Timing of Wage Setting when Firms Invest in R&D.

Juan Carlos Bárcena Ruiz y María Luz Campo Corredera
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5-12-2003

Juan Carlos Bárcena-Ruiz and María Luz Campo*

Departamento de Fundamentos del Análisis Económico I
Universidad del País Vasco

Abstract

In this paper, we analyze the effect that the timing of wage setting (i.e. whether wages are set sequentially or simultaneously) has on the investment in R&D of firms, when that investment increases the productivity of labor, in the context of a Cournot duopoly. Contrary to the result obtained in the literature on wage bargaining, we obtain that unions may choose to set wages simultaneously. This is obtained if the size of the market is small enough and the efficiency of the R&D technology is great enough. It is in this case that firms spend most on R&D. By contrast, when unions choose to set wages sequentially, spending by firms on R&D is at its lowest.

Keywords: Unions, R&D, productivity of labor, wage setting.

JEL Classification: J51, L13, O31.

Corresponding Address: Dpto. de Fundamentos del Análisis Económico I. Universidad del País Vasco. Avda. Lehendakari Aguirre, 83. 48015 Bilbao, Spain. Phone: (34) 94 601 38 33; Fax: (34) 94 601 38 91; e-mail: jepcacom@bs.ehu.es (M.L. Campo), jepharuj@bs.ehu.es (J.C. Bárcena-Ruiz).

* Financial support from Ministerio de Ciencia y Tecnología and FEDER (BEC2003-04430) and UPV (Subvención a Grupos) is gratefully acknowledged. The usual disclaimer applies.
1. Introduction

The interaction between oligopolistic product markets and unionized labor markets has been studied in the literature on wage bargaining in terms of two main bargaining structures: one in which firms negotiate with independent unions at the firm level and one in which each firm bargains with an industry-wide union; in both structures, wage negotiations can take place either simultaneously or sequentially (see, for example, Horn and Wolinsky, 1988; Davidson, 1988; Dobson, 1994; Bárcena-Ruiz, 2003). These studies have been extended to consider the interaction between different union-firm bargaining structures and innovation under oligopoly (see, for example, Tauman and Weiss, 1987; Ulph and Ulph, 1998; Calabuig and González-Maestre, 2002).

The literature that studies the interaction between union-firm bargaining structures and innovation does not consider that wages can be bargained either sequentially or simultaneously, which affects the R&D investments of the firms.\(^1\) To fill this gap in the literature, we analyze how the timing of wage bargaining affects the R&D investments of firms and whether unions prefer to set wages sequentially or simultaneously.

In relation with this last issue, De Fräja (1993), Corneo (1995) and Bárcena-Ruiz and Campo (2000, 2001) show that when firms do not invest in R&D and wage bargaining is decentralized at firm level, unions prefer sequential negotiations. Sequential negotiations lead to higher wages and lower employment than simultaneous bargaining.

The literature is ambiguous in regard to the impact of union rent-seeking behavior on firms’ incentives to invest.\(^2\) Menezes-Filho et al. (1998) argue that most U.S. studies

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\(^1\) Bargaining structures in developed countries differ. In E.U. countries contracts are typically staggered, i.e. different groups bargain at different times (see Layard et al., 1991; Addison and Siebert, 1993). In the U.S., wage negotiations are usually sequential (see Flanagan, 1993). In Japan, wages are negotiated simultaneously in the ‘Spring offensive’ (see Sasajima, 1993).

uncover a negative association between union power and R&D, but the evidence from the few European studies is less compelling. They use micro-econometric evidence in the U.K. from firms and plants, and find that when unions bargain only over wages there is a simple negative relationship between union power and R&D. However, Machin and Wadhwani (1991) provide evidence, for the U.K., that indicates that labor unions can encourage investment. Schnabel and Wagner (1992) show that unions do not appear to have a negative impact on innovative activity in West Germany.

There are papers that find that the effect of unions on investment incentives is negative. In this regard, Grout (1984), Manning (1987) and Van der Ploeg (1987) show that unions can cause underinvestment because employers will be vulnerable to *ex post* exploitation by workers once the capital stock has been accumulated. Ulph and Ulph (1994) show that when unions bargain only over wages then increases in own union power will always reduce the probability of the firm winning a patent race.

However, other papers find that the union effect on investment incentives can be positive. In this regard, Tauman and Weiss (1987) consider the effect of unionization on the adoption of technology in the context of an oligopolistic industry with a small number of firms, some of which are unionized. They show that the higher cost of union labor can induce labor-saving innovation. Ulph and Ulph (1998) show that the presence of a strong union can help a firm to win a patent race. Calabuig and González-Maestre (2002) analyze the effect of union structure on the adoption of innovation considering that workers can set up an independent union in each firm or a single industry-wide union. They show that with a large (small) enough market size, the incentive to innovate is higher under the first (second) type of union.

We consider in our paper that there are two firms that invest in R&D. The investment of each firm increases the productivity of its labor. The cost of R&D is assumed to be quadratic, reflecting the existence of diminishing returns to R&D expenditures (see d’Aspremont and Jacquemin, 1988). The only factor of production is

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3 See, for example, Connolly *et al.* (1986) and Hirsch and Link (1987).
labor and all workers are unionized. To determine the wage set in each firm, we consider
the monopoly-union model (see Booth, 1995). This model assumes that the union chooses
the wage while the firm, once the wage is set by the union, chooses the employment level.
Usually, the timing of wage setting is a long-run decision and, thus, we assume that unions
decide whether wages are set sequentially or simultaneously before firms decide R&D
investments.

The results obtained in this paper are explained by two effects. First, the effect that
arises when unions set wages sequentially (the strategic effect). When firms do not invest
in R&D, the leader union in the sequential wage setting sets a higher wage than the
follower, and both set a higher wage than in the simultaneous game.\(^4\) Secondly, the effect
that the R&D investment of the firms has on the productivity of labor (the productivity
effect). When firms invest in R&D, we obtain that the greater investment in R&D is made
by the firm whose union is the follower in the sequential wage setting (the follower firm);
the lower investment is made by the firm whose union is the leader in the sequential wage
setting (the leader firm).\(^5\) As a result, the productivity of labor is greatest (lowest) in the
follower (leader) firm. Moreover, total expenditure on R&D is greater when wages are set
simultaneously.\(^6\)

When firms do not invest in R&D, the leader union sets a higher wage than the
follower and the number of employees hired by the leader firm is lower than that of the
follower firm since, in this case there is only the strategic effect. However, when firms


\(^5\) We obtain a different result than when firms can decide their investment in R&D, sequentially or
simultaneously (see Madjid et al., 2000). In this case, when both firms invest in R&D, there are no spillovers
and the production cost of the firms is exogenous, it is obtained that the leader makes the greater investment
in R&D and the follower the lower.

\(^6\) This result is consistent with the empirical evidence and helps to explain that evidence in part. In Japan
wages are negotiated simultaneously while in the E.U. and in the U.S. wages are negotiated sequentially. The
GDP share of R&D expenditure in the period 1991-2001 is greater in Japan than in the E.U. and the U.S. For
example, in Japan the GDP share of R&D expenditures in 2001 is 3.09, in the U.S. it is 2.82, and in the E.U.
it is 1.9 (OECD, 2003).
invest in R&D, the wage set by the leader union can be higher or lower than the wage set by the follower union. We obtain that if the efficiency of the technology is low enough, the higher wage is paid to the less productive workers. If the efficiency of the technology is great enough, the more productive workers get the higher wage. Similarly, the number of employees hired by the leader firm can be higher or lower than those hired by the follower firm. We obtain that if the efficiency of the technology is low enough, the firm that has the more productive workers hires the more employees; by contrast, if the efficiency of the technology is high enough the firm with the less productive workers hires more employees. These results depend on whether the productivity effect or the strategic effect is dominant.

The literature on wage bargaining shows that when firms do not invest in R&D unions always prefer to set wages sequentially since they obtain a greater utility than if wages are set simultaneously. By contrast, we show that when firms invest in R&D unions may prefer to set wages simultaneously. This result is obtained if the size of the market is small enough and the efficiency of the R&D technology is great enough, since the productivity effect then dominates the strategic effect. In this case, unions choose to set wages simultaneously and the total expenditure on R&D of the firms is greatest under simultaneous wage setting. By contrast, when unions choose to set wages sequentially, firms’ expenditure on R&D is at its lowest. Therefore, the timing of wage setting chosen by unions can stimulate or reduce total expenditure on R&D by firms.

The rest of the paper is organized as follow. Section 2 states the model. In section 3 we analyze, as a benchmark case, the case in which firms do not invest in R&D. Section 4

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7 Comparing the labor markets of various OECD-member countries we can observe an increasing wage inequality over time for some countries. The literature that analyzes this question (see, for example, Katz and Murphy, 1992; Katz and Autor, 1999; Acemoglu, 1998, 1999; Aghion et al, 1999) points out that there are three main types of inequality: skill-based wage differentials (called college or wage premium), inequality between groups of the same educational level (within group inequality) and age related wage differentials. In this paper we consider the second type of inequality and show that when firms invest in R&D that increases the productivity of labor, the wages set by unions can differ depending on whether wages are set sequentially or simultaneously.
shows the results of the model when firms invest in R&D. Finally, section 5 offers conclusions.

2. The model

We consider a market for a single homogenous good in which there are two firms, \(A\) and \(B\). The industry inverse demand function for the product is:

\[
p = a - q_A - q_B, \quad a > 4r, \quad (1)
\]

where \(p\) is the price and \(q_i\) is the output level of firm \(i\) \((i = A, B)\).

The only factor used in the production process is labor. Firm \(i\) hires \(L_i\) workers with a uniform wage rate \(w_i\), \(i = A, B\). All workers are unionized and there is an independent union in each firm. The utility function of the union of firm \(i\) is:

\[
U_i(w_i, L_i) = (w_i - r)L_i, \quad i = A, B, \quad (2)
\]

where \(r\) is the reservation wage, which can be interpreted as the wage earned in the competitive sector. Unions as well as firms are risk neutral. Unions have the objective of income maximization. To determine the wage set in each firm, we consider the monopoly-union model (see Booth, 1995). This model assumes that the unions set the wage while the firms, once the wage is set by the unions, choose the employment level.

Firm \(i\) invests in R&D which increases the productivity of its labor. We assume, for the sake of simplicity, that there are no R&D externalities. The cost of R&D is assumed to

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8 We assume that \(a > 4r\) to simplify the exposition of the results of the model when comparing the number of employees level hired by the firms under the different wage setting structures. This assumption does not alter the main results of the paper. If we interpret parameter \(a\) as the size of the market, this restriction implies that the size of the market must not be excessively small.
be quadratic, reflecting the existence of diminishing returns to R&D expenditures (see d’Aspremont and Jacquemin, 1988). Therefore, the cost of R&D of firm $i$ is given by:

$$C(x_i) = \frac{\gamma}{2} x_i^2, \gamma > \chi, \ i=A, B, \tag{3}$$

where $\chi = \frac{r\beta^2(1976a + 3129r + \sqrt{3904576a^2 - 11605776ar + 9790641r^2})}{28224}$. Parameter $\gamma$ measures the technology efficiency, so that a low value of the parameter indicates higher efficiency in the R&D technology.

The productive technology in firm $i$ is linear in the amount of labor hired; however, the productivity of labor in firm $i$ depends on its R&D investment:

$$q_i = \frac{L_i}{1 - \beta x_i}, \ i=A, B. \tag{4}$$

From expression (4) we get that $\frac{1}{1 - \beta x_i}$ is the marginal productivity of labor (we denote it as the productivity of labor), for a given value of the investment in R&D, $x_i$. Expression (4) shows that the productivity of labor in firm $i$ increases with its R&D investment. Thus, for a given output level, the labor hired by firm $i$ decreases with its R&D investment ($\frac{dL_i}{dx_i} = -\beta q_i < 0$). Parameter $\beta$ measures how the investment in R&D of firm $i$ affects its number of employees, for a given output level, $q_i$. Thus, for a given $q_i$, the greater the value of this parameter, the lower the number of employees hired by firm $i$.

The profit function of firm $i$ is:

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9 Condition $\gamma > \chi$ assures that $1 - \beta x_i > 0$ ($i=A, B$). This condition also assures that second order conditions hold.
\[ \pi_i = (a - q_i - q_j)q_i - L_i w_i - \frac{\gamma}{2} x_i^2, \quad i \neq j; \quad i, j = A, B, \]  

where, from equation (4), \( L_i = q_i (1 - \beta x_i) \).

We assume that the timing of the wage setting is endogenously determined and is decided by unions since we are assuming the monopoly-union model (and, thus, the wage is set by unions). There is only one production period and unions have to decide whether to set their wage at time \( t=0 \) or at time \( t=1 \). Given that the timing of wage setting is a long-run decision we assume that unions decide whether to set wages sequentially or simultaneously before firms decide their investments on R&D. Therefore, the timing of the game is as follows. In the first stage, unions decide whether to set wages sequentially or simultaneously. In the second stage, firms simultaneously decide their investments on R&D. In the third stage, unions set wages either sequentially or simultaneously. Finally, in the fourth stage, firms make quantity decisions and hire labor. We solve backwards to get a subgame perfect equilibrium.

### 3. Benchmark case: firms do not invest in R&D

Before analyzing the results of the model when firms invest in R&D, we are going to show the results recorded in the relevant literature when unions set wages and firms do not invest in R&D (see De Fraja, 1993; Corneo, 1995; Bárcena-Ruiz and Campo, 2000, 2001). In this case, the game has three stages. In the first stage, unions decide whether to set wages sequentially or simultaneously. In the second stage, unions set wages either sequentially or simultaneously. Finally, in the third stage, firms choose their output and employment levels. We denote the case in which unions set wages simultaneously (sequentially) by superscript SI (SE).

In the third stage, firms choose the output level that maximizes their profits (expression (5)), for \( x_i=0 \). Next, in the second stage, both unions sequentially or simultaneously set the wage that maximizes their utility function (expression (2)). When
unions set wages sequentially, union $i$ sets the wage before union $j$ does. Solving these problems we obtain the following result.

**Lemma 1.** When firms do not invest in R&D, in equilibrium: $w_i^{SE} > w_j^{SE} > w^{SI}$, $q_j^{SE} = L_j^{SE} > q_i^{SI} = L_i^{SE}$, $\pi_j^{SE} > \pi_i^{SI}$, $U_j^{SE} > U_i^{SE} > U^{SI}$.

**Proof.** See Appendix

The result obtained in this lemma is due to the fact that wages are strategic complements and that, in the sequential game, union $i$ sets the wage before union $j$ does (i.e. union $i$ is the leader in the wage setting). When union $i$ sets the wage in the sequential game, this union considers the wage of the other union as given. As a result, union $i$ sets a higher wage than union $j$, and both set higher wage than in the simultaneous game. Thus, firm $i$ looses market share, firm $j$ gains market share and firm $j$ hires more workers than firm $i$ obtaining a higher profit. But the union of firm $j$ will obtain a higher utility than the union of firm $i$ since, in this case, employment outweighs wages in the utility function of unions.

In the first stage, unions decide whether to set wages sequentially or simultaneously. Given that $U_j^{SE} > U_i^{SE} > U^{SI}$, both unions prefer to set wages sequentially. Therefore, when firms do not invest in R&D, we obtain the following result.

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10 We get the usual result, that is, when variables are strategic complements, the leader chooses a higher value of the variable than the follower.

11 It must be noted that if the wage is bargained between firms and unions, where the bargaining power of the firms is $\alpha$ and the bargaining power of the unions is $(1-\alpha)$, we get that $\max\{U_j^{SE}, U_i^{SE}\} > U^{SI}$, where $U_i^{SE} > U_j^{SE}$ if and only if $\alpha>0.2243$. We also obtain that $w_i^{SE} > w_j^{SE} > w^{SI}$ and $L_j^{SE} > L_i^{SE} > L_i^{SE}$ for all $\alpha$. Therefore, if $\alpha$ is low (high) enough, employment has a greater (lower) weight than wages in the utility function of unions.
Lemma 2. When firms do not invest in R&D, in equilibrium both unions set wages sequentially.

4. Results when firms invest in R&D

We first solve the fourth stage of the game, obtaining the equilibrium in the product market. The profit function of firm $i$ is given by expression (5). Solving the first order conditions for profit maximization we obtain the equilibrium output (and employment) levels and profits, as a function of wage rates and R&D investments:

\[
q_i(w_i(x_i, x_j), w_j(x_i, x_j), x_i, x_j) = \frac{a - 2w_i(1 - \beta x_i) + w_j(1 - \beta x_j)}{3},
\]

\[
L_i(w_i(x_i, x_j), w_j(x_i, x_j), x_i, x_j) = \frac{(1 - \beta x_i)(a - 2w_i(1 - \beta x_i) + w_j(1 - \beta x_j))}{3},
\]

\[
\pi_i(w_i(x_i, x_j), w_j(x_i, x_j), x_i, x_j) = \frac{(a - 2w_i(1 - \beta x_i) + w_j(1 - \beta x_j))^2}{9} - \frac{\gamma}{2} x_i^2, i \neq j; i, j = A, B.
\]

(6)

First we analyze the case in which both unions set wages simultaneously.

4.1 Unions set wages simultaneously

In the third stage, unions simultaneously choose the wage that maximizes their utility function (expression (2)):

\[
w_i(w_i(x_i, x_j), x_i, x_j) = \arg \max_{w_i} [(w_i(x_i, x_j) - r) L_i(w_i(x_i, x_j), w_j(x_i, x_j), x_i, x_j)], i \neq j; i, j = A, B.
\]

(7)

where $L_i(w_i(x_i, x_j), w_j(x_i, x_j), x_i, x_j)$ is given by expression (6). Solving the first order condition for (7) we get the wage and employment level of both firms as a function of R&D investments:
\[
    w_i(x_i, x_j) = \frac{5a - 2r(4\beta x_j + \beta x_j - 5)}{15(1 - \beta x_j)}, \quad i \neq j; \quad i, j = A, B. \tag{8}
\]

Equation (8) shows that the wage of firm \( i \) decreases with the rival’s level of R&D, \( x_j \), since it increases the output level of firm \( j \). By contrast, the wage of firm \( i \) increases with its investment in R&D since it increases the productivity of its labor.\(^{12}\)

In the second stage, firm \( i \) chooses R&D investment, \( x_i \), that maximizes its profits. Solving this problem, and substituting in (2), (6) and (8) we get the following result.

**Lemma 3.** When unions set wages simultaneously, the R&D investment of the firms, the wage set by unions, the output and employment levels of the firms, the profit of the firms and the utility of the unions are:

\[
    x_{SI} = \frac{56r \beta (a - r)}{405 \gamma - 56r^2 \beta^2}, \quad w_{SI} = r + \frac{135 \gamma (a - r)}{405 \gamma - 56ar \beta^2}, \quad L_{SI} = \frac{90 \gamma (a - r)(405 \gamma - 56ar \beta^2)}{(405 \gamma - 56r^2 \beta^2)^2},
\]

\[
    q_{SI} = \frac{90(a - r) \gamma}{405 \gamma - 56r^2 \beta^2}, \quad \pi_{SI} = \frac{4 \gamma (a - r)^2 (2025 \gamma - 392r^2 \beta^2)}{(405 \gamma - 56r^2 \beta^2)^2}, \quad U_{SI} = \frac{12150(a - r)^2 \gamma^2}{(405 \gamma - 56r^2 \beta^2)^2}.
\]

We now consider the case in which unions set wages sequentially.

4.2 Unions set wages sequentially

We assume in this case that union \( i \) sets the wage before union \( j \) does. In the third stage, union \( j \) sets the wage bearing in mind that union \( i \) has already set its wage. Thus, the wage set by union \( j \) is given by the solution to the following problem:

\[
    w_j(w_i(x_i, x_j), x_i, x_j) = \arg\max_{w_j} [w_j(x_i, x_j) - r]L_j(w_i(x_i, x_j), w_j(x_i, x_j), x_i, x_j), \quad i \neq j; \quad i, j = A, B, \tag{9}
\]

\[12\] \( \frac{dw_i}{dx_j} = -\frac{2r \beta}{15(1 - \beta x_j)} < 0, \quad \frac{dw_i}{dx_i} = \frac{5a \beta + 2r \beta (1 - \beta x_j)}{15(1 - \beta x_i)^2} > 0, \quad i \neq j; \quad i, j = A, B. \]
where $L_i(w_i(x_i, x_j), w_j(x_i, x_j), x_i, x_j)$ is given by expression (6). Solving the first order condition for (9) we get that:

$$w_j(w_i(x_i, x_j), x_i, x_j) = \frac{a + 2r(1 - \beta x_j) + w_i(1 - \beta x_i)}{4(1 - \beta x_j)}. \quad (10)$$

Union $i$ sets its wage by solving the following maximization problem:

$$w_i(x_i, x_j) = \arg \max_{w_i} [(w_i(x_i, x_j) - r) L_i(w_i(x_i, x_j), w_j(x_i, x_j), x_i, x_j)], \text{ if } i \neq j; \text{ if } i = A, B,$$

where $w_j(w_i(x_i, x_j), x_i, x_j)$ is given by (10) and $L_i(w_i, x_i, x_j)$ is obtained by substituting (10) in (6). The wages set by unions, depending on R&D levels, are:

$$w_i(x_i, x_j) = \frac{5a - r(7\beta x_i + 2\beta x_j - 9)}{14(1 - \beta x_i)}, \quad w_j(x_i, x_j) = \frac{19a - r(7\beta x_j + 30\beta x_j - 37)}{56(1 - \beta x_j)}. \quad (11)$$

We obtain from expression (11) that the wage paid by each firm increases with its investment in R&D since the productivity of its labor increases with this investment.\(^{13}\)

Let

$$C = \frac{a - r}{1274 r^4 \beta^4 - 15315 r^2 \beta^2 \gamma + 42336 \gamma^2}, \quad D = \frac{a - r}{1274 r^4 \beta^4 - 15 r \beta^2 \gamma (343 \alpha + 678 r) + 42336 \gamma^2},$$

and

$$E = \frac{a - r}{1274 r^4 \beta^4 - 3 \beta \gamma (1976 a + 3129 r) + 42336 \gamma^2}. \quad$$

In the second stage, firm $i$ chooses the R&D investment, $x_i$, that maximizes its profits. Solving this problem, and substituting in (2), (6) and (11) we get the following result.

**Lemma 4.** When unions set wages sequentially, the R&D investment of the firms, the wage set by unions, the output and employment levels of the firms, the profit of the firms and the utility of the unions are:

$$\frac{dw_i}{dx_i} = \frac{5a \beta + 2r \beta (1 - \beta x_j)}{14(1 - \beta x_i)^2} > 0, \quad \frac{dw_j}{dx_j} = \frac{19a \beta - 7r \beta (1 - \beta x_i)}{56(1 - \beta x_j)^2} > 0.$$
\[ x_i^{SE} = 49r\beta C(105\gamma - 26r^2\beta^2), \quad x_j^{SE} = 26r\beta C(228\gamma - 49r^2\beta^2), \]
\[ w_i^{SE} = r + 144\gamma D(105\gamma - 26r^2\beta^2), \quad w_j^{SE} = r + 63\gamma E(228\gamma - 49r^2\beta^2), \]
\[ q_i^{SE} = 84\gamma C(105\gamma - 26r^2\beta^2), \quad q_j^{SE} = 42\gamma C(228\gamma - 49r^2\beta^2), \]
\[ L_i^{SE} = \frac{84\gamma}{D}(105\gamma - 26r^2\beta^2)C^2, \quad L_j^{SE} = \frac{42\gamma}{E}(228\gamma - 49r^2\beta^2)C^2, \]
\[ \pi_i^{SE} = \frac{49\gamma}{2}C^2(228\gamma - 49r^2\beta^2)(105\gamma - 26r^2\beta^2)^2, \]
\[ \pi_j^{SE} = 2\gamma C^2(169r^2\beta^2 - 882\gamma)(228\gamma - 49r^2\beta^2)^2, \]
\[ U_i^{SE} = 12096\gamma^2C^2(105\gamma - 26r^2\beta^2)^2, \quad U_j^{SE} = 2646\gamma^2C^2(228\gamma - 49r^2\beta^2)^2. \]

4.3 Comparison between the two cases

Next we compare the results obtained in lemmas 3 and 4. There are two effects that explain the results obtained in this comparison. First, the strategic effect that arises when unions set wages sequentially (the strategic effect). Secondly, the effect of the R&D investment of the firms on the productivity of labor (the productivity effect).

By comparing the investment in R&D of the firms in the two wage setting structures considered, we obtain the following result.

**Proposition 1.** In equilibrium:

i) \( x_j^{SE} > x_i^{SI} > x_i^{SE} \),

ii) \( \frac{\gamma}{2}(x_j^{SE})^2 > \frac{\gamma}{2}(x_i^{SI})^2 > \frac{\gamma}{2}(x_j^{SE})^2 \), \( \frac{\gamma}{2}(x_j^{SI})^2 \), \( \frac{\gamma}{2}(x_i^{SI})^2 \), \( \frac{\gamma}{2}(x_j^{SI})^2 + \frac{\gamma}{2}(x_j^{SE})^2 \), \( \frac{\gamma}{2}(x_i^{SI})^2 + \frac{\gamma}{2}(x_i^{SE})^2 \)

iii) \( \frac{1}{1 - \beta x_j^{SE}} > \frac{1}{1 - \beta x_i^{SI}} > \frac{1}{1 - \beta x_i^{SE}} \).

**Proof.** See Appendix
This proposition shows that the firm whose union is the follower in the sequential wage setting (the follower firm) invests more than the firms do when their unions set wages simultaneously. The investment in this last case is greater than that of the firm whose union is the leader in the sequential wage setting (the leader firm). Therefore, the follower firm makes the greater expenditure on R&D and the leader firm the lower. Finally, total expenditures on R&D is greater when wages are set simultaneously rather than sequentially.

When firms invest in R&D, they decide their investment simultaneously taking into account that wages are set either sequentially or simultaneously. From equations (8) and (11) we obtain that the wage paid by each firm increases with its R&D investment. Therefore, when deciding its R&D investment, each firm takes into account how its investment affects the wage set by unions.

When firms do not invest in R&D (see lemma 1) it is obtained that \( w_j^{SE} > w_j^{SI} > w_j^{SI} \) due to the strategic effect that arises when wages are set sequentially. When firms invest in R&D, this strategic effect means that, in the sequential case, the wage set by the leader union increases more with the R&D investment of its firm than the wage set in the simultaneous case, and this latter wage increases more than the wage set by the follower union with the R&D of its firm. As a result, although the productivity of labor increases with investment in R&D, the leader firm chooses lower investment to avoid the wage paid to its workers increasing excessively. On the other hand, as R&D decisions are strategic substitutes, the follower firm takes advantage of this situation and chooses greater investment.\(^\text{14}\)

\(^{14}\) Madjid et al. (2000) analyze the case in which firms can decide their investment in R&D sequentially or simultaneously. When both firms invest in R&D, there are no spillovers and the production cost of the firms is exogenous, they obtain that: \( x_i^{SE} > x_i^{SI} > x_j^{SE} \). This result is different from that obtained in proposition 1 since the R&D investments of the firms are strategic substitutes and, thus, the leader in the R&D decision invests more than the follower. The investment in the simultaneous case is between these two values. On the other hand, it can be shown that \( x_i^{SE} > x_i^{SI} > x_j^{SE} \) is also obtained if R&D decisions are taken simultaneously but the output of the firms can be chosen either sequentially or simultaneously.
Given that the cost of R&D increases with investment in R&D we get that
\[ \frac{\gamma}{2}(x_j^{SE})^2 > \frac{\gamma}{2}(x_i^{SI})^2 > \frac{\gamma}{2}(x_j^{SE})^2. \]
Therefore, the greater expenditure in R&D is made by the follower firm and the lower expenditure by the leader firm. However, when unions set wages simultaneously, the aggregated expenditure of the firms on R&D is greater than when unions set wages sequentially since the investment of the follower firm is sufficiently greater than that of the leader firm.

This proposition also shows that the productivity of labor depends on the investment in R&D of the firms and, thus, the productivity of labor differs depending on whether wages are set sequentially or simultaneously. Given that \( x_j^{SE} > x_i^{SI} > x_j^{SE} \), the productivity of labor is highest in the follower firm, lowest in the leader firm, and takes an intermediate value when wages are set simultaneously:
\[ \frac{1}{1 - \beta x_j^{SE}} > \frac{1}{1 - \beta x_i^{SI}} > \frac{1}{1 - \beta x_i^{SE}}. \]

Next we compare the wages set by unions in the two wage setting structures. Let \( \gamma_{w1} \) and \( \gamma_{w2} \), respectively, the values of parameter \( \gamma \) such that \( w_j^{SE} = w_i^{SI} \) and \( w_j^{SE} = w_i^{SE} \), \( \gamma_{w1} < \gamma_{w2} \), where:
\[ \gamma_{w1} = \frac{5635ar^2 + 5310r^2 \beta^2 + r^2 \sqrt[5]{(6350645a^2 - 4909212ar + 5639220r^2)}}{30240}. \]

**Proposition 2.** There exists a value of parameter \( \gamma \), \( \gamma_{w2} \), such that in equilibrium:
\[ w_j^{SE} > w_i^{SI} \geq w_j^{SE} \text{ if } \gamma \leq \gamma_{w1}, \quad w_j^{SE} \geq w_i^{SE} \text{ if } \gamma_{w1} < \gamma \leq \gamma_{w2}, \quad \text{and } w_i^{SE} > w_j^{SI} > w_i^{SE} \text{ if } \gamma_{w2} < \gamma. \]

**Proof.** See Appendix

If the efficiency of the technology is low enough, i.e. if parameter \( \gamma \) is great enough \( (\gamma_{w2} < \gamma) \), we obtain the same result as when firms do not invest in R&D (see lemma 1):
$w_j^{SE} > w_j^{SI} > w_i^{SI}$. Given that the R&D investment of firms decreases with parameter $\gamma$, if this parameter is great enough the investment in R&D of the firms is low enough. Thus, the strategic effect dominates the productivity effect. It must be noted that when firms do not invest in R&D, there is only the strategic effect. In this case, the firm with the less productive workers (firm $i$) pays the higher wage.

If the efficiency of the technology is high enough, i.e. if parameter $\gamma$ is low enough ($\gamma \leq \gamma_w^1$), we obtain that $w_j^{SE} > w_j^{SI} \geq w_i^{SE}$. Given that the R&D investment of firms is great enough (since it decreases with parameter $\gamma$), the productivity effect dominates the strategic effect. Given that $x_j^{SE} > x_j^{SI} > x_i^{SE}$, proposition 1 shows that the productivity of labor is greatest in the follower firm and lowest in the leader firm. On the other hand, as the wage increases with the productivity of labor, the higher wage is paid by the follower firm and the lower by the leader firm. In this case, the more productive workers receive the higher wage and the less productive workers receive the lower wage.

When parameter $\gamma$ takes an intermediate value ($\gamma_w^1 < \gamma \leq \gamma_w^2$), we obtain that $w_j^{SE} \geq w_i^{SE} > w_j^{SI}$. In this case, neither of the two effects dominates; these two effects affect firms in different ways. The productivity effect causes the follower union to set the higher wage in this zone. However, the strategic effect makes the leader union set a higher wage than when unions set wages simultaneously ($w_i^{SE} > w_j^{SI}$). The firms spend most on R&D when wages are set simultaneously, and pay the lowest wages. The highest wage is paid by the firm that has the more productive workers.

$$
\begin{align*}
15 \frac{dx_j^{SI}}{d\gamma} &= - \frac{22680r\beta(a-r)}{(405\gamma - 56r^2\beta^2)^2} < 0, \\
\frac{dx_j^{SE}}{d\gamma} &= - \frac{1764r\beta(a-r)(7345r^4\beta^4 - 61152r^2\beta^2\gamma + 123480\gamma^2)}{(1274r^4\beta^4 - 15315r^2\beta^2\gamma + 42336\gamma^2)^2} < 0, \\
and \frac{dx_j^{SE}}{d\gamma} &= - \frac{11466r\beta(a-r)(1043r^4\beta^4 - 9408r^2\beta^2\gamma + 21888\gamma^2)}{(1274r^4\beta^4 - 15315r^2\beta^2\gamma + 42336\gamma^2)^2} < 0 \text{ since } \gamma > \gamma
\end{align*}
$$
Next we compare the labor hired by firms in the two wage setting structures. Let \( \gamma_{L1} \), \( \gamma_{L2} \) and \( \gamma_{L3} \), respectively, be value of parameter \( \gamma \) such that \( L_j^{SE} = L_j^{SI} \), \( L_j^{SE} = L_i^{SE} \) and \( L_i^{SE} = L_i^{SI} \), where \( \gamma_L < \gamma_{L1} < \gamma_{L2} < \gamma_{L3} \).

**Proposition 3.** There are three values of parameter \( \gamma \), \( \gamma_{L1} \), \( \gamma_{L2} \) and \( \gamma_{L3} \), such that in equilibrium: 
- if \( \gamma \leq \gamma_{L1} \),
- if \( \gamma_{L1} < \gamma \leq \gamma_{L2} \),
- if \( \gamma_{L2} < \gamma \leq \gamma_{L3} \),
- if \( \gamma > \gamma_{L3} \).

**Proof.** See Appendix

The result obtained in proposition 3 is illustrated in Figure 1. We show in the Appendix that the number of employees hired by firms increases with parameter \( \gamma \) since the higher the value of this parameter the lower the efficiency of the technology. As a result, the higher the value of \( \gamma \), the lower the investment in R&D and the greater the labor hired by the firms.

Figure 1. Comparison of the labor hired by the firms.
When the firms do not invest in R&D (see lemma 1), it is obtained that \( w_i^{SE} > w_j^{SE} > w_i^{SI} \), which implies that \( L_j^{SE} > L_j^{SI} > L_i^{SE} \). When firms invest in R&D (see proposition 1) we obtain the same result as when they do not invest in R&D if parameter \( \gamma \) is greater than \( \hat{\gamma}_L \). In this case, as the investment of the firms decreases with parameter \( \gamma \), the investment of the firms is low enough. Therefore, the strategic effect dominates the productivity effect. In this case, the firm with the more (less) productive workers hires the higher (lower) number of employees.

When parameter \( \gamma \) is low enough (\( \gamma \leq \gamma_L \)), the result is due to the productivity effect. In this case, as the investment of the firms decreases with parameter \( \gamma \), this investment is great enough. Therefore, given that \( x_j^{SE} > x_i^{SI} > x_i^{SE} \), the productivity of labor is greatest in the follower firm and lowest in the leader firm. As a result, the follower firm (which has the more productive workers) hires less labor lower and the leader firm (which has the less productive workers) hires more: \( L_i^{SE} > L_i^{SI} \geq L_j^{SE} \).

When parameter \( \gamma \) takes an intermediate value, the labor hired by firms depends on both the wage paid by firms and the productivity of labor. The two effects are mixed up, and thus \( L_i^{SE} \geq L_j^{SE} \geq L_i^{SI} \) if \( \gamma_L < \gamma \leq \gamma_L \), and \( L_j^{SE} > L_i^{SE} \geq L_i^{SI} \) if \( \gamma_L \leq \gamma \leq \gamma_L \). In these two zones, the less labor is hired when unions set wages simultaneously. We have seen that the productivity effect (the strategic effect) implies that the leader firm (the follower firm) hires the least (most) labor. Thus, if \( \gamma_L < \gamma \leq \gamma_L \), the productivity effect causes the leader firm to hire the most labor, while the strategic effect makes \( L_j^{SE} > L_i^{SI} \); as a result: \( L_i^{SE} \geq L_j^{SE} > L_i^{SI} \). If \( \gamma_L \leq \gamma \leq \gamma_L \), the strategic effect makes the follower firm hire most labor, while the productivity effect makes \( L_i^{SE} > L_i^{SI} \); as a result: \( L_j^{SE} > L_i^{SE} \geq L_i^{SI} \).

By comparing the output levels and the profits of the firms in the two wage setting structures considered, we obtain the following result.

**Proposition 4.** In equilibrium:
i) \( q_j^{SE} > q_i^{SI} > q_i^{SE} \),

ii) \( \pi_j^{SE} > \pi_i^{SI} > \pi_i^{SE} \).

**Proof.** See Appendix

When firms invest in R&D we obtain the same result as when firms do not invest in R&D, even though the wage set by the follower union may be higher than the wage set by the leader union. From proposition 1 we have that the productivity of labor increases with investment in R&D and, thus, the workers of the follower firm are more productive than the workers of the firms that set wages simultaneously, and the latter are more productive than the workers of the leader firm. Therefore, the strategic effect and the productivity effect reinforce each other and, thus, \( q_j^{SE} > q_i^{SI} > q_i^{SE} \).

Given that \( q_j^{SE} > q_i^{SI} > q_i^{SE} \), the follower firm has the greater market share and the leader firm the smaller. As a result, although the follower firm has the greater total cost of R&D investment and the leader firm the lower (since \( x_j^{SE} > x_i^{SI} > x_i^{SE} \)) and although the follower firm can pay the greater wage, the greater (smaller) market share of the follower (leader) firm means that \( \pi_j^{SE} > \pi_i^{SI} > \pi_i^{SE} \).

Next we compare the utility obtained by unions in the two wage setting structures. Let \( \gamma_U \) denote the value of parameter \( \gamma \) such that \( U_i^{SE} = U_i^{SI} \), where \( \gamma_U = 11.0419r^2\beta^2 \), \( \gamma_U > \mathcal{Z} \).

**Proposition 5.** In equilibrium: \( U_j^{SE} > U_i^{SE} \geq U_i^{SI} \) if (i) \( \gamma_i \leq \gamma \) when \( a < 78.8088r \) and if (ii) \( a \geq 78.8088r \); \( U_j^{SE} > U_i^{SI} > U_i^{SE} \) if \( \gamma < \gamma_U \) when \( a < 78.8088r \).

**Proof.** See Appendix
When the market is small enough \((a<78.8088r)\), if \(\gamma < \gamma_U\) we obtain that the utility of the follower union is greater than that obtained by the unions in the simultaneous case, and this latter utility is greater than that obtained by the leader union \((U_{j SE} > U_{i SI} > U_{i SE})\); if \(\gamma \geq \gamma_U\), we get that the follower union obtains greater utility than the leader and both obtain greater utility than if wages are set simultaneously \((U_{j SE} > U_{i SE} \geq U_{i SI})\). This last result is also obtained if the market is big enough \((a \geq 78.8088r)\) independently of the value of parameter \(\gamma\).

The result shown in this proposition is illustrated in figure 2. When firms do not invest in R&D (see lemma 1) it is obtained that \(w_{i SE} > w_{j SE} > w_{SI}\) and \(I_{j SE} > I_{i SI} > I_{i SE}\). This implies that \(U_{j SE} > U_{i SE} \geq U_{i SI}\) since the employment level outweighs the wage in the utility function of unions. This result is due to the strategic effect. When firms invest in R&D we also obtain that the follower union obtains the greater utility. We have seen in propositions 2 and 3 that the strategic effect causes the leader union to set a greater wage than the follower and the follower firm to hire more labor than the leader. By contrast, the productivity effect causes the follower union to set a higher wage than the leader, and the leader firm to hire more employees than the follower. Therefore, the strategic effect dominates the productivity effect and, thus, the follower union obtains the greater utility.

Figure 2. Illustration of proposition 5.
By comparing the utility obtained by the leader union with that obtained by unions when wages are set simultaneously, we get (see Appendix) that $U^\text{SE}_i > U^\text{SI}$ if $\gamma^\text{U} < \gamma$. Therefore, if parameter $\gamma$ is great enough ($\gamma^\text{U} < \gamma$), the strategic effect dominates since the investment in R&D of the firms is low enough. By contrast, if parameter $\gamma$ is low enough ($\gamma^\text{U} > \gamma$), the productivity effect dominates since the investment of the firms is great enough. It must be noted that $\gamma^\text{U} < \gamma$ if $a \geq 78.8088r$; therefore, if parameter $a$ is great enough the strategic effect dominates implying that $U^\text{SE}_i > U^\text{SI}$. If $\gamma^\text{U} > \gamma (\gamma^\text{U} < \gamma)$ when $a < 78.8088r$, the productivity effect (the strategic effect) dominates and, thus, $U^\text{SE}_i < U^\text{SI} (U^\text{SE}_i \geq U^\text{SI})$.

4.4 Unions decide whether to set wages at $t=0$ or $t=1$

In stage one, unions decide whether to set wages at $t=0$ or $t=1$. If the two unions set wages simultaneously at $t=0$ or $t=1$, they obtain: $U^00=U^11=U^\text{SI}$. If unions set wages sequentially, one sets wages at $t=0$ and the other at $t=1$; they obtain: $U^\text{SE}_i = U^01$ and $U^\text{SE}_j = U^{10}$. Solving this stage we obtain the following result.

**Proposition 6.** In equilibrium, one union sets its wage at $t=0$ and the other at $t=1$ if (i) $\gamma^\text{U} \leq \gamma$ when $a \leq 78.8088r$ and if (ii) $a \geq 78.8088r$; the two unions set wages at $t=1$ if $a < 78.8088r$ when $\gamma < \gamma^\text{U}$.$^{16}$

This proposition shows that if (i) $\gamma^\text{U} \leq \gamma$ when $a \leq 78.8088r$ and if (ii) $a \geq 78.8088r$ there are two equilibria. In each of them, one union sets wages at $t=0$ and the other at $t=1$. This result is also obtained in the literature (see De Fraja, 1993; Corneo, 1995; Bárcena-Ruiz

$^{16}$ We believe that if we consider that the wage is bargained between firm and unions, the result obtained in this proposition holds since it is due to the strategic and productivity effects, and these effects do not change if the wage is bargained between firm and unions.
and Campo, 2000, 2001). Given that $U_j^{SE} > U_i^{SE}$, a coordination problem might arise in the game since each union would like to set its wage after the other union does. This may lead both unions to choose their wage at $t=1$, which reduces the utility of the unions ($U_j^{SE} > U_i^{SE} > U^{SI}$).

On the other hand, if the market is small enough ($a<78.8088r$) when $\gamma<\gamma_U$, both unions set wages at $t=1$ and, thus, they prefer to set wages simultaneously. When firms do not invest in R&D, unions set wages sequentially since there is only the strategic effect. When firms invest in R&D, the productivity effect dominates the strategic effect, which leads unions to set wages simultaneously.

5. Conclusions

The literature that studies the interaction between union-firm bargaining structures and innovation does not consider that wages can be bargained either sequentially or simultaneously, which affects the R&D investments of firms. To fill this gap in the literature, we have analyzed how the timing of wage bargaining affects investment in R&D by firms, in the context of a duopolistic Cournot competition. We have also studied whether unions prefer to set wages sequentially or simultaneously when firms invest in R&D.

The literature on wage bargaining shows that when firms do not invest in R&D, unions prefer to set wages sequentially rather than simultaneously (see, De Fraja, 1993; Corneo, 1995; Bárcena-Ruiz and Campo, 2000, 2001). This result is due to the effect that arises when unions set wages sequentially (the strategic effect). However, when firms invest in R&D a second effect arises since the investment of firms affects the productivity of labor (the productivity effect). We obtain that the firm whose union is the follower in the sequential wage setting (the follower firm) invests more than firms when unions set wages simultaneously; and investment in this latter case is greater than that of the firm whose union is the leader in the sequential wage setting (the leader firm). As a result, the
total expenditure on R&D is greater when wages are set simultaneously and the productivity of labor is greatest (lowest) in the follower (leader) firm.

We show in the paper that when firms invest in R&D unions may prefer to set wages simultaneously. This result is obtained when the market is small enough and the efficiency of the R&D technology is great enough since, in that case, the productivity effect dominates the strategic effect. In that case, unions set wages simultaneously which stimulates total expenditure on R&D by firms, since this expenditure is greater when wages are set simultaneously rather than sequentially. By contrast, when unions choose to set wages sequentially, firms’ expenditure on R&D is at its lowest. Therefore, the structure of wage setting chosen by unions can stimulate or reduce total expenditure on R&D by firms.

One possible extension of this paper would be to consider a different function to relate investment in R&D of the firms to the productivity of labor. We think that the main results are robust to changes in this function since the results of the paper are due to the strategic and productivity effects, and these effects are present independent of the function considered.

Appendix

Proof of lemma 1.

Next we show the results obtained when firms do not invest in R&D. When unions set wages simultaneously, we obtain:

\[ w^{SI} = \frac{a + 2r}{3}, \quad q^{SI} = L^{SI} = \frac{2(a-r)}{9}, \quad \pi^{SI} = \frac{4(a-r)^2}{81}, \quad U^{SI} = \frac{2(a-r)^2}{27}. \]

When unions set wages sequentially we obtain:
Comparing the results obtained in these two cases, we obtain the result shown in lemma 1.

**Proof of proposition 1.**

Comparing the R&D investments of the firms in the two wage setting structures, we get the following results:

\[ w_i^{SE} = \frac{5a + 9r}{14}, \quad w_j^{SE} = \frac{19a + 37r}{56}, \quad q_i^{SE} = L_i^{SE} = \frac{5(a-r)}{24}, \quad q_j^{SE} = L_j^{SE} = \frac{19(a-r)}{84}, \]
\[ \pi_i^{SE} = \frac{25(a-r)^2}{576}, \quad \pi_j^{SE} = \frac{361(a-r)^2}{7056}, \quad U_i^{SE} = \frac{25}{336} (a-r)^2, \quad U_j^{SE} = \frac{361(a-r)^2}{4704}. \]

Comparing the aggregate expenditure of the firms on R&D in the two wage setting structures:

\[ (1002811264r^6 \beta^6 - 15801503928r^4 \beta^4 \gamma + 78966081180r^2 \beta^2 \gamma^2 - 126177158943 \gamma^3) > 0 \]

**Proof of proposition 2.**

Comparing \( w_i^{SE} \) with \( w_i^{SL} \) we get that:
\[ w_i^{SE} - w_i^{SI} = \frac{9(a-r)\gamma(4186ar^3\beta^4 - 16905ar\beta^2\gamma - 15930r^2\beta^2\gamma + 45360\gamma^2)}{(405\gamma - 56ar\beta^2)(1274ar^3\beta^4 - 5145ar\beta^2\gamma - 10170r^2\beta^2\gamma + 42336\gamma^2)}. \] (A.1)

In expression (A.1), \( \frac{9(a-r)\gamma}{405\gamma - 56ar\beta^2} > 0 \) and \( (1274ar^3\beta^4 - 5145ar\beta^2\gamma - 10170r^2\beta^2\gamma + 42336\gamma^2) > 0 \) since \( \gamma > \gamma \). Then, given that \( \gamma > \gamma \), we get that \( w_i^{SE} = w_i^{SI} \) for \( \gamma = \gamma_1, \gamma_1 > \gamma \), where:

\[ \gamma_1 = \frac{5635ar\beta^2 + 5310r^2\beta^2 + r\beta^2\sqrt{5\sqrt{6350645a^2 - 4909212ar + 5639220r^2}}}{30240}. \]

Therefore, \( w_i^{SE} > w_i^{SI} \) if and only if \( \gamma > \gamma_1 \).

Comparing \( w_j^{SE} \) with \( w_j^{SI} \) we get that:

\[ w_j^{SE} - w_j^{SI} = \frac{18(a-r)\gamma(49ar^3\beta^4 - 228ar\beta^2\gamma + 945r^2\beta^2\gamma + 5670\gamma^2)}{(405\gamma - 56ar\beta^2)(1274ar^3\beta^4 - 5928ar\beta^2\gamma - 9387r^2\beta^2\gamma + 42336\gamma^2)}. \] (A.2)

In expression (A.2), \( \frac{18(a-r)\gamma}{405\gamma - 56ar\beta^2} > 0 \), \( (1274ar^3\beta^4 - 5928ar\beta^2\gamma - 9387r^2\beta^2\gamma + 42336\gamma^2) > 0 \) and \( (49ar^3\beta^4 - 228ar\beta^2\gamma + 945r^2\beta^2\gamma + 5670\gamma^2) > 0 \) since \( \gamma > \gamma \). Therefore, \( w_j^{SE} > w_j^{SI}, \forall \gamma > \gamma \).

Comparing \( w_i^{SE} \) with \( w_j^{SE} \) we get that:

\[ w_i^{SE} - w_j^{SE} = -(9(a-r)\gamma(-126\gamma(3307r^4\beta^4 - 20868r^2\beta^2\gamma + 28224\gamma^2) + 73a(1274r^5\beta^6 - 11073r^3\beta^4\gamma + 23940r\beta^2\gamma^2)))((63\gamma(-149r^2\beta^2 + 672\gamma) + 26a(49r^3\beta^4 - 228r\beta^2\gamma))(18\gamma(-565r^2\beta^2 + 2352\gamma) + 49a(26r^3\beta^4 - 105r\beta^2\gamma))). \] (A.3)
The denominator of expression (A.3) is positive since $\gamma > \gamma$. It can be shown that the numerator of (A.3) decreases with parameter $\gamma$ if and only if $\gamma < \gamma_b$, where

$$
\gamma_b = \frac{6935ar\beta^2 + 10434r^2\beta^2 + r\beta^2\sqrt{48094225a^2 + 8920308ar + 38865780r^2}}{42336}.
$$

If $\gamma = \gamma$, then $w_i^{SE} < w_j^{SE}$. If $\gamma = \gamma_b$ then $w_i^{SE} < w_j^{SE}$. If $\gamma$ tends to $\infty$, then $w_i^{SE} > w_j^{SE}$. Then, there exists a value of parameter $\gamma$, $\gamma_{w_2}$, such that $w_i^{SE} > w_j^{SE}$ if and only if $\gamma > \gamma_{w_2}$ (see figure 3). It remains to compare $\gamma_{w_1}$ with $\gamma_{w_2}$. Substituting $\gamma = \gamma_{w_1}$ in $w_i^{SE} - w_j^{SE}$ we get:

$$
\begin{align*}
\gamma_{w_1} & = \frac{1}{(a - r)} \left[ -2606097340a^4 - 13246295217a^3r + 67055393552a^2r^2 - \\
& \quad 56771526825ar^3 + 356331150r^4 + \sqrt{5} \sqrt{48094225a^2 + 8920308ar + 38865780r^2} (a - r) \\
& \quad (462484a^2 - 1500095ar - 7452585r^2) / (8(42532a - 57375r)) \\
& \quad (1132750a^2 - 3836483ar + 36536685r^2)) \end{align*}
$$

Expression (A.4) is negative since both its numerator and its denominator are positive. Therefore, $\gamma_{w_2} > \gamma_{w_1}$ (see figure 3).

Figure 3. Comparison of $w_i^{SE}$ with $w_j^{SE}$
Proof of proposition 3.

We prove first that \( L_i^S \), \( L_i^SE \) and \( L_j^S \) are strictly increasing with \( \gamma \), \( \forall \gamma > \gamma \).

\[
\begin{align*}
\text{i)} \quad & \frac{dL_i^S}{d\gamma} = -\frac{5040(a-r)r\beta^2(56ar^2\beta^2 + 405(a-2r)\gamma)}{(56r^2\beta^2 - 405\gamma)^3} > 0 \text{ since } \gamma > \gamma. \\
\text{ii)} \quad & \frac{dL_i^SE}{d\gamma} = -(588(a-r)r\beta^2(6028568ar^8\beta^8 - 212940r^6(117a + 452r)\beta^6\gamma \\
& - 34398(8897a - 34422r)r^4\beta^4\gamma^2 + 135(15216509a - 35424414r)r^2\beta^2\gamma^3 - \\
& 381024(8575a - 16628r)\gamma^4)}/(1274r^4\beta^4 - 15315r^2\beta^2\gamma + 42336\gamma^2)^3) \quad \text{(A.5)}
\end{align*}
\]

The denominator of (A.5) is positive since \( \gamma > \gamma \). To obtain the sign of the numerator we divide it in two terms:

\[
I = (588(a-r)r\beta^2(6028568ar^8\beta^8 - 212940r^6(117a + 452r)\beta^6\gamma
\]

\[
II = -34398(8897a - 34422r)r^4\beta^4\gamma^2 + 135(15216509a - 35424414r)r^2\beta^2\gamma^3 - \\
& 381024(8575a - 16628r)\gamma^4 = 27\gamma^2(-1274(8897a - 34422r)r^4\beta^4 + \\
& 5(15216509a - 35424414r)r^2\beta^2\gamma - 14112(8575a - 16628r)\gamma^2)
\]

Both \( I \) as \( II \) are negative since \( \gamma > \gamma \). Therefore, \( \frac{dL_i^SE}{d\gamma} > 0 \).

\[
\begin{align*}
\text{iii)} \quad & \frac{dL_j^S}{d\gamma} = -(84(a-r)r\beta^2(39765362ar^8\beta^8 - 280917r^6(933a + 2086r)\beta^6\gamma \\
& - 343980(4016a - 23415r)r^4\beta^4\gamma^2 + 54(263797976a - 692516979r)r^2\beta^2\gamma^3 - \\
& 762048(37544a - 76915r)\gamma^4))//(1274r^4\beta^4 - 15315r^2\beta^2\gamma + 42336\gamma^2)^3).
\end{align*}
\]
The denominator of (A.6) is positive since \( \gamma > \zeta \). To obtain the sign of the numerator, we divide it in two terms:

\[
III = 39765362a r^8 \beta^8 - 280917r^6 (933a + 2086r) \beta^6 \gamma.
\]

\[
IV = -343980(4016a - 23415r)r^4 \beta^4 \gamma^2 + 54(263797976a - 692516979r)r^2 \beta^2 \gamma^3 - \\
762048(37544a - 76915r)\gamma^4) = 54 \gamma^2(-6370(4016a - 23415r)r^4 \beta^4 + \\
(263797976a - 692516979r)r^2 \beta^2 \gamma - 14112(37544a - 76915r)\gamma^2.
\]

Both III as IV are negative since \( \gamma > \zeta \). Therefore, \( \frac{dL_j^{SE}}{d\gamma} > 0 \).

It can be shown that if \( \gamma = \zeta \), then \( L_i^{SE} > L^S > L_j^{SE} \). On the other hand, If \( \gamma \) tends to \( \infty \), \( L_j^{SE} > L^S > L_i^{SE} \). Finally, if \( \gamma = \frac{1}{4} ar\beta^2 \), then \( L_i^{SE} > L_j^{SE} > L^S \). Given these comparisons and that \( L^S, L_i^{SE} \) and \( L_j^{SE} \) are strictly increasing with \( \gamma \), we get (see Figure 1) that there exists a value of \( \gamma \), \( \gamma_{L1} \), such that \( L_j^{SE} = L^S \); there exists a value of \( \gamma \), \( \gamma_{L2} \), such that \( L_i^{SE} = L_j^{SE} \); finally, there exists a value of \( \gamma \), \( \gamma_{L3} \), such that \( L_i^{SE} = L^S \).

Proof of proposition 4.

Comparing the output levels of the firms in the two wage setting structures, we get the following:

i) \( q_j^{SE} - q_i^{SI} = -\frac{12(a - r)\gamma(49r^4 \beta^4 + 717r^2 \beta^2 \gamma + 5670\gamma^2)}{(56r^2 \beta^2 - 405\gamma)(1274r^4 \beta^4 - 15315r^2 \beta^2 \gamma + 42336\gamma^2)} > 0 \), \( \forall \ \gamma > \zeta \).

ii) \( q_i^{SE} - q_j^{SI} = -\frac{6(a - r)\gamma(1274r^4 \beta^4 - 15r^2 \beta^2 \gamma - 39690\gamma^2}{(56r^2 \beta^2 - 405\gamma)(1274r^4 \beta^4 - 15315r^2 \beta^2 \gamma + 42336\gamma^2)} < 0 \), \( \forall \ \gamma > \zeta \).

Therefore: \( q_j^{SE} > q_i^{SI} > q_i^{SE} \).
Comparing the profits obtained by the firms in the two wage setting structures, we get the following:

\[ i) \quad \pi_{j}^{SE} - \pi_{i}^{SI} = -(162(a - r)^2 \gamma^2(3438232r^8 \beta^8 - 40086263r^6 \beta^6 \gamma + \\
6320652r^4 \beta^4 \gamma^2 + 1188027936r^2 \beta^2 \gamma^3 - 3229178400\gamma^4))/ \\
((56r^2 \beta^2 - 405\gamma)^2(1274r^4 \beta^4 - 15315r^2 \beta^2 \gamma + 42336\gamma^2)^2) > 0, \forall \gamma > \gamma. \]

\[ ii) \quad \pi_{i}^{SI} - \pi_{j}^{SE} = (27(a - r)^2 \gamma^2(148823584r^8 \beta^8 - 3250430484r^6 \beta^6 \gamma + \\
26485636140r^4 \beta^4 \gamma^2 - 95945694453r^2 \beta^2 \gamma^3 + 130224477600\gamma^4))/ \\
(2(56r^2 \beta^2 - 405\gamma)^2(1274r^4 \beta^4 - 15315r^2 \beta^2 \gamma + 42336\gamma^2)^2) > 0, \forall \gamma > \gamma. \]

Therefore: \( \pi_{j}^{SE} > \pi_{i}^{SI} > \pi_{i}^{SE} \).

**Proof of proposition 5.**

Comparing the utility obtained by the unions in the two wage setting structures, we get the following:

\[ i) \quad U_{i}^{SI} - U_{j}^{SE} = -(216(a - r)^2 \gamma^2(49r^4 \beta^4 + 717r^2 \beta^2 \gamma + 5670\gamma^2)/ \\
(19159r^4 \beta^4 - 229008r^2 \beta^2 \gamma + 640710\gamma^2))/((56r^2 \beta^2 - 405\gamma)^2(1274r^4 \beta^4 - \\
15315r^2 \beta^2 \gamma + 42336\gamma^2)^2) < 0, \forall \gamma > \gamma. \]

\[ ii) \quad U_{i}^{SE} - U_{j}^{SE} = 378(a - r)^2 \gamma^2(4825r^4 \beta^4 - 18312r^2 \beta^2 \gamma - 11088\gamma^2)/ \\
(1274r^4 \beta^4 - 15315r^2 \beta^2 \gamma + 42336\gamma^2)^2) < 0, \forall \gamma > \gamma. \]

\[ iii) \quad U_{i}^{SE} - U_{j}^{SI} = (54(a - r)^2 \gamma^2(109673564r^8 \beta^8 - 1923956580r^6 \beta^6 \gamma + \\
11014277175r^4 \beta^4 \gamma^2 - 20861064000r^2 \beta^2 \gamma^3 + 1800338400\gamma^4))/ \\
\]
\[
((56r^2\beta^2 - 405\gamma)^2(1274r^4\beta^4 - 15315r^2\beta^2\gamma + 42336\gamma^2)^2).
\]  \hspace{1cm} (A.7)

It can be shown that (A.7) is positive for \(\gamma > \gamma_U\), where

\[
\gamma_U = \frac{306600r^2\beta^2 + 76655\sqrt{14}r^2\beta^2 + r^2\beta^2 \sqrt{70(2364002765 + 631687848\sqrt{14})}}{105840} = 11.0419r^2\beta^2.
\]

Comparing \(\gamma_U\) with \(\gamma\) we get:

\[
\gamma_U - \gamma = \frac{1}{423360}(r(-29640a + 1179465r + 306620r\sqrt{14} + 4r\sqrt{70(2364002765 + 631687848\sqrt{14} - 15\sqrt{3904576a^2 - 11605776a + 9790641r^2})\beta^2}).
\]

The above expression is equal to zero for \(a=78.8088r\), and thus, \(\gamma_U > \gamma\) if and only if \(a<78.8088r\). Therefore: \(U_i^{SE} > U^{SI}\) if \(a>78.8088r\); if \(a<78.8088r\), \(U_i^{SE} > U^{SI}\) if and only if \(\gamma > \gamma_U\).
References


