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On the linearized system of equations for the condensate–normal fluid interaction at very low temperature

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Abstract
The linearization around one of its equilibrium of a system that describes the correlations between the superfluid component and the normal fluid part of a condensed Bose gas in the approximation of very low temperature and small condensate density is studied. A simple and transparent argument gives a necessary and sufficient condition for the existence of global solutions satisfying the conservation of the total number of particles and energy. The global solutions describe the evolution in time of the density of the thermal cloud and, unlike in previous work, that of the condensate's density. Their convergence to a suitable stationary state is also shown and rates of convergence for the normal fluid and superfluid components are obtained.

KEYWORDS
BE condensate, excitations, global existence, rates of convergence

1 | INTRODUCTION

In a uniform condensed Bose gas at temperature below the condensation temperature T_c, the correlations between the superfluid component and the normal fluid part, corresponding to the excitations, may be described by the following equations:

$$\frac{\partial n}{\partial t}(t,p) = Q_3(n_c(t), n(t))(p), \quad t>0,\ p\in\mathbb{R}^3, \tag{1}$$

$$Q_3(n_c,n)(p) = \iint_{(\mathbb{R}^3)^2} [R(p,p_1,p_2) - R(p_1,p,p_2) - R(p_2,p_1,p)]\, dp_1 dp_2, \tag{2}$$

$$\begin{aligned} R(p,p_1,p_2) = {} & |\mathcal{M}(p,p_1,p_2)|^2 [\delta(\omega-\omega_1-\omega_2)\,\delta(p-p_1-p_2)] \\ & \times [n_1 n_2(1+n) - (1+n_1)(1+n_2)n] \end{aligned} \tag{3}$$

(cf. Refs. 1, 2, and 3), where $n(t,p)$ represents the density of particles in the normal gas that at time $t>0$ have momentum p and energy ω, $n_c(t)$ is the density of particles in the condensate at time t and $\mathcal{M}$ is the matrix element of the three-excitation interaction. Equation (1) is complemented with the equation for the fluctuation of the condensate density,

$$\frac{dn_c(t)}{dt} = \int_{\mathbb{R}^3} Q_3(n_c(t), n(t))(p)dp,\ t>0. \tag{4}$$

We consider in this letter the regime of very low temperature and a large density of condensed atoms, where the Bogoliubov dispersion law may be approximated by the phonon dispersion relation (cf. Refs. 1–5). For dimensionless variables, in units that minimize the number of prefactors,

$$\omega = \left(n_c(t)|p|^2 + |p|^4\right)^{1/2} \approx \sqrt{n_c(t)}\,|p|, \tag{5}$$

$$|\mathcal{M}(p,p_1,p_2)|^2 \approx \frac{|p||p_1||p_2|}{n_c(t)}. \tag{6}$$

The resulting equation in its wave turbulence version has been studied in detail in Ref. 4 (see also Ref. 6), where properties of the principal part of the collision integral operator are presented. The nonlinear system (1), (4) has been considered for radial solutions in the mathematical literature in the limit (5), (6) in Ref. 7, where global existence of solutions is obtained under some conditions on the initial data. We consider here, with a slightly different approach, the nonradial close to equilibrium situation, and the long time behavior of the system described by the linearized equations.

A different extreme regime of a uniform condensed Bose gas, for temperatures below but close to the critical temperature T_c, and with small but nonconstant number density of condensed atoms is described in the literature of physics (Refs. 1, 3, 4, and 2). The corresponding linearized equation around an equilibria is considered in Refs. 8 and 9.

It is well known that Equation (1) has a family of equilibria $n_\mu(\omega) = (e^{\beta\omega-\mu}-1)^{-1}$ for $\mu\le 0$ and $\beta>0$. It follows that, for any constant $\varkappa>0$, the pair $(n_0(\omega),\varkappa)$ with $\omega=\sqrt{\varkappa}\,|p|$ is an equilibrium of the system (1), (4). Without any loss of generality, the values $\kappa = \beta = 1$ are taken in all the sequel.

The linearized Equation (1) around the equilibrium n_0 has been studied in Refs. 10 and 11, and 12 in the simplified case of a constant condensate's density n_c.

However, one basic aspect of the nonequilibrium behavior of the system condensate–normal fluid is the evolution itself of the condensate after its formation, to which a great attention has been paid in the physics literature (cf., e.g., Refs. 13, 14, and references therein). It is then important to consider the situation where the condensate's density varies in time together with its thermal cloud. This necessarily leads to consider the coupling of the three collisions operator Q_3 with the differential equation that describes the condensate's density, that is, the system (1),(4), for which a detailed mathematical study is still missing.

We study in this work the linearization of system (1),(4) at a very low temperature in the non-necessarily radial, spatially homogeneous case, and obtain new results on the global existence of solutions and their rate of convergence to equilibrium. That system is particularly interesting because it allows to describe with some detail the main properties and the evolution in time of the densities, of the condensate, and of the thermal cloud, not only that of the thermal cloud as in Ref. 12. There is no, up to now, any other example, even radially symmetric, where the convergence of these two densities to their equilibrium and estimates of the convergence rate have been proved.

The linearization of Equation (1) is deduced using the change of dependent variable

$$n(t) = n_0 + n_0(1 + n_0)F(t) \tag{7}$$

in the collision integral (2) and keeping only linear terms in F in the resulting equation (cf. e.g., Refs. 10–12). Under the approximation (5), the collision manifold in the momentum space in (2) reduces to those p, p_1, and p_2 that are collinear. The resulting equation is then,

$$n_0(p)[1 + n_0(p)]\frac{\partial F}{\partial t}(t, p) = \frac{1}{n_c(t)}\mathscr{L}(F), \tag{8}$$

$$\mathscr{L}(F) = -\Gamma(|p|)\,n_0(|p|)(1 + n_0(|p|))F(t, p) + \int_{\mathbb{R}^3} F(t, p')\,W(p, p')dp', \tag{9}$$

$$\Gamma(x) = \sinh x \int_0^{\infty} \frac{y^2}{\sinh y}\left(\frac{|x-y|^2}{\sinh|x-y|} + \frac{(x+y)^2}{\sinh(x+y)}\right)dy, \tag{10}$$

where the explicit expression of the function W obtained in Ref. 11, is

$$\begin{aligned} W(p, p') = {} & (x - x')^2 H(x - x')n_0(x)(1 + n_0(x'))(1 + n_0(x - x')) \\ & + (x' - x)^2 H(x' - x)n_0(x')(1 + n_0(x))(1 + n_0(x' - x)) \\ & - (x + x')^2 n_0(x + x')(1 + n_0(x))(1 + n_0(x')), \end{aligned} \tag{11}$$

with $x = |p|$, $x' = |p'|$ and H is the Heaviside's function.

When $n_c(t)$ is a constant, an existence and uniqueness result of global solutions $F(t, p)$ to (8) for all data $F_0 \in L^2(d\mu)$, was proved in Ref. 12 where $d\mu \equiv n_0(1 + n_0)dp$. If F_0 also satisfies $\int_{|p|<1} |p|^{-1}|F_0(p)|^2 d\mu < \infty$, long time convergence of the solution to an equilibrium, and a relaxation rate at least of order $t^{-1/2}$ as $t \to \infty$ was also proved.

When the condensate's density $n_c(t)$ is not constant, Equation (8) must be supplemented by the linearization of (4). This leads to the following linearized system for the pair (u, m_c):

$$n_0(p)[1 + n_0(p)]\frac{\partial u}{\partial t}(t, p) = \frac{1}{m_c(t)}\mathscr{L}(u(t))(p), \tag{12}$$

$$\frac{dm_c(t)}{dt} = -\frac{1}{m_c(t)}\int_{\mathbb{R}^3}\mathscr{L}(u(t))(p)dp. \tag{13}$$

The main purpose of this work is then to study the existence, long time behavior, and relaxation rate of global solutions of the system (12), (13) where, unlike in Ref. 12, the condensate's density and the thermal cloud density are varying in time.

It is very natural to rescale the time variable in (12) as follows:

$$\tau = \int_0^t \frac{ds}{m_c(s)}, \tag{14}$$

to get rid of the function m_c in front of the collision integral, obtain the equation

$$n_0(p)[1 + n_0(p)]\frac{\partial v}{\partial \tau}(\tau, p) = \mathscr{L}(v(\tau)), \tag{15}$$

and apply the results in Ref. 12 to obtain a solution $v(\tau)$.

Notice however that system (12), (13) is not reducible to the single Equation (17). The delicate point is indeed to invert the change of variable (14) in order to obtain a solution (u, n_c) of (12), (13) that is global in time. This is possible if and only if the following condition is satisfied,

$$m_c^2(0) > 2\int_{\mathbb{R}^3}(v(\tau, p) - u_0(p))d\mu, \ \forall \tau > 0, \tag{16}$$

(the precise long time asymptotic behavior of v is also used in our proof). Condition (16) is always satisfied when n_c is a positive constant (as in Ref. 12) because in that case, $\int_{\mathbb{R}^3} v(\tau, p)d\mu$ is also a constant in time for all $\tau \geq 0$, and then such a hypothesis is unnecessary.

Our first result shows the global existence, under the necessary and sufficient condition (16), and uniqueness of solutions (u, m_c) to the system (12), (13). These solutions satisfy the conservation of the total number of particles and energy.

In our second result, we show that, as time tends to infinity, $u(t)$ and $m_c(t)$ converge to their corresponding equilibrium and prove a rate of convergence of $t^{-1/2}$ as $t \to \infty$.

These results prove that, under condition (16), system (12), (13) is globally well posed and describes the evolution in time of the densities of the condensate and the thermal cloud in the linearized approximation and their relaxation to equilibrium. A natural extension of the present work would be to consider the nonlinear system (1), (4) as a perturbation of (12), (13), under condition (16), and for initial data close to an equilibrium.

Our two main results and their proofs are presented in the next section.

2 | THE LINEARIZED SYSTEM FOR NONCONSTANT $n_c(t)$

The first result is the following, where $\{Y_{\ell\, m}\}_{\ell,m}$ denotes the spherical harmonics on $\mathbb{S}^2$.

Theorem 1. *Suppose that* $u_0 \in L^2(d\mu)$ *also satisfies*

$$\int_{|p|<1} |u_0(p)|^2\, |p|^{-1} d\mu < \infty \tag{17}$$

and let v *be the solution of Equation (15) with initial data* u_0*. Suppose that* $m_c(0) > 0$ *and (16) are satisfied. Then, there exists a unique pair of functions* (u, m_c) *with*

$$u \in L^\infty\big(0,\infty; L^2(d\mu)\big) \cap C\big([0,\infty); L^2(d\mu)\big); \quad u - u_\infty \in L^2\big(0,\infty; L^2(d\mu)\big), \tag{18}$$

$$u_\infty = \left(\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} c_{\ell\, m} Y_{\ell\, m}\left(\frac{p}{|p|}\right)\right)|p|; \quad c_{\ell\, m} = \int_{\mathbb{R}^3} u_0(p) Y_{\ell\, m}\left(\frac{p}{|p|}\right) d\mu, \tag{19}$$

$$\frac{\partial u}{\partial t} \in L^2\left(0,\infty; L^2\left(\frac{d\mu}{\Gamma(|p|)}\right)\right), \tag{20}$$

$$m_c \in C([0,\infty)),\ \ m_c(t) > 0,\ \ \forall t > 0, \tag{21}$$

satisfying Equation (12) in $L^2(0,\infty; L^2(\frac{d\mu}{\Gamma(|p|)}))$ *and Equation (13) for all* $t > 0$*, such that:*

$$\lim_{t\to 0}\left(|m_c(t) - m_c(0)| + ||u(t) - u_0||_{L^2\left(\frac{d\mu}{\Gamma(|p|)}\right)} + ||u(t) - u_0||_{L^2(d\mu)}\right) = 0. \tag{22}$$

This solution also satisfies the following conservation properties:

$$\frac{dp_c(t)}{dt} + \frac{d}{dt}\int_{\mathbb{R}^3} n_0(p)(1 + n_0(p))u(t,p)dp = 0,\ \ \forall t > 0, \tag{23}$$

$$\frac{d}{dt}\int_{\mathbb{R}^3} n_0(p)(1 + n_0(p))u(t,p)|p|dp = 0,\ \ \forall t > 0. \tag{24}$$

Proof. By Theorem 1.1 of Ref. 12, there exists a unique solution $v(\tau, p)$ to (15) such that

$$v \in L^\infty\big(0,\infty; L^2(d\mu)\big) \cap C\big([0,\infty); L^2(d\mu)\big); \quad v - u_\infty \in L^2\big(0,\infty; L^2(d\mu)\big), \tag{25}$$

$$\lim_{t\to 0}\left(||v(t) - u_0||_{L^2\left(\mathbb{R}^3, \frac{dp}{\Gamma(p)\sinh^4|p|}\right)} + ||v(t) - u_0||_{L^2\left(\mathbb{R}^3, \frac{dp}{\sinh^2|p|}\right)}\right) = 0, \tag{26}$$

$$||v(t) - u_\infty||_{L^2(d\mu)} \le \frac{C}{(1+t)^{1/2}}||u_0 - u_\infty||_{L^2(d\mu)},\ \ \forall t > 0. \tag{27}$$

Let us define now, for all $\tau > 0$,

$$m(\tau) = \int_{\mathbb{R}^3} \mathscr{L}(v(\tau,p))dp, \tag{28}$$

$$q_c(\tau) = \left(m_c(0)^2 - 2\int_0^\tau m(\sigma)d\sigma\right)^{1/2}. \tag{29}$$

By (28) and Equation (15),

$$\int_0^\tau m(\sigma)d\sigma = \int_0^\tau \frac{d}{d\sigma}\int_{\mathbb{R}^3} n_0(1+n_0)v(\sigma)dpd\sigma = \int_{\mathbb{R}^3} n_0(1+n_0)(v(\tau,p)-u_0(p))dp, \tag{30}$$

and it follows from condition (16) that $q_c(\tau)$ is well defined for all $\tau > 0$. The change of variables (14) may now be inverted, defining

$$t = \int_0^\tau q_c(\sigma)d\sigma. \tag{31}$$

Because $q_c(\tau) > 0$ for all $\tau > 0$, the right-hand side of (31) is an increasing function of τ. Moreover, if we denote

$$N_* = \int_{\mathbb{R}^3} n_0(1+n_0)(u_\infty(p)-u_0(p))dp, \tag{32}$$

by the convergence property (27),

$$\begin{aligned} \left|q_c(\tau)^2 - m_c(0)^2 + 2N_*\right| &\le 2\int_{\mathbb{R}^3} |v(\tau,p)-u_\infty(p)|d\mu \\ &\le 2\|v(\tau)-u_\infty\|_{L^2(d\mu)}\|n_0(1+n_0)\|_1^{1/2} \le \frac{C\|u_0-u_\infty\|_{L^2(d\mu)}}{(1+\tau)^{1/2}} \end{aligned} \tag{33}$$

for some numerical constant $C > 0$. Therefore,

$$\lim_{\tau\to\infty} q_c^2(\tau) = m_c^2(0) - 2N_* > 0, \tag{34}$$

using (16) again it follows that the function q_c is not integrable on $(0,\infty)$. Then, given any $t > 0$, there exists a unique $\tau > 0$ satisfying (31). We define then,

$$u(t) = v(\tau), \quad m_c(t) = q_c(\tau). \tag{35}$$

By (31) and Theorem 1.1 in Ref. 12, the equation

$$n_0(1+n_0)\frac{\partial u}{\partial t} = n_0(1+n_0)\frac{\partial v}{\partial \tau}\frac{\partial \tau}{\partial t} = \mathcal{L}(v(\tau))\frac{1}{q_c(\tau)} = \frac{1}{m_c(t)}\mathcal{L}(u(t)) \tag{36}$$

is satisfied in $L^2(0,\infty; L^2(\frac{d\mu}{\Gamma(|p|)}))$. On the other hand, because by (29),

$$\begin{aligned} \frac{dm_c(t)}{dt} &= \frac{dq_c(\tau)}{d\tau}\frac{d\tau}{dt} = -\frac{2m(\tau)}{2q_c(\tau)}\frac{d\tau}{dt} \\ &= -\frac{m(\tau)}{m_c(t)}\frac{d\tau}{dt} = -\frac{1}{m_c(t)}\frac{d}{dt}\int_{\mathbb{R}^3} \mathcal{L}(u(t))dp, \quad \forall t > 0, \end{aligned} \tag{37}$$

Equation (24) is satisfied for all $t > 0$. Properties (25)–(22) and (24) are consequences of properties (15)–(19) of Theorem 1.1 in Ref. 12 applied to the function $v(\tau)$ and, for Properties (21) and (24), that $\partial_t = q_c(\tau)^{-1}\partial_\tau$ with $q_c \in C(0,\infty)$, $q_c(t) > 0$ for all $t > 0$. On the other hand, by Theorem 1.1

in Ref. 12

$$\frac{d}{dt}\int_{\mathbb{R}^3} n_0(1+n_0)u(t,p)dp = \frac{1}{m_c(t)}\frac{d}{d\tau}\int_{\mathbb{R}^3} n_0(1+n_0)v(\tau,p)dp$$
$$= \frac{1}{m_c(t)}\int_{\mathbb{R}^3} n_0(1+n_0)\frac{v(\tau,p)}{\partial\tau}dp$$
$$= \frac{1}{m_c(t)}\int_{\mathbb{R}^3} \mathcal{L}(v(\tau)(p)dp = \frac{1}{m_c(t)}\int_{\mathbb{R}^3} \mathcal{L}(u(t)(p)dp, \tag{38}$$

and by (24), the conservation (23) is satisfied.

If $(\tilde{u}, \tilde{m}_c)$ is another solution satisfying (25)–(22), the function $\tilde{v}(\tau,p) = \tilde{u}(t,p)$ for τ given by (14) would be a solution to (15) satisfying (25)–(21) and (22). It then follows by Theorem 1.1 in Ref. 12 that $\tilde{v}(\tau) = v(\tau)$. Then, because $\tilde{u}(t,p) = \tilde{v}(\tau,p)$ for almost every $p \in \mathbb{R}^3$,

$$\frac{d\tilde{m}_c(t)}{dt} = -\frac{1}{\tilde{m}_c(t)}\int_{\mathbb{R}^3} \mathcal{L}(\tilde{u}(t))(p)dp$$
$$= -\frac{1}{\tilde{m}_c(t)}\int_{\mathbb{R}^3} \mathcal{L}(\tilde{v}(\tau))(p)dp = -\frac{1}{\tilde{m}_c(t)}\int_{\mathbb{R}^3} \mathcal{L}(v(\tau))(p)dp. \tag{39}$$

Moreover $v(t,p) = u(t,p)$ for almost every $p \in \mathbb{R}^3$ and then

$$\frac{d\tilde{m}_c(t)}{dt} = -\frac{1}{\tilde{m}_c(t)}\int_{\mathbb{R}^3} \mathcal{L}(u(t))(p)dp. \tag{40}$$

By the Lipschitz property of right-hand side of Equation (40), we deduce $\tilde{m}_c(t) = m_c(t)$ for all $t > 0$ and this shows the uniqueness of the pair (u, m_c) satisfying (12), (13), and (25)–(22). ■

Our second result shows the convergence of the solution (u, m_c) to an equilibrium and gives an estimate of the relaxation rate.

Theorem 2. *Let u_0 and $m_c(0)$ be as in Theorem 1. Then, there exists a constant $C = C(u_0)$ such that for all $t > 0$,*

$$||u(t) - u_\infty||_{L^2(d\mu)} + \left|m_c^2(t) - m_c^2(0) + 2N_*\right| \leq \frac{C||u_0 - u_\infty||_{L^2(d\mu)}}{\left(1 + \frac{t}{(m_c^2(0) - 2N_*)}\right)^{1/2}}. \tag{41}$$

where N_* is defined in (32).

Proof. By (33) and (35),

$$\left|m_c(t)^2 - m_c(0)^2 + 2N_*\right| \leq \frac{C||u_0 - u_\infty||_{L^2(d\mu)}}{(1 + \tau(t))^{1/2}}, \tag{42}$$

$$\text{where } \tau(t) = \int_0^t \frac{ds}{m_c(s)}, \tag{43}$$

and by (34) and (35),

$$\lim_{t\to\infty} m_c(t) = (m_c^2(0) - 2N_*)^{1/2}. \tag{44}$$

It follows that there exists $t_* > 0$ such that,

$$2m_c(s) \geq (m_c^2(0) - 2N_*)^{1/2},\ \forall s > t_* \tag{45}$$

and

$$\tau(t) \geq \int_{t_*}^{t} \frac{ds}{m_c(s)} > \frac{\sqrt{2}(t - t_*)}{(m_c^2(0) - 2N_*)^{1/2}},\ \forall t > t_*. \tag{46}$$

Then, for some constant $C > 0$,

$$\left| m_c^2(t) - m_c^2(0) + 2N_* \right| \leq \frac{C||u_0 - u_\infty||_{L^2(d\mu)}}{\left(1 + \frac{t}{(m_c^2(0)-2N_*)}\right)^{1/2}},\ \forall t > 0. \tag{47}$$

On the other hand, again by (27) and (46),

$$||u(t) - u_\infty||_{L^2(d\mu)} = ||v(\tau) - u_\infty||_{L^2(d\mu)} \leq \frac{C||u_0 - u_\infty||_{L^2(d\mu)}}{(1 + \tau(t))^{1/2}} \leq \frac{C||u_0 - u_\infty||_{L^2(d\mu)}}{(1 + t)^{1/2}} \tag{48}$$

and (41) follows. ∎

Remark 1.

(i) The nonlocal condition (16) is necessary in order to have a global solution (u, m_c) of (12), (13), satisfying (25)–(22). If such a solution exists indeed, the function m_c must satisfy $m_c^2(t) = m_c^2(0) - 2\int_{\mathbb{R}^3}(v(\tau, p) - u_0(p))d\mu$ for all $t > 0$, where it is defined, and so (16) must be fulfilled.
(ii) It would be interesting to know if there exist initial data u_0 for which as $t \to \infty$, $m_c(t) \to 0$ or $\int_{\mathbb{R}^3} u_\infty(p)d\mu = 0$.

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DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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