On Measuring the sources of Changes in Poverty using the Shapley method. An Application to Europe

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Abstract

The so-called Shapley decomposition approach, is based on the marginal effect on the value of the indicator resulting from eliminating sequentially each of the contributory factors and computing the corresponding marginal change in the statistic. The method then assigns to each factor the average of its marginal contributions in all possible elimination sequences.

The multidimensional Foster, Greer and Thorbecke for $\alpha = 2$ is a function of three determinants, namely the vectors of incidence, the intensity and the inequality. Applying the Shapley decomposition approach, we measure the marginal contribution on FGT_2 of each component.

Using EU-SIlC data for 2008 and 2015 for 28 European Countries, we analyze the change over time in the FGT_2 index and the value of the marginal contributions of the three components.

Keywords: poverty measurement; inequality measurement; Shapley decomposition;

JEL Classification: I30, I32, D63.

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1 Introduction

As Sen mentioned in his seminal work, Sen (1976), any poverty measure should be sensitive to incidence, intensity and inequality among the poor. Most of the poverty measures take into account these three terms, an there exists several indices that can be decomposed in terms of them, see Osberg and Xu (2000) and Aristondo and Ciommi (2016). These decompositions will allow policy makers to know if an increase on poverty is due to, a rise on the number of poor people, an increase on the intensity of them, a growth on the inequality among them, or a combination of the three. In fact, the study of these components could lead policy makers to detect the deterioration in the society derived from for example a poverty decrease with an increase on the headcount ratio compensated with the other two. The decrease of poverty in a country could direct policy makers to decide that the country has no problems related with poverty, when in fact it has problems with the increment of the number of poor people.

Therefore, we think that when poverty need to be measured, we should choose decomposable poverty measures and analyze it and their three underlying components.

However, most of these decompositions are not linear with respect to the three components and the way to measure the contribution of the components to the total poverty change is not obvious. There are several methods to compute these contributions, among others the Shapley value allocation method, see Shapley (1953).

This method has been applied in the poverty and inequality literature, for instance, Sastre and Trannoy (2002), Chakravarty et al. (2008), Chantreuil and Trannoy (2013) and Shorrocks (2013).

In this paper, we want to decompose the poverty change of a poverty measure in terms of the contributions of the incidence, intensity and inequality changes. As mentioned, Shapley decomposition approach, consists in evaluating the impact of each poverty component by eliminating sequentially each of the contributory factors and computing the corresponding marginal change.

However, if we want to apply Shapley method in order to compute the contributions of each factor, we must eliminate sequentially each contributory factor. Following Shapley method the way to eliminate each contributory factor is by equalizing the factors to zero. However, we are proposing to apply the Shapley method where the contribution factors are incidence, intensity and inequality changes between two periods. Therefore, we could annul this changes following different ways as assigning the first period value to the two periods, the second period value or a combination of the two.

In this paper we follow a similar procedure proposed by Kakwani and we propose to eliminate each contributory factor by assigning the arithmetic mean of all the possible combinations that cam be done with the two values, values in time 1 and values in time 0.

Finally, we have provided an example of this proposal for six European countries, computing their poverty change in 2008 and 2015 and the contributions of the three terms' changes.

The rest of the paper is organized as follows, section 2 is devoted to notations and definitions. Section 3 presents the Shapley decomposition and the contributions to the Incidence, Intensity and Inequality of the poor to the Sen and FGT_2 indices, respectively. Finally, section 4 shows an empirical application and section 5 offers some conclusions.

2 Notations and Definitions

We consider a population consisting of $n \geq 2$ individuals. Individual i's income is denoted by $y_i \in \mathrm{R}_{++}$, $i = 1, 2, \cdots, n$. An income distribution is represented by a vector $\boldsymbol{y} = (y_1, y_2, \cdots, y_n) \in \mathrm{R}^n_{++}$. We let $D = \bigcup_{n=1}^{\infty} \mathrm{R}^n_{++}$ represent the set of all finite dimensional income distributions. Without loss of generality, we further assume that the elements of $\mathbf{y}$ are pre-sorted in non-decreasing order, i.e., $y_1 \leq y_2 \leq \cdots \leq y_n$. For any given poverty line $z \in \mathrm{R}_{++}$ and distribution $\boldsymbol{y} \in D$, we define as poor all incomes that satisfy $y_i \leq z$. The set of poor people is denoted by Q. We denote by $n = n(\mathbf{y})$ and $q = q(\mathbf{y}; z)$ for any $\boldsymbol{y} \in D$ the population size and the number of the poor respectively, and by $\mu = \mu(\boldsymbol{y})$ the mean income of $\boldsymbol{y}$. Let $g_i = max\{(z - y_i)/z, 0\}$ be the relative poverty gap ratio of the i-th individual. Then, $\boldsymbol{g} = (g_1, \cdots, g_n)$ and $\boldsymbol{g}_p = (g_1, \cdots, g_q)$ are the relative poverty gap vector and the relative poverty gap vector of the poor, respectively. Note that these vectors are pre-sorted in non-increasing way as; $g_1 \geq g_2 \geq \cdots \geq g_n$.

A censored income vector is obtained by setting $\dot{y}_i = y_i$ if $y_i < z$ and $\dot{y}_i = z$ otherwise. We use the notation $\mathbf{1} = (1, \cdots, 1)$ and $\lambda \cdot \mathbf{1} = (\lambda, \cdots, \lambda)$.

Definition 1. *Let $P = P(\boldsymbol{y}; z) : D \times \mathrm{R}_{++} \to \mathrm{R}_{++}$ be a function whose value indicates the poverty level associated with distribution $\boldsymbol{y} \in D$ and poverty line $z \in \mathrm{R}_{++}$. The properties that a poverty measure could satisfy are, focus, replication invariance, symmetry, scale invariance, translation invariance, monotonicity, S-convexity, normalization and continuity.*

The *focus axiom* is satisfied if poverty does not change when the income of a non poor increase. *Replication invariance* is satisfied if poverty does not change replicating k times the income and

Symmetry is satisfied if poverty does not change permuting the incomes. *Scale Invariance* and *translation invariance* entail that the poverty value does not change multiplying or summing to all the incomes and the poverty line a positive real value, respectively. *Monotonicity* is satisfied if poverty increase by decreasing the income of a poor individual. *S-convexity* is satisfied if poverty decreases by a progressive transfer with at least the recipient being poor [1]. *Normalization* is satisfied if poverty is zero when the set of poor people is empty and and *continuity* is satisfied if the poverty measure is a continuous function on the incomes for any given poverty line.

Inequality measures will also play a role in this work.

Definition 2. *Let $I = I(\boldsymbol{y}) : D \to R_{++}$ be a function whose value indicates the inequality level associated with distribution $\boldsymbol{y} \in D$ which fulfills the properties of Pigou-Dalton transfer principle, normalization, symmetry, replication invariance and continuity.*

The *Pigou-Dalton transfer principle* is satisfied if inequality decreases as a consequence of a progressive transfer. *Normalization* is satisfied if inequality is zero when everybody has the same income. *Symmetry* is satisfied if inequality does not change permuting the incomes. *Replication invariance* is satisfied if inequality does not change replicating k times the income. Finally, *continuity* is satisfied if the inequality measure is a continuous function on the incomes. In addition, we say that an inequality measure is translation invariant (absolute) or scale invariant (relative) if $I(\boldsymbol{y}) = I(\boldsymbol{y} + \lambda \cdot \boldsymbol{1})$ or $I(\boldsymbol{y}) = I(\lambda \cdot \boldsymbol{y})$, respectively.

Now, we will present some poverty and inequality indices that we would need to know throughout this paper.

The *Headcount-ratio* is the first poverty measure introduced in the literature. Given q the number of the poor and n the population size, the *Headcount-ratio* is defined as the percentage of the population whose incomes are under the poverty line:

$$H = H(\boldsymbol{y}; z) = \frac{q}{n} \tag{1}$$

This poverty index only captures the incidence of poverty, moreover the *Headcount ratio* is considered in the literature as the archetypical measure of the incidence of poverty.

Another widely used measure of poverty is the *Income gap ratio*, denoted by M, which repre-

[1] $\mathbf{x} \in D$ is obtained from $\mathbf{y} \in D$ by a progressive transfer with at least the recipient being poor, that is, there exists i and j, $i < j$, such that $x_i - y_i = y_j - x_j = \delta > 0$, $x_i > y_i$ and $x_k = y_k$ for all $k \neq i, j$.

sents the mean among the poor of the poverty gap ratios:

$$M(\boldsymbol{y}; z) = \frac{q}{n}\sum_{i=1}^{n} g_i \tag{2}$$

This poverty index only captures the intensity of poverty, therefore this measure is considered as the archetypical measure of the intensity of poverty.

The above two measures violate the S-convexity axiom and consequently they do not take into account the inequality among the poor individuals. However, there exist numerous poverty measures that take into account for the three indicators: incidence, intensity and inequality among the poor.

In addition, most of them can be decomposed explicitly in terms of these three components. For example the poverty measures that we will use in this work are the poverty measures proposed by Sen (1976), S, and the second index of Foster et al. (1984), FGT_2.

The Sen index is defined as follows:

$$S(\boldsymbol{y}; z) = \frac{2}{q \cdot n}\sum_{i=1}^{q}(q + 0.5 - i)\big(g_i\big) \tag{3}$$

The second FGT index is defined as follows:

$$FGT_2(\boldsymbol{y}; z) = \frac{2}{n}\sum_{i=1}^{q}\big(g_i\big)^2 \tag{4}$$

As mentioned, the *Headcount ratio* and the *Income gap ratio* are considered the archetypical indices to measure incidence and intensity. However, with respect to the inequality term, different inequality measures can be use, see Aristondo (2018). We present the two inequality measures that we will use in this work to decompose the indices S and FGT_2, the Gini index and the Coefficient of variation.

The *Gini index* of the poor individual is defined as follows,

$$G(\boldsymbol{y}_p) = \frac{1}{q\mu(\boldsymbol{y}_p)}\sum_{i=1}^{q}(2i - n - 1)y_i \tag{5}$$

And the *Coefficient of variation* of the poor can be written as follows,

$$CV(\boldsymbol{y}_p) = \frac{\sqrt{\frac{1}{q}\sum_{i=1}^{q}(y_i - \mu(\boldsymbol{y}_p))^2}}{\mu(\boldsymbol{y}_p)} \tag{6}$$

Following Sen (1976) the Sen index can be decomposed in terms of their three components as follows:

$$S(\mathbf{y}; z) = H\big[M + (1 - M)G\big] \tag{7}$$

where $H = H(\mathbf{y}; z)$ is the headcount ratio, $M = M(\mathbf{y}; z)$ is the poverty gap of the poor and $G = G(\mathbf{y}_p)$ is the *Gini index* of the poor people.

On the other hand, FGT_2 can also be decomposed in thre three poverty terms as follows:

$$FGT_2(\mathbf{y}; z) = H\big[M^2 + (1 - M)^2 CV^2\big] \tag{8}$$

where $H = H(\mathbf{y}; z)$ is the headcount ratio, $M = M(\mathbf{y}; z)$ is the poverty gap of the poor and $CV = CV(\mathbf{y}_p)$ is the *Coefficient of variation* of the poor people.

3 Shapley decomposition

3.1 Shapley definition

The Shapley decomposition (1953) is a technique borrowed from game theory, but it has been extended to applied economics by Shorrocks (1999) and Sastre and Trannoy (2002). Let us explain it briefly. Assume that an indicator I is a function of three determinants a, b, c and is written as I(a,b,c). I could be an index of poverty but. more generally, any linear or nonlinear function of the concerned variables is allowable.

there are obviously 3!=6 ways of ordering these three determinants a, b and c:

(a,b,c), (a,c,b), (b,a,c), (b,c,a), (c,a,b), (c,b,a). B1

Each of these three determinants may be eliminated first, second, or third. The respective (marginal) contributions of the determinants a, b, c will hence be a function of all the possible ways in which each of these determinants may be eliminated. let, for example, C(a) be the marginal contribution of a to the indicator I(a,b,c).

If a is eliminated first its contribution to the overall value of the indicator I will be expressed as I(a,b,c)-I(b,c), where I(b,c) corresponds to the case where a is equal to zero. since expression B1 indicates that there are two cases in which a appears first and may thus be eliminated first, we will give a weight of 2/6 to this possibility.

If a is eliminated second, it implies that another determinant has been eliminated first (and been assumed to be equal to 0). ....

We may therefore summarize what we have just explained by stating that the marginal contribution C(a) of the determinant a to the overall value of the indicator is

C(a)=(2/6)[I(a,b,c)-I(0,b,c)]+(1/6)[I(a,0,c)-I(0,0,c)]+(1/6)[I(a,b,0)-I(0,b,0)]+(2/6)[I(a,0,0)]

C(b)=(2/6)[I(a,b,c)-I(a,0,c)]+(1/6)[I(0,b,c)-I(0,0,c)]+(1/6)[I(a,b,0)-I(a,0,0)]+(2/6)[I(0,b,0)]

C(c)=(2/6)[I(a,b,c)-I(a,b,0)]+(1/6)[I(a,0,c)-I(a,0,0)]+(1/6)[I(0,b,c)-I(0,b,0)]+(2/6)[I(0,0,c)]

And then we have that I(a,b,c)=C(a)+C(b)+C(c).

3.2 Shapley decomposition

There exists numerous poverty measures that can be decomposed in terms of the three components of poverty: the incidence, *H*, intensity, *M*, and the inequality among the poor, *I*. In fact, most of the poverty measures have been written as a function of these three terms, $P(H, M, I)$. These decompositions will allow us to know if an increase on poverty is due to an increase in the incidence, an increase in the intensity, an increase in the inequality among the poor or some combination of the three. However, these decompositions are no linear and the contributions of the components cannot be derived from the decompositions explicitly. Therefore, following the Shapley method decomposition we will be able to derive the contributions of these three component changes to the total poverty change.

Let define a poverty measure P that can be decomposed in terms of incidence, intensity and inequality among the poor as follows, $P(H, M, I)$. The poverty change between period $t = 0$ and $t = 1$ is defined as

$$P(H_1, M_1, I_1) - P(H_0, M_0, I_0) = f(H_1 - H_0, M_1 - M_0, I_1 - I_0) \tag{9}$$

For simplicity we define $H_c = H_1 - H_0$, $M_c = M_1 - M_0$, $I_c = I_1 - I_0$. Therefore, we can write the poverty change as an aggregative function of these terms: H_c, M_c, I_c

$$f(H_c, M_c, I_c) \tag{10}$$

Hence, following Shapley method we could obtain the contributions of the three terms to the final function f:

$$f(H_c, M_c, I_c) = c(H_c) + c(M_c) + c(I_c) \tag{11}$$

where the contribution of the incidence change H_c is:

$$c(H_c) = \frac{2}{6}\Big(f(H_c, M_c, I_c) - f(0, M_c, I_c)\Big) + \frac{1}{6}\Big(f(H_c, 0, I_c) - f(0, 0, I_c)\Big)$$
$$+\frac{1}{6}\Big(f(H_c, M_c, 0) - f(0, M_c, 0)\Big) + \frac{2}{6}\Big(f(H_c, 0, 0)\Big) \tag{12}$$

the contribution of the intensity change M_c :

$$c(M_c) = \frac{2}{6}\Big(f(H_c, M_c, I_c) - f(H_c, 0, I_c)\Big) + \frac{1}{6}\Big(f(0, M_c, I_c) - f(0, 0, I_c)\Big)$$
$$+\frac{1}{6}\Big(f(H_c, M_c, 0) - f(H_c, 0, 0)\Big) + \frac{2}{6}\Big(f(0, M_c, 0)\Big) \tag{13}$$

the contribution of the inequality change I_c :

$$c(I_c) = \frac{2}{6}\Big(f(H_c, M_c, I_c) - f(H_c, M_c, 0)\Big) + \frac{1}{6}\Big(f(0, M_c, I_c) - f(H_c, M_c, 0)\Big)$$
$$+\frac{1}{6}\Big(f(H_c, 0, I_c) - f(H_c, M_c, 0)\Big) + \frac{2}{6}\Big(f(0, 0, I_c)\Big) \tag{14}$$

Therefore, this decomposition will allow us to know the contributions of the terms changes to the total poverty change.

However, now we have to think on how we should add the values $H_c = 0$, $M_c = 0$ and $I_c = 0$ to the equations (12), (13) and (14). For example, for $H_c = 0$ we must assume that the headcount ratio have not changed in the period of study. Hence, when computing the contributions we should assume the same value for H_0 and H_1. Nevertheless, we could assume that the two values are H_0, H_1 or one conbitation of the two values.

In this paper, we will follow a similar methodology applied in Kakwani (2000) assuming the two values, H_0 and H_1. Then, we will compute the arithmetic mean of all the combinations. The definitions are shown below.

The $f(H_c, M_c, I_c)$ values when equalizing one variable to zero are:

$$f(0, M_c, I_c) = \frac{1}{2}\Big[\big(P(H_1, M_1, I_1) - P(H_1, M_0, I_0)\big) + \big(P(H_0, M_1, I_1) - P(H_0, M_0, I_0)\big)\Big] \tag{15}$$

$$f(H_c, 0, I_c) = \frac{1}{2}\Big[\big(P(H_1, M_1, I_1) - P(H_0, M_1, I_0)\big) + \big(P(H_1, M_0, I_1) - P(H_0, M_0, I_0)\big)\Big] \tag{16}$$

$$f(H_c, M_c, 0) = \frac{1}{2}\Big[\big(P(H_1, M_1, I_1) - P(H_0, M_0, I_1)\big) + \big(P(H_1, M_1, I_0) - P(H_0, M_0, I_0)\big)\Big] \tag{17}$$

The $f(H_c, M_c, I_c)$ values when equalizing two variables to zero are:

$$f(0, 0, I_c) = \frac{1}{4}\Big[\big(P(H_1, M_1, I_1) - P(H_1, M_1, I_0)\big) + \big(P(H_1, M_0, I_1) - P(H_1, M_0, I_0)\big)$$

$$+\big(P(H_0,M_1,I_1)-P(H_0,M_1,I_0)\big)+\big(P(H_0,M_0,I_1)-P(H_0,M_0,I_0)\big)\Big] \tag{18}$$

$$f(0,M_c,0)=\frac{1}{4}\Big[\big(P(H_1,M_1,I_1)-P(H_1,M_0,I_1)\big)+\big(P(H_1,M_1,I_0)-P(H_1,M_0,I_0)\big)$$
$$+\big(P(H_0,M_1,I_1)-P(H_0,M_0,I_1)\big)+\big(P(H_0,M_1,I_0)-P(H_0,M_0,I_0)\big)\Big] \tag{19}$$

$$f(H_c,0,0)=\frac{1}{4}\Big[\big(P(H_1,M_1,I_1)-P(H_0,M_1,I_1)\big)+\big(P(H_1,M_1,I_0)-P(H_0,M_1,I_0)\big)$$
$$+\big(P(H_1,M_0,I_1)-P(H_0,M_0,I_1)\big)+\big(P(H_1,M_0,I_0)-P(H_0,M_0,I_0)\big)\Big] \tag{20}$$

And finally $f(0,0,0)=0$.

Therefore, it is easy to see that the following proposition is satisfied:

Proposition 1. *The aggregative function f can be decomposed following Shapley method as follows*

$$f(H_c,M_c,I_c)=c(H_c)+c(M_c)+c(I_c) \tag{21}$$

where $c(H_c),c(M_c),c(I_c)$ are defined above.

Proof. The proof is straightforward.

In the previous section we have shown the decompositions in terms of H, M and I of the poverty measures of Sen and the FGT_2. Hence, we can apply the proposed Shapley decomposition to these two measures.

Proposition 2. *Let us consider the Sen and the FGT_2 poverty measures. The poverty change measured by the S and FGT_2 poverty measure can be decomposed as the sum of the contributions of the changes of each poverty component as follows:*

$$S(\boldsymbol{y}_1,z_1)-S(\boldsymbol{y}_0,z_0)=c_S(H_c)+c_S(M_c)+c_S(I_c) \tag{22}$$

$$FGT_2(\boldsymbol{y}_1,z_1)-FGT_2(\boldsymbol{y}_0,z_0)=c_F(H_c)+c_F(M_c)+c_F(I_c) \tag{23}$$

where the components for the Sen poverty index are:

$$c_F(H_c)=\frac{1}{6}\big(H_0-H_1\big)\Big(I_0\big(2M_0+M_1-3\big)+I_1\big(M_0+2M_1-3\big)-3\big(M_0+M_1\big)\Big)$$

$$c_F(M_c)=\frac{1}{6}\big(M_0-M_1\big)\Big(H_0\big(2I_0+I_1-3\big)+H_1(I_0+2I_1-3)\Big)$$

$$c_F(I_c)=\frac{1}{6}\big(I_0-I_1\big)\Big(H_0\big(2M_0+M_1-3\big)+H_1\big(M_0+2M_1-3\big)\Big)$$

and the components for the FGT_2 poverty index are:

$$c_F(H_c) = -\frac{1}{6}\big(H_0 - H_1\big)\Big((I_0)^2\,\big(2(M_0)^2 - 4M_0 + (M_1)^2 - 2M_1 + 3\big)$$

$$+(I_1)^2\,\big((M_0)^2 - 2M_0 + 2(M_1)^2 - 4M_1 + 3\big) + 3\,\big((M_0)^2 + (M_1)^2\big)\Big)$$

$$c_F(M_c) = -\frac{1}{6}\big(M_0 - M_1\big)\Big((I_0)^2(2H_0 + H_1)(M_0 + M_1 - 2) + (I_1)^2(H_0$$

$$+2H_1)(M_0 + M_1 - 2) + 3(H_0 + H_1)(M_0 + M_1)\Big)$$

$$c_F(I_c) = -\frac{1}{6}\big((I_0)^2 - (I_1)^2\big)\Big(H_0\big(2(M_0)^2 - 4M_0 + (M_1)^2 - 2M_1 + 3\big)$$

$$+H_1\big((M_0)^2 - 2M_0 + 2(M_1)^2 - 4M_1 + 3\big)\Big)$$

The formula for each contribution can be seen in the appendix.

4 The empirical application

In this section, we illustrate the methodology developed in the paper with data from twenty eight European Union countries for two different years: 2008 and 2015. We analyze the changes of poverty and the sources of these changes. For this purpose, we use the European Union Survey on Income and Living Conditions (EU-SILC). The variable of interest we use in this application is the *total disposable income*.

Table 1: FGT_2 changes and its contributions.

Country	Δ FGT_2	c(ΔH)	c(ΔM)	c(ΔI)
Netherlands	-0.0217	0.0026 *(-12 %)*	-0.0009 *(4 %)*	-0.0235 *(108 %)*
Latvia	-0.0044	-0.0053 *(122 %)*	-0.0017 *(38 %)*	0.0026 *(-60 %)*
Finland	-0.0037	-0.0018 *(50 %)*	-0.0005 *(14 %)*	-0.0013 *(36 %)*
Slovenia	0.0012	0.0030 *(254 %)*	0.0001 *(11 %)*	-0.0019 *(-165 %)*
Ireland	0.0036	-0.0012 *(-34 %)*	0.0012 *(33 %)*	0.0036 *(101 %)*
Denmark	0.0094	0.0007 *(8 %)*	0.0006*(6 %)*	0.0080 *(86%)*

Note: Own elaboration from EU-SILC data.

We have computed the FGT_2 index and the contributions of the three components, ΔH, ΔM and ΔI, for six European countries: Latvia, Netherlands, Finland, Denmark, Slovenia and Ireland. The results can be seen in table 1. Figure 1 and Figure 2 graph the contributions for the countries with an increase or a decrease in the FGT_2 poverty value, respectively.

If we focus on Figure 1, we have drawn the contributions of the countries for which the poverty has increased: Denmark, Ireland and Slovenia. For these countries we have observed an increment on poverty, hence, a positive contribution of a term means also an increase in this term. However, the sources of the poverty increase are different. For example, Denmark is the unique country for which the three components contribute to the final poverty increase positively. However, for Slovenia the inequality among the poor has decreased with a negative contribution to the total change higher than 160 percent. With respect to Ireland, poverty has also increased but the number of poor people has decreased with a negative contribution of 34 percent.

On the other hand, if we focus on Figure 2, we have drawn the contributions of the countries for which the poverty has decreased: Netherlands, Latvia and Finland. However, the decrease of poverty is not always followed by a decrease of the three components. In this case, a positive contribution of each term will also mean a decrease of the term change.

Although we have had a fall in poverty, the three components have decreased only for Finland. For Latvia the inequality among the poor has worsened with a negative contribution of 60 %. With respect to Netherlands, the Headcount ratio has increased a bit with a low negative contribution of 12 percent.

Figure 1. Contributions of H, M and I to the FGT_2 index for a total poverty increase

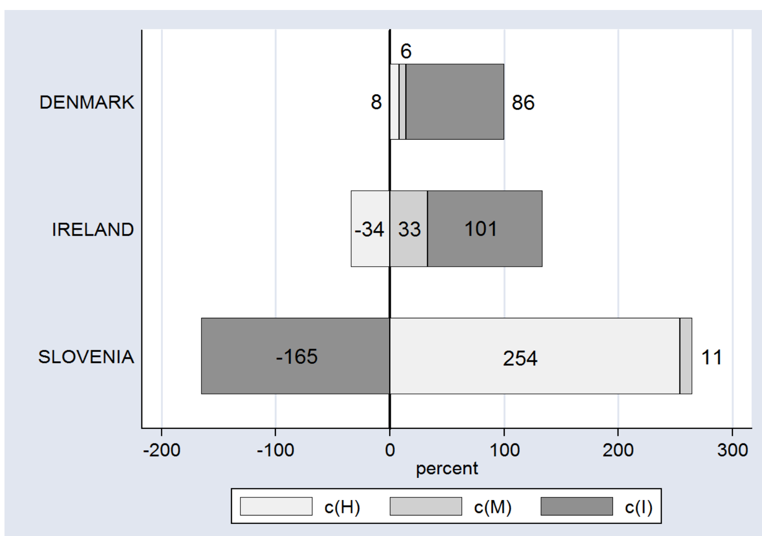
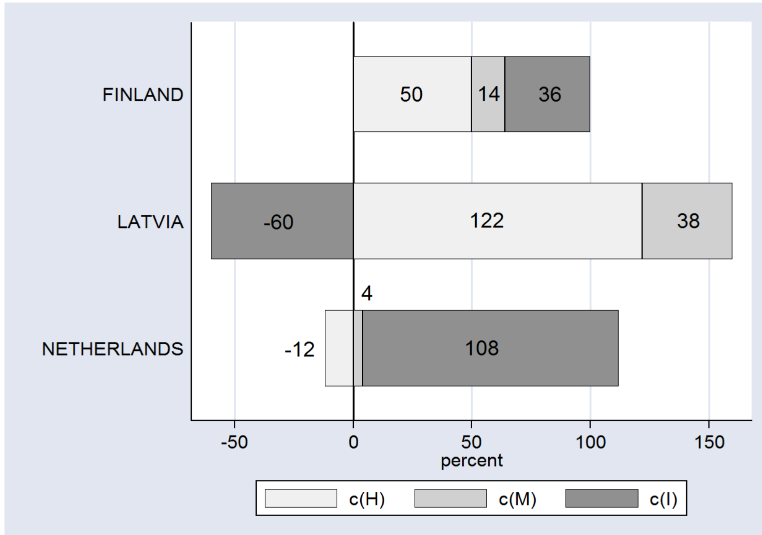


Figure 2. Contributions of H, M and I to the FGT_2 index for a total poverty decrease



5 Conclusions

All the poverty measures should take into account the incidence, intensity and inequality among the poor. In addition, most of them can be decomposed in terms of the three components.

However, usually these decompositions are not written in a linear way. Hence, the contributions of these terms can not be obtained directly from the decompositions. In order to overcome this problem we have applied the Shapley decomposition method. This method has been applied to the total poverty change where the contributory factors are the changes in the incidence, intensity and inequality of the poor. For this purpose, we have defined some new functions in the application of the Shapley method.

This Shapley decomposition has allowed us to know the contribution of each component change to the total poverty change. Hence, we would be able to compute the percentage contributions of incidence, intensity and inequality changes to the total poverty change.

Finally, we have provided an example of this proposal computing the poverty change and their marginal contributions between 2008 and 2015 for six European countries: Latvia, Netherlands, Finland, Denmark, Slovenia and Ireland. We have seen that poverty has increased for Denmark, Ireland and Slovenia and decreased for Netherlands, Latvia and Finland. However, the sources of these changes have been different for each country. For example, the three poverty terms have followed the same trend as poverty only in Denmark and Finland.

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