text{A sequence of random variables } \{X_t\}_t \text{ forms a Markov chain (or process) if: } \text{P}(X_{t+1} = x | X_0, ..., X_t) = \text{P}(X_{t+1} = x | X_t); \text{ that is, if given the present, the future and the past of the sequence are independent. In other words, if all the information of the past history of that variable is captured in the present state } [11] \text{.}
\]
\[ \frac{dS_t}{S_t} = \mu(S_t, t)dt + \sigma(S_t, t)dW_t \] (2)

If the functions \( \mu(S_t, t) = \mu \in \mathbb{R} \), and \( \sigma(S_t, t) = \sigma > 0 \) are constant, we obtain a Geometric Brownian Motion process (GBM) [12]:

\[ \frac{dS_t}{S_t} = \mu dt + \sigma dW_t \] (3)

It is easy to see that the expected value and the variance \(^3\) of the LHS in Eq.(3) are given by: \( E[dS_t/S_t] = \mu dt \), and \( \text{Var}[dS_t/S_t] = \sigma^2 \text{Var}[dW_t] = \sigma^2 dt \). The coefficients \( \mu \) and \( \sigma \), are usually called the drift and the volatility of the process, respectively. Once we have characterized the GBM, it is time to state both a fundamental theorem and one of its corollaries, which will allow us to handle the randomness in the model.

**Theorem. (Feynman-Kac):** Consider the stochastic differential equation (1):

\[ dS_{t'} = a(S_{t'}, t')dt' + b(S_{t'}, t')dW_{t'} \] (4)

Let \( h(\cdot) \) be a function of the stochastic variable \( S_{t'} \). Let \( t' \in [0, t] \) be given, and the final time \( t > 0 \) fixed. Define the expected value of \( h(S_{t'}) \) over the period \([0, t']\) as

\[ \rho(S, t') = E[h(S_{t'})|S_{t'} = S] \] (5)

Then, \( \rho(S, t') \) satisfies the following partial differential equation:

\[ \frac{\partial \rho}{\partial t} + a(S_{t'}, t') \frac{\partial \rho}{\partial S} + \frac{b(S_{t'}, t')^2}{2} \frac{\partial^2 \rho}{\partial S^2} = 0 \] (6)

together with the final condition

\[ \rho(S, t) = h(S), \quad \forall S \] (7)


Equation (6) is known as the Kolmogorov Backward Equation.\(^4\) When we know for sure that the system will be in a certain state \( h(S_{t'}) \) at some certain time in the future \((t' = t)\), the Kolmogorov backward equation describes the probability of being in a state \( S \) at an earlier time \((t' < t)\) [14].

**Corollary.** Consider the stochastic differential equation (3) describing the Geometric Brownian Motion:

\[ dS_{t'} = \mu S_{t'} dt' + \sigma S_{t'} dW_{t'} \] (8)

Let \( h(\cdot) \) be a function of the stochastic variable \( S_{t'} \), and let \( r \) to be a given constant. Let \( t' \in [0, t] \) be given, and the final time \( t > 0 \) fixed. Define the expected discounted value of \( h(S_{t'}) \) over the period \([0, t']\) as

\[ f(S, t') = E[e^{-r(t-t')}h(S_{t'})|S_{t'} = S] \] (9)

Then, \( f(S, t') \) satisfies the following partial differential equation:

\(^3\)From the third property of the Wiener process, it follows that: \( dW_t = \lim_{\Delta t \to 0} \{ W_{t+\Delta t} - W_t \} = W_{t+dt} - W_t \sim N(0, dt) \)

\(^4\)The Kolmogorov Backward Equation is a diffusion type partial differential equation that arises in the theory of continuous-time Markov processes. Even though we arrived at this equation from the Feynman-Kac theorem, it was first studied by the great Russian mathematician Andrey Kolmogorov [15], way before the Feynman-Kac formula was introduced by the mathematician Mark Kac, and the great theoretical physicist Richard Feynman while he was studying path integrals in quantum mechanics [16].
\[
\frac{\partial f}{\partial t} + \mu S \frac{\partial f}{\partial S} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 f}{\partial S^2} = rf
\]

(10)

Together with the final condition

\[f(S, t) = h(S), \quad \forall S\]

(11)

**Proof.** See Shreve (2004) [13].

This last result is of particular interest. Equation (10) is usually known as the Black-Scholes Equation, which is of capital importance in finance and option pricing theory [17]. In general, it is used to model the price evolution of a European option. In this case, instead of modeling options, we will model the price of different FiTs. In all cases, the evolution of the underlying derivative will be governed by the Black-Scholes Equation; however, in our analysis, the terminal condition will depend on the specific FiT we are studying.

In Appendix A, we show that the general solution of Eq.(10) for a given terminal condition \( h(\cdot) \) satisfying Eq.(11), can be expressed as

\[
f(S', t) = \frac{e^{-r(t-t')}}{\sigma \sqrt{2\pi(t-t')}} \int_0^\infty h(x) \left\{ \frac{(\log \left( \frac{x}{S_0} \right) - (\mu - \frac{\sigma^2}{2})(t-t'))}{2\sigma^2(t-t')} \right\} dx
\]

(12)

We are interested in estimating the expected value of the prices of each FiT for different maturity times \( t \), but always starting from the same initial period (\( t' = 0 \)). Thus, we shall always have the same initial value for \( S' = S_0 \) in the previous formula. We can obtain a much simpler way to express equation (12) if we try the following change of variable:

\[
y = \frac{\log \left( \frac{x}{S_0} \right) - (\mu - \frac{\sigma^2}{2})t}{\sigma^2 t} \quad \leftrightarrow \quad x = S_0 \exp \left( \left( \mu - \frac{\sigma^2}{2} \right) t - y\sigma^2 t \right)
\]

(13)

since we can express \( x \) as a function of \( y \), we can easily do the change of function \( \tilde{h}(y) = h(x(y)) \). Hence, we obtain a expression much easier to integrate than the one expressed in equation (12):

\[
f(S_0, t) = \frac{e^{-rt}}{\sqrt{2\pi}} \int_0^\infty \tilde{h}(y) e^{-\frac{y^2}{2}} dy
\]

(14)

This expression will allow us to calculate the discounted expected value of the benefits and costs associated to each of the schemes, by substituting for each policy the corresponding function \( h \) (and hence, the corresponding function \( \tilde{h} \)) that we shall define in section 2.4. In the next section, we describe some features of the Spanish electricity wholesale market.

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5 A European option is a financial derivative which gives the owner of the option the right, but not the obligation, to buy (if it is a “call” option), or sell (if it is a “put” option) a defined quantity of a particular asset, at a specified guaranteed price, on a specified date (maturity or expiration date).
2.2 Electricity market price model

The Iberian Electrical Energy Market (MIBEL) is the joint electricity market for Spain and Portugal. The spot market is regulated by the Spanish division of the Iberian Energy Market Operator (OMIE). This system comprises the day-ahead market, intra-day markets, and the ancillary services market. The weight of the day-ahead market in final electricity prices is usually higher than 80%. For that reason, we will focus on describing how this market works. The day-ahead market is performed once a day to set the amount and price of the electricity traded for each hour of the following day. Under this model, buyers and sellers make their respective offers composed by a specific price and electricity quantity. Bids can be simple or complex. The simple bids only include price and amount of energy, whereas the complex ones, include different technical and economic restrictions. Then, for a given hour, offers are ordered in ascending price order for the supply side, and on descending order for the demand side, being €180.3 the maximum allowed price per MWh. The day-ahead matching price is defined by the intersection of the cumulative aggregate supply and demand curves. Since renewable sources are dependent on the meteorological conditions, their availability is intermittent, affecting electricity prices. This is one of the many reasons why the MIBEL market also includes a series of intra-day markets and flexible adjustment mechanisms, including ancillary services and capacity payments, which take place after the day-ahead market to allow agents to adjust the day-ahead market’s resulting schedule.

Electricity generation sources in the MIBEL market tend to be ordered according to the merit order, that is, bids are in ascending order of production costs (or marginal production costs). This way, since the technologies with the lowest marginal cost (i.e., solar and wind) are matched first, RES penetration tends to reduce electricity prices. This phenomenon is known as the merit-order effect [18]. For non-synchronous generation sources, such as wind, the amount of electricity generated varies greatly depending on the daytime. For example, depending on the location, the wind can blow very differently at nighttime than during daylight hours. On the other hand, since the electricity demand is lower during most of the night, the market price is usually lower. Thus, a more representative measure of the prices received by a wind supplier is usually the Volume-Weighted Average Price (VWAP), where each hour’s price is weighted according to the amount of electricity generated by that technology at that same hour.

As we already discussed, the evolution of electricity market prices will follow a random walk. However, it is possible to model these uncertain prices using the adequate stochastic process in our model. In general, policymakers do not base their decisions on daily electricity price fluctuations. Instead, they focus on more extended periods, such as annual periods. Besides, short-term energy prices exhibit mean reversion. However, the long-term prices exhibit a drift term, which nullifies the mean reversion, removing this way the sensitivity to intra-annual variability. Since renewable energy investment is a long-term investment, long-run price trends are significant, even when analyzing markets with electricity prices having hourly, daily, or monthly fluctuations. These are some of the reasons, together with the possibility of obtaining closed-form analytical solutions, explaining why GBM, with annual timesteps, has been widely used to model long-term electricity prices [19]. Following the same conventional approach, we will use annual timesteps in our model. We will denote the annual VWAP at a period (year) $t$, by $S_t$. As we have already mentioned, we will model it as described in equation (3):

$$dS_t = \mu(Q)S_t dt + \sigma(Q)S_t dW_t$$ (15)

As shown in equation (15), we let the drift $\mu$ and volatility $\sigma$ in this GBM process to depend on the total amount of wind capacity installed ($Q$). Because of the merit-order effect, the electricity market price tends to decrease with the penetration of RES such as wind or solar. Hence, the VWAP, and therefore, the annual percentage drift $\mu$ and volatility $\sigma$, will eventually depend on the installed capacity $Q$. That means that these two parameters will be endogenous to the investor’s optimization problem.
2.3 Game theoretic approach for wind energy investment model

Following the approach proposed by Chang et al. (2013) [20], Farrel et al. (2017) proposed an optimal FiT setting mechanism by modeling the design process as a Stackelberg sequential strategic game, in which the policymaker (leader) moves first choosing the FiT, and then the investors (followers) implement their strategy (investment level) after observing the FiT selected by the leader. In this sequential game, the first mover takes into account the optimal response of the investors and chooses the FiTs that implement the desired level of renewable capacity. We solve the sequential game by backward induction: first, we calculate the best response function of the investors, and then we substitute it in the objective function of the regulator [8].

Let us assume that there are currently $Q_0$ operating units of wind energy (measured in MW) and that the policymaker wishes to incentivize investment for the deployment of at least $Q_I$ additional units. For simplicity, we model the investors’ decision problem as if they were risk-neutral. This assumption may be justified if investors can diversify their risks with investments in other sectors. Under this assumption, their objective is to maximize at time $t = 0$ the total discounted profits of the potential investment. We model the total costs as the sum of the capital costs ($A$) incurred in period $t = 0$, plus the discounted maintenance and operation costs ($O$), which are incurred during the operative life of the installed wind turbines ($T_F$). We assume that the maintenance and operation costs are constant and identical, reflecting the current state of technology during the time the installation takes place. Thus, the total costs per MW ($C$) are calculated as

$$C = A + \sum_{t=1}^{T_F} e^{-rt}O$$

where $r$ is the fixed interest rate (discount rate). For each period, we approximate the expected wind generation $g_t(Q)$ as a concave function of the capacity $Q$, to capture the decrease in generation per unit installed as capacity increases, due to more mediocre site and resource availability:

$$g_t(Q) = Q_{max}uv h (1 - ae^{-\gamma Q})$$

where $Q_{max}$ is the wind deployment potential (the maximum wind energy that is technically feasible to deploy), $u$ is the capacity factor (the actual energy output over a given period divided by the maximum possible energy output over that period), $v$ is the availability factor (the amount of time the installation is operative over a certain period divided by the total time in that period), and $h$ is the number of total hours composing the given period $t$. For simplicity and to get analytical solutions, we use the approximation that $u$, $v$, and $h$ are constant over time. As a consequence, the expected electricity generation will be identical in all periods as well. The parameters $\gamma$ and $a$ are calibrated to match the electricity market data.

Since by the time the investment is made, there are already $Q_0$ units installed, during each period of operation, the number of additional electricity units generated by the deployment of the new extra capacity $Q$, can be expressed as

$$G_t(Q) = g_t(Q_0 + Q) - g_t(Q_0) = Q_{max}uv h a (e^{-\gamma (Q_0+Q)} - e^{-\gamma Q_0})$$

As we already mentioned, in order to facilitate the analytical treatment of the problem, we will assume that investors are risk-neutral. Thus, investors will be indifferent towards exposure to fluctuating market prices, and their decisions will be relative only to the discounted expected prices ($e^{-rt}E[P_t]$) they will perceive, being $P_t$ the payoff of the FiT at time $t$, and $E$ the expectation operator. In general, this
discounted expected price will depend on the VWAP \((S_t)\), which in turns, depends on the total installed capacity \((Q)\). Therefore, investors will try to maximize the expected total profits \((\Pi)\) over \(T_F\) periods by the deployment of \(Q\) new units of wind capacity:

\[
\max_Q \left\{ \Pi = \sum_{t=1}^{T_F} \left( e^{-rt} E[P_t(Q)]G_t(Q) \right) - CQ \right\}
\]

s.t. \(Q \geq 0\) \hspace{1cm} (19)

The First Order Necessary Condition is given by:

\[
\frac{\partial \Pi}{\partial Q} \bigg|_{Q=Q_M} = \sum_{i=1}^{T_F} e^{-rt} \left( E[P_t(Q)] \frac{\partial G_t(Q)}{\partial Q} + \frac{\partial E[P_t(Q)]}{\partial Q} G_t(Q) \right) \bigg|_{Q=Q_M} - C = 0, \quad Q_M \geq 0 \hspace{1cm} (20)
\]

In order to achieve the deployment of at least \(Q_I\) units, the policymaker must set a configuration of the FiT parameters (for the time being, we shall denote these parameters by \(\varphi_i\)) such that when investors solve Eq.(19), they find optimal to invest in the installation of \(Q_M \geq Q_I\) units. The policymaker wishes to minimize the expected total discounted cost of the subsidy \((\Lambda)\), which will have a duration of \(T_1\) years, a duration that does not have to coincide in general with the time in which the installed capacity is operative \((T_1 \leq T_F)\):

\[
\min_{\varphi_i} \left\{ \Lambda = \sum_{i=1}^{T_F} e^{-rt} E[F_t(\varphi_i)]G_t \right\}
\]

where \(F_t\) denotes the cost of supporting the FiT policy during time \(t\). Since the policymaker wishes to minimize costs, and because it is easy to accept that the costs of the subsidy increase as the installed capacity increases, we assume that the policymaker’s problem will be optimal when investors find optimal to deploy just the installation target. Therefore, the overall optimization problem where both investors and regulators solve their respective optimization problems will be achieved when Eq.(19) and Eq.(20) are both evaluated at \(Q_M = Q_I\).

\subsection*{2.4 Expected price of different FiTs}

As we already mentioned in section 1, under the fixed FiT scheme, the policymaker is the one exposed to the risk associated with market price fluctuations, whereas, investors’ profits are fully deterministic. On the other hand, under the constant FiP regime, the policymaker’s costs are deterministic, in contrast with RES investors, whose benefits are now fully exposed to the market price fluctuations. Now, it is time to have a look at the alternative subsidy scheme we will study. If we design different intermediate policies that are a function of the stochastic market price, it is possible to share the risk exposure. Illustrations of how the two proposed alternative schemes work are shown in Figures 4 and 5, where the blue line represents the payoffs received by the investor at each period \(t\), and the shaded red area represents the total final amount of the retribution going to the regulator.

- A Shared Upside subsidy consists of a guaranteed minimum price that the investors receive if the market price is found below this floor. If, on the contrary, the market price exceeds this floor, both

\footnote{The additional required sufficient condition for identifying a local maximum point in \(Q = Q_M\), is given by: \(\frac{\partial^2 \Pi}{\partial Q^2} \bigg|_{Q=Q_M} < 0\). As we shall see in our numerical application, it is not always possible to satisfy both conditions.}
investor and policymaker share the exceeding remuneration according to a predefined share, which may range from 0% to 100%. For example, in the case depicted in Figure 4, the share is 50% each.

- Under a *Cap and Floor* regime, the investor receives a guaranteed minimum price if the market price is lower than this floor, and a maximum price if the market price exceeds the cap. If the market price is higher than the floor but lower than the cap, the investor receives exactly the market price.

![Figure 4: Shared Upside scheme](image1)

![Figure 5: Cap & Floor scheme](image2)

From the investor’s perspective, both of these schemes provide the certainty of minimum payment and some room for receiving additional benefits from high market prices. From the policymaker’s perspective, these tariffs allow them to obtain a split of the benefits, which shall reduce the costs of the subsidy in which the regulator has to incur.

At this point, we have developed all the necessary tools to analytically solve our model for the different subsidy schemes we analyze. In order to solve investor’s problem (20), we have to obtain analytical solutions for the expected discounted payoffs of each FiT at each period \( e^{-rt}E[P_t] \). With the aid of the discounted Feynman-Kac theorem we had already discussed, all we have to do to obtain the desired discounted expected values is to substitute the function \( P_t \) (or instead, \( F_t \) for obtaining the discounted expected costs) of each FiT in the equation (12) in place of the generic function \( h \). In fact, once we obtain for each tariff its function \( h \) (that is, \( P_t \) or \( F_t \)), it is much easier to solve the calculations by substituting in Eq.(14) the associated function \( \tilde{h} \), which we already discussed. The solutions for each tariff, for the periods under the duration of the subsidy \( t \leq T_1 \), are shown below.³ For the rest of the periods \( T_1 < t \leq T_F \), investors will charge the market price \( S_t \) under any scheme, and thus, the policymaker will bear no cost. It is easy to see, that the cost of supporting the FiT policy at each period, will be just the difference between the price that the supplier perceives and the market price at time \( t \):<ref>
\[ F_t = P_t - S_t \] (22)
</ref>

### 2.4.1 No Subsidy

As a consequence of the definition of the GBM in Eq.(3), it turns out that \( S_t \) is log-normally distributed. That is, \( \log S_t \) is normally distributed (with mean and variance given by: \( \log S_0 + (\mu - \sigma^2/2)t \), and \( \sigma^2t \), respectively). An important feature of log-normally distributed variables is that they can take any value between zero and infinity, thus excluding negative prices, another reason why the GBM is appropriate for our model. It can be shown that the expected value of \( S_t \) is given by [21]

\[ E[S_t] = S_0e^{\mu t} \] (23)

³We will not show the derivations of the current formulas, the obtaining of which, is pretty much straightforward integrating equation 14.
Hence, the discounted expected payoffs that an investor perceives at each period without any subsidy are given by

\[ e^{-rt}E[S_t] = S_0e^{(\mu - r)t} \]  \hspace{1cm} (24)

2.4.2 Fixed Tariff

As mentioned in the introduction, a fixed tariff scheme offers a guaranteed payment \((K_A)\), which is totally independent of the electricity market price. Under this scheme, the payoffs that an investor perceives at each period \((t \leq T_1)\):

\[ P_{A,t} = K_A \] \hspace{1cm} (25)

Therefore, policymaker’s costs at time \(t\):

\[ F_{A,t} = K_A - S_t \] \hspace{1cm} (26)

Equation (14) leads to the following pair of solutions:

\[ e^{-rt}E[P_{A,t}] = K_Ae^{-rt} \] \hspace{1cm} (27)

\[ e^{-rt}E[F_{A,t}] = e^{-rt}(K_A - S_0e^{\mu t}) \] \hspace{1cm} (28)

FONC for the investor’s problem described in Eq.(20) leads to the following optimal solution for \(K_A\) (in case there exists an optimal):\(^8\)

\[ K_A^{(opt)} = C - \sum_{t=T_1+1}^{T_T} \left[ S_0e^{(\mu - r)t} \left( t \frac{\partial G_t}{\partial Q} G_t + \frac{\partial G_t}{\partial Q} \right) \right] \bigg|_{Q=Q_I} \]

\[ - \sum_{t=1}^{T_1} e^{-rt} \frac{\partial G_t}{\partial Q} \bigg|_{Q=Q_I} \]

2.4.3 Constant Premium

For a constant premium tariff, the payment that investors receive is based on a constant premium \((X)\) offered above the market price:

\[ P_{B,t}(S_t) = X + S_t \] \hspace{1cm} (30)

Hence, policymaker’s costs at each period are deterministic:

\(^8\)For the four tariffs discussed, we just characterize the singular points where the maximum could be found. However, there is no guarantee that even if we find a feasible singular point, it will be a maximum point. For that purpose, given the specific values of the parameters, we should check in each case whether the second order necessary conditions hold.
\[ F_{B,t}(S_t) = X \] (31)

We find that the expected discounted profits and costs at time \( t \) are given by

\[ e^{-rt} E[P_{B,t}] = e^{-rt}(X + S_0 e^{\mu t}) \] (32)
\[ e^{-rt} E[F_{B,t}] = X e^{-rt} \] (33)

Following the same procedure, we find the optimal solution for the constant premium \( X \):

\[ X^{(opt)} = C - \sum_{t=1}^{T_1} \left[ S_0 e^{(\mu-r)t} \left( \frac{\partial \mu}{\partial Q} G_t + \frac{\partial G_t}{\partial Q} \right) \right] \bigg|_{Q=Q_1} \sum_{t=1}^{T_1} e^{-rt} \frac{\partial G_t}{\partial Q} \bigg|_{Q=Q_1} \] (34)

2.4.4 Shared Upside

The payoffs that an investor receives at \( (t \leq T_1) \) are described by

\[ P_{C,t}(S_t) = \max\{K_C, \omega(S_t - K_C) + K_C\} = \begin{cases} K_C & S_t < K_C \\ \omega(S_t - K_C) + K_C & K_C \leq S_t \end{cases} \] (35)

where \( \omega \in [0, 1] \) represents the share of the market upside received by the investor, and \( K_C \) represents the price floor. Under this scheme, the policymaker’s costs will be

\[ F_{C,t}(S_t) = \max\{K_C - S_t, 0\} + (\omega - 1) \max\{S_t - K_C, 0\} = \begin{cases} K_C - S_t & S_t < K_C \\ (\omega - 1)(S_t - K_C) & K_C \leq S_t \end{cases} \] (36)

If \( \omega \) happened to be equal to one, then we would have a FiT under which the investor charges the market price, but having a guaranteed minimum price, in case the market price is lower than this floor. On the contrary, if \( \omega \) happened to be zero, we would obtain a fixed FiT with a fixed price \( K_C \). As before, we find the expected discounted profits and costs at each period.

\[ e^{-rt} E[P_{C,t}] = e^{-rt} \left( K_C(1 - \omega \Phi(d_2)) + \omega S_0 e^{\mu t} \Phi(d_1) \right) \] (37)
\[ e^{-rt} E[F_{C,t}] = e^{-rt} \left( K_C - S_0 e^{\mu t} + \omega(S_0 e^{\mu t} \Phi(d_1) - K_C \Phi(d_2)) \right) \] (38)

where \( \Phi \) is the cumulative distribution function of the standard normal distribution, and \( d_1 \) and \( d_2 \) are defined as follows:

\[ d_1(K, t) = \frac{\log \left( \frac{S_0}{K} \right) + \left( \mu + \frac{\sigma^2}{2} \right) t}{\sigma \sqrt{t}} \]
\[ d_2(K, t) = \frac{\log \left( \frac{S_0}{K} \right) + \left( \mu - \frac{\sigma^2}{2} \right) t}{\sigma \sqrt{t}} \] (39)
we find the condition for a feasible optimal combination of $K_C$ and $\omega$:

$$K_{C}^{(opt)}(\omega) = C - \sum_{t=1}^{T} \left[ \omega \Phi(d_i(K_{C}^{(opt)}, t)) S_{e} e^{(\mu - r)t} \left( \frac{\partial Q}{\partial Q} G_{t} + \frac{\partial G_{t}}{\partial Q} \right) \right]_{Q=Q_t} - \sum_{t=T+1}^{T} \left[ S_{e} e^{(\mu - r)t} \left( \frac{\partial Q}{\partial Q} G_{t} + \frac{\partial G_{t}}{\partial Q} \right) \right]_{Q=Q_t}$$

As it can be seen in equation (40), since both $d_1$ and $d_2$ depend on $K_C$, we cannot solve the equation for $K_C$ explicitly. Given a certain value for $\omega$, the previous implicit equation may be solved by numerical methods. By performing an iterative procedure, the solution finally converges to the desired optimal value $K_C^{(opt)}$. Conversely, given a value of $K_C$, we could redo the procedure for obtaining an optimal value for $\omega^{(opt)}$. It could be proved, and we will see it later in our practical study, that equation (40) describes a unique locus of efficient pairs of $K_C$ and $\omega$, with a single efficient $\omega$ for each value of $K_C$ (and the other way around). Moreover, it could be proved that there is an inverse relationship between these optimal values of $\omega$ and $K_C$.

### 2.4.5 Cap & Floor

The payoff that an investor receives at $(t \leq T)$ are described by

$$P_{D,t}(S_t) = \max\{K_D, \min\{S_t, \overline{C}\}\} = \begin{cases} K_D & S_t < K_D \\ S_t & K_D \leq S_t < \overline{C} \\ \overline{C} & \overline{C} \leq S_t \end{cases}$$

where $K_D$ and $\overline{C}$ represent the price floor and cap, respectively. Under this scheme, the policymaker’s costs will be

$$F_{D,t}(S_t) = \max\{K - S_t, 0\} + \max\{\overline{C} - S_t, 0\} = \begin{cases} K_D - S_t & S_t < K_D \\ 0 & K_D \leq S_t < \overline{C} \\ \overline{C} - S_t & \overline{C} \leq S_t \end{cases}$$

As always, we find the expected discounted profits and costs at each period:

$$e^{-rt}E[P_{D,t}] = e^{-rt} \left( K_D (1 - \Phi(d_2)) + S_0 e^{\mu t} (\Phi(d_1) - \Phi(d_3)) + \overline{C} \Phi(d_4) \right)$$

$$e^{-rt}E[F_{D,t}] = e^{-rt} \left( K_D (1 - \Phi(d_2)) - S_0 e^{\mu t} (1 - \Phi(d_1)) - S_0 e^{\mu t} \Phi(d_3) + \overline{C} \Phi(d_4) \right)$$

where $d_3$ and $d_4$ are defined as follows:

$$d_3(\overline{C}, t) = \frac{\log \left( \frac{S_0}{\overline{C}} \right) + \left( \mu + \frac{\sigma^2}{2} \right) t}{\sigma \sqrt{t}}$$

$$d_4(\overline{C}, t) = \frac{\log \left( \frac{S_0}{\overline{C}} \right) + \left( \mu - \frac{\sigma^2}{2} \right) t}{\sigma \sqrt{t}}$$

Proceeding as in the previous case, the condition for a feasible optimal combination of $K_D$ and $\overline{C}$:

---

9For simplicity, when computing all the derivatives for both Shared Upside and Cap & Floor schemes, we neglect the variation of $\Phi(d_i(\mu(Q)))$ with respect to $Q$, where $i \in \{1, 2, 3, 4\}$. 

---
functions assuming a second-order polynomial functional form in $Q$, linear, at least in the range of wind deployment we are interested in. For that reason, we estimate both $\mu$ energy penetration. As we will see, the estimated functions are constant; however, in order to be more rigorous, we also estimate how the volatility depends on wind.

In the case of $\mu$, we mentioned earlier, this is a key feature on our model, since $\mu$ is a function of the deployed wind capacity. Since there is no such study done for Spain, we devise a different methodology in order to estimate $\mu$. Instead, for a given value of the cap $C$, we can solve the implicit equation (46) iteratively until it converges to the optimal solution $K_D^{(opt)}$. Conversely, given a value for $K_D$, we could solve it to obtain an optimal value of $C^{(opt)}$. In any case, equation (46) describes the locus of efficient combinations of floor and cap. As in the shared upside regime, there is an inverse relationship between these two policy parameters. A lower floor implies a higher efficient cap, and the other way around. As a consequence, there will be an inverse relationship in investor’s and policymaker’s exposure to market price variability.

### 3 Numerical Application

We apply the mathematical model developed in the previous section to analyze how each of the tariffs under consideration would work in Spain in the years 2013 and 2019. We have chosen these two years because, on the one hand, 2013 was the last year where the government supported RES deployment with FiTs, and on the other hand, 2019 is the last year with available data. First of all, in order to analyze each of the two different scenarios, we have to calibrate the several parameters of the model. In some cases, we find the corresponding values in the literature, but in many other cases, we have to estimate them ourselves. The price we must pay for obtaining analytical solutions is that we must ignore some technical issues. For instance, as we mentioned in section 2.2, electricity prices have to be lower than €180.3/MWh by law. However, this requirement cannot be included in the analytical model, and because of that, and due to the high value of the drift $\mu$ we will use, there is a good chance that in the last periods, this requirement does not hold. Thus, we must clarify that given some of the assumptions and technical limitations present in the methodology, the objective of the following numerical analysis is not to obtain exact figures regarding the benefits and costs that each tariff would entail; this exercise would require a much more thorough and meticulous analysis, and above all, with much more available data. Rather, this exercise’s primary purpose is to show how the intermediate schemes behave compared to the usual FiT in the scenario that we try, within the limitations, to be similar to the Spanish market.

In the case of $\mu$, we are not only interested in a particular value of the annual percentage drift of the VWAP, but rather on its functional form depending on the total amount of capacity deployed. As we mentioned earlier, this is a key feature on our model, since $\mu$ and thus, also its derivative $\frac{d\mu}{dQ}$ are endogenous to the investor’s optimization problem. In their study, Farrel et al. estimated the corresponding functional form in the case of Ireland using the study previously done by Doherty and O’Malley (2011) [22], in which they estimated the projected future wind weighted market prices in Ireland as a function of the deployed wind capacity. Since there is no such study done for Spain, we devise a different methodology in order to estimate $\mu(Q)$. It is worth mentioning that Farrel et al. treated $\sigma$ as a constant; however, in order to be more rigorous, we also estimate how the volatility depends on wind energy penetration. As we will see, the estimated functions $\mu(Q)$ and $\sigma(Q)$ are almost, but not perfectly, linear, at least in the range of wind deployment we are interested in. For that reason, we estimate both functions assuming a second-order polynomial functional form in $Q = Q_0 + Q_I$. 

\[
K_D^{(opt)}(C) = C - \sum_{t=1}^{T_1} \left[ S_0 e^{(\mu - \rho)r} t \left( \Phi(d_1(K_D^{(opt)}, t)) - \Phi(d_2(C, t)) \right) \left( \frac{\partial \mu}{\partial Q} G_1(t) + \frac{\partial G_1}{\partial Q} \right) \right] \bigg|_{Q=Q_t} + \\
\sum_{t=1}^{T_1} \left[ (1 - \Phi(d_2(K_D^{(opt)}, t))) e^{-rt} \frac{\partial G_1}{\partial Q} \right] \bigg|_{Q=Q_t} - \sum_{t=T_1+1}^{T} \left[ S_0 e^{(\mu - \rho)r} t \left( l \frac{\partial \mu}{\partial Q} G_1(t) + \frac{\partial G_1}{\partial Q} \right) \right] \bigg|_{Q=Q_t} \\
- \sum_{t=1}^{T_1} \left[ (1 - \Phi(d_2(K_D^{(opt)}, t))) e^{-rt} \frac{\partial G_1}{\partial Q} \right] \bigg|_{Q=Q_t} - \sum_{t=1}^{T_1} \left[ \Phi(d_1(K_D^{(opt)}, t)) e^{-rt} \frac{\partial G_1}{\partial Q} \right] \bigg|_{Q=Q_t}
\]
3.1 Estimation of how the drift and volatility of the VWAP depend on the installed wind power

For the estimation of these two functions, as well as the estimation of the VWAP, we use a methodology devised by ourselves collecting vast quantities of data and performing some regressions in R. We include the R code used to construct the dataset in Appendix B. We use data from Red Eléctrica de España (REE) and OMIE. The data from REE includes the amount of energy generated by each technology in MWh: wind, solar, hydraulic, nuclear, coal, combined cycle, rest of special regime (biomass, renewable thermal, etc.), fuel/gas, international exchanges and Balearic bond, for every single hour ranging from 01-01-2014 to 12-31-2019 (one file per day) [23]. Our dataset also includes the matching price of electricity at every single hour (price\textsubscript{i}), which is taken from OMIE [24] (one file per day), and the nominal wind power installed up to a given year (power\textsubscript{t}), which is taken from AEE [7]. The aim is to get an estimation of the annual drift (µ) and volatility (σ) as a function of the installed wind power. Let us denote for each year (t) the predicted annual VWAP by p\textsubscript{t}. We are considering discrete annual timesteps, thus: dt ≈ Δt = 1. Hence, recalling the properties followed from equation 3 for the expected value and variance, we get: E[dp\textsubscript{t}/p\textsubscript{t}] = µ, and Var[dp\textsubscript{t}/p\textsubscript{t}] = σ\textsuperscript{2}, respectively. Thus, the GBM parameter estimation is made as follows [25]:

\[
\mu(Q) = \frac{1}{5} \sum_{t=2015}^{2019} \frac{p_t(Q) - p_{t-1}(Q)}{p_{t-1}(Q)}
\]

\[
\sigma(Q) = \sqrt{\frac{1}{4} \sum_{t=2015}^{2019} \left( \frac{p_t(Q) - p_{t-1}(Q)}{p_{t-1}(Q)} - \mu(Q) \right)^2}
\]

First, we need to estimate the VWAP (p\textsubscript{t}) as a function of the total wind power (Q). As a first step, we perform a simple linear regression model for the hourly electricity price using the 52,584 observations in our dataset (one for each hour).

\[
\begin{align*}
\text{price}_i &= \beta_0 + \beta_1 \text{wind}_i + \beta_2 \text{solar}_i + \beta_3 \text{hydraulic}_i + \beta_4 \text{nuclear}_i + \beta_5 \text{coal}_i \\
&+ \beta_6 \text{combined}_i + \beta_7 \text{special}_i + \beta_8 \text{fuelgas}_i + \beta_9 \text{exchanges}_i + \beta_{10} \text{balear}_i \\
&+ \sum_{j=2}^{24} \alpha_j \text{hour}_i^{(j)} + \sum_{m=2}^{7} \chi_m \text{day}_i^{(m)} + \sum_{l=2}^{12} \zeta_{\text{month}}_i^{(l)} + \sum_{k=2015}^{2019} \xi_k \text{year}_i^{(k)} + \epsilon_i
\end{align*}
\]

where \{\text{hour}_i^{(j)}\}_{j=1}^{24} \text{ is a set of dummy variables indicating the hour of the day corresponding to the observation: for example, hour}_i^{(2)} \text{ takes the value one if the observation i corresponds to the hour 2:00, and zero otherwise. In the same way, } \{\text{year}_i^{(k)}\}_{k=2014}^{2019} \text{ indicates the corresponding year: for example, year}_i^{(2019)} \text{ takes the value one if the observation i belongs to the year 2019 and zero otherwise, and so on. Similarly, } \{\text{month}_i^{(l)}\}_{l=1}^{12} \text{ indicates the month, and } \{\text{day}_i^{(m)}\}_{m=1}^{7} \text{ indicates the day of the week (1=Monday, \ldots, 7=Sunday) corresponding to each observation. The obtained results are shown in Table 2.}

As we can see, almost every variable is statistically significant, and in most cases, with p-values lower than the working precision of the machine. Another thing to note is that our OLS model explains 77.3% of the variation of our dependent variable (R\textsuperscript{2} = 0.773). As we can tell from the results, there is an inverse relationship between the hourly price and the wind energy generated. That is not surprising since we already discussed the merit-order effect [26]. The same applies to solar production. On the other hand, at peak hours, production from conventional sources is higher, and so are electricity prices. With the estimated coefficients, we can get a prediction of the hourly prices (price\textsubscript{i}).
Now, let us suppose that for a given observation, instead of having the corresponding wind power installed at that moment \( (\text{power}_i) \), we have a different given power \( (Q_1) \), while everything else remains unchanged. Then, the only difference would be in the wind energy production at that hour \( (\text{wind}_i) \). For the sake of simplicity, and because we are not trying huge wind power differences for the new capacity, assuming a linear relationship with the corresponding new generation might be justified: \[ \text{wind}_i(Q_1) \approx \frac{Q_1}{\text{power}_i} \text{wind}_i(\text{power}_i). \] That is, it seems reasonable to expect that if the wind capacity \( Q_1 \) had been, for example, a 10% higher than the actual capacity \( \text{power}_i \), then the wind production would also have been a 10% higher. Using the parameters we estimated with the regression model in Eq.(49), we predict for an arbitrary installed wind capacity \( Q_1 \), an hourly price for every observation in our dataset:

\[
\hat{\text{price}}_i(Q_1) = \hat{\beta}_0 + \frac{Q_1}{\text{power}_i} \hat{\beta}_1 \text{wind}_i + \hat{\beta}_2 \text{solar}_i + \hat{\beta}_3 \text{hydraulic}_i + \hat{\beta}_4 \text{nuclear}_i + \hat{\beta}_5 \text{coal}_i \\
+ \hat{\beta}_6 \text{combined}_i + \hat{\beta}_7 \text{special}_i + \hat{\beta}_8 \text{fuelgas}_i + \hat{\beta}_9 \text{exchanges}_i + \hat{\beta}_{10} \text{ballear}_i \\
+ \sum_{j=2}^{24} \hat{\alpha}_j \text{hour}_i^{(j)} + \sum_{m=2}^{7} \hat{\chi}_m \text{day}_i^{(m)} + \sum_{l=2}^{12} \hat{\gamma}_l \text{month}_i^{(l)} + \sum_{k=2015}^{2019} \hat{\xi}_k \text{year}_i^{(k)}
\]  

(50)

Once we obtain the predicted hourly prices \( \hat{\text{price}}_i(Q_1) \) for a given capacity of our choice, the next step is to obtain the corresponding annual volume-weighted average price \( p_t(Q_1) \):

\[
p_t(Q_1) = \frac{\sum_{\text{year}_i^{(t)}=1} \hat{\text{price}}_i(Q_1) \cdot \text{wind}_i}{\sum_{\text{year}_i^{(t)}=1} \text{wind}_i}
\]  

(51)

Then, we obtain \( \mu(Q) \) and \( \sigma(Q) \) applying equations (47) and (48), respectively. If we repeat the procedure trying different values for the new wind capacity, we can obtain lists of values \( \{\mu(Q_1)\}_\lambda \) and \( \{\sigma(Q_1)\}_\lambda \) from which we can make a nonlinear fit to estimate its functional form, at least, in the range of \( Q \) in which we are interested. For example, the interval \( Q \in [10000, 40000] \) might be reasonable as an interval of interest. We try different values belonging to this interval. The values \( Q \) that we used and the results for \( \mu(Q) \) and \( \sigma(Q) \) are shown in Table 1. Since the values we obtain in both cases suggest functional forms approximately linear, following Taylor’s theorem, including terms up to second order may be enough to successfully approximate both functions. From these results, we estimate \( \mu(Q) \) and \( \sigma(Q) \) as second order polynomials in \( Q = Q_0 + Q_1 \):

\[
\mu(Q) = 0.069125 + 2.09513 \cdot 10^{-7} Q + 6.83117 \cdot 10^{-13} Q^2 \quad (52)
\]

\[
\sigma(Q) = 0.27021 + 5.31872 \cdot 10^{-7} Q + 1.47851 \cdot 10^{-12} Q^2 \quad (53)
\]

Table 1

Plotting the obtained values shown in Table 1, together with the functional forms that we estimated in Eq.(52) and Eq.(53), we see that we got a very nice fit in both cases.
As we can observe in Figure 6, although the hourly prices, and hence the annual VWAP decrease as the installed wind capacity increases (being consistent with the merit-order effect), the annual drift of the VWAP is an increasing function of the deployed wind capacity. The same thing applies to the annual volatility, as shown in Figure 7.

• Note: It is important to clarify that the development of the methodology we have just discussed has several significant limitations. The most notable one might be the small number of years used to determine the parameters. This is due to the fact that the available data in which the generation by technology is completely broken down begins in 2014. On the other hand, our initial purpose of determining how the amount of installed wind energy affects wholesale electricity prices was to simulate the daily market matching. As we have described in section 2.2, the matching of the market price follows a deterministic method, knowing all the daily sale and purchase offers, and identifying which of those offers come from wind units; we would have a method that would allow us to reliably determine the price of the daily market depending on the amount of the installed wind capacity. Unfortunately, the codes that identify the wind units in OMIE and REE do not coincide at all. Therefore, this alternative methodology cannot be carried out. If this changes in the near future, the parameter determination process would improve substantially.
Table 2: Results of the linear regression model

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>wind</td>
<td>−0.000233*** (0.000019)</td>
</tr>
<tr>
<td>solar</td>
<td>−0.000563*** (0.000046)</td>
</tr>
<tr>
<td>hydraulic</td>
<td>0.000920*** (0.000023)</td>
</tr>
<tr>
<td>nuclear</td>
<td>0.002260*** (0.000054)</td>
</tr>
<tr>
<td>coal</td>
<td>0.001413*** (0.000024)</td>
</tr>
<tr>
<td>combined</td>
<td>0.001001*** (0.000024)</td>
</tr>
<tr>
<td>special</td>
<td>0.001545*** (0.000035)</td>
</tr>
<tr>
<td>fuelgas</td>
<td>0.013194*** (0.000535)</td>
</tr>
<tr>
<td>exchanges</td>
<td>0.001041*** (0.000029)</td>
</tr>
<tr>
<td>boiler</td>
<td>0.002701*** (0.000058)</td>
</tr>
<tr>
<td>year2015</td>
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</tr>
<tr>
<td>year2016</td>
<td>0.603287*** (0.132652)</td>
</tr>
<tr>
<td>year2017</td>
<td>9.966042*** (0.138793)</td>
</tr>
<tr>
<td>year2018</td>
<td>17.950020*** (0.130002)</td>
</tr>
<tr>
<td>year2019</td>
<td>18.105860*** (0.163003)</td>
</tr>
<tr>
<td>hour2</td>
<td>−1.956611*** (0.219181)</td>
</tr>
<tr>
<td>hour3</td>
<td>−3.487866*** (0.222104)</td>
</tr>
<tr>
<td>hour4</td>
<td>−4.102226*** (0.223917)</td>
</tr>
<tr>
<td>hour5</td>
<td>−4.695447*** (0.224572)</td>
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<td>hour6</td>
<td>−4.336743*** (0.223236)</td>
</tr>
<tr>
<td>hour7</td>
<td>−2.927090*** (0.218837)</td>
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<td>hour8</td>
<td>−1.196411*** (0.218881)</td>
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<tr>
<td>hour9</td>
<td>−0.784441*** (0.220414)</td>
</tr>
<tr>
<td>hour10</td>
<td>0.290636 (0.238500)</td>
</tr>
<tr>
<td>hour11</td>
<td>0.813617*** (0.257489)</td>
</tr>
<tr>
<td>hour12</td>
<td>0.767607*** (0.271186)</td>
</tr>
<tr>
<td>hour13</td>
<td>0.892097*** (0.279696)</td>
</tr>
<tr>
<td>hour14</td>
<td>0.860672*** (0.282155)</td>
</tr>
<tr>
<td>hour15</td>
<td>0.300588 (0.274595)</td>
</tr>
<tr>
<td>hour16</td>
<td>−0.664138*** (0.267760)</td>
</tr>
<tr>
<td>hour17</td>
<td>−1.097722*** (0.258639)</td>
</tr>
<tr>
<td>hour18</td>
<td>−0.270835 (0.248453)</td>
</tr>
<tr>
<td>hour19</td>
<td>0.878183*** (0.242426)</td>
</tr>
<tr>
<td>hour20</td>
<td>1.696003*** (0.244304)</td>
</tr>
<tr>
<td>hour21</td>
<td>1.849687*** (0.246245)</td>
</tr>
<tr>
<td>hour22</td>
<td>2.021346*** (0.248302)</td>
</tr>
<tr>
<td>hour23</td>
<td>1.277405*** (0.238016)</td>
</tr>
<tr>
<td>hour24</td>
<td>−0.197902 (0.220333)</td>
</tr>
<tr>
<td>day2</td>
<td>−1.253114*** (0.148240)</td>
</tr>
<tr>
<td>day3</td>
<td>−1.367810*** (0.181420)</td>
</tr>
<tr>
<td>day4</td>
<td>−1.167257*** (0.118563)</td>
</tr>
<tr>
<td>day5</td>
<td>−0.814275*** (0.117670)</td>
</tr>
<tr>
<td>day6</td>
<td>0.32467*** (0.123201)</td>
</tr>
<tr>
<td>day7</td>
<td>−0.290607** (0.144960)</td>
</tr>
<tr>
<td>month2</td>
<td>−5.677778*** (0.163014)</td>
</tr>
<tr>
<td>month3</td>
<td>−2.855130*** (0.169599)</td>
</tr>
<tr>
<td>month4</td>
<td>−1.656027*** (0.177717)</td>
</tr>
<tr>
<td>month5</td>
<td>2.699887*** (0.190722)</td>
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<tr>
<td>month6</td>
<td>1.451456*** (0.183082)</td>
</tr>
<tr>
<td>month7</td>
<td>−1.613633*** (0.182219)</td>
</tr>
<tr>
<td>month8</td>
<td>−0.059674 (0.192902)</td>
</tr>
<tr>
<td>month9</td>
<td>−0.143878 (0.169602)</td>
</tr>
<tr>
<td>month10</td>
<td>4.706360*** (0.162426)</td>
</tr>
<tr>
<td>month11</td>
<td>3.971814*** (0.176132)</td>
</tr>
<tr>
<td>month12</td>
<td>2.020612*** (0.181399)</td>
</tr>
<tr>
<td>Constant</td>
<td>−1.410498** (0.576889)</td>
</tr>
</tbody>
</table>

Observations: 52,584
R²: 0.773808
Adjusted R²: 0.773727
Residual Std. Error: 7.199596 (df = 52528)
F Statistic: 3.267·10^4*** (df = 55, 52528)

Note: "p<0.1; "p<0.05; ""p<0.01
3.2 Calibration

The installation target \( Q_T \) is taken as the difference between the objective set by the Spanish government and the total power installed up to that year \( Q_0 \): in 2011, they established the objective of 35.75 GW for 2020 in Plan de Energías Renovables (PER) [6], while the objective of 50.33 GW for 2030 is defined in Plan Nacional Integrado de Energía y Clima (PNIEC) [27]. On average, wind turbines are operational for 20 years [28]. The duration of the fixed-Fit and constant-FiP given by the Spanish government was 20 years as well [29]. We obtained the VWAP of the initial year \( S_0 \) with the dataset we used for the methodology we have just explained in 3.1, but expanding it, in order to include 2013 too (indeed, we included every hour from 2007 to 2019). For a given year, we took the amount of wind energy generated at every single hour of that year, and the market matching price of that hour. Then, each hour’s price is weighted accordingly to the amount of wind electricity generated at that same hour. The capital and operative costs of onshore and offshore wind energy are different. In general offshore wind-farms are much more expensive both to build and to maintain than the onshore ones. For that reason, we estimate \( A \) as a weighted average of the onshore and offshore wind capital costs, weighted according to the respective proportions of onshore and offshore deployment on the installation target \( Q_T \). We follow the same procedure for estimating the operative costs. For the discount rate \( (r) \), we use the Weighted Average Cost of Capital (WACC). Generally, the risk associated with energy projects which are financed by the government is considerably lower than those which are entirely promoted by the private sector. Thus, the discount rate used in each of the two cases is usually different. WACC, which is a weighted average cost of the firm’s equity and debt [30], does not directly consider the risk involved in the investment. This is why public subsidized RES projects often use this method to discount future cash flows and consider the financial feasibility of the investment. All the calibrated parameters and their respective sources are outlined in Table 3.

Table 3: Calibrated parameters and sources

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 2013</th>
<th>Sources 2013</th>
<th>Value 2019</th>
<th>Sources 2019</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_p )</td>
<td>20</td>
<td>IDAE [2011] [28]</td>
<td>20</td>
<td>IDAE [2011] [28]</td>
</tr>
<tr>
<td>( T )</td>
<td>20</td>
<td>BCE [29]</td>
<td>20</td>
<td>BCE [29]</td>
</tr>
<tr>
<td>( A )</td>
<td>1,400,000/MW</td>
<td>Own estimation (data: PER 2011-2020 [6])</td>
<td>1,150,000/MW</td>
<td>Own estimation (data: PNIEC 2021-2030 [27])</td>
</tr>
<tr>
<td>( Q_o )</td>
<td>0.0354</td>
<td>Own estimation (data: PER 2011-2020 [6])</td>
<td>0.0354</td>
<td>Own estimation (data: PER 2011-2020 [6])</td>
</tr>
<tr>
<td>( Q_{max} )</td>
<td>332,000 MW</td>
<td>IDAE [31]</td>
<td>332,000 MW</td>
<td>IDAE [31]</td>
</tr>
<tr>
<td>( v )</td>
<td>4.67 \times 10^{-4}</td>
<td>Own estimation (data: REE [23])</td>
<td>3.81 \times 10^{-4}</td>
<td>Own estimation (data: REE [23])</td>
</tr>
<tr>
<td>( a )</td>
<td>1.0297</td>
<td>Own estimation (data: REE [23])</td>
<td>1.0285</td>
<td>Own estimation (data: REE [23])</td>
</tr>
<tr>
<td>( p_0 )</td>
<td>0.0091</td>
<td>Own estimation (data: REE [23] &amp; OMIE [24])</td>
<td>0.0091</td>
<td>Own estimation (data: REE [23] &amp; OMIE [24])</td>
</tr>
<tr>
<td>( p_1 )</td>
<td>2.090 \times 10^{-4}</td>
<td>Own estimation (data: REE [23] &amp; OMIE [24])</td>
<td>2.090 \times 10^{-4}</td>
<td>Own estimation (data: REE [23] &amp; OMIE [24])</td>
</tr>
<tr>
<td>( s )</td>
<td>6.39 \times 10^{-4}</td>
<td>Own estimation (data: REE [23] &amp; OMIE [24])</td>
<td>6.389 \times 10^{-4}</td>
<td>Own estimation (data: REE [23] &amp; OMIE [24])</td>
</tr>
<tr>
<td>( q )</td>
<td>0.2708</td>
<td>Own estimation (data: REE [23] &amp; OMIE [24])</td>
<td>0.2708</td>
<td>Own estimation (data: REE [23] &amp; OMIE [24])</td>
</tr>
<tr>
<td>( q_1 )</td>
<td>0.319 \times 10^{-4}</td>
<td>Own estimation (data: REE [23] &amp; OMIE [24])</td>
<td>0.319 \times 10^{-4}</td>
<td>Own estimation (data: REE [23] &amp; OMIE [24])</td>
</tr>
<tr>
<td>( q_2 )</td>
<td>1.479 \times 10^{-4}</td>
<td>Own estimation (data: REE [23] &amp; OMIE [24])</td>
<td>1.479 \times 10^{-4}</td>
<td>Own estimation (data: REE [23] &amp; OMIE [24])</td>
</tr>
<tr>
<td>( r )</td>
<td>0.101</td>
<td>Navarrete et al [32]</td>
<td>0.101</td>
<td>OMIE [24]</td>
</tr>
<tr>
<td>( N_0 )</td>
<td>39.80/MWh</td>
<td>Own estimation (data: REE [23] &amp; OMIE [24])</td>
<td>45.65/MWh</td>
<td>Own estimation (data: REE [23] &amp; OMIE [24])</td>
</tr>
</tbody>
</table>

3.3 Results

3.3.1 Results for the year 2013

First of all, we check for each of the four subsidy schemes, whether the singular points defined by the equations (29), (34), (40) and (46), are maximum points of investor’s problem in Eq.(19) for the given installation target \( Q_T \). As we mentioned in section 2.4, these equations characterize only those points in which first-order necessary conditions are satisfied. Therefore, for concluding whether those points are
maximum or not, we have to check second-order conditions as well. As shown in appendix C, for the parameter values corresponding to the year 2013, the analyzed points in the cases of Fixed Tariff, Shared Upside, and Cap & Floor policies are maximum points for the installation target \( Q = Q_I \). On the contrary, in the case of Constant Premium, the value given by equation (34) for the current parametrization corresponds to a minimum point.

**No subsidy:** Without any subsidy, the expected profits that investors will make according to equation (23) are \( €-2,287.8M \). That is, investors are expected to lose more than 2 billion euros if they only retrieve the market price.

**Fixed Tariff (FT):** In the case of a fixed-Fit, the risk-neutral decision maker will find it optimal to invest in the deployment of exactly \( Q_I \) units of wind energy, if, according to the solution given by equation (29), the policymaker sets a FT of: \( K_{\lambda}^{(opt)} = €87.838/MWh \), which leads to the following expected benefits and costs:

\[
\Pi_{FT} = €607.52 \text{M} \quad \Lambda_{FT} = €2,895.29 \text{M}
\]

**Constant Premium (CP):** As we advanced above, for the given value of the parameters, it does not exist a premium \( X \) for which investors will find optimal to deploy exactly \( Q_I \) new units in this unrestricted optimization problem.\(^{10}\) In this case, equation (34) corresponds to a minimum point. Therefore, we must set another criterion to establish an adequate premium \( X \) both for investors and regulators. Since in Spain, before the removal of the FiTs, the Spanish government was indifferent between offering to a RES investor the option of a FT and of a CP, by choosing an adequate value of the premium \( X \), both the expected investor’s profits and policymaker’s costs are identical to those given by the optimal FT we just set above. The value for that chosen CP would be \( X^\ast = €10.746/MWh \), leading to benefits and costs identical to those of the fixed tariff (€607.52M and €2,895.29M, respectively). Even though both benefits and costs are identical under the two previous policies, as we will see, the risk exposure of investor and policymaker will be totally asymmetric.

**Shared Upside (SU) and Cap & Floor (C&F):** In these two cases, since we have two degrees of freedom when designing each of the FiT, the optimal configuration is now defined by a locus of efficient combinations \((K_C, \omega)\) for the SU, and \((K_D, \overline{C})\) for the C&F regimes. These locus are represented in Figures 8 and 9. In both cases, for each possible value of a price floor \( K \), there exists an optimal value of the other parameter (and the other way around). For any point belonging to this locus, under the corresponding FiT scheme, the investor will find optimal to invest in the installation of exactly \( Q_I \) new units.

As it can be seen in Figures 8 and 9, there is a negative relationship between \( K_C \) and \( \omega \), as well as between \( K_D \) and \( \overline{C} \). This is not a surprise since it is quite intuitive that there must be some trade-off from the policymaker’s perspective: if he or she is willing to offer a higher guaranteed minimum price to the investor, we can expect that he or she will demand a higher share of the potential upside \((1 - \omega)\) for the SU regime, and a lower cap \( \overline{C} \) for the C&F, which would imply more frequent remuneration for him or her (as everything exceeding the cap goes for the regulator). For the given values of the parameters, the feasible values for the price floor ranges from approximately €47 to €87 per MWh under the Shared Upside and from 64 to 87 for the Cap & Floor.\(^{11}\)

\(^{10}\)It could be possible to find an optimal point if we add specific restrictions to the problem. For example, we could bound policymaker’s budget.

\(^{11}\)The feasible values are those fulfilling some sensible requirements, first of all, they must correspond to maximum points. In addition, for the SU tariff, they must fulfill \((0 < \omega < 1)\), whereas for the C&F regime: \((K < \overline{C})\).
It is remarkable that in both cases if the price floor is set at a value €87.838/MWh, under the SU scheme the value of $\omega$ (the efficient share that the investor would perceive of the upside exceeding the floor) becomes zero, and under the C&F, the efficient cap $\bar{C}$ becomes €87.838/MWh as well. In both cases, we would obtain the efficient FT we discussed in the first place, which is a result that supports the consistency of our model.

We wish to represent the expected benefits and costs for the four policies we discussed, as well as the risk exposure. In the case of SU and C&F, the expected benefits and costs depend on the particular point of their respective locus that we use to design the tariff. In other words, depending on the price floor $K$ (as setting the value of $K$ uniquely determines the efficient paired parameter $\omega/\bar{C}$). In order to quantify the different risk exposures of each tariff, we will define the Value at Risk (VaR) as a certain percentile of the distribution of potential benefits(costs) that investors(policymakers) might obtain(incur) under the unpredictable electricity price evolution. In our case, we will set the VaR at the 10th percentile of investor’s benefits as a measure of exposure to under-remuneration, and at the 90th percentile of regulator’s costs as a measure of exposure to cost overrun. Therefore, investors will prefer subsidies offering higher values of VaR, whereas investors will prefer policies offering lower VaR values. We say that investors are exposed to risk if their VaR is lower than their expected profits. In the same way, regulators are exposed to risk when their VaR is higher than the expected costs.

In order to obtain a value for the VaR, we first have to estimate the probability distribution for each tariff of potential profits and costs. For achieving that goal, with the parameters given in Table 3, we simulate the stochastic GBM described in Eq.(15) a great number of times (in our particular case, we performed 500,000 trials). All the simulations are carried out using Wolfram Mathematica. For each trial, we obtain a particular stochastic evolution of the VWAP ($S_t$), and then, we use that value to calculate the profits and costs each FIT would imply, and in the case of SU and C&F, we do it for every feasible price floor. Finally, with the obtained sample of results, and the frequency of each result, we can easily estimate the VaR for investor’s benefits and policymaker’s costs. Next, we show for each tariff, the expected benefits (Figure 10), the expected costs (Figure 11), the investor’s VaR (Figure 12), and the policymaker’s VaR (Figure 13).
First of all, we should notice that if we perform enough trials, the simulation’s expected mean values eventually converge to the expected values already determined analytically. As shown in Figures 10 and 11, the expected benefits and costs of FT and CP policies are higher than those of SU and C&F for any value of $K$. In turn, for both SU and C&F schemes, the higher the value of the floor $K$, the higher the expected benefits and costs. Finally, it should be noted that the SU schemes provide higher expected benefits and costs than the C&F for any value of $K$. Supposing that both participants base their decisions on expected profits(costs), then investors would prefer a FT or CP, whereas policymakers would prefer a SU regime (the one with the lowest expected costs). Moreover, he or she will prefer to set a price floor as low as possible. We have to remark this result: it is possible to design Shared Upside and Cap & Floor schemes that incentivize the same level of investment in renewables than the usual Feed-in Tariffs at a lower cost for the regulator. Recalling that the main reasons why these schemes were abandoned in 2013 were the high government costs they usually involved, these results are of greater significance.

![Figure 10: Expected Profits (2013)](image)

![Figure 11: Expected Costs (2013)](image)

As we can observe in Figures 12 and 13, although FT and CP provide identical benefits and costs, the risk exposure is entirely asymmetrical. As already discussed, in the case of FT, investor’s profits are deterministic. Thus, there is no risk exposure under this scheme for investors, as a consequence, investor’s VaR coincides with the expected profits (€607M). On the contrary, under a CP policy, regulators are not exposed to risk, hence, the policymaker’s VaR coincides with the expected costs (€2,895M). As we can see, there is not any significant difference in the VaR between the two intermediate subsidies. However, policymakers will prefer again to set a price floor as low as possible, and for any value, they would be more protected against cost overruns than under the FT policy.

![Figure 12: Investor’s VaR (2013)](image)

![Figure 13: Policymaker’s VaR (2013)](image)
Since the effectiveness of the Feed-in Tariffs has been attributed to the reduced risk they entail for investors, transferring all the risk to them might be counterproductive. We analyze more in detail some intermediate schemes that share out the risk between both parties. For example, we examine a Shared Upside and a Cap & Floor regimes with an arbitrarily chosen price floor of €67.5/MWh, which according to the previous figures seems to offer an intermediate solution to the opposing interests of the two parties. Thus, the value of the efficient parameters turns out to be: $\omega = 0.634$ and $C = \€229.5/MWh$. Now, we represent the distribution of potential profits and costs obtained after the simulations. In Figures 14 and 15, we represent the PDF and CDF respectively of the distribution of profits we obtained. We do the same in Figures 16 and 17 with the obtained distribution of costs. Finally, we show in Table 4 different percentiles of the distributions, as well as the expected profits and costs (mean of the respective distributions).

It is no coincidence that the PDFs of the distributions of both benefits and costs have a shape that reminds to a skewed Gaussian distribution. As discussed earlier, $S_t$ follows a log-normal distribution (similar to a skewed normal distribution). Hence, profits and costs, which are functions of the stochastic variable $S_t$, follow some similar pattern for every support scheme. Although for each of the four policies the mean value of the distribution of profits is positive, as we can observe in Figure 14, the mode of the distribution for CP, SU and C&F subsidy schemes is found in negative values (around €-11,000M for CP, and €-6,000M for SU and C&F). Except for the FT, whose density distribution is represented by a Dirac delta distribution [34] (vertical arrow), investors are most likely to lose money for all the other schemes. From the CDF shown in Figure 15, it can be seen that unless investors receive a Fixed Tariff subsidy, with a floor $K = 67.5$, there is approximately 65% chance that they will have investment losses under CP and C&F schemes, and 73% chance if they receive a SU type of subsidy. Nevertheless, according to our assumption, since investors are modeled as risk-neutral decision-makers, despite the high probabilities of investment loses, the presence of unlikely but high potential returns outweigh the contribution of the negative earnings when computing the expected values (unlike the left side tail of the distribution, the right tail of the profit PDFs is unbounded).

The preferred option for the policymaker is likely to be the Shared Upside scheme. As shown in Table 4, the expected costs under this subsidy are the lowest among the four options (€2,548M). Furthermore, as can be seen in Figure 17, there is a 30% chance that the regulator would even have negative costs (earnings) under this scheme, whereas for the FT and C&F tariffs there is approximately a 27% and 17% chance respectively.
3.3.2 Results for the year 2019

In the same way as for 2013, for all schemes except for the Constant Premium, the singular points defined by the equations (29), (40) and (46) are maximum points of the investor’s problem in Eq.(19) for the given installation target (see Appendix C).

No subsidy: Without any kind of subsidy, the expected profits that investors will make according to equation (23) are €24,320M. That is, investors are expected to make more than 24 billion euros in the absence of any supporting policy.

Fixed Tariff: In the case of a fixed-FIT, according to the solution given by equation (29), the policymaker sets a FT of $K^{(opt)}_A = €63.458/MWh$, which leads to the following expected benefits and costs:

$$\Pi_{FT} = €1,760 \ M \quad \Lambda_{FT} = €-22,560 \ M$$

Constant Premium: As we already mentioned, given the value of the parameters for 2019, equation (34) corresponds to a minimum point. In this case, however, since even without any subsidy (i.e., which
is the same as offering a premium with zero value) the expected profits are much higher than those of the FT, it does not make any sense to establish a premium that results in expected benefits and costs identical to those of the Fixed Tariff, as we did for 2013. Hence, we shall not discuss this scheme from now on.

**Shared Upside and Cap & Floor:** The loci of efficient combinations for SU and C&F schemes are represented in Figures 18 and 19 respectively. As shown in both figures, there is a negative relationship between \( K_C \) and \( \omega \), as well as between \( K_D \) and \( \bar{C} \), pointing out the same type of trade-off between the efficient combinations of these parameters as the one we discussed for 2013. As we can see, the price floor’s feasible values range from approximately €41.5/MWh to €63.5/MWh under the Shared Upside, and from 5 to 63.5 under the Cap & Floor. Moreover, if the price floor is set at a value €63.458/MWh, under the SU scheme, the value of \( \omega \) becomes zero, and under the C&F, the efficient cap \( \bar{C} \) turns out to be €63.458/MWh as well. Thus, both schemes degenerate to the efficient FT we have discussed, a finding supporting again our model’s consistency.

![Figure 18: Optimal Locus (Shared Upside, 2019)](image)

![Figure 19: Optimal Locus (Cap & Floor, 2019)](image)

We now represent the expected benefits and costs, as well as the risk exposure: the expected benefits (Figure 20), the expected costs (Figure 21), the investor’s VaR (Figure 22), and the policymaker’s VaR (Figure 23).

![Figure 20: Expected Profits (2019)](image)

![Figure 21: Expected Costs (2019)](image)
As shown in Figures 20 and 21, if the investors do not receive any subsidy, their expected benefits are much higher than those under any support scheme. At the same time, for any subsidy policy, the regulator is expected to have negative costs (i.e., profits), whereas, without subsidy, the regulation costs are obviously zero.

In the case of FT, investor’s profits are completely deterministic. As a consequence, under a Fixed Tariff scheme, investor’s VaR coincides with the expected profits (1.76 billion euros). If the policymaker does not care at all about risk, his or her preferred option would be the SU, which implies the lowest expected costs. Moreover, the regulator will prefer to set the price floor as low as possible. As it can be seen in Figures 22 and 23, both investor’s and regulator’s VaR under SU and C&F schemes are almost identical (being slightly higher the one of the C&F for both parties). We observe that the higher the value of the price floor $K$, the higher the VaR, hence, the higher(lower) the risk exposure of the policymaker(investor). For example, we examine a Shared Upside and a Cap & Floor regimes with an arbitrarily chosen price floor of €55/MWh. Hence, the value of the efficient parameters turns out to be: $\omega = 0.138$ (for the SU), and $\overline{C} = 73.35$ (for the C&F). Now, we represent the distribution of potential profits and costs obtained after the simulations for this value of the price floor. In Figures 24 and 25 we represent the PDF and CDF of the distribution of profits we obtained. We do the same in Figures 26 and 27 with the obtained distribution of costs. Finally, we show in Table 5 different percentiles of the distributions, as well as the expected profits and costs (mean of the respective distributions).
As we can observe in Figure 24, except for the FT, the mode of the rest of the schemes is negative. From the CDF shown in Figure 25, it can be seen that, with a floor of €55/MWh for the intermediate schemes, unless investors receive a Fixed Tariff subsidy, there is approximately 65% chance that they will have investment losses under SU, and approximately 43% chance if they do not receive any subsidy, or if they receive a C&F support. Nevertheless, since investors are modeled as risk-neutral decision-makers, despite the considerably high probability of investment losses under no subsidy, the possibility of high potential returns outweighs the presence of such potential investment losses.

The policymaker will prefer to offer the SU scheme rather than the C&F or FT. As shown in Table 5, the expected costs under this subsidy are the lowest among the four options (€-23,170M). Moreover, as can be seen in Figure 27, there is a 60% chance that the regulator would even have negative costs (i.e., earnings) under this scheme, whereas for the FT and C&F regimes, those chances are slightly lower. Furthermore, as we can observe in Figure 26, if market prices are such that costs turn out to be positive, SU policy implies regulation costs approximately 5 to 10 billion lower than those associated to the Fixed Tariff scheme. The expected benefits associated with the C&F (1.48 billion euros) are much lower than those without any subsidy (24.3 billion euros), but so are the potential investment losses. As can be noted in the figures, in the worst possible case for investors under C&F, they would lose approximately 4 billion euros, that is, a 10% of the total investment (38.5 billion euros). On the contrary, without any subsidy, they could potentially lose most of the investment.

<table>
<thead>
<tr>
<th>Investor's Profits</th>
<th>Policymaker's Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>Mean</td>
</tr>
<tr>
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<td>No Subsidy</td>
</tr>
<tr>
<td>1,760.1</td>
<td>-24,320.4</td>
</tr>
<tr>
<td>5th %ile</td>
<td>-23,964.5</td>
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<td>95th %ile</td>
<td>133,502.0</td>
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</table>

Table 5: Percentiles of total potential profits and costs in 2019 (€millions)
3.4 Sensitivity Analysis

Given the values of the parameters that we have been able to predict or estimate, the solutions that we have obtained for the benefits and costs under each subsidy scheme are optimal in expectation. However, the possibility that some parameters that depend on the market might not evolve as expected should be regarded. Knowing the sensitivity of each policy’s expected benefits and costs with respect to deviations in the values of these market-depending parameters can be a fact to consider for both investors and regulators when designing their optimal policies.

Let us consider that once each tariff is designed, with the police parameters \((K, X, \omega, C)\) set, it is revealed that some of our model’s parameters did not evolve as we anticipated. Thus, the expected profits and costs will be compromised. We show below in Table 6 the elasticities \((E)\) of expected profits \((II)\) and costs \((A)\) with respect to some market-depending parameters. Specifically, we analyze deviations in the drift \((\mu)\) and volatility \((\sigma)\) of the GBM, in the discount rate \((r)\), in the total investment cost per installed MW \((C)\), and in the electrical generation in each period \((G_t)\). Where, as usually, elasticities of a variable \(\alpha\) with respect to other variable \(\beta\), are defined as: \(E_{\alpha,\beta} = \frac{\delta \alpha}{\delta \beta}\). This means that if the variable \(\beta\) increases by 1%, then the variable \(\alpha\) increases approximately by an amount of \(E_{\alpha,\beta}\%\). Therefore, lower exposure to unexpected changes of a given parameter will be given by elasticities lower in absolute value. Large elasticities mean that if unexpected changes occur, the expected results can either increase or decrease dramatically. Thus, the closer the elasticities to zero, the safer the investor or regulator will be. For illustration, we briefly discuss some of the obtained results.

<table>
<thead>
<tr>
<th>2019</th>
<th>FT</th>
<th>CP</th>
<th>SU (K=55)</th>
<th>C&amp;F (K=55)</th>
<th>SU (K=67.5)</th>
<th>C&amp;F (K=67.5)</th>
<th>SU (K=80)</th>
<th>C&amp;F (K=80)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profits</td>
<td>(E_{\mu,\mu})</td>
<td>0</td>
<td>25.97</td>
<td>140.19</td>
<td>74.80</td>
<td>30.67</td>
<td>17.21</td>
<td>6.99</td>
</tr>
<tr>
<td>(E_{\mu,\sigma})</td>
<td>0</td>
<td>0</td>
<td>-43.42</td>
<td>6.51</td>
<td>11.00</td>
<td>-1.36</td>
<td>3.00</td>
<td>-0.88</td>
</tr>
<tr>
<td>(E_{\mu,\nu})</td>
<td>-22.35</td>
<td>-30.40</td>
<td>-204.13</td>
<td>-120.28</td>
<td>-62.26</td>
<td>-45.19</td>
<td>-31.46</td>
<td>-27.61</td>
</tr>
<tr>
<td>(E_{\nu,\nu})</td>
<td>-37.95</td>
<td>-37.96</td>
<td>-269.30</td>
<td>-161.61</td>
<td>-88.54</td>
<td>-66.60</td>
<td>-49.47</td>
<td>-44.56</td>
</tr>
<tr>
<td>(E_{\nu,G})</td>
<td>38.95</td>
<td>38.96</td>
<td>270.30</td>
<td>162.61</td>
<td>89.54</td>
<td>67.00</td>
<td>50.47</td>
<td>45.65</td>
</tr>
<tr>
<td>Costs</td>
<td>(E_{\mu,\mu})</td>
<td>-5.45</td>
<td>0</td>
<td>-1.59</td>
<td>-2.10</td>
<td>-3.06</td>
<td>-3.72</td>
<td>-4.55</td>
</tr>
<tr>
<td>(E_{\mu,\sigma})</td>
<td>0</td>
<td>0</td>
<td>0.57</td>
<td>0.38</td>
<td>1.18</td>
<td>-0.15</td>
<td>0.54</td>
<td>-0.16</td>
</tr>
<tr>
<td>(E_{\mu,\nu})</td>
<td>0.90</td>
<td>-0.73</td>
<td>-0.48</td>
<td>-0.34</td>
<td>0.05</td>
<td>0.27</td>
<td>0.61</td>
<td>0.74</td>
</tr>
<tr>
<td>(E_{\nu,\nu})</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(E_{\nu,G})</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2019</th>
<th>FT</th>
<th>NS</th>
<th>SU (K=62)</th>
<th>C&amp;F (K=62)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profits</td>
<td>(E_{\mu,\mu})</td>
<td>2.47</td>
<td>-</td>
<td>1.65</td>
</tr>
<tr>
<td>(E_{\mu,\sigma})</td>
<td>0</td>
<td>-</td>
<td>-0.10</td>
<td>0.33</td>
</tr>
<tr>
<td>(E_{\mu,\nu})</td>
<td>-1.69</td>
<td>-</td>
<td>-0.91</td>
<td>-1.03</td>
</tr>
<tr>
<td>(E_{\nu,\nu})</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(E_{\nu,G})</td>
<td>1.00</td>
<td>-</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 6: Elasticities of expected profits and costs with respect to different parameters

Regarding unexpected changes in \(\mu\), investors under a Constant Premium bear the entire degree of uncertainty compared to those under a Fixed Tariff. For the intermediate schemes, the higher the value of \(K\), the lower the elasticities with respect to the VWAP drift parameter. On the contrary, policy costs are insensitive to such changes under CP schemes, and regulators under FT are the ones who bear the risk, while for the SU and C&F schemes, the higher the value of the price floor, the higher the sensitivity (in absolute value). Expected profits and policy costs are insensitive to changes in volatility under both FT and CP regimes, as the expected value of the GBM \((S_0e^{\mu t})\) does not depend on \(\sigma\). Under a SU
or C&F schemes, a higher value of $K$ (and thus a lower value of $\omega$ or $\overline{C}$) decreases both investor’s and regulator’s sensitivity to changes in the value of $\sigma$. As expected, regulators are insensitive to changes in total installation costs per MW ($C$). As can be seen, the profit’s elasticities with respect to $G$ and $C$ differ exactly in one unit, but with the opposite sign. For both parameters, the higher the value of $K$, the lower (in magnitude) the investor’s sensibility under SU or C&F regimes. For any policy, regulators share the same exposure to changes in the electricity generation at each period $G$.

3.5 Discussion

Our methodology relies on the assumption that investors are risk-neutral, which might be justifiable. When investors are risk-neutral, it is because they can diversify their investments in other sectors. In the best-case scenario, they will have investments negatively correlated with the one we are discussing, so that, if they lose some of the investments in renewables, they could still make money. This assumption allows us to obtain analytical solutions for the optimal expected profits and costs associated with each subsidy scheme. Nevertheless, in light of the results obtained previously, we can push further our analysis. Let us suppose that in the first place, investors seek to maximize expected profits, and policymakers seek to minimize expected regulatory costs. Then, once the optimum is determined for each FiT scheme, risk can be taken into consideration by choosing the most appropriate subsidy type, together with an adequate value of the FiT parameters for SU and C&F designs (i.e., by choosing the proper price floor $K$).

From the results obtained in section 3.3.1, we can conclude that it is unlikely that investors in 2013 decided to invest in the installation of the wind capacity required to meet the government’s objective set in 2011 for the year 2020, without any public subsidy support. Furthermore, the fact that wind energy investment froze after subsidies were eliminated in 2013 strongly supports our findings. Our results suggest that from investors’ perspective, the best way of creating a feasible appetizing investment environment in 2013 was to offer them a Fixed Tariff support scheme, or instead, in order to allow some risk-bearing from investor’s part and relieving some risk exposure from the regulator, offering a Shared Upside or Cap & Floor schemes as close as necessary to the Fixed Tariff (i.e., with the floor $K$ close to the optimal FiT price €87.838/MWh). As we have seen in Figures 10 and 11, the higher the value of the price floor, the higher the expected benefits and the lower the risk exposure to under remuneration the investor has to bear. If investors have to choose between the two intermediate schemes, they will prefer the Cap & Floor subsidy regime with the highest possible price floor. From the policymaker’s perspective, the policy configuration that leads to the lowest expected costs and lower exposure to cost overrun would be the Shared Upside scheme with a price floor as low as possible. Indeed, not only the Shared Upside, but the Cap & Floor scheme also provides better expected results for the policymaker than the traditional subsidies. We cannot stress this finding enough: it is possible to design intermediate schemes that share the risk exposure between regulators and investors, and more importantly, they do it more efficiently, leading to the same level of investment but involving lower expected government costs.

On the contrary, regarding the results obtained in section 3.3.2, we can conclude that an investor in 2019 would decide to invest in the installation of the wind capacity required to meet the government’s objective in 2030 without the need of any public subsidy. For instance, our analysis suggests that a risk-neutral investor seeking to maximize expected profits, will prefer to receive precisely the market price rather than any supporting policy we have considered, contrary to what happened in 2013. This finding is consistent with the fact that in all the energy auctions held in the last couple of years, the discount rate for which the auction winners bid was zero. That is, they were all willing to invest in the installation of wind energy, receiving only the market price as payment. Three main reasons can explain this phenomenon as far as we are concerned. Firstly, the costs associated with the installation of a wind farm have plummeted compared to 2013. It is increasingly cheaper to invest in wind energy deployment. Second, since the annual drift of the market price is endogenous to the installed capacity target and an increasing function of it, as this installation target is so large for the next decade, this parameter increases considerably, and therefore, the market price tends to be higher as well. Moreover, the price of the initial period ($S_0$) is
considerably higher in 2019 than in 2013. Finally, in recent years since the end of the financial crisis, the WACC has been decreasing, and therefore, future payments to investors are discounted at a lower rate.

As we have seen, any of the three schemes analyzed involve negative expected costs (i.e., earnings for the regulator) due to the high market prices. Although this result may seem unrealistic, we must remember that this thesis’s objective is not to obtain completely reliable and accurate results for the future Spanish energy market; rather, our objective is to compare how intermediate schemes behave with respect to the usual FiT. Therefore, it is crucial to point out that, as happened in 2013, the policymaker’s expected costs of the intermediate schemes are again lower than those corresponding to the Fixed-Tariff or to the absence of any subsidy. The same level of investment \(Q_I\) new units of wind power) is encouraged more efficiently, evidence of an indisputable gain compared to the usual FiT schemes.

We have seen how the interests of the policymaker and those of the investor tend to be opposed. Both parties not only have to consider protecting themselves against the risk of low electricity market prices, but they must also consider the risk posed by the possibility that the market-depending parameters for which public subsidies are designed, do not evolve as expected. Thus, the existing trade-off between higher protection against low market prices and higher(lower) expected benefits(costs) that investors(governments) have to face, requires an exhaustive and in-depth analysis which should be handled carefully, and will eventually depend on the particular interest, budget, and the risk that each of the two parties is willing to assume.

### 4 Conclusions

Until they were eliminated in 2013 mainly due to the high costs that they implied for the government, commonly used Feed-in Tariffs have been the main instrument to support investment in renewable energy in Spain. Since then, wind energy investment has been frozen for several years, making it impossible to achieve the installation target set for that regulatory period ending in 2020. Another drawback of the typical FiT schemes is that only one of the parties must assume all the risk. Inspired by the innovation introduced by Farrel et al., by combining sequential game theory, FiT design, and stochastic calculus, we have been able to design and to test intermediate policies that allow risk-sharing between investors and governments, and more importantly, doing it more efficiently.

In this thesis, we have had to devise our own methodology to predict the drift and volatility of the volume-weighted average price of electricity as a function of the amount of installed wind capacity. Even though in our particular case the result can be compromised with a considerable error, due to the fact that in Spain there is not enough openly available data, we believe that this method could be useful in other studies beyond the particular use that we have given to it in this project. Moreover, it could be used for different renewable technologies. Next, we have run stochastic simulations to determine how these schemes would behave in a scenario that tries to approximate the Spanish environment for the years 2013 and 2019. We fully characterized the distribution of potential benefits and costs under each scheme. Finally, we carried out a sensitivity analysis to analyze the effects on the expected benefits and costs under each policy if different market-depending parameters do not evolve as expected.

According to the results we obtained, it is possible to design Shared Upside and Cap & Floor schemes that allow achieving the same level of investment in renewables at a lower cost for the government. Then, investors and regulators can adjust these more efficient schemes to modify the value of the expected costs and benefits, as well as the risk each of the parties has to assume. Besides, our results reflect that there is an unavoidable conflict between the interests of investors and governments, which tend to be opposed. Furthermore, each of the parties faces their own trade-off: in the case of investors, they might have to choose between higher expected profits and lower risk of under remuneration. In turn, regulators might be forced to choose between lower expected costs and lower risk of public cost overruns. Furthermore, we have shown how the sensitivity analysis can offer some interesting insight to both investors and regulators when designing each policy.
Our results have interesting policy implications for future renewable energy regulation. Although the methodology performed may seem complex to manage at first glance, the irruption of technologies such as artificial intelligence, machine learning, and big data, allow this type of analysis to be automated and extended. It would be feasible that such tools that could be programmed nowadays without too many difficulties and outlay, carry out those simulations for all the possible values of the efficient parameters of the FiT, and that given the preferences of the investors, and some technical restrictions, such as, the budget of the government for renewable energy support, would return to the policymaker the optimal tariff and parameter choice that facilitates the desired investment. Also, unlike in Spain, where the parameters of the standard FiT and FiP were reviewed every several years, this automatable procedure would allow updates of the parameters of the FiTs in place much more frequently, which would be an undoubted improvement in efficiency for both parties.

As we have pointed out on several occasions, the purpose of this thesis is not to characterize or predict with accuracy the financial results that a particular policy would have, but rather, to test how each of the analyzed policies behaves compared to the rest. This is because the methodology that we have developed has several limitations in order to consider the obtained figures as reliable. Firstly, the limitations mentioned above in estimating the parameters $\mu$ and $\sigma$. Secondly, the obtainment of analytical solutions to efficient configurations of each tariff does not come without a cost, the high value obtained for the drift of the GBM has the consequence that when we carry out the simulations a large number of times, in some of those trials the legal maximum €180.3 MWh is exceeded. Finally, the various approximations made to obtain closed-form solutions, such as taking the identical costs per MW ($C$) for the entire project, or assuming constant wind generation ($G_t$) over time, will compromise the results.

This thesis demonstrates how the FiT schemes used in Spain until 2013 are improvable in numerous aspects. However, the design of the optimal support scheme for a specific year has not been carried out, given that such a task requires a much longer time and extension than what is understood to comprise this master’s thesis, and especially, a very accurate calibration. Nevertheless, we believe that this exercise can be carried out in the foreseeable future. We propose that the next step in the direction of this research is to give more weight to numerical methods and to play down the importance of obtaining closed-form solutions. By doing so, we believe that some of the mentioned limitations could be overcome, such as the requirement of bounded market prices, and allowing the possibility of relaxing some of the assumptions in which we have based our work. The most remarkable improvement could be to characterize investors’ preferences by a constant relative risk aversion (CRRA) utility function, which would make the obtained results much more general. Another advantage of a fully computational approach could be to include non-constant generation over time ($G_t$) and variable inversion costs ($C$). Furthermore, we believe that a way to overcome the limitations imposed by the obtained values of $\mu$ and $\sigma$ could be instead of considering the VWAP as the market price, to use the wholesale price. Since the VWAP is usually cheaper than the pool price but following similar trends, the wholesale price could be used as an approximation when studying the prices received by investors in renewables. A similar approach is followed by Blazquez et al. (2018) [19]. Finally, the numerically found optimal scheme could be compared to some of the other public support systems for renewables that are or have been commonly used, such as Tradable Green Certificates and Energy Auctions [35]. These tasks are left as open problems for future research.
References


[33] Comisión Nacional de los Mercados y la Competencia (CNMC) [online]. Available: https://www.cnmc.es/2018-11-02-la-cnmc-publica-la metodologia-de-calcu-lo-de-la-tasa-de-retribucion-financiera-de-las


Appendix A: Solving Equation 10

\[
\frac{\partial f}{\partial t'} + \mu S \frac{\partial f}{\partial S} + \frac{\sigma^2 S^2 \partial^2 f}{2 \partial S^2} - rf = 0 \quad f(S,0) = h(S) \quad (54)
\]

First, we define the time to maturity \( \tau = t - t' \). Then we have: \( \frac{\partial}{\partial \tau} = - \frac{\partial}{\partial t'} \). Substituting in Eq.54 and multiplying the whole equation by \( \frac{2}{\sigma^2} \):

\[
- \frac{2}{\sigma^2} \frac{\partial f}{\partial \tau} + \frac{2\mu S}{\sigma^2} \frac{\partial f}{\partial S} + S^2 \frac{\partial^2 f}{\partial S^2} - \frac{2r}{\sigma^2} f = 0 \quad f(S,\tau = t) = h(S) \quad (55)
\]

In order to replace variables \( r \) and \( \mu \), we define \( a = \frac{2r}{\sigma^2} \), and \( b = \frac{2\mu}{\sigma^2} \), which yields to

\[
- \frac{2}{\sigma^2} \frac{\partial f}{\partial \tau} + bS \frac{\partial f}{\partial S} + S^2 \frac{\partial^2 f}{\partial S^2} - af = 0 \quad f(S,\tau = t) = h(S) \quad (56)
\]

We now replace \( S \) and \( \tau \) in the above equation by defining the variables \( u \) and \( v \):

\[
v = \sigma^2 (b - 1)^2 \tau \quad u = (b - 1) \log \left( \frac{S}{K} \right) + v \quad (57)
\]

According to the chain rule to \( u(S,\tau) \) and \( v(S,\tau) \):

\[
\frac{\partial}{\partial S} = \left( \frac{\partial v}{\partial S} \right) \frac{\partial}{\partial v} + \left( \frac{\partial u}{\partial S} \right) \frac{\partial}{\partial u} \quad (58)
\]

\[
\frac{\partial}{\partial \tau} = \left( \frac{\partial v}{\partial \tau} \right) \frac{\partial}{\partial v} + \left( \frac{\partial u}{\partial \tau} \right) \frac{\partial}{\partial u} \quad (59)
\]

From equation 57, solving for \( S \) and \( \tau \), it is straightforward to obtain:

\[
S(u,v) = Ke^{\left( \frac{u}{b-1} \right)} \quad (60)
\]

\[
\tau(u,v) = \frac{2v}{\sigma^2 (b - 1)^2} \quad (61)
\]

We find the Jacobi matrix \((J)\) for the coordinate transformation \((S(u,v), \tau(u,v))\):

\[
J = \left( \begin{array}{cc}
\frac{\partial S}{\partial u} & \frac{\partial S}{\partial v} \\
\frac{\partial \tau}{\partial u} & \frac{\partial \tau}{\partial v}
\end{array} \right) = \left( \begin{array}{cc}
\frac{S}{(b-1)} & -\frac{S}{(b-1)} \\
0 & \frac{2}{\sigma^2 (b - 1)^2}
\end{array} \right)
\quad (62)
\]

From the general properties of the Jacobi matrix, it is easy to prove that the partial derivatives of the inverse transformation \((u(S, \tau), v(S, \tau))\) are given by \( J^{-1} \). That is:
\[
\begin{pmatrix}
\frac{\partial u}{\partial S} & \frac{\partial u}{\partial \tau} \\
\frac{\partial v}{\partial S} & \frac{\partial v}{\partial \tau}
\end{pmatrix} = J^{-1} = \begin{pmatrix}
\frac{b-1}{S} & \frac{\sigma^2(b-1)^2}{2} \\
0 & \frac{\sigma^2(b-1)^2}{2}
\end{pmatrix}
\]

which yields to
\[
\frac{\partial}{\partial S} = \left(\frac{b-1}{S}\right) \frac{\partial}{\partial u}
\]
\[
\frac{\partial}{\partial \tau} = \frac{\sigma^2(b-1)^2}{2} \left(\frac{\partial}{\partial u} + \frac{\partial}{\partial v}\right)
\]

In addition, from Eq.64, we obtain:
\[
\frac{\partial^2}{\partial S^2} = \frac{\partial}{\partial S} \left(\frac{b-1}{S}\right) \frac{\partial}{\partial u} = -\frac{(b-1)}{S^2} \frac{\partial}{\partial u} + \frac{(b-1)^2}{S^2} \frac{\partial^2}{\partial u^2}
\]

Substituting these three last expressions in equation 56, we obtain:
\[
-\frac{\partial f}{\partial v} + \frac{\partial^2 f}{\partial u^2} - \frac{af}{(b-1)^2} = 0
\]

With the additional change of function \(f(S, t') = f(S, t - \tau) \rightarrow g(u, v)\), defined by:
\[
f(S, t - \tau) = e^{-r\tau} g(u, v) = e^{-\frac{av}{(b-1)^2}} g(u, v)
\]

which yields to:
\[
\frac{\partial f}{\partial u} = e^{-\frac{av}{(b-1)^2}} \frac{\partial g}{\partial u} \quad \frac{\partial f}{\partial v} = e^{-\frac{av}{(b-1)^2}} \left(\frac{\partial g(u, v)}{\partial v} - \frac{ag(u, v)}{(b-1)^2}\right)
\]

Substituting \(f\), \(\frac{\partial f}{\partial u}\) and \(\frac{\partial f}{\partial v}\) in 67, finally yields to a very well known PDE: the heat equation.
\[
\frac{\partial^2 g(u, v)}{\partial u^2} = \frac{\partial g(u, v)}{\partial v} \quad g(u, 0) = g_0(u)
\]

In order to solve this equation, we use the Fourier Transform method to solve PDEs. The Fourier transform \((\mathcal{F})\) of the function \(g(u)\) for \(-\infty < u < \infty\) is given by
\[
\mathcal{F}[g(u)] = F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(u) e^{-izu} du
\]

Thus, it is easy to verify that the transformation of the partial derivatives \(\frac{\partial^2 g(u, v)}{\partial u^2}\), and \(\frac{\partial g(u, v)}{\partial v}\) are given by the following formulas
\[
\mathcal{F} \left[ \frac{\partial^2 g}{\partial u^2} \right] = -z^2 \mathcal{F}[g] \quad \mathcal{F} \left[ \frac{\partial g}{\partial v} \right] = \frac{\partial}{\partial v} \mathcal{F}[g]
\]
Hence, denoting \( \mathcal{F}[g] = G(v) \), and \( \mathcal{F}[g_0] = G_0(Z) \), equation 70 becomes

\[
\frac{dG(v)}{dv} = -z^2 G(v) \quad \text{and} \quad G(0) = G_0(z), \tag{74}
\]

Thus, the solution to this problem is obviously given by:

\[
G(v) = G_0(z)e^{-z^2v} \tag{75}
\]

Now, finding the inverse transform \( (\mathcal{F}^{-1}) \), it can be verified that the solution to equation 70 is defined by the following expression:

\[
g(u, v) = \mathcal{F}^{-1}[G_0(z)e^{-z^2v}] = \frac{1}{2\sqrt{\pi}v} \int_{-\infty}^{\infty} g_0(z)e^{-(u-z)^2/v} \, dz \tag{76}
\]

Finally, replacing \( u(S, \tau), v(S, \tau), g_0 \) and substituting \( z = (b - 1) \log \left( \frac{x}{K} \right) \), we obtain the original terminal condition \( h(x) \):

\[
f(S', t) = \frac{e^{-r\tau}}{\sigma \sqrt{2\pi \tau}} \int_{0}^{\infty} h(x) \exp \left( -\frac{\left( \log \left( \frac{x}{S'} \right) - (\mu - \frac{\sigma^2}{2})\tau \right)}{2\sigma^2\tau} \right) \, dx \tag{77}
\]

- All the methods applied during these calculations can be checked out in:

Appendix B: Construction of the dataset (R code)

```r
rm(list=ls())
years=2014:2019
for (a in 1:length(years)){
  year=years[a]
  setwd("/Users/peio/desktop/TFM_ordenado/R")
  n=365
  if (year==2008|year==2012|year==2016|year==2020){n=366}
  h=n*24
  origindate=paste(as.character(year),"-01-01",sep="")
  file.1=paste("/Users/peio/desktop/DATOS/REE/",as.character(year),"/Custom-Report-",as.Date(0,origin = origindate),"-Estructura de generación (MW).csv",sep="")
  datafile.1 = read.csv(file.1, header=F, sep="",)
  datafile.1=datafile.1[-c(1:21),]
  datafile.1=datafile.1[-c(146:163),]
  m1=length(datafile.1[,2])
  m2=length(datafile.1[10,])
  for (j in 2:m2) {
    datafile.1[,j]=as.numeric(paste(datafile.1[,j]))
  }
  if (year>2015) {
    aux8to9=datafile.1[,8]
    aux9to10=datafile.1[,9]
    datafile.1[,8]=datafile.1[,12]+datafile.1[,13]
    datafile.1[,11]=datafile.1[,10]+datafile.1[,11]
    datafile.1[,9]=aux8to9
    datafile.1[,10]=aux9to10
  }
  colnames(datafile.1)=c("Hora","Eolica","Nuclear","Fuel/Gas","Carbón","Ciclo Combinado","Hidraulica","Resto.Reg.Especial","Intercambios","Enlace Balear","Solar")
  datafile.1=datafile.1[,c(1:11)]
}
data_horario.1=data.frame("Hora" = c(1:24),
"Eolica"=c(1:24),
"Nuclear"=c(1:24),
"Fuel/Gas"=c(1:24),
"Carbón"=c(1:24),
"Ciclo Combinado"=c(1:24),
"Hidraulica"=c(1:24),
"Resto.Reg.Especial"=c(1:24),
"Intercambios"=c(1:24),
"Enlace Balear"=c(1:24),
"Solar"=c(1:24))
```

for (i in c(1:24)) {
  inic=6*(i-1)+1
  final=6*i
  data_horario.1[i,1]=i
  data_horario.1[i,2]=mean(datafile.1[,2][c(inic:final)],na.rm = T)
  data_horario.1[i,3]=mean(datafile.1[,3][c(inic:final)],na.rm = T)
  data_horario.1[i,4]=mean(datafile.1[,4][c(inic:final)],na.rm = T)
  data_horario.1[i,5]=mean(datafile.1[,5][c(inic:final)],na.rm = T)
  data_horario.1[i,6]=mean(datafile.1[,6][c(inic:final)],na.rm = T)
  data_horario.1[i,7]=mean(datafile.1[,7][c(inic:final)],na.rm = T)
  data_horario.1[i,8]=mean(datafile.1[,8][c(inic:final)],na.rm = T)
  data_horario.1[i,9]=mean(datafile.1[,9][c(inic:final)],na.rm = T)
  data_horario.1[i,10]=mean(datafile.1[,10][c(inic:final)],na.rm = T)
  data_horario.1[i,11]=mean(datafile.1[,11][c(inic:final)],na.rm = T)
  data_horario.1[i,12]=year
  data_horario.1[i,13]= 1
  data_horario.1[i,14]= 1
}
data.horario=data_horario.1

#-------------------------#
for (k in 1:(n-1)) {
  year=years[a]
  setwd("/Users/peio/desktop/TFM_ordenado/R")
  n=365
  if (year==2008 | year==2012 | year==2016 | year==2020){n=366}
  h=n*24
  origindate=paste(as.character(year),"-01-01",sep="")
  file.1=paste("/Users/peio/desktop/DATOS/REE/",as.character(year),"/Custom-Report-",as.Date(k, origin = origindate),"-Estructura de generación (MW).csv",sep="")
  datafile.1 = read.csv(file.1, header=F, sep="")
  datafile.1=datafile.1[-c(1:21),]
  datafile.1=datafile.1[-c(146:163),]
  m1=length(datafile.1[,2])
  m2=length(datafile.1[10,])
  for (j in 2:m2) {
    datafile.1[,j]=as.numeric(paste(datafile.1[,j]))
  }
  if (year<2015 | (year==2015 & k<120)) {datafile.1[,8]=datafile.1[,8]+datafile.1[,11]}
  if (year>2015 | (year==2015 & k>=120)) {
    aux8to9=datafile.1[,8]
    aux9to10=datafile.1[,9]
    datafile.1[,8]=datafile.1[,12]+datafile.1[,13]
    datafile.1[,11]=datafile.1[,10]+datafile.1[,11]
datafile.1[,9]=aux8to9
datafile.1[,10]=aux9to10

colnames(datafile.1)=c("Hora","Eolica","Nuclear","Fuel/Gas","Carbón","Ciclo Combinado","Hidraulica","Resto.Reg.Especial","Intercambios","Enlace Balear","Solar")
datafile.1=datafile.1[,c(1:11)]

data_horario.1=data.frame("Hora"=c(1:24),
"Eolica"=c(1:24),
"Nuclear"=c(1:24),
"Fuel/Gas"=c(1:24),
"Carbón"=c(1:24),
"Ciclo Combinado"=c(1:24),
"Hidraulica"=c(1:24),
"Resto.Reg.Especial"=c(1:24),
"Intercambios"=c(1:24),
"Enlace Balear"=c(1:24),
"Solar"=c(1:24),
"Year"=c(1:24),
"Month"=c(1:24),
"Day"=c(1:24))

mes.nb=c(31,28,31,30,31,30,31,31,30,31,30,31)
mes.b=c(31,29,31,30,31,30,31,31,30,31,30,31)
cmes.nb=cumsum(mes.nb)-1
cmes.b=cumsum(mes.b)-1


for (i in c(1:24)) {
  inic=6*(i-1)+1
  final=6*i
  data_horario.1[i,1]=i
  data_horario.1[i,2]=mean(datafile.1[,2][c(inic:final)],na.rm=T)
  data_horario.1[i,3]=mean(datafile.1[,3][c(inic:final)],na.rm=T)
  data_horario.1[i,4]=mean(datafile.1[,4][c(inic:final)],na.rm=T)
  data_horario.1[i,5]=mean(datafile.1[,5][c(inic:final)],na.rm=T)
  data_horario.1[i,6]=mean(datafile.1[,6][c(inic:final)],na.rm=T)
  data_horario.1[i,7]=mean(datafile.1[,7][c(inic:final)],na.rm=T)
  data_horario.1[i,8]=mean(datafile.1[,8][c(inic:final)],na.rm=T)
  data_horario.1[i,9]=mean(datafile.1[,9][c(inic:final)],na.rm=T)
  data_horario.1[i,10]=mean(datafile.1[,10][c(inic:final)],na.rm=T)
  data_horario.1[i,11]=mean(datafile.1[,11][c(inic:final)],na.rm=T)
  data_horario.1[i,12]=year

  if((year%in%yearB & 0<k & k<=cmes.b[1])) |
  (!((year%in%yearB) & 0<k & k<=cmes.nb[1])) {data_horario.1[i,13]=1}
  if((year%in%yearB & cmes.b[1]<k & k<=cmes.b[2])) |
  (!((year%in%yearB) & cmes.nb[1]<k & k<=cmes.nb[2])) {data_horario.1[i,13]=2}
  if((year%in%yearB & cmes.b[2]<k & k<=cmes.b[3])) |
  (!((year%in%yearB) & cmes.nb[2]<k & k<=cmes.nb[3])) {data_horario.1[i,13]=3}
  if((year%in%yearB & cmes.b[3]<k & k<=cmes.b[4])) |
  (!((year%in%yearB) & cmes.nb[3]<k & k<=cmes.nb[4])) {data_horario.1[i,13]=4}

  }
(! (year %in% yearB) & cmes.nb[3]<k & k<=cmes.nb[4])) {data_horario .1[i,13]=4}
if((year%in%yearB & cmes.b[4]<k & k<=cmes.b[5]) {data_horario .1[i,13]=5}
if((year%in%yearB & cmes.b[5]<k & k<=cmes.b[6]) {data_horario .1[i,13]=6}
if((year%in%yearB & cmes.b[6]<k & k<=cmes.b[7]) {data_horario .1[i,13]=7}
if((year%in%yearB & cmes.b[7]<k & k<=cmes.b[8]) {data_horario .1[i,13]=8}
if((year%in%yearB & cmes.b[8]<k & k<=cmes.b[9]) {data_horario .1[i,13]=9}
if((year%in%yearB & cmes.b[9]<k & k<=cmes.b[10]) {data_horario .1[i,13]=10}
if((year%in%yearB & cmes.b[10]<k & k<=cmes.b[11]) {data_horario .1[i,13]=11}
if((year%in%yearB & cmes.b[11]<k & k<=cmes.b[12]) {data_horario .1[i,13]=12}

} data.horario=rbind(data.horario, data_horario.1)

file.1=paste("/Users/peio/desktop/DATOS/Omie/", as.character(year) 
,"/marginalpdbc_",gsub("-", ",", as.Date(0, origin = origindate)), ",1", sep="")
datafile.omie = read.csv(file.1, header=F, sep=";")
colnames(datafile.omie)=c("Año" ,"Mes","Dia","Hora","Precio_PT","Precio_ES","nas")
datafile.omie = datafile.omie[as.na(datafile.omie[,c(2:25),c(1:6)])
datafile.omie = datafile.omie[-25,]
OMIE.data=datafile.omie

for (j in 1:(n-1)) {
  file.1=paste("/Users/peio/desktop/DATOS/Omie/",as.character(year) 
,"/marginalpdbc_",gsub("-", ",",as.Date(j, origin = origindate)), ",1",sep="")
datafile.omie = read.csv(file.1, header=F, sep=";")
colnames(datafile.omie)=c("Año" ,"Mes","Dia","Hora","Precio_PT","Precio_ES","nas")
datafile.omie = datafile.omie[c(2:25),c(1:6)]
datafile.omie = datafile.omie[-25,]
OMIE.data=rbind(OMIE.data, datafile.omie)
}

OMIE.data$Precio_ES[which(is.na(OMIE.data$Precio_ES))]=0
row.names(OMIE.data)=as.character(c(1:h))
row.names(OMIE.data)=as.character(c(1:h))
if (year<=2009){
  OMIE.data$Precio_PT=OMIE.data$Precio_PT*10
  OMIE.data$Precio_ES=OMIE.data$Precio_ES*10
}
data.horario=OMIE.data$Precio_ES

write.csv(data.horario, paste("Datos_2014horarios", as.character(year), sep=""), row.names = FALSE)
rm(list=ls())
years=2014:2019

year=years[1]
setwd("/Users/peio/desktop/TFM_ordenado/R")
file.1=paste("Datos_2014horarios",as.character(year),sep="")
datafile.1 = read.csv(file.1, header=T, sep="",)
data.horario=datafile.1

for (a in 2:length(years)) {
  year=years[a]
  file.1=paste("Datos_2014horarios",as.character(year),sep="")
datafile.1 = read.csv(file.1, header=T, sep="",)
data.horario=rbind(data.horario,datafile.1)
}

write.csv(data.horario,"Datos_2014horarios", row.names = FALSE)

week=rep(c(rep(c(3,24),
            rep(c(4),24),
            rep(c(5),24),
            rep(c(6),24),
            rep(c(7),24),
            rep(c(1),24),
            rep(c(2),24)),313)[1:52584]

for (l in 1:52584) {
  data.horario$Day[l]=week[l]
}

data.horario$`2014`=0
data.horario$`2015`=0
data.horario$`2016`=0
data.horario$`2017`=0
data.horario$`2018`=0
data.horario$`2019`=0

data.horario$`h1`=0
data.horario$`h2`=0
data.horario$`h3`=0
data.horario$`h4`=0
data.horario$`h5`=0
data.horario$h6=0
data.horario$h7=0
data.horario$h8=0
data.horario$h9=0
data.horario$h10=0
data.horario$h11=0
data.horario$h12=0
data.horario$h13=0
data.horario$h14=0
data.horario$h15=0
data.horario$h16=0
data.horario$h17=0
data.horario$h18=0
data.horario$h19=0
data.horario$h20=0
data.horario$h21=0
data.horario$h22=0
data.horario$h23=0
data.horario$h24=0

data.horario$d1=0
data.horario$d2=0
data.horario$d3=0
data.horario$d4=0
data.horario$d5=0
data.horario$d6=0
data.horario$d7=0

data.horario$m1=0
data.horario$m2=0
data.horario$m3=0
data.horario$m4=0
data.horario$m5=0
data.horario$m6=0
data.horario$m7=0
data.horario$m8=0
data.horario$m9=0
data.horario$m10=0
data.horario$m11=0
data.horario$m12=0

years=2014:2019
m=1:12
d=1:7
h=1:24

for (l in 1:52584) {
  for (j in 1:6) {
    if(data.horario$Year[l]==years[j]) {data.horario[l,(15+j)]=1}
  }
  for (j in 1:24) {
    if(data.horario$Hora[l]==h[j]) {data.horario[l,(21+j)]=1}
  }
}
```r
for (j in 1:7) {
  if(data.horario$Day[1]==d[j]){data.horario[1,(45+j)]=1}
}
for (j in 1:12) {
  if(data.horario$Month[1]==m[j]){data.horario[1,(52+j)]=1}
}
}
write.csv(data.horario,"Datos_2014horarios_panel", row.names = FALSE)

# References:
# [1]
# OMIE. Hourly prices Spanish day-ahead market (marginalpdbc_YYYYMMDD.txt).
# https://www.omie.es/es/file-access-list?parents%5B0%5D=/&parents%5B1%5D=
# Mercado%20Diario&parents%5B2%5D=Precios&dir=Precios%20horarios%20del%20mercado
# %20horarios%20del%20mercado&realdir=marginalpdbc
# (Last time accessed: 23 July, 2020)

# [2]
# Red Eléctrica de España (REE). Hourly electricity generation:
# https://demanda.ree.es/visiona/peninsula/demanda/tablas/
# (Last time accessed: 23 July, 2020)
```
Appendix C: Minimum or Maximum?

We check for each of the four subsidy schemes, whether the singular points defined by the equations (29), (34), (40) and (46), are maximum points of (19) for the installation target $Q_I$. For both 2013 and 2019, the analyzed points in Fixed Tariff, Shared-Upside, and Cap & Floor policies are maximum points for the installation target $Q = Q_I$. On the contrary, in the case of Constant Premium, the values given by equation (34) for the parametrization of both years, correspond to minimum points. Furthermore, we see how for SU and C&F schemes, the singular points can either be maximum or minimum depending on the particular value of the efficient price floor ($K$) chosen.

2013

Fixed Tariff

Solution of Eq.(29): $K_A = 87.838$

![Figure A1: FT 2013](image)

Constant Premium

Solution of Eq.(34): $X = 8.1963$
Shared Upside

$K_C$ ranging from 55 to 85

Cap & Floor

$K_D$ ranging from 55 to 85
2019

Fixed Tariff

Solution of Eq.(29): $K_A = 63.4575$
Constant Premium

Solution of Eq. (34): $X = -43.2159$

![Graph showing solution of Eq. (34)](image)

Figure A6: CP 2019

Shared Upside

$K_C$ ranging from 25 to 65

![Graph showing $K_C$ ranging from 25 to 65](image)

Figure A7: SU 2019
Cap & Floor

$K_D$ ranging from 5 to 65

Figure A8: C&F 2019