Results in Engineering 16 (2022) 100685

Contents lists available at ScienceDirect

Results in Engineering

journal homepage: www.sciencedirect.com/journal/results-in-engineering

Classical and intelligent methods in model extraction and stabilization of a dual-axis reaction wheel pendulum: A comparative study

Yüksel Ediz Bezci [a], Vahid Tavakol Aghaei [b], Batuhan Ekin Akbulut [a], Deniz Tan [a], Tofigh Allahviranloo [c], Unai Fernandez-Gamiz [d], Samad Noeiaghdam [e,f,*]

[a] Bahçeşehir University, Mechatronics Engineering, Beşiktaş, Istanbul, Turkey
[b] Istinye University, Electrical and Electronics Engineering, Istanbul, Turkey
[c] Istinye University, Faculty of Engineering and Natural Sciences, Istanbul, Turkey
[d] Nuclear Engineering and Fluid Mechanics Department, University of the Basque Country UPV/EHU, Nieves Cano 12, 01006, Vitoria-Gasteiz, Spain
[e] Industrial Mathematics Laboratory, Baikal School of BRICS, Irkutsk National Research Technical University, Irkutsk, 664074, Russia
[f] Department of Applied Mathematics and Programming, South Ural State University, Lenin Prospect 76, Chelyabinsk, 454080, Russia

ARTICLE INFO

Keywords:
Dual-axis reaction wheel pendulum
Fuzzy Logic
Fuzzy controller
Intelligent control
State feedback

ABSTRACT

Controlling underactuated open-loop unstable systems is challenging. In this study, first, both nonlinear and linear models of a dual-axis reaction wheel pendulum (DA-RWP) are extracted by employing *Lagrangian* equations which are based on energy methods. Then to control the system and stabilize the pendulum's angle in the upright position, fuzzy logic based controllers for both $x - y$ directions are developed. To show the efficiency of the designed intelligent controller, comparisons are made with its classical optimal control counterparts. In our simulations, as proof of the reliability and robustness of the fuzzy controller, two scenarios including noise-disturbance-free and noisy-disturbed situations are considered. The comparisons made between the classical and fuzzy-based controllers reveal the superiority of the proposed fuzzy logic controller, in terms of time response. The simulation results of our experiments in terms of both mathematical modeling and control can be deployed as a baseline for robotics and aerospace studies as developing walking humanoid robots and satellite attitude systems, respectively.

1. Introduction

Control systems on variants of inverted pendulum models have drawn the attention of many researchers because of their applicability in spacecrafts, marine vessels and robotics [18,20,26,48]. Among the different types of the inverted pendulum models, the so called "reaction wheel" pendulums, which inherit nonlinear and unstable structure, are good examples as benchmarks for examining their behavior in terms of applying different linear and nonlinear control methods [40].

There are numerous approaches to designing a control rule capable of stabilizing the reaction wheel pendulum utilizing all sensory feedback information. Due to its single-input single-output nature, a basic proportional-integral-derivative (PID) controller is clearly insufficient for this purpose. In this regard, for a successful control all state variables required to be stabilized at the same time [34]. Due to the nonlinear structure of the plant, a properly designed control mechanism is inevitable. There are advanced strategies built specifically for nonlinear systems. Fuzzy control [47], has recently emerged as an alternative approach to classical control methods for handling complicated processes which incorporates the benefits of the expert knowledge into the classical control algorithms. The primary advantages of fuzzy controllers can be highlighted as their applicability to nonlinear systems with complicated mathematical models as well as their flexibility to be programmed to exploit heuristic principles based on experts' experiences.

As the application areas for reaction wheel systems, some research has concentrated on using them to stabilize walking humanoid robots by providing efficiency in their biped walking [9,10,45], in addition to using them as modeling tools. Such devices have been proposed as balancing aids for people [24]. They are less confined by joint restrictions than upper-body link motion, and as a result, they represent

* Corresponding author. Industrial Mathematics Laboratory, Baikal School of BRICS, Irkutsk National Research Technical University, Irkutsk, 664074, Russia.
E-mail addresses: yukselediz.bezci@bahcesehir.edu.tr (Y.E. Bezci), vahid.aghaei@istinye.edu.tr (V. Tavakol Aghaei), batuhanekin.akbulut@bahcesehir.edu.tr (B.E. Akbulut), deniz.tan@bahcesehir.edu.tr (D. Tan), tofigh.allahviranloo@istinye.edu.tr (T. Allahviranloo), unai.fernandez@ehu.eus (U. Fernandez-Gamiz), noiagdams@susu.ru, snoei@istu.edu (S. Noeiaghdam).

https://doi.org/10.1016/j.rineng.2022.100685
Received 31 May 2022; Received in revised form 16 September 2022; Accepted 3 October 2022
Available online 14 October 2022

the upper flexibility limit with angular-momentum based inertial actuation. By using reaction wheel, underwater vehicles can be controlled without propellants in roll, pitch and yaw [1,23]. This can result in a significant reduction in propellant requirements for the vehicle. As with underwater applications, reaction wheels can also be used in spacecraft and satellites to stabilize satellites in earth orbit [37]. With this system, it is possible to achieve efficient satellite stabilization.

Many works have focused on modeling and control of reaction wheels due to their nonlinear yet simple structure [14,28–30,33]. Among the control approaches [17], used fuzzy logic controllers, sliding mode control is applied on the works done by Refs. [19,21] and full state feedback as well as PID controllers have been performed in Refs. [7,32, 41]. For an enhanced stabilizing action in one axis [43], proposed using two reaction wheels instead of the usual one reaction wheel and controlled the system by using a nonlinear Proportional Integral (PI) and sliding mode control. As a different idea [8], hire an adaptive control algorithm to tackle the problem of stabilization of the reaction wheel pendulum on the upright position when mounted on a moving platform. All the mentioned studies have been focused on the single-axis reaction wheel pendulum. In the current paper, our concentration will be on the model derivation and control of DA-RWP. Similar to ours, there are other studies based on DA-RWPs, as an instance [44], developed a physical system and conducted experimental evaluations in real-life where pole placement technique was applied as the stabilizing controller. In another study done by Ref. [5], a learning based control algorithm was developed using deep neural networks where a performance comparison was made with a nonlinear model predictive controller.

With the advent of fuzzy logic theory, the capacity to handle unpredictable problems is improved. Fuzzy reasoning's fundamental properties such as being less model dependent, robust, and easily adaptable to modeling language rules, as well as being simple to incorporate expert information into the control law enables it to handle a variety of uncertainties and noises involved [16]. Thanks to these properties, researches have applied fuzzy logic to different control problems incorporating stabilization of the Inverted Pendulum [2,13], controlling n-link robot manipulators [35], autonomous vehicle control [4], navigation of unmanned aerial vehicles and GPS systems [15,31, 42], control of hydraulic servo systems [22] power distribution and photovoltaic systems [12,39], state feedback of fractional ordered systems [38], and ship engine vibration control [36].

Recently [25], used s fuzzy controller for the attitude control of a reaction wheel system. However, the structure of the fuzzy control differs in the number of inputs for the fuzzy control in which they have only used error to obtain the control signal. Another difference is that, their mathematical structure is simplified, whereas we have used a nonlinear model of the reaction wheel system, making the control problem a challenging one. Finally, they did not compare their results with optimal control methods, while our proposed fuzzy controller was compared with classical methods as PID, LQR and state feedback.

In the current study first, we contribute to the derivation of the nonlinear equations of motion of a DA-RWP system with the aid of the *Euler* − *Lagrange* method accompanied by the mechanical design of the system, then in order to better recognize system's linear behavior, the equations of motion are transformed into a state-space model. To stabilize the system, some well-known modern control strategies like PID state-feedback, and linear quadratic regulator (LQR) are designed. As another contribution, a fuzzy PID controller is designed to control the angle deviations of the DA-RWP in which fuzzy controller's scaling gains are learned using GA. One of the main advantages of our fuzzy controller is its simple structure which can be theoretically explored and verified. Unlike [11,17] where a single axis reaction wheel is used, in the current work a different and simple structure for the fuzzy controller design is developed where our design can reduce the complexity of the controller. Simulations of the motion equations are done in the Matlab's Simulink environment in two scenarios. First, system is being simulated and controlled under an ideal noise-free and disturbance-free environment. Then, to increase the validity of the methods in real-time, noise and a pulse disturbance is injected to the system dynamics. The simulations reveal that the designed fuzzy based controller outperforms the classical ones in terms of the stabilization performance of the system. One of the main advantages and contributions of the current comparative study based on the results, is to provide a tutorial-based comparison between the available commonly used classical control algorithms with an intelligent fuzzy based method for the engineering students.

According to the provided discussions, the contributions of the current paper can be summarized as following:

- A fuzzy logic-based control is proposed to control a nonlinear model of a dual-axis reaction wheel system across both x and y axes.
- The proposed fuzzy controller is robust in terms of noise and external disturbance applied to the system.
- The performance of the proposed fuzzy controller has been analyzed and compared with the existing results of its optimal classical counterparts i.e. PID, LQR and state feedback, which exhibited superiority over them.
- The simulation results of our experiments can be used as a baseline for designing satellite systems [6] based on reaction wheel in terms of both mathematical modeling and intelligent control.
- A mechanical CAD design of the system is developed for our future work to establish a physical prototyping purposes.

The reminder of the paper is arranged as follow: After having a brief literature review in 1, the paper proceeds with the mechanical design and derivation of the dynamical equations of the system in 2.3 briefly discusses the classical control methodologies as well as a fuzzy type control algorithm applied to the system. The related simulation results for the control performance of the system will be given in 4. Paper will be concluded by final observations and future directions regarding the current work in 5.

2. Nonlinear model extraction and 3D design of dual-axis reaction wheel pendulum

To analyze the behavior of the dual-axis reaction wheel pendulum in the simulation environment, deriving accurate mathematical formulations is inevitable. Furthermore, for future studies related to the construction of the physical model of the system under investigation, a mechanical design based on computer aided programs taking into account all the details of the design procedure, can be perceived.

2.1. Equations of motion

The mathematical model of the system is important for understanding the dynamic characteristics of the wheel and pendulum. Modeling was simplified and assumed that the pendulum rotates on a single axis. Since the second reaction wheel is perpendicular to the rotation axis, it does not affect the system and that is why a reduced model can be assumed. The computer aided design (CAD) of the system along with a free body diagram representation of the system which has been sketched by using *SolidWorks* is given in Fig. 1.

Dynamics of the system are obtained by assuming the parameters specified below:

- O_1: pendulum's mass-center
- O_2: wheel's mass-center
- m_p: pendulum's mass
- m_w: wheel's mass
- L_1: length from O_1 to the coordinate's origin (O)
- L_2: length from O_2 to the coordinate's origin (O)
- θ: pendulum's angle from $y-$ axis
- φ: wheel's angle from $y-$ axis

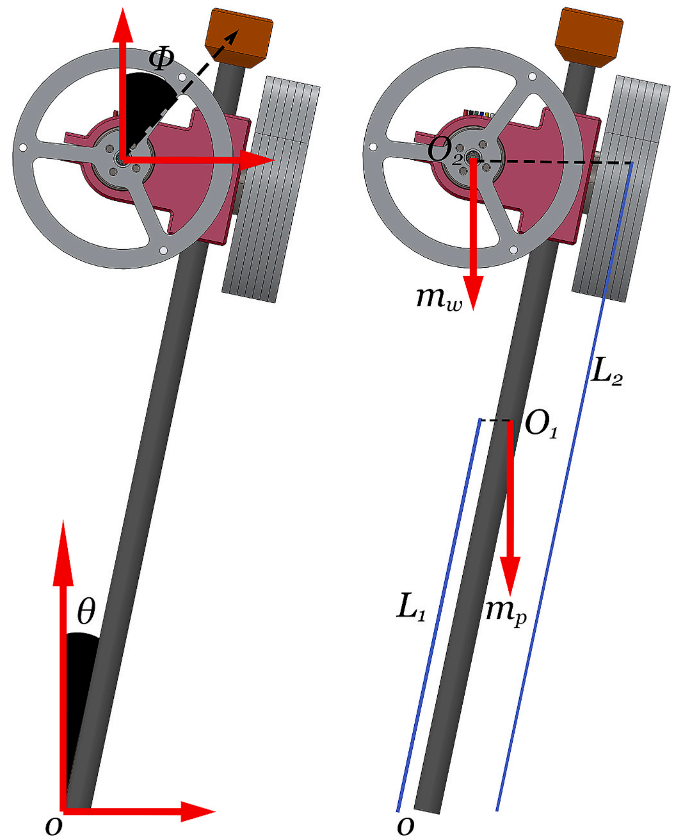


Fig. 1. CAD design of the dual-axis reaction wheel pendulum.

- •τ: motor torque applied on wheel
- •I_p: pendulum's rotational inertia about O_1
- •I_w: wheel's rotational inertia about O_2

The *Lagrangian mechanics* can be used to find the motion equations of the system based on some known general coordinates q_i. This method makes use of the energy change of an object moving in a state space with a general representation given in Eq. (1)

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = 0, \tag{1}$$

Here $\mathcal{L}$ identified as *Lagrangian*, is calculated as the difference between the total kinetic and potential energies of a system:

$$\mathcal{L} = KE - PE, \tag{2}$$

where KE and PE stand for system's total kinetic and potential energies, respectively. The pendulum's kinetic energy KE_p and wheel's KE_w can be defined as:

$$KE_p = \frac{1}{2}\left(m_p L_1^2 + I_p\right)\dot{\theta}^2, \tag{3}$$

$$KE_w = \frac{1}{2} mw L_2^2 \dot{\theta}^2 + \frac{1}{2}, I_w(\dot{\theta} + \dot{\varphi})^2, \tag{4}$$

The total potential energy for the DA-RWP is obtained as:

$$PE = \left(m_p L_1 + m_w L_2\right) g\cos\theta, \tag{5}$$

According to the given kinetic and potential energies, now Lagrangian can be formulated as:

$$\mathcal{L} = \frac{1}{2}\left(m_p L_1^2 + m_w L_2^2 + I_p + I_w\right)\dot{\theta}^2 + I_w \dot{\theta}\dot{\varphi} + \frac{1}{2} I_w \dot{\varphi}^2 - \left(m_p L_1 + m_w L_2\right) g\cos\theta, \tag{6}$$

For the ease of calculations in Eq. (6), let us assume the following parameters:

$$m_0 = m_p L_1^2 + m_w L_2^2 + I_p + I_w, \tag{7a}$$

$$\widetilde{m} = m_p L_1 + m_w L_2, \tag{7b}$$

The obtained Lagrangian function in Eq. (6) is substituted in Eq. (1) and the resulting differential equations are calculated in terms of θ and φ as:

$$m_0 \ddot{\theta} + I_w \ddot{\varphi} - \widetilde{m} g \sin\theta = 0, \tag{8}$$

$$I_w \ddot{\theta} + I_w \ddot{\varphi} = \tau, \tag{9}$$

Luckily, the only nonlinear component in the resulting differential equations is $\sin\theta$ and by assuming that $\theta \approx 0$ it can be simplified as $\sin\theta \approx \theta$ and the linearized dynamical equations can be obtained without the need for *Jacobian* linearization method.

$$m_0 \ddot{\theta} + I_w \ddot{\varphi} - \widetilde{m} g \theta = 0, \tag{10}$$

$$I_w \ddot{\theta} + I_w \ddot{\varphi} = \tau, \tag{11}$$

By the help of *Cramer's* rule, $\ddot{\theta}$ and $\ddot{\varphi}$ are left alone and the equations of motion are obtained as below:

$$\ddot{\theta} = \frac{I_w \widetilde{m} g \theta}{m_0 I_w - I_w^2} - \frac{I_w \tau}{m_0 I_w - I_w^2}, \tag{12}$$

$$\ddot{\varphi} = \frac{-I_w \widetilde{m} g \theta}{m_0 I_w - I_w^2} + \frac{m_0 \tau}{m_0 I_w - I_w^2}, \tag{13}$$

2.2. Mechanical CAD design

The 3D design of the system shown in Fig. 1 has been done using the *Solidworks* software. The perceived design consists of a pendulum, two motors, two reactional wheels to generate the required momentum, two motor shaft hubs for connecting the wheels to the motor, a mounting bracket to connect motors to the pendulum's stick, and an inertial measurement unit sensor (IMU) fixed on the pendulum's top part to get accurate sensory data. The wheel part consists of two different parts to change the weight of the wheel and to try different scenarios in the system. The wheels have a radius of 42.5 mm and its depth increases by 3 mm in each added part, so the weight of the system increases by 15 gr. The material of the pendulum's stick is thought to be built from aluminum with a length of 30 cm in a cylindrical shape having a radius of 5 mm. The fixing part can be stuck to the desired point on the stick to adjust system's mass-center. The total weight of the system is 540 gr. The numerical quantities for the mechanical design of the system are provided in Table 1. This configuration will be considered as a base

Table 1
Numerical values of the designed system.

Parameter	Value	Unit
m_p	0.04	*kg*
m_w	0.25	*kg*
L_1	0.15	*m*
L_2	0.267	*m*
I_p	3×10^{-4}	$kg.m^2$
I_w	9.03×10^{-5}	$kg.m^2$
G	9.81	m/s^2
K_m	5.5×10^{-3}	$(N.m)/\sqrt{w}$
τ_{max}	0.4	*kg.cm*

study for our future work in the prototyping of the physical system.

3. Classical and intelligent control

3.1. PID control design

A common control algorithm employed in most of the industrial applications, is the PID controller. The working principle of the PID is to decline the error $e(t)$ between the given reference input to the system and its measured output by regulating the control signal $u(t)$ where the control signal is given as:

$$u(t) = k_p e(t) + k_i \int e(t)dt + k_d \dot{e}(t), \tag{14}$$

The control signal can be adjusted using tunable gains defined in Eq. (14) as proportional k_p, integral k_i and derivative k_d gains. For the DA-RWP, the given reference trajectory is the upright position.

3.2. Mechanism of the state-feedback controller

A strategy employed in control theory to set system's closed-loop poles at predetermined positions, is known as full state-feedback. To apply state-feedback the so called state-space representation is used. With the state-space method, an $n-th$ order differential equation is transformed into n first order ones, making it easier to represent it in a matrix form. The general form of a state-space model of a dynamical system is defined as:

$$\dot{x} = Ax + Bu, \tag{15}$$

where for a single input system $x^{n\times 1}$ is the state vector, $A^{n\times n}$ is a matrix, $B^{n\times 1}$ is a vector, and u is the input to the system. For the DA-RWP, we define the state variables as

$$x = [\theta \;\; \dot{\theta} \;\; \dot{\varphi}]^T,$$

The state-space model is then created by using equations (12) and (13) considering state variables. For this, the equations of motion should be transferred into the form given in equation (15) as a matrix representation. Using the state vector, its derivative can be obtained as:

$$\dot{x}_1 = \dot{\theta} = x_2, \tag{16a}$$

$$\dot{x}_2 = \ddot{\theta} = \frac{I_w \widetilde{m} g \theta}{m_0 I_w - I_w^2} - \frac{I_w \tau}{m_0 I_w - I_w^2}, \tag{16b}$$

$$\dot{x}_3 = \ddot{\varphi} = \frac{-I_w \widetilde{m} g \theta}{m_0 I_w - I_w^2} + \frac{m_0 \tau}{m_0 I_w - I_w^2}, \tag{16c}$$

Now rearranging equation (16) in the matrix form and taking torque τ as the multiplication of the motor torque constant and the control signal ($\tau = K_m u$), will result in the following representation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{I_w \widetilde{m} g}{m_0 I_w - I_w^2} & 0 & 0 \\ \frac{-I_w \widetilde{m} g}{m_0 I_w - I_w^2} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{I_w K_m}{m_0 I_w - I_w^2} \\ \frac{m_0 K_m}{m_0 I_w - I_w^2} \end{bmatrix} u, \tag{17}$$

Consequently, in the body of the state space model A and B matrices are defined as:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{I_w \widetilde{m} g}{m_0 I_w - I_w^2} & 0 & 0 \\ \frac{-I_w \widetilde{m} g}{m_0 I_w - I_w^2} & 0 & 0 \end{bmatrix}, \tag{18}$$

$$B = \begin{bmatrix} 0 \\ -\frac{I_w K_m}{m_0 I_w - I_w^2} \\ \frac{m_0 K_m}{m_0 I_w - I_w^2} \end{bmatrix}, \tag{19}$$

The applicability of a state-feedback control is possible if the system is controllable (full rank), so first this condition should be satisfied. For our case, the controllability matrix is constructed as

$$\mathcal{C} = [B \;\; AB \;\; A^2B], \tag{20}$$

Calculating the rank of the matrix in (20) reveals that it is a full rank of 3 matrix.The control signal for the state-feedback is calculated as:

$$u = -kx, \tag{21}$$

Here $k^{1\times n}$ is the feedback gain. We can determine the stabilizing gain vector either by pole placement (putting the closed loop poles in desired pole locations) or the *Ackermann's* formula given in Eq. (22).

$$k = [0 \;\; 0 \;\; 1]\mathcal{C}^{-1}\psi(A), \tag{22}$$

where the feedback closed-loop has its desired characteristic equation as $\psi(.)$. By substituting equation (21) in 15 system's new sate-space description can be rewritten as:

$$\dot{x} = (A - Bk)x, \tag{23}$$

where the eigenvalues of this closed loop state feedback should be equal to the desired closed loop pole locations. By simply using coefficient matching between two equations, the state feedback control gain can be found. Four our case, we have adopted the pole-placement method.

3.3. LQR control design

The Linear Quadratic Regulator (LQR) is a control method which provides a set of optimal feedback gains for the system by minimizing a given cost function [3]:

$$J = \int_0^\infty (x^T Q x + u^T R u)\, dt, \tag{24}$$

where weight matrices $Q \geq 0$ and $R > 0$ inject information related to the importance and effect of each state and control signal in J. By using LQR the need for choosing specific set of desired closed loop poles are eliminated. The optimal control gains for the state-feedback control defined in equation (21) can be obtained as:

$$k = R^{-1}B^T P, \tag{25}$$

where the solution of the algebraic Riccati equation is equal to P:

$$0 = A^T P + PA + Q - PBR^{-1}B^T P, \tag{26}$$

3.4. Fuzzy PID controller design

A fuzzy logic controller (FLC), can typically contain a set of expert knowledge in terms of logical rules in which the control effort can be obtained based on a fuzzy decision making process [27]. Generally, these rules can be represented in the form of "If-Then" rules as given in Eq. (27).

$$\text{If } \mathcal{X}_1 \text{ is } \mathcal{A}_1 \ldots \text{ and } \mathcal{X}_n \text{ is } \mathcal{A}_n, \text{ Then } \mathcal{O} \text{ is } \mathcal{B}, \tag{27}$$

where $\mathcal{X}_j$ is the input, $\mathcal{A}_j$ is the fuzzy set and $\mathcal{O}$ is the output for the corresponding output fuzzy set $\mathcal{B}$. The form of the fuzzy controller that is used in this work, is analogous to the conventional PID controller with the difference that for the fuzzy-based PID control, a PD is hired in the input of the FLC and an integrator in the output. In Fig. 2 a schematic representation of the typical fuzzy-based PID type control is shown,

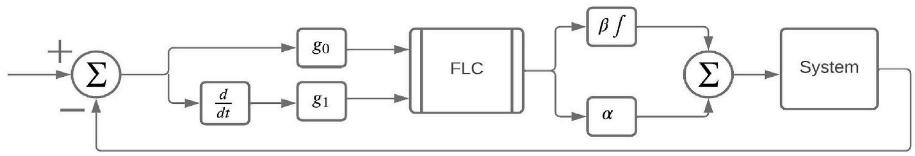


Fig. 2. Schematic of formation of a PID-type FLC.

where g_0, g_1, α, and β compose the tuning gains for the input-output relations of the FLC.

The inputs to the FLC are taken as the error and change of the error for the pendulum's angle θ. For each input and output variables 5 triangular equidistant membership functions (MF) are used. The reference input and actual output in Fig. 2 are $r = 0$ and θ, respectively. Therefore, the angle's error and its derivative (error change with time) are:

$$e = -\theta \quad \text{and} \quad \dot{e} = -\dot{\theta}, \tag{28}$$

In order to embed the expert knowledge in controlling the angle of the two-axis reaction wheel pendulum considering equation (28), some linguistic variables are defined as **Z, PS, PL, NS** and **NL**. To digest the working principle of the designed FLC mechanism let us describe some possible scenarios:

- When the error is **Z** (zero), angle is very close to the upright position.
- When the error is **PS** (positive small), the angle is a bit to the left of the upright position.
- When the error is **NL** (negative large), the angle is far away from the upright position towards the right.
- When the change in error is **NS** (negative small), the angle is slowly moving away from the vertical position in a clockwise direction.

By considering these linguistic explanations a rule-base can be constructed for the control of the two-axis reaction wheel pendulum by combining various situations for both the error and its derivative. For our problem, this rule-base combination have been summarized in 2.

In order to calculate a crisp control output the center of gravity method is used. This method returns a precise crisp value depending on the fuzzy set's center of gravity [46].:

$$u = \frac{\Sigma_j c_j \int \mu_j}{\Sigma_j \int \mu_j}, \tag{29}$$

where $j = 1, 2, \ldots, M$ represents the total number of rules, c_j is the center of the output membership functions and μ_j is the truth value for each rule. The truth value for each rule considering inputs as error and error derivative is obtained as:

$$\mu = \mu_{A_1}(e) \times \mu_{A_2}(\dot{e}), \tag{30}$$

4. Simulated scenarios

In the following section, DA-RWP's behavior will be investigated under the control algorithms mentioned in 3 based on simulation studies in Matlab for two cases as.

- noise-free and disturbance-free
- affected by noise and disturbance

For both conditions, pendulum's angle in $x - y$ axis will be

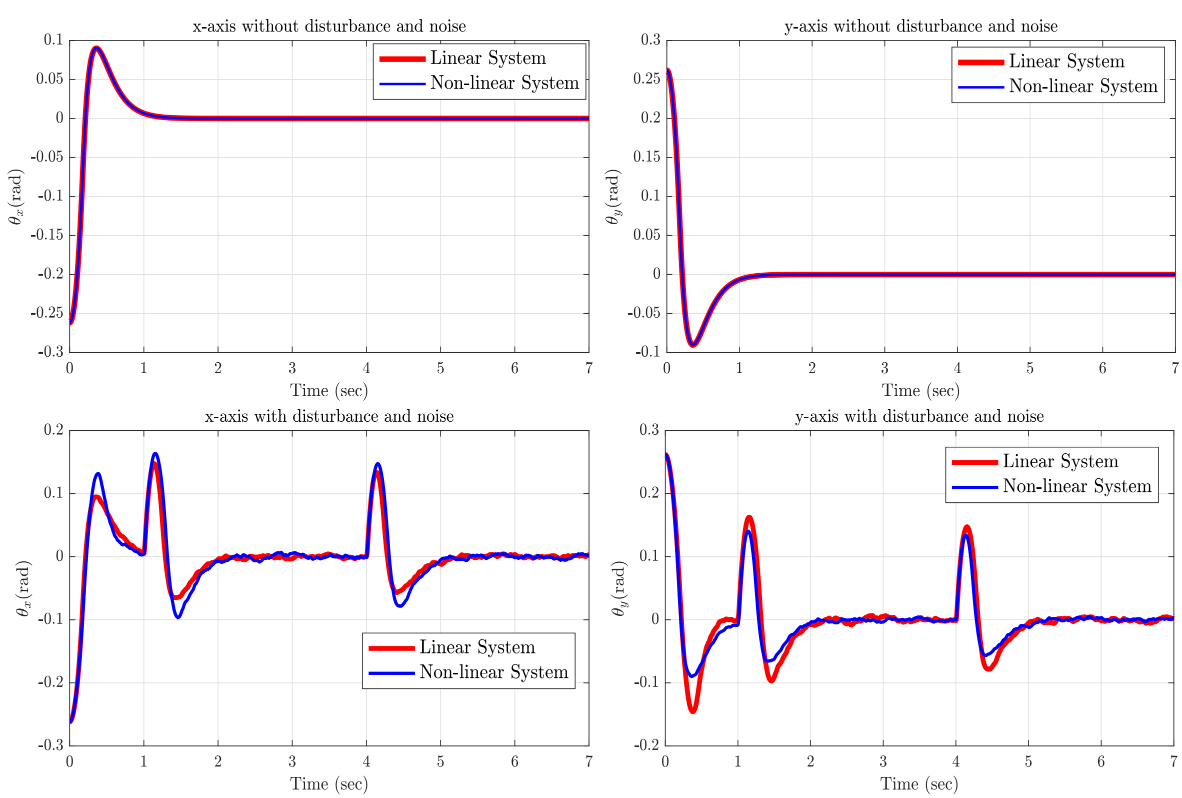


Fig. 3. Stabilizing the pendulum angle in the $x - y$ direction for both the nonlinear and linear system using PID.

controlled.

In the first simulation demonstrated in Fig. 3, for angle to be kept in an upright position both the non-linear and linear models of the system are used by applying a PID controller. The angle has been controlled in both x and y directions where its initial condition set to start from opposite directions. In the first scenario, PID controller has been applied to a noise-free and disturbance free case and according to the simulations, the angle reaches zero around 1.6 (sec). In the second scenario, a band limited noise as well as a pulse disturbance, which is applied on 1^{st} and 4^{th} seconds, affect the system and as depicted in the figure, PID becomes successful in stabilizing the pendulum angle in both directions. Another fact is that, in a noise-disturbance free environment, angle performance for both the linear and non-linear systems can track each other, however when the system is being subjected to noise and pulse disturbances the angle performances for linear and non-linear models are not exactly identical. Regardless of the successes of the PID controller in keeping the pendulum's angle vertically, if the angular velocity of the wheel itself is being analyzed, it will be observed from Fig. 4 that the wheel is turning with a constant angular velocity resulting in drawing more power from the system moving the motor speed towards its maximum value.The corresponding PID control parameters for these simulations are given as 3.

In the second simulation, angle of the pendulum in $x-y$ direction is stabilized using a state-feedback control over the DA-RWP's linear model, where the corresponding stabilizing gain vector given in 31, has been designed by pole-placement method assuming that the resulting closed-loop pole locations are: $s_{1,2} = -5.2850 \pm 5.5440i$, and $s_3 = -26.425 - 27.72i$.

$$k = [-7.1428 \quad -1.4935 \quad -0.0037], \tag{31}$$

As it can be observed from Fig. 5 that the designed state feedback controller can successfully stabilize the pendulum's angle in the upright position for both the noise-disturbance-free and affected by noise-disturbance scenarios. when it comes to the performance of wheel's angular velocity, unlike the PID controller, state feedback control handles it well and stabilizes it at zero for a noise-disturbance-free case and around zero when affected by noise-disturbance as can be seen from Fig. 6.

Another optimal control method that is used for simulating the performance of the pendulum's angle for both the linear and non-linear system is the LQR method. The gain parameters for this method are obtained by using the LQR function in MATLAB where matrices obtained in equations (18) and (19) are used. Moreover, weight matrices in equation (24) are chosen as $Q = \text{diag}([1\ 1\ 1])$ and $R = 1$. The resulting gain vector is calculated as:

$$k = [-2587.8134 \quad -423.4889 \quad -1], \tag{32}$$

Similar to our previous simulations, pendulum's angle for both linear and non-linear systems is initialized from opposite directions. As can be seen from Fig. 7, the pendulum's angle reaches equilibrium position for both linear and non-linear systems using the designed LQR controller for both the created scenarios.

In Fig. 8 the behavior of the wheel's velocity is illustrated where for the noise and disturbance free cases the LQR performs well, whereas, injecting noise and disturbance to the system makes the velocity control a challenging problem by encountering undesirable overshoots/undershoots in the response.

Finally, in contrast to classical control methodologies, to improve the performance of the stabilization problem of the reaction wheel pendulum system, simulation results of PID type FLCs are provided. A Matlab Simulink structure for the FLCs has been given in Fig. 10. The stabilization of the non-linear model of the reaction wheel's angle can be seen in Fig. 9a where for the noise and disturbance free case, it manages to completely remove the overshoots in the angle response, as opposed to that of its classical counterparts shown in the previous results. On the other hand, when encountering with the noisy states accompanied by disturbance, its performance is superior to that of PID and LQR controllers. When compared with the state-feedback control's performance shown in Fig. 5, their trend is similar. However, it should be noted that the state feedback controller is applied on a linear model of the system whereas for the FLC, the non-linear system is considered. Shown in Fig. 9b, the related simulations for the wheel's angular speed are given under the designed PID-type FLC.

For the error and error derivative variables of the angle which are

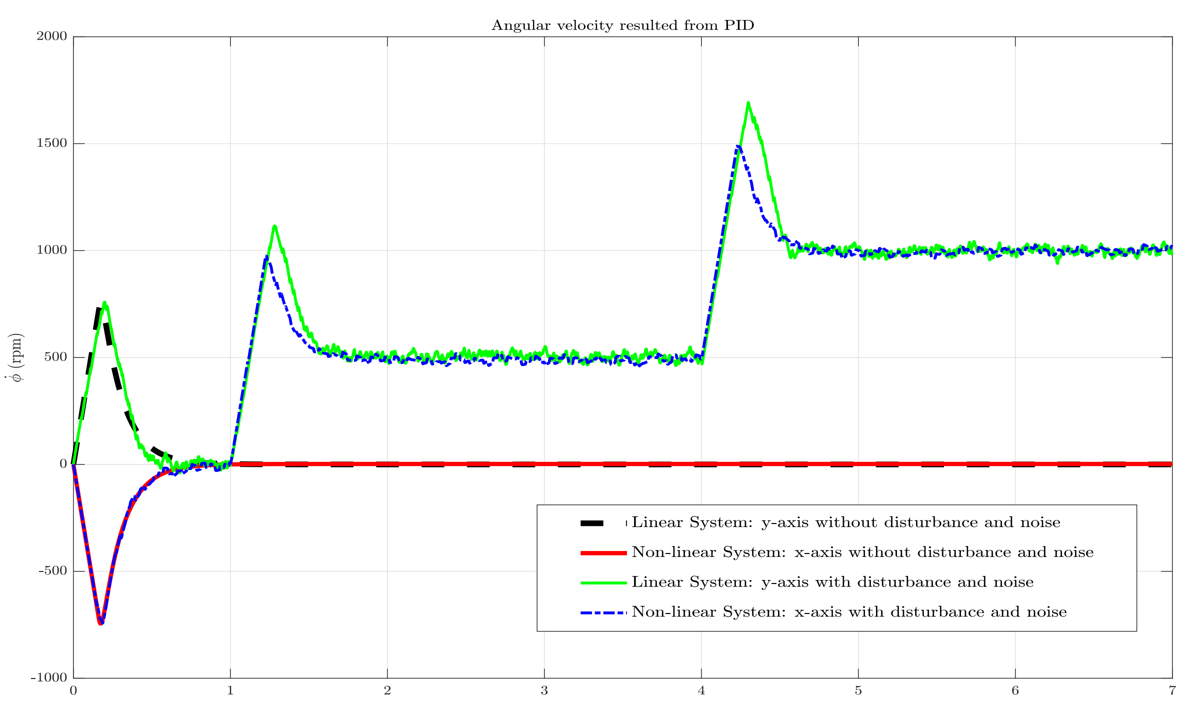


Fig. 4. Wheel's angular velocity under PID controller.

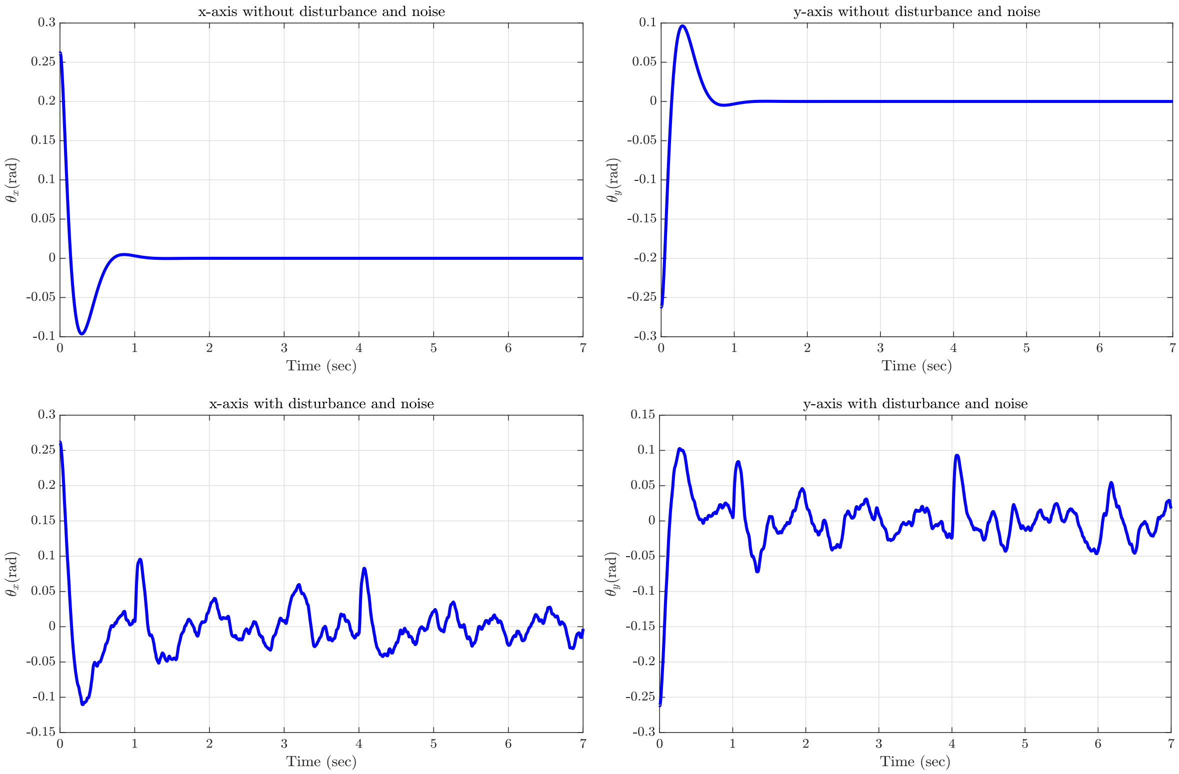


Fig. 5. Stabilizing the pendulum's angle in $x - y$ direction using a state-feedback control for the linear system.

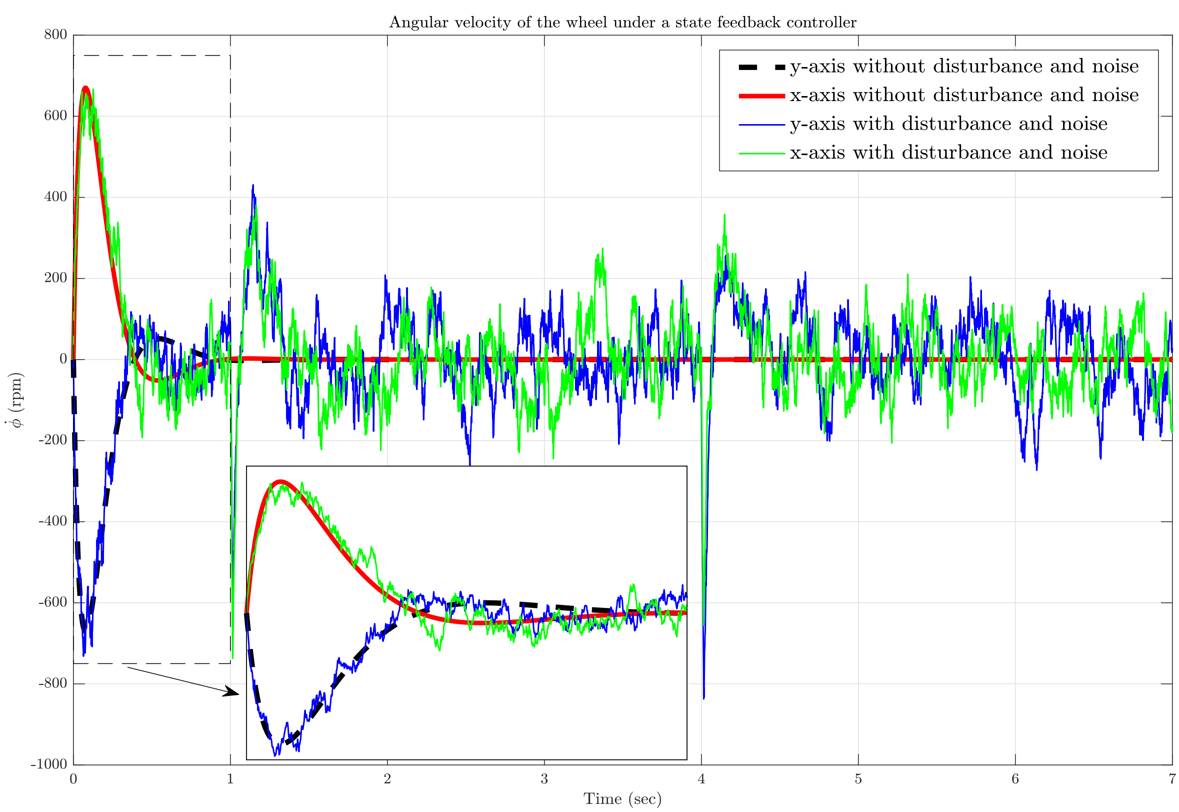


Fig. 6. Behavior of the wheel's angular velocity under the designed state-feedback control for the linear model.

assumed to be the inputs of the FLC, 5 membership functions (MF) have been used as shown in Fig. 12a, where the outer left and right ones are *Z* and *S*-shaped MFs, respectively. And the remaining MFs are triangular ones. For the output MFs, 5 equidistant triangular MFs have been used as depicted in Fig. 12b. The scaling gains of the FLC (g_0, g_1, α and β) are obtained using the genetic algorithm toolbox of the Matlab where the

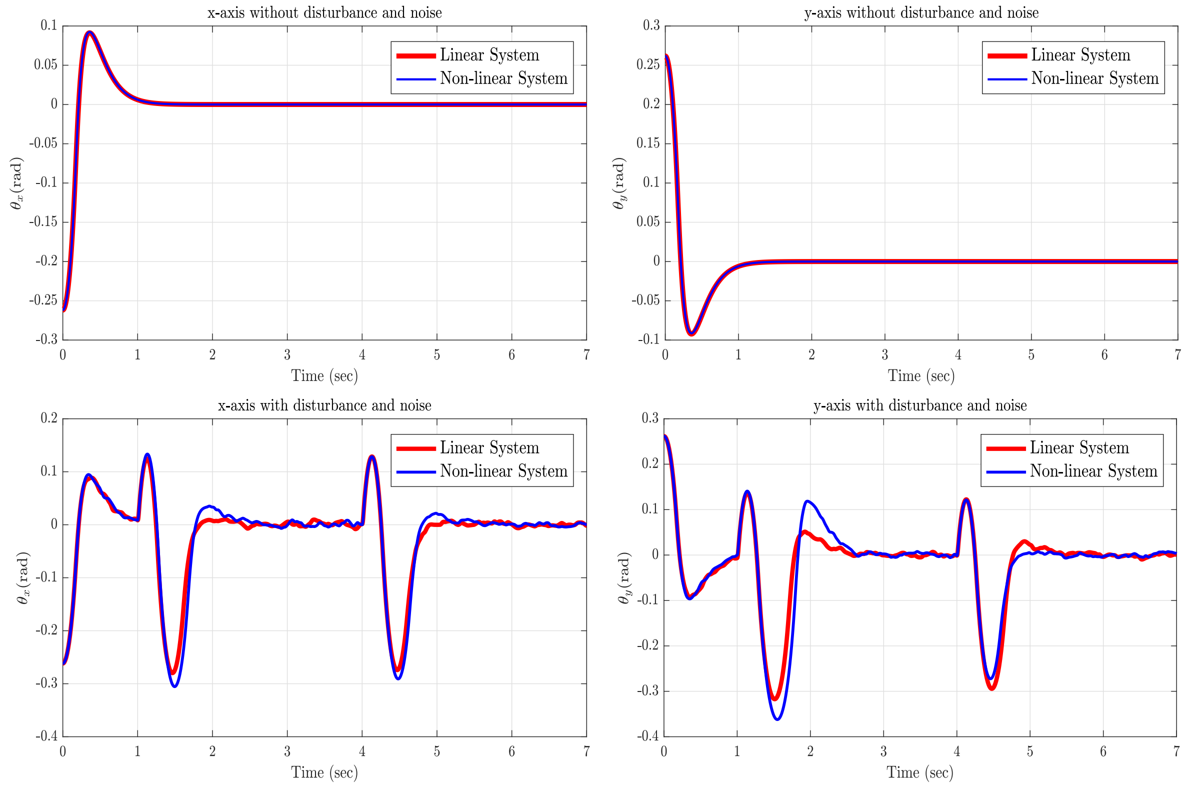


Fig. 7. Pendulum' angle under the designed LQR controller for both linear and non-linear system.

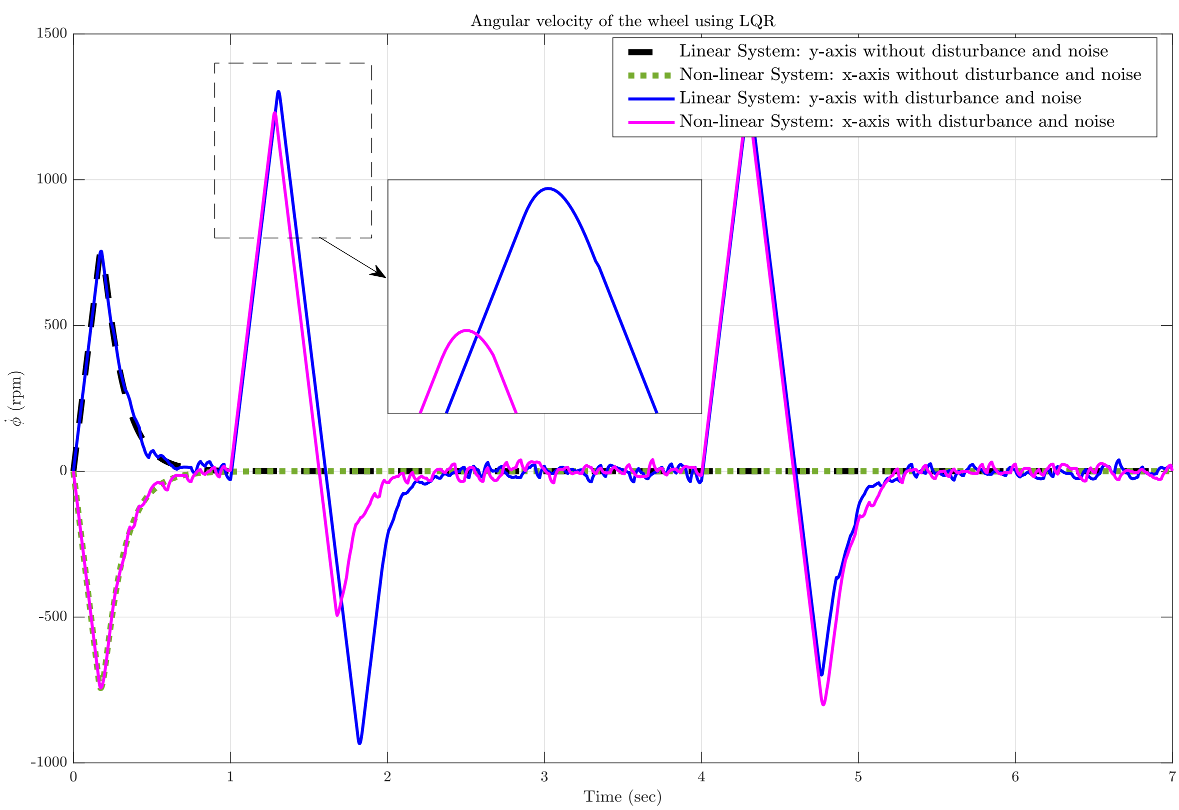
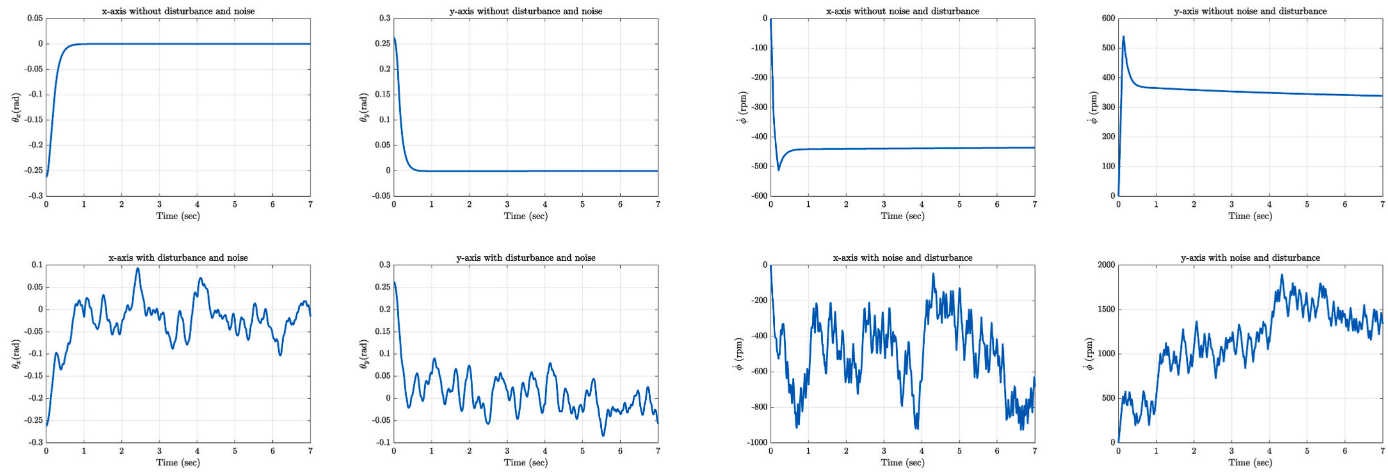


Fig. 8. Wheel's angular velocity under an LQR control.



(a) Pendulum's angle for the non-linear model (b) Wheel's angular velocity

Fig. 9. Response of the designed PID-type FLC for (a) pendulum's angle, and (b) wheel's angular speed.

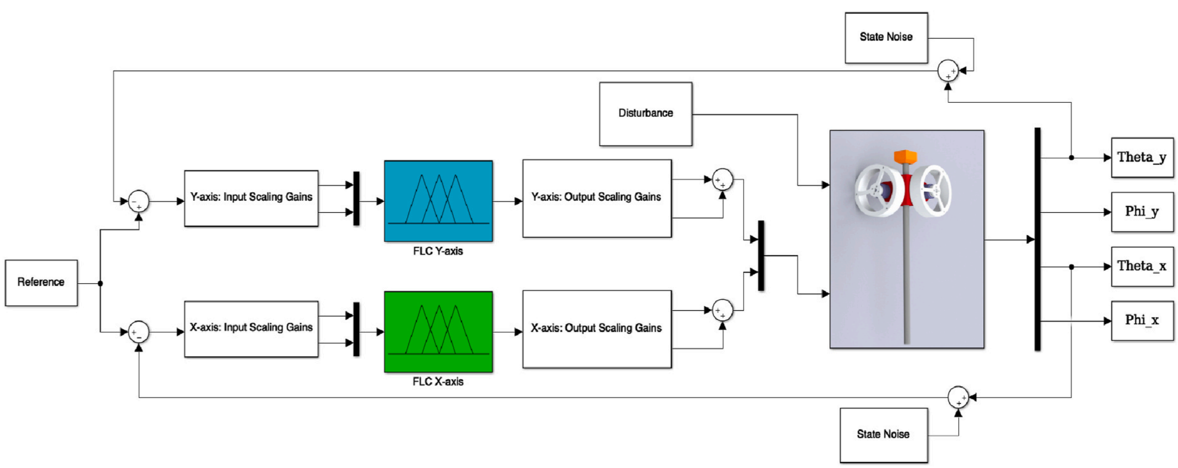


Fig. 10. Closed loop structure of the FLC for the dual axis reaction wheel pendulum.

cost function tor be optimized is given as the Integral Time Square Error (ITSE) function:

$$\mathcal{ITSE} = \int_0^T \gamma e^2(\gamma) d\gamma, \tag{33}$$

For the $x-y$ axes these gains are summarized in Table 4.

By applying the rule-base outlined in Table 2 to the FLC, the resulting nonlinear *control surface* is obtained and shown in Fig. 11 (see Table 3).

The ultimate purpose of tuning is to create the nonlinear structure that the fuzzy controller implements. All of the key fuzzy controller parameters affect this nonlinear surface. Consider Fig. 11 which is plotted versus *error* and *derivative of the error* as its corresponding inputs.

Table 2
Expert fuzzy rules for controlling the DA-RWP.

	Change in error				
Error	NL	NS	Z	PS	PL
NL	PL	PL	PL	PS	Z
NS	PL	PL	PS	Z	NS
Z	PL	PS	Z	NS	NL
PS	PS	Z	NS	NL	NL
PL	Z	NS	NL	NL	NL

Table 3
Stabilizing gains for the PID controller.

PID gains	Values
k_p	2669.032
k_i	8020.288
k_d	218.107

Table 4
Scaling gains of the designed FLC learned using GA.

Scaling gains	$x-$ axis	$y-$ axis
g_0	14	10
g_1	9	5.5
α	200	100
β	8	15

The surface provides a compact representation of the information, though it is constrained by the fact that visualizing the surface in case there are more than two inputs is hard. The surface slope can be changed by modifying the values of g_0 and g_1. Increasing the values of g_0 and g_1 corresponds to the rise in the proportional and derivative gains of the used PID controller, respectively. Moreover, changes in the α and β

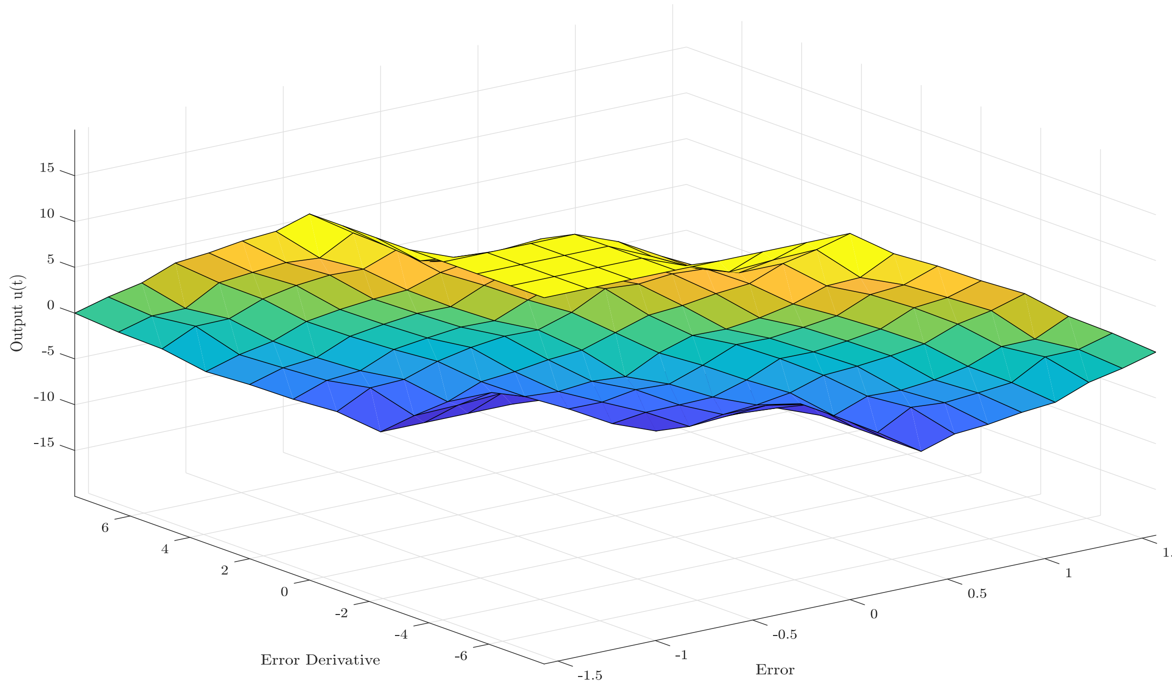
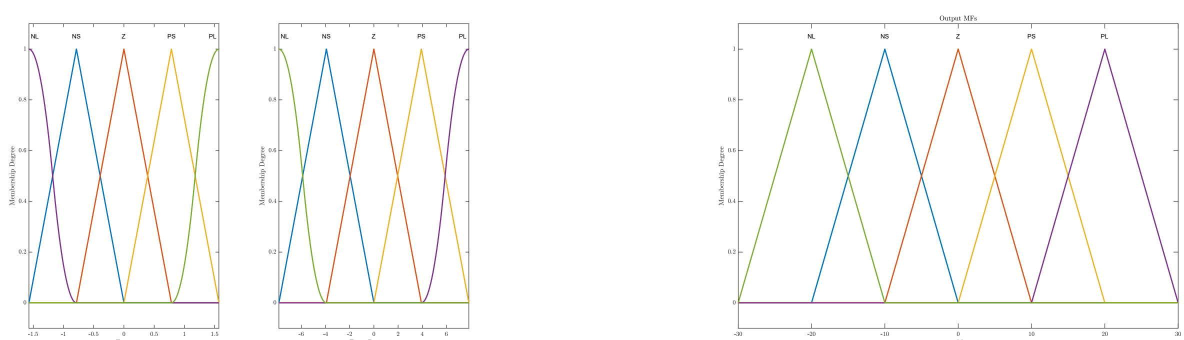


Fig. 11. Control surface of the FLC.



(a) Input MFs of the FLC

(b) Output MFs of the FLC

Fig. 12. FLC's MFs for (a) the error, error derivative, and (b) control signal.

values can rescale the vertical axis of the control surface, sometimes increasing them in a way that results in the saturation of the generated control signal, as needed to limit the control effort. It should be noted that the so-called *Mamdani*-based fuzzy inference mechanism is used to design the decision making part of the FLC.

5. Conclusions and future observations

In this study, first mathematical and mechanical designs for a dual-axis reaction wheel pendulum are obtained and then some classical and intelligent control methods are applied to stabilize the pendulum's angle in an upright position. For the classical control algorithms, PID, state-feedback, and LQR controllers are used. On the other side, as an intelligent method, a fuzzy logic controller has been designed for this purpose. To compare the dynamical response of the pendulum's angle and also wheel's angular speed, corresponding simulations have been performed. Simulations are done in two scenarios where in the first concept an ideal noise-and disturbance-free environment is considered whereas in the second case the mathematical model of the system has been affected by external disturbances and noise. When comparing the obtained results, it is observed that the designed intelligent fuzzy logic controller outperforms its classical counterparts in terms of the stabilization performance and is more robust when encountered with unexpected noise and disturbances, which is considered to be the main contribution of the paper. The results of our experiments can then be used as baseline for the robotics and aerospace engineering domains, specifically in controlling the walking humanoid robots and satellite attitude control systems, in terms of mathematical modeling and control design methods.

As a future concern, authors will concentrate on building a physical system based on the mechanical designs of the current study and accordingly validating the simulations on the experimental setup.

Credit author statement

Yüksel Ediz Bezci Conceptualization; Formal analysis; Investigation; Methodology; Validation; Software; Roles/Writing – original draft; Vahid Tavakol Aghaei Conceptualization; Formal analysis; Investigation; Software; Methodology; Validation; Writing – original draft; Batuhan Ekin Akbulut Conceptualization; Formal analysis;

Methodology; Resources; Software; Validation; Visualization; Writing – review & editing. Deniz Tan Conceptualization; Data curation; Formal analysis; Software; Resources; Validation; Writing – review & editing. Tofigh Allahviranloo Conceptualization; Data curation; Formal analysis; Software; Supervision; Validation; Writing – review & editing. Unai Fernandez-Gamiz Formal analysis; Funding acquisition; Methodology; Resources; Validation; Samad Noeiaghdam Formal analysis; Investigation; Validation; Methodology; Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgments

The work of U.F.-G. was supported by the government of the Basque Country for the ELKARTEK21/10 KK-2021/00014 and ELKARTEK22/85 research programs, respectively.

References

[1] Modeling and station-keeping control of an underwater vehicle manipulator system through reactions wheels, in: Dynamics, Vibration, and Control of ASME International Mechanical Engineering Congress and Exposition 7B, 2021, p. 11, https://doi.org/10.1115/IMECE2021-69031. URL https://doi.org/10.1115/IMECE2021-69031. V07BT07A001.
[2] Vahid Tavakol Aghaei, Ahmet Onat, Ibrahim Eksin, Mujde Guzelkaya, Fuzzy pid controller design using q-learning algorithm with a manipulated reward function, in: 2015 European Control Conference (ECC), 2015, pp. 2502–2507, https://doi.org/10.1109/ECC.2015.7330914.
[3] Brian D.O. Anderson, John B. Moore, Optimal Control: Linear Quadratic Methods, Prentice-Hall, Inc., USA, 1990, 0136385605.
[4] Ersin Armagan, Tufan Kumbaa2sar, A fuzzy logic based autonomous vehicle control system design in the torcs environment, in: 2017 10th International Conference on Electrical and Electronics Engineering (ELECO), 2017, pp. 737–741.
[5] Daulet Baimukashev, Nazerke Sandibay, Bexultan Rakhim, Huseyin Atakan Varol, Matteo Rubagotti, Deep Learning-Based Approximate Optimal Control of a Reaction-Wheel-Actuated Spherical Inverted Pendulum, in: 2020 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM), 2020, pp. 1322–1328, https://doi.org/10.1109/AIM43001.2020.9158920.
[6] P. Baldi, M. Blanke, P. Castaldi, N. Mimmo, S. Simani, Adaptive ftc based on control allocation and fault accommodation for satellite reaction wheels, in: 2016 3rd Conference on Control and Fault-Tolerant Systems (SysTol), 2016, pp. 672–677, https://doi.org/10.1109/SYSTOL.2016.7739826.
[7] Daniel J. Block, Karl Johan Åström, Mark W. Spong, The Reaction Wheel Pendulum, Synthesis Lectures on Control and Mechatronics, Morgan & Claypool Publishers, 2007, ISBN 9781598291940, https://doi.org/10.2200/S00085ED1V01Y200702CRM001. URL https://doi.org/10.2200/S00085ED1V01Y200702CRM001.
[8] Alexey A. Bobtsov, Anton A. Pyrkin, Sergey A. Kolyubin, Adaptive stabilization of a reaction wheel pendulum on moving lego platform, in: 2009 IEEE Control Applications, (CCA) Intelligent Control, (ISIC), 2009, pp. 1218–1223, https://doi.org/10.1109/CCA.2009.5281045.
[9] Travis L. Brown, James P. Schmiedeler, Energetic effects of reaction wheel actuation on underactuated biped robot walking, in: 2014 IEEE International Conference on Robotics and Automation (ICRA), 2014, pp. 2576–2581, https://doi.org/10.1109/ICRA.2014.6907228.
[10] Travis L. Brown, James P. Schmiedeler, Reaction wheel actuation for improving planar biped walking efficiency, IEEE Trans. Robot. 32 (5) (2016) 1290–1297, https://doi.org/10.1109/TRO.2016.2593484.
[11] Víctor D. Correa-Ramírez, Didier Giraldo-Buitrago, Andrés Escobar-Mejía, Fuzzy Control of an Inverted Pendulum Driven by a Reaction Wheel Using a Trajectory Tracking Scheme, 2017.
[12] Illia Diahovchenko, Petrichenko Roman, Lubov Petrichenko, Anatolijs Mahnitko, Pavlo Korzh, Michal Kolcun, Zsolt Čonka, Mitigation of transformers' loss of life in power distribution networks with high penetration of electric vehicles, Results Eng. 15 (2022), 100592, https://doi.org/10.1016/j.rineng.2022.100592. ISSN 2590-1230, URL https://www.sciencedirect.com/science/article/pii/S2590123022002626.
[13] Ahmad M. El-Nagar, Mohammad El-Bardini, M. Nabila, El-Rabaie, Intelligent control for nonlinear inverted pendulum based on interval type-2 fuzzy pd controller, Alex. Eng. J. 53 (1) (2014) 23–32, https://doi.org/10.1016/j.aej.2013.11.006. ISSN 1110-0168, URL https://www.sciencedirect.com/science/article/pii/S111001681300118X.
[14] Manuel A. Estrada, Jesús Mendoza-Avila, Jaime A. Moreno, Leonid Fridman, Homogeneous control and finite-time stabilization of a reaction wheel pendulum system, in: Proceedings of the Congreso Nacional de Control Automático, 2017.
[15] Changhong Fu, A. Miguel, Olivares-Méndez, Ramon Suarez-Fernandez, Pascual Campoy Cervera, Monocular visual-inertial slam-based collision avoidance strategy for fail-safe uav using fuzzy logic controllers - comparison of two cross-entropy optimization approaches, J. Intell. Rob. Syst. 73 (1–4) (2014) 513–533. URL, http://dblp.uni-trier.de/db/journals/jirs/jirs73.html#FuOSC14.
[16] Changhong Fu, Andriy Sarabakha, Erdal Kayacan, Christian Wagner, Robert John, M. Jonathan, Garibaldi, Input uncertainty sensitivity enhanced nonsingleton fuzzy logic controllers for long-term navigation of quadrotor uavs, IEEE ASME Trans. Mechatron. 23 (2) (2018) 725–734, https://doi.org/10.1109/TMECH.2018.2810947.
[17] Xiao Gang Ruan, Yu feng Wang, The modelling and control of flywheel inverted pendulum system, in: 2010 3rd International Conference on Computer Science and Information Technology, vol. 6, 2010, pp. 423–427, https://doi.org/10.1109/ICCSIT.2010.5563931.
[18] Mukhtar Fatihu Hamza, Hwa Jen Yap, Imtiaz Ahmed Choudhury, Abdulbasid Ismail Isa, Aminu Yahaya Zimit, Tufan Kumbasar, Current development on using rotary inverted pendulum as a benchmark for testing linear and nonlinear control algorithms, Mech. Syst. Signal Process. 116 (2019) 347–369, https://doi.org/10.1016/j.ymssp.2018.06.054. ISSN 0888-3270, URL https://www.sciencedirect.com/science/article/pii/S0888327018303923.
[19] V.M. Hernandez, H. Sira-Ramirez, Generalized pi control for swinging up and balancing the inertia wheel pendulum, in: Proceedings of the 2003 American Control Conference, ume 4, 2003, pp. 2809–2814, https://doi.org/10.1109/ACC.2003.1243748, vols. 4, 2003.
[20] Jin-Gu Kang, Real time balancing control of 2 wheel robot using a predictive controller, J. Korea Soc. Comput. Info. 19 (2014) 11–16.
[21] Nasir Khalid, Y. Attaullah, Memon. Output feedback stabilization of an inertia wheel pendulum using sliding mode control, in: 2014 UKACC International Conference on Control (CONTROL), 2014, pp. 157–162, https://doi.org/10.1109/CONTROL.2014.6915132.
[22] Mojtaba Ahmadieh Khanesar, Okyay Kaynak, Recurrent interval type-2 neuro-fuzzy control of an electro hydraulic servo system, in: 2016 IEEE 14th International Workshop on Advanced Motion Control (AMC), 2016, pp. 593–600, https://doi.org/10.1109/AMC.2016.7496414.
[23] Jinseong Lee, Brian Free, Shyline Santana, Derek A. Paley, State-feedback control of an internal rotor for propelling and steering a flexible fish-inspired underwater vehicle, in: 2019 American Control Conference (ACC), 2019, pp. 2011–2016, https://doi.org/10.23919/ACC.2019.8814908.
[24] Dustin Li, Heike Vallery, Gyroscopic assistance for human balance, in: 2012 12th IEEE International Workshop on Advanced Motion Control (AMC), 2012, pp. 1–6, https://doi.org/10.1109/AMC.2012.6197144.
[25] V.F. Shishlakov, D.O. Yakimovskii, M.V. Burakov, I.G. Krivolapchuk, Fuzzy control of spacecraft reaction wheel, Gyroscopy Navigat. 10 (4) (2019) 339–345, https://doi.org/10.1134/S2075108719040072.
[26] Takaaki Matsumoto, Koichi Kita, Suzuki Ren, Atsushi Oosedo, Kenta Go, Yuta Hoshino, Atsushi Konno, Masaru Uchiyama, A hovering control strategy for a tail-sitter vtol uav that increases stability against large disturbance, in: ICRA, IEEE, 2010, pp. 54–59. URL, http://dblp.uni-trier.de/db/conf/icra/icra2010.html#MatsumotoKSOGHKU10.
[27] M. Mizumoto, Realization of pid controls by fuzzy control methods, Fuzzy Set Syst. 70 (2) (1995) 171–182. ISSN 0165-0114. doi:https://doi.org/10.1016/0165-0114(94)00215-S. URL https://www.sciencedirect.com/science/article/pii/016501149400215S. Modern Fuzzy Control.
[28] Oscar Danilo Montoya, Walter Gil-González, Nonlinear analysis and control of a reaction wheel pendulum: lyapunov-based approach, Eng. Sci. Technol. Int. J. 23 (1) (2020) 21–29, https://doi.org/10.1016/j.jestch.2019.03.004. ISSN 2215-0986, https://www.sciencedirect.com/science/article/pii/S221509861831574X. URL.
[29] Oscar Danilo Montoya, Víctor Manuel Garrido, Walter Gil-González, C. Orozco-Henao, Passivity-based control applied of a reaction wheel pendulum: an ida-pbc approach, in: 2019 IEEE International Autumn Meeting on Power, Electronics and Computing (ROPEC), 2019, pp. 1–6, https://doi.org/10.1109/ROPEC48299.2019.9057105.
[30] Oscar Danilo Montoya, Walter Gil-González, Juan A. Dominguez-Jimenez, Alexander Molina-Cabrera, and Diego A. Giral-Ramírez. Global stabilization of a reaction wheel pendulum: a discrete-inverse optimal formulation approach via a control lyapunov function. Symmetry, 12(11), 2020. ISSN 2073-8994. doi: 10.3390/sym12111771. URL https://www.mdpi.com/2073-8994/12/11/1771.
[31] Hossein Nourmohammadi, Jafar Keighobadi, Fuzzy adaptive integration scheme for low-cost sins/gps navigation system, Mech. Syst. Signal Process. 99 (2018) 434–449, https://doi.org/10.1016/j.ymssp.2017.06.030. ISSN 0888-3270, URL https://www.sciencedirect.com/science/article/pii/S0888327017303436.
[32] Manuel Olivares, Albertos Pedro, On the linear control of underactuated systems: the flywheel inverted pendulum, in: 2013 10th IEEE International Conference on Control and Automation (ICCA), 2013, pp. 27–32, https://doi.org/10.1109/ICCA.2013.6564905.
[33] Manuel Olivares, Albertos Pedro, Linear control of the flywheel inverted pendulum, ISA (Instrum. Soc. Am.) Trans. 53 (5) (2014) 1396–1403, https://doi.org/10.1016/j.isatra.2013.12.030. ISSN 0019-0578, URL https://www.sciencedirect.com/science/article/pii/S0019057813002401. ICCA 2013.
[34] Owczarkowski Adam, Dariusz Horla, Joanna Zietkiewicz, Introduction of feedback linearization to robust lqr and lqi control – analysis of results from an unmanned

bicycle robot with reaction wheel, Asian J. Control 21 (2) (2019) 1028–1040, https://doi.org/10.1002/asjc.1773, https://onlinelibrary.wiley.com/doi/pdf/10.1002/asjc.1773. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/asjc.1773.

[35] Yingnan Pan, Peihao Du, Hong Xue, Hak-Keung Lam, Singularity-free fixed-time fuzzy control for robotic systems with user-defined performance, IEEE Trans. Fuzzy Syst. 29 (8) (2021) 2388–2398, https://doi.org/10.1109/TFUZZ.2020.2999746.

[36] Do Xuan Phu, Seung-Bok Choi, Vibration control of a ship engine system using high-load magnetorheological mounts associated with a new indirect fuzzy sliding mode controller, Smart Mater. Struct. 24 (2) (dec 2014), 025009, https://doi.org/10.1088/0964-1726/24/2/025009. URL https://doi.org/10.1088/0964-1726/24/2/025009.

[37] Afshin Rahimi, Krishna Dev Kumar, Hekmat Alighanbari, Fault isolation of reaction wheels for satellite attitude control, IEEE Trans. Aero. Electron. Syst. 56 (1) (2020) 610–629, https://doi.org/10.1109/TAES.2019.2946665.

[38] Nguyen Thi Kim Son, Hoang Thi Phuong Thao, Tofigh Allahviranloo, Hoang Viet Long, State feedback control for fractional differential equation system in the space of linearly correlated fuzzy numbers, Fuzzy Set Syst. (2022), https://doi.org/10.1016/j.fss.2022.06.022. ISSN 0165-0114, URL https://www.sciencedirect.com/science/article/pii/S0165011422003013.

[39] Ricsa Alhassane Soumana, Michael Juma Saulo, Christopher Maina Muriithi, New control strategy for multifunctional grid-connected photovoltaic systems, Results in Engineering 14 (2022), 100422, https://doi.org/10.1016/j.rineng.2022.100422. ISSN 2590-1230, URL https://www.sciencedirect.com/science/article/pii/S2590123022000925.

[40] Mark W. Spong, Corke Peter, Rogelio Lozano, Nonlinear control of the reaction wheel pendulum, Automatica 37 (11) (2001) 1845–1851, https://doi.org/10.1016/S0005-1098(01)00145-5. ISSN 0005-1098, URL https://www.sciencedirect.com/science/article/pii/S0005109801001455.

[41] Vahid Tavakol Aghaei, Buse ilayda Kömürcü, Didar Saka, Bahar Aydoğan, Gizem Kizilca, Şeda Erener, Modeling and simulation of the control performance of a reaction wheel pendulum, in: 2021 29th Signal Processing and Communications Applications Conference, SIU, 2021, pp. 1–4, https://doi.org/10.1109/SIU53274.2021.9477808.

[42] Marco Antonio Simoes Teixeira, Flavio Neves-Jr, Anis Koubaa, Lucia Valeria Ramos De Arruda, Andre Schneider De Oliveira, A quadral-fuzzy control approach to flight formation by a fleet of unmanned aerial vehicles, IEEE Access 8 (2020) 64366–64381, https://doi.org/10.1109/ACCESS.2020.2985032.

[43] João Francisco Silva Trentin, Samuel Da Silva, Jean Marcos De Souza Ribeiro, Hanspeter Schaub, Inverted pendulum nonlinear controllers using two reaction wheels: design and implementation, IEEE Access 8 (2020) 74922–74932, https://doi.org/10.1109/ACCESS.2020.2988800.

[44] Abdullah Turkmen, M. Yusuf Korkut, Mustafa Erdem, Omer Gonul, Volkan Sezer, Design, implementation and control of dual axis self balancing inverted pendulum using reaction wheels, in: 2017 10th International Conference on Electrical and Electronics Engineering (ELECO), 2017, pp. 717–721.

[45] Katsuyoshi Tsujita, Kazuo Tsuchiya, An experimental study on motion control of a biped locomotion machine using reaction wheels, in: Oussama Khatib, John Kenneth Salisbury Jr. (Eds.), ISER, Volume 223 of Lecture Notes In Control And Information Sciences, Springer, 1995, ISBN 3-540-76133-0, pp. 558–565.

[46] Ester Van Broekhoven, Bernard De Baets, Fast and accurate center of gravity defuzzification of fuzzy system outputs defined on trapezoidal fuzzy partitions, Fuzzy Set Syst. 157 (7) (apr 2006) 904–918, https://doi.org/10.1016/j.fss.2005.11.005. ISSN 0165-0114, URL https://doi.org/10.1016/j.fss.2005.11.005.

[47] José Velásquez, M. Kevin, Passino, Fuzzy fault tolerant control for smart lights, J. Intell. Fuzzy Syst. 28 (6) (2015) 2605–2620.

[48] Pengfei Zhang, Zhengxing Wu, Huijie Dong, Min Tan, Junzhi Yu, Reaction-wheel-based roll stabilization for a robotic fish using neural network sliding mode control, IEEE ASME Trans. Mechatron. 25 (4) (2020) 1904–1911, https://doi.org/10.1109/TMECH.2020.2992038.