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by

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Turnout Intention and Social Networks*

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Abstract

How can networking affect the turnout in an election? We present a simple model to explain turnout as a result of a dynamic process of formation of the intention to vote within Erdös-Renyi random networks. Citizens have fixed preferences for one of two parties and are embedded in a given social network. They decide whether or not to vote on the basis of the attitude of their immediate contacts. They may simply follow the behavior of the majority (followers) or make an adaptive local calculus of voting (Downsian behavior). So they either have the intention of voting when the majority of their neighbors are willing to vote too, or they vote when they perceive in their social neighborhood that elections are "close". We study the long run average turnout, interpreted as the actual turnout observed in an election. Depending on the combination of values of the two key parameters, the average connectivity and the probability of behaving as a follower or in a Downsian fashion, the system exhibits monostability (zero turnout), bistability (zero turnout and either moderate or high turnout) or tristability (zero, moderate and high turnout). This means, in particular, that for a wide range of values of both parameters, we obtain realistic turnout rates, i.e. between 50% and 90%. Keywords: Turnout, Social Networks, Adaptive Behavior.

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1 Introduction

Two basic questions concerning the turnout in elections are: "Who vote?" and "Why do people vote?" Empirical research answers the first question by pointing out a list of individual characteristics that influence participation. The results suggest that non participation is positively correlated with low education level, social or geographical isolation (Matsusaka 1995), being a newcomer or immigrant (Jackson 2003), having a low income (Lijphart 1997), and being young (Johnson 2002; Gimpel, Morris and Armstrong 2004). Moreover, those persons who voted in the previous election are more likely to vote in the next: voting is thus a habit (Green and Shachar 2000; Plutzer 2002; Gerber, Green and Shachar 2003). The effect of external characteristics such as the electoral system or the closeness of the race between candidates has also been studied. It appears that voters are more likely to turn out under proportional electoral systems than under majority systems (Geys 2006). Closeness also matters (Geys 2006; Shachar and Nalebuff 1999) although the evidence is patchy (Matsusaka 1993; Matsusaka and Palda 1993).

The influence of social networks is well known in political science and sociology (Lazarsfeld, Berelson and Gaudet 1948; Berelson, Lazarsfeld and McPhee 1954). The recent empirical findings concerning what McClurg (2004) calls "behavioral contagion" and Blais and Young (1999) call "perceived pressure to vote" can be summarized as follows. People whose neighbors and friends usually vote are more likely to participate (Kenny 1992; McClurg 2004). Interpersonal discussion influences political participation (Huckfeldt and Sprague 1995). The effect of social interaction on participation is contingent on the amount and the quality of political discussion that occurs within the social network (McClurg 2003, 2006). Moderately informed voters tend to imitate their neighbors' voting behavior (Johnson 2002). The contagion effect occurs among spouses (Nickerson 2008), but weaker ties or even casual interactions may also determine political behavior patterns (Huckfeldt, Beck, Dalton and Levine 1995). Publicizing participation increases the turnout (Gerber, Green and Larimer 2008). Political disagreement within the network tends to dampen turnout (Mutz 2002; Gimpel, Dyck and Shaw 2004), although Nir (2005) distinguishes between isolation within one's own opinion environment and the balance of exposure to two conflicting points of views.

At the theoretical level, Downs (1957) uses rational choice to question the turnout in large elections: Why do so many people vote given that in marginal terms the cost of voting is larger than its benefits? Indeed the benefit of voting depends on the voter's probability of being decisive, which is extremely low in large electorates. Since then many explanations have been given, among them a sense of duty (Riker and Ordeshook 1968) and the objective of minimizing regret (Ferejohn and Fiona 1974). Game theoretical models (Palfrey and Rosenthal 1983, 1985; Ledyard 1984) have been proposed, as have group-based models of mobilization (Uhlaner 1989; Shachar and Nalebuff 1999). For a review of these models, see Feddersen (2004) or Geys (2006). More recently, network theory has sought to explain turnout by contagion through social networks: groups of voters can convince their nearest neighbors to go and vote. Amaro de Matos and Barros (2004) and Fowler (2005) show that if people imitate their neighbors' behavior, a small group of people with strong feelings about voting can bring about a massive turnout by a "domino" or "cascade" effect. Fowler and Smirnov (2005) propose a model where individuals base their decision to abstain or participate on what the most satisfied neighbors did in the previous period, and find that the result is a large turnout.

In this paper we bridge the gap between rational theory and network theory by combining adaptive calculus of voting and imitation within social networks. We assume that individuals decide whether or not to vote on the basis of the influence of their social neighbors (e.g. family, friends, coworkers, etc.): while some individuals simply follow the observed majority behavior (imitation or contagion effect), others tend to turn out if they perceive that elections are "close" (local adaptive calculus of voting effect).

In more detail, we propose the following model. Two parties compete in an election. Each citizen has a given preference for one party or the other and that preference does not change during the relevant period. Instead, before the election takes place, the decision that evolves is the intention to participate. Citizens are embedded in a random social network and dynamically make their choice to vote or not depending on what they observe within their social neighborhoods. We study the long-run emerging average behavior which, as discussed in the next section, can be interpreted as the actual turnout observed in the election.

We represent the social structure as a fixed random network. Nodes are citizens and links (ties) are social relationships. More specifically, we consider undirected Erdös-Renyi random networks. In this type of network all ties have the same probability of being present and for a large number of nodes the connectivity distribution is approximately Poisson. This implies that the network can be fully characterized by the average connectivity (e.g. average number of links per node or average degree). This feature is therefore one of the two parameters of our model. Its magnitude depends on who is really influential when one decides whether to participate or not. Do people discuss politics only with friends, with friends and family, with friends, family and co-workers, etc? Naturally, it may also depict different kinds of society, some of which are more densely connected than others.

Given the pattern of interactions, citizens form their intention to vote. Agents have limited information and are backward adaptive learners. They only have access to local information - that which they can gather in their immediate neighborhood -; and use the past as a guide for making their current decision (see Kanazawa 1998 or Fowler and Smirnov 2005). That is, whenever a citizen updates his/her intention whether or not to turn out, say at time t, he/she takes into account his/her neighbors' intention to participate at $t-1$ as well as their given political preferences.[1] We consider two possible behaviors - imitation and adaptive calculus of voting - which may reflect differences in political awareness (see Beck, Dalton, Greene and Huckfeldt 2002). The probability of adopting one of these behaviors is the second parameter of our model. If the citizen acts as a "follower," he/she decides to vote if the majority of his/her neighbors are willing to vote too. If the citizen is a "local Downsian" he/she decides to vote if he/she may be "decisive" in his/her social neighborhood. In this case, if a large majority of his/her voting neighbors have preferences for his/her preferred party or his/her opposed party he/she will not vote. Not voting if one is isolated in an enemy neighborhood is empirically found by Gimpel, Dyck and Shaw 2004. The "local Downsian" agent only votes if his/her neighborhood is divided; he/she does not vote if he/she feels that either party may win by a very large majority.

We use two complementary approaches to find the long-run turnout equilibrium, i.e. the average turnout that remains stable through time. The results obtained by an analytical approximation (the so-called mean-field technique) are confirmed by Monte Carlo simulations. The interplay between the two key parameters, average connectivity and probability of being a follower results in a rich long-run behavior. The model often does not predict a unique stable equilibrium: the system may exhibit bistability with zero and high or moderate turnout and tristability with zero, moderate and high rates of turnout.

The rest of the paper is organized as follows. In Section 2 we present the framework and the assumptions. In Section 3 we highlight the main

[1] It should be clear that the dynamic process of turnout formation occurs just before an election. Therefore step t is an arbitrary time unit each time (i.e. it does not represent different election days).

results and compare them to real election data. The model is analytically developed in Section 4 and complemented by Monte Carlo simulations in Section 5. Finally, in Section 6 we sum up and discuss possible avenues for future research. For the sake of clarity we relegate some mathematical derivations to the Appendix.

2 The model: evolution of turnout intention

Two parties, A and B, compete in an election. Before the election takes place, citizens are involved in a dynamic process of formation of turnout intention. Let N denote the set of agents or citizens with the right to vote (indexed by $i = 1, ..., n$). Each agent i usually interacts with a small group, thus we model the pattern of interactions as a (exogenously given and fixed) random social network G à la Erdös-Renyi. In this network, agents are nodes and links (ij) represent social relationships or political discussions. Ties are undirected (thus if i is connected to j, so is j with respect to i) and any two agents have the same probability of being connected. We denote $N_i = \{j \neq i, j \in N : ij \in G\}$, the set of agents with whom i is connected or i's neighbors. The connectivity of i (number of neighbors) is denoted by $k_i = |N_i|$. As G belongs to the Erdös-Renyi family of networks, for a large n, the connectivity is approximately Poisson distributed and there are no correlations; in particular the clustering coefficient (i.e. the average probability of two *neighbors* of any agent i themselves being connected) is very small. All this implies that for a sufficiently large number of realizations and a large n, the network can be fairly described by its average connectivity $\langle k \rangle = \sum_{i \in N} k_i/n$, the unique parameter of the Poisson distribution.

Each agent has two basic characteristics: his/her preference and his/her turnout intention. Agent i's preference does not change during the relevant period and is denoted by $u_i \in \{A, B\}$. This means that if agent i votes, he/she chooses the candidate of party A (B) whenever $u_i = A$ (B). We assume that preferences are uniformly distributed among the population and there are exactly $n/2$ agents of each type. What does change over time is the intention of agent i to vote or not. Let $v_{i,t} = 1(0)$ denote the intention of agent i to turnout (or not) at time t. We assume that initially a proportion of the population is willing to vote and its distribution (uniform) is independent of the network structure and the distribution of preferences. Agents are bounded rational and at each t consider the information they know from $t - 1$. More specifically, the information that they can gather is the characteristics of their neighbors; therefore at time t each agent i knows

u_j and $v_{j,t-1}$ for every $j \in N_i$.

The turnout intention dynamics is as follows. At each time $t = 1, ..$ one agent is random uniformly chosen to update his/her turnout intention. With probability p, the chosen agent behaves as a "follower" and is willing to vote if a majority of his/her neighbors are also willing to vote. With probability $1-p$, the chosen agent behaves as a "Downsian" agent: he/she is willing to vote if he/she thinks that elections are more or less close. Close elections are understood as a 40%-60% division between neighbors who are considering voting. Note that as citizens only have access to information within their neighborhood, so this adaptive calculus of voting is done at the local level.

If agent i is chosen to update his/her turnout behavior at time t, he/she uses the information gathered at period $t-1$, namely, the number of voting neighbors, which we denote by $x_{i,t-1}$:

$$x_{i,t-1} = \sum_{j \in N_i} v_{j,t-1}, \tag{1}$$

and the number of voting neighbors with identical preferences, which we denote by $y_{i,t-1}$:

$$y_{i,t-1} = \sum_{j \in N_i : u_j = u_i} v_{j,t-1}. \tag{2}$$

Therefore, formally, if agent i behaves as a "follower", his/her turnout intention at time t will be:

$$v_{i,t} = \begin{cases} 1 \quad if \quad x_{i,t-1} \geq 0.5 k_i \\ \\ 0 \qquad\qquad \text{otherwise;} \end{cases}$$

while if agent i behaves in a "Downsian" fashion:

$$v_{i,t} = \begin{cases} 1 \quad if \quad 0.4 x_{i,t-1} \leq y_{i,t-1} \leq 0.6 x_{i,t-1} \\ 0 \qquad\qquad\qquad \text{otherwise.} \end{cases}$$

If agent i has no neighbors ($k_i = 0$) or no one in his/her neighborhood is willing to vote ($x_{i,t-1} = 0$), we assume that he/she simply copies his/her own past behavior, i.e. $v_{i,t} = v_{i,t-1}$.

Under the assumption that $v_{1,t}, v_{2,t}, ..$ are i.i.d. random variables, the average turnout intention at time t, $\sum_{i \in N} v_{i,t}/n$, for n large approximates the expected turnout intention at time t, i.e. $\sum_{i \in N} v_{i,t}/n \xrightarrow{P} \langle v \rangle_t$. The equilibrium turnout v is approximated by the long run value of $\langle v \rangle_t$, that is $\langle v \rangle_t \xrightarrow{P} v$, or in other words, when $\langle v \rangle_t$ remains stable over time, $v = \langle v \rangle_t = \langle v \rangle_{t-1}$ (see Section 4 for more specific details).[2]

The long-run turnout intention obtained v represents, or can be interpreted as, the actual turnout that would be observed on election day. We hence study the behavior of v as a function of the following parameters:

- $p \in [0, 1]$, the probability of being a follower or the fraction of time for which any individual behaves as a follower.
- $\langle k \rangle = 5, 6, ..25$, the average connectivity of the network. We assume $\langle k \rangle \geq 5$ because for $\langle k \rangle < 5$, the fraction of isolated individuals is too great to allow the initial behavior to be updated. (This is similar to Fowler 2005, where $\langle k \rangle$ is assumed to be between 4 and 20).

3 Qualitative and quantitative results

We adopt two different complementary approaches to solve the model. We approximate analytically the long run average turnout via mean-field techniques (Section 4). The approximation obtained is then confirmed and complemented by Monte Carlo simulations (Section 5). In this section, we outline the intuition of the dynamics underlying our main results and compare the theoretical levels of turnout against real election data.

Let us start with the extreme case where only follower or imitation behavior prevails in the population ($p = 1$). Any agent is willing to vote if a majority of his/her neighbors intends to turnout. This does not depend much on the connectivity but rather on the average turnout in the previous period. For an initial turnout weakly larger than 50%, agents are likely to start out as willing to vote, and so on in the following periods, which should increase the turnout. Contagion thus spreads participation through the whole population and a very high turnout can be expected in the long run. Indeed we show that in this case the equilibrium turnout is 100%. Similarly if the initial turnout is smaller than 50%, non participation should spread through the population, and the equilibrium turnout should be 0%.

[2] $\xrightarrow{P}$ denotes convergence in probability. Note that assuming that $v_{1,t}, v_{2,t}, ...$ are i.i.d. random variables allow us to invoke the weak law of large numbers. That is, for any $\delta > 0$: $\sum_{i \in N} v_{i,t}/n \xrightarrow{P} \langle v \rangle_t = \lim_{n \to \infty} \Pr\left(\left|\sum_{i \in N} v_{i,t}/n - \langle v \rangle_t\right| > \delta\right) = 0$. An analogous argument lies behind $\langle v \rangle_t \xrightarrow{P} v$, for random variables $\langle v \rangle_1, \langle v \rangle_2, ...$

More generally, the qualitative effect of follower behavior on turnout is to reinforce the prevailing conditions.

Now let us focus on the other extreme case, i.e. when there are only Downsian agents ($p = 0$). The connectivity matters for the calculus of voting. The probability of close division at local level depends on the number of voting neighbors and also on the preferences of those neighbors. If we assume that supporters of A and B are uniformly distributed, the more voting neighbors there are, the larger the probability of close division is (though the parity effect plays a role). Qualitatively, if average connectivity increases the number of voting neighbors should also increase, and thus the equilibrium of turnout should be higher. Note however that an equilibrium turnout equal to 100% cannot arise in the presence of pure Downsian behavior. The intuition is simple. Assume that initially all agents are willing to vote. As preferences are uniformly distributed within the population an agent is more likely to find that elections are "close" in his/her neighborhood. However, this is *not certain* for *all* agents, and some will change their intention towards non participation. By contrast a null turnout can be an equilibrium: if no one is willing to vote, no one can make his/her local calculus of voting and thus all agents maintain their prevalent behavior. In sum, the effect of Downsian behavior is less obvious than the effect of follower behavior, but there seems to be a positive relation between connectivity and turnout.

Now let us consider the interplay between the two types of behavior, depending on the initial turnout and the connectivity. We observe two kinds of self-reinforcing dynamics, one yielding low turnout and the other high turnout:

- **Low Turnout: the "Vicious Cycle":** Consider a situation with an initial turnout below 50% and/or low connectivity so that Downsian agents are likely not to vote. As a consequence followers will not vote either, which decreases the number of voting neighbors. If the Downsian agents face smaller subsets of voting neighbors they tend to vote less, followers continue to reinforce this behavior, and so on. In the long run, followers are likely not to vote at all, while the turnout of Downsian agents may be moderate or null. The process thus stabilizes around zero turnout or, under some conditions, at a positive but moderate turnout (below 50%).

- **High Turnout: the "Virtuous Cycle":** Next, consider a situation with an initial turnout above 50% and/or high connectivity so that Downsians are likely to vote, and push the average voting above 50%.

In this case, followers are likely to vote, Downsians have many voting neighbors, which increases the probability of a close division, and thus vote, followers reinforce this outcome, etc. The long run outcome will be a high turnout.

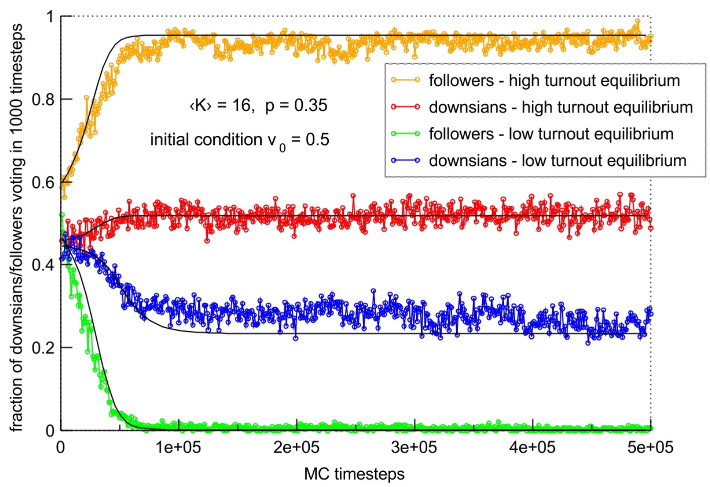

FIGURE 1. Evolution of turnout intention (v) conditional on behavior. These are two realizations for $n = 5 \times 10^3$, $\langle k \rangle = 16$, $p = 0.35$ and $T = 5 \times 10^5$. Each 1000 steps, we calculate the fraction of times a Downsian (follower) agent votes. Black lines are theoretical predictions cf. Section 4, (4) and (5).

Of course, both cycles are mediated by the probability of an agent being a follower, and a cycle may start because followers reinforce an increasing or decreasing turnout or because Downsian agents increase or decrease overall participation. Moreover both cycles can be observed on networks with identical numbers of agents, parameters, and initial turnout. In Figure 1 we show an example of the evolution over time of the probability of voting conditional on each type of behavior. These are two realizations for a network of $n = 5 \times 10^3$ agents and average connectivity $\langle k \rangle = 16$; a probability of being a follower of $p = 0.35$, and an initial turnout of 50%.[3] Every 1000

[3]The maximum number of MC timesteps is $T = 5 \times 10^5$ and the dynamics includes a

steps we plot the fraction of times that a Downsian agent votes and a follower votes. As the average connectivity is medium-high, Downsian agents tend to vote, but the key is whether or not they are able to maintain an average turnout above 50%. If so a virtuous cycle starts, and if not a vicious cycle starts. For the realization that leads to the low turnout equilibrium, the Downsian turnout falls below 50%, the followers have no incentive to vote, and thus the vicious cycle operates. In the high turnout equilibrium, initially driven by Downsian agents, the followers tend to vote en masse, and the virtuous cycle starts. As we show below, this is not an exception. There are many combinations of parameters that lead to non uniqueness of the turnout equilibrium.

Now let us focus on the levels of the equilibrium turnout and compare them with the turnout that we observe in real elections. The international Institute for Democracy and Electoral Assistance (International IDEA) has a voter turnout website on which statistics are available on political participation.[4] From these data, it can be said than more than half of the countries listed have turnouts of between 60% and 80%, and four fifths have turnouts of between 50% and 90%. The countries with turnouts of more than 90% are usually countries where voting is compulsory (such as Australia and Belgium).

The question is whether our model can lead to equilibria compatible with real data, that is with turnout rates between 50% and 90%. The answer is yes, but not for all values of our parameters. In Figure 2 we plot the regions of pairs $(p, \langle k \rangle)$ with which we obtain realistic turnout rates. These rates are one of the two or three equilibria that the model predicts, specifically the highest one (see Sections 4 and 5). Thus, it should be clear that these realistic turnout rates could arise if the dynamic process of turnout formation stabilizes around the highest equilibrium.

- A turnout rate of between 80% and 90% can be obtained for all connectivity levels $\langle k \rangle$. Roughly speaking, the more densely connected is the network is, the lower the probability of being a follower p needs to be, but at the same time, the range of possible values of p increases. For instance, if $\langle k \rangle = 5$, p should be between 0.75 and 0.82; while if $\langle k \rangle = 25$, it should be between 0.43 and 0.7.

- Something similar occurs with a turnout of between 70% and 80%.

small noise $\varepsilon = 0.001$, the probability that the selected node does anything (i.e. the agent chooses randomly between voting or not). As we will explain in Section 5, this noise is included for technical reasons related to simulations.

[4] See http://www.idea.int/vt/index.cfm

The difference is that all the possible values of p are smaller than in the previous case.

- A turnout of between 60% and 70% can be obtained for low levels of probability of being a follower and connectivity levels greater than 5.
- Finally, a turnout of between 50% and 60% can be obtained for very low levels of probability of being a follower and connectivity levels greater than 9. Note that due to the parity effect, for $\langle k \rangle = 11, 13$ and 15 the theoretical turnout is always greater than 60%, and that for $\langle k \rangle = 10, 12, 14, 16$ and 17, the turnout in this region is greater than 55%, meaning that only for $\langle k \rangle \geq 18$, would turnouts of close to 50% be observed.

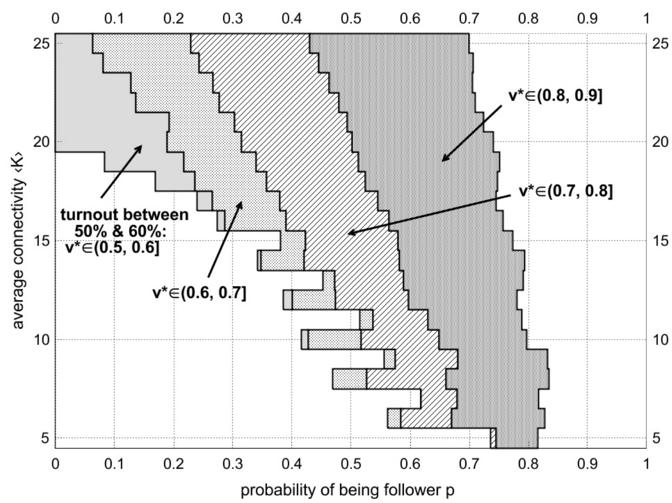

FIGURE 2. Regions where the predicted turnout rates are realistic. The four regions depict the combinations of average connectivity $\langle k \rangle$ and probability of being follower p that yield realistic long run turnout rates. The non-monotonic shape of the borders is due to the fact that $\langle k \rangle \in \mathbb{N}$ and the subsequent parity effect.

For realistic turnout rates to be obtained, the probability cannot take extreme values: for $p = 1$, we have either 0% or 100% participation, while for $p = 0$ the turnout is below 60%, even for high connectivity. Realistic levels of turnout are obtained either for a large proportion of follower behavior and not very densely connected networks, or for more connected networks and a larger fraction of Downsian behavior.

4 Mean-field approximation

Our aim in this section is to obtain an analytical approximation of the equilibrium turnout, i.e. the long-run state of the dynamics. As a first step, we approximate the evolution in time of the expected turnout intention $\langle v \rangle_t$. The idea is simple. We assume that the binary variables $v_{i,t}$ are i.i.d. Bernoulli random variables with a probability of success of $E(v_{i,t}) = \langle v \rangle_t$, therefore for n large the average turnout at time t converges in probability to the expected value $\langle v \rangle_t$, i.e. $\sum_{i \in N} v_{i,t}/n \xrightarrow{P} \langle v \rangle_t$. In this context, $\langle v \rangle_t$ is the approximated probability that *any* agent i is willing to vote at time t.

The probability that an agent i is willing to vote at time t depends on his/her behavior at time t (either Downsian or follower) and his/her local information, which in turn depends on what his/her neighbors did at time $t-1$. Recalling the mean-field basic hypothesis, we assume that all neighbors intended to vote at time $t-1$ with probability $\langle v \rangle_{t-1}$ and that i has $k_i \approx \langle k \rangle$ neighbors, where $\langle k \rangle$ is the average connectivity of the network.[5]

If i behaves as a follower, what matters is the fraction of neighbors who were willing to vote at time $t-1$ ($x_{i,t-1}/k_i$). If i behaves as a Downsian, he/she cares about the fraction of voting neighbors with the same preferences as himself/herself ($y_{i,t-1}/x_{i,t-1}$). Our assumptions imply that $x_{i,t-1} \approx x_{t-1}$ and $y_{i,t-1} \approx y_{t-1}$ for all i. They also allow us to interpret x_{t-1} as a random variable with binomial distribution $\left(\langle v \rangle_{t-1}, \langle k \rangle\right)$; and y_{t-1}, as a random variable with binomial distribution $(1/2, x_{t-1})$.

The probability that any agent i is willing to vote at time t is hence approximated as:

$$\langle v \rangle_t \approx p \;\Pr\left(x_{t-1} \geq 0.5 \langle k \rangle\right) + (1-p) \;\Pr\left(0.4 x_{t-1} \leq y_{t-1} \leq 0.6 x_{t-1}\right); \quad (3)$$

where the probability of voting *conditional* on behavior as a follower (in the first term) can be computed as:

[5] Here we assume that the average connectivity is a natural number, while it may be a real number.

$$\begin{aligned}
& \Pr\left(x_{t-1} \geq 0.5\left\langle k\right\rangle\right) \\
= & \sum_{l=\lceil 0.5\langle k\rangle\rceil}^{\langle k\rangle} \Pr\left(x_{t-1}=l\right) \\
= & \sum_{l=\lceil 0.5\langle k\rangle\rceil}^{\langle k\rangle} \binom{\langle k\rangle}{l} \left\langle v\right\rangle_{t-1}^{l} \left(1-\left\langle v\right\rangle_{t-1}\right)^{\langle k\rangle-l}, \qquad (4)
\end{aligned}$$

(where $\lceil z\rceil$ denotes the ceiling integer value of z, i.e., the smallest integer larger than z) and the probability of turnout conditional on Downsian behavior (second term of (3)) as:

$$\begin{aligned}
& \Pr\left(0.4x_{t-1} \leq y_{t-1} \leq 0.6x_{t-1}\right) \\
= & \sum_{l=1}^{\langle k\rangle} \Pr(x_{t-1}=l) \Pr\left(\lceil 0.4x_{t-1}\rceil \leq y_{t-1} \leq \lfloor 0.6x_{t-1}\rfloor \,|x_{t-1}=l\right) \\
= & \sum_{l=1}^{\langle k\rangle} \Pr(x_{t-1}=l) \sum_{m=\lceil 0.4l\rceil}^{\lfloor 0.6l\rfloor} \Pr(y_{t-1}=m) \\
= & \sum_{l=1}^{\langle k\rangle} \binom{\langle k\rangle}{l} \left\langle v\right\rangle_{t-1}^{l} \left(1-\left\langle v\right\rangle_{t-1}\right)^{\langle k\rangle-l} \sum_{m=\lceil 0.4l\rceil}^{\lfloor 0.6l\rfloor} \binom{l}{m} \frac{1}{2^{l}}, \qquad (5)
\end{aligned}$$

where $\lfloor z\rfloor$ denotes the floor integer value of z, i.e., the largest integer smaller than z. Substituting the probabilities into (3) we obtain $\left\langle v\right\rangle_t$ as a function of $\left\langle v\right\rangle_{t-1}$. As discussed above, we are interested in the long-run emergent behavior, which is therefore approximated by the asymptotically stable solutions of (3).

"Long-run" solutions: existence and stability

In the long run, $\left\langle v\right\rangle_t = \left\langle v\right\rangle_{t-1} = v$, thus given $\langle k\rangle$ and p, the turnout intention v meets:

$$\begin{aligned}
v \;=\; & p \sum_{l=\lceil 0.5\langle k\rangle\rceil}^{\langle k\rangle} \binom{\langle k\rangle}{l} v^{l} \left(1-v\right)^{\langle k\rangle-l} \\
& + (1-p) \left[\sum_{l=1}^{\langle k\rangle} \binom{\langle k\rangle}{l} v^{l} \left(1-v\right)^{\langle k\rangle-l} \sum_{m=\lceil 0.4l\rceil}^{\lfloor 0.6l\rfloor} \binom{l}{m} \frac{1}{2^{l}}\right] \qquad (6)
\end{aligned}$$

The right hand side of (6) is a function of v, $f(v)$, thus by definition any fixed point v^* satisfies condition (6). Fixed points reflect long-run behavior as long as they are asymptotically stable. Therefore we require that v^* to meet the additional condition $|f'(v^*| \langle k \rangle, p)| < 1$ (that is, the absolute value of the slope of f evaluated at the fixed point should be smaller than 1).

First, we address two results that can be easily shown analytically.

Proposition 1 $v^* = 0$ *is always an asymptotically stable solution for any choice of parameters.*

Proof. See the Appendix. ■

The intuition is simple. If at some point in time no one has the intention to vote, followers follow this behavior and Downsian agents are not able to update, so they keep their past behavior. The equilibrium turnout is then equal to zero.

One might wonder whether the other extreme, that is, a 100% turnout, may also be a stable equilibrium. The answer is yes, but only if all voters are followers. Formally:

Proposition 2 *If* $v^* = 1$ *is an asymptotically stable solution then* $p = 1$*.*

Proof. See the Appendix. ■

These two propositions together imply that for $p = 1$ there is bistability, that is, we have two different (although extreme and unrealistic) equilibrium turnouts. This means that depending on the initial conditions either or both of these equilibria may emerge.[6]

For other values of p, the solutions of $f(v) - v = 0$ have to be found numerically, as the degree of the polynomial ($\langle k \rangle \geq 5$) is too high for analytical solutions to be obtained. We fix $\langle k \rangle$ and show the typical bifurcation diagrams taking p as the bifurcation parameter. The function $v^*(p)$ describes branches of fixed points and the bifurcation diagram presents all those branches in the (p, v^*) space. When for a value of p, say p_0, several branches come together, the point (p_0, v^*) is said to be a bifurcation point and p_0, its bifurcation value. As Figure 3 depicts, in the diagrams of our model there can be one or two bifurcation points, at which saddle-node or fold bifurcations emerge.[7]

[6] It also implies that there must be at least one more fixed point between 0 and 1 which is not asymptotically stable. See Figure 3.

[7] In a saddle-node or fold bifurcation, two branches of fixed points emerge. One of them is stable, the other unstable.

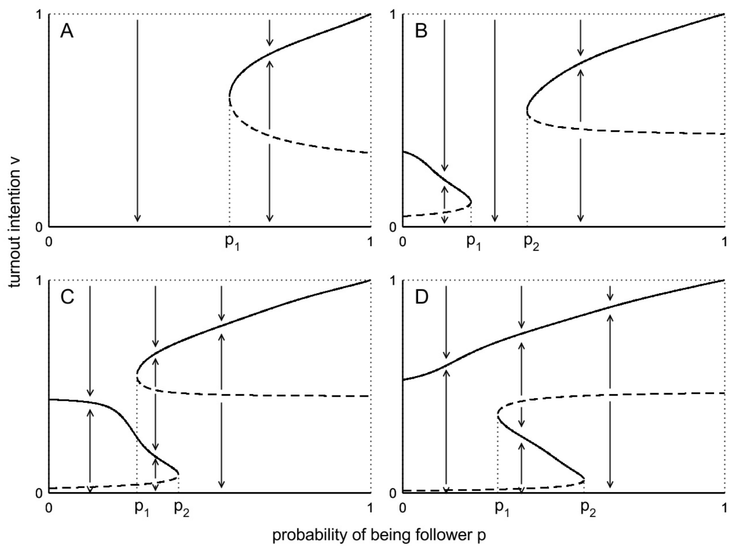

FIGURE 3. Bifurcation Diagrams, $v^*(p)$.
Solid lines depict stable fixed points; dashed, unstable fixed points. p_1, p_2 are bifurcation values. Panels: A, $\langle k \rangle = 6$; B, $\langle k \rangle = 12$; C, $\langle k \rangle = 16$; D, $\langle k \rangle = 22$.

An important conclusion that can be derived is that, depending on the combination of p and $\langle k \rangle$, the system may exhibit monostability, bistability or tristability. That is, for some pairs $(p, \langle k \rangle)$, the model predicts multiple equilibria. This does *not* mean, however, that all the equilibria are equally likely to emerge: the equilibrium observed depends on the initial conditions, i.e. the initial fraction of population willing to vote (v_0). The arrows in Figure 3 describe the basins of attraction of each equilibrium.

Consider for example the diagram of Figure 3-C, for p between p_1 and p_2, where the model predicts tristability (zero, moderate and high turnout). If the initial condition lies strictly above the upper dashed line, we will observe only the high turnout equilibrium. If the initial condition lies approximately *on* the upper dashed line, both the high and moderate long-run turnouts are likely to be observed (in this situation we can say that there exists "true" multistability). Similarly, if v_0 lies strictly below the upper dashed and strictly above the lower dashed line, only the moderate turnout will emerge; if it lies *on* the bottom dashed line, again two equilibria are possible (moderate and zero turnout); and, finally, if it lies strictly below the lower dashed line we will observe only the zero turnout equilibrium.

The stability zones in the $(p, \langle k \rangle)$ space are shown in Figure 4. The critical probabilities are the aforementioned bifurcation values. We observe that:

- If the connectivity is low ($\langle k \rangle \in [5, 9] \cap \mathbb{N}$), there is a critical probability p_1 such that for $p < p_1$ there is a unique zero turnout equilibrium; while for $p > p_1$ the system exhibits bistability, with either zero or high turnout. An example is given in Figure 3-A ($\langle k \rangle = 6$).

- If the connectivity is intermediate ($\langle k \rangle \in [10, 14] \cap \mathbb{N}$), there are two critical values, p_1 and p_2, ($0 < p_1 < p_2 < 1$). For $p < p_1$ the system exhibits bistability, with either zero or moderate turnout. For $p_1 < p < p_2$ there is a unique zero turnout equilibrium, and for $p > p_2$ the system exhibits bistability, either zero or high turnout. The typical diagram is shown in Figure 3-B ($\langle k \rangle = 12$).

- Finally, if the connectivity is high ($\langle k \rangle \in [15, 25] \cap \mathbb{N}$), there are two critical values, p_1 and p_2, ($0 < p_1 < p_2 < 1$). For $p < p_1$ we have either zero or moderate turnout (bistability). For $p_1 < p < p_2$ turnout can be zero, moderate or high (tristability), and for $p > p_2$ there can be either zero or high turnout (bistability). In this case, there are two kinds of bifurcation diagram (cf. Figure 3-C for $\langle k \rangle = 16$ and Figure 3-D for $\langle k \rangle = 22$).

Remark Although there is no difference between the two bottom panels of Figure 3 (C and D) in terms of stability zones, an important difference emerges if we study the effect of gradual variations of p. Starting from one extreme of the range of p (either 0 or 1), we let the system stabilize at the corresponding fixed point and gradually vary the value of p (either up or down, respectively) until another steady state is reached, then change p again and so forth. Thus for example, when $\langle k \rangle$ is very high (between 18 and 25, as in Figure 3-D), if we start from $p = 1$ and vary p downwards, we move along the upper stable branch (i.e. without "jumps"). But when $\langle k \rangle$ is medium-high (between 15 and 17, as in Figure 3-C), we observe a discontinuity or discrete jump at p_1.

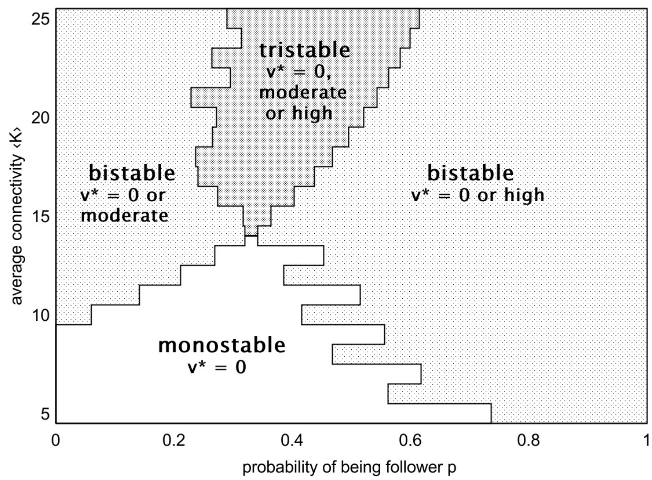

FIGURE 4. Stability zones. Interplay between average connectivity $\langle k \rangle$ and follower's probability (p). The non-monotonic shape of the borders is due to the fact that $\langle k \rangle \in \mathbb{N}$ and its parity effect on the location of the bifurcation points.

We conclude by discussing what would happen if we varied some parameters that we have kept constant so far, namely the size of the majority and how "closeness" is defined. Imagine that a follower is willing to vote if

$$x_{i,t-1} \geq rk_i \text{ where } r \in [\frac{1}{2}, 1];$$

while a Downsian agent is willing to vote if

$$\left(\frac{1}{2} - \beta\right) x_{i,t-1} \leq y_{i,t-1} \leq \left(\frac{1}{2} + \beta\right) x_{i,t-1}, \text{ with } \beta \in \left[0, \frac{1}{2}\right].$$

Roughly, increasing (decreasing) r (β) reduces the average turnout and, depending on $\langle k \rangle$, changes the bifurcation diagram. In particular, when $\langle k \rangle$ is large, instead of a diagram like Figure 3-D, we would have one similar to Figure 3-C or 3-B because when either r is close to 1 or β is close to 0, we are introducing a bias towards non-voting behavior, the effect of which is similar to reducing connectivity $\langle k \rangle$. The opposite occurs when r (β) is reduced (increased), i.e., r and β are close to 1/2.

5 Simulations

In this section we run simulations to supplement the analysis. The aim is to check whether the long run solution of the mean-field approximation describes, at least qualitatively, the long-run state of the model.

In one realization (or run), an Erdös-Renyi network of 5×10^3 nodes and given average connectivity $\langle k \rangle$ is created. Starting with an *initial condition of* 50% *of voting citizens* (i.e. $v_0 = 0.5$), the behavior evolves for $T = 5 \times 10^5$ timesteps.[8] The turnout of each realization is the average over the last 2×10^3 timesteps. This is repeated 50 times (with the same values of p and $\langle k \rangle$), and the fixed points v^* are the average over all runs.[9] For technical reasons we have introduced the possibility of an agent with a low probability (ε) randomly choosing whether to participate or not. This is done to prevent the possibility of being stuck in the extreme turnout of 0%, although this is very unlikely given $v_0 = 0.5$.[10]

In order to test our analytical approximation, we reproduce the bifurcation diagram for each $\langle k \rangle \in [5, 25] \cap \mathbb{N}$. According to the typical behavior of the *simulated* diagrams, we present four groups, from low to high average connectivity (see Figures 5-8).

[8] This maximum number of timesteps is quite safe, as can be observed in Figure 1. In order to determine it, we have proceeded as usual: we previously ran several realizations starting from $T = 10^5$ and observe whether the dynamics tended to stabilize around any particular value. Then, we have increased T until the stabilization was evident (depending on $\langle k \rangle$, this occurred around $T = 3 \times 10^5$).

[9] Whenever the dynamics ended in different fixed points, we obtained 30 additional realizations in order to have more observations to calculate both averages.

[10] In other words, we introduce ε to restore ergodicity.

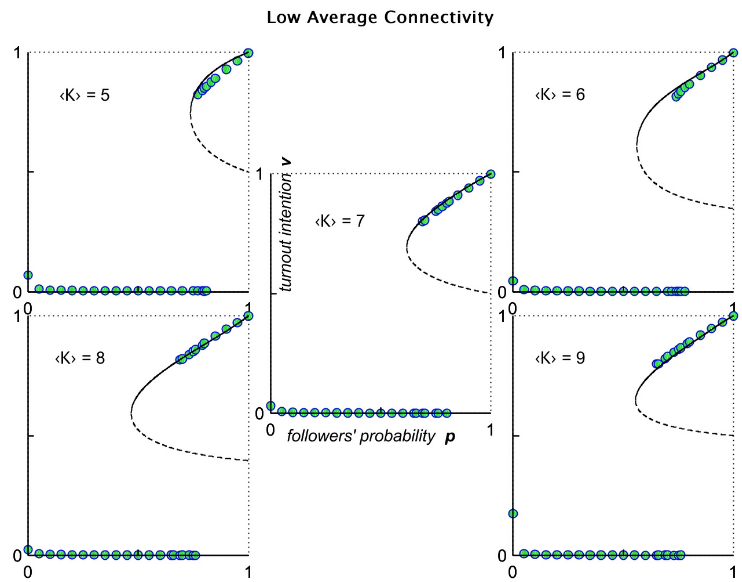

FIGURE 5. Turnout intention (v) against the probability of being follower (p).

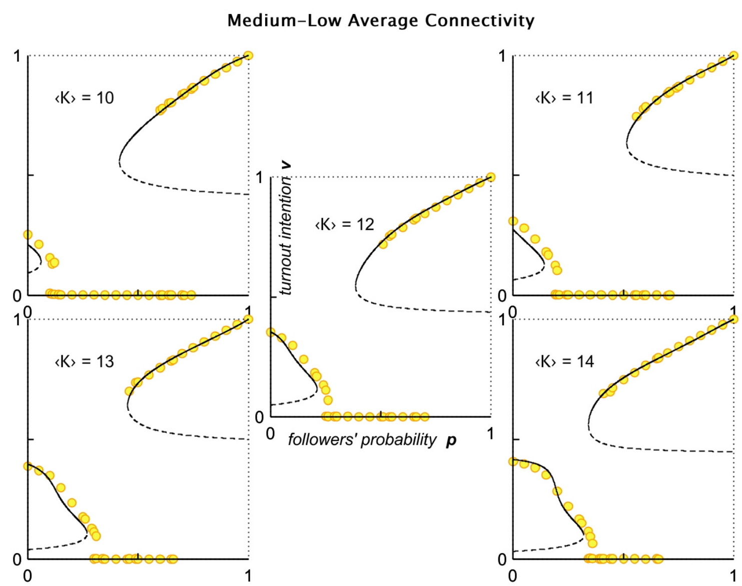

FIGURE 6. Turnout intention (v) against probability of being follower (p). Solid lines are theoretical predictions; circles, MC simulations ($v_0 = 0.5$, $n = 5 \times 10^3$, $\varepsilon = 0.001$, $T = 5 \times 10^5$). Each circle is the average of 50 runs; each run, of the last 2×10^3 timesteps.

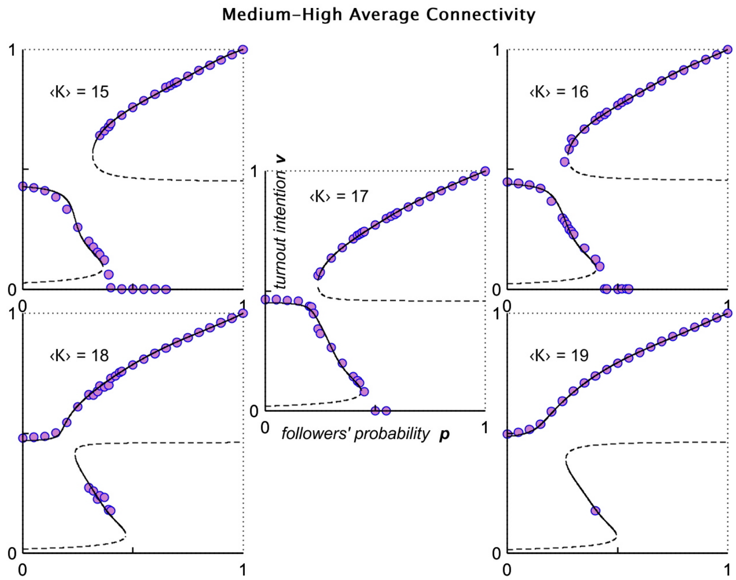

FIGURE 7. Turnout intention (v) against the probability of being follower (p).

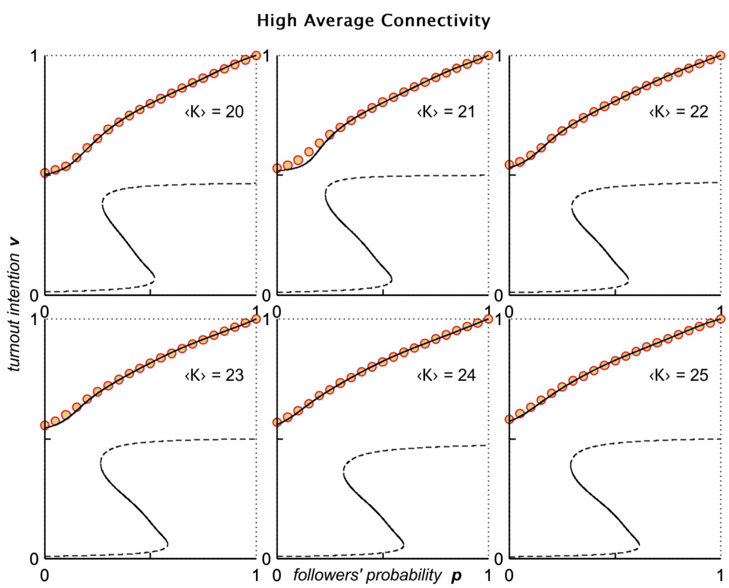

FIGURE 8. Turnout intention (v) against probability of being follower (p). Solid lines are theoretical predictions; circles, MC simulations ($v_0 = 0.5$, $n = 5 \times 10^3$, $\varepsilon = 0.001$, $T = 5 \times 10^5$). Each circle is the average of 50 runs; each run, of the last 2×10^3 timesteps.

Two aspects of the simulations can be stressed:

- First, the mean-field approximation describes the actual long-run turnout intention quite accurately. Almost all the simulated turnout points (circles) correspond to stable fixed points of condition (6) (solid lines). Recall that not *all* stationary points arise as an outcome of the simulated dynamic process because initial conditions matter (see Section 4).

- Second, with the initial condition of 50%, for some combinations of parameters the system exhibits what we call "true" multistability. This is observed for medium-low connectivity and a relatively large p, and for medium-high connectivity and intermediate values of p (cf. Figures 6 and 7).

In order to see whether other equilibria (stable fixed points) could be observed, we modified the initial conditions. Recall that for the extreme case when there are only followers ($p = 1$) the initial condition completely determines the equilibrium turnout: if the initial turnout is smaller than 50%, by contagion we end up with a null turnout. By contrast the initial condition has almost no effect in the case of purely Downsian behavior ($p = 0$). The reason is simple: on the one hand, when the average connectivity is small ($5 \leq \langle k \rangle \leq 9$) the only possible equilibrium for $p = 0$ is zero turnout for all initial conditions. On the other hand, even if there are two possible equilibria (zero and moderate) for $\langle k \rangle \geq 10$ the initial condition would have to be extremely low for the zero turnout equilibrium to emerge. Consequently, for almost all the range of initial conditions we observe the positive turnout equilibrium. Indeed, this occurs approximately for all $p \leq$

0.35.[11] See Figure 9 for some examples.

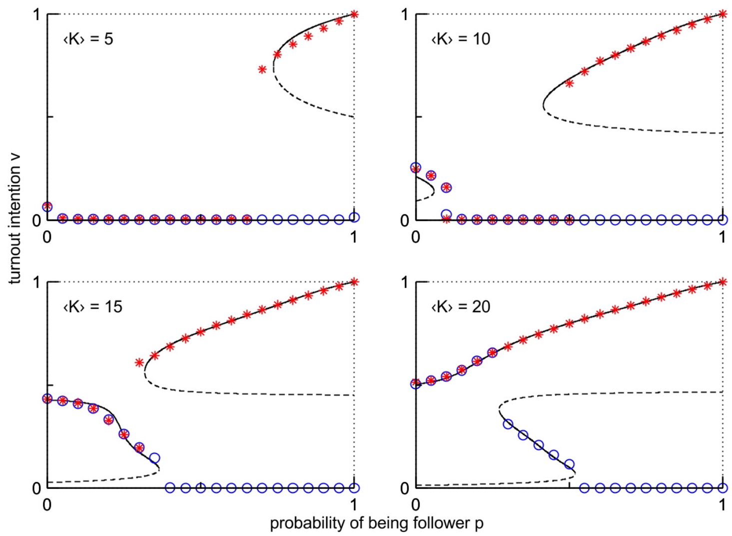

FIGURE 9. Turnout intention (v) against probability of being follower (p). Solid lines are theoretical predictions; circles ($v_0 = 0.3$) and stars ($v_0 = 0.7$) are MC simulations ($n = 5 \times 10^3$, $\varepsilon = 0.001$, $T = 5 \times 10^5$). Each symbol is the average of 50 runs; each run, of the last 2×10^3 timesteps.

6 Conclusions

We present a simple model to explain turnout rates under the assumption that before any election takes place, i.e. during the campaign, citizens dynamically form their intention to vote as a consequence of their social interactions. The pattern of interactions is fixed and modeled as a Erdös-Renyi random network and hence it can be characterized by its average connectivity. Individuals may simply follow the majority behavior (and vote or not) or they may behave as "local adaptive Downsian agents." In the latter case they tend to vote if they perceive that the election is "close" in their neighborhood.

We study the long-run average turnout intention, which in this model represents the turnout observed in an election. When all agents behave as

[11] As explained in the previous section in the analysis of Figure 3, for the zero turnout equilibrium to emerge initial conditions would have to lie below the lower dashed line (unstable fixed points). These unstable branches are, in turn, very close to zero.

pure followers, long-run turnout rates are very unrealistic (either all vote or no-one does) and connectivity plays no role. The introduction of Downsian behavior then has two interesting effects. On the one hand the resulting turnout rates are in general more realistic; on the other hand those outcomes depend on the average connectivity of the network. Depending on the combination of values of the two key parameters (average connectivity and the probability of being a follower/Downsian), the system exhibits monostability (zero turnout), bistability (zero turnout and either moderate or high turnout) or tristability (zero, moderate and high turnout). When there is more than one possible equilibrium, different initial conditions converge to different stable stationary states. In some cases, the *same* initial condition yields different equilibria, so turnout eventually becomes unpredictable. Interestingly, for a wide range of the parameters values this model predicts realistic turnout rates, i.e. comparable to the average turnout observed in the real-world elections.

In our model, citizens have fixed preferences for two parties and their decision is whether to vote or not. There is no correlation among neighbors' preferences. This is not consistent with voting literature, where it is found that citizens tend to segregate in groups of identical preferences: citizens with identical political preference are more likely to be connected. In our setup this segregation would only decrease turnout. To see why, consider an agent with Downsian behavior who shares the same preference with all his/her neighbors. Then he/she will never vote. This would induce a trend of non participation that would spread through the whole population by contagion. The fact that segregation depresses turnout was found previously in Fowler and Smirnov (2005). Further research could include the coevolution of preferences and turnout intention, perhaps assuming that a fraction of the population does not have clear preferences for one particular party or may change its preference. This would reflect what was found by Zuckerman, Valentino and Zuckerman (1994): half of the electorate switch their decision at least one over three ballots. Differences between supporters of the two parties could also be introduced, that is, the initial fraction of supporters may differ for one party or the other.

Appendix

Proof of Proposition 1

As the RHS of (6) as a function of v is a polynomial of degree $\langle k\rangle$ with an independent coefficient of zero, $f(v)-v=0$ has always at least one solution, which is $v^*=0$, whatever the values of $\langle k\rangle$ and p. To check its stability we consider $f'(v)$:

$$\begin{aligned} & f'(v) \\ = \; & p\sum_{l=\lceil 0.5\langle k\rangle\rceil}^{\langle k\rangle}\binom{\langle k\rangle}{l}\left(\begin{array}{c} lv^{l-1}(1-v)^{\langle k\rangle-l} \\ -v^{l}(\langle k\rangle-l)(1-v)^{\langle k\rangle-l-1}\end{array}\right) \\ & +(1-p)\sum_{l=1}^{\langle k\rangle}\binom{\langle k\rangle}{l}\left(\begin{array}{c} lv^{l-1}(1-v)^{\langle k\rangle-l} \\ -v^{l}(\langle k\rangle-l)(1-v)^{\langle k\rangle-l-1}\end{array}\right)\sum_{m=\lceil 0.4l\rceil}^{\lfloor 0.6l\rfloor}\binom{l}{m}\frac{1}{2^{l}}\end{aligned}$$

If we expand this expression, in all terms except the first one preceded by $(1-p)$ v or a power of v appears. This term (recall that $l=1$) is:

$$(1-p)\langle k\rangle\left((1-v)^{\langle k\rangle-1}-v(\langle k\rangle-1)(1-v)^{\langle k\rangle-2}\right)\sum_{m=\lceil 0.4\rceil}^{\lfloor 0.6\rfloor}\binom{1}{m}\frac{1}{2}$$

but then, in this case, $\sum_{m=\lceil 0.4l\rceil}^{\lfloor 0.6l\rfloor}\binom{l}{m}\frac{1}{2^{l}}=0$ (if i has only one voting neighbor, he/she never votes). Thus, for $v=0$, $f'=0<1$. Actually, $v^*=0$ is an asymptotically super-stable fixed point. ∎

Proof of Proposition 2

First assume that $p=1$. Then, v^* satisfies:

$$v=\sum_{l=\lceil 0.5\langle k\rangle\rceil}^{\langle k\rangle}\binom{\langle k\rangle}{l}v^{l}(1-v)^{\langle k\rangle-l}.$$

In all the terms except the last one $(1-v)$ or a power of $(1-v)$ appears. The last term is simply $v^{\langle k\rangle}$. Thus, $v^*=1$ is a fixed point as $\left[\sum_{l=\lceil 0.5\langle k\rangle\rceil}^{\langle k\rangle}\binom{\langle k\rangle}{l}v^{l}(1-v)^{\langle k\rangle-l}-v\right]_{v=1}=1^{\langle k\rangle}-1=0$. Now consider $f'(v)$:

$$f'(v) = \sum_{l=\lceil 0.5\langle k\rangle \rceil}^{\langle k\rangle} \binom{\langle k\rangle}{l} \left(l v^{l-1} (1-v)^{\langle k\rangle - l} - v^{l} (\langle k\rangle - l)(1-v)^{\langle k\rangle - l - 1} \right).$$

In all terms except those for $l = \langle k\rangle - 1$ and $l = \langle k\rangle$ $(1-v)$ or a power of $(1-v)$ appears. The terms for $l = \langle k\rangle - 1$ and $l = \langle k\rangle$ are (after simplifying)

$$\langle k\rangle \left((\langle k\rangle - 1) v^{\langle k\rangle - 2} (1-v) - v^{\langle k\rangle} \right) + \langle k\rangle v^{\langle k\rangle - 1}$$

$$= \langle k\rangle (\langle k\rangle - 1) v^{\langle k\rangle - 2} (1-v) - \langle k\rangle v^{\langle k\rangle} + \langle k\rangle v^{\langle k\rangle - 1}.$$

Thus, for $v = 1$ all terms are zero except for those in which $(1-v)$ does not appear, but $f'(1) = -\langle k\rangle + \langle k\rangle = 0$ and $v^* = 1$ is an asymptotically super-stable fixed point.

Next, assume that $v^* = 1$ and $p < 1$. If $v \to 1$, in condition (6) all the terms except those for $l = \langle k\rangle$ are zero, so the condition becomes (after rearranging):

$$(1-p)\left(1 - \sum_{m=\lceil 0.4\langle k\rangle \rceil}^{\lfloor 0.6\langle k\rangle \rfloor} \binom{\langle k\rangle}{m} \frac{1}{2^{\langle k\rangle}} \right) = 0$$

as $\sum_{m=\lceil 0.4\langle k\rangle \rceil}^{\lfloor 0.6\langle k\rangle \rfloor} \binom{\langle k\rangle}{m} \frac{1}{2^{\langle k\rangle}} < 1$, it must be the case that $p = 1$, otherwise $v^* = 1$ would not be a fixed point. Contradiction. ■

References

[1] Amaro de Matos, J. and P. P. Barros, 2004, Social norms and the paradox of elections' turnout, *Public Choice* **121**, 239-255.

[2] Beck, P. A., R. J. Dalton, S. Greene, and R. Huckfeldt, 2002, The Social Calculus of Voting: Interpersonal, Media, and Organizational Influences on Presidential Choices, *American Political Science Review,* **96(1)**, 57-73.

[3] Berelson, B., P. F. Lazarsfeld, and W. N. McPhee, 1954, *Voting,* Chicago, University of Chicago Press.

[4] Blais, A., and R. Young, 1999, Why do people vote? An experiment in rationality, *Public Choice* **99**, 39-55.

[5] Downs, A., 1957, *An economic theory of democracy*, New York: Harper and Row.

[6] Feddersen, T. J., 2004, Rational Choice Theory and the Paradox of Not Voting, *Journal of Economic Perspectives* **18(1)**, 99-112.

[7] Ferejohn, J., and M. Fiona, 1974,The paradox of not voting: A decision theoretic analysis, *American Political Science Review* **68**, 525-536.

[8] Fowler, J. H., 2005, Turnout in a Small World, in Alan Zuckerman, ed., The Social Logic of Politics: Personal Networks as Contexts for Political Behavior, Temple University Press, 269-287.

[9] Fowler, J. H., and O. Smirnov, 2005, Dynamic parties and social turnout: an Agent-Based Model, *American Journal of Sociology* **110** (4), 1070-94.

[10] Gerber, A. S., D. P. Green, and R. Shachar, 2003, Voting May Be Habit-Forming: Evidence from a Randomized Field Experiment, *American Journal of Political Science* **47(3)**, 540-550.

[11] Gerber, A. S., D. P. Green, and C. W. Larimer, 2008, Social Pressure and Voter Turnout: Evidence from a Large-Scale Field Experiment, *American Political Science Review* **102(1)**, 33-48.

[12] Geys, B., 2006, 'Rational' Theories of Voter Turnout: A Review, *Political Studies Review* **4**, 16-32.

[13] Gimpel, J. G., J. J. Dyck, and D. R. Shaw, 2004, Registrants, voters and turnout variability across neighborhoods, *Political Behavior* **26(4)**, 343-375.

[14] Gimpel, J. G., I. L. Morris, and D. R. Armstrong, 2004, Turnout and the local age distribution: examining political participation across space and time, *Political Geography* **23**, 71-95.

[15] Green, D. and R. Shachar, 2000, Habit Formation and Political Behaviour: Evidence of Consuetude in Voter Turnout, *British Journal of Political Science* **30**, 561-573.

[16] Huckfeldt, R., P. A. Beck, R. J. Dalton, and J. Levine, 1995, Political environments, cohesive social groups, and the communication of public opinion, *American Journal of Political Science* **39(4)**, 1025-1054.

[17] Huckfeldt, R., and J. Sprague, 1995, Citizens, Politics, and Social Communication: Information and Influence in an Election Campaign, Cambridge University Press.

[18] Jackson, R. A., 2003, Differential Influences on Latino Electoral Participation, *Political Behavior* **25**, 339-366.

[19] Johnson, M., 2002, Revisiting the Ties that Bind: A Rational-Psychological Perspective on Social Connectedness, Mimeo, Rice University.

[20] Kanazawa, S., 1998, A Possible Solution to the Paradox of Voter Turnout, *The Journal of Politics* **60**, 974-995.

[21] Kenny, C. B., 1992, Political participation and effects from the social environment, *American Journal of Political Science* **36(1)**, 259-267.

[22] Lambiotte, R., and S. Redner, 2008, Dynamics of non-conservative voters, arXiv: 0712.0364v3 [physics.data-an].

[23] Lazarsfeld, P. F., B. Berelson, and H. Gaudet, 1948, *The People's Choice,* New York, Columbia University Press.

[24] Ledyard, J., 1984, The pure theory of two candidates elections, *Public Choice* **44(1)**, 7-41.

[25] Lijphart, A., 1997, Unequal participation: Democracy's unresolved dilemma, *American Political Science Review,* **91(1)**, 1-14.

[26] Matsusaka, J. G., 1993, Election closeness and voter turnout: Evidence from California ballot propositions, *Public Choice* **76**, 313-334.

[27] Matsusaka, J. G., 1995, Explaining voter turnout patterns: An information theory, *Public Choice* **84**, 91-117.

[28] Matsusaka, J. G., and F. Palda, 1993, The Downsian voter meets the ecological fallacy, *Public Choice* **77**, 855-878.

[29] McClurg, S. D., 2003, Social Networks and Political Participation: The Role of Social Interaction in Explaining Political Participation, *Political Research Quarterly* **56**, 449-464.

[30] McClurg, S. D., 2004, Indirect mobilization: the social consequences of party contacts, *American Politics Research* **32**, 406-443.

[31] McClurg, S. D., 2006, The electoral relevance of political talk: examining disagreement and expertise effects in social networks on political participation, *American Journal of Political Science* **50** (3), 737-754.

[32] Mutz, D. C., 2002, The consequences of cross-cutting networks for political participation, *American Journal of Political Science* **46** (4), 838-855.

[33] Nickerson, D. W., 2008, Is Voting Contagious? Evidence from Two field Experiments, *American Journal of Political Science* **102** (1), 49-57.

[34] Nir, L., 2005, Ambivalent social networks and their consequences for participation, *International Journal of Public Opinion Research* **17** (4), 422-442.

[35] Palfrey, T., and H. Rosenthal, 1983, A strategic calculus of voting, *Public Choice* **41**, 7-53.

[36] Palfrey, T., and H. Rosenthal, 1985, Voter Participation and Strategic Uncertainty, *American Political Science Review* **79**, 62-78.

[37] Plutzer, E., 2002, Becoming a Habitual Voter: Inertia, Resources, and Growth in Young Adulthood, *American Political Science Review* **96**, 41-56.

[38] Riker, W. H., and P. C. Ordeshook, 1968, A theory of the calculus of voting, *American Political Science Review* **62**, 25-43.

[39] Shachar, R., and B. Nalebuff, 1999, Follow the leader: Theory and evidence on political participation, *American Economic Review* **89**, 525-547.

[40] Uhlaner, C. J., 1989, Rational Turnout: The Neglected Role of Groups, *American Journal of Political Science* **33**, 390-422.

[41] Zuckerman, A. S., N. A. Valentino, and E. W. Zuckerman, 1994, A Structural Theory of Vote Choice: Social and Political Networks and Electoral Flows in Britain and the United States, *Journal of Politics* **56**, 1008-1033.