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MULTIMARKET COMPETITION AND WELFARE EFFECTS OF PRICE DISCRIMINATION

by

Iñaki Aguirre

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University of the Basque Country

Multimarket Competition and Welfare Effects of Price discrimination

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Iñaki Aguirre[*]

University of the Basque Country

Abstract

The paper investigates the effects on welfare of price discrimination when a multimarket seller faces competition in one of its two markets. With respect to uniform pricing, price discrimination changes competition in such a way, that even with linear demands, price discrimination can be welfare-improving, both under strategic substitutes and strategic complements.

JEL Classification: L13, L41.

Key words: Price discrimination, multimarket competition, welfare analysis.

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1. Introduction

A well-known result in the economics of monopolistic third degree price discrimination is that a move from uniform pricing to third degree price discrimination reduces welfare if total output decreases. Robinson (1933) shows that if a monopolist faces two independent linear demand curves, the use of price discrimination will not affect industry output but will reduce welfare. Schmalensee (1981) proves this conjecture assuming nonlinear demand curves, perfectly separated markets and constant marginal cost.[1] Varian (1985) extends the result by allowing imperfect arbitrage, so that demand in any market can depend on prices in other markets, and by allowing marginal cost to be constant or increasing.[2] Using a revealed-preference argument, Schwartz (1990) generalizes the result to the case in which marginal cost is decreasing (see also Bertoletti, 2004, for more recent analysis).[3]

Theoretical literature on the welfare effects of price discrimination has mainly focused on the case of final good monopolies. As Katz (1987) claims, monopoly is precisely a market structure where antidiscrimination legislations do not apply. For instance, the Robinson-Patman Act concerns harm to competition, but in the case of a final good monopoly there is no competition among either sellers or buyers.[4] Despite the empirical relevance and the importance for the competition policy, there are not many works analyzing the effects of price discrimination on competition and welfare in oligopolistic frameworks. Notable exceptions, where discriminating oligopolists are discussed, are the papers by Neven and Phlips (1985),

[1] Ippolito (1980) by decomposing the change in social welfare into two effects provides an earlier, easier and more intuitive proof. See, also Aguirre (2011).

[2] It is important to point out that the latter result depends on the assumption that all markets are served under both pricing regimes. Some authors have shown that when there are two potential markets price discrimination may lead, by opening markets, to a Pareto welfare improvement. Hausman and MacKie-Mason (1988) show that if the marginal cost is constant or falling, then price discrimination results in a Pareto improvement if it serves to open new markets. Even when price discrimination does not open new markets Hausman and MacKie-Mason (1988) and Nahata *et al*. (1990) have shown that price discrimination can result in a Pareto improvement by lowering prices in all markets.

[3] Aguirre, Cowan and Vickers (2010) find sufficient conditions for third-degree price discrimination to increase social welfare related to the shape of inverse and direct demand functions.

[4] On the other hand, some recent empirical works have analyzed price dispersion in oligopolistic markets. Evidence of price discrimination is found by Shepard (1991) for the (Massachusetts) retail gasoline market, by Borenstein and Rose (1994) for the U.S. airline industry, and by Verboven (1996) for the European car market. In these works, neither cost differences nor peak-load pricing seem to be the most plausible explanations for the observed price differences.

Holmes (1989), Corts (1998), Dastidar (2006), Stole (2007) and Adachi and Matsushima (2011).[5]

Neven and Phlips (1985) state that whenever demand has a different price elasticity in different markets, oligopolists will tend to price discriminate exactly in the same way as the discriminating monopolist would. They consider a multimarket Cournot duopoly, with homogeneous product, and conclude that allowing duopolists to discriminate between submarkets leads to a welfare loss.[6] Holmes (1989) also studies a discriminating duopoly, but firms produce differentiated products and compete in prices. In his model, equilibrium price differentials can be accounted for both by differences in cross-price elasticity as well as differences in industry-demand elasticity. In fact, as this author (and Borenstein, 1985) shows price discrimination may increase as a market moves from monopoly to imperfect competition. He compares total industry output when the firms can discriminate between two markets with that output under uniform pricing. What determines which regime has a larger output is the sum of an adjusted-concavity condition and an elasticity-ratio condition.

Adachi and Matsushima (2011) show that price discrimination can improve social welfare (especially) if firms' brands are substitutes in the market where the discriminatory price is higher and are complements in the market where it is lower, but it never improves in the reverse case. They verify, however, that consumer surplus is never improved by price discrimination: welfare improvement by price discrimination is solely the result of an increase in the firms' profits.

We consider a multimarket seller facing competition in one of its two markets, and show that the pricing policy, price discrimination or uniform pricing, of the multimarket established

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[5] See also Borenstein (1985) for an analysis of price discrimination in monopolistically competitive environments. In a setting of spatial competition DeGraba (1987) shows that the use or non use of price discrimination by a national firm can affect nonprice decisions made by local firms that compete with the national firm. A related literature is that of spatial price discrimination (an intermediate case between first and third-degree price discrimination). See, for example, Lederer and Hurter (1986) and Thisse and Vives (1988). See also Stole (2007) and Liu and Serfes (2010) for excellent surveys on price discrimination in oligopoly.

[6] Neven and Phlips consider linear demands and the total output, as occurs under monopoly, is unchanged by price discrimination.

firm meaningfully affects competition in the duopolistic market, both under strategic substitutes and strategic complements.[7] Under price discrimination, and just like a monopolist, the multimarket firm charges more in the market with the lowest elasticity of demand; the qualification being that a monopolist faces market demands whereas the multimarket seller faces a residual demand in the duopolistic market, given the rival´s price. Following Robinson’s (1933) terminology, let us call one market the “strong” market and the other the “weak” market.[8] Under price competition, when the duopolistic market is weak, price discrimination makes the multimarket firm more aggressive (by reducing prices) in price competition and the rival also reacts more aggressively. As a consequence, there is a fall in the profit of the rival in the duopolistic market and the effect on the total profit of the multimarket seller is ambiguous (given that its profits in the monopolistic market increase). If the duopolistic market is strong, price discrimination leads to a moderation of price competition and the profits of the firms are higher than those under uniform pricing. [9] Under quantity competition, when the duopolistic market is weak, price discrimination makes the multimarket firm more aggressive (by increasing its output) and the rival reacts being less aggressive. As a consequence, there is a fall in the profit of the rival in the duopolistic market and the total profit of the multimarket seller increase. If the duopolistic market is strong, price discrimination makes the multimarket firm less aggressive (by decreasing its output) and the rival reacts more aggressively. As a consequence the profit of the rival increases and the effect on the total profit of the multimarket seller is ambiguous.

[7] Note that this is a fit setting given that a common feature of most cases under antidiscrimination litigation is that competition varies across markets. See Hausman and MacKie-Mason (1988) for an interesting case in which Du Pont’s adversary (Akzo) charged that third-degree price discrimination practices were a misuse of a patent. See other cases in, for example, Scherer and Ross (1990) or Varian (1989).

[8] Under monopoly, a strong (weak) market is defined as a market in which, at the profit maximizing uniform price, the market elasticity indicates that the profit in that market could be increased by increasing (decreasing) the price in that market. That is, in a strong (weak) market the discriminatory price is at least as great as (not greater than) the profit maximizing uniform price. In our case, what determines if the market is strong or weak is the elasticity of the multimarket seller’s residual demand, not the market demand. It must be pointed out that a market may be strong under monopoly but weak under duopoly. Many works in the literature on price discrimination have used Robinson's terminology. See, for example, Schmalensee (1981), Holmes (1989), Malueg (1993), Layson (1994) or, more recently, Aguirre, Cowan and Vickers (2010)..

[9] Note that when the duopolistic market is strong uniform pricing would be optimal only if the objective of the multimarket seller were to deter entry or to induce exit. If this is the case and if there are not legal restrictions to price discriminate, the multimarket seller might commit itself to uniform pricing, for instance, by signing most-favoured-customer clauses with its consumers (see Aguirre 2000).

Varian (1985), (1989) formulates a general test for a move from uniform pricing to price discrimination to be welfare improving. Following his analysis, we obtain upper and lower bounds on welfare change when a move is made by the multimarket firm from uniform pricing to price discrimination. These bounds on welfare change provide necessary and sufficient conditions for price discrimination to increase social welfare. Our main results are obtained by assuming linear demand functions. We show that price discrimination reduces welfare if the duopolistic market is weak, and that, if the duopolistic market is strong, it is satisfied the necessary condition for price discrimination to lead to a welfare improvement.. These results contrast with those obtained for the monopoly case: under linear demands price discrimination reduces welfare because the total output does not change.

Our findings provide an interesting competition policy implication. Section 2(b) of the Robinson-Patman Act permits a seller to rebut the *prima facie* presumption of illegality by showing that its discriminatory price was quoted "in good faith to meet an equally low price of a competitor" (see, for example, Scherer and Ross, 1990, p. 514). However, if linearity of demand is not a bad approximation, we might expect the impact of price discrimination on welfare to be negative when the duopolistic market is weak.

Several authors have stressed that the elimination of price discrimination can be particularly dangerous if it leads to the closure of markets.[10] We analyze the effects of price discrimination on social welfare when the multimarket seller does not serve the weak market under uniform pricing. If the duopolistic market is *weak*, price discrimination increases welfare given that uniform pricing leads to a monopolization of the weak market by the rival. If the duopolistic market is *strong*, price discrimination yields a Pareto improvement.

This paper is organized as follows. Section 2 develops the basic model. In section 3, we analyze the effects of price discrimination on price and quantity competition and market structure. Section 4 analyzes the welfare effects of price discrimination and discusses

[10] See, for example, Robinson (1933), Varian (1985) and Hausman and MacKie-Mason (1988). Layson (1994) derives conditions that determine when price discrimination will induce service to a market that otherwise would not be served under uniform pricing.

implications for antitrust policy. Section 5 offers concluding remarks.

2. The model

Consider a multimarket firm, firm *A*, which sells one product in two separated markets. In market 1 firm *A* is a monopolist and it faces competition from firm *B* in market 2. Denote by $u_2(x_A, x_B) + y$ the utility function (separable and linear in the numeraire good) of a representative consumer in market 2. Suppose that $u_2(x_A, x_B)$ is a differentiable strictly concave function on R_+^2, which is strictly monotone in a nonempty bounded region X_2. The maximization of the representative consumer yields an inverse demand system $p_2(x_A, x_B)$, $i = A, B,$ where x_i is the amount of good x produced by firm i ($i = A, B$), which is twice-continuously differentiable in the interior of X_2. Inverse demands are downward sloping $\partial p_i / \partial x_i < 0$, $i = A, B,$ and we assume that firms sell substitutes $\partial p_i / \partial x_j < 0$, $j \neq i, i =, A, B$. The inverse demand system can be inverted to yield a demand system $x_i = D_i(p_A, p_B)$, $i = A, B.$ The bounded region in price space where demands are positive is denoted by P_2. The demand system is twice-continuously differentiable in the interior of P_2. Direct demands are downward sloping $\partial D_i / \partial p_i < 0$ and $\partial D_i / \partial p_j > 0$, $j \neq i, i =, A, B$; that is, the "own effect" is greater than the "cross effect". We assume that $\left|\partial p_i / \partial x_i\right| > \left|\partial p_i / \partial x_j\right|$ and $\left|\partial D_i / \partial p_i\right| > \left|\partial D_i / \partial p_j\right|$, $j \neq i, i =, A, B$. Denote by $p_1(x_1)$, the inverse demand, which is twice-continuously differentiable in the interior of X_1 (arising from maximizing the strictly concave quasi-linear utility of a representative consumer) and $D_1(p_1)$ the demand in market 1, assumed to be downward sloping. Marginal production costs are constant and identical for both firms (and both markets), $c_A = c_B = c > 0$.

Firm *i*'s profits in market 2 in terms of prices are $\pi^i(p_A, p_B) = (p_i - c)D_i(p_A, p_B)$, $i = A, B,$ and in terms of quantities, $\pi^i(x_A, x_B) = [p_i(x_A, x_B) - c]x_i, i = A, B.$ The incumbent's profit function in market 1 is $\pi^1(x_1) = [p_1(x_1) - c]x_1$. We assume that either firm in market 2 can make positive profits even when the rival's price is the marginal cost. This ensures interior solutions in equilibrium. Moreover, we assume that $\pi_{ii}^i < 0$, $\pi_{ij}^i > 0$, $\pi_{ii}^i + \left|\pi_{ij}^i\right| < 0$, $j \neq i, i =, A, B$, and $\pi_{11}^1 < 0$. These assumptions ensure that Bertrand and Cournot reaction functions are well behaved and that there exist unique Bertrand and Cournot equilibria. Furthermore, the incumbent's profit function in market 1 is concave and the monopoly output (and price) is well defined.

Some of the main results of the paper will be obtained by considering the following linear inverse demands and demands systems:

$$\begin{aligned} p_1(x_1) &= \alpha_1 - \beta_1 x_1 \\ p_A(x_A, x_B) &= \alpha - \beta x_A - \gamma x_B \\ p_B(x_A, x_B) &= \alpha - \beta x_B - \gamma x_A, \end{aligned} \tag{1}$$

$$\begin{aligned} D_1(p_1) &= a_1 - b_1 p_1 \\ D_A(p_A, p_B) &= a - bp_A + cp_B \\ D_B(p_A, p_B) &= a - bp_B + cp_A, \end{aligned} \tag{2}$$

with $a_1 > b_1 c$ and $a > (b-d)c$, where c is the marginal cost. Assumptions $a_1 > b_1 c$ and $a > (b-d)c$ ensure that under price discrimination all markets are served.

In section 3, we analyze the effects of price discrimination on the competition between firms (in terms of price levels, outputs and profits) and compare them with those arising under uniform pricing, and Section 4 provides the welfare implications.

3. The effects on competition of the multimarket seller's price policy

3.1. *Price discrimination*

First, we assume that the incumbent firm is allowed to price discriminate. Firm *A* is a monopolist in market 1 and a duopolist in market 2. Denote by $\pi_T(p_1, p_A, p_B)$ the total profit of firm *A*, where $\pi_T(p_1, p_A, p_B) = \pi_1(p_1) + \pi_A(p_A, p_B)$. The profit maximization problem of the multimarket seller, firm *I*, is:

$$\max_{p_1, p_A} \ (p_1 - c)D_1(p_1) + (p_A - c)D_A(p_A, p_B).$$

The first-order conditions are:

$$\frac{\partial \pi_T}{\partial p_1} = \frac{d\pi_1}{dp_1} = D_1(p_1) + (p_1 - c)\frac{dD_1(p_1)}{dp_1} = 0, \tag{3}$$

$$\frac{\partial \pi_T}{\partial p_A} = \frac{\partial \pi_A}{\partial p_A} = D_A(p_A, p_B) + (p_A - c)\frac{\partial D_A(p_A, p_B)}{\partial p_A} = 0. \tag{4}$$

The first-order condition for the profit maximization problem of firm *B* is:

$$\frac{\partial \pi_B}{\partial p_B} = D_B(p_A, p_B) + (p_B - c)\frac{\partial D_B(p_A, p_B)}{\partial p_B} = 0. \tag{5}$$

Note that as the markets are separated, the price decisions of the multimarket seller for the two markets are independent. In market 1, from condition (3), firm *A* sets the monopoly price level, p_1^m, sells the monopoly output, $x_1^m = D_1(p_1^m)$, and obtains monopoly profits, π_1^m. Conditions (4) and (5) implicitly define the best response functions of firm *A* and *B*, $R_A(p_B)$ and $R_B(p_A)$. A Bertrand equilibrium is a pair of prices, p_A^* and p_B^*, such that $p_A^* = R_A(p_B^*)$ and $p_B^* = R_B(p_A^*)$. The equilibrium outputs are $x_A^* = D_A(p_A^*, p_B^*)$ and $x_B^* = D_B(p_A^*, p_B^*)$. Denote by π_A^* and π_B^* equilibrium profits in market 2. The total profit of firm *A* under price discrimination is $\pi_T^D = \pi_1^m + \pi_A^*$.

It is useful to rewrite first-order conditions in terms of price-cost mark up. Conditions (3) and (4) can be written, at the equilibrium prices, as:

$$\frac{p_1^m - c}{p_1^m} = \frac{1}{\varepsilon_1(p_1^m)} \ , \tag{6}$$

$$\frac{p_A^* - c}{p_A^*} = \frac{1}{\varepsilon_A^R(p_A^*, p_B^*)} \ , \tag{7}$$

where $\varepsilon_1(p_1) = -D_1'(p_1)p_1 / D_1(p_1)$ and $\varepsilon_A^R(p_A, p_B) = -[\partial D_A(p_A, p_B)/\partial p_A]p_A / D_A(p_A, p_B)$ are the demand elasticity of market 1 (in absolute value) and the elasticity of the multimarket seller's residual demand in market 2, respectively.[11] Optimal pricing implies that the multimarket seller should charge the lowest price in the market with the most elastic demand.

3.2. *Uniform pricing*

Now assume that the multimarket seller is restricted to selling at a single price (because of, for instance, price discrimination is banned). Under uniform pricing the total profit function of the incumbent is $\pi_T(p, p, p_B) = (p-c)D_1(p) + (p-c)D_A(p, p_B)$. The first order condition is given by:

$$\frac{\partial \pi_T}{\partial p} = D_1(p) + (p-c)\frac{dD_1(p)}{dp_1} + D_A(p, p_B) + (p-c)\frac{\partial D_A(p, p_B)}{\partial p_A} = 0 \ , \tag{8}$$

which implicitly defines the reaction function of the multimarket seller under uniform pricing,

[11] Note that, as the products of the firms are substitutes in market 2, each firm would be charging too low a price from the point of view of the industry profits. If both products were produced by a monopolist, the Lerner index for each product would exceed the inverse of the own elasticity of demand. In particular, the price-cost markup of product i, $i = A, B,$ would be $\frac{p_i - c}{p_i} = \frac{1}{\varepsilon_i^R} + \frac{(p_j - c)D_j \varepsilon_{ij}}{R_i \varepsilon_i^R}$, where $\varepsilon_i^R = -(\partial D_i / \partial p_i)(p_i / D_i)$ is the elasticity of the residual demand (the own elasticity of demand), $\varepsilon_{ij} = (\partial D_j / \partial p_i)(p_i / D_j)$ is the cross-elasticity of demand for product j with respect to the price of product i, and $R_i = p_i D_i$ is the revenue associated with product i. This is a special case of the multiproduct monopolist's pricing problem analyzed by Tirole (1988).

$R_A^u(p_B)$. It is easy to check that this reaction function also has a slope of less than one in absolute value. Denote by p^u and p_B^u the equilibrium prices under uniform pricing. The output of firm I in market 1 is $x_1^u = D_1(p^u)$ and the equilibrium outputs in market 2 are $x_A^u = D_A(p^u, p_B^u)$ and $x_E^u = D_E(p^u, p_E^u)$. Let π_1^u be the equilibrium profit of firm A in market 1 and denote by π_A^u and π_B^u equilibrium profits in market 2. The total profit of firm A under uniform pricing is $\pi_T^u = \pi_1^u + \pi_A^u$.

First-order condition under uniform pricing, (7), can be expressed as:

$$\frac{p^u - c}{p^u} = \frac{D_1(p^u) + D_A(p^u, p_B^u)}{D_1(p^u)\varepsilon_1(p^u) + D_A(p^u, p_B^u)\varepsilon_A^R(p^u, p_B^u)}. \qquad (9)$$

We can write the price-cost margin as a weighted average of the inverse elasticities:

$$\frac{p^u - c}{p^u} = \mu \frac{1}{\varepsilon_A^R(p^u, p_B^u)} + (1-\mu)\frac{1}{\varepsilon_1(p^u)},$$

with $\mu = \dfrac{D_A(p^u, p_B^u)\varepsilon_A^R(p^u, p_B^u)}{D_1(p^u)\varepsilon_1(p^u) + D_A(p^u, p_B^u)\varepsilon_A^R(p^u, p_B^u)}$.

Thus the mark-up satisfies:

$$\min\left\{\frac{1}{\varepsilon_1(p^u)}, \frac{1}{\varepsilon_A^R(p^u, p_B^u)}\right\} \le \frac{p^u - c}{p^u} \le \max\left\{\frac{1}{\varepsilon_1(p^u)}, \frac{1}{\varepsilon_A^R(p^u, p_B^u)}\right\}.$$

Hence, with respect to uniform pricing, price discrimination increases the price in the less price sensitive market and reduces it in the other.[12]

[12] It must be stressed that elasticities differ between uniform price and price discrimination policies, given that elasticities generally depend on the price levels (which change with the pricing policy).

3.3. *Comparison of results*

Firstly, we shall analyze the effects of the pricing policy of the multimarket seller on its strategic behaviour in the duopolistic market. In the next Lemma, which follows the lines of theorem 1 in Nahata *et al*. (1990), we state the relation between the reaction function of the multimarket seller under price discrimination with that under uniform pricing.

Lemma 1. The concavity of the profits functions implies that:

$$\min\left\{R_A(p_B), p_1^m\right\} \leq R_A^u(p_B) \leq \max\left\{R_A(p_B), p_1^m\right\}.$$

Proof. Note that, given a rival's price p_B, for $p < \min\left\{R_A(p_B), p_1^m\right\}$, the multimarket seller's profit function under uniform pricing $\pi_T(p, p_B) = \pi_1(p) + \pi_A(p, p_B)$ is an increasing function of *p*, provided that the profit functions $\pi_1(p_1)$ and $\pi_A(p_A, p_B)$ are concave and achieve a maximum at p_1^m and $R_A(p_B)$, respectively. Similarly, for $p > \max\left\{R_A(p_B), p_1^m\right\}$, $\pi_I(p, p_B)$ is decreasing. Therefore, if $R_A^u(p_B)$ maximizes $\pi_T(p, p_B)$ then it is satisfied that $R_A^u(p_B) \in \left[\min\left\{R_A(p_B), p_1^m\right\}, \max\left\{R_A(p_B), p_1^m\right\}\right]$.Q.E.D.

The concavity of the profit functions (Lemma 1) and the fact that the reaction functions are upward sloping allow us to state the following relation between the equilibrium prices.

Lemma 2. (i) $\min\left\{p_A^*, p_1^m\right\} \leq p^u \leq \max\left\{p_A^*, p_1^m\right\}$.

(ii) If $p_A^* \geq p^u$ then $p_B^* \geq p_B^u$ and if $p_A^* \leq p^u$ then $p_B^* \leq p_B^u$ (and vice versa).

Proof. See Appendix.

Part (i) of Lemma 2 states that the multimarket seller under price discrimination raises the price at one market and lowers it at the other, and part (ii) is a direct consequence of competition with strategic complements. Following Robinson's (1933) terminology, let us call one market the *strong* market and the other the *weak* market. The duopolistic market, market 2, is *strong* (*weak*) if the discriminatory price in that market is at least as great as (no greater than) the profit maximizing uniform price; that is, market 2 is *strong* (*weak*) if $\varepsilon_A^R(p^u, p_B^u) < (>) \varepsilon_1(p^u)$.[13] Similarly, a market is strong (weak) if the multimarket firm's discriminatory price is higher (lower) in that market than in the other market.

Lemma 2 implies that when the duopolistic market is weak $p_1^m \geq p^u \geq p_A^*$ and $p_B^* \leq p_B^u$, and when the duopolistic market is strong $p_A^* \geq p^u \geq p_1^m$ and $p_B^* \geq p_B^u$. In order to understand these results and to state the effects of the multimarket seller's pricing policy on profits, it is useful to consider a move from uniform pricing to price discrimination.

Marginal profits in market 1 and 2 of the multimarket seller at the equilibrium prices under uniform pricing (p^u, p_B^u) can be expressed, respectively, as:

$$\frac{d\pi_1(p^u)}{dp_1} = D_1(p^u)\left[1 - \frac{p^u - c}{p^u}\varepsilon_1(p^u)\right], \tag{10}$$

$$\frac{\partial \pi_A(p^u, p_B^u)}{\partial p_A} = D_A(p^u, p_B^u)\left[1 - \frac{p^u - c}{p^u}\varepsilon_A^R(p^u, p_B^u)\right]. \tag{11}$$

By substituting the price-cost margin from (9) and omitting arguments, we get:

$$\frac{d\pi_1(p^u)}{dp_1} = D_1\left[\frac{D_A(\varepsilon_A^R - \varepsilon_1)}{D_A\varepsilon_A^R + D_1\varepsilon_1}\right], \tag{12}$$

$$\frac{\partial \pi_A(p^u, p_B^u)}{\partial p_A} = D_A\left[\frac{D_1(\varepsilon_1 - \varepsilon_A^R)}{D_A\varepsilon_A^R + D_1\varepsilon_1}\right]. \tag{13}$$

The signs of (12) and (13) depend on the relation between elasticities. When the duopolistic

[13] It must be pointed out that we use a duopoly-defined measure; that is, under another market structure the classification of a market as strong or weak might differ. For example, market 2 may be strong under monopoly but weak under duopoly. As we will show, this fact may lead the multimarket seller to engage in a price discrimination policy that results in the lowest price precisely in the market where the maximization of the industry profits requires the highest. As a consequence, price discrimination may reduce the multimarket seller's profit. Other types of competition, different from simultaneous price competition, might also affect the strong-weak ranking between markets.

market is *weak* $\varepsilon_A^R(p^u, p_B^u) > \varepsilon_1(p^u)$ then $\frac{d\pi_1(p^u)}{dp_1} > 0$ and $\frac{\partial \pi_A(p^u, p_B^u)}{\partial p_A} < 0$. and thus, given that profit functions are concave (Lemma 1 and 2), $p_1^m > p^u$ and $p^u = R_A^u(p_B^u) > R_A(p_B^u)$. Note that price discrimination makes the multimarket seller become *more aggressive* in market 2; with strategic complements, the rival's optimal response to this more aggressive play by the multimarket firm is to become more aggressive, and firm *B* reacts also by decreasing its price.[14] Therefore, $p_1^m > p^u > p_A^*$ and $p_B^u > p_B^*$.[15] As a consequence, firm *A* increases its output and the market share of firm *B* decreases (the decrease in the price of firm *A* is higher than the decrease in the price of firm *B*). The effect of price discrimination on the multimarket seller's total profits is ambiguous; although its profits in market 1 increase, they can decrease in market 2. It is easy to check, by using the first order condition, that $d\pi_B\left(R_B(p_A), p_A\right)/dp_A > 0$, and therefore, price discrimination results in a decrease in the rival's profit, $\pi_B^* < \pi_B^u$.

When the duopolistic market is *strong* $\varepsilon_A^R(p^u, p_B^u) < \varepsilon_1(p^u)$ then $\frac{d\pi_1(p^u)}{dp_1} < 0$ and $\frac{\partial \pi_A(p^u, p_B^u)}{\partial p_A} > 0$ thus, $R_A(p_B^u) > R_A^u(p_B^u) = p^u > p_1^m$. In this case price discrimination makes the multimarket seller become *less aggressive* (by raising its price) in market 2. With strategic complements, the rival's optimal response to a less aggressive play by firm *A* is to be less aggressive; that is, firm *B* reacts also by increasing its price.[16] Therefore, $p_A^* > p^u > p_1^m$ and $p_B^* > p_B^u$. As a consequence, firm *A* reduces its output and the market share of firm *B* is increased; note that, as reaction functions have slope less than one, the increase in the price of firm *A* is higher than the increase in the price of firm *B*. Note that $d\pi_A\left(p^u, R_B(p^u)\right)/dp_A > 0$ and, therefore, this moderation in price competition results in an increase in the profits of both firms; $\pi_A^* > \pi_A^u$ and $\pi_B^* > \pi_B^u$. In market 1 the multimarket seller reduces its price, $p_1^m < p^u$,

[14] It is easy to check that price discrimination shifts inward the reaction curve of the multimarket seller, $R_A^u(p_B) > R_A(p_B)$ at least for $p_B \in \left[\min\left\{p_B^*, p_B^u\right\}, \max\left\{p_B^*, p_B^u\right\}\right]$. Given that the rival's reaction function is upward sloping, price discrimination reduces the price of both firms in the duopolistic market.

[15] We are now assuming that both markets are served by the multimarket firm under uniform pricing and that elasticities differ across markets and therefore, the relation between prices holds with strict inequalities.

[16] Price discrimination shifts outward the reaction curve of the multimarket seller, $R_A^u(p_B) < R_A(p_B)$ for $p_B \in \left[\min\left\{p_B^*, p_B^u\right\}, \max\left\{p_B^*, p_B^u\right\}\right]$. Given that the rival's reaction function is upward sloping, price discrimination leads to an increase in the price of both firms in the duopolistic market.

and therefore, its sales and profits in this market go up, $\pi_1^m > \pi_1^u$. Therefore, when price discrimination makes the incumbent less aggressive, its total profits are increased: $\pi_T^* > \pi_T^u$. The following proposition summarizes the above results:

Proposition 1. Under Bertrand competition (strategic complements), with respect to uniform pricing:

(i) If the duopolistic market is *weak* $(\varepsilon_A^R(p^u, p_B^u) > \varepsilon_1(p^u))$ price discrimination reduces the profits of the single-market firm, and the effect on the total profit of the multimarket seller is ambiguous given that its profits in market 1 increase.

(ii) If the duopolistic market is *strong* $(\varepsilon_A^R(p^u, p_B^u) < \varepsilon_1(p^u))$ price discrimination increases the profits of both firms.[17]

In order to illustrate how the ability to price discriminates can reduce the profit of the multimarket seller when the duopolistic market is weak, we shall consider the linear demands case (see, (2)). The change in the profit of the multimarket seller due to a move from uniform pricing to price discrimination is given by:

$$\Delta\pi_T = \frac{A}{4b_1\Gamma^2(2b-d)^2}\Big\{\Big[(4b^2-d^2)^2+16b_1b^3\Big]A$$

$$+4b_1d(2b-d)\big[(2b+d)d[a-(b-d)c]+2bd(a_1-b_1c)\big]\Big\}, \tag{14}$$

where $A = 2ab_1 - 2a_1b + a_1d + b_1dc$ and $\Gamma = \left[4b(b+b_1) - d^2\right]$. It is easy to check that if market 2 is weak $(\varepsilon_A^R(p^u, p_B^u) > \varepsilon_1(p^u))$, then $A < 0$, whereas if market 2 is strong $(\varepsilon_A^R(p^u, p_B^u) < \varepsilon_1(p^u))$, then $A > 0$. Note that when the duopolistic market is strong, price discrimination increases the profit of the multimarket seller (proposition 1).[18] The next

[17] If $\varepsilon_A^R(p^u, p_B^u) = \varepsilon_1(p^u)$ the market outcome would be the same under both pricing policies.

[18] Note that if market 2 is strong under duopoly, it will also be strong under monopoly. Therefore, the direction of the price discrimination policy (higher prices in market 2) is correct from the point of view of the industry profits. As a consequence, the multimarket seller's profit increases with price discrimination.

Corollary allows us to understand under what conditions price discrimination can reduce the profit of the multimarket seller.

Corollary 1. If the profits of the multimarket firm are reduced by price discrimination, then this firm would be practising a type of price discrimination contrary to that which maximizes the industry profit.

That is, market 2 is weak under competition (from the point of view of the multimarket seller) whereas, under multiproduct monopoly, market 2 would be strong. It is easy to check that price discrimination reduces the multimarket seller's profit if the following condition is satisfied:

$$\frac{2b_1}{(2b-d)} < \frac{(a_1-b_1c)}{[a-(b-d)c]} < \frac{2b_1}{(2b-d)} \frac{(4b^2-d^2)^2+8b_1b(2b^2-d^2)+2[4b(b+b_1)-d^2]d^2}{(4b^2-d^2)^2+8b_1b(2b^2-d^2)} \qquad (15)$$

The left side inequality of (15) defines market 2, the duopolistic market, as weak,[19] and the right side inequality of (15) implies that discrimination reduces profits. It is straightforward to check that if condition (15) is satisfied then $\{[a_1-b_1c]/[a-(b-d)c]\}<b_1/(b-d)$, but this implies that market 2 would be strong for a monopolist; that is $p_I^m > p_1^m$, where p_A^m denotes the price charged by a multiproduct monopolist in market 2 under price discrimination.[20] Therefore, the competition between firms transforms a strong market into a weak one. Thus, each firm under competition would be charging too low a price from the point of view of the industry profits (firms do not take into account that products are substitutes and that an increase in the price of one product raises the demand for the other product).

The following proposition summarizes the results under quantity competition.

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[19] The equilibrium prices under price discrimination are $p_A^* = p_B^* = (a+bc)/(2b-d)$ and $p_1^m = (a_1+b_1c)/2b_1$. Thus, if the duopolistic market is weak, $p_1^m > p_A^*$, then $\{[a_1-b_1c]/[a-(b-d)c]\} > 2b_1/(2b-d)$, or equivalently $A<0$.

[20] The optimal prices of the multimarket monopolist under price discrimination are $p_A^m = p_B^m = [a+(b-d)c]/2(b-d)$ and $p_1^m = (a_1+b_1c)/2b_1$; therefore, if $\{(a_1-b_1c)/[a-(b-d)c]\} < b_1/(b-d)$) then $p_A^m > p_1^m$.

Proposition 2. Under Cournot competition (strategic substitutes), with respect to uniform pricing:

(i) If the duopolistic market is *weak* price discrimination reduces the profits of the single-market firm, and increases the total profit of the multimarket seller.

(ii) If the duopolistic market is *strong* price discrimination increases the profits of the single-market firm and the effect on the total profit of the multimarket seller is ambiguous given that its profits in market 1 increase.

We next provide the intuition for proposition 2 for the case of homogeneous product. Banning price discrimination (that is, uniform pricing) imposes a constraint $p_A(x_A + x_B) = p_2(x_2)$ to the multimarket firm. From first order condition of the constraint maximization problem we get (omitting arguments):

$$\frac{p^u - c}{p^u} = \frac{x_A^u + x_1^u}{x_A^u \varepsilon_2 + x_1^u \varepsilon_1}. \tag{16}$$

The multimarket firm's marginal profits in market 1 and 2 at the equilibrium outputs under uniform pricing are:

$$\frac{d\pi_1(x_1^u)}{dx_1} = \frac{p^u x_1^u}{\varepsilon_1 x_1^u + \varepsilon_2 x_2^u}(s_A^u \varepsilon_2 - \varepsilon_1), \tag{17}$$

$$\frac{d\pi_A(x_A^u, x_B^u)}{dx_A} = \frac{p^u x_2^u}{\varepsilon_1 x_1^u + \varepsilon_2 x_2^u}(\varepsilon_1 - s_A^u \varepsilon_2), \tag{18}$$

where $x_2^u = x_A^u + x_B^u$ is the total output and $s_A^u = x_A^u / x_2^u$ is the multimarket seller's share in market 2. When the duopolistic market is *weak* then $\partial\pi_A(x_A^u, x_B^u)/\partial x_A > 0$ and

$d\pi_1(x_1^u)/dx_1 < 0.$ Under price discrimination the multimarket firm becomes more aggressive in market 1 and under strategic substitutes, the rival reacts being less aggressive: therefore, $x_A^* > x_B^u$, $x_B^* < x_B^u$ and $x_1^m < x_1^u$. As a consequence the profits of the rival firm decreases with price discrimination and the multimarket seller's profits increases.

When the duopolistic market is *strong* then $\partial\pi_A(x_A^u, x_B^u)/\partial x_A < 0$ and $d\pi_1(x_1^u)/dx_1 > 0.$ Under price discrimination the multimarket firm becomes less aggressive in market 1 and under strategic substitutes, the rival reacts being more aggressive: therefore, $x_A^* < x_A^u$, $x_B^* > x_B^u$ and $x_1^m > x_1^u$. As a consequence the profits of the rival firm increases with price discrimination and the effect on the total profit of the multimarket seller is ambiguous given that its profits in market 1 increase.

3.4. *Market opening under price discrimination*

In the above analysis we have assumed that the multimarket firm pricing policy (or the legal status of price discrimination) does not affect its decision on whether to serve markets. The following corollary analyzes some potential effects of the pricing policy on the market structure.

Corollary 2. (i) If the duopolistic market is *weak*, uniform pricing may lead to a monopolization of this market by the rival. (ii) If the duopolistic market is *strong*, uniform pricing may induce the multimarket seller not to serve the weak market (market 1).

Note that the uniform price can be so high that no purchases are made in markets that would otherwise be profitably served under price discrimination. When the duopolistic market is weak, if $\pi_1^m > \pi_1(p^u) + \pi_A(p^u, p_B^u)$ then under uniform pricing the multimarket seller would not serve market 2 and would fix a price p_1^m. As a consequence, firm *E* would remain as a monopolist in market 2 and price at p_2^m. Suppose that the duopolistic market is strong, if market 1 is not served under uniform pricing then the equilibrium prices would be p_A^* and p_E^*. Therefore, uniform pricing *closes* market 1 when $\pi_I^* > \pi_1(p^u) + \pi_A(p^u, p_B^u)$; the opening of market 1 does not offset the profit loss in market 2.

In the next section, we shall analyze the effects of the pricing policy (price discrimination or uniform pricing) of the multimarket seller on social welfare taking into account the above effects on the competition derived from one or the other policy.

4. Effects on social welfare

It is well known that, even in the pure monopoly case, a move from uniform pricing to price discrimination has in general ambiguous consequences on social welfare. The differences in price between markets, due to price discrimination, entail a loss of efficiency because the marginal valuations of output are not the same across buyers. As a consequence a necessary condition for price discrimination to increase social welfare is that output increases. In this work, the existence of competition in one market introduces additional effects that make the welfare question more difficult to answer. A move by the multimarket seller from price discrimination to uniform pricing actually makes marginal utilities equal across its buyers. However, the strategic effect derived from the pricing policy change leads to marginal valuations of the products of both firms which are not equal. On the other hand, the multimarket firm pricing policy (or the legal status of price discrimination) may affect its decision on whether to serve markets. We address the issue by establishing upper and lower bounds on welfare change. Our analysis is based on the general test for welfare improvement proposed by Varian (1985), (1989).

Consider an aggregate utility function of the form $u(x_1, x_A, x_B) + y$,[21] where x_i, $i = A, B,$ are the product varieties consumed in market 2, x_1 is the consumption in market 1 and y is the money to be spent on other goods. We assume that u is concave and differentiable. The inverse demand functions are given by $p_j(x_1, x_A, x_B) = \partial u(x_1, x_A, x_B) / \partial x_j$, $j = 1, A, B.$

[21] We wish to use the classical measure of social welfare as consumers' plus producers' surplus, and the most general preference structure for which this is possible is that of quasi-linear utility. This class of preferences satisfies the condition that not only does consumers' surplus serve as a legitimate measure of individual welfare, but also individual consumers' utility functions can be added up to form a social utility function, thus the aggregate consumers' surplus is also meaningful. See Varian (1985).

Consider two configurations of output, (x_1^0, x_A^0, x_B^0) and (x_1^1, x_A^1, x_B^1), with associated prices (p_1^0, p_A^0, p_B^0) and (p_1^1, p_A^1, p_B^1). By using the concavity of the aggregate utility function we obtain:

$$u(x_1^1, x_A^1, x_B^1) \leq u(x_1^0, x_A^0, x_B^0) + \frac{\partial u(x_1^0, x_A^0, x_B^0)}{\partial x_1}(x_1^1 - x_1^0)$$

$$+\frac{\partial u(x_1^0, x_A^0, x_B^0)}{\partial x_A}(x_A^1 - x_A^0) + +\frac{\partial u(x_1^0, x_A^0, x_B^0)}{\partial x_B}(x_B^1 - x_B^0). \tag{19}$$

By rearranging and using the definition of inverse demand function we have:

$$\Delta u \leq p_1^0 \Delta x_1 + p_A^0 \Delta x_A + p_B^0 \Delta x_B, \tag{20}$$

where $\Delta u = u(x_1^1, x_A^1, x_B^1) - u(x_1^0, x_A^0, x_B^0)$ and $\Delta x_j = x_j^1 - x_j^0,\ j = 1, A, B.$ Using a similar argument, we get:

$$\Delta u \geq p_1^1 \Delta x_1 + p_A^1 \Delta x_A + p_B^1 \Delta x_B. \tag{21}$$

Under constant marginal cost the change in total cost is $\Delta C = c\Delta x_1 + c\Delta x_A + c\Delta x_B.$ Since $\Delta W = \Delta u - \Delta C$, we obtain upper and lower bounds on welfare change:

$$(p_1^0 - c)\Delta x_1 + (p_A^0 - c)\Delta x_A + (p_B^0 - c)\Delta x_B \geq \Delta W \geq (p_1^1 - c)\Delta x_1 + (p_A^1 - c)\Delta x_A + (p_B^1 - c)\Delta x_B. \tag{22}$$

If (p^u, p^u, p_B^u) are the prices when the multimarket seller engages in uniform pricing and (p_1^m, p_A^*, p_B^*) the discriminatory prices, then:

$$(p^u - c)(\Delta x_1 + \Delta x_A) + (p_B^u - c)\Delta x_B \geq \Delta W \geq (p_1^m - c)\Delta x_1 + (p_A^* - c)\Delta x_A + (p_B^* - c)\Delta x_B. \tag{23}$$

The upper bound implies that a necessary condition for welfare to increase is that the sum of weighted output changes is positive, where the weights are the price-cost margins under uniform pricing. Notice that if firm A and B sold independent products, we would obtain the traditional result according to which an increase in total industry output is a necessary condition for price discrimination to be welfare improving. The lower bound gives a sufficient condition for welfare to increase under price discrimination, namely, that the sum of the weighted output changes is positive, where the weights are the equilibrium price-cost margins under price discrimination.

In order to stress the relevance of the above bounds on welfare, we shall next consider the case of linear demands. Assume that the inverse demands and direct demands systems are given by (1) and (2).[22] The following lemma provides an exact measure of the welfare change under linear demands.

Lemma 3. Under linear demands and constant marginal cost, the welfare change is the average between the upper and the lower bounds: $\Delta W = \frac{1}{2}UB + \frac{1}{2}LB.$

Proof. See Appendix.

Note that, under linear demands and constant marginal cost, a necessary and sufficient condition for price discrimination to be welfare improving is that the upper bound is higher than the lower bound in absolute value.[23]

The changes of the output in market 1 and the output of firms A and B in market 2, which are due to a move from uniform pricing to price discrimination, are given by:

[22] The aggregate utility function is given by:

$$u(x_1, x_A, x_B) = \alpha_1 x_1 - \frac{1}{2}\beta_1 x_1^2 + \alpha(x_A + x_B) - \frac{1}{2}(\beta x_A^2 + 2\gamma x_A x_B + \beta x_B^2) + y.$$

The maximization of the representative consumers in markets 1 and 2 yields the demand system where $a_1 = \frac{\alpha_1}{\beta_1}, b_1 = \frac{\alpha_1}{\beta_1}, a = \frac{\alpha}{\beta+\gamma}, b = \frac{\beta}{\beta^2-\gamma^2}$ and $d = \frac{\gamma}{\beta^2-\gamma^2}$.

[23] It must be pointed out that this result only depends on the linearity of demands (and on the assumption of constant marginal cost) and it holds for arbitrary configurations of prices. In particular, it holds both under competition in one market or under pure monopoly.

$$\Delta x_1 = \frac{(4b^2 - d^2)A}{2\Gamma(2b-d)}, \quad \Delta x_A = -\frac{(4b^2 - 2d^2)A}{2\Gamma(2b-d)}, \quad \Delta x_B = \frac{2bdA}{2\Gamma(2b-d)}, \tag{24}$$

where $A = 2ab_1 - 2a_1 b + a_1 d + b_1 dc$ and $\Gamma = \left[4b(b+b_1) - d^2\right]$. The multimarket seller's total output change is:

$$\Delta x_1 + \Delta x_A = \frac{d^2 A}{2\Gamma(2b-d)}. \tag{25}$$

It is easy to check that the upper bound (*UB*) and the lower bound (*LB*) on welfare change are given by:

$$UB = \frac{dA}{2\Gamma^2(2b-d)}\left\{\left[4b(b+b_1) + (4b+d)d\right]\left[a-(b-d)c\right] + 4bd(a_1 - b_1 c\right\}, \tag{26}$$

$$LB = \frac{(2b+d)A}{4b_1\Gamma(2b-d)^2}\left\{-(2b-d)A + 2b_1 d\left[a-(b-d)c\right]\right\}. \tag{27}$$

The next proposition states the effects on social welfare of price discrimination with respect to uniform pricing.

Proposition 3. (i) If the duopolistic market is *weak* ($\varepsilon_A^R(p^u, p_B^u) > \varepsilon_1(p^u)$ and therefore $p_1^m > p_I^*$), price discrimination reduces social welfare. (ii) If the duopolistic market is *strong* ($\varepsilon_A^R(p^u, p_B^u) < \varepsilon_1(p^u)$ and therefore $p_1^m < p_I^*$), it is satisfied the necessary condition for price discrimination to increase social welfare.

Proof. It is easy to check that if market 2 is weak ($\varepsilon_A^R(p^u, p_B^u) > \varepsilon_1(p^u)$), then $A < 0$, whereas if market 2 is strong ($\varepsilon_A^R(p^u, p_B^u) < \varepsilon_1(p^u)$), then $A > 0$. Thus, if market 2 is weak, the upper bound on welfare change is negative (see (26)) and, consequently, price discrimination reduces welfare. Note that, from (24) and (25), $\Delta x_1 + \Delta x_A < 0$ and $\Delta x_B < 0$, and therefore the two terms of the upper bound (see condition (23)) are negative. If market 2 is strong then the upper bound (26) is positive; note that $\Delta x_1 + \Delta x_A > 0$ and $\Delta x_B > 0$. Therefore, the necessary condition for price discrimination to increase welfare is satisfied. Q.E.D.

Quantity competition

The output changes due to a move from uniform to price discrimination are given by:

$$\Delta x_1 = \frac{(4\beta^2 - \gamma^2)C}{2\Phi(2\beta+\gamma)}, \quad \Delta x_A = -\frac{4\beta^2 C}{2\Phi(2\beta+\gamma)}, \quad \Delta x_B = \frac{2\beta\gamma C}{2\Phi(2\beta+\gamma)}, \tag{28}$$

where $C = 2\alpha\beta - 2\alpha_1\beta - \alpha_1\gamma$ and $\Phi = 4\beta^2(\beta+\beta_1) - \gamma^2(2\beta+\beta_1)$.

It is easy to check that if market 1 is weak (strong) then $C < 0$ ($C > 0$). The upper bound (*UB*) on welfare change can be written:

$$UB = \frac{\beta\gamma C}{2\Phi^2(2\beta+\gamma)}\left[4\alpha\beta^2(\beta-\gamma) + 4\alpha\beta\beta_1(\beta-\gamma) + \gamma^2(\alpha\beta_1 + \alpha_1\gamma\right]. \tag{29}$$

The sign of this expression depends on the sign of *C*. If market 1 is weak ($C < 0$) then the upper bound is negative and hence price discrimination reduces welfare. If market 1 is strong ($C > 0$) then the neccessary condition for price discrimination to increase welfare is satisfied. The lower bound is:

$$LB = \frac{(2\beta-\gamma)C}{4\Phi(2\beta+\gamma)^2}\left[-2\beta C + 2\alpha_1\gamma(2\beta+\gamma)\right]. \tag{30}$$

The following proposition summarizes the results under quantity competition.

Proposition 4. (i) If the duopolistic market is *weak* price discrimination reduces social welfare. (ii) If the duopolistic market is *strong* it is satisfied the necessary condition for price discrimination to increase social welfare.

To understand proposition 3 and 4, it could be useful to make a comparison with the results under multiproduct monopoly. Consider a multimarket firm which is a monopolist in the two markets and produces two product varieties: *A* and *B*. Product *A* is sold in both markets and product *B* is only sold in market 2. We shall next show that, with linear demands, a move from uniform pricing to price discrimination does not change the total output of products *A* and *B*; that is, $\Delta x_1^m = -\Delta x_A^m$ and $\Delta x_B^m = 0$ (where superscript m denotes monopoly). A move from uniform pricing to price discrimination leads to the following changes in prices:

$$\Delta p_E^m = -\frac{(\partial D_B / \partial p_A) + (\partial D_A / \partial p_B)}{2(\partial D_B / \partial p_B)} \Delta p_A^m = \frac{d}{b} \Delta p_A^m, \tag{31}$$

$$\Delta p_A^m = -\frac{dD_1 / dp_1}{\partial D_A / \partial p_A} \Delta p_1^m - \frac{(\partial D_B / \partial p_A) + (\partial D_A / \partial p_B)}{2(\partial D_B / \partial p_B)} \Delta p_B^m = -\frac{b_1}{b} \Delta p_1^m + \frac{d}{b} \Delta p_B^m. \tag{32}$$

The maximization of the profits of market 2 requires that the price of product *j* be adjusted in response to a change in the price of product *i*, taking into account that goods are substitutes and, thus, that the externality between them is internalized (condition (31) and second term of (32)). The first term in (32) represents the standard price adjustment implied by price discrimination. Under linear demands we can express the changes in the outputs as: $\Delta x_1 = (dD_1 / dp_1)\Delta p_1 = -b_1 \Delta p_1$. and $\Delta x_i = (\partial D_i / \partial p_i)\Delta p_i + (\partial D_i / \partial p_j)\Delta p_j = -b\Delta p_i + d\Delta p_j$, $i, j = A, B,\ j \neq i$. Therefore, given (31) and (32), we get $\Delta x_1^m = -\Delta x_A^m$ and $\Delta x_B^m = 0$. Note that, if we consider a single-product monopoly, the price adjustment to the difference in elasticity between markets would be $\Delta p_2^m = -(b_1 / b_2)\Delta p_1^m$, and the total output would be unchanged, $\Delta x_1^m = -b_1 \Delta p_1^m = -\Delta x_2^m$. In any case, we obtain the standard result according to which under pure monopoly and linear demands price discrimination reduces welfare because the total output (of each product) does not change.

When the multimarket seller meets competition in one market, market 2, results change drastically. First note that the existence of a competitor in market 2 makes the total output of each product change with the pricing policy. The reason for this is that firms maximize their own profit function without taking into account the effect of their price choices on the rival's demand. The changes in the prices of the rival, Δp_B, and the multimarket firm, Δp_A and Δp_1, satisfy:

$$\Delta p_B = \frac{dR_B(p_A)}{dp_A}\Delta p_A = -\frac{(\partial D_B / \partial p_A)}{2(\partial D_B / \partial p_B)}\Delta p_A = \frac{d}{2b}\Delta p_A, \tag{33}$$

$$\begin{aligned}\Delta p_A &= -\frac{dD_1 / dp_1}{\partial D_A / \partial p_A}[\Delta p_1 - \frac{dR_A^u(p_B)}{dp_B}\Delta p_B] + \frac{dR_A^u(p_B)}{dp_B}\Delta p_B \\ &= -\frac{dD_1 / dp_1}{\partial D_A / \partial p_A}\Delta p_1 + \frac{dR_A(p_B)}{dp_B}\Delta p_B = -\frac{dD_1 / dp_1}{\partial D_A / \partial p_A}\Delta p_1 - \frac{\partial D_A / \partial p_B}{2\partial D_A / \partial p_A}\Delta p_B \\ &= -\frac{b_1}{b}\Delta p_1 + \frac{d}{2b}\Delta p_B.\end{aligned} \tag{34}$$

Condition (33) reflects the rival reaction to a change in the multimarket seller's price according to its reaction function. Given that it does not take into account the effect on the demand of product *A*, the change in the rival's price, Δp_B, is too low (in absolute value) from the point of view of the industry profit (but not necessarily from a welfare point of view). The first term in (34) is the price adjustment implied by price discrimination, similar to that under multiproduct monopoly (see (32)), and the second one is the adjustment, following its reaction function, to the change in the rival's price. Note that, by comparing (31)-(32) with (33)-(34), given a change in the price of market 1, the changes in the prices of product *A* and *B* in market 2 are lower (in absolute value) under competition than under monopoly; as a consequence, the output of each firm changes with the multimarket seller's pricing policy. The output change of the multimarket seller in market 1 and 2, its total output change and the change in the rival's output are: $\Delta x_1 = -b_1 \Delta p_1$, $\Delta x_A = b_1 \Delta p_1 + (d/2)\Delta p_B$, $\Delta x_1 + \Delta x_A = (d/2)\Delta p_B$ and $\Delta x_B = (d/2)\Delta p_A$ (at the equilibrium prices we obtain the same output changes as in (24) and (25)). Note that sign (Δx_A) = - sign (Δx_B), sign ($\Delta x_1 + \Delta x_A$) = sign (Δx_B), and that the change in the output of market 1 is greater than the one in the total output of market 2: $|\Delta x_A + \Delta x_B| < |\Delta x_1|$. If the duopolistic market is weak, then both firms reduce prices, $\Delta p_A < 0$ and $\Delta p_B < 0$, the total output of each firm is reduced and, as a consequence, price discrimination reduces social welfare. When the duopolistic market is strong then both firms increase prices, $\Delta p_A > 0$ and $\Delta p_B > 0$, the total output of each firm increases and, as a consequence, it is satisfied the necessary condition for price discrimination to lead to a welfare improvement..

We now turn to the lower bound (26). Notice that if the duopolistic market, market 2, is strong, then $\Delta x_1 > 0$ and $\left|\Delta x_A + \Delta x_B\right| < \Delta x_1$. However, in the lower bound, the change in the total output in market 2 receives more weight than the output change of market 1. Therefore the sign of lower bound is not determined. The lower bound provides a sufficient condition for price discrimination to increase welfare, namely, $2b_1 d[a-(b-d)c] > (2b-d)A$. We can rewrite this condition as:

$$\frac{\varepsilon_1(p_A^*) - \varepsilon_A^R(p_A^*, p_B^*)}{\varepsilon_1(p_A^*)} < \frac{2\varepsilon_{AB}(p_A^*, p_B^*)}{2\varepsilon_A^R(p_A^*, p_B^*) - \varepsilon_{AB}(p_A^*, p_B^*)},$$

where $\varepsilon_{AB} = (\partial D_B / \partial p_A)(p_A / D_B)$ is the cross-price elasticity. Note that if the elasticity difference between market 1 and 2 under price discrimination is low enough, then the sufficient condition for price discrimination to increase social welfare is satisfied. Another equivalent form to write this condition is: $(p_1^m - c)/(p_A^* - c) > (p_A^* - c)/(p_A^m - c)$ where $(p_A^m - c)$ is the price-cost mark up under multimarket monopoly for product *A*. On the other hand, from Lemma 3, even if the lower bound were negative price discrimination could increase welfare if the upper bound is higher than the lower bound in absolute value.

Market opening under price discrimination and price competition

In the above analysis we have assumed that both markets are served under uniform pricing; that is, the multimarket seller sells in both markets. The following proposition analyzes the effects on social welfare when price discrimination makes the multimarket seller open the weak market.

Proposition 5. When the multimarket seller only serves the weak market under price discrimination: (i) If the duopolistic market is *weak* price discrimination increases welfare, given that uniform pricing leads to a monopolization of the weak market by the rival. (ii) If the duopolistic market is *strong* price discrimination yields a Pareto improvement by opening the weak market.

Proof. Note that for case (i) the lower bound on welfare change is $(p_A^* - c)\Delta x_A + (p_B^* - c)\Delta x_B$ given that $\Delta x_1 = 0$. As firm *A* does not serve market 2 under uniform pricing, then $\Delta x_A = x_A^*$ and $\Delta x_B = x_B^* - x_2^m$ because firm *B* is a monopolist in market 2 under uniform pricing. If the Bertrand equilibrium is symmetric (this is not strictly necessary), $p_A^* = p_B^*$, the lower bound can be written as $(p_A^* - c)(x_A^* + x_B^* - x_2^m)$, and this in general will be positive given that a duopoly produces more than a monopoly. (ii) When the duopolistic market is strong, and if the weak market is not served under uniform pricing, then the lower bound is positive: $(p_1^m - c)\Delta x_1 > 0$ given that $\Delta x_A = \Delta x_B = 0$ and $\Delta x_1 = x_1^m$. In fact, price discrimination yields a Pareto improvement because it benefits consumers in the weak market (market 1), benefits the multimarket seller, but does not harm either consumers or the rival in market 2. Q.E.D.

Social and private incentives to price discriminate

Our results allow us to understand under what conditions a multimarket seller would choose to engage in price discrimination or engage in uniform pricing and also the welfare implications of such behaviour. If the duopolistic market is strong, the multimarket seller will choose to price discriminate (proposition 1) and, under linear demands, price discrimination can be welfare improving (proposition 2). When the duopolistic market is weak, price discrimination increases the profit of the multimarket seller in one market but this can be reduced in the other market. Therefore, the choice of pricing policy is not clear, but it is possible, for example in the linear case, to find situations in which the multimarket seller chooses not to price discriminate. From a social welfare point of view price discrimination would reduce welfare under linear demands (proposition 2). Therefore, private incentives to price discriminate may be compatible with social incentives. The next Corollary summarizes the results.

Corollary 3. Under linear demands and price competition, if the multimarket seller uses a uniform pricing policy, society will prefer this choice. If society benefits from price discrimination, the multimarket firm will make this choice.

These results contrast with those under monopoly: a monopolist (single or multiproduct) never benefits from uniform pricing and price discrimination reduces social welfare under linear demands. In fact, as the following remark states, private incentive to price discriminate is opposed to social incentive.

Remark 1. Under linear demand and (single or multiproduct) monopoly, the welfare loss due to price discrimination is equal to a half of the profit gain: $\Delta W^m = -\frac{1}{2}\Delta\pi^m$.[24]

When the multimarket firm only serves the weak market under price discrimination, then this pricing policy increases both the multimarket seller's profit and social welfare. As several authors have emphasized for the case of monopoly, the banning of price discrimination is particularly harmful if it leads to the closure of markets. But it should also be noted that, under uniform pricing, a multimarket seller is more likely to abandon the weak market when it is a duopolist in this market instead of a monopolist.

On the "meeting competition defence"

The linear demand analysis serves to illustrate some perverse effects arising from the Robinson-Patman Act.[25] Assume that the multimarket seller engages in price discrimination, and imagine that the Federal Trade Commission initiates a case against this firm under section 2 of the Robinson-Patman Act (which says that it is unlawful "to discriminate in price between different purchases of commodities of like grade and quality"). The Act permits the multimarket seller to rebut the presumption of illegality by showing that its discriminatory price was quoted "in good faith to meet an equally low price of a competitor". As the following proposition states, for the case of linear demands, this defence may allow price discrimination to occur in situations in which it would reduce welfare.

[24] Varian's (1985) result that the welfare loss is equal to the profit gain in the linear demand case is incorrect as Layson (1988) shows. In the Appendix, we provide an alternative proof based on Lemma 3.
[25] See Schwartz (1986) for a review of perverse effects of the Robinson-Patman Act.

Proposition 6. (i) If the duopolistic market is weak, the “meeting competition” defence (if it were successful) allows price discrimination precisely when it reduces welfare. (ii) If the duopolistic market is strong, though price discrimination can be welfare improving, the “meeting competition” defence cannot be invoked.

This defence could be used successfully (in an economic sense) if the duopolistic market were weak but not if it were strong. However, if linearity of demand is not a bad approximation, we might expect the impact of price discrimination on welfare to be negative when the duopolistic market is weak (see propositions 3 and 4). Therefore, the banning of price discrimination would imply a welfare improvement. When the duopolistic market is strong the “good faith meeting” defence is unsuccessful but to allow price discrimination, precisely in this case, can increase social welfare. Note that the above conclusions depend on both markets being served under uniform pricing (see proposition 5).

5. Concluding remarks

With respect to the welfare analysis of third-degree price discrimination, this paper offers some contributions. The existence of a competitor in one market makes price discrimination by a multimarket seller welfare improving in settings, linear demands, where price discrimination would reduce welfare if the multimarket seller were a monopolist in both markets. The banning of price discrimination is particularly harmful when it leads to some markets not being served by the multimarket seller: not only may it lead to the closure of markets but also to market monopolization by the rival. The paper also illustrates some perverse effects arising from the Robinson-Patman Act. We show that the “meeting competition” defence may be used successfully (in an economic sense) precisely in cases where price discrimination would reduce welfare.

Results do not depend on the type of competition in the duopolistic market. We have shown that results under quantity competition are similar the qualification being that strategic variables become strategic substitutes. This fact makes the strategic effects of price discrimination, in comparison with uniform pricing, different from those under price competition. However, it is possible to reproduce similar welfare effects, in particular for the linear demands case.

It has been argued that price discrimination encourages entry because a firm which is established in one market can enter a new market which may need to set a low price without having to lower the price in its home market.[26] However, it has also been argued that price discrimination may deter entry: a multimarket incumbent can credibly threaten to respond more aggressively to a single-market entrant when the incumbent can make targeted price cuts directed against the entrant (see Katz, 1987). The findings of this paper allow us to show that both arguments are compatible. Note that to argue that price discrimination encourages entry is the same as saying that price discrimination may serve to open markets, and therefore, it may lead to a welfare improvement, even in the Pareto sense (proposition 5). Throughout the paper we have assumed that the rival firm is a well established firm, but the model also allows us to reach conclusions when this firm is a potential entrant and it has to decide whether to enter or not. If the duopolistic market is weak, the multimarket seller's behaviour is more aggressive under price discrimination than under uniform pricing (see proposition 1), and the former policy might deter entry by reducing the entrant's profits.[27] However, it must be pointed out that when the duopolistic market is strong, the results are reversed: price discrimination facilitates entry.[28]

[26] The U.S. Department of Justice (1977) (see Katz, 1987) and Hausman and MacKie-Mason (1988), for example, present variants of this argument.
[27] The effects on entry of for example banning price discrimination would be rather similar to those arising when an incumbent firm follows a most-favoured-customer pricing policy. See Aguirre (2000).
[28] Of course, the consequences of entry deterrence on social welfare are ambiguous since the size of the entry cost might make entry socially undesirable.

APPENDIX

Proof of Lemma 2

(i) $\min\left\{p_A^*, p_1^m\right\} \le p^u \le \max\left\{p_A^*, p_1^m\right\}$

(ii) If $p_A^* \ge p^u$ then $p_B^* \ge p_B^u$ and if $p_A^* \le p^u$ then $p_B^* \le p_B^u$ (and vice versa).

Part (ii) of Lemma 2 is obvious given that the reaction functions are upward sloping. If $p_A^* \ge p^u$ then $p_B^* = R_B(p_A^*) \ge R_B(p^u) = p_B^u$ and if $p_A^* \le p^u$ then $p_B^* = R_B(p_A^*) \le R_B(p^u) = p_B^u$. If $p_B^* \ge p_B^u$ then $p_A^* = R_A(p_B^*) \ge R_A^u(p_B^u) = p^u$ given that $p_B^* \ge p_B^u$ implies $R_B\left(R_A(p_B^*)\right) \ge R_B\left(R_A^u(p_B^u)\right)$. We next demonstrate part (i) by showing that

(a) if $p_B^* \ge p_B^u$ then $\min\left\{p_A^*, p_1^m\right\} \le p^u \le \max\left\{p_A^*, p_1^m\right\}$ and

(b) if $p_B^* \le p_B^u$ then $\min\left\{p_A^*, p_1^m\right\} \le p^u \le \max\left\{p_A^*, p_1^m\right\}$.

(a) If $p_B^* \ge p_B^u$ then $p_A^* = R_A(p_B^*) \ge R_A^u(p_B^u) = p^u$. We know from Lemma 1 that the concavity of the profit functions implies that $\min\left\{R_A(p_B), p_1^m\right\} \le R_A^u(p_B) \le \max\left\{R_A(p_B), p_1^m\right\}$. So given the rival's prices p_B^* and p_B^u we have that $\min\left\{R_A(p_B^*), p_1^m\right\} \le R_A^u(p_B^*) \le \max\left\{R_A(p_B^*), p_1^m\right\}$ and $\min\left\{R_A(p_B^u), p_1^m\right\} \le R_A^u(p_B^u) \le \max\left\{R_A(p_B^u), p_1^m\right\}$. As $p_B^* \ge p_B^u$ and the reaction function is upward sloping then $\max\left\{R_A(p_B^u), p_1^m\right\} \le \left\{R_A(p_B^*), p_1^m\right\}$ and therefore $\min\left\{R_A(p_B^u), p_1^m\right\} \le R_A^u(p_B^u) = p^u \le \max\left\{R_A(p_B^u), p_1^m\right\} \le \max\left\{R_A(p_B^*), p_1^m\right\}$ which proves that $p^u \le \max\left\{p_A^*, p_1^m\right\}$. We next show that $\min\left\{R_A(p_B^u), p_1^m\right\} = p_1^m$; if this occur then $\min\left\{R_A(p_B^*), p_1^m\right\} = p_1^m$ and this implies $\min\left\{p_A^*, p_1^m\right\} \le p^u$. Suppose, on the contrary, that $\min\left\{R_A(p_B^u), p_1^m\right\} = R_A(p_B^u)$; then, as $\min\left\{R_A(p_B^u), p_1^m\right\} \le \min\left\{R_A(p_B^*), p_1^m\right\}$ there would be two possibilities: (1) $\min\left\{R_A(p_B^*), p_1^m\right\} = R_A(p_B^*)$ and (2) $\min\left\{R_A(p_B^*), p_1^m\right\} = p_1^m$. Case (1) would imply that $R_A(p_B^u) \le R_A^u(p_B^u) = p^u \le R_A(p_B^*) \le R_A^u(p_B^*) \le p_1^m$ and case (2) would lead to $R_A(p_B^u) \le R_A^u(p_B^u) = p^u \le p_1^m \le R_A^u(p_B^*) \le R_A(p_B^*)$. But these cases cannot occur (except for the trivial case $p_1^m = p^u = p_A^*$ and $p_B^* = p_B^u$. The best response functions $R_A(p_B)$ and $R_A^u(p_B)$ are upward sloping (with slopes less than one) and as can be easily proved $d[R_A(p_B) - R_A^u(p_B)]/dp_B > 0$, thus implying that $R_A(p_B)$ and $R_A^u(p_B)$ cross once at most.

(b) If $p_B^* \le p_B^u$ then $p_A^* = R_A(p_B^*) \le R_A^u(p_B^u) = p^u$. From Lemma 1, the concavity of the profit functions implies that $\min\left\{R_A(p_B), p_1^m\right\} \le R_A^u(p_B) \le \max\left\{R_A(p_B), p_1^m\right\}$. So given the rival's prices p_B^* and p_B^u we have that $\min\left\{R_A(p_B^*), p_1^m\right\} \le R_A^u(p_B^*) \le \max\left\{R_A(p_B^*), p_1^m\right\}$ and $\min\left\{R_A(p_B^u), p_1^m\right\} \le R_A^u(p_B^u) \le \max\left\{R_A(p_B^u), p_1^m\right\}$. As $p_B^* \le p_B^u$ and the best response function is upward sloping then $\max\left\{R_A(p_B^u), p_1^m\right\} \ge \left\{R_A(p_B^*), p_1^m\right\}$ and therefore $\min\left\{R_A(p_B^*), p_1^m\right\} \le \min\left\{R_A(p_B^u), p_1^m\right\} \le R_A^u(p_B^u) = p^u \le \max\left\{R_A(p_B^u), p_1^m\right\}$ which proves that $p^u \ge \min\left\{p_A^*, p_1^m\right\}$. We next show that $\max\left\{R_A(p_B^u), p_1^m\right\} = p_1^m$; if this occur then $\max\left\{R_A(p_B^*), p_1^m\right\} = p_1^m$ and this implies $\max\left\{p_A^*, p_1^m\right\} \ge p^u$. Suppose, on the contrary, that $\max\left\{R_A(p_B^u), p_1^m\right\} = R_A(p_B^u)$; then, as $\max\left\{R_A(p_B^u), p_1^m\right\} \ge \max\left\{R_A(p_B^*), p_1^m\right\}$ there would be two possibilities: (1)' $\max\left\{R_A(p_B^*), p_1^m\right\} = R_A(p_B^*)$ and (2)' $\max\left\{R_A(p_B^*), p_1^m\right\} = p_1^m$. Case (1)' would imply that $p_1^m \le R_A^u(p_B^*) \le R_A(p_B^*) \le R_A^u(p_B^u) \le R_A(p_B^u)$ and case (2) would lead to $R_A(p_B^*) \le R_A^u(p_B^*) \le p_1^m \le R_A^u(p_B^u) = p^u \le R_A(p_B^u)$. Once again these conditions cannot be satisfied by the equilibrium prices. Q.E.D.

Proof of Lemma 3

The aggregate utility function is given by:

$$u(x_1, x_A, x_B) + y = u_1(x_1) + u_2(x_A, x_B) + y$$

$$= \alpha_1 x_1 - \frac{1}{2}\beta_1 x_1^2 + \alpha(x_A + x_B) - \frac{1}{2}(\beta x_A^2 + 2\gamma x_A x_B + \beta x_B^2) + y$$

It is easy to check that the consumer surplus in market 1 and 2 is, respectively:

$$S_1(x_1) = u_1(x_1) - p_1(x_1)x_1 = \frac{1}{2}\beta_1 x_1^2,$$

$$S_2(x_A, x_B) = u_2(x_A, x_B) - p_A(x_A, x_B)x_A - p_B(x_A, x_B)x_B = \frac{1}{2}\beta x_A^2 + \gamma x_A x_B + \frac{1}{2}\beta x_B^2.$$

Consider two configurations of output, (x_1^0, x_A^0, x_B^0) and (x_1^1, x_A^1, x_B^1), with associated prices (p_1^0, p_A^0, p_B^0) and (p_1^1, p_A^1, p_B^1). The changes of the consumer surplus in market 1 and 2, which are due to a move from (x_1^0, x_A^0, x_B^0) to (x_1^1, x_A^1, x_B^1), are, respectively:

$$\Delta S_1 = \frac{1}{2}\beta_1(x_1^1 + x_1^0)\Delta x_1,$$

$$\Delta S_2 = \frac{1}{2}\beta(x_A^1 + x_A^0)\Delta x_A + \frac{1}{2}\beta(x_B^1 + x_B^0)\Delta x_B + \gamma(x_A^1\Delta x_B + x_B^1\Delta x_A),$$

By using inverse demands and rearranging, we get:

$$\Delta S_1 = \frac{1}{2}\alpha_1\Delta x_1 - \frac{1}{2}p_1^0\Delta x_1 + \frac{1}{2}\beta_1 x_1^1\Delta x_1,$$

$$\Delta S_2 = \frac{1}{2}\alpha(\Delta x_A + \Delta x_B) - \frac{1}{2}p_A^0\Delta x_A - \frac{1}{2}p_B^0\Delta x_B$$
$$+\frac{1}{2}\beta x_A^1\Delta x_A + \frac{1}{2}\beta x_B^1\Delta x_B + \gamma x_A^1\Delta x_B - \frac{1}{2}\gamma x_B^0\Delta x_B + \frac{1}{2}\gamma x_B^0\Delta x_A.$$

The changes of profits in market 1 and 2 are, respectively:

$$\Delta\pi_1 = (p_1^1 - c)x_1^1 - (p_1^0 - c)x_1^0$$

$$\Delta\pi_2 = \Delta\pi_A + \Delta\pi_B = (p_A^1 - c)x_A^1 - (p_A^0 - c)x_A^0 + (p_B^1 - c)x_B^1 - (p_B^0 - c)x_B^0$$

By adding and subtracting $(p_1^0 - c)x_1^1$ to the first expression and $(p_A^0 - c)x_A^1$ and $(p_B^0 - c)x_B^1$ to the second and rearranging, we get:

$$\Delta\pi_1 = (p_1^0 - c)\Delta x_1 + \Delta p_1 x_1^1$$

$$\Delta\pi_2 = (p_A^0 - c)\Delta x_A + (p_B^0 - c)\Delta x_B + \Delta p_A x_A^1 + \Delta p_B x_B^1$$

Given that $\Delta p_1 = -\beta_1\Delta x_1$ $\Delta p_A = -(\beta\Delta x_A + \gamma\Delta x_B)$ and $\Delta p_B = -(\beta\Delta x_B + \gamma\Delta x_A)$, we obtain:

$$\Delta\pi_1 = (p_1^0 - c)\Delta x_1 - \beta_1 x_1^1\Delta x_1 \qquad \text{(A1)}$$

$$\Delta\pi_2 = (p_A^0 - c)\Delta x_A + (p_B^0 - c)\Delta x_B - \beta x_A^1\Delta x_A - \gamma x_A^1\Delta x_B - \beta x_B^1\Delta x_B - \gamma x_B^1\Delta x_A. \qquad \text{(A2)}$$

The changes in social welfare in market 1 and 2 are, respectively:

$$\Delta W_1 = \Delta S_1 + \Delta\pi_1 = \frac{1}{2}(p_1^0 - c)\Delta x_1 + \frac{1}{2}(\alpha_1 - c - \beta_1 x_1^1)\Delta x_1 \qquad \text{(A3)}$$

$$\begin{aligned}\Delta W_2 = \Delta S_2 + \Delta\pi_2 &= \frac{1}{2}(p_A^0 - c)\Delta x_A + \frac{1}{2}(p_B^0 - c)\Delta x_B \\ &+ \frac{1}{2}(\alpha - c - \beta x_A^1 - \gamma x_B^1)\Delta x_A + \frac{1}{2}(\alpha - c - \beta x_B^1 - \gamma x_A^1)\Delta x_B\end{aligned} \qquad \text{(A4)}$$

Given that $p_1^1 = \alpha_1 - \beta_1 x_1^1$, $p_A^1 = \alpha - \beta x_A^1 - \gamma x_B^1$ and $p_B^1 = \alpha - \beta x_B^1 - \gamma x_A^1$, we obtain:

$$\Delta W_1 = \frac{1}{2}(p_1^0 - c)\Delta x_1 + \frac{1}{2}(p_1^1 - c)\Delta x_1$$

$$\Delta W_2 = \frac{1}{2}(p_A^0 - c)\Delta x_A + \frac{1}{2}(p_B^0 - c)\Delta x_B + \frac{1}{2}(p_A^1 - c)\Delta x_A + \frac{1}{2}(p_B^1 - c)\Delta x_B$$

Therefore, the total change of social welfare is:

$$\Delta W = \Delta W_1 + \Delta W_2 = \frac{1}{2}UB + \frac{1}{2}LB.$$ Q.E.D.

Proof of Remark 1. Welfare effects of price discrimination under monopoly

Denote by p_1^m, p_A^m and p_B^m the monopoly prices under price discrimination and let p^s and p_B^s be the monopoly prices under uniform pricing (that is, when the monopolist is constrained to sell product A at the same price, p^s, in market 1 and 2). Let (x_1^m, x_A^m, x_B^m) and (x_1^s, x_A^s, x_B^s) be the associated outputs. It is easy to show that under linear demands price discrimination reduces welfare because the total output does not change; $\Delta x_1 + \Delta x_A = 0$ and $\Delta x_B = 0$. Therefore, the upper bound on the welfare change is zero and the lower bound is negative; as a consequence, given the above results, $\Delta W^m = (1/2)LB < 0$. It is useful to rewrite the welfare change in terms of the change of the monopoly profits. The welfare change can be expressed, from (A3) and (A4), as:

$$\Delta W^m = \Delta W_1^m + \Delta W_2^m = \frac{1}{2}(\alpha_1 - c - \beta_1 x_1^m)\Delta x_1 + \frac{1}{2}(\alpha - c - \beta x_A^m - \gamma x_B^m)\Delta x_A.$$

If outputs (x_1^m, x_A^m, x_B^m) maximize the industry profits then

$$\Delta W^m = \frac{1}{2}\beta_1 x_1^m \Delta x_1 + \frac{1}{2}(\beta x_A^m + \gamma x_B^m)\Delta x_A.$$

Given (A1) and (A2), the change of the monopoly profit is: $\Delta\pi^m = \Delta\pi_1^m + \Delta\pi_2^m = -\beta_1 x_1^m \Delta x_1 - (\beta x_A^m + \gamma x_B^m)\Delta x_A$. Thus $\Delta W^m = -(1/2)\Delta\pi^m$. It is straightforward to obtain the same result under single-product monopoly. Q.E.D.

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