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Thermal enhancement and numerical solution of blood nanofluid flow through stenotic artery

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The blood flow through stenotic artery is one of the important research area in computational fluid mechanics due to its application in biomedicine. Aim of this research work is to investigate the impact of nanoparticles on the characteristics of human blood flow in a stenosed blood artery. In under consideration problem Newtonian fluid is assumed as human blood. Newtonian fluid flows through large blood vessels (more than 300 µm). The constitutive equations together with the boundary conditions are diminished to non-dimensional form by using boundary layer approximation and similarity transfiguration to attain the solution of velocity and temperature distribution of blood flow through arterial stenosis numerically with the help of Matlab bvp4c. The results for physical quantities at cylindrical surface are calculated and their effects are also presented through tables. The heat transfer rate increases throughout the stenosed artery with the concentration of copper nanoparticle. Velocity curve decreases by increasing the values of flow parameter and nanoparticle volume fraction. Temperature curve increases due to increase in the values of nanoparticle volume fraction and decrease in Prandtl number.

Abbreviations

(x, r)	Coordinates (m)
R	Radius of artery (m)
L_0	Length (m)
R_0	Width of unblocked region (m)
λ	Maximum height of stenosis
T_0	Temperature of fluid (C^0)
T, θ	Dimensional and dimensionless temperature (C^0)
ϕ	Nanoparticles volume fraction
ψ	Stream function
ν_f	Kinematics viscosity of fluid $(\mathrm{m}^2/\mathrm{s})$
ρ_f	Density of fluid $\left(\mathrm{kg/m}^3\right)$
ρ_{nf}	Density of nanofluid $\left(\mathrm{kg/m}^3\right)$
C_f	Skin friction
η	Similarity index
(u, v)	Velocity components (m/s)
μ_{nf}	Viscosity of nanofluid (Pa/s)
μ_f	Viscosity of fluid (Pa/s)
$\left(C_p\right)_{nf}$	Specific heat of nanofluid (J/kg K)
$\left(C_p\right)_f$	Specific heat of fluid (J/kg K)
T_1	Temperature on boundary (C^0)
k_{nf}	Thermal conductivity of nanofluid (Wm/K)

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k_{bf}	Thermal conductivity of base fluid
f, ϵ	Non-dimensional measures of stenosis
γ	Flow parameter
Pr	Prandtl number
Nu	Nusselt number
τ_w	Wall shear stress
q_w	Heat flux

Today, due to modern lifestyle, smoking, high level of blood cholesterol and possibly a genetic problem we can see that arteries are obstructed. In arterial system the most common disease is hardening and contraction of the walls of blood vessels. In medical sciences it is known as arterial stenosis. Stenosis arteries are a constriction or narrowing of inner surface of arteries which consequently reduced the fluid flow passing to the other organs and tissues. The flow of blood in arterial stenosis is very significant discussion for the understanding of circulatory disorders due to the fact that main cause of many cardiovascular diseases are associated to the mechanical behavior of the blood vessel walls and nature of blood movement.

Biological fluids demonstrate all important information for the monitoring and diagnosis of numerous diseases, as well as for the foundation of the correct treatment. Examples of biological fluids involve blood; lymph, which is produced by the filtration of blood plasma through tissues; synovial fluid in joints; and the vitreous fluid of the eye. One of the important biofluid is blood, just like other bio-fluids. Experientially, it has been demonstrated that blood is the mixture of white blood cells (WBCs), red blood cells (RBCs) and platelets. Blood follows Newtonian nature when flowing through larger area arteries i.e., when the flow shear rate is high and is treated as non-Newtonian when flowing through area of smaller arteries, veins and in the downstream of the stenosis i.e., when the shear rate is below. The most earliest and basic paper about blood flow is of Ref.[1] in which he proposed that the boundary asymmetry can be a major element in the progression and development of arterial disease. Many other researcher[2–6] also analyzed the mathematical models of blood flow under different effects.

The latest and expanded field in the evolution of diagnostics and therapeutics is blood-mediated nanoparticle delivery. To increase function of nanoparticles in biological systems, their surface chemistry, size and shape can be managed. This enables blood clearance profile, transition of immune system interactions and reciprocity with target cells thereby supporting efficacious delivery of contents inside tissues or cells. Heat conduction ability of fluids like blood, water, ethylene glycol, engine oil and polymer solutions is low in the comparison of solids. Thermal conductivity of these fluids enhances by mixing the solid particles of higher thermal conductivity to these fluids. Sheikholeslami and Ebrahimpour[7] studied the addition of nanoparticles to enhance the thermal ability of linear Fresnel solar system. Sheikholeslami et al.[8] analyzed thermal behavior for parabolic solar system with addition of nanofluid. Awais et al.[9] analyzed the addition of gyrotactic effects in bioconvection to stabilize the nanoparticles.

For different biomedical uses a large number of nanoparticles have been increased and some of them have shown great potential in imaging and treatment of diseases[10–18]. These are water, oils, lubricants, organic liquids (e.g., refrigerants, ethylene, triethylene glycols, etc.), biofluids, and polymeric mixture. The nanoparticles used in nanofluids are typically formed of metal oxides (silica, titania, zirconia and alumina), metal nitrides (SiN, AIN), metal carbides (SiC), chemically stable metals (copper, gold, aluminum), different types of carbon (graphite, fullerene, carbon nanotubes, diamond), and functionalization of nanoparticles. It is not a simple liquid–solid mixture; the collection of freely steady suspension for long duration without resulting in any chemical changings in the base fluid is the most important criterion of nanofluid.

Awan et al.[19] presented the non-Newtonian fluid flow study through parallel plates by adding nanofluid and radiation effects. Qureshi et al.[20] described the magnetic field effects during the peristaltic nanofluid flow through flexible tube. Awan et al.[21] analyzed nanofluid flow with heat transfer, radiation and magnetohydrodynamic effects and obtained solution numerically by using backward difference solver. Awan et al.[22] described heat transfer rate and entropy generation peristaltic fluid flow by adding nanoparticles and magnetic effects. Different researchers[9,23–33] paid attention to mathematically analyze the fluid flow problems under addition of nanoparticles and different effects to enhance the heat transfer rate. Awais et al.[34] investigated the impact of heat transportation over porous disk and calculated their solution numerically. Hussain et al.[35] discussed the applications of nanoparticles through diseased artery. Samad Khan et al.[36] studied non-Newtonian nanofluid flow through stretching surface and computed the heat dissipation by the phenomenon of joule heating and viscous dissipation. Awais et al.[37] presented solution methodology based on artificial neural network for MHD non-Newtonian fluid flow problem through stretching sheet. Awais et al.[38] discussed peristaltic motion with magnetic field and entropy generation with addition of copper as nanoparticle and water as base fluid.

With the above motivation, an attempt to develop a mathematical model to investigate the properties of the blood flow in the presence of arterial stenosis with the addition of nanoparticles is made. The nanofluid is considered as a mixture of copper and base fluid blood. Equations that govern the problem are solved and solution is attained numerically. Results for velocity and temperature profile are obtained by using MATLAB software. The physical features of distinct emerging parameters have been explained by plotting the graphs.

Physical modeling

In under discussion problem we assumed that incompressible two dimensional flow of blood behaves as Newtonian fluid through stenosed artery of $\frac{L_0}{2}$ length and the flow field is steady. The coordinates are chosen in this manner that fluid flow towards $x - axis$ and $r - axis$ is considered normal to the fluid flow.

Properties	Nanofluid
Density	$\rho_{nf} = \rho_f\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)$
Viscosity	$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$
Heat capacity	$(\rho C_p)_{nf} = (\rho C_p)_f\left((1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)$
Thermal conductivity	$\frac{k_{nf}}{k_f} = \frac{k_s+2k_{bf}-2\phi(k_{bf}-k_s)}{k_s+2k_{bf}+\phi(k_{bf}-k_s)}$

Table 1. Thermophysical properties of nanofluid[41].

Properties	Blood	Cu
$\rho(\text{kg/m}^3)$	1063	8933
$C_p(\text{Jkg}^{-1}\text{K}^{-1})$	3594	385
$k(\text{Wm}^{-1}\text{K}^{-1})$	0.492	400

Table 2. Base fluid (blood) and nanoparticles experimental values[42].

Consider two dimensional flow of blood in the presence of stenosed region of cosine shape constraint having unblocked area width $2R_0$, radius of the artery is $R(x)$ and the maximum stenosed region height is λ. Profile for stenosed region is chosen as

$$\begin{array}{ll} R(x) = R_0 - \frac{\lambda}{2}\left(1+\cos\left(\frac{4\pi x}{L_0}\right)\right), & -\frac{L_0}{4} < x < \frac{L_0}{4} \\ R(x) = R_0 & \mathit{Otherwise}. \end{array} \tag{1}$$

With all above supposition, the steady boundary layer equations governing the flow and transfer of heat for Newtonian nanofluid are described as[39,40]:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0, \tag{2}$$

$$\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial r}\right)u = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial}{r\partial r}\left(r\frac{\partial u}{\partial r}\right), \tag{3}$$

$$\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial r}\right)T = \frac{k_{nf}}{(\rho C_p)_{nf}}\frac{\partial}{r\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{4}$$

together with boundary conditions

$$\left.\begin{array}{l} u = 0, v = 0 \text{ and } T = T_1 \text{ at } r = R, \\ \frac{\partial u}{\partial r} = 0, \frac{\partial T}{\partial r} = 0 \text{ at } r = 0. \end{array}\right\}, \tag{5}$$

where viscosity is μ_{nf} and $(\rho C_p)_{nf}$ represents the heat capacity of fluid and density is ρ_{nf} which are described in Tables 1 and 2.

The continuity Eq. (1) can be satisfied by introducing stream function ψ for u and v such that

$$u = r^{-1}\partial\psi/\partial r, v = -r^{-1}\partial\psi/\partial x. \tag{6}$$

Then Eqs. (3) and (4) becomes

$$\frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial}{\partial x}\left(\frac{1}{r}\frac{\partial\psi}{\partial r}\right) - \frac{1}{r}\frac{\partial\psi}{\partial x}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi}{\partial x}\right) = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial}{r\partial r}\left(\frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r}\right), \tag{7}$$

$$\left(\frac{1}{r}\frac{\partial\psi}{\partial r}\right)\frac{\partial T}{\partial x} - \left(\frac{1}{r}\frac{\partial\psi}{\partial x}\right)\frac{\partial T}{\partial r} = \frac{k_{nf}}{(\rho C_p)_{nf}}\frac{\partial}{r\partial r}\left(r\frac{\partial T}{\partial r}\right). \tag{8}$$

To transform the Eqs. (7) and (8) into ordinary differential equations, we have assumed the following transfiguration

$$u = \frac{u_0 x}{L_0} F^{'}(\eta), v = -\frac{R}{r}\sqrt{\frac{u_0 \nu_f}{L_0}} F(\eta), \eta = \frac{r^2 - R^2}{2R}\sqrt{\frac{u_0}{\nu_f L_0}}, \theta(\eta) = \frac{T - T_0}{T_1 - T_0}, \tag{9}$$

$$\psi = \sqrt{\frac{u_0 x^2 \nu_f}{L_0}} RF(\eta),$$

where $x = \frac{\tilde{x}}{L_0}$ and after applying similarity transformation the Eqs. (7) and (8) finally becomes:

$$\frac{1}{(1-\phi)^{2.5}\left((1-\phi) + \phi\frac{\rho_s}{\rho_f}\right)}\left[(1+2\gamma\eta)F^{'''} + 2\gamma F''\right] + FF^{''} - F'^2 = 0, \tag{10}$$

$$\frac{k_s + 2k_{bf} - 2\phi\left(k_{bf} - k_s\right)}{Pr\left(k_s + 2k_{bf} + \phi\left(k_{bf} - k_s\right)\right)\left((1-\phi) + \phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)}\left[(1+2\gamma\eta)\theta'' + 2\gamma\theta'\right] + F\theta' - F^{'}\theta = 0. \tag{11}$$

The non-dimensional form of Eq. (1) is

$$\begin{aligned} f &= 1 - \tfrac{\epsilon}{2}(1 + \cos(4\pi\tilde{x})), \; -\tfrac{1}{4} < \tilde{x} < \tfrac{1}{4} \\ f &= 1 \qquad\qquad\qquad\qquad\qquad Otherwise, \end{aligned} \tag{12}$$

where $f = \frac{R(x)}{R_0}$ and $\epsilon = \frac{\lambda}{R_0}$ is the non-dimensional measure of stenosis in reference artery.

The non-dimensional form of boundary conditions on stenosed artery are

$$\begin{aligned} &F(0) = 0, F^{'}(0) = 0, \theta(0) = 1 \text{ at } \eta = 0, \\ &F''(\eta) = 0, \theta'(\eta) = 0 \text{ at } \eta = f. \end{aligned} \tag{13}$$

In the above equation non dimensional parameters are Prandtl number $Pr = k_f/(\mu C_p)_f$, flow parameter $\gamma = \sqrt{\nu_f L_0/u_0 R^2}$ and ϕ represented solid nanoparticle volume fraction.

The significant quantities i.e., heat transfer coefficient Nu_x and skin friction coefficient C_f of flow field are described:

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho_f U_w^2}, \tag{14}$$

$$Nu = \frac{x q_w}{k_f (T_w - T_\infty)}, \tag{15}$$

where shear stress τ_w is

$$\tau_w = \mu_{nf}\left.\frac{\partial u}{\partial r}\right|_{r=R}, \tag{16}$$

and heat flux is

$$q_w = -k_{nf}\left.\frac{\partial T}{\partial r}\right|_{r=R}, \tag{17}$$

Non-dimensional form of Eqs. (14) and (15) takes the form

$$Re_x^{1/2} C_f = \frac{1}{(1-\phi)^{2.5}} F^{''}(0), \tag{18}$$

$$Re_x^{-1/2} Nu_x = -\frac{k_{nf}}{k_f}\theta^{'}(0), \tag{19}$$

where local Reynolds number represented by $Re_x^{-1/2}$.

Numerical solution

The Eqs. (7) and (8) having nonlinearity are changed into ODE's with the association of appropriate conversions. Their solution is obtained numerically by using bvp4c in MATLAB. Bvp4c is a finite difference code that uses the three stage Lobatto IIIa formula. Result for temperature and velocity curves are plotted to show the impact of distinct values of non-dimensional quantities.

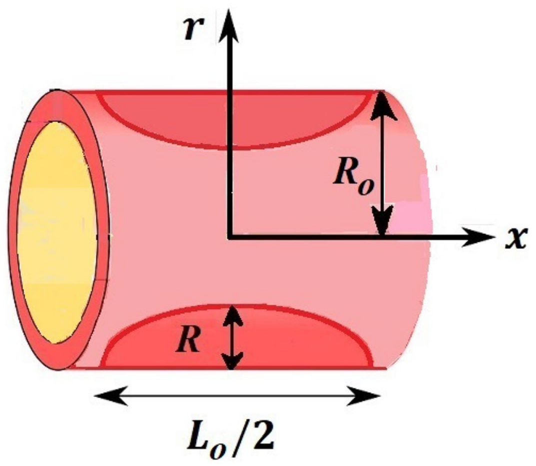

Figure 1. Geometrical structure of artery with stenosis.

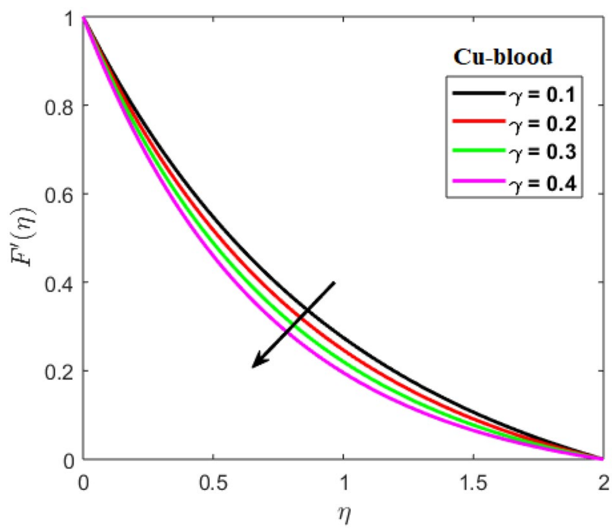

Figure 2. Results of γ on velocity.

Graphical results and discussion

Shah and Kumar[43] solved the problem governing equation and plotted the expression to study the consequences of the work. The part of body that received its blood from diseased artery will not be able to get enough oxygen and energy which is important for that area of body and because of that lack the affected area cells will die in that part, causing the heart attack. Generally, it is difficult to find the analytical solution of biological phenomena presented in Fig. 1 and it shows geometrical structure of diseased artery. In results and discussion section a detail investigation of the consequences of emerging parameters is to be analyzed. Normal artery and diseased blood artery is shown in graphical abstract. Figure 2 exhibit results for the flow parameter $\gamma = 0.1, 0.2, 0.3, 0.4$ impact on velocity curve. Velocity curve decreases due to increment in the value of γ. Figure 3 is plotted the outcomes of ϕ on curve of velocity. The velocity profile goes down owing to increment in $\phi = 0.001, 0.03, 0.07, 0.2$. It is observed that due to addition of nanoparticles we can slow down the flow. Figure 4 describe the variations of temperature curve for γ. The curve goes up by increasing $\gamma = 0.1, 0.2, 0.3, 0.4$. Consequences of Pr on curve of temperature are shown in Fig. 5. Curve of temperature goes up by diminishing the values of $Pr = 6.0, 5.0, 4.3, 3.6$. Figure 6 shows the results of ϕ on temperature curve. Temperature curve rises by enhancing the $\phi = 0.001, 0.03, 0.07, 0.1$ values. Nusselt number variations by changing γ and Pr values are presented in Fig. 7 and the curve increases by rising Pr values. Consequences of skin friction curve are shown in Fig. 8 and from results it is observed that the curve gradually goes down. Table 1 describes thermophysical properties of nanoparticles. Table 2 describes the nanoparticles and blood (base fluid) values. The results of γ and Pr on heat transfer coefficient are presented in Table 3. It is noted that the heat transfer coefficient values rises due to enhancement in Pr values. Table 4 presented the results of γ and ϕ on skin friction and from results it can be concluded that when solid nanoparticles volume fraction ϕ and γ accelerates then the values of Skin friction coefficient also goes up.

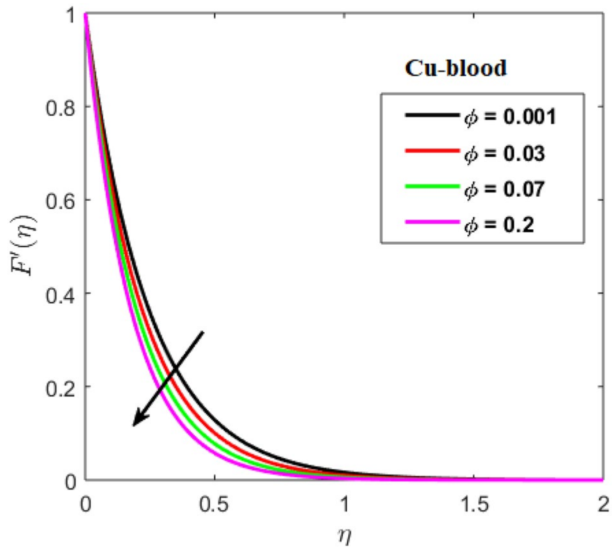

Figure 3. Results of ϕ on velocity.

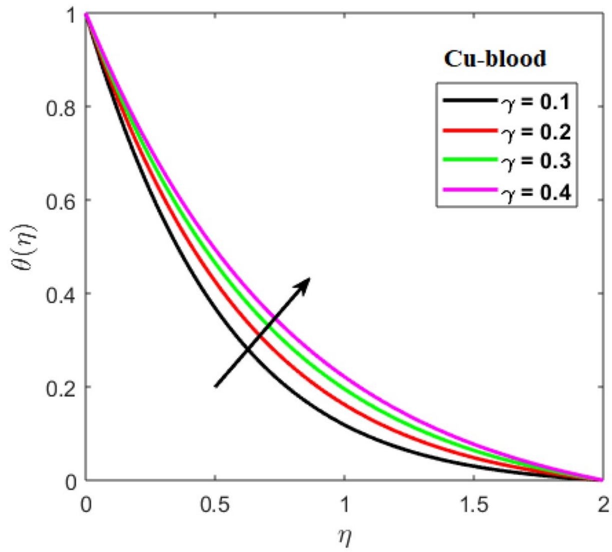

Figure 4. Impact of γ on temperature.

Conclusion

In present paper, we considered Newtonian characteristics of blood and the solution of the discussed problem is obtained numerically by using bvp4c. The nanofluid is considered as a mixture of copper and blood. Influence of parameters on temperature and velocity of blood is studied with the help of graphs and tables. To predict the cause of the atherosclerosis the mathematical modelling and numerical solution plays an important role and also assumed analysis summarize that nanoparticles technique could be an auspicious therapeutic approach against arterial diseases. Therefore, the present study is able to define some prime features of interest in biomedical applications. Main observations are:

(1) It is observed that velocity curve decreases by increasing the values of $\gamma = 0.1, 0.2, 0.3, 0.4$ and nanoparticle volume fraction $\phi = 0.001, 0.03, 0.07, 0.2$.

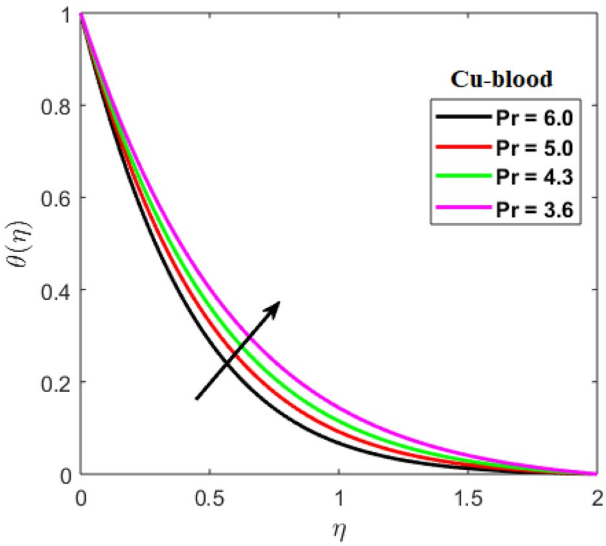

Figure 5. Profile of *Pr* for temperature.

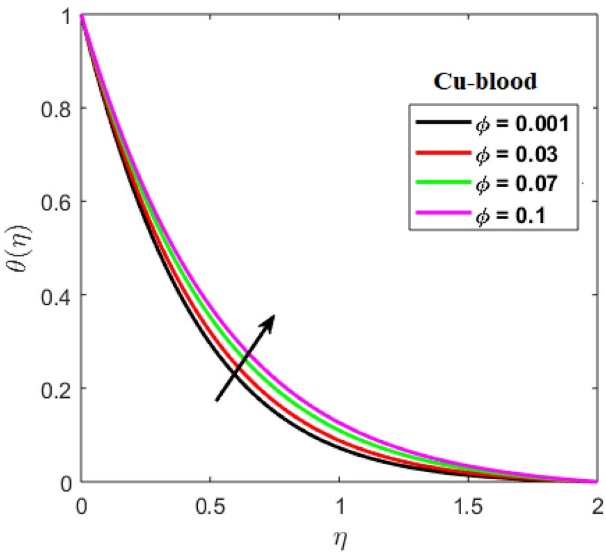

Figure 6. Results of ϕ on temperature.

(2) Blood flow performance can be improved by managing the properties of nanoparticles.
(3) As the nanoparticle concentration $\phi = 0.001, 0.03, 0.07, 0.1$ increases, the temperature and flow of blood increases positively.
(4) By decreasing the values of $Pr = 6, 5, 4.3, 3.6$, the temperature curve increases.
(5) Nusselt number graph is greatly influenced with the variation in the $Pr = 4.0, 5.0, 6.0$ values.
(6) Increase in nanoparticle values lessen the Skin friction curve.
(7) The Cu nanoparticles as a drug are efficient to minimize the hemodynamics of stenosis.

Future application will undoubtedly concentrate on personalized medicine taking into account the specific features of individual patient.

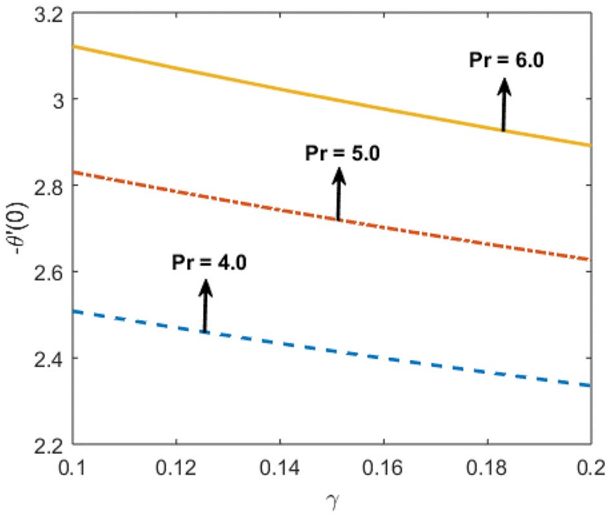

Figure 7. γ and *Pr* variations on heat transfer coefficient.

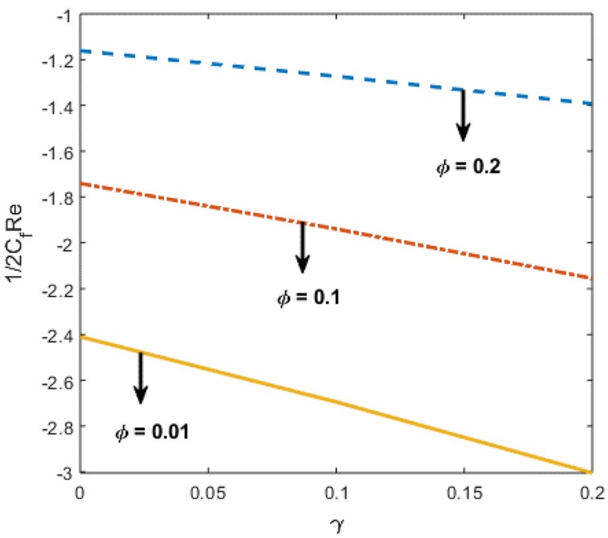

Figure 8. Results of γ and ϕ on Skin friction coefficient.

γ	Pr	$-\frac{k_{nf}}{k_f}\theta'(0)$
0.1	4.0	2.52255
0.12		2.48385
0.14		2.44730
0.1		2.52255
	5.0	2.84664
	6.0	3.13990

Table 3. Nusselt number $-\frac{k_{nf}}{k_f}\theta'(0)$ with respect to γ and Pr.

γ	ϕ	$\frac{1}{2}C_f Re$
0.1	0.01	− 3.12100
0.12		− 3.06967
0.14		− 3.02118
0.1		− 3.12100
	0.1	− 2.81082
	0.2	− 2.42815

Table 4. Skin friction coefficient values with respect to γ and ϕ.

Data availability

The datasets used during the current study available from the corresponding author on reasonable request.

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References

1. Young, D. F. Effect of a time-dependent stenosis on flow through a tube. *J. Eng. Ind. Trans. Am. Soc. Mech. Eng.* **90**, 248–254 (1968).
2. Malik, Y. M., Hussain, A. & Nadeem, S. Flow of a non-newtonian nanofluid between coaxial cylinders with variable viscosity. *Z. Naturforsch.* **67**, 255–261. https://doi.org/10.5560/ZNA.2012-0018 (2012).
3. Hussain, A., Sarwar, L., Akbar, S. & Nadeem, S. Mathematical model for blood flow through the stenosed channel. *Phys. Scr.* **95**, 2 (2019).
4. Tanveer, A., Salahuddin, T., Khan, M., Malik, M. Y. & Alqarni, M. Theoretical analysis of non-Newtonian blood flow in a microchannel. *Comput. Methods Progr. Biomed.* **191**, 105280 (2019).
5. Sankar, D. S. & Yatim, Y. Comparative analysis of mathematical models for blood flow in tapered constricted arteries. *Abstr. Appl. Anal.* **34**, 235960 (2012).
6. Shah, S. R. & Kumar, R. Mathematical modeling of blood flow with the suspension of nanoparticles through a tapered artery with a blood clot. *Front. Nanotechnol.* https://doi.org/10.3389/fnano.2020.596475 (2020).
7. Sheikholeslami, M. & Ebrahimpour, Z. Thermal improvement of linear Fresnel solar system utilizing Al2O3-water nanofluid and multi-way twisted tape. *Int. J. Therm. Sci.* **176**, 107505 (2022).
8. Sheikholeslami, M., Said, Z. & Jafaryar, M. Hydrothermal analysis for a parabolic solar unit with wavy absorber pipe and nanofluid. *Renew. Energy* **188**, 922–932 (2022).
9. Awais, M., Awan, S. E., Raja, M. A. Z. & Shoaib, M. Effects of gyro-tactic organisms in bio-convective nano-material with heat immersion, stratification, and viscous dissipation. *Arab. J. Sci. Eng.* **46**(6), 5907–5920 (2021).
10. Rizwana, A. H. & Nadeem, S. MHD oblique stagnation point flow of copper-water nanofluid with variable properties. *Phys. Scr.* **94**, 125808 (2019).
11. Rizwana, A. H. & Nadeem, S. Series solution of unsteady MHD oblique stagnation point flow of copper-water nanofluid flow towards Riga plate. *Heliyon* **6**(10), e04689 (2020).
12. Ijaz, S. & Nadeem, S. Examination of nanoparticles as a drug carrier on blood flow through catheterized composite stenosed artery with permeable walls. *Comput. Methods Progr. Biomed.* **1339**, 83–94 (2016).
13. Sheikholeslami, M., Jafaryar, M., Gerdroodbary, M. B. & Alavi, A. H. Influence of novel turbulator on efficiency of solar collector system. *Environ. Technol. Innov.* **26**, 102383 (2022).
14. Sheikholeslami, M. & Farshad, S. A. Nanoparticles transportation with turbulent regime through a solar collector with helical tapes. *Adv. Powder Technol.* **33**(3), 103510 (2022).
15. Sheikholeslami, M. Numerical investigation of solar system equipped with innovative turbulator and hybrid nanofluid. *Sol. Energy Mater. Sol. Cells* **243**, 111786 (2022).
16. Sheikholeslami, M. Analyzing melting process of paraffin through the heat storage with honeycomb configuration utilizing nanoparticles. *J. Energy Storage* **52**, 104954 (2022).
17. Sheikholeslami, M. Numerical analysis of solar energy storage within a double pipe utilizing nanoparticles for expedition of melting. *Sol. Energy Mater. Sol. Cells* **245**, 111856 (2022).
18. Sheikholeslami, M. & Ebrahimpour, Z. Nanofluid performance in a solar LFR system involving turbulator applying numerical simulation. *Adv. Powder Technol.* **33**(8), 103669 (2022).
19. Awan, S. E. *et al.* Numerical computing paradigm for investigation of micropolar nanofluid flow between parallel plates system with impact of electrical MHD and Hall current. *Arab. J. Sci. Eng.* **46**(1), 645–662 (2021).
20. Qureshi, I. H. *et al.* Influence of radially magnetic field properties in a peristaltic flow with internal heat generation: Numerical treatment. *Case Stud. Therm. Eng.* **26**, 101019 (2021).
21. Awan, S. E., Raja, M. A. Z., Mehmood, A., Niazi, S. A. & Siddiqa, S. Numerical treatments to analyze the nonlinear radiative heat transfer in MHD nanofluid flow with solar energy. *Arab. J. Sci. Eng.* **45**(6), 4975–4994 (2020).
22. Awan, S. E. *et al.* Numerical treatment for dynamics of second law analysis and magnetic induction effects on ciliary induced peristaltic transport of hybrid nanomaterial. *Front. Phys.* **9**, 631903 (2021).
23. Parveen, N. *et al.* Thermophysical properties of chemotactic microorganisms in bio-convective peristaltic rheology of nano-liquid with slippage, Joule heating and viscous dissipation. *Case Stud. Therm. Eng.* **27**, 101285 (2021).
24. Awan, S. E., Khan, Z. A., Awais, M., Rehman, S. U. & Raja, M. A. Z. Numerical treatment for hydro-magnetic unsteady channel flow of nanofluid with heat transfer. *Results Phys.* **9**, 1543–1554 (2018).
25. Raja, M. A. Z. *et al.* Integrated intelligent computing application for effectiveness of Au nanoparticles coated over MWCNTs with velocity slip in curved channel peristaltic flow. *Sci. Rep.* **11**(1), 1–20 (2021).
26. Awais, M., Bibi, M., Raja, M. A. Z., Awan, S. E. & Malik, M. Y. Intelligent numerical computing paradigm for heat transfer effects in a Bodewadt flow. *Surf. Interfaces* **26**, 101321 (2021).
27. Awan, S. E., Raja, M. A. Z., Awais, M. & Shu, C. M. Intelligent Bayesian regularization networks for bio-convective nanofluid flow model involving gyro-tactic organisms with viscous dissipation, stratification and heat immersion. *Eng. Appl. Comput. Fluid Mech.* **15**(1), 1508–1530 (2021).
28. Raja, M. A. Z., Awan, S. E., Shoaib, M. & Awais, M. Backpropagated intelligent networks for the entropy generation and joule heating in hydromagnetic nanomaterial rheology over surface with variable thickness. *Arab. J. Sci. Eng.* **47**(6), 7753–7777 (2022).

29. Awan, S. E., Raja, M. A. Z., Awais, M. & Bukhari, S. H. R. Backpropagated intelligent computing networks for 3D nanofluid rheology with generalized heat flux. In *Waves in Random and Complex Media*, 1–31 (2022).
30. Awais, M. *et al.* Effects of variable transport properties on heat and mass transfer in MHD bioconvective nanofluid rheology with gyrotactic microorganisms: Numerical approach. *Coatings* **11**(2), 231 (2021).
31. Awais, M., Kumam, P., Ali, A., Shah, Z. & Alrabaiah, H. Impact of activation energy on hyperbolic tangent nanofluid with mixed convection rheology and entropy optimization. *Alex. Eng. J.* **60**(1), 1123–1135 (2021).
32. Awais, M., Raja, M. A. Z., Awan, S. E., Shoaib, M. & Ali, H. M. Heat and mass transfer phenomenon for the dynamics of Casson fluid through porous medium over shrinking wall subject to Lorentz force and heat source/sink. *Alex. Eng. J.* **60**(1), 1355–1363 (2021).
33. Awais, M. *et al.* Heat transfer in nanomaterial suspension (CuO and Al2O3) using KKL model. *Coatings* **11**(4), 417 (2021).
34. Awais, M. *et al.* Numerical analysis of MHD axisymmetric rotating Bodewadt rheology under viscous dissipation and ohmic heating effects. *Sci. Rep.* **12**(1), 1–17 (2022).
35. Hussain, A. *et al.* Heat transfer analysis and effects of (silver and gold) nanoparticles on blood flow inside arterial stenosis. *Appl. Sci.* **12**(3), 1601 (2022).
36. Samad Khan, A., Abrar, M. N., Uddin, S., Awais, M. & Usman, I. Entropy generation due to micro-rotating Casson's nanofluid flow over a nonlinear stretching plate: Numerical treatment. In *Waves in Random and Complex Media*, 1–16 (2022).
37. Awais, M. *et al*. Hall effect on MHD Jeffrey fluid flow with Cattaneo–Christov heat flux model: An application of stochastic neural computing. In Complex & Intelligent Systems, 1–25 (2022).
38. Awais, M. *et al.* Endoscopy applications for the second law analysis in hydromagnetic peristaltic nanomaterial rheology. *Sci. Rep.* **12**(1), 1–14 (2022).
39. Vajravelu, K., Prasad, K. V. & Santhi, S. R. Axisymmetric magneto-hydrodynamic (MHD) flow and heat transfer. *Appl. Math. Comput.* **219**, 3993–4005 (2012).
40. Elgazery, N. S. Flow of non-Newtonian magneto-fluid with gold and alumina nanoparticles through a non-Darcian porous medium at a non-isothermal stretching cylinder. *J. Egypt. Math. Soc.* **27**, 39. https://doi.org/10.1186/s42787-019-0017-x (2019).
41. Abbas, N., Nadeem, S. & Saleem, A. Computational analysis of water based Cu-Al2O3/H2O flow over a vertical wedge. *Adv. Mech. Eng.* **12**(11), 1–10 (2020).
42. Ahmed, A. & Nadeem, S. Effects of magnetohydrodynamics and hybrid nanoparticles on a micropolar fluid with 6-types of stenosis. *Results Phys.* **7**, 4130–4139 (2017).
43. Shah, S. R. & Kumar, R. Mathematical modeling of blood flow with the suspension of nanoparticles through a tapered artery with a blood clot. *Front. Nanotechnol.* **2**, 596475 (2020).

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Author contributions
A.H. supervised and reviewed the manuscript. L.S. wrote the manuscript and prepared the figures. U.F.G. critically revised as well as analyzed the abstract section. He is expert in this area. Without his contribution, the abstract was not up to the level as suggested by reviewers. S.A. improved the introduction part as mentioned by the reviewer. A.R. revised the abstract and conclusion section. She did all these modifications and helped using order to give the paper required form. E.-S.M.S. explained the numerical method used in the simulation properly and also revised graphical results and discussion section critically he is expert in this technique. These authors are expert in their fields and they critically observed and improved the paper as per suggestions of the reviewers. Without their contribution it was not possible for us to modify the paper according to the suggestions of the reviewers. We categorically explained the work done by each above mentioned experts in order to improve the paper. Their contribution is of such an importance that cannot be ignored. Also, this is unethical thing to not add the names of authors who contributed a lot in the revision of manuscript.

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Competing interests
The authors declare no competing interests.

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