PARTIAL PRIVATIZATION OF STATE HOLDING CORPORATIONS*

by

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ABSTRACT

We consider a state holding corporation with two plants that may produce complementary or substitute goods and that competes with one or two private firms. We find that the government partially privatizes the two plants of the state holding corporation and is indifferent between selling them partially to a single investor and to different investors. However, in the former case the government retains a greater (smaller) stake in the state corporation if goods are substitutes (complements).

1 INTRODUCTION

One of the issues analyzed by the literature on mixed oligopoly is the decision by governments whether to privatize a single public firm (see, for example, De Fraja and Delbono, 1989, 1990; Corneo and Jeanne, 1994).[1] These papers have been extended to consider, among other factors, partial privatization of public firms (Matsumura, 1998; Lin and Matsumura, 2012), strategic privatization under international trade (Bárcena-Ruiz and Garzón, 2005*a*, 2005*b*), sequential privatization of public firms (Matsumura and Shimizu, 2010), privatization when the public firm is as efficient as private firms (Bárcena-Ruiz, 2012), privatization under an interdependence payoff structure (Matsumura and Okamura, 2015), and privatization with vertically related markets (Wu et al., 2016; Shuai and Tomaru, 2016).

The papers cited above usually assume that the public firm produces a single good at a single production plant. However, in practice governments own firms that produce various types of goods at various production plants, and they are mainly organized as state holding corporations (see Kumar, 1992).[2] As far as we know, the theoretical literature on mixed oligopoly has hardly analyzed the privatization of state holding corporations (henceforth referred to as state corporations). One exception is the paper by Bárcena-Ruiz and Garzón (2017), who consider a state corporation with two plants that produce differentiated goods. They study whether the government wants to privatize the state corporation, and if so whether the two plants are sold to different private investors or to a single investor. They assume that if a plant of the state corporation is privatized it is fully sold to private investors. This has happened in many cases of privatization of public firms integrated into state corporations.[3] However, on other occasions state corporations partially privatize their firms. This issue has not been

[1] The OECD (2005) points out that in the EU governments are the largest shareholders in many partially privatized firms. State control is also significant in Japan, China, Indonesia, Korea, Malaysia, Singapore, and Thailand. In many industries in these countries there is interaction between private and public firms, as in the markets for cars, ships, and steel manufactures (see De Fraja, 2009).

[2] Holdings comprising domestic public firms have been set up by European governments such as, for example, the Sociedad Estatal de Participaciones Industriales -SEPI- in Spain (see www.sepi.es), the Istituto per la Ricostruzione Industriale -IRI- in Italy (see Cafferata, 2010), the Agence des Participations de l'État -APE- in France (www.economie.gouv.fr), and the State-owned Assets Supervision and Administration Commission of the State Council (SASAC) in China (http://www.sasac.gov.cn).

[3] Examples of fully privatized Spanish public firms include the telecommunications firm Telefónica, the shipbuilding firm Iza and the insurance company Musini (see www.sepi.es).

studied by the relevant literature, so in order to fill this gap this paper analyzes the decision whether to partially privatize state corporations.

Our paper relates to the literature on partial privatization that began with the seminal paper by Matsumura (1998). He considers a mixed duopoly where a public firm and a private firm compete and finds partial privatization in equilibrium under moderate conditions. That paper has been extended to analyze factors that affect partial privatization of public firms. Among other factors, the literature has considered partially foreign-owned private firms (Han and Ogawa, 2008), foreign investment in partially privatized firms (Lin and Matsumura, 2012), cross-ownership of firms (Jain and Pal, 2012; Chai and Karasawa-Ohtashiro, 2015), trade policies (Chao and Yu, 2006; Long and Stähler, 2009), product differentiation (Fujiwara, 2007; Lu and Poddar, 2007), free entry (Matsumura and Kanda, 2005; Wang and Chen, 2010), endogenous timing of decisions (Bárcena-Ruiz and Garzón, 2010), environmental problems (Kato, 2006; Ohori, 2006), and merger problems (Bárcena-Ruiz and Garzón, 2003; Mendez-Naya, 2008). However, this literature has not considered partial privatization of state corporations that produce more than one type of goods at more than one production plant.

State corporations are usually multiproduct, multiplant firms that produce different products, which may be substitutes or complements. Therefore, we consider an industry made up by a state corporation and a private sector. The state corporation owns two production plants each of which produces a differentiated good, and those goods may be substitutes or complements. The private sector comprises two private plants that produce differentiated goods. Those plants may be owned by different investors (henceforth uniplant firms) or by a single investor (henceforth the multiplant firm).[4] The government may partially privatize the two plants of the state corporation, so it has two options: It may sell a percentage of the ownership of both plants to a single private investor or it may sell a percentage of the ownership of each plant to a different private investor. Thus, the optimal degree of privatization of the state corporation depends on

[4] Multiproduct firms are omnipresent in modern economies (Eckel and Neary, 2010). Literature on this issue has analyzed, for example, market structure (Shaked and Sutton, 1990), product choice and the determinants of product variety (Anderson and De Palma, 2006), the effect of firm heterogeneity on industry profitability and welfare (Symeonidis, 2009), international trade and productivity (Bernard et al., 2010), and environmental policies implemented by governments (Bárcena-Ruiz and Garzón, 2014).

whether goods are substitutes or complements and on whether private firms and the state corporation are uniplant or multiplant. This analysis cannot be made assuming a single-plant public firm that competes with uniplant private firms, which is the usual model studied by papers analyzing partial privatization.

We find evidence supporting the idea that the analysis conducted in the paper is important. The China FAW Group Corporation (FAW) is an example of a firm producing substitute goods. FAW is a state-owned automotive corporation that produces several brands of cars at different subsidiaries. Two of these brands are Tianjin FAW and FAW Car, in which the Chinese government owns stakes of 73.38 % and 55% respectively.[5] An example of a public firm that produces complementary goods is China Petroleum & Chemical Corporation (Sinopec Corp), a Chinese state-owned corporation. It has several business lines such as oil refining, the exploration, production, transportation, and marketing of oil and natural gas. In 2014 this corporation partially privatized one of its subsidiaries, Sinopec Marketing (currently called Sinopec Chemical Sales Company), which specializes in the marketing, sales, and logistics of Sinopec petrochemical products. The Chinese government owns more than 70% of the shares in this subsidiary.[6]

We find in the paper that the government partially privatizes the two plants of the state corporation but is indifferent between selling them to a single private investor or to different private investors. When goods are complements, the government keeps a larger stake in the state corporation if its plants are sold to different private investors than if they are sold to a single private investor. Moreover, the government retains a larger stake if private firms are multiplant than if they are uniplant. If goods are substitutes the contrary result is obtained. If goods are independent in demand the government keeps the same stake in the state corporation in all cases. From this it can be concluded that the stake that the government retains in the state corporation depends

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5 See http://www.tjfaw.com/index.php/DQBG.shtml (for Tianjin FAW), and http://www.4-traders.com/FAW-CAR-CO-LTD-6495758/company/ (for FAW Car).

6 See http://www.sinopecgroup.com/group/Documents/StockImportFile/2014/452bab44-2b99-4660-a6a3-1ff32e70d38b.pdf. Another example is China State Shipbuilding Corporation (CSSC), a state corporation that has many subsidiaries. In 2015 one of its core subsidiaries, Guangzhou Shipyard International Company (GSCI), whose activity mainly covers ship-building, offshore marine construction and heavy machine manufacturing, sold 4.83% of its shares to the private firm Yangzhou Kejin Shipbuilding. Now GSCI is known as Offshore & Marine Engineering Company (COMEC) and the Chinese government owns more than 60% of the shares in this company (http://comec.cssc.net.cn).

on the type of goods produced by the state corporation and the private firms, and on whether private firms are uniplant or multiplant.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the privatization of the state corporation assuming uniplant private firms. Section 4 extends the analysis by considering a multiplant private firm. Section 5 compares the two cases, and Section 6 concludes.

2 THE MODEL

We consider an economy made up of a public sector and a private sector that produce differentiated goods, denoted by 1 and 2, which may be substitutes or complements. The public sector comprises a state holding corporation, denoted by firm *A*, whose objective function is social welfare if it is fully public. It owns two plants producing goods 1 and 2, denoted by 1*A* and 2*A* respectively. The private sector may comprise two uniplant private firms or a single multiplant firm with two plants. We denote the private uniplant firms and the plants of the private multiplant firm which produce good 1 and good 2 by 1*B* and 2*B* respectively. The objective function of a uniplant private firm is its own profit and the objective function of the multiplant private firm is the joint profits of its two plants.

On the consumption side, there is a continuum of consumers of the same type. The representative consumer maximizes $U(q_1, q_2) - p_1 q_1 - p_2 q_2$, where p_i is the price of good i, $q_i = q_{iA}+q_{iB}$ is the quantity of good i and, q_{ik} is the output produced by firm or plant ik, i=1, 2; k=*A*, *B*. The function $U(q_1, q_2)$ is assumed to be quadratic, strictly concave and symmetric in q_1 and q_2:

$$U(q_1, q_2) = (q_1+q_2) - ((q_1)^2+2bq_1q_2+(q_2)^2)/2, \; -1<b<1.$$

The inverse demand functions are given by:

$$p_i = 1 - (q_{iA}+q_{iB}) - b(q_{jA}+q_{jB}), \; i\neq j; \; i,j=1,2; \; -1<b<1, \qquad (1)$$

where goods are substitutes if $b>0$, complements if $b<0$, and independent in demand if $b=0$. Following De Fraja and Delbono (1989) and Bárcena-Ruiz and Garzón (2005*a*), we assume that firms have identical technologies, represented by the following quadratic cost function:

$$C(q_{ik}) = cq_{ik}^2/2,\ i = 1, 2;\ k = A, B.$$

Therefore, the profit function of plant or firm *ik* is:

$$\pi_{ik} = p_i\ q_{ik} - cq_{ik}^2/2,\ i=1, 2;\ k = A, B. \tag{2}$$

The profit of multiplant firm *k* is the joint profit of the two plants:

$$\pi_k = \pi_{1k}+\pi_{2k},\ k = A, B. \tag{3}$$

The producer surplus is the sum of the profits of firms and is given by $PS = \pi_{1A} + \pi_{2\mathrm{A}} + \pi_{1B} + \pi_{2B}$. The consumer surplus is given by:

$$CS = ((q_{1A}+q_{1B})^2 + 2b(q_{1A}+q_{1B})(q_{2A}+q_{2B}) + (q_{2A}+q_{2B})^2)/2. \tag{4}$$

The government aims to maximize social welfare, i.e. the sum of the producer surplus and the consumer surplus:

$$W = CS + PS. \tag{5}$$

To increase social welfare the government can sell off part of the plants of the state corporation to private investors. In this case the government has two options: to sell to a single private investor or to different private investors. The state corporation is then jointly owned by the public and private sectors. We assume that the government owns β percent of the shares and the private investor owns the remaining $(1-\beta)$ percent, so the partially privatized firm maximizes the weighted average of social welfare and firm profits (see Matsumura, 1998). Therefore, if the government sells part of the two plants

of the state corporation to the same investor it owns the same stake in the two plants and thus the objective function of the firm is given by:

$$V = \beta W + (1-\beta)(\pi_{1A}+\pi_{2A}),\ 0\leq\beta\leq 1. \tag{6}$$

When the government sells part of each plant to a different private investor, the objective function of plant *i* is given by:

$$V_i = \beta_i W + (1-\beta_i)\ \pi_{iA},\ 0\leq\beta_i\leq 1,\ i=1,\ 2. \tag{7}$$

Clearly, if β_i=1 plant *iA* of the state corporation remains public, and if β_i=0 plant *iA* is fully privatized.

To analyze the government's decision on the optimal degree of partial privatization of the state corporation we propose a two-stage game. In the first stage the government decides what percentage of the shares in the state corporation it will sell to private investors. In this case the government has two options: sell off part of both plants to a single private investor or sell part of each plant to a different investor. In the second stage the firms make production decisions simultaneously. We solve the game by backward induction to obtain a subgame perfect equilibrium. We consider two cases: there are two uniplant private firms in the first case and a single multiplant private firm in the second.

Henceforth we assume that *c*=1 to simplify the presentation of results. It can be shown that results are robust to changes in this parameter.

3 UNIPLANT PRIVATE FIRMS

Denote the case where private firms are uniplant by superscript *U*. The government may partially privatize the two plants of the state corporation, selling shares in each plant to a different private investor (denoted by the superscript *D*) or selling shares in the state corporation to a single private investor (denoted by superscript *S*).

To explain the results that appear in the different cases we describe two effects. The first is the internalization effect: a multiplant firm internalizes how the output of one of

its plants affects that of its other plant. When such a firm produces substitute goods (b>0) it takes on board that its two plants compete with each other, which encourages it to reduce the output of its plants. Thus, with substitute goods multiplant firms produce less than uniplant firms. With complementary goods (b<0) the opposite result is obtained since a multiplant firm takes on board that its two plants cooperate, so the output of one plant increases with that of the other plant. Thus, with complementary goods multiplant firms produce more than uniplant firms. The second effect is the output effect: it is due to the fact that private firms have a different objective function than the state corporation. The state corporation produces more than private firms since it takes consumer surplus into account. If the state corporation is semipublic its production decreases with the stake owned by the private sector, 1-β.

3.1 Single Private Investor

In this case the two plants of the state corporation are sold off in part to a single private investor. In the second stage of the game semipublic firm *A* chooses the output levels q_{1A} and q_{2A} that maximize its objective function, given by expression (6). Private firm *iB* sets the output level q_{iB} that maximizes its profit, given by expression (2). Solving these problems, we obtain the following first order conditions:

$$\begin{aligned} 1 - q_{iA}(3-\beta) - b(2-\beta)q_{jA} - q_{iB} - bq_{jB} = 0, \\ 1 - q_{iA} - 3q_{iB} - b\left(q_{jA} + q_{jB}\right) = 0,\ i \neq j;\ i, j = 1, 2. \end{aligned} \tag{8}$$

From (8) we obtain the following output of firms and social welfare as a function of β:

$$\begin{aligned} q_{iA} = \frac{2}{8+b^2(1-\beta)-3\beta+b(7-4\beta)},\ \ q_{iB} = \frac{2+b-\beta(1+b)}{8+b^2(1-\beta)-3\beta+b(7-4\beta)}, \\ W = \frac{(40+44b+12b^2+b^3-2(1+b)(14+b(8+b))\beta+(1+b)^2(4+b)\beta^2)}{(8+b^2(1-\beta)-3\beta+b(7-4\beta))^2},\ i = 1, 2. \end{aligned} \tag{9}$$

In the first stage of the game the government chooses the optimal value of β that maximizes social welfare, given by expression (9). Solving this problem, we obtain the following result.

Lemma 1: Under uniplant private firms, when the government sells part of the state corporation to a single private investor, in equilibrium:

$$\beta^{SU}=\frac{4+b}{5+b},\ q_{iA}^{SU}=\frac{5+b}{2(7+6b+b^2)},\ q_{iB}^{SU}=\frac{3+b}{2(7+6b+b^2)},\ \pi_{iA}^{SU}=\frac{(5+b)(7+3b)}{8(7+6b+b^2)^2},\ \pi_{iB}^{SU}=\frac{3(3+b)^2}{8(7+6b+b^2)^2},$$

$$CS^{SU}=\frac{(1+b)(4+b)^2}{(7+6b+b^2)^2},\ PS^{SU}=\frac{(31+20b+3b^2)}{2(7+6b+b^2)^2},\ W^{SU}=\frac{9+2b}{2(7+6b+b^2)},\ i=1,2.$$

From Lemma 1 we obtain that the government partially privatizes the state corporation, $0<\beta^{SU}<1$, for all values of parameter b. This last parameter affects the optimal value of β. Specifically, the percentage of shares that remains public increases with b ($\partial\beta^{SU}/\partial b>0$). These results are explained below.

In the first stage of the game, the government chooses the right stake in the state corporation, β, taking into account how the output effect influences market competition and social welfare. Independently of the value of parameter b, as β increases market competition becomes stronger due to the output effect, so the consumer surplus increases while the producer surplus decreases. When β is low market competition is weak so the government chooses a greater β to increase the consumer surplus and social welfare. When β is high market competition is strong, so the government chooses a smaller β to increase the producer surplus and social welfare. The effect of the consumer and producer surpluses balance for a value of β between 0 and 1, so the government partially privatizes the state corporation setting $\beta=\beta^{SU}$, $0<\beta^{SU}<1$, for all values of parameter b.[7]

Next we explain why β^{SU} increases with parameter b ($\partial\beta^{SU}/\partial b>0$), the degree to which goods are substitutes or complements. Parameter b affects β^{SU} in two ways. First, as b increases goods 1 and 2 become less complementary or close substitutes, which decreases the output of the state corporation due to the internalization effect. Private firms are uniplant, so they do not take into account the internalization effect. Second,

[7] Compared with partial privatization, full privatization of the state corporation ($\beta=0$) is not optimal since the privatized firm maximizes profits, which strongly reduces the output of industry and thus the consumer surplus and social welfare. Keeping the state corporation fully public ($\beta=1$) is not optimal since it maximizes social welfare, so competition in the product market is very high, which reduces the producer surplus and social welfare.

the total output of each of the two goods decreases as b increases.[8] As a result of the above, when b increases the total output of each of the two goods decreases strongly with b, for a given β, which reduces the consumer surplus and increases the producer surplus. Moreover, as the production of the semipublic state corporation increases with β, due to the output effect, the government chooses a greater β to counterbalance the effect on market competition of a greater b. This means that the percentage of shares that remains public increases as goods become less complementary or closer substitutes.

Finally, $q_{iA}^{SU} > q_{iB}^{SU}$ and $\pi_{iA}^{SU} > \pi_{iB}^{SU}$ for all values of parameter b. This is because firm A is semipublic and therefore takes into account the consumer surplus, which increases with the output of the firms. As a result firm A produces more than private firms and obtains greater market share and profits.

3.2 Different Private Investors

In the second stage of the game, private firm iB sets the output level q_{iB} that maximizes its profit, given by expression (2). In this case part of each plant of the state corporation is sold to a different private investor. Each semipublic firm chooses the output level q_{iA} that maximizes its objective function given by expression (7). Solving these problems we obtain the following first order conditions:

$$\begin{aligned} 1 - q_{iB} - b\left(q_{jA} + q_{jB}\right) - q_{iA}(3 - \beta_i) &= 0, \\ 1 - q_{iA} - 3q_{iB} - b\left(q_{jA} + q_{jB}\right) &= 0, \; i \neq j; \; i, j = 1, 2. \end{aligned} \tag{10}$$

From (10) we obtain the following output of the firms and social welfare as a function of β_1 and β_2:

$$q_{iA} = \frac{-2(8 - b(4 - \beta_j) - 3\beta_j)}{b^2(4 - \beta_i)(4 - \beta_j) - (8 - 3\beta_i)(8 - 3\beta_j)}, \quad q_{iB} = \frac{-(2 - \beta_i)(8 - b(4 - \beta_j) - 3\beta_j)}{b^2(4 - \beta_i)(4 - \beta_j) - (8 - 3\beta_i)(8 - 3\beta_j)}, \; i \neq j; \; i, j = 1, 2,$$

[8] The inverse demand function of good i is given by $p_i = 1 - q_i - bq_j$, where $q_i = q_{iA} + q_{iB}$ and $q_j = q_{jA} + q_{jB}$; thus, for a given q_j, if b increases $1 - bq_j$ decreases so the market size of good i also decreases. This means that q_i decreases if b increases.

$$W = ((b^3(4-\beta_1)^2(4-\beta_2)^2 - 2b^2(\beta_1(48\beta_2 - 7{\beta_2}^2 - 96) + {\beta_1}^2(14 - 7\beta_2 + {\beta_2}^2) + 2(96 - 48\beta_2 + 7{\beta_2}^2)) + b({\beta_1}^2(104\beta_2 - 15{\beta_2}^2 - 152) + 8\beta_1(132 - 90\beta_2 + 13{\beta_2}^2) - 8(192 - 132\beta_2 + 19{\beta_2}^2)) + 2(\beta_1(672\beta_2 - 111{\beta_2}^2 - 928) + {\beta_1}^2(154 - 111\beta_2 + 18{\beta_2}^2) + 2(640 - 464\beta_2 + 77{\beta_2}^2))))/\ (b^2(4-\beta_1)(4-\beta_2) - (8-3\beta_1)(8-3\beta_2))^2. \quad (11)$$

In the first stage of the game the government chooses the optimal value of β_1 and β_2 that maximizes social welfare, given by expression (11). Solving this problem, we obtain the following result.

Lemma 2: Under uniplant private firms, when the government sells part of each plant of the state corporation to a different private investor, in equilibrium:

$$\beta_i^{DU} = \frac{4}{5+b},\ q_{iA}^{DU} = \frac{5+b}{2(7+6b+b^2)},\ q_{iB}^{DU} = \frac{3+b}{2(7+6b+b^2)},\ \pi_{iA}^{DU} = \frac{(5+b)(7+3b)}{8(7+6b+b^2)^2},\ \pi_{iB}^{DU} = \frac{3(3+b)^2}{8(7+6b+b^2)^2},$$

$$CS^{DU} = \frac{(1+b)(4+b)^2}{(7+6b+b^2)^2},\ PS^{DU} = \frac{31+20b+3b^2}{2(7+6b+b^2)^2},\ W^{DU} = \frac{9+2b}{2(7+b(6+b))},\ i = 1, 2.$$

Lemma 2 shows that the state corporation is partially privatized by selling its plants to different private investors ($0<\beta_i^{DU}<1$), so there are two semipublic uniplant firms. Given the symmetry of the model, in equilibrium the government retains the same stake in each plant of the state corporation. The percentage of shares that remains public, β_i^{DU}, decreases with b ($\partial\beta_i^{DU}/\partial b < 0$).

In this case all firms are uniplant, so there is no internalization effect. Independently of the value of parameter *b*, the output effect means that market competition becomes stronger as β_i increases so, as in Lemma 1, the consumer surplus increases while the producer surplus decreases. When β_i is low (high), market competition is weak (strong) so the government chooses a greater (smaller) β_i to increase the consumer (producer) surplus and social welfare. As a result, the government partially privatizes the state corporation by selling each plant to a different private investor, setting $\beta_i = \beta_i^{DU}$, $0<\beta_i^{DU}<1$, for all values of parameter *b*.

As seen in Lemma 1, as *b* increases the total output of each of the two goods decreases. There is no internalization effect since all firms are uniplants so, for a given β, the total output of each of the two goods decreases less with *b* than in Lemma 1. As a

result, market competition is greater in this case. Due to the output effect, the stake in the semipublic firms retained by the government becomes smaller with b to reduce market competition $(\partial \beta_i^{DU} / \partial b < 0)$. Finally, $q_{iA}^{DU} > q_{iB}^{DU}$ and $\pi_{iA}^{DU} > \pi_{iB}^{DU}$ for all values of parameter b since firm iA is semipublic and takes consumer surplus into account.

3.3 Comparison of Results

We first compare the degree of privatization of the two plants of the state corporation when they are sold to a single investor with that which results when they are sold to different investors. From Lemmas 1 and 2 the following is obtained.

Proposition 1: Under uniplant private firms, in equilibrium: $\beta^{SU} > \beta_i^{DU}$ if goods are substitutes (b>0), $\beta^{SU} < \beta_i^{DU}$ if goods are complements (b<0), and $\beta^{SU} = \beta_i^{DU}$ if goods are independent in demand (b=0).

The internalization effect is present only when the two plants of the state corporation are sold in part to a single private investor. Thus, given a stake-holding in the state corporation by the government, when goods are substitutes production and market competition are lower if the two plants are sold in part to a single private investor due to the internalization effect. Thus, in that case, due to the output effect, the government retains a greater percentage of the shares in the state corporation ($\beta^{SU} > \beta_i^{DU}$) to increase production. However, when goods are complements production and market competition are lower if each plant is sold off in part to a different private investor due to the internalization effect. Thus, in that case, the output effect implies that the government retains a greater stake in the state corporation ($\beta^{SU} < \beta_i^{DU}$) to increase production. Finally, if goods are independent in demand (b=0) there is no internalization effect in the case of a multiplant firm so a multiplant firm produces the same output as uniplant firms, which means that $\beta^{SU} = \beta_i^{DU}$.

By comparing the welfare levels shown in Lemmas 1 and 2 the following result is obtained.

Proposition 2: Under uniplant private firms, in equilibrium, $W^{SU}=W^{DU}$.

Proposition 2 shows that when private firms are uniplant the government is indifferent between selling part of the two plants of the state corporation to a single investor and selling part of each plant to a different private investor. The intuition of this result is the following. Proposition 1 shows that if goods are substitutes (complements) the government retains a greater (smaller) stake in the state corporation when the plants are partially sold to a single private investor than when they are sold to different private investors. Thus, in both cases the government chooses the optimal stake in the state corporation taking into account how it affects market competition, thus offsetting the internalization effect. This stake makes the firms produce the same output in both cases ($q_{iA}^{SU} = q_{iA}^{DU}$, $q_{iB}^{SU} = q_{iB}^{DU}$), so the consumer and producer surpluses and welfare are also the same ($CS^{SU} = CS^{DU}$, $PS^{SU} = PS^{DU}$, $W^{SU} = W^{DU}$).[9]

Next we compare the results obtained in this section with those obtained by Bárcena-Ruiz and Garzón (2017), who assume that partial privatisation is not possible. They show that the government does not privatize the state corporation if market competition (measured by the number of private firms) is low, both plants are sold to a single private investor if market competition takes an intermediate value when goods are complements, and each plant is sold to a different private investor otherwise.

Bárcena-Ruiz and Garzón (2017) do not permit partial privatization, so the output effect is different in their model since the state corporation always maximizes social welfare. In our model this effect permits the government to choose the right degree of market competition by choosing which part of the state corporation is sold, thus offsetting the internalization effect. As a result we find that the government is indifferent between selling part of the state corporation to a single private investor and to different private investors.[10] When partial privatization is not possible the government cannot choose the right degree of market competition so it is not indifferent

[9] This is not possible when the government has to fully privatize the state corporation (as in Bárcena-Ruiz and Garzón, 2017), so welfare is not equal in both cases under full privatization.

[10] As noted in Footnote 11, the main results of this paper hold when n is greater than 1.

between selling the state corporation to a single private investor and to different private investors. In that case the internalization effect plays an important role and explains the decision taken by the government.[11]

4 MULTIPLANT PRIVATE FIRMS

Up to now we have considered that the state corporation competes in the product market with uniplant private firms. However, in modern economies multiplant firms are omnipresent and state corporations may compete with multiplant private firms rather than with uniplant private firms. Thus, we now consider that the state corporation competes in the product market with a multiplant private firm that owns two plants producing differentiated goods. We denote this case by superscript *M*.

We now compare the degree of privatization of the two plants of the state corporation when they are sold to a single investor and when they are sold to different investors. From Lemmas A1 and A2 (see Appendix) the following is obtained.

Proposition 3: Under a multiplant private firm, in equilibrium: $\beta^{SM} > \beta_i^{DM}$ if goods are substitutes ($b>0$), $\beta^{SM} < \beta_i^{DM}$ if goods are complements ($b<0$), and $\beta^{SM} = \beta_i^{DM}$ if goods are independent in demand ($b=0$). Finally, $\partial \beta_i^{DM} / \partial b < 0$, and $\partial \beta^{SM} / \partial b < 0$ if $b<0$ and $\partial \beta^{SM} / \partial b > 0$ if $b>0$.

The explanation of this result is similar to that given in Proposition 1 so we omit it. The explanation of how β^{SM} and β_i^{DM} vary with b is relegated to the Appendix. The main difference is that there is one multiplant private firm rather than two uniplant private firms. Due to the internalization effect, a multiplant private firm produces less (more) with substitute (complementary) goods than uniplant private firms.

[11] It can be shown that the result of the comparison also applies to the case with multiplant private firms.

By comparing the welfare levels shown in Lemmas A1 and A2 the following result is obtained.

Proposition 4: Under a multiplant private firm, in equilibrium: $W^{SM}=W^{DM}$.

The explanation of this result is similar to that given in Proposition 2 so we omit it.

5 Comparison of Results

By comparing Propositions 1 and 3 the following result is obtained.

Proposition 5: In equilibrium: if $b<0$ we obtain that $\beta_i^{DM} > \beta_i^{DU} > \beta^{SM} > \beta^{SU}$, if $b>0$ we obtain that $\beta^{SU} > \beta^{SM} > \beta_i^{DU} > \beta_i^{DM}$, and if $b=0$ we obtain that $\beta^{SU}=\beta^{SM}=\beta_i^{DU}=\beta_i^{DM}$.[12]

This proposition shows that when goods are complements ($b<0$) the government keeps a larger stake in the state corporation if its plants are sold to different private investors than if they are sold to a single private investor. Moreover, the government retains a larger stake if private firms are multiplant than if they are uniplant. If goods are substitutes ($b>0$) the contrary result is obtained. Finally, if goods are independent in demand ($b=0$) the government keeps the same stake in the state corporation in all cases. From this it can be concluded that the stake that the government retains in the state corporation depends on the type of goods produced by the firms and on whether private firms are uniplant or multiplant.

When goods are complements the internalization effect means that the two plants of the state corporation produce more if they are sold in part to a single private investor than if they are sold in part to different private investors, independently of whether the private firms are uniplant or multiplant. Thus, the output effect implies that the government retains a lower stake in the state corporation in the first case ($\beta_i^{Dl} > \beta^{Sl}$,

[12] It can be shown that the main result obtained in this proposition holds if it is assumed that there are n uniplant or multiplant private firms competing in the product market. The state corporation is never fully privatized even when n is high, and the degree of privatization increases with n. This is a well known result in the relevant literature (see, for example, Fujiwara, 2007).

l=U, M). Moreover, the private sector produces more when there is a private multiplant firm than when there are two private uniplant firms because of the internalization effect. This causes semipublic plants or firms to produce less in the first case, so the government retains a larger stake in the state corporation due to the output effect. Thus, whoever buys the state corporation, if goods are complements the government retains a greater percentage of the shares in the state corporation if private firms are multiplant than if they are uniplant: $\beta_i^{DM} > \beta_i^{DU} > \beta^{SM} > \beta^{SU}$.

When goods are substitutes the internalization effect causes the two plants of the state corporation to produce less if they are sold in part to a single private investor than if they are sold in part to different private investors, independently of whether private firms are uniplant or multiplant. Thus, due to the output effect the government retains a greater stake in the state corporation in the first case ($\beta_i^{Dl} < \beta^{Sl}$, *l=U, M*). Moreover, due to the internalization effect, the private sector produces less when there is a private multiplant firm than when there are two private uniplant firms. This encourages partially privatized plants or firms to produce more in the first case, so the government retains a smaller stake in the state corporation due to the output effect. Thus, whoever buys the state corporation, if goods are substitutes the government retains a lower percentage of the shares in the state corporation if private firms are multiplant than if they are uniplant: $\beta^{SU} > \beta^{SM} > \beta_i^{DU} > \beta_i^{DM}$. Finally, if goods are independent in demand there is no internalization effect, so $\beta^{SU} = \beta^{SM} = \beta_i^{DU} = \beta_i^{DM}$.

6 CONCLUSIONS

The theoretical literature on mixed oligopoly has hardly analyzed the privatization of state corporations. One exception is the paper by Bárcena-Ruiz and Garzón (2017), who assume that if a plant of the state corporation is privatized it is fully sold to private investors. However, there is evidence showing that state corporations often partially privatize their firms. In this paper we analyze a government's decision on whether to partially privatize a state holding corporation with two plants that may produce complementary or substitute goods. The state holding corporation competes with two private plants, which may belong to different firms or to a multiplant firm. To privatize

the state holding corporation the government may sell off part of its two plants to a single private investor or to different private investors.

The result obtained in this paper helps to understand the different degrees to which state holding corporations have been privatized by governments in practice depending on whether the goods produced by firms are substitutes or complements. We find in the paper that the government partially privatizes the two plants of the state corporation but is indifferent between selling them to a single private investor or to different private investors.

When goods are complements the government keeps a larger stake in the state corporation if its plants are sold to different private investors than if they are sold to a single private investor. However, social welfare in equilibrium is the same in the two cases and thus the government is indifferent between them. Moreover, the government retains a larger stake if private firms are multiplant than if they are uniplant.

If goods are substitutes the contrary result is obtained. The government keeps a smaller stake in the state corporation if its plants are sold to different private investors than if they are sold to a single private investor. Social welfare in equilibrium is the same in the two cases so the government is indifferent between them. Moreover, the government retains a smaller stake if private firms are multiplant than if they are uniplant.

Finally, if goods are independent in demand the government keeps the same stake in the state corporation in all cases considered in this paper. From this it can be concluded that the stake that the government retains in the state corporation depends on the type of goods produced by the state corporation and the private firms, and on whether private firm are uniplant or multiplant.

One possible extension of the paper is to consider that the fraction of the public firm that is sold to the private sector is bought by a private firm already in the market. We leave this for future research.

APPENDIX: UNIPLANT PRIVATE FIRM

Proof of Lemma 1. From (1) to (6), we obtain that $V = q_{jA}\left(1 - bq_{iB} - q_{jB}\right) - ({q_{iA}}^2 + {q_{jA}}^2)(3 - \beta)/2 + ((1 - q_{iB})q_{iB} + (1 - bq_{iB})q_{jB} - {q_{jB}}^2)\beta + q_{iA}(1 - q_{iB} - b(2q_{jA} + q_{jB} -$

$\beta q_{jA}))$ and $\pi_{iB}=q_{iB}(1-q_{iA}-q_{iB}-b(q_{jA}+q_{jB}))-{q_{iB}}^2/2$, $i\neq j$; i, j=1, 2. In the second stage of the game semipublic firm A chooses the q_{1A} and q_{2A} that maximize V and private firm iB sets the q_{iB} that maximizes π_{iB}, i=1, 2. Solving these problems we obtain the first order conditions given by expression (8). It is easy to see that second order conditions from these problems hold. From (8) we obtain social welfare as a function of β given by expression (9).

In the first stage of the game the government chooses the optimal value of β that maximizes social welfare, given by expression (9). Given that W is a continuous function in β, with $0\leq\beta\leq1$, the solution to the above problem may be an interior solution (i.e. such that $\partial W/\partial\beta=0$) or a corner solution, whichever generates greater welfare. Solving this problem, we obtain the following first order condition: $4+b(1-\beta)-5\beta=0$, so the interior solution is $\beta^{SU}=(4+b)/(5+b)$ with $W^{SU}=(9+2b)/(2(7+6b+b^2))$. It can be shown that welfare for $\beta=0$ and $\beta=1$, respectively, is $W(\beta=0)=(40+44b+12b^2+b^3)/(8+b(7+b))^2$ and $W(\beta=1)=(16+9b)/(5+3b)^2$. Finally, W^{SU}-$W(\beta=0)=(4+5b+b^2)^2/(2(7+6b+b^2)(8+7b+b^2)^2)>0$ and W^{SU}-$W(\beta=1)=(1+b)^2/(2(5+3b)^2(7+6b+b^2))>0$, so β^{SU} is an overall maximum.

Proof of Lemma 2. From (1) to (5) and (7) we obtain that $V_i=q_{iA}(1-q_{iB}-b(q_{jA}+q_{jB}))-{q_{iA}}^2(3-\beta_i)/2-({q_{jA}}^2+{q_{iB}}^2-(1-q_{jB})q_{jB}-q_{jA}(1-bq_{iB}-q_{jB})-q_{iB}(1-bq_{jB}))\beta_i$ and $\pi_{iB}=q_{iB}(1-q_{iA}-q_{iB}-b(q_{jA}+q_{jB}))-{q_{iB}}^2/2$, $i\neq j$; i, j=1, 2. In the second stage of the game semipublic firm iA chooses the q_{iA} that maximizes V_i and private firm iB sets the q_{iB} that maximizes π_{iB}, i=1, 2. Solving these problems we obtain the first order conditions given by expression (10). It is easy to see that second order conditions from these problems hold. From (10) we obtain social welfare, W, as a function of β given by expression (11).

In the first stage of the game the government chooses the optimal value of β_1 and β_2 that maximizes W, given by expression (11). Given that W is a continuous function in β_1 and β_2, with $0\leq\beta_1\leq1$ and $0\leq\beta_2\leq1$, the solution to the above problem may be an interior solution or a corner solution. Solving this problem, we obtain the following first order conditions: $(8-b(4-\beta_j)-3\beta_j)((5\beta_i-4)(8-3\beta_j)^2+b^3\beta_i(4-\beta_j)^2-b^2(8(8-8\beta_j+$

$\beta_j{}^2) + \beta_i(16 - 4\beta_j + \beta_j{}^2)) + b(\beta_i(148\beta_j - 21\beta_j{}^2 - 208) + 4(64 - 52\beta_j + 7\beta_j{}^2))) = 0$, $i \neq j$; i, j=1, 2. These first order conditions equal to zero for three solutions: $\beta_i^{DU} = 4/(5 + b)$, i=1, 2; β_1=$4(14 - 8b - 2b^2 + b^3)/(21 - 12b - 2b^2 + b^3)$ and $\beta_2 = 4(2 - b)/(3 - b)$; β_2=$4(14 - 8b - 2b^2 + b^3)/(21 - 12b - 2b^2 + b^3)$ and $\beta_1 = 4(2 - b)/(3 - b)$. For $\beta_i^{DU} = 4/(5 + b)$, i=1, 2, we obtain that W^{DU}= $(9 + 2b)/(2(7 + b(6 + b)))$>0. For both the second and third solutions we obtain that $W = -(7 - b)(1 - b)/(4b^2) < 0$ so we reject them.

Welfare for β_1 =0 is the following: $W(\beta_1 = 0) = (640 + 4b^3(4 - \beta_2)^2 - 464\beta_2 + 77\beta_2{}^2 - b(384 - 264\beta_2 + 38\beta_2{}^2) - b^2(96 - 48\beta_2 + 7\beta_2{}^2))/(4(16 - b^2(4 - \beta_2) - 6\beta_2)^2)$. Welfare for β_1 =1 is the following: $W(\beta_1 = 1) = (9b^3(4 - \beta_2)^2 - b(632 - 440\beta_2 + 63\beta_2{}^2) - 2b^2(110 - 55\beta_2 + 8\beta_2{}^2) + 2(506 - 367\beta_2 + 61\beta_2{}^2))/(40 - 3b^2(4 - \beta_2) - 15\beta_2)^2$. Next we compare welfare in the interior solution with welfare at the corners:

i) W^{DU}- $W(\beta_1 = 0) = 128 - 128b + 32b^2 - \beta_2(208 - 168b + 16b^2 + 8b^3) + \beta_2{}^2(109 - 52b - 16b^2 + 4b^3 + b^4)/(4(7 + 6b + b^2)(16 - b^2(4 - \beta_2) - 6\beta_2)^2)$. The denominator of this expression is positive. The numerator is strictly convex in β_2 and is positive at its minimum when β_2=$4(26 - 21b + 2b^2 + b^3)/(109 - 52b - 16b^2 + 4b^3 + b^4)$. Therefore W^{DU}-$W(\beta_1 = 0)$ is positive for all values of b and β_2.

ii) W^{DU}- $W(\beta_1 = 1)$=$(232 - 96b - 32b^3 + 8b^4 - \beta_2(524 - 248b - 48b^2 - 8b^3 + 4b^4) + \beta_2{}^2(317 - 132b - 74b^2 + 12b^3 + 5b^4))/(2(7 + 6b + b^2)(40 - 12b^2 - 15\beta_2 + 3b^2\beta_2)^2)$. The denominator of this expression is positive. The numerator is strictly convex in β_2 and is positive at its minimum when β_2 = $2(131 - 62b - 12b^2 - 2b^3 + b^4)/(317 - 132b - 74b^2 + 12b^3 + 5b^4)$. Therefore, W^{DU}- $W(\beta_1 = 1)$ is positive for all values of b and β_2.

Due to symmetry we obtain a similar result for $\beta_2 = 0$ and for $\beta_2 = 1$. Therefore, $\beta_i^{DU} = 4/(5 + b)$, i=1, 2, is a global maximum.

Proof of Proposition 1. $\beta^{SU} - \beta_i^{DU} = (4 + b)/(5 + b) - 4/(5 + b)$ =$b/(5 + b)$, which is positive if goods are substitutes (b>0), negative if goods are complements (b<0), and zero if goods are independent in demand (b=0).

APPENDIX: MULTIPLANT PRIVATE FIRM

The proof in this case is similar to the case of uniplant private firms, so we provide only a schematic outline of the proof here.

i) Single private investor. In this case the two plants of the state corporation are sold in part to a single private investor. In the second stage of the game semipublic firm *A* chooses the output levels q_{1A} and q_{2A} that maximize its objective function, given by expression (6). Private firm *B* sets the output levels q_{1B} and q_{2B} that maximize its profit, given by expression (2) and (3). Solving these problems, we obtain the following first order conditions:

$$1 - q_{iB} - b(q_{jB} + q_{jA}(2 - \beta)) - q_{iA}(3 - \beta)=0,$$
$$1 - q_{iA} - 3q_{iB} - b\left(q_{jA} + 2q_{jB}\right) = 0,\ i\neq j;\ i, j=1, 2.$$

It can be shown that second order conditions hold. From the above first order conditions we obtain the following output of the firms and social welfare as a function of β:

$$q_{iA} = \frac{2+b}{8+5b(2-\beta)-3\beta+b^2(3-2\beta)},\ \ q_{iB} = \frac{2+b-\beta(1+b)}{8+5b(2-\beta)-3\beta+b^2(3-2\beta)},$$
$$W = \frac{(2(2+b)^2(5+4b)-2(1+b)(2+b)(7+5b)\beta+(1+b)^2(4+3b)\beta^2)}{(8+5b(2-\beta)-3\beta+b^2(3-2\beta))^2},\ i=1, 2.$$

In the first stage of the game the government chooses the optimal value of β that maximizes social welfare. Given that *W* is a continuous function in β, with $0\leq\beta\leq1$, the solution to the above problem may be an interior solution (i.e. such that $\partial W/\partial\beta = 0$) or a corner solution. Solving this problem, we obtain the following first order condition: $4 + b^2(1 - \beta) - 5\beta + b(4 - 5\beta) = 0$, so the interior solution is $\beta^{SM}=(2 + b)^2/(5 + 5b + b^2)$ and thus $W^{SM}=(9 + 8b + b^2)/(14 + 22b + 10b^2 + b^3)$. It can be shown that welfare for $\beta=0$ and $\beta=1$, respectively, is $W(\beta=0)=2(5 + 4b)/(4 + 3b)^2$ and $W(\beta=1)=(16 + 21b + 8b^2 + b^3)/(5 + 5b + b^2)^2$. Finally, it can be shown that W^{SM}-$W(\beta=0)=(2 + 3b + b^2)^2/((4 + 3b)^2(14 + 22b + 10b^2 + b^3)) >0$ and W^{SM}-$W(\beta=1)=(1 + b)^4/((5 + 5b + b^2)^2(14 + 22b + 10b^2 + b^3))>0$, so β^{SM} is an overall maximum.

Lemma A1: Under multiplant private firms, when the government partially sells the state corporation to a single private investor, in equilibrium:

$$\beta^{SM} = \frac{(2+b)^2}{5+5b+b^2},\ q_{iA}^{SM} = \frac{5+5b+b^2}{14+22b+10b^2+b^3},\ q_{iB}^{SM} = \frac{3+2b}{14+22b+10b^2+b^3},$$

$$\pi_{iA}^{SM} = \frac{(5+5b+b^2)(7+9b+3b^2)}{2(14+22b+10b^2+b^3)^2},\ \pi_{iB}^{SM} = \frac{(3+2b)^3}{2(14+22b+10b^2+b^3)^2},\ CS^{SM} = \frac{(1+b)(8+7b+b^2)^2}{(14+22b+10b^2+b^3)^2},$$

$$PS^{SM} = \frac{62+134b+103b^2+32b^3+3b^4}{(14+22b+10b^2+b^3)^2},\ W^{SM} = \frac{(9+8b+b^2)}{14+22b+10b^2+b^3},\ i = 1, 2.$$

It can be shown that β^{SM} increases (decreases) with b when goods are substitutes (complements). The explanation is similar to that provided in the previous lemmas, so we omit it.

ii) Different private investors. In this case, each plant of the state corporation is sold in part to a different private investor. In the second stage of the game, private firm *iB* sets the output level q_{iB} that maximizes its profit, given by expression (2). Each semipublic firm chooses the output level q_{iA} that maximizes its objective function given by expression (7). Solving these problems, we obtain the following first order conditions:

$$1 - q_{iB} - b(q_{jA} + q_{jB}) - q_{iA}(3 - \beta_i) = 0,$$

$$1 - q_{iA} - 3q_{iB} - b(q_{jA} + 2q_{jB}) = 0,\ i \neq j;\ i, j = 1, 2.$$

It can be shown that second order conditions hold. From the above first order conditions we obtain the following output of the firms and social welfare as a function of β_1 and β_2:

$$q_{iA} = \frac{(2+b)(8+b^2-3\beta_j-b(7-2\beta_j))}{b^4+b^2(\beta_i(11-4\beta_j)-11(3-\beta_j))+(8-3\beta_i)(8-3\beta_j)},$$

$$q_{iB} = \frac{(b^2(2-\beta_j)+(8-3\beta_j)(2-\beta_i)+b(\beta_j(5-2\beta_i)+6\beta_i-14))}{b^4+b^2(\beta_i(11-4\beta_j)-11(3-\beta_j))+(8-3\beta_i)(8-3\beta_j)},\ i \neq j;\ i, j = 1, 2,$$

$$W = (b^7 - 2b^6 + b^5(\beta_1(22 - 9\beta_2) + 22\beta_2 - 59) + b^4(92 + 2{\beta_1}^2(1 - \beta_2) -$$

$$41\beta_2 + 2{\beta_2}^2 + \beta_1(26\beta_2 - 2{\beta_2}^2 - 41)) + b^3(1020 - 673\beta_2 + 101{\beta_2}^2 +$$

$$\beta_1(454\beta_2 - 71{\beta_2}^2 - 673) + {\beta_1}^2(101 - 71\beta_2 + 12{\beta_2}^2)) +$$

$$2\left(\beta_1\left(672\beta_2 - 111{\beta_2}^2 - 928\right) + {\beta_1}^2\left(154 - 111\beta_2 + 18{\beta_2}^2\right) + 2\left(640 - 464\beta_2 + 77{\beta_2}^2\right)\right) +$$

$$b^2(1078\beta_2 - 173{\beta_2}^2 - 1560 + {\beta_1}^2(125\beta_2 - 20{\beta_2}^2 - 173) + \beta_1(1078 - 770\beta_2 + 125{\beta_2}^2)) +$$

$$b(\beta_1{}^2(127\beta_2 - 21\beta_2{}^2 - 180) - 4(416 - 290\beta_2 + 45\beta_2{}^2) + \beta_1(1160 - 804\beta_2 + 127\beta_2{}^2))))/$$
$$(b^4 + b^2(\beta_1(11 - 4\beta_2) - 11(3 - \beta_2)) + (8 - 3\beta_1)(8 - 3\beta_2))^2.$$

In the first stage of the game the government chooses the optimal values of β_1 and β_2 that maximize social welfare. Given that W is a continuous function in β_1 and β_2, with $0 \leq \beta_1 \leq 1$ and $0 \leq \beta_2 \leq 1$, the solution to the above problem may be an interior solution or a corner solution. Solving this problem, we obtain the following interior solution: $\beta_i^{DM} = (4 + 3b)/(5 + 5b + b^2)$, i=1, 2, and thus $W^{DM} = (9 + b(8 + b))/(14 + b(22 + b(10 + b)))$. Finally, it can be shown that W^{DM}- $W(\beta_i = 0)$>0 and W^{DU}- $W(\beta_i = 1)$>0 for all b, so β_i^{DM}, i=1, 2, is an overall maximum.

Lemma A2: With a multiplant private firm, when the government sells each plant of the state corporation to a different private investor, in equilibrium:

$$\beta_i^{DM} = \frac{4+3b}{5+5b+b^2},\ q_{iA}^{DM} = \frac{5+5b+b^2}{14+22b+10b^2+b^3},\ q_{iB}^{DM} = \frac{3+2b}{14+22b+10b^2+b^3},$$
$$\pi_{iA}^{DM} = \frac{(5+5b+b^2)(7+9b+3b^2)}{2(14+22b+10b^2+b^3)^2},\ \pi_{iB}^{DM} = \frac{(3+2b)^3}{2(14+22b+10b^2+b^3)^2},\ CS^{DM} = \frac{(1+b)(8+7b+b^2)^2}{(14+22b+10b^2+b^3)^2},$$
$$PS^{DM} = \frac{62+134b+103b^2+32b^3+3b^4}{(14+22b+10b^2+b^3)^2},\ W^{DM} = \frac{9+b(8+b)}{14+b(22+b(10+b))},\ i = 1, 2.$$

It can be shown that β_i^{DM} decreases with b ($\partial \beta_i^{DM}/\partial b < 0$). The explanation is similar to that provided in the previous lemmas, so we omit it.

Proof of Proposition 3. $\beta^{SM} - \beta_i^{DM} = b(1 + b)/(5 + 5b + b^2)$, which is positive if goods are substitutes (b>0), negative if goods are complements (b<0), and zero if goods are independent in demand (b=0).

Proof of Proposition 5. $\beta^{SU} - \beta^{SM} = b/((5 + b)(5 + 5b + b^2))$, $\beta^{SM} - \beta_i^{DU} = b(4 + 5b + b^2)/((5 + b)(5 + 5b + b^2))$ and $\beta_i^{DU} - \beta_i^{DM} = b(1 + b)/((5 + b)(5 + 5b + b^2))$, which are positive if goods are substitutes (b>0), negative if goods are complements (b<0), and zero if goods are independent in demand (b=0). Therefore, $\beta^{SU} > \beta^{SM} > \beta_i^{DU} > \beta_i^{DM}$ if b>0 and $\beta^{SU} < \beta^{SM} < \beta_i^{DU} < \beta_i^{DM}$ if b>0.

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