A Metamodeling Technique for Variable Geometry Trusses Design via Equivalent Parametric Macroelements.

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Abstract

A metamodeling methodology is applied to reduce the large computational cost required for static and dynamic analyses in the design of Variable Geometry Trusses (VGT's). This methodology drastically reduces the size of complex and large nonlinear finite element models, established as per the Equivalent Parametric Macroelement (EPM) concept. Using this methodology, submodels of finite elements within a complete FE model of the VGT are substituted by groups of fewer elements called EPM's. Some of the physical properties of elements forming the EPM are chosen as parameters; therefore, after optimization of given equivalence criteria, these parameters are calculated to adjust EPM behavior as best as possible to that of the submodel. Equivalence criteria are established according to elastic energy and inertial property conservation, and the optimization process is based on Nonlinear Least Square Minimization and Genetic Algorithms as initial value seeder. Static and dynamic validations were done to demonstrate the method performance.

1 Introduction

Finite Element Method is one of the most powerful and widespread method to perform most kind of mechanical analysis and design, so it has become a very valuable tool to perform static, dynamic and kinematic analyses on deformable bodies and assemblies in all mechanical engineering fields. Furthermore, the increasing power of computers have made possible to manage huge finite element models with millions of degrees of freedom, obtaining the solutions in relatively small periods of time. Nevertheless, it must be pointed out that one of the main goals is to reduce the computing effort as much as possible while keeping a sufficient level of accuracy. In this sense, the diverse methods for FE model reduction have not lost yet their topicality. There is a great amount of cases [1-5] in which a good model reduction has to be performed in order to get solutions with a reasonable cost, especially when a great amount of load cases have to be managed.

When a Finite Element model reduction has to be performed, any of the following methods can be used, depending on the nature of the analysis, size of the model and results to be reported: condensation [6-9], macroelements [10-13], substructures [14-16] and also metamodels. Metamodeling is a general process of creating an abstraction of an underlying phenomenon over a given domain, creating a “model of the model”. In the general scope of metamodeling, a response may be evaluated via a physical experiment or a computer simulation at a number of points in the domain. The common feature of all approaches is that the actual response is known at a finite number of points; therefore, a metamodel of the domain is created and used as a surrogate for the original model. In the past two decades, a great amount of work on metamodeling based on the interaction of high- and low fidelity numerical models have been developed; a recent review of the state of the art can be found in [17]. Toropov and Markine [18] considered a metamodel as a tuned low fidelity model and suggested three approaches to tune it: linear and multiplicative biparametric metamodels, use of correction functions [19], and the use of low fidelity model inputs as tuning parameters [20]. The procedure presented in this paper can be classified within this last approach in which a deeper understanding of the process being modeled is required. However, unlike in conventional metamodel building where responses of low and high fidelity models themselves are matched, in this paper the matching properties refer to individual subsystems of the high and low fidelity models. So, a new method is developed combining techniques of macroelements and metamodels which allows the replacement of a group of elements within a FE model, with a more simple and reduced one, called Equivalent Parametric Macroelement or EPM, being possible to tackle with internal nonlinearities such as great displacements and rotations and contact relations. This replacement is done taking into account certain equivalence criteria which is to be developed deeply in section 2. In section 3 an application is presented to show the advantage of replacing some repeated groups of the whole model by EPM's in the design of a VGT. Finally a concluding section is reported to summarize the work.

2 Equivalent Parametric Macroelement Technique

2.1 Definition of an EPM

In this work, a model reduction is performed substituting groups of elements (from now on "submodels") with the so-called EPM's, reducing so the total amount of dof's of the model. As in any reduction technique, sufficient equivalence must be fulfilled between the two entities to be considered valid; such equivalence can be both static and dynamic; in the static case, elastic potential energy equivalence is proposed and in the dynamic case equivalent point mass concepts are taken into account.

As mentioned before, an EPM substitutes a submodel but it is composed also by elements commonly used such as beams, plates, shells, solid elements… The number of dof of the elements forming the EPM will be much smaller than that of the substituted submodel. As a general rule, the more elements forming the EPM, the more accurate the substitution will be; on the other hand the fewer elements forming the EPM, the higher the computational saving. Another important feature the EPM has to fulfill is the kinematic equivalence (in addition to the static and dynamic one), i.e. the possible rigid-solid movements must be taken into account; therefore, the convenient amount of dof's must be left free inside the elements forming the EPM to achieve the desired kinematic equivalence.

To justify the use of EPM's, the systems in whose models they are included would have a high number of dof's and some type of nonlinearity thus rendering useless methods as condensation, substructures and macroelements in a direct way. One representative type of such systems is the case of the modular Variable Geometry Trusses (VGT's). These Multi-degree-of-freedom systems have two features making the use of EPM's very interesting: on the one hand, in this systems some submodels are repeated in many modules of the VGT; on the other hand, when mechanical analyses have to be performed in VGT's, a lot of positions must be taken into account and therefore, many analysis in different positions have to be performed. In section 4, the application of EPM's for VGT's is described.

2.2 Submodel Substitution Procedure via EPM's

Once it has been decided which submodel must be substituted by an EPM, its design process begins. In step one, the total amount of elements forming the EPM must be decided, as well as the constraints among the dof's in order to be kinematically equivalent; this step is called "topological-kinematical design". Afterwards, the stiffness properties must be determined in step two called "static design". The last step, called "dynamic design", equivalent point mass distributions are decided to conserve the mass and inertial properties.

2.2.1 *Topological-kinematical design.* The first condition to topologically-kinematically design an EPM is to ensure the correct transmission of forces between it and the rest of the model. This fact made limit the use of some simple elements like rod-type ones (bi-articulated deformable bars without moment transmission capability) for example in locations where moment transmission is needed. As a second condition, the same mobility as the substituted submodel would be required for the EPM. In practice this means that if a rotation or a translation is present within the submodel, this type of movement must also be present among the elements forming the EPM.

2.2.2 *Static design.* The aim at this stage is to fix the stiffness properties of the EPM to be equivalent to those of the submodel. To achieve this, some properties of the elements forming the EPM have to be chosen as parameters to be optimized according to certain equivalence criterion. The parameters can be elastic properties of the elements such as the Young Modulus of the material or geometrical properties (not having interference with those chosen in the topological-kinematical design stage) such as the section area of a beam element or the thickness of a shell element. The equivalence criterion adopted in this work was the equality between stored elastic energy of the submodel and that of the elements of the EPM in some selected displacement cases, being careful with the scope of the real range of deformations and displacements required for the model. In general, the displacement cases for which the equivalence is to be arranged will be much greater than the number of parameters chosen for the EPM. Therefore, a least square methodology will be arranged to fulfill the equivalence criterion as best as possible.

Problem arrangement. The connection dof number *h* of the submodel with the rest of the model will be named as “master dof’s” and have to be present in both the submodel and the MEP. The different *n* displacement cases are stored in the following vectors:

$$\{\delta\}_i = [\delta_1 \quad \delta_2 \quad \dots \quad \delta_h]_i^T \tag{1}$$

with i=1,2,…,n. These n vectors can be stored in a $[\delta]_{h\times n}$ matrix. The elastic energy stored by the submodel will be denoted as U_i and it only will be a function of the corresponding displacement case, i.e. $U_i = U_i(\{\delta_i\})$. All the submodel elastic energies can be put in the following vector:

$$\{U\} = [U_1 \quad U_2 \quad \dots \quad U_n]^T \tag{2}$$

This vector is calculated only once, and represents the elastic behavior of the submodel in the range of displacements given by (1) from i=1 to n. The selected parameters in the EPM would also be stored in the vector $\{A\}$:

$$\{A\} = [A_1 \quad A_2 \quad \dots \quad A_p]^T \tag{3}$$

In the same way, the elastic energy stored by the EPM will be denoted as V_i, and apart from being function of the associated displacement case, it is also function of the unknown parameters (3), i.e. $V_i = V_i(\{\delta_i\}, \{A\})$. All the EPM elastic energies can be arranged in the following vector:

$$\{V\} = [V_1 \quad V_2 \quad \dots \quad V_n]^T \tag{4}$$

At this point, the equivalence criterion requires the following equation system to be solved over $\{A\}$:

$$\{U([\delta])\} = \{V([\delta], \{A\})\} \tag{5}$$

This equation system with n equations and p unknowns is nonlinear due the type of elastic energy dependence on the parameters. Besides, the system is highly overconstrained because it should be usual to take a lot of displacement cases to cover the total operational range of the submodel, i.e. $n >> p$. In order to solve (5) a Nonlinear Least Squares Method (NLSM) is used.

Matrix formulation of the NLSM. Iterative process. Due to the nonlinearity of (5), its linearized form has to be solved iteratively. For an iteration *k*, the linearized system can be formulated in matrix form as follows:

$$[T]^k \cdot \{A\}^{k+1} = \{B\}^k \tag{6}$$

where:

$$\begin{aligned} [T]^k &= \left[\frac{\partial V}{\partial A}\right]^k \\ \{B\}^k &= [T]^k \cdot \{A\}^k + \{U\} - \{V\}^k \end{aligned} \tag{7}$$

Multiplying the two terms of (6) by the transpose of [*T*], the following *n*×*n* linear system is achieved, from where the value of {*A*} is obtained in the iteration *k*+1:

$$[T]^{k^T} \cdot [T]^k \cdot \{A\}^{k+1} = [T]^{k^T} \cdot \{B\}^k \tag{8}$$

This process is followed until certain stop criteria are fulfilled. The error can be understood in terms of variations between vectors {*U*} and {*V*} or in variations of unknown vector {A}. For the first case we define the error as the average quadratic difference between those vectors in the iteration *k*:

$$\varepsilon^k = \sqrt{\frac{\sum_{i=1}^{n} \left(U_i - V_i^k\right)^2}{n}} \tag{9}$$

And then the following stop criterion can be arranged, being *q* the desired stop tolerance:

$$\left|\frac{\varepsilon^{k+1} - \varepsilon^k}{\varepsilon^k}\right| < q \tag{10}$$

For the second case, the difference norm between vectors $\{A\}^{k+1}$ and $\{A\}^k$ can be proposed, being q' another stop tolerance:

$$\left\|\{A\}^{k+1} - \{A\}^k\right\| = \sqrt{\sum_{j=1}^{p} \left(A_j^{k+1} - A_j^k\right)^2} < q' \tag{11}$$

Additional remarks. Some comments should be made here about two topics. In the one hand, a correct estimation of the initial values of the parameters might be done. Initial values far from the true values of the parameters can lead to undesired local minima. To achieve good enough initial values, the use of Genetic Algorithms is encouraged.

Another important question is the appropriate control of the variables. In fact, when the value of some of the variables has little influence on the EPM elastic energy V, these variables can take values far from those having any real physical significance. In this case, special control of these variables is proposed, established upon the values of matrix $[T]$. In fact, as this matrix is formed by the derivatives of $\{V\}$ with respect to $\{A\}$, in case any value of $[T]$ is very small, then it means that the influence of the variable is small in the variation of the function.

2.2.3 *Dynamic design.* In the case of Guyan condensation [6], calculation of the Guyan matrix $[G]$ is established upon the equality of both elastic energy and kinetic energy along the vibration modes. In the case of the EPM's an elastic energy least square conservation criterion has been taken into account to perform the static equivalence. Therefore, the most logical step to dynamically design the EPM would be to solve some type of least square minimization of the difference between the kinetic energy of the submodel and that of the EPM along the movement of vibration modes, something like equation (5) but in terms of kinetic energy instead of elastic energy. The kinetic energy of the submodel would be a function of nodal velocity cases given by movements along natural modes; on the other hand, the kinetic energy of the EPM would also be function of the vector of variables/parameters; in the case of dynamic design, the parameters would be the values of point masses located at nodes of the EPM. With the application of a NLSM the values of the masses would be calculated and the complete static/dynamic model would be achieved.

Nevertheless, for a general submodel where nonlinearities can appear, the concept of vibration mode has no direct interpretation. In fact, instead of vibration modes, the concept of nonlinear normal mode should be applied. However, the development of kinetic energy in the movement along nonlinear normal modes would be computationally intensive; therefore the most logical way would be to leave both the submodel and the EPM in free nonlinear movement, for some cases of representative initial conditions; in this case the equivalence criterion for example would be the equality of amplitudes in the movement (supposing non-damped movement).

Instead, a simpler dynamic equivalence can be developed, imposing rigid-solid dynamic equivalence. In this sense, unknown point masses are placed in the nodes of the EPM and rigid-solid equivalence conditions are imposed: the mass conservation equation:

$$\sum_{i=1}^{n} m_i = M \tag{12}$$

The three Center of Mass conservation equations:

$$\frac{\sum_{i=1}^{n} m_i \cdot x_i}{M} = X_G \quad \frac{\sum_{i=1}^{n} m_i \cdot y_i}{M} = Y_G \quad \frac{\sum_{i=1}^{n} m_i \cdot z_i}{M} = Z_G \tag{13}$$

And the six inertia matrix conservation equations:

$$\begin{aligned} &\sum_{i=1}^{n} m_i \cdot (y_i^2 + z_i^2) = I_{xx} \quad && \sum_{i=1}^{n} m_i \cdot x_i \cdot y_i = I_{xy} \\ &\sum_{i=1}^{n} m_i \cdot (x_i^2 + z_i^2) = I_{yy} \quad && \sum_{i=1}^{n} m_i \cdot y_i \cdot z_i = I_{yz} \\ &\sum_{i=1}^{n} m_i \cdot (x_i^2 + y_i^2) = I_{zz} \quad && \sum_{i=1}^{n} m_i \cdot x_i \cdot z_i = I_{xz} \end{aligned} \tag{14}$$

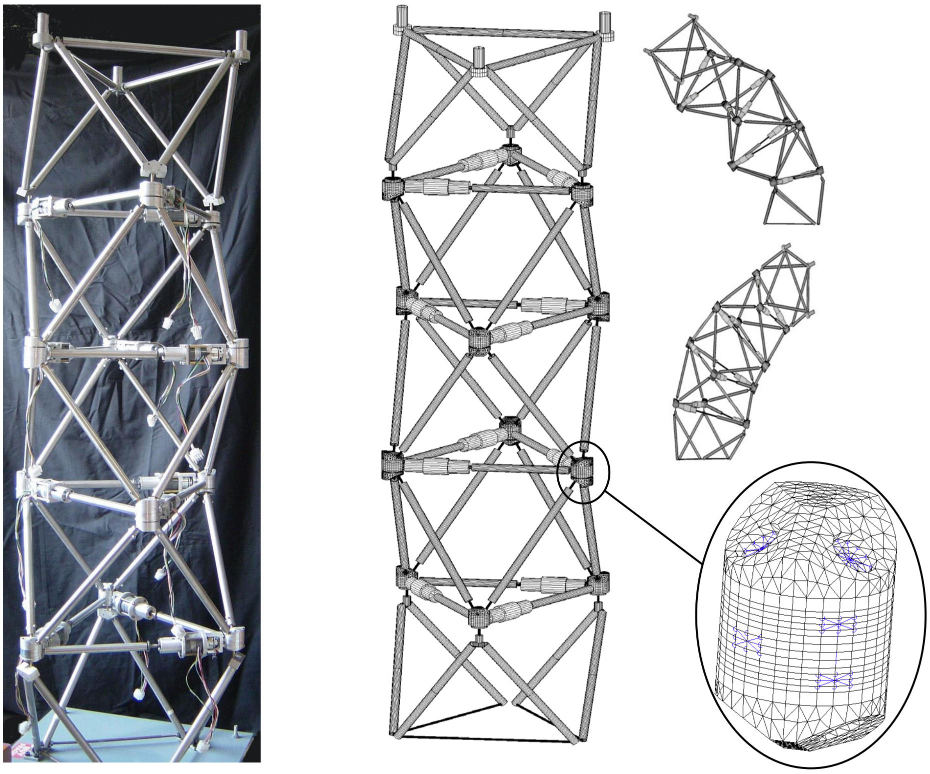
There are a total of ten linear equivalence equations. If ten point mass locations are selected, a 10×10 linear equation system might be solved. If fewer locations were available in the EPM, the Linear Least Squares Method must be used to fit the solution properly. In addition, if rigid solid movements are possible within the submodel, some different p positions must be taken and then $10p$ equations would be available to solve by means the Least Squares Method. Furthermore, if the local dynamic properties of the submodel are not important from the global view point, a unique point mass could be taken into account in the gravity center of the submodel, whose value might be the total mass, evidently.

3 Using the EPM concept in the design of VGT's

Variable geometry structures are multi-degree-of-freedom systems capable of modifying their geometry to adapt to different loads and working conditions. This is possible because some of the elements comprising them can vary their length; these elements are called actuators. Study of these structures dates back to the 80's [21]. A specific type within this group is based on spatial truss type structures known as Variable Geometry Trusses (VGT) [22-26].

The authors have developed a five-module VGT prototype [27], where the geometry of the main module is established upon the octahedral shape. The prototype is shown in Fig. 1a. Each module is a parallel kinematic mechanism with actuators set on the horizontal planes. These planes are joined together using fixed length bars called longerons. The joint between these bars and the actuators is made by special joints. These joints have also been developed and patented by the workgroup.

When mechanical analyses (static or/and dynamic) are to be performed in a VGT it must frequently be done on successive positions. It means that finite element models have to be built for each position. In Fig. 1b some FEM models are shown corresponding to different positions which have been automatically generated from a kinematic framework.



a) Prototipe **b) Finite Element Models**

Fig. 1 Five-module VGT (MBAD)

It can be observed that most of the degrees of freedom are concentrated in the joints. The rest of the solids have been properly modeled with a small number of elements, and therefore a small number of dof's. In Fig. 1b the FEM model of a joint is shown in greater detail, which has about 10.000 degrees of freedom. In itself this is not a great number of dof's; however contacts are defined between all its components; this makes model complexity much higher and analysis time increases enormously. If in addition we multiply it by the twelve joints present in the model and all the desired positions the calculation time can become unviable. In the following lines an EPM is to be developed to substitute the joints of the VGT.

3.1 Topological-kinematical Design

The choice of the type of elements is free as far as connections with the rest of the model and internal mobility are preserved. In this case, the defined EPM to replace the finite element submodel of the joint is formed by eight 3D beam elements with circular section and nine nodes, as shown in Fig. 2. Each of these nodes has six dof's, three translations (u_x, u_y, u_z) and three rotations (θ_x, θ_y, θ_z). The beams are arranged so nodes 1, 3, 6 and 7 correspond to the central points of the joint spherical bearings and nodes 8 and 9 to the link points of the rings with actuator bars. It can be appreciated node 9 really corresponds to the point midway between the two link points of the double ring. Nodes 2, 4 and 5 are internal and have no physical correspondence with any of the joints. Node 4 is located on the central point belonging to the rotation axis of the joint rings. In Fig. 2 one can appreciate the correspondence of the EPM nodes with the joint submodel.

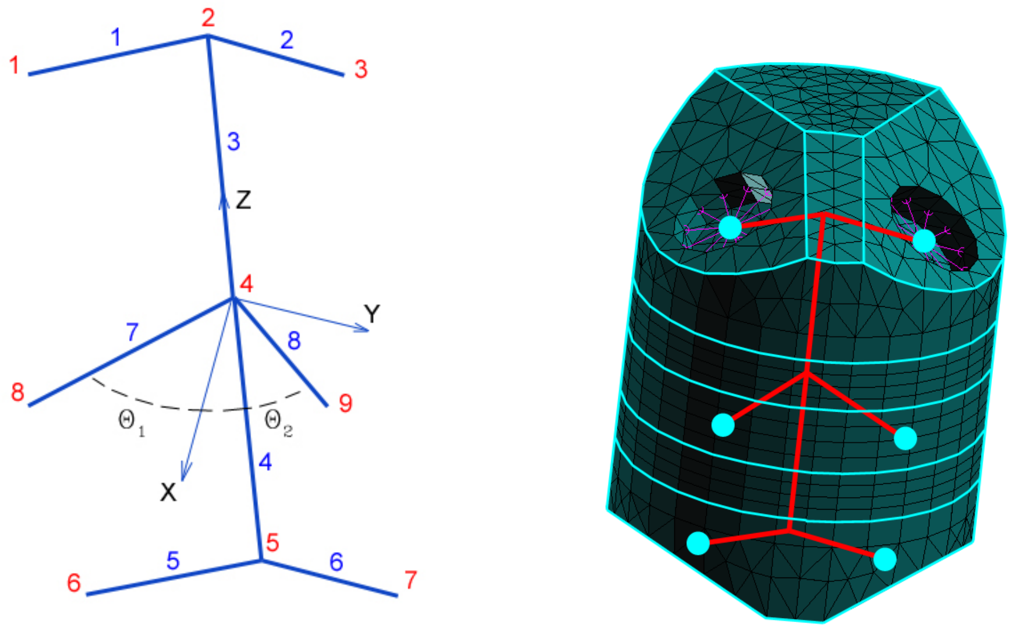


Fig. 2 EPM chosen and its correspondence with the joint geometry

To arrange the static design of the EPM, its stiffness matrix must be calculated first. In addition, it is necessary to reduce the stiffness matrix to the dof's of the connection nodes because the displacement cases are to be defined only on them for both the real joint and the EPM. The EPM stiffness matrix is obtained via the sum of expanded matrices of each element, once expressed in the global system. The axes of this global system are *XYZ* shown in Fig. 2. As the EPM consists of 9 nodes each with 6 dofs, the final matrix is 54 x 54. The element matrices are 12 x 12, since 3D beam type elements were used. As already mentioned, the EPM must be kinematically equivalent to the submodel. The rings round the joint body can rotate in relation to the central axis. These rotation angles have been defined as θ_1 and θ_2. The rotation condition must be included in the EPM. This has been done releasing the rotation dof regarding the central axis on the nodes corresponding to elements 7 and 8. These elements would correspond to the rings in the submodel. The fact of releasing dofs on a node is equal to including the null force transmission condition in the same direction. The node undergoing this situation is number 4, where four elements concur numbered 3, 4, 7 and 8. Elements 7 and 8, as already mentioned, would correspond to the rings and elements 3 and 4 would represent the central axis of the real joint. Therefore, contribution of the moment in direction *Z* on node 4 of elements 7 and 8 must be cancelled. Nonetheless, effort continues being transmitted in that direction on this node, although only between elements 4 and 5. Introducing this condition means the stiffness matrix of elements 7 and 8 suffer certain modifications directly influencing the EPM stiffness matrix. Once the previous steps have been executed to achieve kinematic equivalence, the EPM global stiffness matrix is obtained via coupling of all the matrices of each element in global coordinates. At this point, the EPM has been fully defined, which depends on the physical properties (E, υ, G) and secondary dimensions of the elements comprising the same, such as the section or radii. Any of these variables may be selected as a parameter.

Besides, as already mentioned, it is necessary to reduce the stiffness matrix to the dof's of the connection nodes. If connection nodes are assimilated as master nodes in the nomenclature of condensation, the condensed stiffness matrix $[K]_r$ should be calculated. Therefore the formula to calculate the elastic energy becomes:

$$V = \frac{1}{2} \cdot \{\delta\}_m^T \cdot [k]^r \cdot \{\delta\}_m \tag{15}$$

At this point, the procedure to obtain the EPM strain energy has been explained. Input variables are the displacements of the master dof's. However, the EPM parameters must be defined and adjusted to achieve equivalence with the submodel. Here, the radii of the 3D beam elements with circular section have been chosen. Total number of parameters is 8, likewise the number of elements. The stiffness matrix coefficients depend on these parameters and constants of the materials. It must not been forgotten that the two angles θ_1 and θ_2 also influence the matrix *[K]*, and likewise *[K]^r*.

3.2 Static Design

To statically design the EPM, 60 displacement cases have been chosen, for which vectors *{δ}* of the connection degrees of freedom are created randomly; these displacements are divided into three sets of 20 cases each one, corresponding to the maximum angle between bars 7 and 8 (2x44=88º), minimum (2x21=42º) and mean (2x32,5=65º). The magnitudes of these displacements were chosen so they were higher than expected for the usual working conditions of the real joint. Thus, correct EPM behavior under normal conditions is guaranteed. Random function is centered at zero with values between -10^{-5} and 10^{-5} meters.

The calculation of vector $\{U\}$ has been performed with a Finite Element Analysis package. This vector can be obtained in different ways; the most common form is to apply (16), being *{F}* the vector comprising components of reaction forces associated to master dof's where displacements are applied. Repeating (16) for all displacement cases, the vector *{U}* is obtained. As already mentioned this vector is only calculated once.

$$U_i = \frac{1}{2} \cdot \{\delta\}_i^T \cdot \{F\}_i \tag{16}$$

The material used is aluminum and the NLSM application conditions are:

Number of macroelement parameters:	$p = 8$
Number of master dofs:	$h = 18$
Number of displacement cases:	$n = 60$
Increased percentage in the sensitivity matrix calculation:	$m = 0.01$
Maximum Number of iterations:	$k\_max = 15$
Value of admissible q:	$q = 0.5$ %
Maximum variation in parameter control:	$f = 10$ %

The search for initial approximation $\{A\}^0$ was carried out executing a simple Genetic Algorithm code with standard values of the parameters, but with variable mutation rate. After 400 generations the values obtained for initial approximation (expressed in millimeters) were the following:

$$\{A\}^0 = [8.989 \quad 8.999 \quad 25.000 \quad 25.000 \quad 10.917 \quad 10.822 \quad 4.971 \quad 2.810]^T$$

These are the values initially defining the EPM; now the elastic strain energy V_i can be calculated for each displacement case. By calculating these values for the 60 cases, vector *{V}* is formed. Now, the degree of equivalence can be assessed between the EPM and the submodel, using the Eq. (9), which obtained a value of $\varepsilon = 0.0292$. Fig. 3 represents the quadratic difference values of the *U* and *V* energies per displacement case *i*. This figure confirms the *V* values are considerably similar to those of *U*.

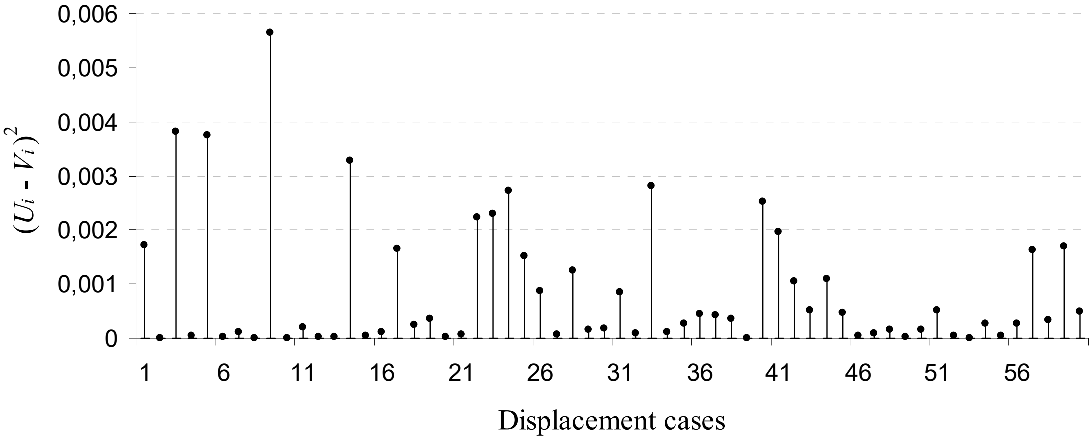


Fig. 3 Difference between U and V energies for the 60 displacement cases

Taking this initial approximation, the method described in the previous section is applied to refine parameter values. Fig. 4 shows error evolution (ε) after 10 iterations. One can see how it diminishes on each iteration, although each time with less rate of decrease. Complete equivalence is impossible to achieve, since the EPM has considerably fewer elements than the submodel; the purpose here is to approximately represent the mechanical behavior of the submodel. In iteration k=10 the iterative process stop requirement is met: relative error q is now under 0.5 %.

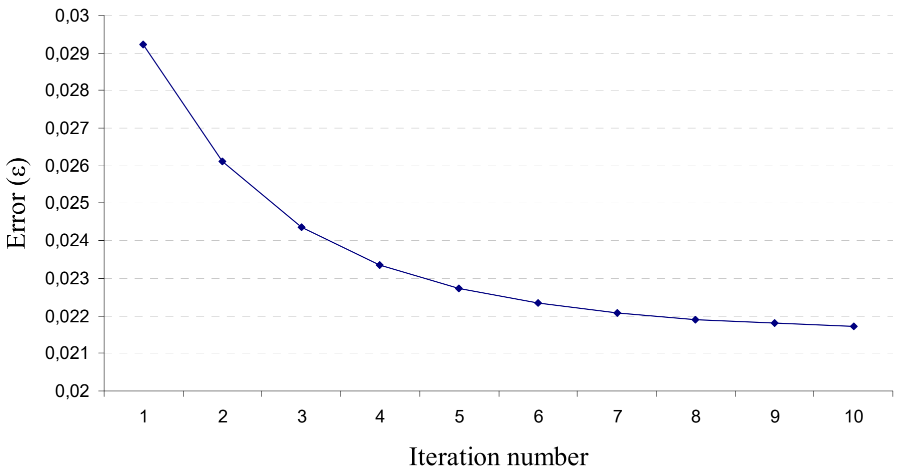


Fig. 4 Error evolution

In these 10 iterations the error has reduced from 0.0292 to 0.0217, i.e. equal to a reduction of 25.64 %. Therefore, application of the method has considerably improved the initial solution. The values of the parameters after the NLS optimization:

$$\{A\}^{0} = [9.510 \quad 9.691 \quad 24.687 \quad 58{,}949 \quad 10.704 \quad 10.346 \quad 4.417 \quad 2.290]^{T}$$

The number of displacement cases in the resolution of this specific case was set at n=60. With a view to fully demonstrating the EPM validity obtained, 10 displacement groups were chosen. Group 1 was used to solve this example and the other 9 groups created randomly, and the error calculated for each. The mean value is 0.0236, with a maximum deviation of 10%. With these data the value of n=60 is considered sufficient and the EPM obtained valid for any displacement combination.

3.3 Some Words on Dynamic Design

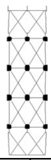
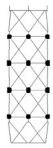
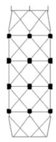
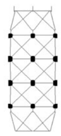
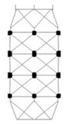
In the case of VGT applications dynamic behavior is governed by the crossbeams and the motor-bars. The mass distribution of the joints was not borne in mind and joints are characterized dynamically with point masses at the center of gravity, without practically any loss in accuracy (in this case, of course). The only drawback of this approach is the impossibility of detecting local modes. If a more detailed dynamic design is wanted to reach, the steps given in section 2.2.3 can be applied in the same way as for the static design, and without the necessity of resorting to the simplification mentioned.

3.4 Validation of the Reduced Model

In this subsection a validation is made from both static and dynamic points of view. In the case of static validation, a set of static analyses has been performed in various positions of the VGT and for various values of the applied forces, and for both models: the reduced and the complete. In the dynamic validation a linear modal analysis has been done again for both models; in the case of the complete nonlinear model all the contact regions have been bonded at this stage of the research in order not to calculate the nonlinear modes. However, this dynamic validation could be based for instance on forward nonlinear dynamics and therefore the contact regions would not have to be bonded to perform the comparison between the low and high fidelity models.

3.4.1 *Static validation.* Five positions have been considered for the validation on a folding operation of the VGT. The force is vertical and is applied to the upper horizontal triangle of the VGT. In table 1 the values of the displacements of this triangle have been measured in millimeters. A small variation is observed between the displacements of the original model and those of the reduced one. Error evolution always remains under 0.8% for any load case.

Table 1 Values of maximum displacement (mm) in complete and reduced models

Load N	Position 1 		Position 2 		Position 3 		Position 4 		Position 5 	
	compl	reduc	compl	reduc	compl	reduc	compl	reduc	compl	reduc
10	1.515	1.527	2.155	2.171	3.064	3.085	4.422	4.452	6.639	6.682
20	1.782	1.795	2.531	2.548	3.593	3.616	5.178	5.210	7.764	7.809
30	2.050	2.063	2.907	2.925	4.122	4.146	5.935	5.968	8.888	8.936
40	2.317	2.331	3.283	3.302	4.651	4.677	6.691	6.726	10.013	10.062
50	2.584	2.559	3.659	3.679	5.180	5.207	7.447	7.484	11.138	11.189
60	2.851	2.867	4.035	4.056	5.710	5.738	8.203	8.242	12.262	12.316
70	3.119	3.135	4.411	4.433	6.239	6.268	8.960	9.000	13.387	13.443
80	3.386	3.403	4.787	4.810	6.768	6.799	9.716	9.758	14.512	14.570
90	3.653	3.671	5.163	5.187	7.297	7.329	10.472	10.516	15.636	15.697
100	3.920	3.939	5.539	5.564	7.826	7.860	11.228	11.273	16.761	16.824

Belonging to computational effort, the calculations has been performed on a Pentium 4 with 2.66 GHz and 512 MB of RAM memory. The OS is Windows XP SP2. In table 2 computation time (in seconds) has been recorded for the complete model and the reduced one. The computations have been made for one position and for one and ten load cases. It can be observed that the computation time is two orders of magnitude smaller in the reduced model than the complete one. Furthermore, if *p* positions have to be analyzed for the structure, the saving would be *p* times greater.

Table 2 Computation time (s) with complete and reduced models

Load cases	Time (s)	
	Complete	Reduced
1	304	4
10	801	7

3.4.2 *Dynamic validation.* In Fig. 5, the evolution of the frequencies of different vibration modes are compared between reduced and complete models for the first position remarked in Table 1. Until frequency 12 there is no appreciable error in the calculation of their values; the differences observed in the higher frequencies is due to local effects in the joints, which is not possible to detect with the dynamic modeling adopted in this work.

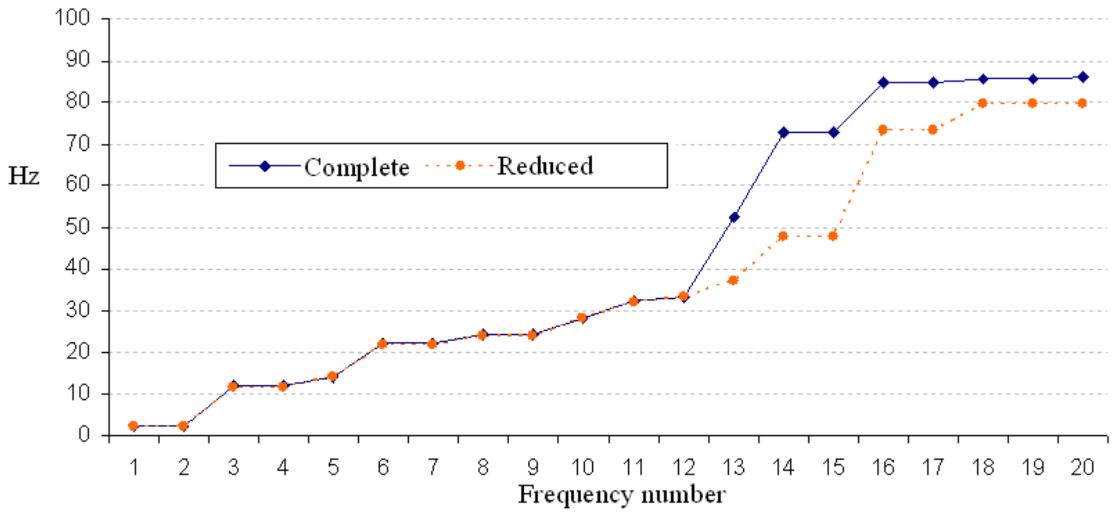


Fig. 5 Evolution of the value of natural frequencies (Position 1)

4 Concluding Remarks

In this work an original methodology has been developed to drastically reduce the size of some complex and large nonlinear finite element models, arranged from the concept of Equivalent Parametric Macroelement (EPM). An EPM is an assembly of a small number of finite elements of which some physical properties are taken as parameters, and whose aim is to replace submodels with nonlinearities in a whole model to save computational effort maintaining a good accuracy.

To determine the values of the parameters that will made the EPM capable of successfully replacing the cited submodels, some equivalence criteria have been established. From the point of view of static equivalence, the equality of elastic energy has been developed, leading to a nonlinear set of overconstrained equations to be solved by Nonlinear Least Squares methodology. Dealing with dynamic equivalence, some work still remains to be done as for instance the inclusion of contact nonlinearities; however, an inertial conservation technique has been developed theoretically in this paper which leads to a linear set of overconstrained equations to be solved by Linear Least Squares methodology.

An example has been developed to demonstrate EPM calculation viability and its efficiency to save computational effort. The case of a VGT joint has been presented where nonlinearities like contacts and rigid-solid movements occur. Some validation tests have been developed to research how good the reduced model fits the static and dynamic behavior of the complete finite element model. The results in the static case are very promising because in the example developed in this work the computational time spent on the model with EPM's is about two orders of magnitude less than the time necessary to analyze the complete model, maintaining the results under a percentage error of 0.8%. In the dynamic case the values of the twelve smaller natural frequencies have been calculated without appreciable error and a deviation is detected in higher frequencies due to local effects in the EPM which can be overcome if necessary with a more detailed mass distribution within the EPM.

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Figure Legends:

Fig. 1 Five-module VGT (MBAD)

Fig. 2 EPM chosen and its correspondence with the joint geometry

Fig. 3 Difference between U and V energies for the 60 displacement cases

Fig. 4 Error evolution

Fig. 5 Evolution of the value of natural frequencies (Position 1)

Tables:

Table 1 Values of maximum displacement (mm) in complete and reduced models

Table 2 Computation time (s) with complete and reduced models