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On the Soliton Solutions for the Stochastic Konno–Oono System in Magnetic Field with the Presence of Noise

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Abstract: In this study, we consider the stochastic Konno–Oono system to investigate the soliton solutions under the multiplicative sense. The multiplicative noise is considered firstly in the Stratonovich sense and secondly in the Itô sense. Applications of the Konno–Oono system include current-fed strings interacting with an external magnetic field. The F-expansion method is used to find the different types of soliton solutions in the form of dark, singular, complex dark, combo, solitary, periodic, mixed periodic, and rational functions. These solutions are applicable in the magnetic field when we study it at the micro level. Additionally, the absolute, real, and imaginary physical representations in three dimensions and the corresponding contour plots of some solutions are drawn in the sense of noise by the different choices of parameters.

Keywords: stochastic Konno–Oono system; soliton solutions; Stratonovich sense; Itô sense; F-expansion method

MSC: 15B51; 60H40

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1. Introduction

The magnetic field is a profile that is used as a tool to specify how the magnetic force is distributed around and inside magnetic objects. The motional induction effect, which results from the movement of the conducting crust via the Earth's magnetic field, produces an electromagnetic (EM) field during an earthquake. On the other hand, intriguing natural phenomena, such as the seismic sea waves, are created as a result of these earthquakes. These waves' height and wavelength range are quite important. In particular, these waves provide enormous power that can be transformed into a new type of energy that will become essential in the coming years. Therefore, taking into account such natural difficulties is crucial in mathematical physics. Nonlinear partial differential equations can be used to represent the majority of natural occurrences (NPDEs) [1–3]. NPDEs act a key role in describing complex natural phenomena [4]. The details of its solutions are frequently discussed. The study of solitary solutions in the magnetic field is very important because it helps improve our understanding of physical phenomena, such as the chiral soliton lattice and the nematic liquid crystal in the magnetic field, among other things [5,6].

Every mathematical model has a random motion at the micro level instead of a linear motion where it physically appears. Therefore, several researchers use multiplicative white noise in the mathematical model to add randomness [7–11]. Differential equations are referred to as stochastic differential equations when this phrase appears. During the last decade, when the epidemic starts, the classical models fails to describe the true behavior of the disease dynamics. So, stochastic models are more suitable as compared to classical models. Different researchers are working on the solutions of stochastic partial differential equation [12–14]. In this study, we consider the stochastic coupled Konno–Oono (K–O) system in the form [15]

$$\phi_{xt} - 2\phi\psi = \nu F(\phi), \tag{1}$$

$$\psi_t + 2\phi\phi_x = 0, \tag{2}$$

where ϕ and ψ are functions of x and t, ν is the noise strength, $F(\phi)$ is the noise term.

So, we study the two cases of multiplicative noise, as follows [16]:

1. $F(\phi) = \phi_x \cdot \beta_t$ is taken for the Stratonovich sense;
2. $F(\phi) = \phi_x \beta_t$ is taken for the Itô sense.

We are limited to the case that noise is a constant in space. Applications of the Konno–Oono equation system for current-fed strings interacting with an external magnetic field have been studied [17–19], as well as the parallel transport of each curve point along the direction of time where the connection is magnetic. Many researchers investigated the coupled Konno–Oono equation, such as Mahmoud A. E. A., who investigated with the help of the unified solver technique [20]; Jalil M. et al. used the extended trial equation method [19]; Montri T. et al. used the extended simplest equation method [19]; Mirhosseini-Alizamini S. M. used the new modified extended direct algebraic method [21]; Kang-Jia W. used the simplified extended tanh-function method and variational direct method [22,23]; and some others are considered the Konno–Oono equation system for the sake of solitary wave solutions. Wael W. M. [15] considered the stochastic version of the coupled Konno–Oono model and investigated the solitary wave solution by using the generalized G'/G-expansion method. He found the solutions in the form of trigonometric, hyperbolic and rational solutions, but we extract the different forms of solutions, such as dark, singular, complex dark, combo, solitary, periodic, mixed periodic, and rational functions. In this study, we use the stochastic coupled model and investigate the soliton solution with the help of the F-expansion method. This technique provides us the solutions in the forms of dark, singular, complex dark, combo, solitary, periodic, mixed periodic, and rational functions. This has not been used before to investigate the solutions of the Konno–Oono system. In this study, this system is under consideration by the noise in two senses: the first one is the Stratonovich sense, and secondly the Itô sense. These results are new and very beneficial for the researcher when they consider the problem at the micro level. In the next section, we consider the K–O system with Stratonovich sense to investigate the soliton solution with the help of the F-expansion method [24–26].

2. Wiener Process

Suppose a non-differentiable Wiener process β_t with the following properties [27]:

$$\lim_{\Delta t \to 0} \Delta\beta_t = 0; \tag{3}$$

$$\lim_{\Delta t \to 0} \frac{(\Delta\beta_t)^n}{\Delta t} = \begin{cases} 1, & n = 2 \\ 0, & n = 3, 4, \cdots \end{cases} \tag{4}$$

Definition 1. *Stochastic process $(\beta_t)_{t \leq 0}$ is said to be a Brownian motion if the following conditions are satisfied:*

- *β_t is a continuous functionif $t \leq 0$.*
- *$\beta_0 = 0$.*
- *For $\tau_1 < \tau_2$, $\beta_{\tau_2} - \beta_{\tau_1}$ is independent.*

- $\beta_{\tau_2} - \beta_{\tau_1}$ *has a Gaussian distribution* $\kappa(0, \tau_2 - \tau_1)$.

$\beta_t = \frac{d\beta}{dt}$ is the time derivative of Wiener process $\beta(t)$.

3. Soliton Solutions for K–O System with Stratonovich Sense

In this section, we find the exact solutions for the K–O system with the Stratonovich sense.

$$\phi_{xt} - 2\phi\psi = \nu\phi_x \cdot \beta_t, \tag{5}$$
$$\psi_t + 2\phi\phi_x = 0. \tag{6}$$

So, by choosing the wave transformation in the noise, such as [7,28,29],

$$\phi(x,t) = U(\rho)e^{[\nu\beta(t) - \nu^2 t]}, \quad \psi(x,t) = V(\rho), \quad \text{where} \quad \rho = x - ct$$

where U and V are deterministic functions, and c is the speed of light. β_t is the time derivative of Wiener process $\beta(t)$. By substituting this transformation, Equations (5) and (6) are converted to the SDEs, such as

$$-cU'' + \nu\beta_t - \nu^2 U' - 2UV = \nu U' \cdot \beta_t, \tag{7}$$
$$-cV' + 2UU' e^{[\nu\beta(t) - \nu^2 t]} = 0. \tag{8}$$

Multiply (-2) by Equation (7) and obtain

$$2cU'' - 2\nu U'\beta_t + 4UV = -2\nu U' \cdot \beta_t - 2\nu^2 U'. \tag{9}$$

Conversion between Itô and Stratonovich integrals, such as

$$\nu U' \cdot \beta_t = \nu U' \beta_t + \nu^2 U', \tag{10}$$

putting Equation (10) into Equation (9), obtain

$$cU'' + 2UV = 0. \tag{11}$$

Now, we take the expectation on both sides of Equation (8), and we obtain

$$-cU' + 2UU'\mathbb{E}(e^{2\nu\beta(t)}) = 0. \tag{12}$$

Here, we use the conditional expectation (conditioned by the filtration generated by Wiener process), and $\mathbb{E}(e^{2\nu\beta(t)})$ is identity element; for more detail, see [30–32]. So, Equation (12) takes the form

$$-cU' + 2UU' = 0, \tag{13}$$

and integrating Equation (13) with respect to ρ, we obtain

$$V = \frac{1}{c}(U^2 + \gamma), \tag{14}$$

where γ is the constant of integration. Now putting Equation (14) into Equation (11), we obtain

$$c^2 U'' + 2U^3 + 2\gamma U = 0. \tag{15}$$

Now, we suppose that the solution is in the polynomial form from the improved F-expansion method as follows:

$$U(\rho) = \delta_0 + \sum_{i=-M}^{N} \delta_1 \Omega^i(\rho), \tag{16}$$

where δ_0 and δ_i are constants that are determined later. The N is a positive integer that is determined with the help of the homogeneous balancing principle. So, $\Omega(\rho)$ satisfies the Riccati equation as follows:

$$\Omega^{'}(\rho) = P + Q\Omega(\rho) + R\Omega^2(\rho), \tag{17}$$

where P, Q and R are constants. Substituting $N = 1$ in Equation (16), we obtain

$$U(\rho) = \delta_0 + \delta_1\Omega(\rho) + \delta_{-1}\Omega^{-1}(\rho). \tag{18}$$

Inserting Equation (18) in Equation (15) by the help of Equation (17), we obtain the infinite series in $\Omega^i(\rho)$. Setting all the same powers of $\Omega^i(\rho)$ equal to zero yields a system of equations. To find the values of the constant by the aid of Wolfram Mathematica 11.1 version, we gain the three cases, namely,

Case 1. When we take $P = 0$ we obtained

$$\delta_0 = i\sqrt{\gamma}, \delta_1 = \frac{2i\sqrt{\gamma}R}{Q}, \delta_{-1} = 0, c = -\frac{2\sqrt{\gamma}}{Q},$$

Case 2. When we take $Q = 0$ we obtained

$$\delta_0 = 0, \delta_1 = \frac{i\sqrt{\gamma}\sqrt{R}}{\sqrt{2}\sqrt{P}}, \delta_{-1} = \frac{i\sqrt{\gamma}\sqrt{P}}{\sqrt{2}\sqrt{R}}, c = -\frac{\sqrt{\gamma}}{\sqrt{2}\sqrt{P}\sqrt{R}},$$

Case 3. When we take $P = 0$, $Q = 0$ but $R \neq 0$, then we obtained

$$\delta_0 = 0, \delta_1 = -icR, \delta_{-1} = -\frac{i\gamma}{3cR},$$

Now, by substituting these values in Equation (18), then by the help of the wave transformation, we obtain the different types of soliton, trigonometric and rational solutions of Equation (5) as follows:

Family-I: When $P = 0$, $Q = 1$ and $R = -1$, then $\Omega(\rho) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{\rho}{2}\right)\right]$. So, we obtain the soliton solutions of Equation (5) as follows:

$$\phi_1(x,t) = \left[i\sqrt{\gamma} - 2i\sqrt{\gamma}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(2\sqrt{\gamma}t + x)\right) + \frac{1}{2}\right)\right]e^{[\beta\nu t - \nu^2 t]}. \tag{19}$$

Now from Equation (14), we obtain the solution of Equation (6) as

$$\psi_1(x,t) = -\frac{1}{2\sqrt{\gamma}}\left(\gamma + \left(i\sqrt{\gamma} - 2i\sqrt{\gamma}\left(\frac{1}{2}\tanh\left[\frac{1}{2}(2\sqrt{\gamma}t + x)\right] + \frac{1}{2}\right)\right)^2\right). \tag{20}$$

Family-II: When $P = 0$, $Q = -1$ and $R = 1$, then $\Omega(\rho) = \left[\frac{1}{2} - \frac{1}{2}\coth\left(\frac{\rho}{2}\right)\right]$. So, we obtained the Soliton solutions of Equation (5) as follow,

$$\phi_2(x,t) = \left[i\sqrt{\gamma} - 2i\sqrt{\gamma}\left(\frac{1}{2} - \frac{1}{2}\coth\left(\frac{1}{2}(x - 2\sqrt{\gamma}t)\right)\right)\right]e^{[\beta\nu t - \nu^2 t]}, \tag{21}$$

Now from Equation (14), we obtain the solution of Equation (6) as

$$\psi_2(x,t) = \frac{1}{2\sqrt{\gamma}}\left(\gamma + \left(i\sqrt{\gamma} - 2i\sqrt{\gamma}\left(\frac{1}{2} - \frac{1}{2}\coth\left(\frac{1}{2}(x - 2\sqrt{\gamma}t)\right)\right)\right)^2\right). \tag{22}$$

Family-III: When $P = \frac{1}{2}$, $Q = 0$ and $R = -\frac{1}{2}$, then $\Omega(\rho) = \left[\tanh(\rho) \pm i\,\mathrm{sec}\,h\right]$ or $\Omega(\rho) = \left[\coth(\rho) \pm \mathrm{csc}\,h\right]$. So, we obtain the soliton solutions of Equation (5) as follows:

$$\phi_3(x,t) = \left[\frac{\sqrt{\gamma}}{\sqrt{2}\left(\tanh\left(x - i\sqrt{2}\sqrt{\gamma}t\right) + i\mathrm{sech}\left(x - i\sqrt{2}\sqrt{\gamma}t\right)\right)} - \frac{\sqrt{\gamma}\left(\tanh\left(x - i\sqrt{2}\sqrt{\gamma}t\right) + i\mathrm{sech}\left(x - i\sqrt{2}\sqrt{\gamma}t\right)\right)}{\sqrt{2}}\right]e^{[\beta\nu t - \nu^2 t]}, \tag{23}$$

Now, from Equation (14), we obtain the solution of Equation (6) as

$$\psi_3(x,t) = -\frac{i}{\sqrt{2}\sqrt{\gamma}}\left(\gamma + \left(\frac{\sqrt{\gamma}}{\sqrt{2}\left(\tanh\left(x - i\sqrt{2}\sqrt{\gamma}t\right) + i\mathrm{sech}\left(x - i\sqrt{2}\sqrt{\gamma}t\right)\right)} - \frac{\sqrt{\gamma}\left(\tanh\left(x - i\sqrt{2}\sqrt{\gamma}t\right) + i\mathrm{sech}\left(x - i\sqrt{2}\sqrt{\gamma}t\right)\right)}{\sqrt{2}}\right)^2\right). \tag{24}$$

or

$$\phi_4(x,t) = \left[\frac{\sqrt{\gamma}}{\sqrt{2}\left(\coth\left(x - i\sqrt{2}\sqrt{\gamma}t\right) + \mathrm{csch}\left(x - i\sqrt{2}\sqrt{\gamma}t\right)\right)} - \frac{\sqrt{\gamma}\left(\coth\left(x - i\sqrt{2}\sqrt{\gamma}t\right) + \mathrm{csch}\left(x - i\sqrt{2}\sqrt{\gamma}t\right)\right)}{\sqrt{2}}\right]e^{[\beta\nu t - \nu^2 t]}, \tag{25}$$

Now, from Equation (14), we obtain the solution of Equation (6) as,

$$\psi_4(x,t) = -\frac{i}{\sqrt{2}\sqrt{\gamma}}\left(\gamma + \left(\frac{\sqrt{\gamma}}{\sqrt{2}\left(\coth\left(x - i\sqrt{2}\sqrt{\gamma}t\right) + \mathrm{csch}\left(x - i\sqrt{2}\sqrt{\gamma}t\right)\right)} - \frac{\sqrt{\gamma}\left(\coth\left(x - i\sqrt{2}\sqrt{\gamma}t\right) + \mathrm{csch}\left(x - i\sqrt{2}\sqrt{\gamma}t\right)\right)}{\sqrt{2}}\right)^2\right). \tag{26}$$

Family-IV: When $P = 1$, $Q = 0$ and $R = -1$, then $\Omega(\rho) = \tanh[\rho]$ or $\Omega(\rho) = \coth(\rho)$. So, we obtain the soliton solutions of Equation (5) as follows:

$$\phi_5(x,t) = \left[\frac{\sqrt{\gamma}\coth\left(x - \frac{i\sqrt{\gamma}t}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\sqrt{\gamma}\tanh\left(x - \frac{i\sqrt{\gamma}t}{\sqrt{2}}\right)}{\sqrt{2}}\right]e^{[\beta\nu t - \nu^2 t]}, \tag{27}$$

Now from Equation (14), we obtain the solution of Equation (6) as

$$\psi_5(x,t) = -\frac{i\sqrt{2}}{\sqrt{\gamma}}\left(\gamma + \left(\frac{\sqrt{\gamma}\coth\left(x - \frac{i\sqrt{\gamma}t}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\sqrt{\gamma}\tanh\left(x - \frac{i\sqrt{\gamma}t}{\sqrt{2}}\right)}{\sqrt{2}}\right)^2\right), \tag{28}$$

or

$$\phi_6(x,t) = \left[\frac{\sqrt{\gamma}\tanh\left(x - \frac{i\sqrt{\gamma}t}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\sqrt{\gamma}\coth\left(x - \frac{i\sqrt{\gamma}t}{\sqrt{2}}\right)}{\sqrt{2}}\right] e^{[\beta\nu t - \nu^2 t]}, \tag{29}$$

Now from Equation (14), we obtain the solution of Equation (6) as

$$\psi_6(x,t) = -\frac{i\sqrt{2}}{\sqrt{\gamma}}\left(\gamma + \left(\frac{\sqrt{\gamma}\tanh\left(x - \frac{i\sqrt{\gamma}t}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\sqrt{\gamma}\coth\left(x - \frac{i\sqrt{\gamma}t}{\sqrt{2}}\right)}{\sqrt{2}}\right)^2\right). \tag{30}$$

Family-V: When $P = \frac{1}{2}$, $Q = 0$ and $R = \frac{1}{2}$, then $\Omega(\rho) = \left[\tan(\rho) + \sec\right]$ or $\Omega(\rho) = \left[\cot(\rho) + \csc(\rho)\right]$. So, we obtain the trigonometric solutions of Equation (5) as follows:

$$\phi_7(x,t) = \left[\frac{i\sqrt{\gamma}\left(\tan\left(\sqrt{2}\sqrt{\gamma}t + x\right) + \sec\left(\sqrt{2}\sqrt{\gamma}t + x\right)\right)}{\sqrt{2}} + \frac{i\sqrt{\gamma}}{\sqrt{2}\left(\tan\left(\sqrt{2}\sqrt{\gamma}t + x\right) + \sec\left(\sqrt{2}\sqrt{\gamma}t + x\right)\right)}\right] e^{[\beta\nu t - \nu^2 t]}, \tag{31}$$

Now, from Equation (14), we obtain the solution of Equation (6) as

$$\psi_7(x,t) = -\frac{i\sqrt{2}}{\sqrt{\gamma}}\left(\gamma + \left(\frac{\sqrt{\gamma}}{\sqrt{2}\left(\tan\left(x - \frac{i\sqrt{\gamma}t}{\sqrt{2}}\right) + \sec\left(x - \frac{i\sqrt{\gamma}t}{\sqrt{2}}\right)\right)} - \frac{\sqrt{\gamma}\left(\tan\left(x - \frac{i\sqrt{\gamma}t}{\sqrt{2}}\right) + \sec\left(x - \frac{i\sqrt{\gamma}t}{\sqrt{2}}\right)\right)}{\sqrt{2}}\right)^2\right), \tag{32}$$

or

$$\phi_8(x,t) = \left[\frac{i\sqrt{\gamma}}{\sqrt{2}\left(\csc\left(\sqrt{2}\sqrt{\gamma}t + x\right) - \cot\left(\sqrt{2}\sqrt{\gamma}t + x\right)\right)} + \frac{i\sqrt{\gamma}\left(\csc\left(\sqrt{2}\sqrt{\gamma}t + x\right) - \cot\left(\sqrt{2}\sqrt{\gamma}t + x\right)\right)}{\sqrt{2}}\right] e^{[\beta\nu t - \nu^2 t]}, \tag{33}$$

Now, from Equation (14), we obtain the solution of Equation (6) as

$$\psi_8(x,t) = -\frac{1}{\sqrt{2}\sqrt{\gamma}}\left(\gamma + \left(\frac{i\sqrt{\gamma}}{\sqrt{2}\left(\cot\left(\sqrt{2}\sqrt{\gamma}t + x\right) + \csc\left(\sqrt{2}\sqrt{\gamma}t + x\right)\right)} + \frac{i\sqrt{\gamma}\left(\cot\left(\sqrt{2}\sqrt{\gamma}t + x\right) + \csc\left(\sqrt{2}\sqrt{\gamma}t + x\right)\right)}{\sqrt{2}}\right)^2\right). \tag{34}$$

Family-VI: When $P = -\frac{1}{2}$, $Q = 0$ and $R = -\frac{1}{2}$, then $\Omega(\rho) = \left[\sec(\rho) - \tan(\rho)\right]$ or $\Omega(\rho) = \left[\csc(\rho) - \cot(\rho)\right]$. So, we obtain the trigonometric solutions of Equation (5) as follows:

$$\phi_9(x,t) = \left[\frac{i\sqrt{\gamma}\left(\sec\left(x-\sqrt{2}\sqrt{\gamma}t\right)-\tan\left(x-\sqrt{2}\sqrt{\gamma}t\right)\right)}{\sqrt{2}} + \frac{i\sqrt{\gamma}}{\sqrt{2}\left(\sec\left(x-\sqrt{2}\sqrt{\gamma}t\right)-\tan\left(x-\sqrt{2}\sqrt{\gamma}t\right)\right)}\right]e^{[\beta\nu t-\nu^2 t]}, \tag{35}$$

Now, from Equation (14), we obtain the solution of Equation (6) as

$$\psi_9(x,t) = \frac{1}{\sqrt{2}\sqrt{\gamma}}\left(\gamma + \left(\frac{i\sqrt{\gamma}\left(\sec\left(x-\sqrt{2}\sqrt{\gamma}t\right)-\tan\left(x-\sqrt{2}\sqrt{\gamma}t\right)\right)}{\sqrt{2}} + \frac{i\sqrt{\gamma}}{\sqrt{2}\left(\sec\left(x-\sqrt{2}\sqrt{\gamma}t\right)-\tan\left(x-\sqrt{2}\sqrt{\gamma}t\right)\right)}\right)^2\right), \tag{36}$$

or

$$\phi_{10}(x,t) = \left[\frac{i\sqrt{\gamma}}{\sqrt{2}\left(\csc\left(x-\sqrt{2}\sqrt{\gamma}t\right)-\cot\left(x-\sqrt{2}\sqrt{\gamma}t\right)\right)} + \frac{i\sqrt{\gamma}\left(\csc\left(x-\sqrt{2}\sqrt{\gamma}t\right)-\cot\left(x-\sqrt{2}\sqrt{\gamma}t\right)\right)}{\sqrt{2}}\right]e^{[\beta\nu t-\nu^2 t]}, \tag{37}$$

Now from Equation (14), we obtain the solution of Equation (6) as

$$\psi_{10}(x,t) = \frac{1}{\sqrt{2}\sqrt{\gamma}}\left(\gamma + \left(\frac{i\sqrt{\gamma}}{\sqrt{2}\left(\csc\left(x-\sqrt{2}\sqrt{\gamma}t\right)-\cot\left(x-\sqrt{2}\sqrt{\gamma}t\right)\right)} + \frac{i\sqrt{\gamma}\left(\csc\left(x-\sqrt{2}\sqrt{\gamma}t\right)-\cot\left(x-\sqrt{2}\sqrt{\gamma}t\right)\right)}{\sqrt{2}}\right)^2\right). \tag{38}$$

Family-VII: When $P = 1$, $Q = 0$ and $R = 1$, then $\Omega(\rho) = \left[\tan(\rho)\right]$. So, we obtain the trigonometric solutions of Equation (5) as follows:

$$\phi_{11}(x,t) = \left[\frac{i\sqrt{\gamma}\tan\left(\frac{\sqrt{\gamma}t}{\sqrt{2}}+x\right)}{\sqrt{2}} + \frac{i\sqrt{\gamma}\cot\left(\frac{\sqrt{\gamma}t}{\sqrt{2}}+x\right)}{\sqrt{2}}\right]e^{[\beta\nu t-\nu^2 t]}, \tag{39}$$

Now, from Equation (14), we obtain the solution of Equation (6) as

$$\psi_{11}(x,t) = -\frac{\sqrt{2}}{\sqrt{\gamma}}\left(\gamma + \left(\frac{i\sqrt{\gamma}\tan\left(\frac{\sqrt{\gamma}t}{\sqrt{2}}+x\right)}{\sqrt{2}} + \frac{i\sqrt{\gamma}\cot\left(\frac{\sqrt{\gamma}t}{\sqrt{2}}+x\right)}{\sqrt{2}}\right)^2\right). \tag{40}$$

Family-VIII: When $P = -1$, $Q = 0$ and $R = -1$, then $\Omega(\rho) = \left[\cot(\rho)\right]$. So, we obtain the trigonometric solutions of Equation (5) as follows:

$$\phi_{12}(x,t) = \left[\frac{i\sqrt{\gamma}\tan\left(x-\frac{\sqrt{\gamma}t}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{i\sqrt{\gamma}\cot\left(x-\frac{\sqrt{\gamma}t}{\sqrt{2}}\right)}{\sqrt{2}}\right]e^{[\beta\nu t-\nu^2 t]}, \tag{41}$$

Now, from Equation (14), we obtain the solution of Equation (6) as

$$\psi_{12}(x,t) = -\frac{\sqrt{2}}{\sqrt{\gamma}}\left(\gamma + \left(\frac{i\sqrt{\gamma}\tan\left(\frac{\sqrt{\gamma}t}{\sqrt{2}} + x\right)}{\sqrt{2}} + \frac{i\sqrt{\gamma}\cot\left(\frac{\sqrt{\gamma}t}{\sqrt{2}} + x\right)}{\sqrt{2}}\right)^2\right). \quad (42)$$

Family-IX: When $P = 0$, $Q = 0$ and $R \neq 0$, then $\Omega(\rho) = \left[-\frac{1}{R\rho + A}\right]$. So, we obtain the rational solutions of Equation (5) as follows:

$$\phi_{13}(x,t) = \left[\frac{i\gamma(-A - R(x-ct))}{3cR} - \frac{icR}{A + R(x-ct)}\right]e^{[\beta\nu t - \nu^2 t]}, \quad (43)$$

Now, from Equation (14), we obtain the solution of Equation (6) as

$$\psi_{13}(x,t) = \frac{\gamma + \left(\frac{i\gamma(-A-R(x-ct))}{3cR} - \frac{icR}{A+R(x-ct)}\right)^2}{c}. \quad (44)$$

4. Soliton Solutions for K–O System with Itô Sense

In this section, we find the exact solutions for the K–O system with the Stratonovich sense.

$$\phi_{xt} - 2\phi\psi = \nu\phi_x\beta_t, \quad (45)$$

$$\psi_t + 2\phi\phi_x = 0. \quad (46)$$

So, by choosing the wave transformation in the noise, such as

$$\phi(x,t) = U(\rho)e^{[\nu\beta(t) - \nu^2 t]}, \quad \psi(x,t) = V(\rho), \text{where} \quad \rho = x - ct$$

where U and V are deterministic function, c is the speed of light. By substituting this transformation into Equations (5) and (6) when $F(\phi) = \phi_x\beta_t$, we converted to the stochastic ODEs, such as

$$-cU'' + \nu\beta_t - \nu^2 U' - 2UV = \nu U'\beta_t, \quad (47)$$

$$-cV' + 2UU'e^{[\nu\beta(t) - \nu^2 t]} = 0. \quad (48)$$

We take the expectation $\mathbb{E}(e^{2\nu\beta(t)})$ for the Equation (48), integrating once to obtain V as

$$V = \frac{1}{c}(U^2 + \gamma). \quad (49)$$

Now, substituting Equation (49) into (47), we obtain

$$-cU'' + c\nu U' + 2U^3 + 2\gamma U = 0. \quad (50)$$

Substituting $N = 1$ in Equation (16), we obtain

$$U(\rho) = \delta_0 + \delta_1\Omega(\rho) + \delta_{-1}\Omega^{-1}(\rho). \quad (51)$$

Inserting Equation (51) in Equation (50) with the help of Equation (17), we obtain the infinite series in $\Omega^i(\rho)$. Setting all the same powers of $\Omega^i(\rho)$ equal to zero yields a system of equations. To find the values of the constant with the aid of Wolfram Mathematica 11.1 version, we gain the three cases, namely:

Case 1. When we take $P = 0$, we obtain

$$\delta_0 = -\frac{\sqrt{-c^2Q^2 + cv^2Q - 2\gamma}}{\sqrt{6}}, \delta_1 = 0, \delta_{-1} = \frac{\sqrt{-c^2Q^2 + cv^2Q - 2\gamma}\left(-c^2Q^2 + cv^2Q + 4\gamma\right)}{3\sqrt{6}cR(cQ - v^2)},$$

Case 2. When we take $Q = 0$, we obtain

$$\delta_0 = -\frac{\sqrt{-c^2PR - \gamma}}{\sqrt{3}}, \delta_1 = 0, \delta_{-1} = \frac{cP\sqrt{-c^2PR - \gamma}}{\sqrt{c^2PR + \gamma}}, v = -\frac{\sqrt{2}\sqrt{c^2PR - 2\gamma}\sqrt[4]{c^2PR + \gamma}}{3^{3/4}c\sqrt{P}\sqrt{R}},$$

Case 3. When we take $P = 0$, $Q = 0$ but $R \neq 0$, then we obtain

$$\delta_0 = \frac{i\sqrt{\gamma}}{\sqrt{3}}, \delta_1 = 0, \delta_{-1} = \frac{4i\gamma^{3/2}}{3\sqrt{3}cv^2R},$$

Now, by substituting these values in Equation (51), then by the help of wave transformation, we obtain the different types of soliton, trigonometric and rational solutions of Equation (6) as follows:

Family-I: When $P = 0$, $Q = 1$ and $R = -1$, then $\Omega(\rho) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{\rho}{2}\right)\right]$. So, we obtain the soliton solutions of Equation (45) as follows:

$$\phi_{14}(x,t) = \left[-\frac{\sqrt{-c^2 + cv^2 - 2\gamma}}{\sqrt{6}} - \frac{\left(-c^2 + cv^2 + 4\gamma\right)\sqrt{-c^2 + cv^2 - 2\gamma}}{3\sqrt{6}c(c - v^2)\left(\frac{1}{2}\tanh\left(\frac{1}{2}(x - ct)\right) + \frac{1}{2}\right)}\right]e^{[\beta vt - v^2 t]}, \quad (52)$$

Now from Equation (49), we obtain the solution of Equation (46) as,

$$\psi_{14}(x,t) = \frac{1}{c}\left(-\frac{\sqrt{-c^2 + cv^2 - 2\gamma}}{\sqrt{6}} - \frac{\left(-c^2 + cv^2 + 4\gamma\right)\sqrt{-c^2 + cv^2 - 2\gamma}}{3\sqrt{6}c(c - v^2)\left(\frac{1}{2}\tanh\left(\frac{1}{2}(x - ct)\right) + \frac{1}{2}\right)}\right)^2 + \frac{\gamma}{c}. \quad (53)$$

Family-II: When $P = 0$, $Q = -1$ and $R = 1$, then $\Omega(\rho) = \left[\frac{1}{2} - \frac{1}{2}\coth\left(\frac{\rho}{2}\right)\right]$. So, we obtain the soliton solutions of Equation (45) as follows:

$$\phi_{15}(x,t) = \left[-\frac{\sqrt{-c^2 + cv^2 - 2\gamma}}{\sqrt{6}} - \frac{\left(-c^2 + cv^2 + 4\gamma\right)\sqrt{-c^2 + cv^2 - 2\gamma}}{3\sqrt{6}c(c - v^2)\left(\frac{1}{2} - \frac{1}{2}\coth\left(\frac{1}{2}(x - ct)\right)\right)}\right]e^{[\beta vt - v^2 t]}, \quad (54)$$

Now, from Equation (49), we obtain a solution of Equation (46) as

$$\psi_{15}(x,t) = \frac{1}{c}\left(\frac{\sqrt{-c^2 - cv^2 - 2\gamma}\left(-c^2 - cv^2 + 4\gamma\right)}{3\sqrt{6}c(-c - v^2)\left(\frac{1}{2} - \frac{1}{2}\coth\left(\frac{1}{2}(x - ct)\right)\right)} - \frac{\sqrt{-c^2 - cv^2 - 2\gamma}}{\sqrt{6}}\right)^2 + \frac{\gamma}{c}. \quad (55)$$

Family-III: When $P = \frac{1}{2}$, $Q = 0$ and $R = -\frac{1}{2}$, then $\Omega(\rho) = \left[\tanh(\rho) \pm i\,\mathrm{sec}\,h\right]$ or $\Omega(\rho) = \left[\coth(\rho) \pm \mathrm{csc}\,h\right]$. So, we obtain the soliton solutions of Equation (45) as follows:

$$\phi_{16}(x,t) = \left[\frac{c\sqrt{-\frac{c^2}{2} - \gamma}}{2\sqrt{\frac{c^2}{2} + \gamma}(-\coth(ct - x) - \mathrm{csch}(ct - x))} - \frac{\sqrt{-\frac{c^2}{2} - \gamma}}{\sqrt{3}}\right] \quad (56)$$

$$e^{\left[-\frac{2\beta t\sqrt{\frac{c^2}{2} - 2\gamma}\sqrt[4]{\frac{c^2}{2} + \gamma}}{3^{3/4}c} - \frac{4t\left(\frac{c^2}{2} - 2\gamma\right)\sqrt{\frac{c^2}{2} + \gamma}}{3\sqrt{3}c^2}\right]}, \quad (57)$$

Now, from Equation (49), we obtain a solution of Equation (46) as

$$\psi_{16}(x,t) = \frac{\gamma}{c} + \frac{1}{c}\left(-\frac{\sqrt{\frac{c^2}{4}-\gamma}}{\sqrt{3}} + \frac{c\sqrt{\frac{c^2}{4}-\gamma}}{2\sqrt{\gamma-\frac{c^2}{4}}(-\tanh(ct-x)+i\operatorname{sech}(ct-x))}\right)^2, \tag{58}$$

or

$$\phi_{17}(x,t) = \left[-\frac{\sqrt{\frac{c^2}{4}-\gamma}}{\sqrt{3}} + \frac{c\sqrt{\frac{c^2}{4}-\gamma}}{2\sqrt{\gamma-\frac{c^2}{4}}(-\tanh(ct-x)+i\operatorname{sech}(ct-x))}\right] \tag{59}$$

$$e^{\left[-\frac{2\beta t\sqrt{\frac{c^2}{2}-2\gamma}\sqrt[4]{\frac{c^2}{2}+\gamma}}{3^{3/4}c}-\frac{4t\left(\frac{c^2}{2}-2\gamma\right)\sqrt{\frac{c^2}{2}+\gamma}}{3\sqrt{3}c^2}\right]}, \tag{60}$$

and now from Equation (49), we obtain a solution of Equation (46) as

$$\psi_{17}(x,t) = \frac{\gamma}{c} + \frac{1}{c}\left(-\frac{\sqrt{\frac{c^2}{4}-\gamma}}{\sqrt{3}} + \frac{c\sqrt{\frac{c^2}{4}-\gamma}}{2\sqrt{\gamma-\frac{c^2}{4}}(\coth(ct-x)\pm\operatorname{csch}(ct-x))}\right)^2. \tag{61}$$

Family-IV: When $P=1$, $Q=0$ and $R=-1$, then $\Omega(\rho)=\tanh[\rho]$ or $\Omega(\rho)=\coth(\rho)$. So, we obtain the soliton solutions of Equation (45) as follows:

$$\phi_{18}(x,t) = \left[-\frac{\sqrt{\frac{c^2}{4}-\gamma}}{\sqrt{3}} - \frac{c\sqrt{\frac{c^2}{4}-\gamma}\coth(ct-x)}{2\sqrt{\gamma-\frac{c^2}{4}}}\right] e^{\left[\frac{8t\left(-\frac{c^2}{4}-2\gamma\right)\sqrt{\gamma-\frac{c^2}{4}}}{3\sqrt{3}c^2}+\frac{2i\sqrt{2}\beta t\sqrt{-\frac{c^2}{4}-2\gamma}\sqrt[4]{\gamma-\frac{c^2}{4}}}{3^{3/4}c}\right]}, \tag{62}$$

and now from Equation (49), we obtain a solution of Equation (46) as

$$\psi_{18}(x,t) = \frac{1}{c}\left(-\frac{\sqrt{c^2-\gamma}}{\sqrt{3}} - \frac{c\sqrt{c^2-\gamma}\coth(ct-x)}{\sqrt{\gamma-c^2}}\right)^2 + \frac{\gamma}{c}. \tag{63}$$

or

$$\phi_{19}(x,t) = \left[-\frac{\sqrt{\frac{c^2}{4}-\gamma}}{\sqrt{3}} - \frac{c\sqrt{\frac{c^2}{4}-\gamma}\tanh(ct-x)}{2\sqrt{\gamma-\frac{c^2}{4}}}\right] e^{\left[\frac{8t\left(-\frac{c^2}{4}-2\gamma\right)\sqrt{\gamma-\frac{c^2}{4}}}{3\sqrt{3}c^2}+\frac{2i\sqrt{2}\beta t\sqrt{-\frac{c^2}{4}-2\gamma}\sqrt[4]{\gamma-\frac{c^2}{4}}}{3^{3/4}c}\right]}, \tag{64}$$

now from Equation (49), we obtain the solution of Equation (46) as

$$\psi_{19}(x,t) = \frac{1}{c}\left(-\frac{\sqrt{c^2-\gamma}}{\sqrt{3}} - \frac{c\sqrt{c^2-\gamma}\tanh(ct-x)}{\sqrt{\gamma-c^2}}\right)^2 + \frac{\gamma}{c}. \tag{65}$$

Family-V: When $P=\frac{1}{2}$, $Q=0$ and $R=\frac{1}{2}$, then $\Omega(\rho)=\Big[\tan(\rho)+\sec(\rho)\Big]$ or $\Omega(\rho)=\Big[\csc(\rho)-\cot(\rho)\Big]$. So, we obtain the trigonometric solutions of Equation (45) as follows:

$$\phi_{20}(x,t) = \left[\frac{c\sqrt{-\frac{c^2}{4}-\gamma}}{2\sqrt{\frac{c^2}{4}+\gamma}(\tan(\rho)+\sec(\rho))} - \frac{\sqrt{-\frac{c^2}{4}-\gamma}}{\sqrt{3}}\right] e^{-\frac{2\sqrt{2}\beta t\sqrt{\frac{c^2}{4}-2\gamma}\sqrt[4]{\frac{c^2}{4}+\gamma}}{3^{3/4}c}-\frac{8t\left(\frac{c^2}{4}-2\gamma\right)\sqrt{\frac{c^2}{4}+\gamma}}{3\sqrt{3}c^2}}, \tag{66}$$

now from Equation (49), we obtain the solution of Equation (46) as,

$$\psi_{20}(x,t) = \frac{1}{c}\left(\frac{c\sqrt{-\frac{c^2}{4}-\gamma}}{2\sqrt{\frac{c^2}{4}+\gamma}(\cot(ct-x)-\csc(ct-x))} - \frac{\sqrt{-\frac{c^2}{4}-\gamma}}{\sqrt{3}}\right)^2 + \frac{\gamma}{c}, \tag{67}$$

or

$$\phi_{21}(x,t) = \left[\frac{c\sqrt{-\frac{c^2}{4}-\gamma}}{2\sqrt{\frac{c^2}{4}+\gamma}(\csc(\rho)-\cot(\rho))} - \frac{\sqrt{-\frac{c^2}{4}-\gamma}}{\sqrt{3}}\right] e^{\left[-\frac{2\sqrt{2}\beta t\sqrt{\frac{c^2}{4}-2\gamma}\sqrt[4]{\frac{c^2}{4}+\gamma}}{3^{3/4}c} - \frac{8t\left(\frac{c^2}{4}-2\gamma\right)\sqrt{\frac{c^2}{4}+\gamma}}{3\sqrt{3}c^2}\right]}, \tag{68}$$

now from Equation (49), we obtain the solution of Equation (46) as,

$$\psi_{21}(x,t) = \frac{1}{c}\left(\frac{c\sqrt{-\frac{c^2}{4}-\gamma}}{2\sqrt{\frac{c^2}{4}+\gamma}(\tan(ct-x)+\sec(ct-x))} - \frac{\sqrt{-\frac{c^2}{4}-\gamma}}{\sqrt{3}}\right)^2 + \frac{\gamma}{c}. \tag{69}$$

Family-VI: When $P = -\frac{1}{2}$, $Q = 0$ and $R = -\frac{1}{2}$, then $\Omega(\rho) = \left[\sec(\rho) - \tan(\rho)\right]$ or $\Omega(\rho) = \left[\csc(\rho) + \cot(\rho)\right]$. So, we obtain the trigonometric solutions of Equation (45) as follows:

$$\phi_{22}(x,t) = \left[\frac{c\sqrt{-\frac{c^2}{4}-\gamma}}{2\sqrt{\frac{c^2}{4}+\gamma}(\cot(\rho)+\csc(\rho))} - \frac{\sqrt{-\frac{c^2}{4}-\gamma}}{\sqrt{3}}\right] e^{\left[-\frac{2\sqrt{2}\beta t\sqrt{\frac{c^2}{4}-2\gamma}\sqrt[4]{\frac{c^2}{4}+\gamma}}{3^{3/4}c} - \frac{8t\left(\frac{c^2}{4}-2\gamma\right)\sqrt{\frac{c^2}{4}+\gamma}}{3\sqrt{3}c^2}\right]}, \tag{70}$$

and now from Equation (49), we obtain the solution of Equation (46) as,

$$\psi_{22}(x,t) = \frac{1}{c}\left(-\frac{\sqrt{-\frac{c^2}{4}-\gamma}}{\sqrt{3}} - \frac{c\sqrt{-\frac{c^2}{4}-\gamma}}{2\sqrt{\frac{c^2}{4}+\gamma}(\tan(ct-x)+\sec(ct-x))}\right)^2 + \frac{\gamma}{c}, \tag{71}$$

or

$$\phi_{23}(x,t) = \left[\frac{c\sqrt{-\frac{c^2}{4}-\gamma}}{2\sqrt{\frac{c^2}{4}+\gamma}(\tan(ct-x)+\sec(ct-x))} - \frac{\sqrt{-\frac{c^2}{4}-\gamma}}{\sqrt{3}}\right] e^{\left[-\frac{2\sqrt{2}\beta t\sqrt{\frac{c^2}{4}-2\gamma}\sqrt[4]{\frac{c^2}{4}+\gamma}}{3^{3/4}c} - \frac{8t\left(\frac{c^2}{4}-2\gamma\right)\sqrt{\frac{c^2}{4}+\gamma}}{3\sqrt{3}c^2}\right]}, \tag{72}$$

and now from Equation (49), we obtain the solution of Equation (46) as,

$$\psi_{23}(x,t) = \frac{1}{c}\left(-\frac{\sqrt{-\frac{c^2}{4}-\gamma}}{\sqrt{3}} - \frac{c\sqrt{-\frac{c^2}{4}-\gamma}}{2\sqrt{\frac{c^2}{4}+\gamma}(-\cot(ct-x)-\csc(ct-x))}\right)^2 + \frac{\gamma}{c}. \tag{73}$$

Family-VII: When $P = 1$, $Q = 0$ and $R = 1$, then $\Omega(\rho) = \left[\tan(\rho)\right]$. So, we obtain the trigonometric solutions of Equation (5) as follows:

$$\phi_{24}(x,t) = \left[-\frac{\sqrt{-c^2-\gamma}}{\sqrt{3}} - \frac{c\sqrt{-c^2-\gamma}\cot(ct-x)}{\sqrt{c^2+\gamma}}\right] e^{\left[-\frac{\sqrt{2}\beta t\sqrt{c^2-2\gamma}\sqrt[4]{c^2+\gamma}}{3^{3/4}c} - \frac{2t\left(c^2-2\gamma\right)\sqrt{c^2+\gamma}}{3\sqrt{3}c^2}\right]}, \tag{74}$$

now from Equation (49), we obtain the solution of Equation (46) as

$$\psi_{24}(x,t) = \frac{1}{c}\left(\frac{c\sqrt{-\frac{c^2}{4}-\gamma}\cot(ct-x)}{2\sqrt{\frac{c^2}{4}+\gamma}} - \frac{\sqrt{-\frac{c^2}{4}-\gamma}}{\sqrt{3}}\right)^2 + \frac{\gamma}{c}. \tag{75}$$

Family-VIII: When $P=-1$, $Q=0$ and $R=-1$, then $\Omega(\rho) = \left[\cot(\rho)\right]$. So, we obtain the trigonometric solutions of Equation (5) as follows:

$$\phi_{25}(x,t) = \left[\frac{c\sqrt{-c^2-\gamma}\tan(ct-x)}{\sqrt{c^2+\gamma}} - \frac{\sqrt{-c^2-\gamma}}{\sqrt{3}}\right]e^{\left[\frac{\sqrt{2}\beta t\sqrt{c^2-2\gamma}\sqrt[4]{c^2+\gamma}}{3^{3/4}c} - \frac{2t\left(c^2-2\gamma\right)\sqrt{c^2+\gamma}}{3\sqrt{3}c^2}\right]}, \tag{76}$$

and now from Equation (49), we obtain the solution of Equation (46) as,

$$\psi_{25}(x,t) = \frac{1}{c}\left(\frac{c\sqrt{-c^2-\gamma}\tan(ct-x)}{\sqrt{c^2+\gamma}} - \frac{\sqrt{-c^2-\gamma}}{\sqrt{3}}\right)^2 + \frac{\gamma}{c}. \tag{77}$$

Family-IX: When $P=0$, $Q=0$ and $R\neq 0$, then $\Omega(\rho) = \left[-\frac{1}{R\rho+A}\right]$. So, we obtain the rational solutions of Equation (5) as follows:

$$\phi_{26}(x,t) = \left[\frac{4i\gamma^{3/2}(-A-R(x-ct))}{3\sqrt{3}c\nu^2 R} + \frac{i\sqrt{\gamma}}{\sqrt{3}}\right]e^{[\beta\nu t-\nu^2 t]}, \tag{78}$$

and now from Equation (49), we obtain the solution of Equation (46) as,

$$\psi_{26}(x,t) = \frac{1}{c}\left(\gamma + \left(\frac{i\sqrt{\gamma}}{\sqrt{3}} - \frac{2ic\gamma^{3/2}(-A-ct+x)}{(c^2-2\gamma)\sqrt{c^2+\gamma}}\right)^2\right). \tag{79}$$

5. Graphical Discussion

Here, we demonstrate the physical interpretation of the results constructed above in the presence of noise. These plots demonstrate the different soliton behaviors to illustrate the noise in the physical interpretation of the solutions that are extracted with the help of the F-expansion method. The absolute, real, and imaginary representations are presented for some solutions with the help of of Wolfram Mathematica 11.1 version. In the presence of noise ν, we plotted some graphs: Figure 1 is plotted for $\phi_1(x,t)$ when we choose $\gamma = 1.2$ and $\nu = 0.9981$, Figure 2 when $\gamma = 1.9$ and $\nu = 0.9871$, Figure 3 when $\gamma = 2.9$ and $\nu = 0.9$, Figure 4 when $\gamma = 0.9$ and $\nu = 0.99$, Figure 5 when $\gamma = 1.9$ and $\nu = 0.8$, Figure 6 when $\gamma = 1.2$ and $\nu = 0.8$, Figure 7 when $\gamma = 2.2$, $c = 1.7$ and $\nu = 0.981$, Figure 8 when $\gamma = 0.9$, $c = 1.5$ and $\nu = 0.71$, and Figure 9 when $\gamma = 2.9$, and $\nu = 0.9$. For the function ψ, there is no influence of noise here for $\nu = 0$, and Figures 10 and 11 are plotted when $\gamma = -2.2$, $\gamma = -1.2$, $\gamma =$. Figure 12 is drawn when $\gamma = 0.52$, and $c = 1.9$. Figure 13 is drawn when $\gamma = 3.2$. Here, the selection of parameters is different because γ is an integrating constantand c is the speed of light. However, the ν is the control parameter of the noise; if we choose the smaller value of ν, the influence of noise in the plots is low and it does not show the mush spikes.The solutions are extracted successfully in the form of the dark, singular, complex dark, combo, solitary, periodic, mixed periodic, and rational functions found in the presence of noise. The Konno–Oono system is a coupled model to show the random behavior of the magnetic waves we involve noise in $F(\phi)$. These wave structures are very helpful for the dynamical study of the Konno–Oono system. When we see the problem at the micro level, the physical phenomena of the magnetic field appear randomly. At the moment, this study is helpful for researchers. So, these wave structures are very beneficial in the study of the magnetic field when we consider the problem at the micro level. The noise or randomness is clearly shown in the plots by the different choices of parameters (Algorithm 1).

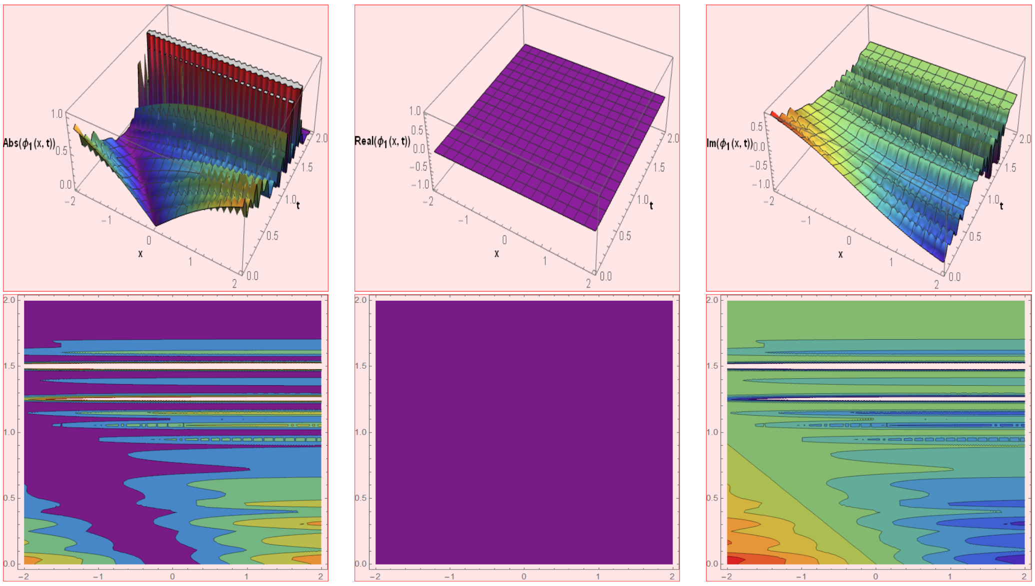

Figure 1. The absolute, real, and imaginary behavior in the shape of 3D and their corresponding contour plots are dispatched for $\phi_1(x,t)$.

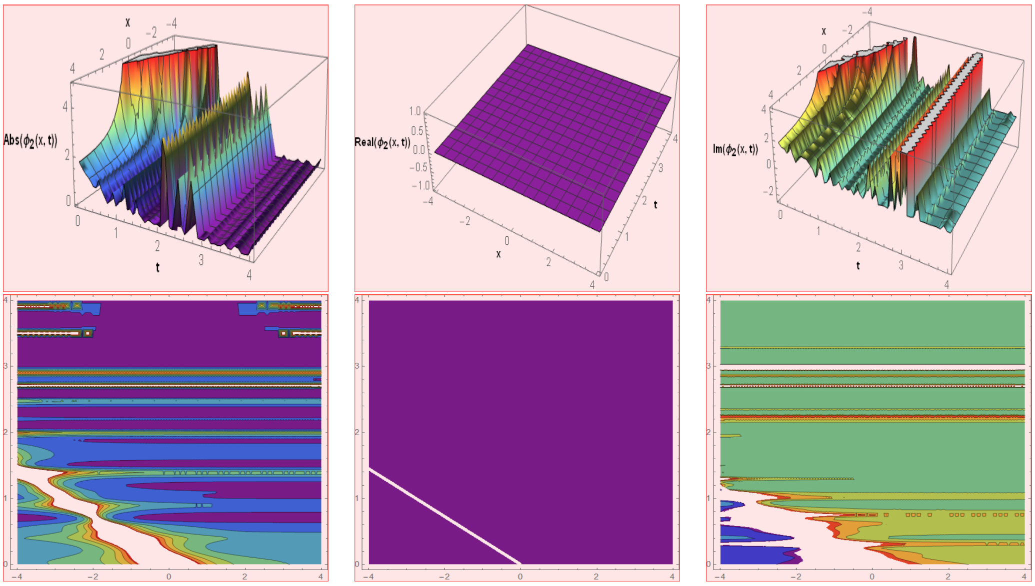

Figure 2. The absolute, real, and imaginary behavior in the shape of 3D and their corresponding contour plots are dispatched for $\phi_2(x,t)$.

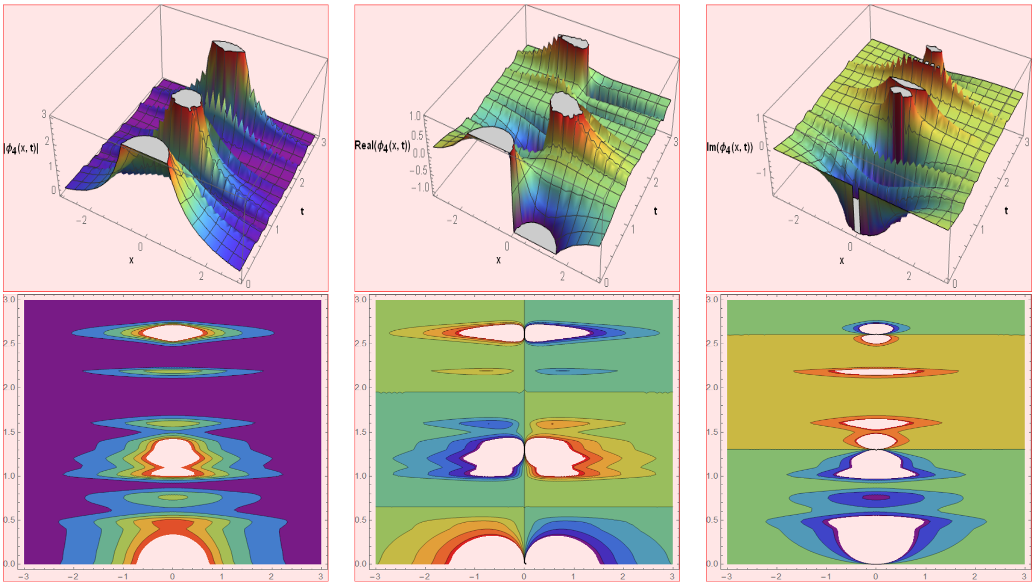

Figure 3. The absolute, real, and imaginary behavior in the shape of 3D and their corresponding contour plots are dispatched for $\phi_4(x,t)$.

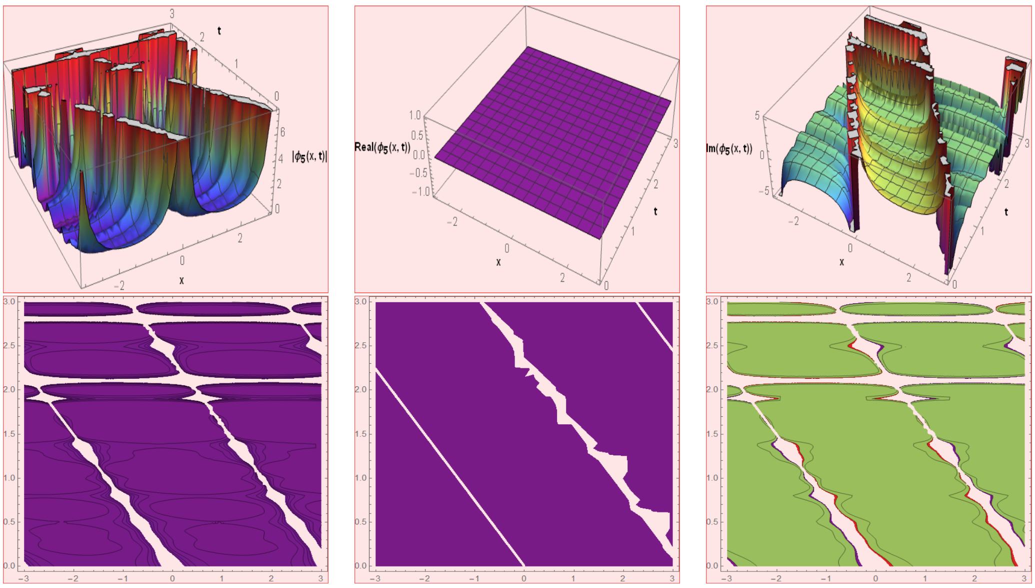

Figure 4. The absolute, real, and imaginary behavior in the shape of 3D and their corresponding contour plots are dispatched for $\phi_5(x,t)$.

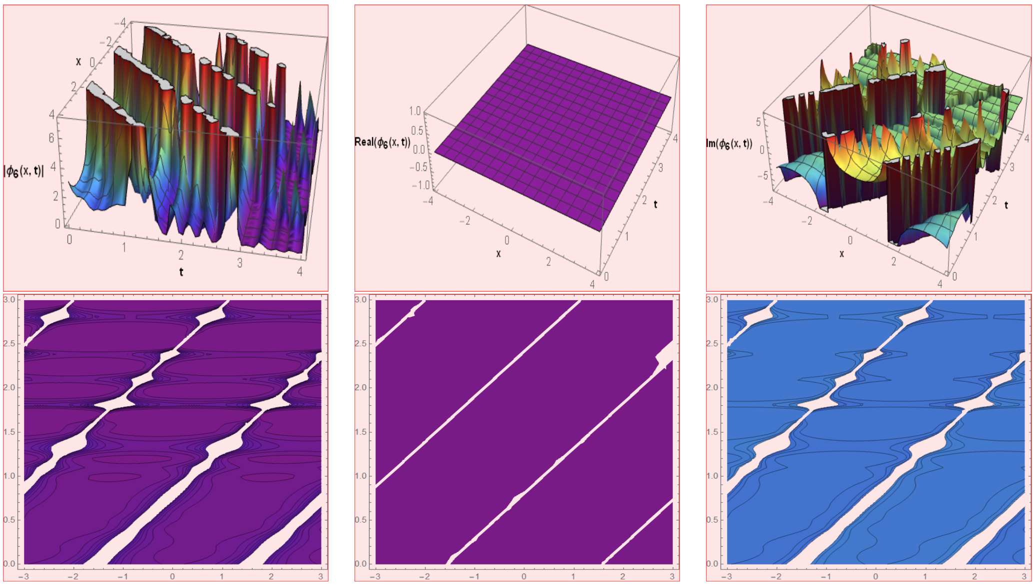

Figure 5. The absolute, real, and imaginary behavior in the shape of 3D and their corresponding contour plots are dispatched for $\phi_6(x,t)$.

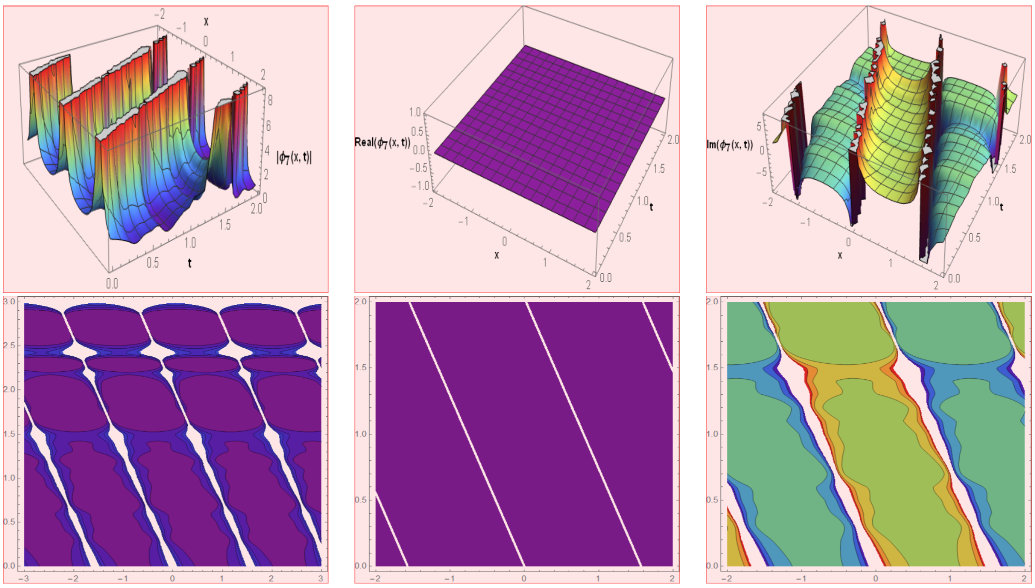

Figure 6. The absolute, real, and imaginary behavior in the shape of 3D and their corresponding contour plots are dispatched for $\phi_7(x,t)$.

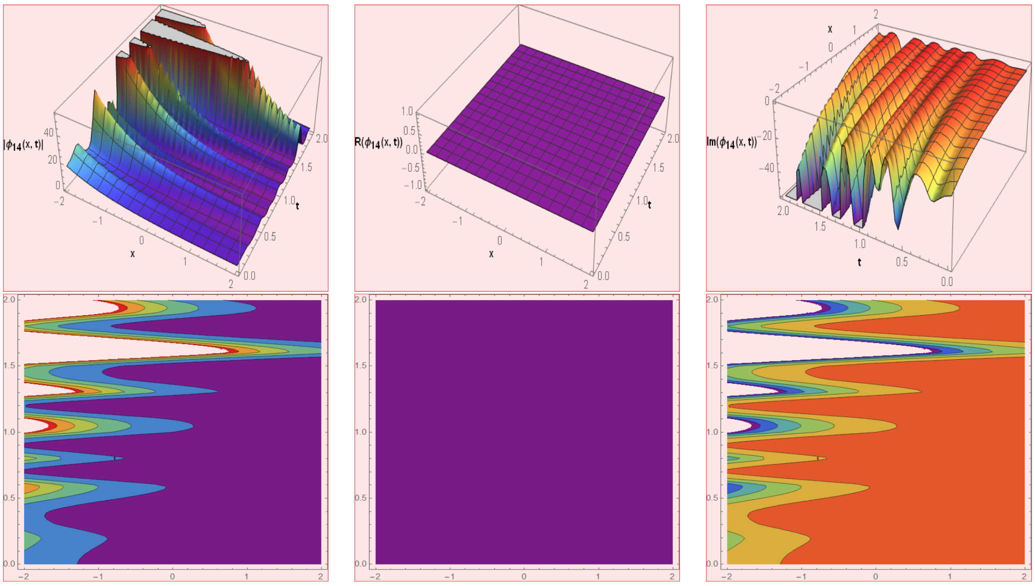

Figure 7. The absolute, real, and imaginary behavior in the shape of 3D and their corresponding contour plots are dispatched for $\phi_{14}(x,t)$.

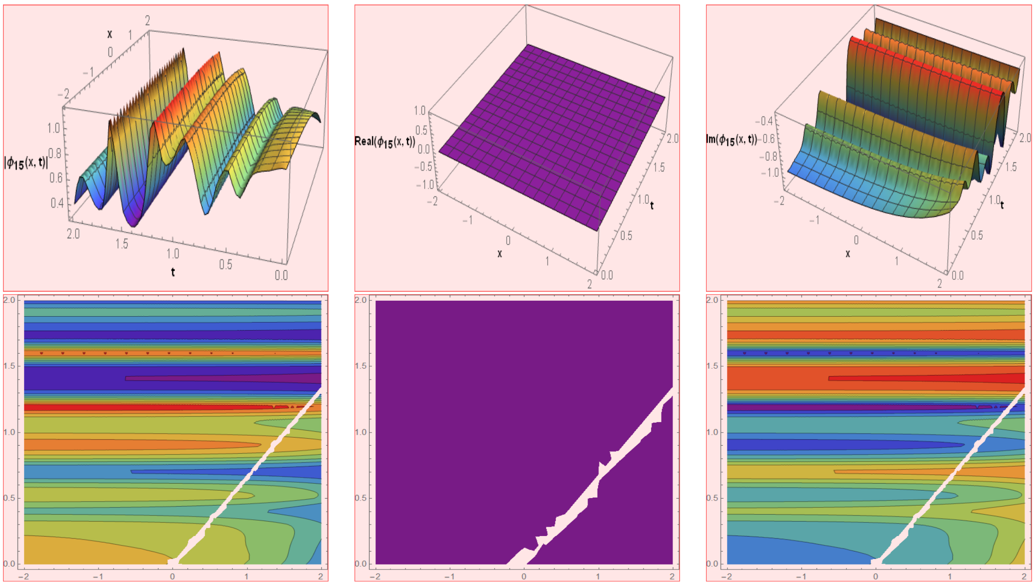

Figure 8. The absolute, real, and imaginary behavior in the shape of 3D and their corresponding contour plots are dispatched for $\phi_{15}(x,t)$.

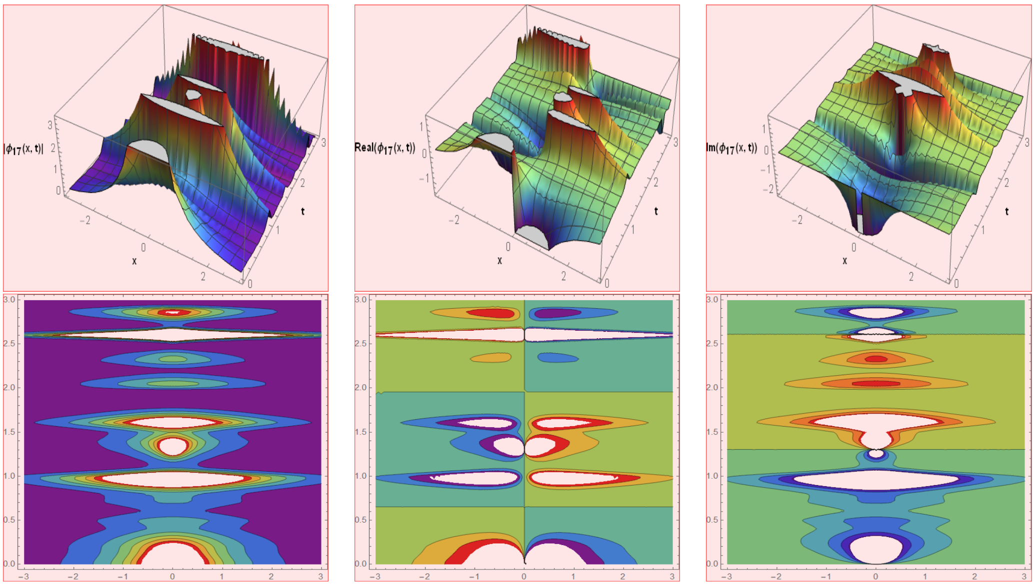

Figure 9. The absolute, real, and imaginary behavior in the shape of 3D and their corresponding contour plots are dispatched for $\psi_{17}(x,t)$.

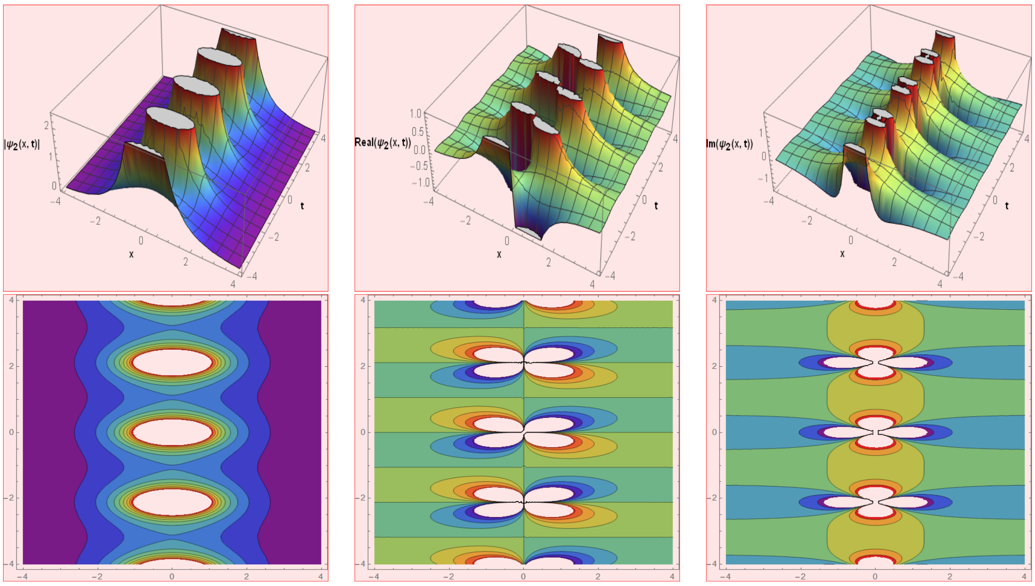

Figure 10. The absolute, real, and imaginary behavior in the shape of 3D and their corresponding contour plots are dispatched for $\psi_2(x,t)$.

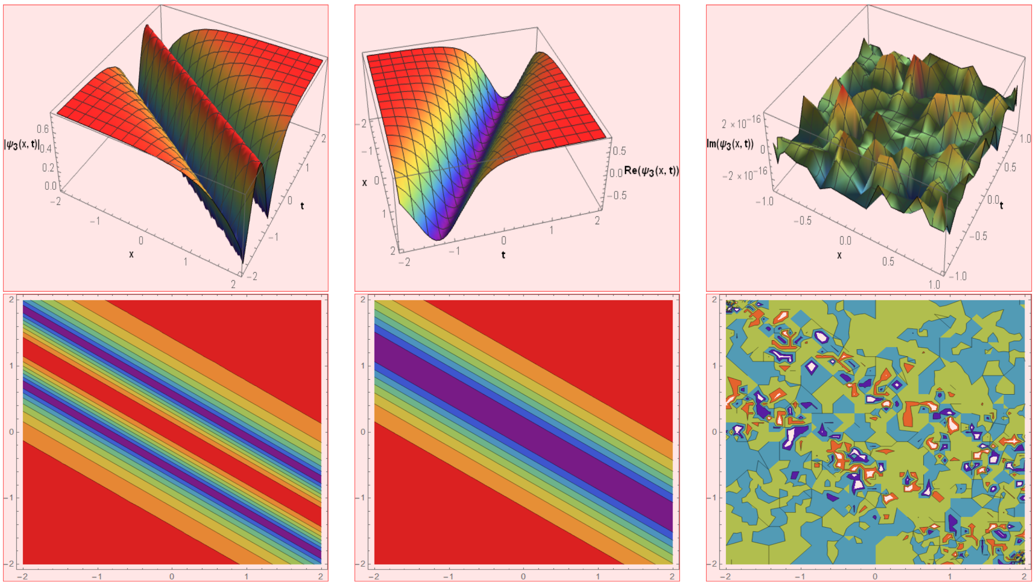

Figure 11. The absolute, real, and imaginary behavior in the shape of 3D and their corresponding contour plots are dispatched for $\psi_3(x,t)$.

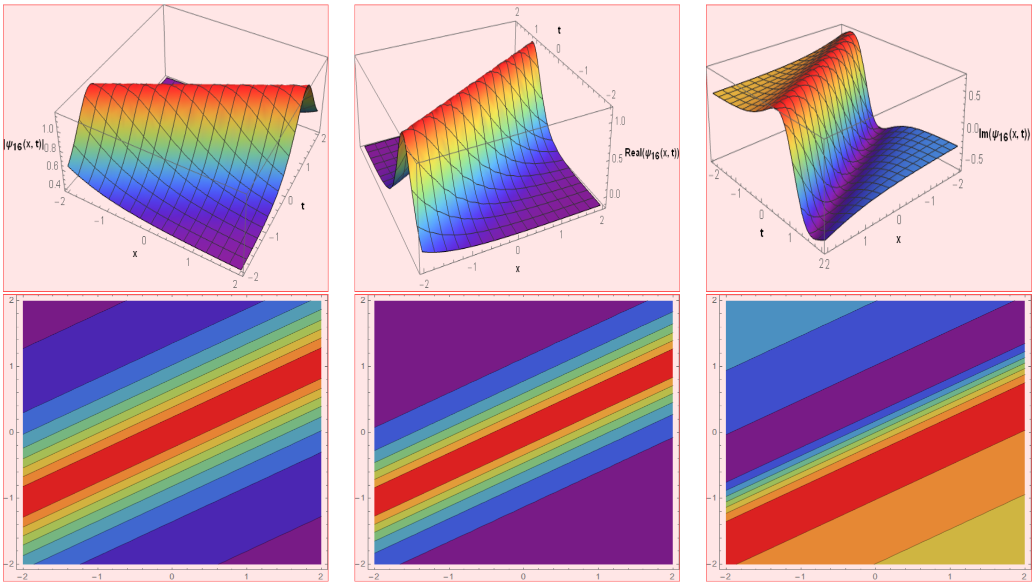

Figure 12. The absolute, real, and imaginary behavior in the shape of 3D and their corresponding contour plots are dispatched for $\psi_{16}(x,t)$.

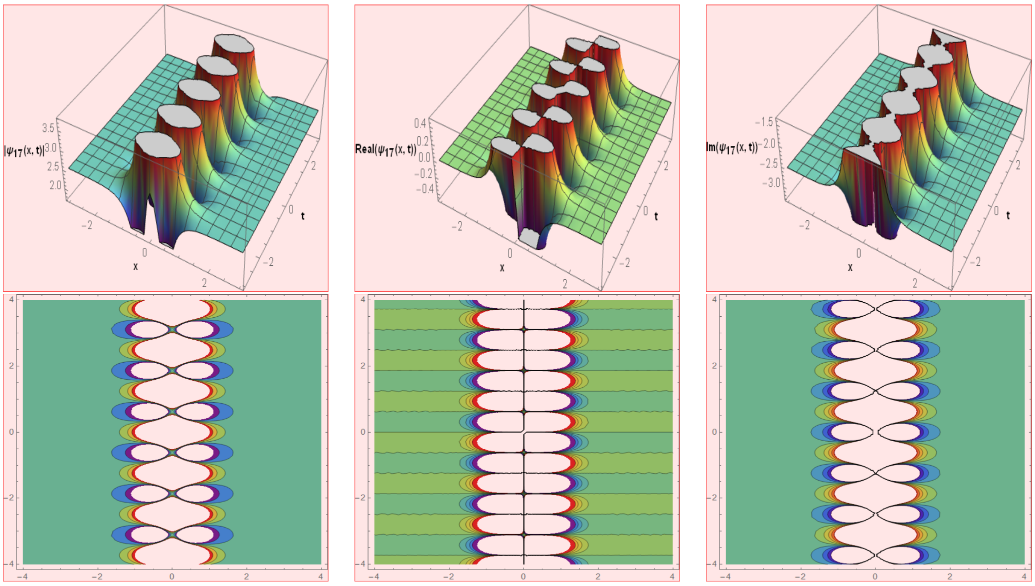

Figure 13. The absolute, real, and imaginary behavior in the shape of 3D and their corresponding contour plots are dispatched for $\psi_{17}(x, t)$.

Algorithm 1 Pseudocode for WhiteNoise

```
procedure MY PROCEDURE
    Define the function U(x, t)
    Input all the constants ν, γ
    Input time
    Input space
    noise = Interpolation[ Normal[RandomFunction[ WhiteNoiseProcess[ν], 0, input
time * samplesPerSec]][1]]
    Compute the function
```

6. Conclusions

This study deals with the stochastic Konno–Oono system with multiplicative noise under the Stratonovich sense and secondly the Itô sense. This model is applicable in the magnetic field. The stochastic wave structures are constructed with the help of the F-expansion technique. The different types of soliton solutions in the form of dark, singular, complex dark, combo, solitary, periodic, mixed periodic, and rational functions are found in the presence of noise. When we see the problem at the micro level, the physical phenomena of the magnetic field appear randomly. At the moment, this study helps researchers. So, these wave structures are very beneficial in the study of the magnetic field when we consider the problem at the micro level. The noise or randomness is clearly shown in the plots by the different choices of parameters. Additionally, the absolute, real, and imaginary physical representation in three dimensions and their corresponding contour plots of some solutions are drawn in the sense of noise by the different choices of parameters.

Author Contributions: Formal analysis, T.S.S.; Investigation, M.Z.B.; Resources, N.A.; Writing—original draft, N.S.; Writing—review & editing, A.A.; Project administration, M.D.l.S. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest: The authors declare no conflict of interest.

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