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Estimating the Volatility of Flights and Risk of Saturation of Airspaces in the European Core Area: A Methodological Proposal

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Abstract: Despite having some fluctuations and the impact of the COVID-19 crisis, the demand for flights had a general growing trend for the past years. As the airspace is limited, efforts to better manage the total number of flights are noteworthy. In addition, volatility (i.e., unpredicted changes) in the number of flights has been observed to be increasing. Efforts to improve flight forecasting are thus necessary to improve air traffic efficiency and reduce costs. In this study, volatility in the number of flights is estimated based on past trends, and the outcomes are used to project future levels. This enables risk situations such as having to manage unexpectedly high numbers of flights to be predicted. The methodological approach analyses the Functional Airspace Block of Central Europe (FABEC). Based on the number of flights for 2015–2019, the following are calculated: historic mean, variance, volatility, 95th percentile, flights per hour and flights per day of the week in different time zones in six countries. Due to the nature of air traffic and the overdispersion observed, this study uses counting data models such as negative binomial regressions. This makes it possible to calculate risk measures including expected shortfall (ES) and value at risk (VaR), showing for each hour that the number of flights can exceed planned levels by a certain number. The study finds that in Germany and Belgium at 13:00 h there is a 5% worst-case possibility of having averages of 683 and 246 flights, respectively. The method proposed is useful for planning under uncertainties. It is conducive to efficient airspace management, so risk indicators help Air Navigation Service Providers (ANSPs) to plan for low-probability situations in which there may be large numbers of flights.

Keywords: air traffic management; number of flights; uncertainty; negative binomial regression; risk measures; risk of airspace saturation

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1. Introduction

Due to the growing number of flights, increasing delays and high-cost pressure on the whole aviation system, the provision of Air Navigation Services (ANS) has recently drawn increasing attention from both academics and policy decision-makers. A major challenge regarding ANS provision is "planning under uncertainties", e.g., as a result of volatile traffic demand in terms of movement numbers and flow patterns, which can significantly influence resource planning and allocation. Several factors could cause or increase volatility, e.g., weather, strikes, geopolitical factors, airline decisions and unexpected economic downturns [1].

Volatile traffic affects ANS planning on multiple time scales and operational levels [2]. Changes in traffic demand and flow patterns have a direct influence on pre-tactical and strategic capacity planning and on resulting delays, their associated costs and safety. Thus, traffic volatility and the associated airspace risk have become a daily concern for Europe's ANSPs and Functional Airspace Blocks (FABs), posing a complex challenge due to the size and extent of the problem. So much so that FABEC launched an interactive platform in 2018 to discuss this topic as part of an initiative addressing new developments in air traffic flow management. In the aftermath of the COVID-19 pandemic, the importance of addressing this issue has become even clearer: Although traffic in FABEC started to recover in 2021, the increase in demand was slow and accompanied by extreme volatility, as unplanned flights put pressure on airspace capacity and staff resources.

Changes in traffic demand are not a new phenomenon. But there are at least two reasons that explain why this issue is taken seriously and why a more in-depth analysis of its causes and consequences is being undertaken [3]. On the one hand, volatility has shifted from being an isolated phenomenon to affecting the entire aviation system. On the other hand, recent years have been characterised by increasingly wide variations in the volume of flights and routes (and a high level of volatility in the rate of recovery of traffic levels across Europe following the COVID-19 pandemic).

Figure 1 shows changes in air traffic relative to the base year of 1970 for different countries. Note that, in all cases, the tendency is for a steady increase as years go by. Surprisingly, in 2019, air traffic fell just prior to the general spread of the COVID-19 pandemic. COVID-19 affected the mobility of European air transport with a reduction of as much as 89% in the number of flights [4] and created an uncertain future for the aviation industry [5]. In any event, it must be noted that, in general, the air traffic pattern shows tremendous changes, mean reversion and jumps.

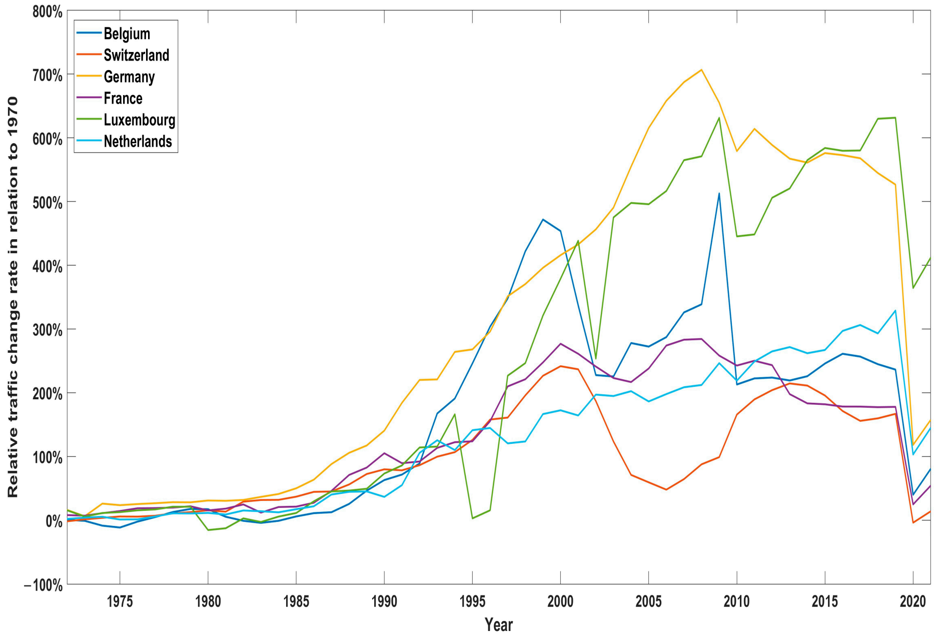

Figure 1. Rate of change in air traffic relative to 1970. Source: Prepared by authors using data from ref. [6].

ANSPs are constantly recruiting new personnel (controllers) and adapting capacity to demand to provide more flexibility. These solutions have proved effective in the short term [7], but as activity in the industry becomes even more unpredictable [8,9], it is increasingly clear that new ways of setting targets, assessing performance plans in terms of profitability and ultimately measuring the impact of volatility on ANSP operations need to be found. Providing flexible air traffic services therefore requires new thinking to minimise the impact of volatility on the travelling public, while at the same time providing the capacity to meet demand in the short- and long term. This means that efforts to understand what volatility is and propose ways to measure and estimate it are especially welcome in this topic.

The complexity and scope of the industry offer a number of research opportunities from a variety of perspectives. For example, studies on volatility have focused on applying a few metrics [10] or [11] realise a survey on Artificial Intelligence (AI) in order to maintain aviation safety. In the analysis of air traffic flows and delays, there is one part that can be predicted (deterministic) and another that cannot (stochastic). It is common to focus only on the deterministic part. However, when it comes to the aviation sector, non-predictable information (strikes, weather, etc.) is even more important than in other sectors. The study by [12] focuses on volatility (and on changes in it) by applying a stochastic modelling method to estimate future air traffic, delays and the cost of future delays in Germany to quantify risk and its significance for the delivery of cost-effective services.

In this study, we propose negative binomial regression models as a way of estimating volatility in the number of flights in the FABEC area, which compromises the airspace of France, Benelux, Germany and Switzerland and is regarded as the core area of Europe. By estimating the necessary parameters, we can obtain the volatility in the number of flights depending on the days of the week and the hours. That is, we propose a method for reducing and using the uncertainty associated with the number of flights per hour so as to contribute to better planning of ANS. We use hourly traffic data for 2015–2019.

As far as we know, this is a novel methodology that has never before been applied for estimating volatility in the aviation sector. By estimating volatility parameters, we are able to draw up simulations that reveal the full distribution of the number of flights for all FABEC countries. The distributions are very useful in understanding the likelihood and risk of the number of flights exceeding a given threshold number.

The paper is structured as follows: Section 2 reviews the literature and presents the approach. Section 3 provides the data used for the analysis. Section 4 sets out the counting data model, which is a negative binomial regression model. Results and conclusions are given in Sections 5 and 6, respectively.

2. State of the Art

Air Traffic Management (ATM) needs to be improved in the wake of significant growth and variations in traffic [13]. A new regulatory framework could enhance safety, cost and flight efficiency; an elastic economic regulatory system could also enable capacity and react faster to changes in demand. Such a new ATM system would enhance the Green Deal measures [14].

Single European Sky ATM Research (SESAR) was created in 2008 due to increasing air traffic and rational delays since 2000. Its fourth pillar establishes the management of air transport capacity. In 2018, the Airspace Architecture Study (AAS) [15] identified possible solutions for capacity and demand imbalance such as arrival management and improved aviation meteorology. In [16], two main challenges in the Main Plan (MP) are foreseen: environmental concerns and a mismatch between traffic demand and ATM capacity.

Delays can be due to various reasons including weather conditions [17], ground delays, runway queues and capacity constraints [18], and delays are a major source of direct and opportunity costs [19]. A review of different approaches to flight delay prediction and how this problem is addressed is presented in [20]. They compare the prediction models used, such as operational research [21], machine learning [22], Bayesian network approach [23], probabilistic models, statistical analysis, a super statistical approach [24] and ensemble methods and select representative algorithms [25]. A novel predictive model applying graphs to sequence learning architecture is studied in [26]. Authors in [27] affirm that comparing flight schedules and flight plans is a very useful way of locating flight delay occurrences and modelling flight delays.

Air traffic network efficiency depends on strikes [28] and arrival processes [29] among other factors. The Arrival Manager (AMAN) seeks to improve the flow when capacity constraints exist, so the system needs reliable assessment and estimations of delay and capacity. All this translates into additional miles flown due to cancellations, delays or rerouting of scheduled flights, increasing horizontal flight distance and thus affecting fuel consumption, environmental factors and costs to customers and airlines [30]. An analysis of the insights on the estimated climate costs of the aviation sector due to air management for 2018 and 2019 is presented in [31] and found them to be as high as 1 bn EUR. Other authors study the expected costs for airspace users as a result of Air Traffic Management (ATFM) regulations [32].

ANSPs, airlines, planners and regulators manage imbalances between short- and long-term demand and air capacity in different FABs. Recently, the Eurocontrol Network Operations Plan (NOP) realised that traffic flow predictions are not as accurate as they used to be because of sharp peaks in demand, which make it difficult to apply enough capacity. This situation is exacerbated in core areas of Europe such as FABEC, where 60% of airlines are flying longer and more expensively [2]. As a result, a new term has become very familiar in ATM: volatility. This refers to unexpected changes in the number of flights. It seems to be a useful indicator that can further understanding of the balance between demand and capacity, i.e., traffic variations in time and space. Volatility depends on seasonality, weather forecasts, the closing of airspace due to geopolitical decisions, strikes, airline decisions, unexpected Air Traffic Charges (ATC), service charges and economic cycles. A fuzzy cognitive map of 39 concepts to analyse the links between them and estimate the causes and effects of volatility is drawn in [1]. Guerra et al. conducted a literature review of volatility in air transport, identified factors that influence it and suggested strategies for addressing it [33]. However, they do not mention volatility as a measure of flight fluctuation. The reference [34] shows that the path and cycle approach is a reasonable way of modelling this hotspot problem.

Volatility seems to be an emerging topic in ATM and one that affects Air Space Management (ASM), planning, air security, environmental issues and airport management. The literature mentions various topics directly and indirectly affected by flight volatility such as the foregoing, but as far as we are aware, there is no clear definition of the concept of "volatility in a number of flights" and therefore, no clearly identified model for predicting such volatility.

3. Materials: The Data

In the current study, volatility means a fluctuation in output, not in resources. In terms of ANSPs, output includes metrics, e.g., flight hours, flights, movements at airports and flight distances. So in this case, we are particularly interested in variations in the number of flights.

Eurocontrol offers a number of public and semi-public data sources, e.g., the ATM Cost-Effectiveness (ACE) data [35] and Performance Review Unit (PRU) data [36]. However, each of these datasets covers different temporal and operational levels. They also differ in regard to the years available. The key criterion for this study is the granularity of the data. Data on a daily basis is not sufficient to address short-term volatility, so the PRU database cannot be used. However, there is no publicly available data source for hourly flights. As an alternative, however, ANSPs have access to trajectory data, which can be analysed, for example, via the Network Strategic Tool (NEST) [37]. NEST provides both the actual and planned numbers of flights.

The dataset used for short-term volatility was obtained by using a NEST assessment [38]. Hourly data are available for 2015–2019. Overall, the dataset contains 89 units. However, airspaces often contain overlapping areas. For example, the airspace ED contains all areas connoted to Germany, including parts of Maastricht UAC. However, flights are also available for EDCC (German Area Control Centres (ACCs)) and EDYYCC (Maastricht ACC in German airspace). Finally, there are units with a "CTA" suffix, so the _CC and

_CTA airspaces may be subsets, but this is not the case for all observations. As a result, the database was split up in advance to avoid double counting. The differences between LP and LPCC airspace are illustrated in Figure 2.

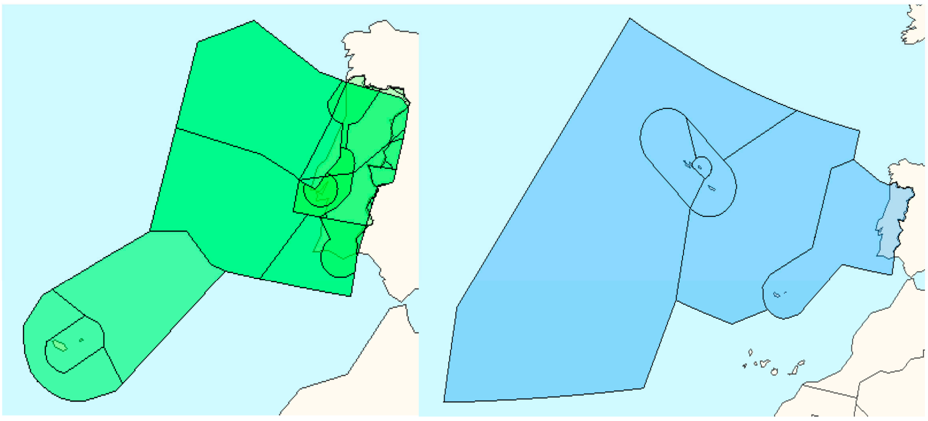

Figure 2. Portuguese airspace according to NEST—LP (**left**) and LPCC (**right**).

The times indicated are in UTC/GMT and refer to the time stamp when a flight enters the unit.

Note that the accuracy of these data is limited in 3 dimensions: time (1 min), vertical distance (400 ft) (or 1000 ft in the climb/descent-phase) and lateral distance (10 NM).

A preliminary analysis of the actual data presented in Tables 1–3 shows a clear overdispersion in many hour ranges. That is, the variance is well above the mean in all cases. Data for Germany are used to illustrate the analysis, but the comments and findings can be generalised to all FABEC countries, as shown below.

Table 1. Historic mean, variance and risk for Germany (EDCC) depending on the hour.

Time	Values				
	Mean	Volatility	Variance	95th Percentile	Variance/Mean
1	40.0	17.2	295.9	72.0	7.4
2	43.4	18.3	336.5	74.0	7.8
3	78.0	39.9	1593.0	143.0	20.4
4	127.0	35.8	1282.9	179.0	10.1
5	286.5	146.6	21,479.1	451.0	75.0
6	430.8	144.0	20,745.5	600.0	48.2
7	488.6	70.8	5010.8	587.0	10.3
8	509.1	92.2	8504.6	630.0	16.7
9	574.1	101.0	10,195.5	715.2	17.8
10	604.0	66.7	4452.9	695.0	7.4
11	588.2	64.1	4109.0	681.0	7.0
12	562.6	76.3	5814.4	662.0	10.3
13	554.1	66.7	4443.6	650.0	8.0
14	558.4	80.0	6405.5	670.0	11.5
15	550.6	59.1	3488.9	639.0	6.3
16	543.0	100.4	10,081.1	677.0	18.6
17	556.9	77.4	5991.8	669.0	10.8
18	523.2	66.4	4402.6	606.0	8.4
19	475.4	58.9	3469.4	547.0	7.3
20	469.2	82.5	6808.9	577.2	14.5
21	416.1	62.3	3879.9	504.0	9.3
22	228.0	55.2	3043.6	322.0	13.3
23	129.8	28.2	794.3	177.0	6.1
24	69.6	21.0	439.7	107.0	6.3

Table 2. Mean number of flights in Germany depending on day of the week.

Time	Weekday						
	Mon	Tue	Wed	Thur	Fri	Sat	Sun
1	50.9	37.4	39.1	38.5	40.4	34.3	39.2
2	41.7	48.1	51.0	49.1	51.7	31.4	30.5
3	58.6	93.8	92.2	93.9	96.4	58.5	52.4
4	108.4	137.7	137.2	138.6	143.1	117.3	106.3
5	284.3	295.6	296.9	299.1	299.2	280.8	248.1
6	452.3	443.2	442.0	448.2	450.2	419.5	358.8
7	519.1	507.3	511.3	510.1	510.5	464.9	395.0
8	531.3	518.8	521.7	529.9	536.3	489.6	433.9
9	599.7	569.7	587.1	593.0	608.9	558.7	499.1
10	626.6	595.8	608.9	612.1	627.2	600.9	554.2
11	598.7	562.9	585.6	595.2	619.2	586.6	567.3
12	567.6	531.5	559.9	565.3	593.3	554.0	564.6
13	552.6	528.6	547.9	555.7	582.1	544.4	565.3
14	557.9	537.7	562.5	563.6	592.3	525.9	567.1
15	549.0	535.6	556.9	556.8	589.4	506.7	557.3
16	549.7	532.6	557.6	556.8	578.0	471.9	552.1
17	561.4	558.0	574.0	580.4	580.4	469.1	572.8
18	534.5	520.5	542.3	545.4	548.9	426.4	542.4
19	486.1	469.7	493.9	490.0	503.3	379.9	503.2
20	481.4	475.2	484.6	494.5	491.6	365.1	490.2
21	425.0	411.2	428.0	433.6	438.4	337.1	437.9
22	230.9	222.6	232.6	241.8	249.4	188.1	230.0
23	131.8	130.7	134.9	137.1	136.9	118.1	119.0
24	66.3	66.4	68.6	71.2	69.4	71.1	73.8

Table 3. Historic mean, variance and risk for FABEC countries between 9 and 12 h.

Country	Time	Values				
		Mean	Volatility	Variance	95th Percentile	Variance/Mean
Germany	9	574.1	101.0	10,195.5	715.2	17.8
	10	604.0	66.7	4452.9	695.0	7.4
	11	588.2	64.1	4109.0	681.0	7.0
	12	562.6	76.3	5814.4	662.0	10.3
Belgium	9	196.1	32.0	1026.5	240.0	5.2
	10	198.4	18.7	350.7	225.2	1.8
	11	191.8	25.3	640.7	227.0	3.3
	12	197.4	30.3	917.1	241.0	4.6
France	9	560.7	111.4	12,417.2	714.0	22.1
	10	576.8	90.0	8108.2	706.0	14.1
	11	579.5	92.7	8593.2	715.0	14.8
	12	585.5	111.3	12,377.3	739.2	21.1
Netherlands	9	206.5	32.7	1071.2	252.0	5.2
	10	216.3	21.5	464.3	246.0	2.1
	11	224.9	33.1	1098.4	270.2	4.9
	12	217.4	22.8	518.5	250.0	2.4
Switzerland	9	197.6	41.6	1729.5	258.0	8.8
	10	216.4	37.5	1404.5	272.0	6.5
	11	212.6	29.8	889.4	258.0	4.2
	12	208.8	35.0	1223.1	259.0	5.9
Luxembourg	9	12.2	4.5	20.7	20.0	1.7
	10	12.2	4.6	20.7	20.0	1.7
	11	11.6	4.4	19.7	19.0	1.7
	12	13.9	5.1	26.2	22.0	1.9

Note that the variance depends on the time of the flight (from 1–24 h), with 10 h being the time when traffic is heaviest. However, a look at the 95th percentile shows that the highest risk occurs at 9 h, given that there are more than 715 flights in 5% of cases.

Table 1 also shows that there are big differences between volatility and the variance/mean ratio depending on the hour. Note that neither variance nor volatility values take into consideration whether the values on the database are high or low, but the ratio does take this into consideration. This is why the use of the ratio is recommended. However, from the point of view of airspace saturation, more attention must be paid to volatility during peak traffic hours, i.e., when the 95th percentile shows high values, because in such situations the risk of saturation is much higher.

Table 2 presents the mean air traffic figures per hour and per day of the week in Germany. It illustrates that time and day of the week patterns may exist, such as substantial increases in flights between 6 h and 19:00 h. Big differences may arise between different days of the week.

Table 3 shows some data indicators for all the countries in the FABEC area for times from 9:00 to 12:00. These are the hours with the greatest air traffic for all these countries except Belgium, where the peak is at 16 h. A clear overdispersion can be noted.

4. Methods: Modelling Efforts and Calibration

When there is a need to model a variable such as air traffic, which takes integer or zero values, the use of counting data models seems appropriate [39–41]. These are specific models for situations in which the dependent variable is either integer positive or zero, i.e., it cannot, by nature, have a negative sign. In such cases, variance is a function of the expected value. In this specific case, the expected value depends on the time, day of the week, seasonality and trend.

The most widely used counting regression models are the Poisson and the negative binomial. Using these models, highly accurate forecasts can be made of the expected number of events (number of flights in this case) and of volatility. In this case, volatility is caused by the uncertainty in the number of flights in each hour. For instance, some authors [42] have used the lineal Poison autoregressive (PAR) model as an alternative to the Poisson model to analyse the impacts of laws and climate on annual road traffic accidents. This alternative method may be applied in future work for comparative purposes.

Note that in the Poisson model, the variance is equal to the expected value, which has its limitations. However, the negative binomial model is applied in situations in which there is overdispersion, i.e., when the variance is greater than the mean. Therefore, an initial analysis of the data is required before it can be decided which of the two models fits better for the purpose of this paper, although there are more precise statistics for making this decision. The preliminary data analysis in Section 3 suggests that a negative binomial model is more adequate for analysing air traffic.

This model is an extension of Poisson regression for cases in which the variance is greater than the mean value. As stated, this is the case of the data used here.

In a counting regression model such as the negative binomial, the expected value is as shown in Equation (1).

$$E(Y|X) = \exp(\beta_1 + \beta_2 t + AC(t) + WC(t) + DC(t)) = \mu \tag{1}$$

where Y stands for the number of flights and X represents the independent variables. The calculation of expected value includes:

(a) A constant β_1.
(b) A trend $\beta_2 t$, where t is the time in years.
(c) A yearly cycle $AC(t)$ with its seasonal components: annual, semi-annual, quarterly, etc.
(d) A weekly cycle $WC(t)$ depending on the days of the week. These are dummy variables.
(e) A daily cycle $DC(t)$ based on the hours with their seasonal components in a similar way to the yearly cycle.

The yearly cycle is modelled as in Equation (2).

$$AC(t) = \sum_{j=1}^{j=5} [\beta_{1+2j} \sin(2j\pi t) + \beta_{2+2j} \cos(2j\pi t)] \tag{2}$$

The weekly cycle is modelled as in Equation (3) using six dummy variables.

$$WC(t) = \beta_{13}D_1(t) + \beta_{14}D_2(t) + \beta_{15}D_3(t) + \beta_{16}D_4(t) + \beta_{17}D_5(t) + \beta_{18}D_6(t) \tag{3}$$

where the dummy variable $D_1(t) = 1$ if the day is Monday and $D_1(t) = 0$ for other days of the week. When all dummy variables are zero, it is Sunday.

The hourly cycle is modelled as in Equation (4).

$$DC(t) = \sum_{j=1}^{j=5} [\beta_{17+2j} \sin(2j\pi\tau)/24 + \beta_{18+2j} \cos(2j\pi\tau)/24] \tag{4}$$

where the variable τ indicates the hour and takes a value between 1 and 24. There are 28 parameters for calculating the expected value in the overall model, depending on trend, annual cycle, weekday and time. Table 4 presents the parameter values calculated for Germany, where the betas correspond to the constant, trend, yearly cycle parameters, weekly parameters and daily cycle parameters as specified in Equation (1) to Equation (4), also the alpha value is calculated.

In this regression model, the variance is calculated using Equation (5).

$$\mathrm{Var}(Y \mid X) = (1 + \alpha \times \mu) \times \mu \tag{5}$$

where α is a value calculated by the regression. Figure 3 shows the variance values as functions of α and μ.

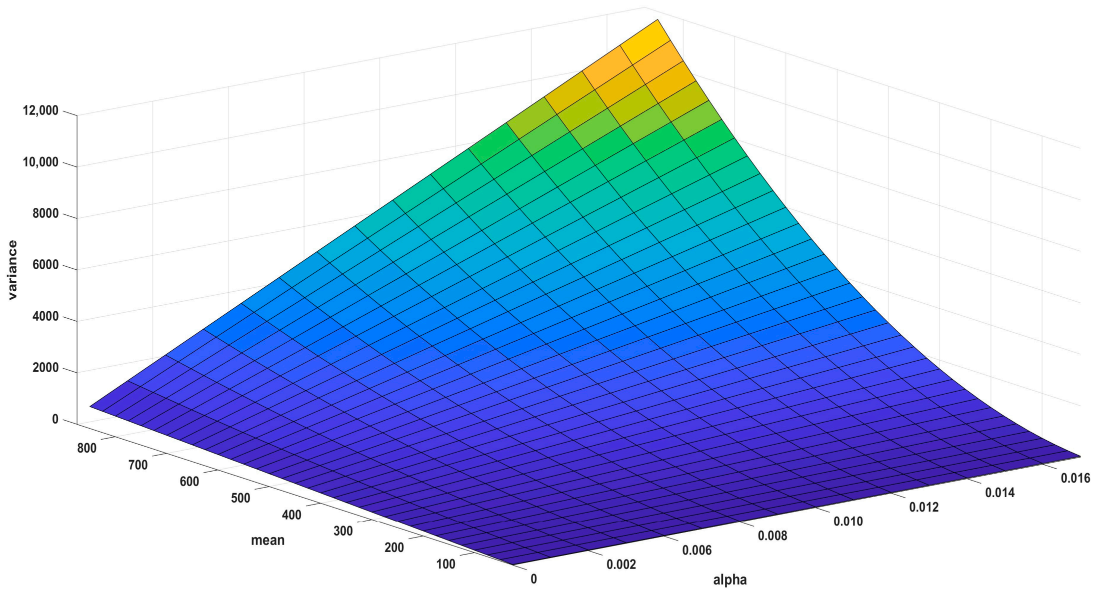

Figure 3. The variance values as functions of α and μ.

Table 4. Negative binomial parameters for Germany.

Dependent Variable: Number of Flights						
Beta		Coefficient	Standard Deviation	z Value	*p* Value	Significance
β_1	Constant	5.563280	0.003409	1632	<0.0001	***
β_2	Trend	0.030876	0.000680	45.43	<0.0001	***
β_3	Yearly cycle	−0.0628108	0.001338	−46.93	<0.0001	***
β_4	Yearly cycle	−0.185169	0.001478	−125.3	<0.0001	***
β_5	Yearly cycle	0.004903	0.001338	3.664	0.0002	***
β_6	Yearly cycle	−0.0252754	0.001390	−18.18	<0.0001	***
β_7	Yearly cycle	0.021829	0.001334	16.36	<0.0001	***
β_8	Yearly cycle	−0.00780915	0.001389	−5.621	<0.0001	***
β_{10}	Yearly cycle	0.011339	0.001382	8.206	<0.0001	***
β_{11}	Yearly cycle	−0.0123824	0.001351	−9.167	<0.0001	***
β_{12}	Yearly cycle	−0.00649979	0.001367	−4.754	<0.0001	***
β_{13}	Weekly cycle	0.066178	0.003786	17.48	<0.0001	***
β_{14}	Weekly cycle	0.060554	0.003828	15.82	<0.0001	***
β_{15}	Weekly cycle	0.093100	0.003728	24.97	<0.0001	***
β_{16}	Weekly cycle	0.099576	0.003777	26.36	<0.0001	***
β_{17}	Weekly cycle	0.122761	0.003790	32.39	<0.0001	***
β_{18}	Weekly cycle	−0.0478952	0.003816	−12.55	<0.0001	***
β_{19}	Daily cycle	−0.366208	0.001467	−249.7	<0.0001	***
β_{20}	Daily cycle	−0.979481	0.001491	−656.9	<0.0001	***
β_{21}	Daily cycle	−0.436885	0.001492	−292.8	<0.0001	***
β_{22}	Daily cycle	−0.429011	0.001464	−293.1	<0.0001	***
β_{23}	Daily cycle	−0.242307	0.001535	−157.8	<0.0001	***
β_{24}	Daily cycle	−0.111641	0.001407	−79.33	<0.0001	***
β_{25}	Daily cycle	−0.0974446	0.001428	−68.23	<0.0001	***
β_{26}	Daily cycle	−0.00667475	0.001485	−4.494	<0.0001	***
β_{27}	Daily cycle	0.009193	0.001274	7.218	<0.0001	***
β_{28}	Daily cycle	0.003709	0.001525	2.432	0.015	**
alpha		0.038011	0.000587	64.7	<0.0001	***

Note: Asterisks denote significance at the following levels: *** 0.1%; ** 1%; standard deviations QML.

The value of parameter α is obtained by regressing the model in Equation (1). The calculation process can be found in text books as given in [41].

With $E(Y \mid X) = \mu$ the variance can be obtained using Equation (5). These estimates enable future numbers of flights to be simulated and the full distribution to be obtained. Risk measures can then be proposed to cater for situations in which a given number of flights might be exceeded.

The measures of risk calculated are the well-known ES and VaR [43]. The VaR (95%) is the 95th percentile and is the number of flights that is only exceeded in 5% of cases. This is usually estimated for the 95% percentile, which illustrates the exact point above which the low probability (5%) zone of having an unmanageable number of flights is entered. The ES shows the mean number of flights in that zone or for the 5% of worst cases. Both risk measures are employed to better understand what may happen in the unfavourable tail of the distribution. This is why both ES and VaR are used as risk measures under uncertain conditions.

The method also enables correlations to be estimated between numbers of flights in different countries so as to provide an understanding of the links between different airspaces, so it can be used to develop new airspace regulations.

5. Results and Discussion

The method proposed enables us to estimate the alpha values, which can be shown here as indicators of volatility. The higher these parameters are, the higher the volatility is. As explained above, once the value for alpha is obtained, it is easy to run simulations and obtain the full distribution of the number of flights per hour and per country analysed. In addition, correlations can be estimated between the numbers of flights in different countries in an effort to better understand the interrelationship of airspaces at given times.

For the following calculations, Equation (6) is used. This is a reduced version of Equation (1), where the weekly cycle is erased and calculations are performed per hour. This enables an alpha value to be calculated for each hour and country. However, note that the method proposed can be used with a higher level of disaggregation for each country, hour and day of the week and even for other time periods such as months.

$$E(Y|X) = \exp(\beta_1 + \beta_2 t + AC(t) + DC(t)) = \mu \tag{6}$$

Table 5 presents the values for Belgium as an illustration. The values for the other countries are shown in Appendix A. These variables for some hours are not statistically different from zero, which usually happens when the alpha value is small. Note that when alpha is zero, we are in the case of Poisson regression as there is no overdispersion. The Poisson model is nested in the negative binomial model.

Table 5. Alpha values for Belgium.

Hour	Alpha Value	Standard Deviation	z	*p*-Value	
9	0.002131	0.000939	2.2690	0.0233	**
10	0.000636	0.000704	0.9029	0.3666	
11	0.001390	0.000762	1.8240	0.0681	*
12	0.000833	0.000638	1.3040	0.1922	
13	0.000550	0.000542	1.0150	0.3101	
14	0.001786	0.000434	4.1120	0.0000	***
15	0.000973	0.000340	2.8630	0.0042	***
16	0.001247	0.000309	4.0320	0.0001	***

Note: Asterisks denote significance as follows: *** 0.1% level; ** 1%; * 5%.

To calculate risk indicators, an expected figure must first be calculated, and then, the full distributions of flights must be obtained. There are various ways of estimating the expected value. In this paper, we use Equation (1) for a given day, i.e., 15 December 2023. Any other day could be used for this purpose, or expected values could be estimated by other methods. Once the expected value is calculated for a given time and day, we use the estimated alpha values to obtain the variance by applying Equation (5). Note that variance is defined as the square of volatility.

Table 6 presents the 95th percentile risk measures and the ES (95%) (i.e., the mean of the 5% of worst cases) for each hour and country. These risk measures are calculated using negative binomial simulations for 1,000,000 values. A mean of simulated values is shown in each case to control for the correct execution of the simulation.

The values above show that in the cases of Germany and Belgium, for instance, there is a 5% chance of average numbers of flights being 683 and 246 at 13:00 h. Knowing these risk indicators can help the authorities to plan for low-probability situations in which large numbers of flights may arise. Being prepared for low-probability, high-impact situations is tantamount to good risk-averse planning.

Table 6. Risk measures.

Country	Hour	Values		Simulated		
		Mean	**Alpha**	**Mean Simulated**	**95th Percentile**	**ES (95%)**
Belgium	9	179.34	0.002131	179.36	206	222
	10	208.33	0.000636	208.31	234	249
	11	191.57	0.001390	191.55	218	233
	12	192.45	0.000833	192.44	217	232
	13	205.38	0.000550	205.37	231	246
	14	196.22	0.001786	196.22	223	240
	15	178.67	0.000973	178.67	203	217
	16	178.43	0.001247	178.44	203	218
Germany	9	575.20	0.004061	575.22	649	693
	10	661.44	0.002289	661.44	730	771
	11	437.26	0.002441	437.29	488	517
	12	620.27	0.002760	620.23	689	730
	13	589.18	0.001910	589.14	648	683
	14	570.50	0.002354	570.49	632	668
	15	572.65	0.001920	572.64	630	665
	16	514.56	0.003658	514.58	579	618
Netherlands	9	206.89	0.001006	206.88	233	249
	10	254.77	0.001083	254.77	285	303
	11	236.42	0.001936	236.41	267	286
	12	276.40	0.000917	276.36	307	326
	13	226.71	0.000384	226.71	253	268
	14	222.44	0.000000	222.43	247	262
	15	237.46	0.000000	237.48	263	278
	16	203.65	0.000689	203.66	229	244
France	9	489.19	0.004175	489.16	554	593
	10	538.82	0.003457	538.83	605	644
	11	572.54	0.003336	572.50	641	682
	12	552.46	0.004130	552.51	624	667
	13	547.64	0.002843	547.60	610	648
	14	522.88	0.004047	522.85	591	632
	15	528.73	0.003186	528.78	592	630
	16	529.25	0.002619	529.25	589	624
Switzerland	9	175.15	0.002057	175.13	201	216
	10	196.31	0.000874	196.32	222	237
	11	229.38	0.000929	229.38	257	274
	12	209.94	0.001874	209.95	239	256
	13	211.18	0.001850	211.19	240	257
	14	169.67	0.004436	169.68	199	216
	15	182.47	0.002289	182.46	209	226
	16	180.39	0.002777	180.38	208	225
Luxembourg	9	38.50	0.000000	38.50	49	55
	10	31.79	0.000000	31.78	41	47
	11	41.36	0.000000	41.36	52	59
	12	28.83	0.009319	28.83	39	46
	13	41.74	0.018811	41.75	57	66
	14	33.82	0.016448	33.83	46	55
	15	44.03	0.008168	44.02	57	66
	16	41.91	0.000000	41.89	53	59

6. Conclusions

Volatility in the number of flights poses a serious challenge to airspace management in all countries in the FABEC area. Every year, great efforts are put into trying to understand this and learning to anticipate how the number of flights may fluctuate at given times and on given days of the year.

The analysis presented in this paper presents a sound mathematical method for estimating volatility in the near future based on past data. This volatility may differ depending on various factors, such as country, time and day of the week. This estimation effort then makes it possible to obtain the full distribution of expected flights at a given future time and, consequently, to estimate risk indicators in the form of ES and VaR. These indicators highlight situations in which the number of flights may exceed a given threshold and come to pose a risk for anyone who has to manage the airspace.

The method is applied using actual data to estimate expected future flight numbers and volatilities for each period of interest in each country. Using the expected flights and variance calculated, the full distributions are obtained for those periods and the risk measures are calculated. As far as we know, this is the first time this method has been applied to estimate the volatility of the number of flights, taking into account that the number must be an integer or zero and that volatility can vary with certain factors such as the time and day of the week. The results shown here should enable airspace to be managed in a safer, economically optimal fashion.

The method proposed can also be applied in three other ways:

(a) By combining different methods to estimate expected flights and using the methodology presented here to estimate volatility and risk measures.
(b) By using a higher level of disaggregation in time periods such as months combined with minutes, hours and days of the week.
(c) By linking volatility and risk with welfare, employment and CO_2 emissions.

Availability of data and the possibility of making those data public are thus the only limiting factors in undertaking a much more in-depth, detailed analysis.

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Nomenclature

Abbreviations:

AAS	Airspace Architecture Study
ACCs	Area Control Centres
ACE	ATM Cost-Effectiveness
AI	Artificial Intelligence
AMAN	Arrival Manager
ANS	Air Navigation Services
ANSP	Air Navigation Service Providers

ASM	Air Space Management
ATC	Air Traffic Charges
ATFM	Air Traffic Flow Management
ATM	Air Traffic Management
COVID-19	Corona virus 2019
CTA	Controlled airspace
ED	All areas connoted to Germany
EDCC	German Area Control Centres
EDYYYC	Maastricht ACC in German Space
ES	Expected shortfall
FAB	Functional Airspace Block
FABEC	Functional Airspace Block Central Europe
GMT	Greenwich Mean Time
LP	All areas connoted to Portugal
MP	Main Plan
NEST	Network Strategic Tool
NM	Nautical Mile
NOP	Network Operation Plan
PRU	Performance Review Unit
SESAR	Single European Sky ATM Research
UAC	Upper Area Control
UTC	Universal Coordinated Time
VaR	Value at Risk

Counting model variables and parameters:

$E(Y\|X)$	Expected number of flights
Y	Number of flights
X	Independent variables
β_{1-28}	Negative binomial parameters
$AC(t)$	Yearly cycle
$WC(t)$	Weekly cycle
$DC(t)$	Daily cycle
D_{1-6}	Dummy variables
τ	Hour values between 1 and 24
QML	Quasi Maximum Likelihood

Appendix A

All Tables in Appendix A show the authors' own calculations.

Table A1. Alpha values for Belgium.

Hour	Alpha Value	Standard Deviation	z	*p*-Value	
9	0.002131	0.000939	2.2690	0.0233	**
10	0.000636	0.000704	0.9029	0.3666	
11	0.001390	0.000762	1.8240	0.0681	*
12	0.000833	0.000638	1.3040	0.1922	
13	0.000550	0.000542	1.0150	0.3101	
14	0.001786	0.000434	4.1120	0.0000	***
15	0.000973	0.000340	2.8630	0.0042	***
16	0.001247	0.000309	4.0320	0.0001	***

Note: Asterisks denote significance as follows: *** 0.1% level; ** 1%; * 5%.

Table A2. Alpha values for Germany.

Hour	Alpha Value	Standard Deviation	z	p-Value	
9	0.004061	0.001554	2.6130	0.0090	***
10	0.002289	0.001170	1.9570	0.0504	*
11	0.002441	0.001143	2.1350	0.0328	**
12	0.002760	0.000892	3.0950	0.0020	***
13	0.001910	0.000652	2.9280	0.0034	***
14	0.002354	0.000469	5.0200	0.0000	***
15	0.001920	0.000297	6.4700	0.0000	***
16	0.003658	0.000192	19.0700	0.0000	***

Note: Asterisks denote significance as follows: *** 0.1% level; ** 1%; * 5%.

Table A3. Alpha values for the Netherlands.

Hour	Alpha Value	Standard Deviation	z	p-Value	
9	0.001006	0.000831	1.2110	0.2258	
10	0.001083	0.000759	1.4270	0.1536	
11	0.001936	0.000889	2.1780	0.0294	**
12	0.000917	0.000633	1.4480	0.1475	
13	0.000384	0.000531	0.7226	0.4699	
14	0.000000	Poisson			
15	0.000000	0.000172	0.0003	0.9998	
16	0.000689	0.000377	1.8300	0.0673	*

Note: Asterisks denote significance as follows: ** 1% level; * 5%.

Table A4. Alpha values for France.

Hour	Alpha Value	Standard Deviation	z	p-Value	
9	0.004175	0.001376	3.0340	0.0024	***
10	0.003457	0.001239	2.7900	0.0053	***
11	0.003336	0.001194	2.7940	0.0052	***
12	0.004130	0.001079	3.8270	0.0001	***
13	0.002843	0.000652	4.3590	0.0000	***
14	0.004047	0.000541	7.4740	0.0000	***
15	0.003186	0.000345	9.2320	0.0000	***
16	0.002619	0.000324	8.0830	0.0000	***

Note: Asterisks denote significance as follows: *** 0.1% level.

Table A5. Alpha values for Switzerland.

Hour	Alpha Value	Standard Deviation	z	p-Value	
9	0.002057	0.000876	2.3480	0.0189	**
10	0.000874	0.000641	1.3630	0.1729	
11	0.000929	0.000635	1.4640	0.1433	
12	0.001874	0.000622	3.0140	0.0026	***
13	0.001850	0.000485	3.8110	0.0001	***
14	0.004436	0.000584	7.5930	0.0000	***
15	0.002289	0.000296	7.7330	0.0000	***
16	0.002777	0.000315	8.8080	0.0000	***

Note: Asterisks denote significance as follows: *** 0.1% level; ** 1%.

Table A6. Alpha values for Luxembourg.

Hour	Alpha Value	Standard Deviation	z	*p*-Value	
9	0.000000	Poisson			
10	0.000000	0.000000	0.0929	0.9260	
11	0.000000	0.000000	1.5420	0.1231	
12	0.009319	0.002768	3.3670	0.0008	***
13	0.018811	0.002395	7.8530	0.0000	***
14	0.016448	0.002329	7.0640	0.0000	***
15	0.008168	0.001951	4.1870	0.0000	***
16	0.000000	0.000000	0.1025	0.9184	

Note: Asterisks denote significance as follows: *** 0.1% level.

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