Partial privatization in an international mixed oligopoly under product differentiation[*]

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Abstract

We consider an international mixed market that comprises two countries, each of which owns one public firm and one private firm. As a benchmark case, we consider a single country that owns all four firms. In both cases, governments decide whether to partially privatize their public firms or not. Under an international mixed market privatization decisions are driven by strategic reasons, while in the benchmark case they are driven by efficiency reasons. We find that whether governments privatize more or less in the former case than in the latter depends on the type of goods produced by the firms (homogeneous, independent in demand, complements and substitutes). We also find that social welfare may be greater under an international mixed market than in the benchmark case. Finally, under an international mixed market there is more privatization when each public firm produces the same good as the domestic private firm than when they produce different goods.

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1 Introduction

In today's economy public firms compete with private firms in the product market in many industries. Examples include the markets for cars, ships, natural gas, airlines, rails, and steel manufactures, and in the provision of services such as telecommunications, banking, home loans, health care, hospitals, life insurance, broadcasting and education (De Fraja 2009; Haraguchi et al. 2018). This is especially so in Europe, where governments own firms in various sectors of the economy. For most of the second half of the 20th century, countries such as France, Norway, and West Germany had large state-owned enterprise sectors (Parker 1998; Parker and Saal 2003). State control is also significant in other countries such as Canada, Japan, China, Indonesia, Korea, Malaysia, Singapore, and Thailand (Chang 2007; Ishida and Matsushima 2009).[1]

In the last few decades there has been a wave of privatization of public firms worldwide. European countries been fully or partially privatizing some of their state corporations since the 1980's (Megginson and Netter 2001; Parker and Saal 2003).[2] There is substantial evidence of private firms competing in the product market with partially privatized firms (semipublic firms). Cases in point include the automobile industry, where Volkswagen (VW) and Renault, two semipublic firms, compete with private firms. VW is partly owned by the government of Lower Saxony, which holds a 11.8 percent stake in it (www.volkswagenag.com). Similarly, the French government owns a 15 percent stake in Renault (www.renault.com).[3]

There is competition between semipublic and private firms in a huge range of industries, leading the literature on partial privatization to become richer and more diverse (see Section 2). This literature usually assumes one public firm and a single country, so

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[1] Examples of public firms include Électricité de France, Korea Development Bank, Areva and the United States Postal Service (Lee et al. 2018).

[2] The OECD (2005) points out that in the EU governments are the largest shareholders in many partially privatized firms.

[3] The Japanese government partially privatized Japan Post, Postal Bank, and Kampo in 2015 (Haraguchi, et al. 2018). Since 1978 China has also implemented a privatization policy in order to reform its public firms (Bai et al. 2009). For more examples and new advances in the analysis of partial privatization see Heywood and Ye (2009), Bose et al. (2014) and Chen (2017).

the decision to partially privatize is taken by the government for efficiency reasons. However, in practice public and private firms from a given country usually compete in international mixed markets with firms from other countries.[4] A partially privatized public firm takes the consumer surplus into account, so the lower the percentage of the shares in the semipublic firm that is publicly-owned, the lower its output and market competition is. Thus, when a public firm is partially privatized it produces less, which affects the output and profits of rival firms. Those rival firms may be domestic and foreign owned, public and private, and may compete in the same or in related international markets. This means that governments may strategically choose the degree to which they privatize their public firms to influence market competition (and, thus, social welfare in their country). However, few papers have analyzed this issue (see Section 2). Moreover, those few papers assume that firms only produce a homogeneous good but in practice they may also produce heterogeneous goods. Thus, the fact that competition within homogeneous product markets coexists with competition across differentiated markets is ignored. As a result, the literature has not analyzed how the type of goods produced by firms influences the strategic decision whether to partially privatize public firms that compete in an international mixed market. The main objective of this paper is to fill that gap.

We consider an international mixed market that comprises two countries each of which has one domestic public firm, which can be partially privatized, and one domestic private firm. There are two differentiated goods, which may be independent in demand, substitutes or complements. Each firm produces only one good and firms sell their products in both countries. In this case each government can strategically sell off part of its public firm to domestic private investors to increase social welfare. Seeking to understand this strategic choice by governments, we take as a benchmark the case of a single country with two regions and one government. Each region has one public firm and one private firm, so in this case the government's decisions on whether to partially privatize its public firms are

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[4] For example, VW and Renault compete with each other but also with domestic and foreign private firms in the European market.

driven by efficiency reasons. In each country (region) the public firm and the private firm may produce the same good or different goods, so there are three possible cases.[5]

In Case 1 the firms in each country (region) produce the same good. An example that fits this case is the broadcasting of images and audio by means of radio waves, cable television networks, satellite television or the Internet. In many European countries there is a public broadcaster that competes with private television channels.[6] The types of programs that they broadcast are very similar and can thus be considered as homogeneous products. They compete with foreign private and public television channels through cable, satellite and the Internet. The programs offered by those foreign channels are substitutes for domestic television channels when the contents are similar, but complements if the programs are used to learn languages and in the case of thematic television channels.

In Case 2 the public firm in one country (region) and the private firm in the other produce the same good. A case in point is the following: The Ducati and Bugatti motorcycle brands are owned by the VW group (a German semipublic corporation). They compete with French private motorcycle producers such as Peugeot and Orcal. The French semipublic firm Renault also produces cars that compete with the German private car-makers BMW and Mercedes.[7] Cars and motorcycles are imperfect substitute products, but they can also be complementary.

Finally, in Case 3 the two public firms produce the same good. An example can be found in international passenger transport: Trenitalia and SNCF are high-speed rail operators owned by the Italian and French governments, respectively. They have been competing on international lines between France and Italy since 2010. Rail transport also

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[5] In the real world there are mixed oligopolies in many industries where public, semipublic and private firms may compete. These firms produce a great variety of products, which makes it complicated to design a model that cover all possible cases. We therefore focus on the most representative cases.

[6] Some countries have privatized part of their public television channels. For example, France privatized TF1 in 1987. Another country that has privatized public channels is Chile. In addition, there are semipublic corporations such as Orange and Deutsche Telekom that offer television channels.

[7] This example has limitations since Renault also competes with VW in the market for cars. It is difficult to find an example that fits perfectly with this case due to the wide variety of products that firms manufacture.

competes with bus transport, which is managed by private firms in both countries. Buses and trains are imperfect substitutes.[8]

We first compare the results obtained under strategic privatization with those obtained when firms are privatized by efficiency reasons. We find that comparison crucially depends on the type of goods produced by the firms.[9] When the firms in each country (region) produce the same good, there is more privatization with a single country than with two countries for goods which are close substitutes; however, if goods are complements or weak substitutes the opposite result is obtained. Moreover, if goods are close substitutes social welfare is greater with two countries although in that case the decision to partially privatize is taken for strategic reasons. When the public firm in one country (region) and the private firm in the other produce the same good, there is more privatization with a single country, and social welfare is no lower in that case. When the two public firms produce the same good the public firms are never privatized for goods which are complements and independent in demand, regardless of whether there is one country or two. However, if the goods are substitutes there is more privatization with a single country and social welfare is no lower in that case.

Next we compare the results obtained when there is strategic interaction between governments, i.e. when there is an international mixed market with two countries. We find that for both substitute and complementary goods there is more privatization when each public firm produces the same good as the domestic private firm than when they produce different goods. When the public firm in each country produces a different good from the private firm, the public firms remain fully public if the goods are complements and independent in demand. If the goods are weak substitutes there is less privatization when each public firm produces the same good as the foreign private firm. However, if

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[8] Another example is given by competition among telecommunications firms in the European market. They offer mobile phone, land line, Internet and television services. Deutsche Telekom and Orange are corporations partially owned by the German and French governments, respectively. They compete with private operators such as Telefonica Germany, SFR and Bouyagues Telecom. The services offered by Deutsche Telekom and Orange can be either complements or substitutes for the services provided by the private operators.

[9] A related paper is that by Bárcena-Ruiz and Garzón (2020), who analyze mergers between two local public firms when the two goods sold in the market are independent in demand, substitutes or complements.

the goods are close substitutes there is less privatization when the public firms produce the same good.

Finally, when there is no strategic interaction between countries we find that there is more privatization if the public firms produce different goods than if they produce the same good.

The analysis conducted in this paper enables economic policy recommendations to be made to governments. First, the policy of privatizating public firms has to take several factors into account: Whether firms compete on international markets or on the domestic market, and what type of goods they produce. We show that the degree to which public firms are privatized depends crucially on these factors. On the other hand, this study serves as a grounding for making economic policy recommendations as to whether countries in a free trade area (such as the EU) should centralize the privatization decisions of their public firms in a supranational authority (or, at least, create directives that restrict the decisions of countries). We show that if the firms in each country produce a homogeneous good, creating such a supranational authority would increase the joint welfare of the countries, except when the goods that they produce are close substitutes. If the firms in each country produce heterogeneous goods, they are never worse off if there is a supranational authority. The conclusion reached is that the type of goods produced by firms in different countries is crucial for recommending the creation of supranational authorities in charge of privatization policies.

The rest of the paper is organized as follows. Section 2 reviews the literature on privatization under a mixed oligopoly. Section 3 presents the model. Sections 4, 5, and 6 analyze partial privatization of public firms in Cases 1, 2, and 3, respectively, both when there is an international mixed market with two countries and when there is a single country. Section 7 compares the results in the three cases, and Section 8 concludes.

2 Review of the literature

The idea of a mixed oligopoly dates back several decades. The pioneering papers on the topic are those of Merrill and Schneider (1966) and Harris and Wiens (1980). Since then there has been numerous papers that analyze competition between public and private firms in mixed markets. However, given the wave of privatization that has taken place

worldwide in the last few decades, one of the topics most analyzed by the theoretical literature on mixed oligopoly is the decision by governments whether to fully privatize a single public firm (De Fraja and Delbono 1989, 1990; Corneo and Jeanne 1994; Pal and White 1998). These papers have been extended to analyze how different factors affect privatization, e.g. sequential decisions (Matsumura and Shimizu 2010), interdependence payoff structure (Matsumura and Okamura 2015), price and quantity competition (Matsumura and Ogawa 2012; Haraguchi and Matsumura 2016), collusion between firms (Correia-da-Silva and Pinho 2018), vertically related markets (Lee 2006; Shuai and Tomaru 2017; Bárcena-Ruiz and Garzón 2018a), switching costs (Dong and Bárcena-Ruiz 2016), endogenous timing of decisions (Tomaru and Saito 2010; Lee and Xu 2018), environmental policy (Bárcena-Ruiz and Garzón 2006; Dong et al. 2019), long-run competition (Anderson et al. 1997; Matsumura and Kanda 2005), endogenous product differentiation (Matsushima and Matsumura 2003, Matsumura and Matsushima 2003), and R&D competition (Poyago-Theotoky 1998; Ishibashi and Matsumura 2006).

In the wake of the global wave of privatization of public firms, there is substantial evidence of private firms competing in the product market with partially privatized firms. Given that evidence, an increasing number of papers in the theoretical literature on mixed oligopoly have analyzed the partial privatization of public firms. The literature on partial privatization began with the seminal paper by Matsumura (1998), which has been extended to analyze different factors that affect the partial privatization of public firms. The factors considered in this literature include foreign investment in partially privatized firms (Lin and Matsumura 2012), cross-ownership of firms (Jain and Pal 2012; Cai and Karasawa-Ohtashiro 2015), trade policies (Chao and Yu 2006; Long and Stähler 2009; Cato and Matsumura 2105), free entry (Matsumura and Kanda 2005; Wang and Chen 2010; Cato and Matsumura 2012; Lee et al. 2018), endogenous timing of decisions (Bárcena-Ruiz and Garzón 2010; Escrihuela-Villar and Gutiérrez-Hita 2019), R&D competition (Heywood and Ye 2009), environmental problems (Kato 2006; Ohori 2006; Fridman 2018), mergers (Bárcena-Ruiz and Garzón 2003; Mendez-Naya 2008; Beladi et al. 2013; Colombo 2016), consumer social responsibility (Kim et al. 2019), and product differentiation (Lu and Poddar 2007; Fujiwara 2006, 2007; Kumar and Saha 2008; Haraguchi et al. 2018).

The publications cited above analyze partial privatization on the assumption that there is one public firm and a single country so the decision to partially privatize is taken for efficiency reasons.[10] However, public and private firms from a given country usually compete in international mixed markets with public and private firms from other countries. This means that governments may partially privatize their public firms for strategic reasons, but few papers have analyzed this issue.[11] In this regard, Han and Ogawa (2008) consider a mixed oligopoly model with two countries each of which owns public and private firms producing a homogeneous good that compete in an international market. They find that governments partially privatize their public firms and that the extent of privatization in an international mixed market with two countries is smaller than in a mixed market with a single country. Lee et al. (2013) extend that paper by considering tariff policies.

The papers cited above assume that all firms produce a homogeneous good, but in today's economy public and private firms may also produce heterogeneous goods.[12] In this regard, Haraguchi et al. (2018) study the optimal degree of privatization in a mixed oligopoly in which one public firm competes with n private firms in the same market and with m private firms in the neighboring market. They consider that competition within homogeneous product markets coexist with competition across two differentiated

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[10] One exception is the paper by Kato (2006), who analyzes the decision to partially privatize in a single country where several public firms compete with several private firms. In this case, this decision is taken for efficiency reasons since the analysis is restricted to one country.

[11] The decision of governments on whether to fully privatize their public firms for strategic reasons has been analyzed in several papers. Bárcena-Ruiz and Garzón (2005a) consider a single market comprising two countries, each with one public firm and n private firms that produce a homogeneous good using quadratic cost functions. Bárcena-Ruiz and Garzón (2005b) consider a similar framework but assume that firms have constant marginal costs of production and that the public firms are less efficient than the private firms. Czerny et al. (2014) consider a spatial competition model and investigate the effect of port privatization in a setting with two ports located in different countries. Xu et al. (2016) study the effect of the excess burden of taxation on the privatization of public firms in a two-country, two-mixed-markets model.

[12] Cremer et al. (1991) and Anderson et al. (1997) were the first to analyze product differentiation in mixed oligopolies, using the Hotelling model and monopolistic competition, respectively. Later, Matsumura and Matsushima (2004), Matsumura et al. (2009) and Bárcena-Ruiz and Casado-Izaga (2018) use a spatial competition model to analyze the importance of product differentiation.

markets. However, the decision as to whether to partially privatize for strategic reasons is not analyzed since they consider only one country and one public firm. In this paper we extend the model of Haraguchi et al. (2018) to consider strategic interaction between governments.[13]

3 The Model

We consider an international mixed market with two countries denoted by *I* and *II*. Country *k* has a domestic public firm and a domestic private firm denoted by *kA* and *kB*, respectively, *k = I, II*. As a benchmark we consider the case of a single country with two regions, *I* and *II*, and one government. Each region has one public firm and one private firm. In both cases there are two differentiated goods, denoted by 1 and 2, that may be substitutes, complements or independent in demand. Each firm produces only one good. The firms located in each country (region) may produce the same good or different goods, so there are three possible cases. In Case 1 the firms in a given country (region) produce the same good, which is different from that produced by the firms in the other country (region); in Case 2 the domestic public firm in one country (region) and the private firm in the other country (region) produce the same good; and in Case 3 the two public firms produce the same good.

Table 1 shows the goods, *i* and *j*, that are produced by the firms of each country (region) in the different cases, where $i \neq j; i, j = 1, 2$.

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[13] A related paper is that by Dong et al. (2018), who analyze partial privatization of a state holding corporation with two plants that produce differentiated goods which may be substitutes or complements, assuming a single country. They consider a public firm with two plants and just one country, so there is no partial privatization for strategic reasons. This paper is extended by Bárcena-Ruiz et al. (2019) assuming a partially foreign-owned private sector.

		Case 1	Case 2	Case 3
Country (region) I	Domestic public firm:	good i	good i	good i
	Domestic private firm:	good i	good j	good j
Country (region) II	Domestic public firm:	good j	good j	good i
	Domestic private firm:	good j	good i	good j

Table 1 Types of goods produced by the firms in the three cases

On the consumption side, there is a continuum of consumers of the same type, and the representative consumer maximizes $U(q_1, q_2) - p_1 q_1 - p_2 q_2$, where p_i is the price of good i and q_i is the amount of good i, $i = 1, 2$. The function $U(q_1, q_2)$ is assumed to be quadratic, strictly concave and symmetric in q_1 and q_2. Specifically, we assume that $U(q_1, q_2) = a (q_1 + q_2) - ((q_1)^2 + 2bq_1 q_2 + (q_2)^2)/2$, $-1<b<1$. Thus, industry inverse demand functions are:

$$p_i = a - q_i - bq_j, \; i \neq j; \; i, j = 1, 2, \tag{1}$$

where if $b>0$ goods 1 and 2 are substitutes, if $b<0$ they are complements, and if $b=0$ the demands for the goods are independent.

Following De Fraja and Delbono (1989) and Bárcena-Ruiz and Garzón (2005*a*), we assume that firms have the same technology represented by the following quadratic cost function: $C(q_i^{km}) = c(q_i^{km})^2$, $i = 1, 2$; $k = I, II$; $m=A, B$. Therefore, the profit function of firm km, which produces good i, is:

$$\pi_i^{km} = p_i \, q_i^{km} - c(q_i^{km})^2, \; i = 1, 2; \; k = I, II; \; m=A, B. \tag{2}$$

In the case of an international mixed market the producer surplus in country k, denoted by PS^k ($k = I, II$), comprises the profit of the domestic firms. Products 1 and 2 are heterogeneous, so the consumer surplus is given by $CS = (q_1)^2/2 + bq_1q_2 + (q_2)^2/2$. We assume that each country has half of the consumer population so the consumer surplus in country k, $k = I, II$, is: $CS^k = CS/2$ (see Duval and Hamilton, 2002; Bárcena-Ruiz and Garzón, 2014). The social welfare function considered by country k comprises the consumer surplus of domestic consumers, CS^k, and the producer surplus of domestic firms, PS^k:

$$W^k = CS^k + PS^k, \; k = I, II. \tag{3}$$

To increase social welfare each government can sell off part of its public firm to domestic private investors, so it is then jointly owned by the public and private sectors. We assume that government k owns β^k percent of the shares and private investors own the remaining $(1-\beta^k)$ percent, so the partially privatized firm maximizes the weighted average of social welfare and firm profits (Matsumura, 1998). Therefore, if the government sells part of the public firm to domestic investors, the objective function of this firm is given by:

$$V^{kA} = \beta^k W^k + (1 - \beta^k)\, \pi_i^{kA},\; 0 \leq \beta^k \leq 1,\; k = I, II;\; i=1, 2, \tag{4}$$

where if $\beta^k = 1$ firm kA remains fully public, and if $\beta^k = 0$ firm kA is fully privatized. As usual, the objective function of each private firm is its profit, given by (2).

In the benchmark case the social welfare function of the country, W, includes the consumer surplus, CS, and the producer surplus, PS: $W = CS + PS$, where $CS = (q_1)^2/2 + bq_1q_2 + (q_2)^2/2$, and the producer surplus comprises the sum of the profit of the four firms. If the government sells part of public firm kA to domestic investors, the objective function of this firm is given by:

$$V^{kA} = \beta^k (W) + (1 - \beta^k)\, \pi_i^{kA},\; 0 \leq \beta^k \leq 1,\; k = I, II;\; i=1, 2. \tag{5}$$

To analyze the governments' decisions on whether or not to partially privatize their public firms, we propose a two-stage game. In the first stage, each government decides what percentage of the shares in the public firm it will sell to domestic private investors in the case of two countries; in the case of a single country, the government decides what percentage of the shares in the public firms it will sell to domestic private investors. In the second stage, firms make production decisions simultaneously. The equilibrium concept used is the subgame perfect equilibrium solved by backward induction. To simplify the presentation of results we assume that $c=1$.[14]

To explain the findings in the paper two facts must be taken into account: First, semipublic firms take the consumer surplus into account when taking production decisions, which leads them to produce more than private firms. Thus, the greater the percentage of the shares in the semipublic firm that is publicly-owned, the greater its

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[14] It can be shown that the main results of the paper hold for values of c other than 1.

output and market competition are. Second, the output of each firm is affected by the output of the others. When firms produce substitute goods they compete in the product market, and this competition is stronger as goods are more substitutes. In addition, in this case if a given firm increases its production the other firms reduce theirs. When firms produce complementary goods we may interpret that they cooperate since if a given firm produces more the other firms reacts by also producing more.

To analyze the strategic behavior of governments when they decide whether or not to partially privatize their public firms in the three cases considered, we compare the results when there are two countries with those when there is a single country. In the first case, governments take decisions on partial privatization for strategic reasons while in the second they take them for efficiency reasons.

4 Case 1: The firms in each country or region produce a homogeneous good

In this case there are two public firms (IA, IIA) and two private firms (IB, IIB). Firms IA and IB located in country (region) I produce good i and firms IIA and IIB located in country (region) II produce good j, so $q_i = q_i^{IA} + q_i^{IB}$ and $q_j = q_j^{IIA} + q_j^{IIB}$, $i \neq j$; $i, j = 1, 2$. Thus, from (1), industry inverse demand functions are:

$$p_i = a - (q_i^{IA} + q_i^{IB}) - b(q_j^{IIA} + q_j^{IIB}), i \neq j; i, j = 1, 2.$$

4.1 An international mixed market with two countries

In an international mixed market with two countries the government of each country strategically chooses its stake in the domestic public firm. In the second stage of the game, private firm kB sets the output level q_i^{kB} that maximizes its profit function, given by expression (2), $i=1, 2, k= I, II$. Semipublic firms IA and IIA choose the output levels q_i^{IA} and q_j^{IIA} that maximize their objective functions, V^I and V^{II}, respectively, given by expression (4). Solving these problems, we obtain the output of firms and social welfare as a function of β^I and β^{II} (these expressions are relegated to the Appendix). It can be shown from the first order conditions in outputs that if firms produce a homogeneous good or if they produce substitute goods then the production of a given firm decreases as

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the production of the others increase. However, if firms produce complementary goods, the production of a given firm increases with that of the others.[15]

In the first stage of the game the optimal value of β^k that maximizes the social welfare of country k given by (3) is chosen, $k = I, II$. Let subscript $C1$ denote Case 1 when there are two countries. Due to the symmetry of the model, we denote by β_{C1} the stake that each government retains in its public firm ($\beta_{C1} = \beta_{C1}^k = \beta_{C1}^l, k \neq l; k, l = I, II$). In addition, CS_{C1}, PS_{C1} and W_{C1} denote the total consumer surplus, the total producer surplus and the joint welfare of the two countries, respectively.[16] Let $H_{C1} = (7396 + 1548b - 863b^2 + 144b^3 + 64b^4)^{1/2}$. Solving the above problems, we obtain the following result:

For $b \neq 0$, $\beta_{C1} = \frac{3(104 + 21b - 8b^2 - H_{C1})}{5(19 + 3b - 4b^2)}$, $CS_{C1} = \frac{4a^2(1+b)(86 + 9b - 32b^2 + H_{C1})^2}{25(86 - 19b - 41b^2 - 8b^3 + (1+b)H_{C1})^2}$,

$PS_{C1} = PS_{C1}^k + PS_{C1}^l = 8a^2(14792 + 774b - 17183b^2 + 1056b^3 + 3463b^4 - 192b^5 - 64b^6 + (172 - 9b + 22b^2 - 15b^3 - 8b^4)H_{C1})/(25(86 - 19b - 41b^2 - 8b^3 + (1+b)H_{C1})^2)$, and $W_{C1} = W_{C1}^k + W_{C1}^l = \frac{2a^2(344 + 1962b + 1223b^2 - 144b^3 - 64b^4 + (104 - 9b - 8b^2)H_{C1})}{900(43 + 29b + 4b^2)}$, $k \neq l; k, l = I, II$.

For $b = 0$, $\beta_{C1} \in [0, 1]$ and $W_{C1} = W_{C1}^k + W_{C1}^l = \frac{12a^2}{25}$, $k \neq l; k, l = I, II$.

In view of the above we obtain the following result.

Proposition 1 *When there are two countries and the firms in each country produce a homogeneous good that is different from that produced by the firms of the other country, in equilibrium:* $0 < \beta_{C1} < 1$ *for* $b \neq 0$, *where* $\partial\beta_{C1}/\partial b > 0$.[17]

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[15] The same result is obtained in Cases 2 and 3 considered in the paper.

[16] The equilibrium results obtained in the Cases 2 and 3 are defined in a similar way.

[17] We find that $\beta_{C1} \in [0, 1]$ for $b = 0$. When $b=0$ the firms in each country produce the same good, which is independent in demand from that produced by the firms in the other country. As a result, there is no strategic behavior by governments, so domestic welfare does not depend on the stake that each government holds in its domestic public firm, which means that $\beta_{C1} \in [0, 1]$ for $b=0$. This result relies on the assumptions made in the model. Thus, for example, it does not hold if countries have different sizes or if firms have different costs.

We find in this case that governments partially privatize their public firms ($0 < \beta_{C1} <$ 1) for $b \neq 0$. This result is explained by two effects that go in opposite directions. First, each government internalizes the idea that the domestic public firm and the domestic private firm produce the same good. Thus, each government partially privatizes its public firm to reduce its output, which benefits its domestic private firm. This effect leads each government to seek to reduce β. Second, goods i and j can be substitutes or complements, so each government behaves strategically to influence the production of its public firm. If goods are substitutes, each government encourages its public firm to produce more to gain market share and welfare at the expense of the rival country. In this case each government seeks to increase β. If goods are complements, each government provides incentives to its public firm to produce more, which makes the firms of the other country produce more. In this case each government seeks also to increase β. The first effect dominates which means that governments partially privatize their public firms. The second effect keeps the public firm from being fully privatized. Moreover, as b increases, the goods produced in the two countries become less complementary or closer substitutes so each government encourages its public firm to gain market share at the expense of the other country. As a result, the stakes in the semipublic firms retained by the governments increases with b ($\partial \beta_{C1}/\partial b > 0$).

Next, we consider the benchmark case of a single country.

4.2 A single country

We now assume one country with two regions, *I* and *II*, that has two public firms (*IA*, *IIA*) and two private firms (*IB*, *IIB*). Firms *IA* and *IB* produce good i and firms *IIA* and *IIB* produce good j, where $i \neq j$; $i, j = 1, 2$.

In the second stage of the game, private firm kB sets the output level q_i^{kB} that maximizes its profit function, given by expression (2), $i=1, 2$, $k= I, II$. Semipublic firm kA chooses the output level q_i^{kA} that maximizes the weighted average of the social welfare of the country and its profit, given by expression (5). Solving these problems, we obtain the output of

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firms and social welfare as a function of β^I and β^{II}; to simplify the presentation of results, these expressions are relegated to the Appendix.

In the first stage of the game the optimal values of β^I and β^{II} that maximize the welfare of the country are chosen. Let subscript $S1$ denote Case 1 when there is a single country. Solving the above problems, we obtain the following result:

$$\beta_{S1} = \frac{9}{11+2b}, \quad CS_{S1} = \frac{a^2(1+b)(19+4b)^2}{(43+29b+4b^2)^2}, \quad PS_{S1} = \frac{2a^2(271+134b+16b^2)}{(43+29b+4b^2)^2},$$

$$W_{S1} = \frac{a^2(21+4b)}{(43+29b+4b^2)}.$$

In view of the above we obtain the following result.

Proposition 2 *When there is a single country and the firms in each region produce the same good, in equilibrium:* $0 < \beta_{S1} < 1$*, where* $\partial\beta_{S1}/\partial b < 0$*.*

In this case we find that each public firm is partially privatized $(0<\beta_{S1}<1)$. This result is explained by two effects that go in opposite directions: On the one hand, firms kA and kB produce homogeneous goods, so the greater the output of the public firm, kA, the lower the output of the private firm kB. As a result, the government partially privatizes the public firms (which means reducing β_{S1}) to encourage them to produce less so as to get the private firms to produce more. On the other hand, the government internalizes the idea that goods i and j are complements (substitutes) and the output of good i increases (decreases) with that of good j, so the government tries to increase (reduce) its stake in the semipublic firm. The first effect dominates the second when goods i and j are complements and reinforces it when they are substitutes. Moreover, for a given stake in the public firm, as parameter b increases market competition becomes stronger, since the goods are less complementary or closer substitutes. The government internalizes this idea and to reduce market competition the stake in the semipublic firms retained by the government becomes smaller with b $(\partial\beta_{S1}/\partial b < 0)$.

4.3 Comparison of results

Comparing the results obtained in Propositions 1 and 2, we obtain the following.

Proposition 3 *In equilibrium: for $b \neq 0$, $\beta_{C1} > \beta_{S1}$ if and only if $b > 0.5870$. Moreover, $W_{S1} > W_{C1}$ if and only if $b < 0.8484$.*

Proposition 3 shows that, when the firms in each country or region produce the same good and the goods are complements and weak substitutes ($-1 < b < 0.5870$) there is more privatization with two countries than with just one. This is because with two countries partial privatization decisions are taken for strategic reasons, while with a single country they are taken for efficiency reasons (i.e. $\beta_{C1} < \beta_{S1}$). When the goods are close substitutes ($b > 0.5870$), with two countries strategic behavior by governments leads them to privatize less than with a single country (i.e. $\beta_{C1} > \beta_{S1}$).

The explanation of the above result is the following: If the goods are complements, when there is a single country the government internalizes the idea that goods i and j are complements so the output of the firms producing good i increases with that of the firms producing good j. However, when there are two countries their governments do not internalize that goods are complements since they act strategically. As a result, the percentage of the shares in the public firms that is sold to private investors is lower in the former case (i.e. $\beta_{C1} < \beta_{S1}$). If the goods are substitutes, with two countries the government of each country uses its privatization policy strategically to gain market share and welfare at the expense of the other country. This effect becomes stronger the closer substitutes goods i and j are, so β_{C1} increases with b. With a single country, the government internalizes the idea that goods i and j are substitutes and that market competition is strong, so β_1 decreases with b. This means that $\beta_{C1} > \beta_{S1}$ if b is sufficiently large ($b > 0.5870$) and that $\beta_{C1} < \beta_{S1}$ if b is sufficiently small ($b < 0.5870$).

Comparing the joint welfare obtained by the two countries when governments act strategically (W_{C1}), with that obtained when partial privatization decisions are taken by efficiency reasons (W_{S1}), reveals that $W_{S1} > W_{C1}$ if and only if $b < 0.8484$. This result is mainly driven by the consumer surplus. It can be checked that $CS_{S1} > CS_{C1}$ if and only if $b < 0.8484$. When $b < 0.5870$ the stake owned by the government in the public firms is

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greater in the case of a single country (i.e. $\beta_{C1} < \beta_{S1}$), so the output of the public firms and thus the consumer surplus is also greater. When $b > 0.5870$ that stake is lower in the case of a single country (i.e. $\beta_{C1} > \beta_{S1}$), but the output of the private firms means that the consumer surplus is greater in this case if $b < 0.8484$. Only when the goods are close substitutes ($b > 0.8484$) is the result reversed so that the consumer surplus and, thus, the joint social welfare is greater with two countries.

5 Case 2: The public firm of a given country (region) produces the same good that the private firm of the other country (region)

In this case public firm *IA* and private firm *IIB* produce good i and the other firms produce good j, so $q_i = q_i^{IA} + q_i^{IIB}$ and $q_j = q_j^{IIA} + q_j^{IB}$, $i \neq j$; $i, j = 1, 2$. Thus, from (1), industry inverse demand functions are:

$$p_i = a - (q_i^{IA} + q_i^{IIB}) - b(q_j^{IIA} + q_j^{IB}),\ i \neq j;\ i, j = 1, 2.$$

5.1 An international mixed market with two countries

In the second stage of the game, private firm kB sets the output level q_i^{kB} that maximizes its profit function, given by expression (2), $i = 1, 2$, $k = I, II$. Semipublic firms *IA* and *IIA* choose, respectively, the output levels q_i^{IA} and q_j^{IIA} that maximize their objective functions, given by expression (4). Solving these problems, we obtain the output of firms and social welfare as a function of β^I and β^{II} (these expressions are relegated to the Appendix). In the first stage of the game government k chooses the optimal value of β^k, $k = I, II$, that maximizes the social welfare of country k, given by expression (3). Let subscript $C2$ denote Case 2 with two countries. Let $H_{C2} = (10609 + 1768b - 3812b^2 - 680b^3 + 340b^4 + 64b^5)^{1/2}$. Solving these problems, we obtain the following result:

$$\beta_{C2} = min\{\frac{3(127+10b-16b^2-4b^3-H_{C2})}{69+20b+8b^2-5b^3-2b^4}, 1\},$$

$$W_{C2} = W_{C2}^k + W_{C2}^l = 8a^2\ (-11092 + 1767b + 27759b^2 + 6284b^3 - 7786b^4 -$$

$$2728b^5 + 384b^6 + 284b^7 + 32b^8 + (334 + 103b - 145b^2 - 40b^3 + 30b^4 + 6b^5)H_{C2})/$$

$$(3(103 - 63b - 32b^2 + 18b^3 + 4b^4) + (1 + b)(3 + 2b)H_{C2})^2, k \neq l; \ k, \ l = I, II.$$

In view of the above we obtain the following result.

Proposition 4 *When there are two countries and each public firm produces the same good as the foreign private firm, in equilibrium: $\beta_{C2} = 1$ for $b \leq 0.1912$, and $0 < \beta_{C2} < 1$ with $\partial\beta_{C2}/\partial b < 0$ for $b > 0.1912$.*

Proposition 4 shows that the two public firms remain fully public (i.e. $\beta_{C2} = 1$) for $b \leq$ 0.1912. Otherwise each government partially privatizes its public firm ($0 < \beta_{C2} < 1$); in that case, the percentage of shares that remains public decreases with b ($\partial\beta_{C2}/\partial b < 0$). These results are explained below.

In this case, each public firm produces the same good as the foreign private firm, and thus goods can be complementary or substitutes of those produced by the other firms. When the goods are complements ($b<0$), governments choose $\beta_{C2}=1$ because each public firm is encouraged to produce more for two reasons: To gain market share at the expense of the foreign private firm and to benefit the domestic private firm. If the goods are weak substitutes ($0 < b \leq 0.1912$), gaining market share at the expense of the foreign firm leads governments to choose $\beta_{C2} = 1$.[18] If the goods are sufficiently close substitutes ($b > 0.1912$), gaining market share harms the domestic private firm. This leads governments to partially privatize their public firms in order to encourage them to produce less (i.e. $0 < \beta_{C2} < 1$), where β_{C2} becomes lower as the goods are more substitutes ($\partial\beta_{C2}/\partial b < 0$).

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[18] This result is in sharp contrast with that of Matsumura (1998), who finds that the public firms are partially privatized under moderate conditions. Matsumura and Kanda (2005) also find that the public firm remains fully public in a free entry market, and Sato and Matsumura (2019) obtain the same result assuming dynamic privatization. However, the mechanism considered in our paper is different from theirs since we consider strategic privatization.

5.2 A single country

We now assume one country that has two public firms (*IA*, *IIA*) and two private firms (*IB*, *IIB*). Let subscript *S*2 denote Case 2 when there is a single country. We obtain the same result as in Proposition 2 since there is no strategic interaction between governments and public firms produce heterogeneous goods, as in Case 1. Thus, $\beta_{S2} = \beta_{S1}$ and $W_{S2} = W_{S1}$.

5.3 Comparison of results

Comparing the results in Propositions 2 and 4 reveals the following.

Proposition 5 *In equilibrium:* $\beta_{C2} > \beta_{S2}$ *for all values of parameter b. Moreover,* $W_{S2} > W_{C2}$ *except for* b = -0.6115 *where* $W_{S2} = W_{C2}$.

Proposition 5 shows that, with two countries, strategic behavior by governments leads them to privatize less than with a single country, when privatization decisions are taken for efficiency reasons (i. e. $\beta_{C2} > \beta_{S2}$). This is because each public firm produces the same good as the private firm in the other country, but that good may be a complement or a substitute of that produced by the domestic private firm. This leads public firms to act strategically seeking to gain market share at the expense of the other country, so governments privatize less than with a single country. This strategic behavior implies excessive market competition, which reduces social welfare, so $W_{S2} > W_{C2}$ except for b= -0.6115 where $W_{S2} = W_{C2}$.

6 Case 3: Both public firms produce a homogeneous good

In this case the two public firms produce good *i* and the two private firms produce good *j*, so $q_i = q_i^{IA} + q_i^{IIA}$ and $q_j = q_j^{IB} + q_j^{IIB}$, $i \neq j$; $i, j = 1, 2$. Thus, from (1), industry inverse demand functions are:

$$p_i = a - (q_i^{IA} + q_i^{IIA}) - b(q_j^{IB} + q_j^{IIB}), i \neq j; i, j = 1, 2.$$

6.1 An international mixed market with two countries

In the second stage of the game, private firm kB sets the output level q_i^{kB} that maximizes its profit function, given by expression (2), $i=1, 2, k= I, II$. Semipublic firm kA chooses the output level q_i^{kA} that maximizes its objective function, given by expression (4). Solving these problems, we obtain the output of firms and social welfare as a function of β^I and β^{II} (these expressions are relegated to the Appendix). In the first stage of the game, government k chooses the optimal value of β^k, $k = I, II$, that maximizes the social welfare of country k, given by expression (3). Let subscript $C3$ denote Case 3 with two countries. Let $H_{C3} = (30625 - 24200b - 3384b^2 + 6320b^3 - 816b^4 - 320b^5 + 64b^6)^{1/2}$. Solving the above problems, we obtain the following result:

$$\beta_{C3} = \min\left\{\frac{225 - 96b - 20b^2 + 8b^3 - H_{C3}}{10(5-2b)}, 1\right\}, W_{C3} = W_{C3}^k + W_{C3}^l =$$

$$\frac{16a^2\left(b(10225 - 13088b + 5190b^2 - 232b^3 - 280b^4 + 48b^5) + (175 - 203b + 70b^2 + 6b^3)H_{C3}\right)}{25(25 - 4b - 20b^2 + 8b^3 + H_{C3})^2}, k \neq l; k, l = I, II.$$

In view of the above we obtain the following result.

Proposition 6 *When there are two countries and both public firms produce the same good, in equilibrium:* $\beta_{C3} = 1$ *for* $b \leq 0$, *and* $0 < \beta_{C3} < 1$ *with* $\partial\beta_{C3}/\partial b < 0$ *for* $b > 0$.

Proposition 6 shows that public firms remain fully public ($\beta_{C3} = 1$) when the goods are complements or independent in demand, and are partially privatized ($0<\beta_{C3}<1$) if the goods are substitutes. This is because the two public firms produce the same good which can be a substitute or a complement of that produced by the private firms. When the goods are complements, each public firm tries to gain market share at the expense of the rival public firm, which benefits the domestic private firm, so governments choose $\beta_{C3}=1$. If the goods are substitutes, competition between the public firms harms the domestic private firm, which leads them to produce less. This means that $0 < \beta_{C3} < 1$, where β_{C3} becomes smaller as the goods are more substitutes ($\partial\beta_{C3}/\partial b < 0$).

6.2 A single country

We now assume a single country with two regions that has two public firms (*IA*, *IIA*) and two private firms (*IB*, *IIB*). Public firms produce good *i* and private firms produce good *j*. In the second stage of the game, private firm *kB* sets the output level q_i^{kB} that maximizes its profit function, given by expression (2), *i*=1, 2, *k*= *I*, *II*. Semipublic firm *kA* chooses the output level q_i^{kA} that maximizes the weighted average of the social welfare of the country and its profit, given by expression (5). Solving these problems, we obtain the output of firms and social welfare as a function of β^I and β^{II} (these expressions are relegated to the Appendix). In the first stage of the game the optimal values of β^I and β^{II} that maximize the welfare of the country are chosen. Let subscript *S3* denote Case 3 with a single country. Solving the above problems, we obtain the following result:

$$\beta_{S3} = min\{\frac{(5-2b)^2}{25-12b}, 1\}, W_{S3} = \frac{a^2(49-24b)}{4(25-6b^2)}.$$

In view of the above we obtain the following result.

Proposition 7 *When there is a single country and both public firms produce the same good, in equilibrium:* $\beta_{S3} = 1$ *for* $b \leq 0$; $0 < \beta_{S3} < 1$ *for* $b > 0$, *where* $\partial \beta_{S3}/\partial b < 0$.

Proposition 7 shows that public firms remain fully public ($\beta_{S3} = 1$) when the goods are complements or independent in demand, and are partially privatized ($0<\beta_{S3}<1$) if the goods are substitutes. This is because the two public firms produce the same good, which is a substitute or a complement of the good produced by the private firms. When the goods are complements, given that the public firms take into account the welfare of the country, they are encouraged to produce more since it benefits private firms, so β_{S3}=1. If the goods are substitutes, competition between the public firms harms the private firms, so public firms are encouraged to produce less. This means that they are partially privatized ($0<\beta_{S3}<1$), where β_{S3} becomes smaller as the goods become closer substitutes ($\partial \beta_{S3}/\partial b < 0$).

6.3 Comparison of results

Comparing the results in Propositions 6 and 7 reveals the following.

Proposition 8 *In equilibrium: $\beta_{C3} = \beta_{S3} = 1$ for $b \leq 0$, and $1 > \beta_{C3} > \beta_{S3} > 0$ for $b > 0$. Moreover, $W_{S3} > W_{C3}$ except for $b = 0$ where $W_{S3} = W_{C3}$.*

Proposition 8 shows that, when both public firms produce the same good, the public firm remains fully public in both cases if the goods are complements and independent in demand. However, if the goods are substitutes with two countries strategic behavior by governments leads them to privatize less than with a single country. This is because the two public firms produce the same good, which may be a complement or a substitute of that produced by private firms. This leads public firms owned by the different governments to act strategically to gain market share and welfare at the expense of the other country, so governments privatize less than with a single country. This strategic behavior implies excessive market competition, which reduces social welfare, so $W_{S3} > W_{C3}$ except for $b=0$ where $W_{S3} = W_{C3}$.

7 Comparison of the results in the three cases

In this section we compare the results obtained in Cases 1 to 3. First we compare the results obtained with a single country.

Proposition 9 *When there is a single country: $\beta_{S3} = 1 > \beta_{S1} = \beta_{S2} > 0$ if $b \leq 0$ and $1 > \beta_{S3} > \beta_{S1} = \beta_{S2} > 0$ if $b > 0$.*

The result obtained in Proposition 9 shows that in the case of a single country the type of good produced by the public firms affects the privatization policy of the government. We find that there is more privatization if the public firms produce heterogeneous goods (Cases 1 and 2) than if they produce the same good (Case 3).

As shown above, when the firms in each region produce the same good (Case 1) and when each public firm produce the same good than the private firm of the other region (Case 2) the government partially privatize its public firms when goods are complements. However, when both public firms produce the same good (Case 3), the public firms remain fully public. As a result, when the goods are complements there is more privatization in Cases 1 and 2 than in Case 3. When the goods are substitutes the public firms are partially privatized in all cases. However, there is more privatization in Cases 1 and 2 than in Case 3. In all three cases the greater the output of the public firms is the lower the output of the private firms is. In Cases 1 and 2 each public firm produces the same good as one private firm and an imperfect substitute good of the other, but in Case 3 public firms produce an imperfect substitute for the goods of both private firms. This implies that the government privatizes more in Cases 1 and 2 than in Case 3 to encourage public firms to produce less.

Next, we compare the results obtained under an international mixed market.

Proposition 10 *In the case of two countries:* $\beta_{C3} = \beta_{C2} = 1 > \beta_{C1} > 0$ *if b<0;* $\beta_{C1} \in [0,1]$ *and* $\beta_{C3} = \beta_{C2} = 1$ *if b=0;* $\beta_{C2} = 1 > \beta_{C3} > \beta_{C1} > 0$ *if $0.1912 \geq b > 0$; $1 > \beta_{C2} > \beta_{C3} > \beta_{C1} > 0$ if $0.7172 > b > 0.1912$; $1 > \beta_{C2} = \beta_{C3} > \beta_{C1} > 0$ if b=0.7172; and $1 > \beta_{C3} > \beta_{C2} > \beta_{C1} > 0$ if b>0.7172.*

The result obtained in Proposition 10, illustrated in Figure 1, indicates that the type of good produced by the public firms affects the privatization policy of governments when there are two countries. When the goods are complements and substitutes, governments privatize more if each public firm produces the same good as the domestic private firm (Case 1) than if they produce different goods (Cases 2 and 3). In Cases 2 and 3, the public firms remain fully public when the goods are complements or independent in demand. However, whether there is less privatization in Case 2 or 3 when the goods are substitutes depends on the degree to which goods are substitutes: $\beta_{C2} > \beta_{C3}$ if b<0.7172 and $\beta_{C3} > \beta_{C2}$ if $b > 0.7172$.
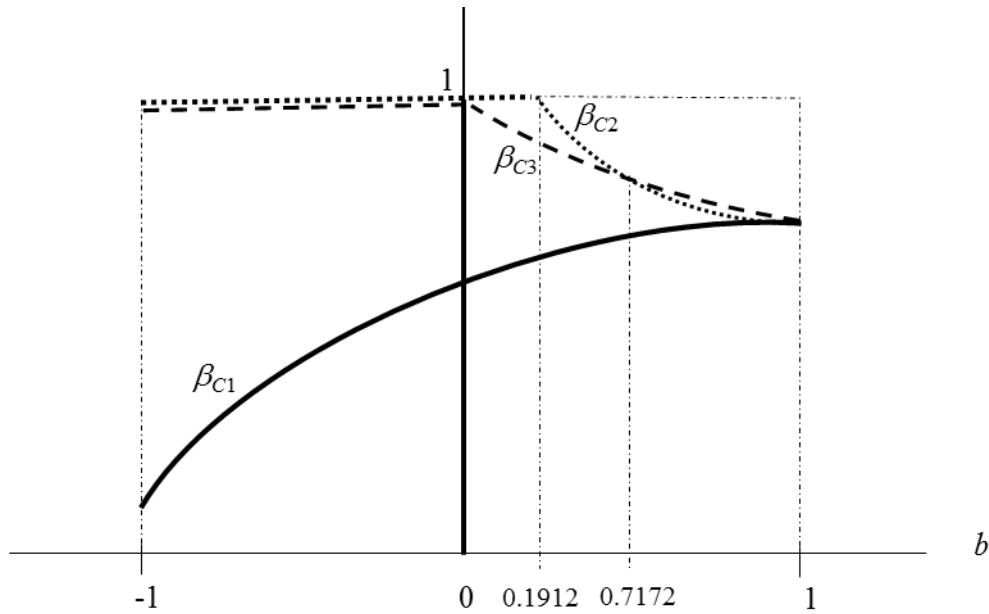
Fig. 1 Comparison of results with two countries

We consider first that goods are complements. In Case 1, the public and private firms in each country produce the same good, so governments partially privatize their public firms to encourage them to produce less and thus not to damage domestic private firms. In Cases 2 and 3, the public and private firms in each country produce complementary goods. Thus, the public firms remain fully public to encourage them to produce more in order to gain market share and welfare at the expense of the other country and to benefit the domestic private firm. As a result, when the goods are complements there is more privatization if the public and private firms produce the same good in each country (Case 1) than if they produce different goods (Cases 2 and 3).

When the goods are substitutes there is more privatization in Case 1 than in the other cases. In all three cases each public firm tries to gain market share at the expense of the rival country, which makes them behave more aggressively and produce more. In Case 1 each public firm produces the same good as the domestic private firm, but in Cases 2 and 3 they produce imperfect substitute goods. Thus, in Case 1 governments encourage their public firms to produce less than in Cases 2 and 3, so there is more privatization in Case 1.

Next we compare privatization when each public firm produces the same good as the foreign private firm (Case 2) and when both public firms produce the same good (Case 3) for substitute goods. In Case 2 each public firm produces the same good as the foreign private firm. If the goods are weak substitutes ($b \leq 0.1912$), the desire to gain market share at the expense of the foreign firm leads governments to encourage their public firms to produce more, so they choose $\beta_{C2} = 1$. However, if the goods are sufficiently close substitutes ($b > 0.1912$), gaining market share harms the domestic private firm. This leads each government to encourage its public firm to produce less, so β_{C2} becomes lower as the goods become closer substitutes. In Case 3 the two public firms produce the same good, which is an imperfect substitute of the good produced by the private firms. As the goods are substitutes, market competition between the public firms harms the domestic private firm, which leads them to produce less, so β_{C3} becomes lower as the goods become closer substitutes. Both β_{C2} and β_{C3} decreases as the goods become closer substitutes but β_{C3} decreases more smoothly than β_{C2} for the following reason: In Case 3, when $b=0$ competition in the market for the good produced by the public firms is strong. As b increases, public firms start to compete with private firms, but competition increases slightly. Thus, β_{C3} decreases smoothly. In Case 2, when $b=0$ competition in the market for each good is lower than in Case 3 in the market of the good produced by the two public firms. Thus, when the goods are weak substitutes ($b<0.1912$) public firms in Case 2 remain fully public ($\beta_{C2}=1$), since competition among firms is not strong enough. When $b>0.1912$, as b increases competition among firms increases more strongly in Case 2 than in Case 3. This is because in Case 2 one semipublic firm produces each good while in Case 3 the two private firms produce the same good. Thus, as b increases, β_{C2} decreases more with b than β_{C3}, so, $\beta_{C3} < \beta_{C2}$ if $b < 0.7172$ and $\beta_{C3} > \beta_{C2}$ if $b > 0.7172$.

8 Conclusions

In Europe and Asia governments own firms in various sectors of the economy. Waves of privatization have led to the partial privatization of public firms in many countries, but a large percentage of firms in different sectors of industry remain publicly owned. This situation has sparked research on the partial privatization of public firms.

The literature on partial privatization usually assumes one public firm and a single country, so it analyzes decisions to partially privatize for efficiency reasons. However, in practice public and private firms from different countries compete in international mixed markets, so governments may act strategically when taking decisions concerning partial privatization to gain welfare at the expense of other countries. The few papers that analyze partial privatization of public firms which compete in international mixed markets assume that all firms produce a homogeneous good, but public and private firms may well produce substitute or complementary goods. Those papers are therefore ignoring the fact that competition within a homogeneous product market coexists with competition across differentiated markets. As a result, they do not analyze how the type of goods produced by firms affects the partial privatization of public firms in international mixed markets. The main objective of this paper is to provide such an analysis.

We find that the privatization policy of governments depends on the type of good produced by firms. When the firms in each country (region) produce the same good there is more (less) privatization with a single country than with two countries for goods which are close substitutes (complements and weak substitutes). Social welfare is greater with two countries if the goods are close substitutes. When each public firm produces the same good as the private firm of the other country or region there is more privatization with a single country, and welfare is no lower in that case. When both public firms produce the same good the public firm is never privatized for goods which are complements and independent in demand; if the goods are substitutes there is more privatization with a single country. Moreover, social welfare with one country is no lower than with two countries, regardless of whether goods are substitutes or complements.

When there is an international mixed market with two countries, so that privatization policies are driven by strategic reasons, we find that there is more privatization when each public firm produces the same good as the domestic private firm, independently of the type of goods produced by firms. When each domestic public firm produces a different good from the domestic private firm, the public firms remain fully public if the goods are complements. If the goods are weak substitutes, there is less privatization when each public firm produces the same good as the foreign private firm. However, if the goods are close substitutes there is less privatization when the public firms produce the same good.

The analysis undertaken in this paper can be extended in several ways. First, private firms may care about corporate social responsibility. Second, governments may sell part of their public firms to foreign investors. Thirdly, there may be *n* private firms in each country, which affects market competition. These are open problems that are left for future research.

Appendix

Case 1: An international mixed market.

$$q_i^{kA} = a(b^2\beta^k(12 - 5\beta^l) - 5(6 - \beta^k)(6 - \beta^l) + 2b(6 - 5\beta^k)(6 - \beta^l))/G_1,$$

$$q_i^{kB} = a(12b(6 - \beta^l) - 5(6 - \beta^k)(6 - \beta^l) - b^2\beta^k(12 - 5\beta^l))/G_1,$$

$$W^k = ((2a^2(75(6 - \beta^k)^2(6 - \beta^l)^2 - b^4(\beta^k)^2(12 - 5\beta^l)^2 - 10b(6 - \beta^k)^2(6 - \beta^l)(21 - 5\beta^l) + 6b^3\left(12 - \beta^k(13 - 5\beta^k)\right)(6 - \beta^l)(12 - 5\beta^l) - 6b^2(9\left(192 - \beta^k(244 - 67\beta^k)\right) + 15\left(60 - \beta^k(74 - 17\beta^k)\right)\beta^l - 5(15 - \beta^k(23 - 5\beta^k))(\beta^l)^2)))/(G_1)^2,$$ where $G_1 = b^2(12 - 5\beta^k)(12 - 5\beta^l) - 25(6 - \beta^k)(6 - \beta^l)$, $i \neq j$; $i, j = 1, 2$; $k \neq l$; $k, l = I, II$.

Case 1: A single country.

$$q_i^{kA} = 3a(15 - 4\beta^l - b(6 - \beta^l))/G_1, \quad q_i^{kB} = a(3 - \beta^k)(15 - 4\beta^l - b(6 - \beta^l))/G_2,$$

$$W^I + W^{II} = a^2(2b^3(6 - \beta^k)^2(6 - \beta^l)^2 + (\beta^k)^2(2853 - 1464\beta^l + 160(\beta^l)^2) - 6\beta^k(4185 - 2160\beta^l + 244(\beta^l)^2) + 9(5400 - 2790\beta^l + 317(\beta^l)^2) - 6b^2((\beta^k)^2(30 - 10\beta^l + (\beta^l)^2) + 6(144 - 48\beta^l + 5(\beta^l)^2) - 2\beta^k(144 - 48\beta^l + 5(\beta^l)^2)) - 6b(\beta^k(897\beta^l - 1854 - 85(\beta^l)^2) + (\beta^k)^2(174 - 85\beta^l + 8(\beta^l)^2) + 6(630 - 309\beta^l + 29(\beta^l)^2)))/ (2(G_2)^2),$$ where $G_2 = b^2(6 - \beta^k)(6 - \beta^l) - (15 - 4\beta^k)(15 - 4\beta^l)$, $i \neq j$; $i, j = 1, 2$; $k \neq l$; $k, l = I, II$.

Case 2: An international mixed market.

$$q_i^{kA} = a(3(6 + \beta^k)(10 - \beta^l) - 7b^2\beta^k\beta^l + 2b^3\beta^k\beta^l - 4b(18 + \beta^k(3 - 2\beta^l)))/G_3,$$

$$q_j^{kB} = a(3(10 - \beta^k)(6 - \beta^l) - 3b^2\beta^k\beta^l + 2b^3\beta^k\beta^l - 2b(36 - (6 - \beta^k)\beta^l))/G_3,$$

$$W^k = (2a^2(18(72(5-2b)^2(3+b) - 12(14+b)(5-2b)\beta^k - (73-16(5-b)b)(\beta^k)^2) -$$

$$6(36(19-b)(5-2b) + 12(-45 + b(6 + b(48 - b(23 + 4(2-b)b)))))\beta^k +$$

$$(1-b)(3-2b)(b(22+b) - 14)(\beta^k)^2)\beta^l + (18(77 - 8(5-b)b) - 6(1-b)(3-2b)$$

$$(13 + (15-b)b)\beta^k - (3-2b)^2(1-b)^2(1 - b(3 + b(3+b)))(\beta^k)^2)(\beta^l)^2))/(G_3)^2, \quad \text{where}$$

$$G_3 = 9(10 - \beta^k)(10 - \beta^l) + 4b^4\beta^k\beta^l - b^2(144 + 13\beta^k\beta^l), i \neq j; \; i, j = 1, 2; \; k \neq l; \; k, l = I, II.$$

Case 3: An international mixed market.

$$q_i^{kA} = a(5-2b)(6 + \beta^k - \beta^l)/3G_4, \quad q_j^{kB} = a(10 - 4b - \beta^k - \beta^l)/G_4,$$

$$W^k = (a^2(5400 + 288b^3 - 73(\beta^k)^2 - 1140\beta^l + 77(\beta^l)^2 - 8\beta^k(105 - 13\beta^l) + 4b(20(\beta^k)^2 -$$

$$630 + \beta^k(69 - 10\beta^l) + 129\beta^l - 10(\beta^l)^2) - 8b^2(72 + 2(\beta^k)^2 + 3\beta^l - (\beta^l)^2 - \beta^k(3 + \beta^l))))/$$

$$(9(G_4)^2), \text{ where } G_4 = (5(10 - \beta^k - \beta^l) - 8b^2), i \neq j; \; i, j = 1, 2; \; k \neq l; \; k, l = I, II.$$

Case 3: A single country.

$$q_i^{kA} = a(5-2b)(3 - \beta^l)/G_5, \quad q_j^{kB} = a(15 - \beta^k(4 - \beta^l) - 4\beta^l - b(6 - \beta^k - \beta^l))/G_5,$$

$$W^I + W^{II} = (a^2(5400 - 2790\beta^l + 317(\beta^l)^2 - 8b^3(6 - \beta^k - \beta^l)^2 - 4b^2(144 + 5(\beta^k)^2 -$$

$$6\beta^k(8 - \beta^l) - 48\beta^l + 5(\beta^l)^2) + (\beta^k)^2(317 - 146\beta^l + 12(\beta^l)^2) - 2\beta^k(1395 - 697\beta^l +$$

$$73(\beta^l)^2) - 4b(630 - 309\beta^l + 29(\beta^l)^2 + (\beta^k)^2(29 - 11\beta^l) - \beta^k(309 - 144\beta^l +$$

$$11(\beta^l)^2))))/(2(G_5)^2), \text{ where } G_5 = 75 - 12b^2 + 5\beta^k\beta^l - 2(10 - b^2)(\beta^k + \beta^l), i \neq j; \; i, j = 1, 2;$$

$$k \neq l; \; k, l = I, II.$$

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