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Large size FFTs over time-varying channels

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In orthogonal frequency division multiplexing (OFDM) signals, the usage of larger sized FFTs reduces the guard interval percentage, and therefore, increases the data throughput reducing the data overhead. In addition, for the same pilot pattern, the distance between adjacent pilots is smaller, which will improve the channel estimation. Nevertheless, up to now, they have not been considered for delivering mobile services as the Inter Carrier Interference (ICI) due to the Doppler Effect is very critical. The main objective of this work is to show that taking advantage of the latest improvements in error correction techniques, it is perfectly feasible to use large sized FFTs for time-varying channels. Furthermore, through this letter there is also a theoretical estimation for quantifying the loss due to the ICI, and finally, several simulation results that reinforce the idea that large OFDM symbols are suitable for mobile channels.

Introduction: Due to its advantages OFDM has been used as the transmission multiplexing technique for the most important broadcasting standards, such as DVB family, ISDB-T or CMBB. Nevertheless, it is very sensitive to the channel time domain selectivity, which can lead to inter-carrier interference [1]. The performance degradation due to this distortion depends, on the one hand, on the maximum Doppler Frequency, and on the other hand, on the existing spacing between OFDM carriers. This spacing is inversely proportional to the FFT size, that is to say, the smaller the FFT, the bigger the subcarrier spacing, and therefore, the more ICI that the system can withstand. In fact, this is the main reason why the mobile broadcasting standards are only defined for short symbol lengths.

This work proves the suitability of using large sized FFTs for mobile channels. Apart from that, it offers two more contributions: first, it theoretically estimates the impact of the ICI degradations on the receiving thresholds, and second, the LDPC performance loss that may occur due to a mismatch in the noise power estimation is shown.

System Model: First of all, the system model approach for the Doppler noise analysis is presented. The transmitter applies an N-point IFFT to an $x[k]$ data block of QAM-symbols, where k represents the sub-channel modulated symbol:

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} x[k]e^{j\frac{2\pi}{N}nk} \tag{1}$$

At the receiver, perfect time and frequency synchronization are assumed, and thus, the channel output spectrum, $y[k]$, can be defined by the following expression [2]:

$$y\left[k\right] = \frac{1}{N}\sum_{m=0}^{N-1} X[k]H\left[k-m,m\right] + w\left[k\right] \tag{2}$$

where $H\ [k,m]$ describes the double Fourier transform of the channel impulse response (CIR) and $w[k]$ is the additive white Gaussian noise (AWGN) with zero mean and N_0 standard deviation. The previous expression can be simplified assuming that $\mathbf{Z}$ denotes the matrix defining the circular-shifted convolution, and thus, the received signal spectrum $\mathbf{y}$ can be represented as the matrix multiplication of $\mathbf{Z}$ and the transmitted symbol spectrum $\mathbf{x}$, plus the additive noise given by $\mathbf{w}$. It must be borne in mind that the channel matrix $\mathbf{Z}$ gathers two different sources of distortion: the multipath environment and the Doppler Effect due to the mobile scenario. Specifically, the main diagonal, $\mathbf{Z}^{\mathbf{d}}$, is related to the channel attenuation due to the multipath fading; and $\mathbf{Z}^{\mathbf{ICI}}$, which is composed of the rest of $\mathbf{Z}$ matrix sub-diagonals and super-diagonals, is connected to the ICI due to the Doppler Effect.

$$\mathbf{y} = \mathbf{Zx} + \mathbf{w} = \left(\mathbf{Z}^{\mathbf{d}} + \mathbf{Z}^{\mathbf{ICI}}\right)\mathbf{x} + \mathbf{w} \tag{3}$$

Furthermore, taking into account that the CIR is changing within a symbol duration, the $\mathbf{Z}$ matrix diagonal can be expressed as the mean value of the channel impulse response, whereas the other sub-diagonals just contribute as noise [3]. Therefore, if least square (LS) channel estimation is performed at the receiver, the following expression is obtained:

$$\mathbf{H_{est}} = \frac{\mathbf{y}}{\mathbf{x}} = \mathbf{H_{ave}} + \mathbf{w_{ICI}} + \mathbf{w_{AWGN}} \tag{4}$$

where the first term in the summation accounts for the CIR mean value, the second one for the ICI and the third one for the common AWGN.

Doppler Noise Theoretical Modelling: The distortion introduced by the Doppler Effect is twofold; it does not only introduce an additional noise, but also reduces the signal power, which is converted into ICI. In other words, due to the loss of orthogonality, at the receiver some part of the conveyed signal power is turned into ICI. Thus, it is clear that the higher the ICI is, the more the performance degrades. Providing that the Doppler noise can be treated as another type of noise, the theoretical performance loss can be obtained from (5) and (6).

$$K_{ICI}(dB) = 10\log_{10}\left[1 - 10^{\frac{\left(\left(\frac{C}{N}\right)^{\min\_theor} + \Delta_{ICI}\right)}{10}}\right] \tag{5}$$

$$\left(\frac{C}{N}\right)^{\min\_real} = \left(\frac{C}{N}\right)^{\min\_theor} - K_{ICI} + 10\log_{10}\left(1 + 10^{\left(\frac{\Delta_{ICI}}{10}\right)}\right) \tag{6}$$

The correction factor K_{ICI} calculates the impact of the inter-carrier interference in the received signal, and in Eq. (7), the power division is added to this degradation. Both expressions use Δ_{ICI}, which is the power relation between the multipath component and the Doppler noise, i.e., the relative power of the ICI.

Once the theoretical boundaries have been set up, the next step is to estimate the approximated values of the Δ_{ICI} component in order to see how deep its impact could be. As an example, the TU-6 channel model is selected, which is supposed to be one of the most used channels for testing mobile performance.

Fig.1 demonstrates the Δ_{ICI} of OFDM modulation for different Doppler rates. A carrier frequency of 700 MHz is assumed, which is the worst case scenario for mobile channels. The ICI relation is given for different $f_d{\cdot}T_u$ values, which represent the relation between the maximum Doppler frequency f_d and the symbol duration T_u. The upper thick line, SNR_{min}, represents the overall noise power that an OFDM signal with an LDPC R=1/5 code can withstand. It can be seen that the blue line, corresponding to an 8K signal at 75 km/h, is more than 20 dB below the signal power. Therefore, the ICI power is negligible if compared to the AWGN. This is the main reason why the 8K size is perfectly suitable for mobile scenarios. Furthermore, comparing the different ICI powers, it can be seen that there is about 6 dB increase when doubling the Doppler frequency. If the worst case is taken, that is, the purple line representing a 32K signal at 150 km/h, it can be seen that it is about 8 dB below the receiving threshold. Therefore, it seems that, if a strong error correction code is used, it is feasible to use large FFT sized signals for mobile scenarios. The main advantages of using larger sized FFTs are, first, that the data throughput is increased; and second, that for the same pilot and data carrier ratio, larger FFT means that the distance between adjacent pilots is smaller. This will improve the channel estimation accuracy, which will lead to better signal cancellation and system performance. Although this proves that large FFT sizes can be used, it is important to figure out what the actual performance loss will be. In order to do that, the theoretical expressions defined in (5) and (6) can be used. To do so, the Δ_{ICI} value is obtained as an approximated mean value based on multiple simulations.

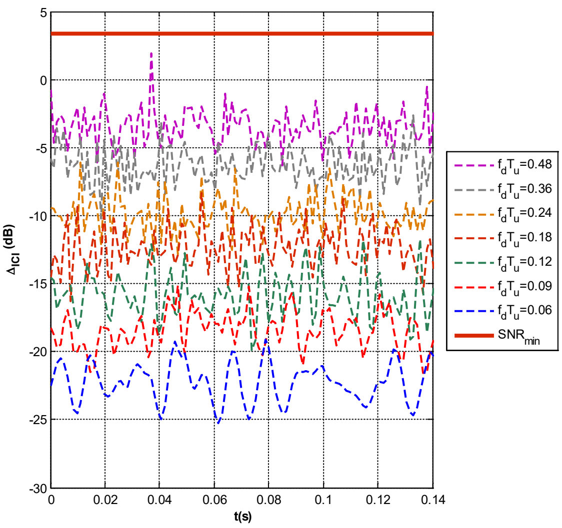

Fig. 1. *OFDM ICI for different Doppler frequencies*

The results are shown in Table I. It has been assumed that in the case of a nearly stationary TU-6, i.e., ICI free case, the minimum receiving threshold SNR^{min_theor} is about -3.1 dB for a code rate of 1/5. Based on those values and applying the mathematical formulation defined in (5) and (6), the theoretical approximated receiving threshold is calculated.

Table I: Approximated calculation of the receiving SNR (dB) based on the possible estimation of the ICI power

$f_d{\cdot}T_u$	0.06	0.09	0.12	0.18	0.24	0.36	0.48
Δ_{ICI}	-21.8	-18.0	-15.6	-12.2	-9.0	-6.0	-3.5
SNR_{est}	-3.1	-3.1	-3.0	-2.8	-2.5	-1.7	-0.6

According to the theoretical calculations, it seems reasonable to use large FFT sizes for mobile scenarios, although a performance loss must be expected. For instance, for the worst case considered, 32K and 150 km/h or $f_d{\cdot}T_u=0.48$, this loss could be up to 3 dB. However, for the same FFT length but at 75 km/h or $f_d{\cdot}T_u=0.36$, the performance loss is supposed to be about just 1.5 dB, when compared to the smallest Doppler contribution ($f_d{\cdot}T_u=0.06$).

Simulations Results: In the previous section it has been theoretically proved that using very robust code rates, large sized FFTs can be applied for mobile scenarios. The main objective of this section is to prove through simulations that the previous assumption is feasible. In order to test the real Doppler impact over existing broadcasting standards, an LDPC coding/decoding stage has been included in the system model presented before. The LDPC decoding should be performed using soft decision values or metrics, which are also known as Log Likelihood-Ratios (LLR). The LLR reliability depends on the channel estimation accuracy and overall noise power as shown in (7):

$$LLR_i\left(I,Q_i \middle/ x_i\right) = \frac{1}{2\pi N_o} e^{\frac{\left(I_i-\rho_I I_{x_i}\right)^2-\left(Q_i-\rho_Q Q_{x_i}\right)^2}{2N_o}} \qquad (7)$$

where (I,Q) and (I_x,Q_x) represent the transmitted and received in-phase and quadrature pairs, respectively. As seen before, if the channel time variability is high, the ICI power value should be taken into account, as its value is close to the existing Gaussian noise. If not considered, there might be an additional performance loss, not only due to the increase of Doppler related noise, but also due to a mismatch in the LLR calculation.

In Table II, the results for different mobile user cases are shown. Each scenario has three results: in the first one, the Gaussian noise is the only noise power taken into account for the LDPC decoding (Case I), whereas for the second case, the LDPC input is feed with the sum of both distortion noises (Case II). The last column is based on the theoretical formula presented previously. It can be seen that the second option improves the performance of the LDPC decoding, showing a performance loss for the worst case of just 2 dB. What is more, for the 115 km/h case, a 32K signal just suffers 1 dB loss, which makes it completely feasible for real implementation. Furthermore, it must be taken into account that the TU-6 is a very challenging channel and that such high speeds are not normally expected in an urban environment. Finally, it is important to note that these practical values are pretty close to the theoretical assumption.

Table II: Minimum receiving SNR threshold for different mobile scenarios.

	50 Hz (75 km/h)			75 Hz (115 km/h)			100 Hz (155 km/h)		
	I	II	Th	I	II	Th	I	II	Th
8k	-3.1	-3.1	-3.1	-3.1	-3.1	-3.1	-3.0	-3.1	-3.0
16k	-3.0	-3.0	-3.0	-2.7	-2.9	-2.8	-2.6	-2.7	-2.5
32k	-2.6	-2.7	-2.5	-1.8	-2.1	-1.7	-0.2	-1.1	-0.6

Conclusions: The main contribution of this letter is to show that using robust error correction codes, the large FFT OFDM signals are quite robust to the Doppler effects. In addition, a theoretical formulation for estimating the performance loss due to the ICI has been presented, and finally, it has been proved that the addition of the ICI to the overall noise estimation increases the LDPC decoding performance, thanks to the more accurate noise estimation. As a result, large FFTs might be implemented for mobile channels, if the approach of using ICI power error corrections is considered.

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