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A chemical engineering application on hyperbolic tangent flow examination about sphere with Brownian motion and thermophoresis effects using BVP5C

N. Vedavathi^a, G. Dharmiah^b, S. Noeiaghdam^{c,d,*}, U. Fernandez-Gamiz^e

^a Department of Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeswaram, India

^b Department of Mathematics, Narasaraopeta Engineering College, Yellamanda, Narasaraopet, India

^c Industrial Mathematics Laboratory, Baikals School of BRICS, Irkutsk National Research Technical University, Irkutsk, 664074, Russia

^d Department of Applied Mathematics and Programming, South Ural State University, Lenin Prospect 76, Chelyabinsk, 454080, Russia

^e Nuclear Engineering and Fluid Mechanics Department, University of the Basque Country UPV/EHU, Nieves Cano, 12, 01006, Vitoria-Gasteiz, Spain

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ABSTRACT

Brownian motion and thermophoresis impacts are discussed in relation to a tangent hyperbolic fluid encircling a sphere subject to a convective boundary condition and a Biot number. Concentration boundary conditions involving a wall normal flow of zero nanoparticles are an unexplored area of research. The governing non-linear BVP is transformed into a higher-order non-linear ODE using similarity transformations. Following equations were numerically solved for various values of emerging parameters using the matlab function `bvp5c`. Calculated values for velocity, concentration, temperature, the skin friction coefficient, Sherwood and Nusselt numbers are all shown, tabulated for analysis. Laminar boundary layer flow and heat transfer from a sphere in two-dimensional nano fluid is the novelty of the current work. The Weissenberg number decreases the velocity boundary layer thickness. The Biot number parameter lowers the field's temperature and speed.

1. Introduction

Non-Newtonian fluids have numerous practical uses in many industrial processes; therefore their boundary layer flow from a sphere has been studied for a long time. Non-Newtonian fluids have more uses in today's industries than Newtonian fluids do. A few additions may improve the efficiency with which such fluids are used. The efficiency with which heat is removed from a product during production directly correlates to the quality of that product. Since the early 2000s, researchers have paid extraordinary attention to the flow of nanofluids due to their many potential applications in cutting-edge, high-tech sectors. Today's technologically advanced businesses look for fluids with very high thermal conductivity coefficients to speed up the cooling of their goods and machinery. The thermal conductivity coefficient of metals is well-known to be much higher than that of convective heat transfer liquids. Thus, increasing thermal conductivity requires changing the coefficient of thermal conductivity of a convective fluid. Buongiorno [1] investigated the nano fluid convective transport by building a model which includes the contributions of Nb and Nt. The natural nano fluid convective BLF through a surface has been explored by ([2–4]) using the Buongiorno model. More discussions on fluid and

* Corresponding author. Industrial Mathematics Laboratory, Baikals School of BRICS, Irkutsk National Research Technical University, Irkutsk, 664074, Russia.
E-mail addresses: vedavathi@kluniversity.in (N. Vedavathi), dharmia.g2007@gmail.com (G. Dharmiah), snoei@istu.edu, noiagdams@susu.ru (S. Noeiaghdam), unai.fernandez@ehu.eus (U. Fernandez-Gamiz).

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nanofluid flow can be found in Refs. [18–25,34,35].

In virtue of their wide range of industrial uses, non-Newtonian fluids’ heat transfer and BLF have recently attracted the attention of many scientists. Thus, thermal radiation impact on the 2-D mixed convection flow of a tangential hyperbolic liquid approaching stagnancy point has been explored by Hayat et al. [5]. Furthermore, the MHD tangential hyperbolic flow via a caricatured cylinder studied by Malik et al. [6]. The BLF of hyperbolic tangent fluid behind a vertically stretched cylinder investigated by Naseer et al. [7]. Heat transmission in a Hyperbolic Tangent liquid via a stretched cylinder proposed by Salahuddin et al. [8], and was affected by the magnetic field and the varying thermal conductivity. The influence of heat source/sink on Hyperbolic Tangent nano liquid across a caricatured cylinder analyzed by Salahuddin et al. [9].

The curvature of the containers used is an important factor in maximizing thermal performance in multifarious chemicle engineering. In addition to spherical geometries, torus geometries, wavy surfaces, cylindrical geometries, cone geometries, ellipsoid geometries, and oblate spheroids are all commonplace in process systems, with the latter being particularly well-suited for chemical storage and batch reactor processing. As a result, scientists and engineers in chemicle engineering have focused a lot of effort on studying heat transfer from spherical objects, using both experimental and computational methods to examine the phenomenon for Newtonian and non-Newtonian fluids. Applying hot-film anemometry methods, Amato and Chi [10] investigated spontaneous convected from heated spheres in water for Rayleigh numbers. Experimentations on power-law liquids convective free isothermal sphere performed by Liew and Adelman [11]. Using hot-film anemometric, Amato and Chi [12] and Churchill [13] conducted additional empirical studies of aqueous polymer solutions.

Mass transport of a tangential hyperbolic liquid away from a sphere focused of this investigation. Author is aware of no prior research focusing on BLF tangent hyperbolic fluid with the passive control nanoparticles. This paper thus examined the cumulative impact of factors involved in mass and heat transmission. Using similarity variables, the governing BLEs were reduced to a two-point BVP that was then numerically solved using bvp5c in Matlab. Shooting as a technique has potential for future use in a wide range of technological and physical problems [26–29].

2. Mathematical model for tangent hyperbolic fluid

Steady, incompressible, laminar Tangent Hyperbolic liquid flow from a sphere, is intended, and plotted in Fig. 1. Tangent hyperbolic fluid constitutive equation Akbar et al. [14] is

$$\tau = [\eta_\infty + (\eta_0 + \eta_\infty)\tanh(\Gamma\dot{\Omega})^n]\dot{\Omega} \tag{1}$$

$$\text{Where } \dot{\Omega} = \sqrt{\frac{1}{2}\sum_i\sum_j\dot{\Omega}_{ij}\dot{\Omega}_{ji}} = \sqrt{\frac{\Pi}{2}} \tag{2}$$

Here $\Pi = \frac{1}{2}\text{tr}(\text{grad}V + (\text{grad}V)^T)^2$

Furtherly, $\eta_\infty = 0$ & $\Gamma\dot{\Omega} \ll 1$ is assumed for the tangent hyperbolic fluid which represents shear thinning characteristics. Hence, final formal of equation (1) is

$$\tau = \eta_0\dot{\Omega}[(\Gamma\dot{\Omega})^n] = \eta_0\dot{\Omega}[(1 + \Gamma\dot{\Omega} - 1)^n] = \eta_0\dot{\Omega}[1 + n(\Gamma\dot{\Omega} - 1)] \tag{3}$$

Tangent hyperbolic fluid model with nano particle equations are Yih [15], Abdul et al. [17]:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \tag{4}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \sqrt{2}vn\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\Gamma + g\beta\sin\left(\frac{x}{a}\right)(T - T_\infty) + \nu(1 - n)\frac{\partial^2 u}{\partial y^2} \tag{5}$$

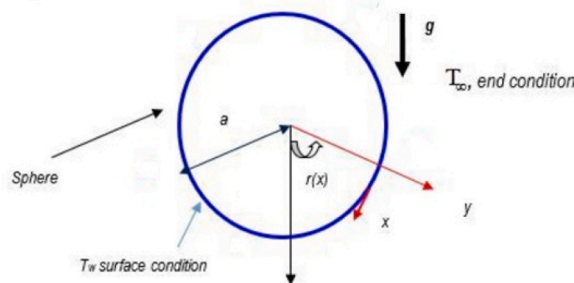


Fig. 1. Geometry of flow problem.

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{6}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \tag{7}$$

The corresponding boundary conditions are ([15,17,32,33])

$$u = 0, v = 0, -k \frac{\partial T}{\partial y} = h_w(T_w - T), D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} = 0 \quad \text{at } y = 0 \tag{8}$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty \tag{9}$$

The stream function ψ is defined as:

$$ru = \frac{\partial(r\psi)}{\partial y}, rv = -\frac{\partial(r\psi)}{\partial x} \tag{10}$$

Introducing non-dimensional quantities are [30,31]:

$$\varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}; Le = \frac{\alpha_D}{D_B}; Nt = \frac{D_T(\rho C)_p(T_w - T_\infty)}{(\rho C)_f \nu T_\infty}; Nb = \frac{D_B(\rho C)_p(C_w - C_\infty)}{(\rho C)_f \nu}; \xi = \frac{x}{a}$$

$$We = \frac{\sqrt{2}\nu\Gamma_x Gr^{\frac{3}{2}}}{a^3}; Pr = \frac{\nu}{\alpha}; f = \frac{\psi Gr^{-\frac{1}{2}}}{\xi \nu}; \eta = \frac{y Gr^{\frac{1}{2}}}{a} \tag{11}$$

3. The nomenclature explains all the terms

The transformed boundary layer equations:

$$\xi \left(f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} \right) = (1 - n)f''' + nWe f'' f''' - f' 2 + (1 + \xi \cot \xi) f f'' + \frac{\sin \xi}{\xi} \theta \tag{12}$$

$$\xi \left(f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right) = \frac{1}{Pr} \theta'' + (1 + \xi \cot \xi) f \theta' \tag{13}$$

$$\xi \left(f' \frac{\partial \varphi}{\partial \xi} - \varphi' \frac{\partial f}{\partial \xi} \right) = \varphi'' + (1 + \xi \cot \xi) Pr Le f \varphi' + \frac{Nt}{Nb} \theta'' \tag{14}$$

The changed dimensionless boundary conditions are as follows:

$$f = 0, f' = 0, \theta = 1 + \gamma^{-1} \theta', Nb \varphi' + Nt \theta' = 0 \quad \text{at } y = 0 \tag{15}$$

$$f' \rightarrow 0, \theta \rightarrow 0, \varphi \rightarrow 0 \quad \text{as } y \rightarrow \infty \tag{16}$$

With the above transformations, the following expressions can be used to describe the sphere surface Cf, Nu and Sh are:

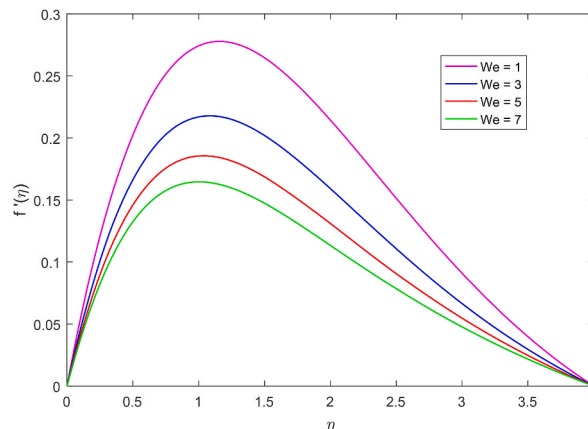


Fig. 2. Stimulate of 'We' on f'.

$$Gr^{-\frac{3}{4}}Cf = (1 - n)\xi f''(\xi, 0) + \frac{n}{2}We\xi(f''(\xi, 0))^2 \tag{17}$$

$$Gr^{-\frac{1}{4}}Nu = -\theta'(\xi, 0) \tag{18}$$

$$Gr^{-\frac{1}{4}}Sh = -\phi'(\xi, 0) \tag{19}$$

4. Numerical solution

System of nonlinear ODEs (12) through (14) with BCs (15)–(16) are solved by plugging numbers into bvp5c in matlab. First, Eqs. (12)–(14) are changed into a set of coupled 1st order equations system so that bvp5c can be used from Matlab. The numeral outcomes are found for distinct values of parameters which were involved here.

5. Interpretation of results

In this section, the impacts of different parameters on concentration, temperature and velocity are shown in tables and graphs. The section shows impacts of distinct parameters on concentration, temperature and velocity. These effects were shown in Figs. 2–10 & Tables 2 and 3 as graphs also tables, respectively.

Fig. 2 shows an affect graphs of velocity for distinct values of the Weissenberg number. Fig. 2 machinations what happens to speed when the Weissenberg number We is changed. When the values of Weissenberg number go up, the speed slows down and the hydrodynamic boundary layer gets thinner. Physically, when the Weissenberg number goes up, the resistance to fluid motion goes down. This means that the flow field and VBL thickness go down as well. Fig. 3 machinations, how the Biot number affects the speed profiles. When the Biot number goes up, both the speed and BLT go down. Fig. 4 shows how the tangential coordinate affects the velocity profiles. When the values of go up, both the speed and BLT get better. Fig. 5 shows how the Biot number affects the way temperatures change over time. When the Biot number goes up, both the temperature and BLT go down. Physically, the Biot number is the ratio of surface convection to surface conduction inside a body. Fig. 6 shows how Prandtal affects the way temperatures change over time. When the values of Pr go up, the temperature and thickness of the boundary layer go down. Fig. 7 shows how the concentration graph changes as the power-law index changes. For some values of n, the graph of concentration goes up, and then it starts to get close to zero. Fig. 8 shows a graph of how concentration changes as Nt changes. As the value of Nt goes up, the graph of concentration goes down. Fig. 9 shows a graph of how the concentration changes with respect to Nb. As the value of Nb goes up, the concentration graph goes up as well. Fig. 10 shows the concentration graph as a function of the Lewis number. As the value of the Lewis number goes up, the concentration graph goes up.

Table 1 shows that the numerical values of Nu in this article for different values of ξ when γ = n = We = 0 are in accorded with reported findings in Ref. [16]. Also, Table 2 shows how the Cf and Nu were calculated for different values of the controlling parameters. It has been found that Pr, γ, and Weissenberg number all make the skin friction coefficient go down, while n and ξ make it go up. Also, the Nusselt number goes down as the parameters ξ, γ and We go down, but it goes up as Pr and n go up. In Table 3, you can see how the Sherwood number is calculated for distinct values of the controlling parameters. It is found that the Sherwood number goes down as Nb goes up and up as Le and Nt go up.

6. Conclusions

The main points of the investigation can be summed up as follows:

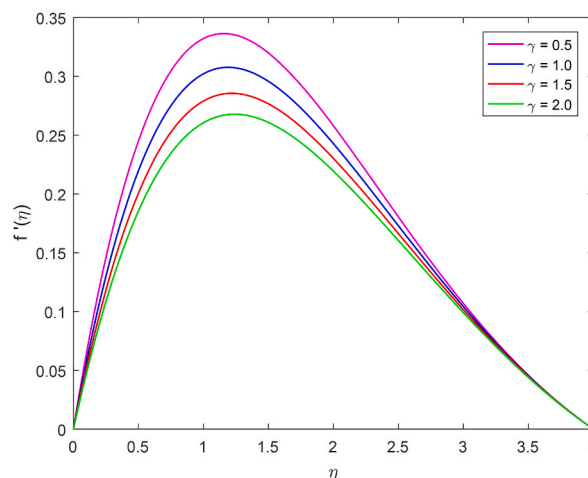


Fig. 3. Stimulate of ‘γ’ on f’.

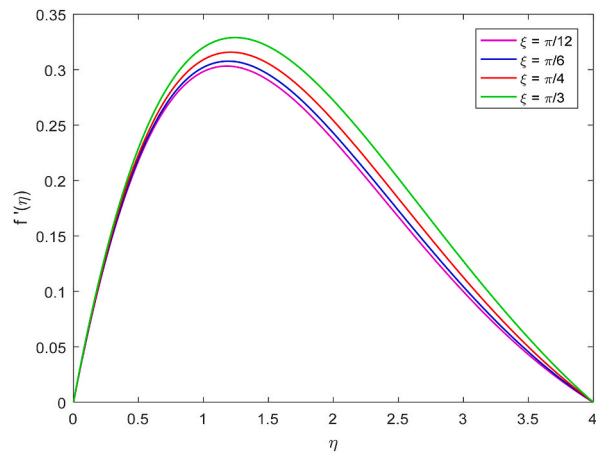


Fig. 4. Stimulate of ' ξ ' on f' .

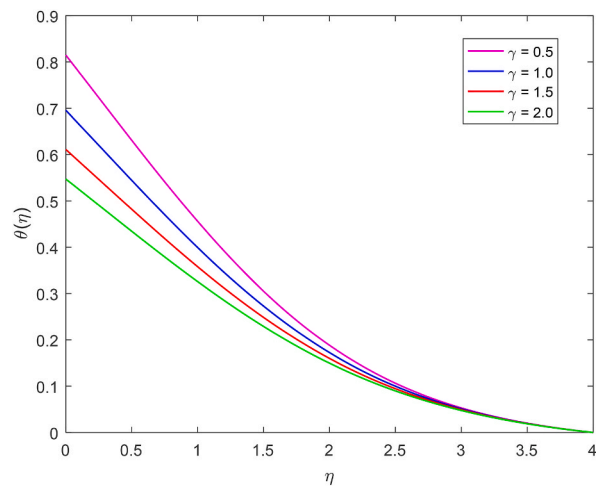


Fig. 5. Stimulate of ' γ ' on θ .

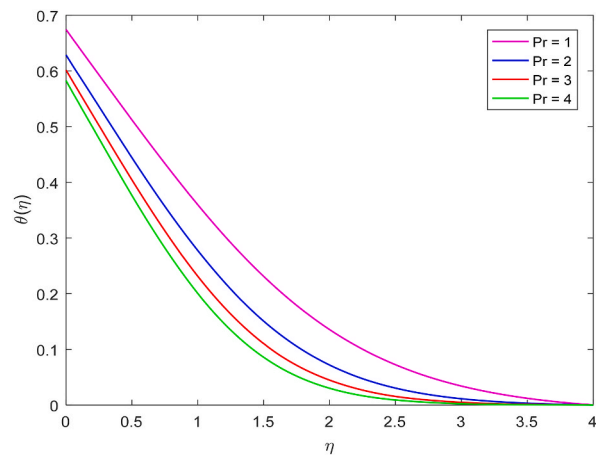


Fig. 6. Stimulate of ' Pr ' on θ .

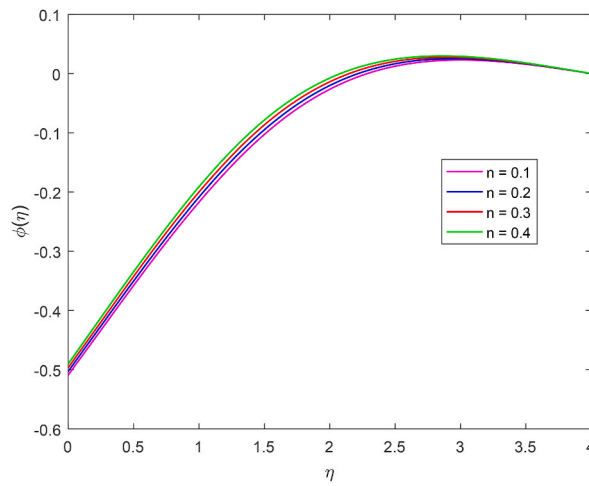


Fig. 7. Stimulate of 'n' on ϕ .

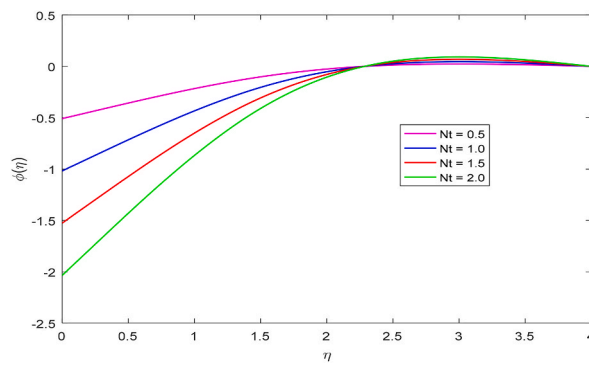


Fig. 8. Stimulate of 'Nt' on ϕ .

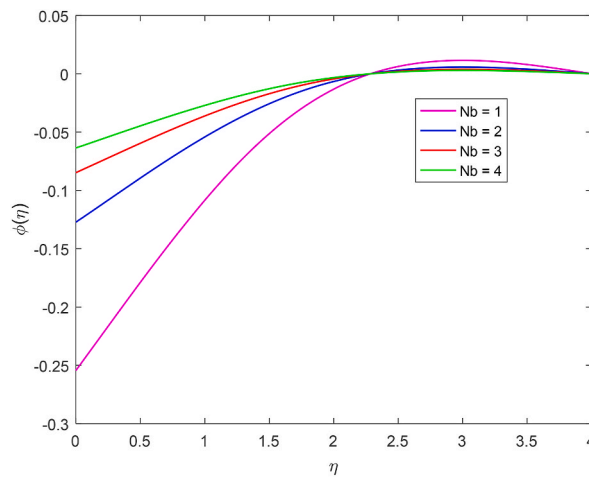


Fig. 9. Stimulate of 'Nb' on ϕ .

1. The Weissenberg number decreases the velocity boundary layer thickness.
2. The Biot number parameter lowers the field's temperature and speed.
3. Both the C_f and Nu go down when the Weissenberg number goes up.
4. As the thermophoresis parameter goes up, heat transfer goes up in a straight line.

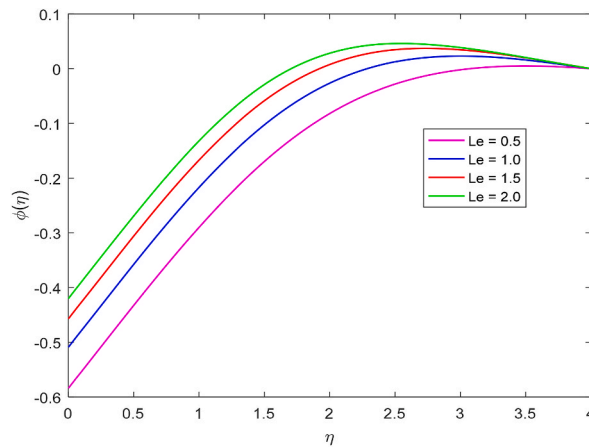


Fig. 10. Stimulate of 'Le' on φ.

Table 1
Comparison of *Nu* values with [16] when $\gamma = n = We = 0$.

ξ	Ref [16] with Pr = 0.7	Present Results	Ref [16] with Pr = 7.0	Present Results
0^0	0.4574	0.4571	0.9581	0.9590
10^0	0.4563	0.4562	0.9559	0.9555
20^0	0.4532	0.4537	0.9496	0.9501
30^0	0.4480	0.4483	0.9389	0.9380

Table 2
Cf & *Nu* values for various parameters.

ξ	Pr	γ	n	We	Cf	<i>Nu</i>
$\pi/12$					0.6031	0.3060
$\pi/6$					0.6076	0.3041
$\pi/4$					0.6156	0.3008
$\pi/3$					0.6283	0.2955
	1				0.5672	0.3256
	2				0.4825	0.3713
	3				0.4344	0.3984
	4				0.4017	0.4174
		0.5			0.6850	0.3698
		1.0			0.6076	0.3041
		1.5			0.5502	0.2592
		2.0			0.5054	0.2263
			0.1		0.6076	0.3041
			0.2		0.6355	0.3071
			0.3		0.6647	0.3101
			0.4		0.6951	0.3133
				1	0.6076	0.3041
				2	0.5942	0.3035
				3	0.5821	0.3030
				4	0.5711	0.3025

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Author contributions

Formal analysis, G. Dharmiah, N. Vedavathi and U. Fernandez-Gamiz; Investigation, G. Dharmiah, U. Fernandez-Gamiz and S. Noeiaghdam; Methodology, G. Dharmiah and U. Fernandez-Gamiz; Project administration, S. Noeiaghdam; Resources, G. Dharmiah and N. Vedavathi; Software, N. Vedavathi; Supervision, S. Noeiaghdam; Validation, N. Vedavathi and S. Noeiaghdam; Writing – original draft, N. Vedavathi, G. Dharmiah; Writing – review & editing, G. Dharmiah, N. Vedavathi, U. Fernandez-Gamiz and S. Noeiaghdam.

Table 3
Sh values for various parameters.

Le	Nb	Nt	Sh
0.5			0.3041
1.0			0.3042
1.5			0.3043
2.0			0.3044
	0.1		1.5207
	0.3		0.5669
	0.5		0.3041
	0.7		0.2172
		0.1	0.0608
		0.3	0.1825
		0.5	0.3041
		0.7	0.4258

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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NOMENCLATURE

a : - radius of sphere
 f : - non dimensional stream function
 g : - acceleration due to gravity
 k : - thermal conductivity of fluid
 $r(x) = a \sin(\frac{x}{a})$: - radial distance from symmetrical axis to surface of the sphere
 V : - velocity vector
 x : - stream wise coordinate
 y : - transverse coordinate
 We : - Weissenberg number
 n : - power law index
 γ : - Biot number
 Pr : - Prandtl number
 ξ : - Tangential co-ordinate
 Nt : - thermophoresis parameter
 Nb : - Brownian motion parameter
 Le : - Lewis number
 Cf : - skin-friction coefficient
 TBL : - Thermal Boundary Layer
 VBL : - Velocity Boundary Layer
 BL : - Boundary Layer
 D_B : - Brownian diffusion coefficient
 D_T : - thermophoresis diffusion coefficient
 Gr : - Grashof number
 T : - temperature of the fluid
 α : - thermal diffusivity
 η : - the dimension less radial co-ordinate
 ρ : - density of non-newtonian fluid
 μ : - dynamic viscosity
 I : - second variant strain sensor
 Γ : - time dependent material constant
 w : - conditions at the wall(sphere surface)
 ∞ : - free stream conditions
 θ : - non dimensional temperature
 τ : - the extra stress tensor
 η_{∞} : - the infinity shear rate viscosity
 η_0 : - the zero shear rate viscosity
 β : - the coefficient of thermal expansion