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Steven P. Cassou & Jesús Vázquez

*New Keynesian Model Features that Can
Reproduce Lead, Lag and Persistence Patterns*

New Keynesian model features that can reproduce lead, lag and persistence patterns[*]

Steven P. Cassou[†]
Kansas State University

Jesús Vázquez[‡]
Universidad del País Vasco

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Abstract

This paper uses a new method for describing dynamic comovement and persistence in economic time series which builds on the contemporaneous forecast error method developed in den Haan (2000). This data description method is then used to address issues in New Keynesian model performance in two ways. First, well known data patterns, such as output and inflation leads and lags and inflation persistence, are decomposed into forecast horizon components to give a more complete description of the data patterns. These results show that the well known lead and lag patterns between output and inflation arise mostly in the medium term forecasts horizons. Second, the data summary method is used to investigate a rich New Keynesian model with many modeling features to see which of these features can reproduce lead, lag and persistence patterns seen in the data. Many studies have suggested that a backward looking component in the Phillips curve is needed to match the data, but our simulations show this is not necessary. We show that a simple general equilibrium model with persistent IS curve shocks and persistent supply shocks can reproduce the lead, lag and persistence patterns seen in the data.

JEL Classification: E31, E32, E37
Keywords: New Keynesian, output and inflation comovement, inflation persistence, forecast errors

[†]Corresponding author - Department of Economics, 327 Waters Hall, Kansas State University, Manhattan, KS, 66506 (USA), (785) 532-6342, Fax:(785) 532-6919, email: scassou@ksu.edu.

[‡]Fundamentos del Análisis Económico II, Facultad de Ciencias Económicas y Empresariales, Universidad del País Vasco, Av. Lehendakari Aguirre 83, 48015 Bilbao (SPAIN), Phone: +34 946013779, Fax: +34 946017123, email: jesus.vazquez@ehu.es.

1 Introduction

The relationship between output and inflation has long been of interest in the monetary economics literature. Today, some consensus has formed about some important issues. For instance, the general view is that this relationship is at most weak in the long-run, reflecting a sort of classical dichotomy between nominal and real variables. On the other hand, the short run seems to be well described by some variant of the New Keynesian Phillips Curve (NKPC).[1] Yet despite the emerging consensus for using the NKPC to model the short run, there remains considerable disagreement about what form it should take. Numerous studies have shown that the strictly forward looking NKPCs are unable to replicate many of the empirical patterns found in the data.[2] However, one limitation of many of these studies is that they have focused on models consisting of just a single NKPC equation (i.e. the aggregate supply) and have overlooked the aggregate demand side of the economy and its interaction with the NKPC. This focus on single equation NKPC models often results in misleading conclusions.[3] Less attention has been placed on more fully specified general equilibrium

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[1]The fact that output and inflation should be connected in the short-run does not imply the existence of a stable relationship between the two variables since both the sources of economic shocks and the way central banks monitor policy may change over time and across countries. See Walsh (2003, Chapter 1) and references therein for a summary of the main empirical regularities found in the monetary economics literature.

[2]The original NKPCs were founded on contracting ideas from Taylor (1980) and Calvo (1983). However, performance issues, such as the lack of inflation persistence, led to the introduction of backward looking terms to these basic rational agent models. Furher and Moore (1995) and Furher (1997, 2006) advocate a contracting idea from Buiter and Jewett (1981) to motivate the backward looking term, while Galí and Gertler (1999) use an empirical motivation for the backward term along with a marginal cost structure substituted for the output gap. Numerous papers, including Coenen and Wieland (2005), Rudd and Whelan (2006) have explored the merits of these formulations. More recently, Ireland (2007), Lansing (2007), Cogley and Sbordone (2008) and others have explored models which add learning, unit roots or near unit roots to the discusssion.

[3]For instance, as noted in Mankiw (2003, pp. 66-67), it is hard to identify the parameters featured in the NKPC when inflation-push shocks are relatively more important than output gap shocks. Put differently, the chances of identifying the NKPC model parameters are higher when output gap shocks dominate. Indeed, many of the papers relying on single equation NKPC models assume an exact relationship between current inflation, expected inflation and output gap (i.e. there are no inflation-push shocks), and the source of variation comes from the output gap (i.e. the variable driving in the NKPC). In addition, from a policy perspective, the single equation NKPC models are often associated with the possibility for an immediate and costless disinflation policy because current inflation is entirely determined by the expected path of future output gaps and if the central bank could commit to setting the path of future output gaps equal to zero, adjustment would occur immediately (Galí and Gertler (1999, pp. 203)). However, this argument does not hold when output

models.[4] This paper fills this gap by investigating the short run performance of the NKPC in a small-scale general equilibrium model with several sources of persistence. We use a general equilibrium structure that is rich enough so as to reproduce the key statistical features seen in the data that are not easily matched in single equation NKPC models, but also is simple enough, in contrast with the recent medium-scale models as in Smets and Wouters (2003, 2007), so that it is possible to understand exactly which model features are necessary to match the dynamic patterns of output and inflation data.

This paper contributes to our understanding of the output and inflation relationship in three important ways. First, it provides a new statistical method, that builds on techniques developed in den Haan (2000). This extension is designed to shed light on lead and lag comovements of the data. It not only identifies the lead and lag empirical regularities, but it also shows whether they are part of the short term or long term forces driving the data. Second, the paper uses these statistical techniques to describe the lead, lag and contemporaneous comovement between output and inflation as well as inflation persistence. This description is particularly useful for the output and inflation application here where so much of the debate has centered on whether the NKPC is able to replicate dynamic patterns seen in the data. Third, a small-scale New Keynesian model (NKM) with a rich set of modeling features is described and then studied to see which of these features are important for generating the actual patterns. These model features include, a consumer utility function with generalized habit persistence, a hybrid NKPC à la Galí and Gertler (1999), a monetary policy rule that incorporates inflation, output and output growth as suggested by Smets and Wouters (2007) and persistence in the IS curve and the NKPC shock processes.

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gap and inflation-push shocks are modelled so that they are highly persistent as part of the economic structure. Under such a structure, it is not feasible for a central bank to make a commitment about the path of future output gaps.

[4]Some exceptions include Rotemberg and Woodford (1997), Christiano, Eichenbaum and Evans (2005), Keen (2009), and Smets and Wouters (2003, 2007) which propose medium-scale general equilibrium models with many equations and, in some cases, numerous sources for variation.

The statistical method is used in two important ways. First, lead, lag and persistence patterns of the data are described. Early work by Fuhrer and Moore (1995) documented these patterns and thus set the mark which most studies of the NKPC have sought to achieve. Our method refines the typical data summary to decompose the lead, lag and persistence patterns into forecast horizons, thus allowing one to judge whether the data patterns are more short term or long term in nature. We find a hump shape in our lead and lag diagrams which show that these patterns are arising from medium term components rather than short- or long-term components. Second, we use this data description in a fitting exercise which calibrates our rich NKM to the data and thus shows which of the modeling features are important for achieving data matches.

Our calibration experiments find several important results. First, we find that there are several ways to reproduce the lead and lag patterns between output and inflation. The key features of the model that are needed to achieve this dimension of fit are: 1) the model needs to have both demand equations and supply equations with their own stochastic elements; 2) the model needs to get the relative proportions for the supply and demand shock variances just right; and 3) the model needs to get the relative persistence for the supply and demand shocks just right. It is shown that these requirements can be satisfied, at least qualitatively, with a variety of alternative demand shock and persistence specifications. The intuition for this structure is relatively easy to understand from the impulse response functions provided below. The demand shocks produce a positive lead of output over inflation when the effects of these shocks are more persistent in inflation than output, and the supply shocks produce a negative lead of inflation over output when the effects of supply shocks last longer in output than inflation. By balancing these two dynamic features with the right variances for the demand and supply shocks and the proper persistence levels of the model, the lead and lag patterns between output and inflation can be reproduced.

A second stylized fact of the data is that both inflation and output are highly persistent, but inflation is more so. Reproducing this fact is more difficult. To

achieve this feature of the data jointly with the lead and lag patterns, we find it necessary to have the right balance (size and persistence) between IS and inflation-push shocks. Most importantly, we find that persistence of the IS shock is key to the fit. Without this persistence, the backward looking term in the hybrid NKPC becomes positive, and the implied lead, lag and persistence patterns fall short of the IS shock specification in reproducing the actual data patterns. This perhaps explains the difficulty that single equation models, without a demand side to the economy, have had in achieving persistence in inflation.

The rest of the paper is organized as follows. Section 2 is designed as a data section. It first reviews some of the data features described by Fuhrer and Moore (1995) and others. It then describes the extension of Den Haan's (2000) method to analyze lead and lag comovements in the data and applies the method to the U.S. post-war output and inflation time series. Section 3 introduces our small-scale NKM with three key building blocks, an IS curve, a hybrid NKPC and a monetary policy rule. This model is designed to have a rich set of demand and supply shock structures as well as several sources of persistence. At the same time, the model is simple enough to clearly understand what are the key features necessary to produce the statistical patterns seen in the data. In Section 4, we follow a calibration approach designed to uncover which of the model features are needed to reproduce the output and inflation dynamics. Section 5 concludes.

2 Leads, lags and persistence in output and inflation data

In this section we investigate the lead, lag and persistence patterns in the output and inflation data for the U.S. The section is broken into three subsections. In the first subsection, we begin by reviewing some of the findings from Furher and Moore (1995) which have become well known stylized facts for the literature in this area. Next we describe an extension to the forecast error correlation methods in den Haan (2000) which allows a more complete picture of the data movements. Finally, we

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apply this new method to the output and inflation data, and see how it provides a richer summary of the dynamic movements of these two data series than the Furher and Moore (1995) approach.

2.1 Review of lead, lag and persistence measurements

In order to understand our new forecast error correlation approach, it is helpful to first review some of the more familiar lead and lag facts first described in Fuhrer and Moore (1995) and emphasized by Galí and Gertler (1999) and many others later on. Fuhrer and Moore (1995) used a trivariate VAR to summarize the data on output gap, inflation and short-term interest rates. For the output gap they used (the log of) deviations of per capita nonfarm business output from a linear-fitted trend, for inflation they used the annualized growth rate in the implicit deflator for the nonfarm business output and for the short-term interest rate they used the 3-month Treasury bill rate.

Our analysis has four small differences from theirs. First, our plots only include the output and inflation data series and leave out the short-term interest rate plots. We left out the short-term interest rate plots to keep things simple and focus on the output and inflation dynamics, which are the main focus of most of the liturature in this area. Second, for our analysis, output gap is obtained by implementing a Hodrick and Prescott (1997) filter to the data.[5] Third, we consider the Fed funds rate as the short-term interest rate and fourth, we use a larger sample period (1965:1 to 2008:4) which also includes data from the last fifteen years that was not available to Fuhrer and Moore (1995).

The results of our calculations are provided in Figure 1. This figure shows that these small differences in the calculations have little effect on the dynamic patterns in the data. The autocorrelation functions for output and inflation, as well as the lead and lag correlation patterns, are almost identical to what was found by Fuhrer and

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[5]This filter is designed to extract so called business cycle frequencies, that is frequencies corresponding to 2 and 8 year cycles, from the data. The result of this filter is a detrended data series which could also be interpreted as a measure of output gap.

Moore (1995). In particular, the diagonal elements in Figure 1 show that inflation is quite persistent, whereas as the output gap has somewhat less persistence. On the other hand, the off-diagonal elements together show the familiar lead and lag pattern of output and inflation, where output leads inflation when there is a positive correlation and inflation leads output when there is a negative correlation. Thus, a high level of output anticipates a high level of inflation about five quarters later (upper-right graph), while a high level of inflation is followed by a lower level of output about ten quarters later (lower-left graph).[6] These correlations are interpreted as follows. The negative correlation of lagged inflation with current output is sometimes interpreted as indicating that high (low) inflation rates generally leads to tight (loose) monetary policy which reduces (increases) future output, while the positive current and future correlations are interpreted as indicating that high (low) output levels put (reduce) inflationary pressures on the economy leading to higher (lower) future inflation.

2.2 A new method for measuring leads, lags and persistence

In den Haan (2000) a new methodology for assessing the comovement of economic variables was developed.[7] The method makes use of forecast errors for assessing comovement and is attractive for several reasons. First, the method does not require any modeling assumptions, such as a VAR ordering or structural assumptions on the error terms, to be applied. Second, it does not require that the data be detrended or that the variables in the model have identical orders of integration.[8]

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[6] An almost identical empirical lead-lag pattern can also be obtained by using a univariate approach instead of a VAR approach. For instance, Galí and Gertler (1999) and Smets and Wouters (2007), following a univariate approach, report similar lead-lag patterns. These authors also plot leads and lags in the same diagram. Their depiction of the lead-lag pattern exhibits the well known S-shaped pattern with lagged values of inflation exhibiting negative correlations with current output, and current and future values of inflation exhibiting positive correlations with current output.

[7] In addition to den Haan (2000), other applications of this approach include den Haan and Sumner (2004) and María-Dolores and Vázquez (2008).

[8] Avoiding detrending of the data is useful because den Haan (2000, p. 5) argues that the negative correlation between output and prices often found in the data could be an artifact of common detrending procedures used to make the data stationary. Moreover, Fuhrer and Moore (1995) devoted several pages to discussing the order of integration of output, inflation and interest rates and nonconclusive evidence was found.
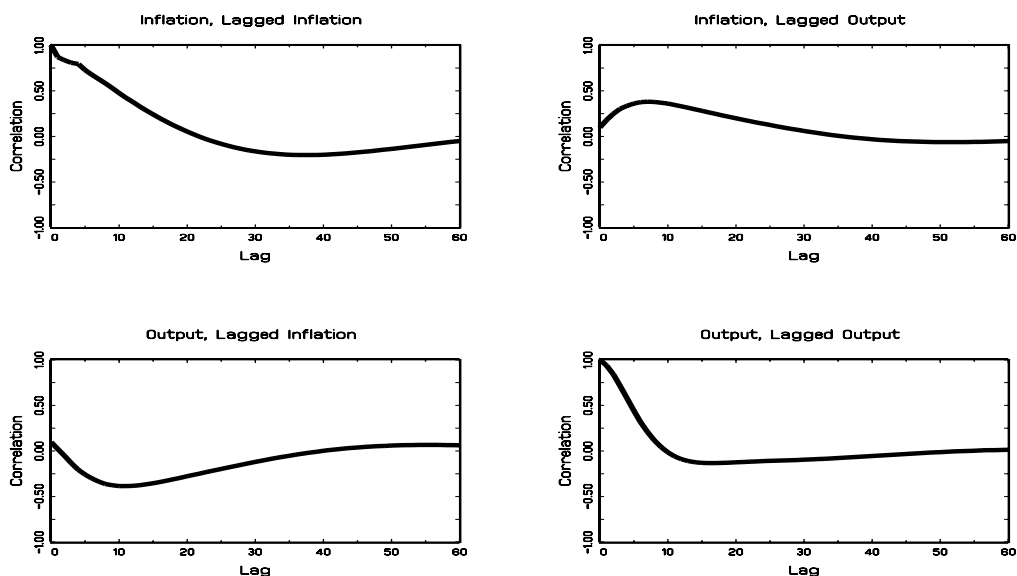
Figure 1: Autocorrelation and lead/lag pattern of output and inflation based on a VAR

Another salient feature of the den Haan (2000) approach is the interpretation for the sources of fluctuations. As in typical VAR methods, the fluctuations in both the data and thus in the forecast errors originate from some underlying structural shocks which could be associated with the various variables in the model. However, the method does not need to identify exactly which structural shocks play a role in any particular equation and can be left unspecified. One simply envisions that all of the structural shocks play some role in each of the model variables and the comovements in the observed data are shaped by the importance of these structural shocks in the variables for which comovements are being investigated, but sorting out which of the structural shocks are important is not necessary.[9]

The focus in den Haan (2000) was on contemporaneous comovements of the economic variables, but for our investigation, we are interested in more than just that.

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[9]One limitation of this approach is that it does not provide standard impulse response functions which show the responses of each endogenous variable to alternative structural shocks. However, den Haan (2000) views this as a positive feature as he notes that such standard impulse response analysis requires an identification structure which is often the subject of some dispute.

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Here we extend this methodology to look at not only the contemporaneous comovements, but also lead and lag comovements and autocorrelation functions in order to analyze inflation and output persistence. This provides a more complete description of the data dynamics. Such lead and lag and persistence analyses are familiar to readers of the modern dynamic macroeconomic literature. However, the technique here provides a broader format for describing the data dynamics than the approach used in the macroeconomic literature as well.

We begin by running a VAR of the form

$$X_t = \mu + Bt + Ct^2 + \sum_{l=1}^{L} A_l X_{t-l} + \varepsilon_t \tag{1}$$

where A_l is an $N \times N$ matrix of regression coefficients, μ, B, and C are N-vectors of constants, ε_t is an N-vector of innovations, and the total number of lags included is equal to L. The ε_t are assumed to be serially uncorrelated, but the components of the vector can be correlated with each other. For the application here, we run trivariate VARs, so $N = 3$. Also, following popular forecasting practice, we let $L = 4$, so there is one full year worth of lags in the VAR.

From this VAR, forecast errors can be computed for alternative forecast horizons. A particular N-vector of forecast errors can then be viewed as the cyclical component of X_t determined by a particular forecast horizon K. Thus, the forecast errors associated with short-term horizons would tend to capture more of the high-frequency components of the data whereas long-term forecast errors would tend to emphasize relatively more low-frequency components. Each of these forecast errors, or cyclical components, obtained from the different equations at various forecast horizons can then be used to compute contemporaneous correlations for the forecast errors from the different equations at various forecast horizons as in den Haan (2000).

In our analysis, we extend this approach by further using these forecast errors to compute cross correlations at various leads and lags as well as autocorrelation functions. In particular, the cross correlations are computed by matching forecast errors for the same forecast horizon where each forecast is based on an information

set that differs with a specific number of leads or lags. These calculations provide a more complete dynamic perspective of comovement than the alternative approaches suggested by Fuhrer and Moore (1995), Galí and Gertler (1999) and den Haan (2000) by not only showing how the data comove at leads and lags, but also by showing how the data comove at leads and lags at alternative forecast horizons. These alternative forecast horizons thus tell us if the lead and lag patterns are arising due to more short term or more long term components of the data. In the next subsection, we show how this system of lead and lag correlations between forecast errors can be plotted against the forecast horizon to conveniently assess the lead and lag structure of the data.

2.3   New insights into the data comovements

This subsection is broken down into two smaller sections in order to keep the discussion clear. We begin by looking at the lead and lag results between output and inflation. Next, the persistence of inflation and output is discussed.

2.3.1   Lead and lag relationships between output and inflation

Figure 2 presents a set of six diagrams for the forecast error correlations between output and inflation. One common element in all the diagrams is the contemporaneous correlation which is plotted at various forecast horizons in each diagram by a dashed line. Each of the six diagrams then has a lead-lag pair in which a contemporaneous forecast error for output is matched with a lead (thick solid line) or a lag (thin solid line) forecast error for inflation. The upper left diagram has a lead-lag pair in which the correlations are for inflation eight quarters, or two years, ahead or behind output, while the upper right diagram has a lead-lag pair corresponding to six quarters, the middle left diagram has a lead-lag pair corresponding to four quarters, the middle right has a lead-lag pair corresponding to three quarters, the lower left has a lead-lag pair corresponding to two quarters and the lower right has a lead-lag pair corresponding to one quarter. A useful comparison of these diagrams can be made with the off-diagonal graphs in Figure 1 by noting that if one focuses on the

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lead lines in Figure 2 and one moves upward through the diagrams (i.e. one moves through the diagrams with progressively longer leads), it is the same type of exercise as moving from the origin to the right in the upper-right diagram of Figure 1, while if one focuses on the lag lines in Figure 2 and one moves upward through the diagrams (i.e. moves through the diagrams with progressively longer lags), it is the same type of exercise as moving from the origin to the right in the lower-left graph in Figure 1.

Interpreting the diagrams borrows insights from both the Fuhrer and Moore (1995) and Galí and Gertler (1999) approach and the den Haan (2000) approach. As in Fuhrer and Moore (1995) and Galí and Gertler (1999), places where the lead correlation is higher than the contemporaneous correlation, one would interpret output as leading inflation. Furthermore, as in den Haan (2000), the horizontal axis represents the forecast horizon and provides information about whether the correlation occurs in the short run or long run. Situations in which the lead line exceeds the contemporaneous line toward the right edge of the diagram would indicate that output leads inflation at longer forecast horizons. Because alternative filters used in the literature (for instance, the Hodrick and Prescott filter used by Galí and Gertler (1999) or the linear-trend filter used by Fuhrer and Moore (1995)) are often set to isolate so called business cycle frequencies, our diagrams have as their highest forecast horizon 32 quarters (i.e. 8 years). We use forecast horizons as low as one quarter so the left side of the diagrams consists of short term correlations. These correlations are typically low because of the high percentage of noise at short term forecast horizons.

To be more concrete about the actual results, let us start by walking through the middle right diagram in Figure 2. To conduct this analysis, it is important to recognize that the lead plot in essence decomposes the single three quarter correlation value in the upper right diagram of Figure 1, the lag plot in essence decomposes the single three quarter correlation value in the lower left diagram in Figure 1, and the contemporaneous correlation plot in essence decomposes the contemporaneous correlation value which is the left edge value of both the upper right and lower left

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diagrams in Figure 1.

First notice that the contemporaneous correlation plot in Figure 2 is relatively low and ranges between 0.1 and 0.25 over all the forecast horizons as emphasized by María-Dolores and Vázquez (2008). These values are in line with the contemporaneous correlation displayed on the left edge of the upper right and lower left diagrams in Figure 1. Next note that both the lead and lag lines are close to zero for the first 4 quarters. This is because the population moment (i.e. the population correlation) between the inflation forecast error four quarters ahead (or behind) and the current forecast error for output is zero. As one moves past the four quarter horizon, the lead line moves up positively and the lag line moves down negatively. These results indicate that at all forecast horizons, (i) high values of output lead to high values of inflation three quarters later, and (ii) high values for lagged inflation anticipate low values for output three quarters later. Both results are consistent with those displayed in Figure 1. What is new here is that the lead and lags have been broken down by forecast horizons. Since the forecast horizons are loosely related to frequencies, with short-term forecast horizon errors emphasizing high frequencies and the long term horizons emphasizing low frequencies, we see that the rising lead line and the falling lag line tells us that the positive lead and negative lag values in Figure 1 are due mostly to medium and longer term (low) movements (frequencies). Since the lead plot has a hump shape to it, we see that the medium term movements are somewhat more important than the long term movements for producing the lead of output over inflation. Similarly, since the lag plot has a cup shape to it, we see that the medium term movements are somewhat more important than the long term movements for producing the lag of output over inflation. Looking at the other diagrams in Figure 2 shows similar results with the curves spreading out for the medium term forecast horizons, but still maintaining a sizable lead or lag for the longer forecast horizons. These also indicate the values in Figure 1 are mostly due to the medium term movements, but the longer term movements also have a role.

It is also useful to note that, loosely speaking, the correlations in Figure 1 are

recovered as the forecast horizon approaches infinity.[10]   So looking at the middle left diagram in Figure 2 (with 1-year leads and lags) and focusing on the right edge of the lead and lag lines, we see that the right edge of the lead is somewhat higher than the right edge of the lead in the middle right diagram (with a 3-quarter lead) and the right edge of the lag is somewhat lower than the right edge of the lag -in the middle left diagram (with a 3-quarter lag). Recognizing this shows that Figure 2 also captures the S-shaped pattern described above in the discussion of Figure 1. What Figure 2 shows is that this S-shaped pattern not only exists at the aggregate, but it is also true across all forecast horizons and that the S-shape is due mostly to the medium term frequencies.

2.3.2   Persistence of inflation and output

Figure 3 shows the autocorrelation functions for inflation and output. These functions have been computed using the forecast error decomposition and thus are useful for understanding whether the autocorrelations are due to short term or long term components of the data as well.

To understand how these plots are calculated, first focus on the solid line in each of the diagrams of Figure 3. These plots correspond to the first order autocorrelation value of inflation and output from the standard Box-Jenkins calculations, only here the first order autocorrelations correspond to the cyclical component of the variable associated with the forecast horizon enumerated on the horizontal axis. Each point in these plots is computed by computing a vector of n-step ahead forecast errors and then using this vector to compute the first order autocorrelation for the n-step ahead forecast horizon. The other plots are computed in a similar way. Each point in the two lag plot is computed by computing a vector of n-step ahead forecast errors and then using this vector to compute the second order autocorrelation for the n-step

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[10]The "loosely speaking" qualification is important when considering situations when some variable in the VAR is non-stationary. In this case, the correlation coefficient of the forecast errors might not converge to the unconditional correlation coefficient of the two time series as K goes to infinity, but it can be estimated consistently, as shown in den Haan (2000), since forecast errors are stationary for a fixed horizon K.

ahead forecast horizon.

One useful point of reference to the Fuhrer and Moore (1995) calculations (i.e. the diagonal diagrams in Figure 1) is to note that the Box-Jenkins autocorrelation value is, loosely speaking, recovered as the forecast horizon approaches infinity.[11] This means that the right edge values of these diagrams are approximately equal to the values that Fuhrer and Moore (1995) compute in their plots. Focusing on this right edge, we see that the autocorrelation function for inflation falls off much more slowly than the correlation function for output. This illustrates the well known inflation persistence observation which so many NKM seek to match. By comparing the first order autocorrelation of inflation in Figure 1 (0.87) and Figure 3 (0.76) we observe that our cyclical component based on long-term forecast errors is less persistent than inflation itself. The intuition for this is simple. Our measures of cyclical inflation remove any linear and quadratic trend from actual inflation data thus reducing the long term correlation. These lower correlation results are in line with those described in Cogley and Sbordone (2008, p. 2111, Table 1) who also removed trends using a different technique.

Also of interest is to note that, as in the lead and lag analysis above, the forecast errors at different horizons can be interpreted as capturing more or less of the short term or long term components of the data, with the short term forecast horizons capturing more of the short term components of the data and the long term forecast horizons capturing more of the long term components of the data. Using this insight, we see that neither the inflation or output autocorrelation plots exhibit the hump shapes seen in the lead and lag analysis. This means that both the inflation and output persistence observed in the data is due to medium and long term components of the data.

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[11] Again, the "loosely speaking" qualification is important when considering situations when some variable in the VAR is non-stationary. In this case, the autocorrelation of the forecast horizon does not converge to the Box-Jenkings autocorrelation as the forecast horizon goes to infinity, but it can be estimated consistently for a fixed horizon K.
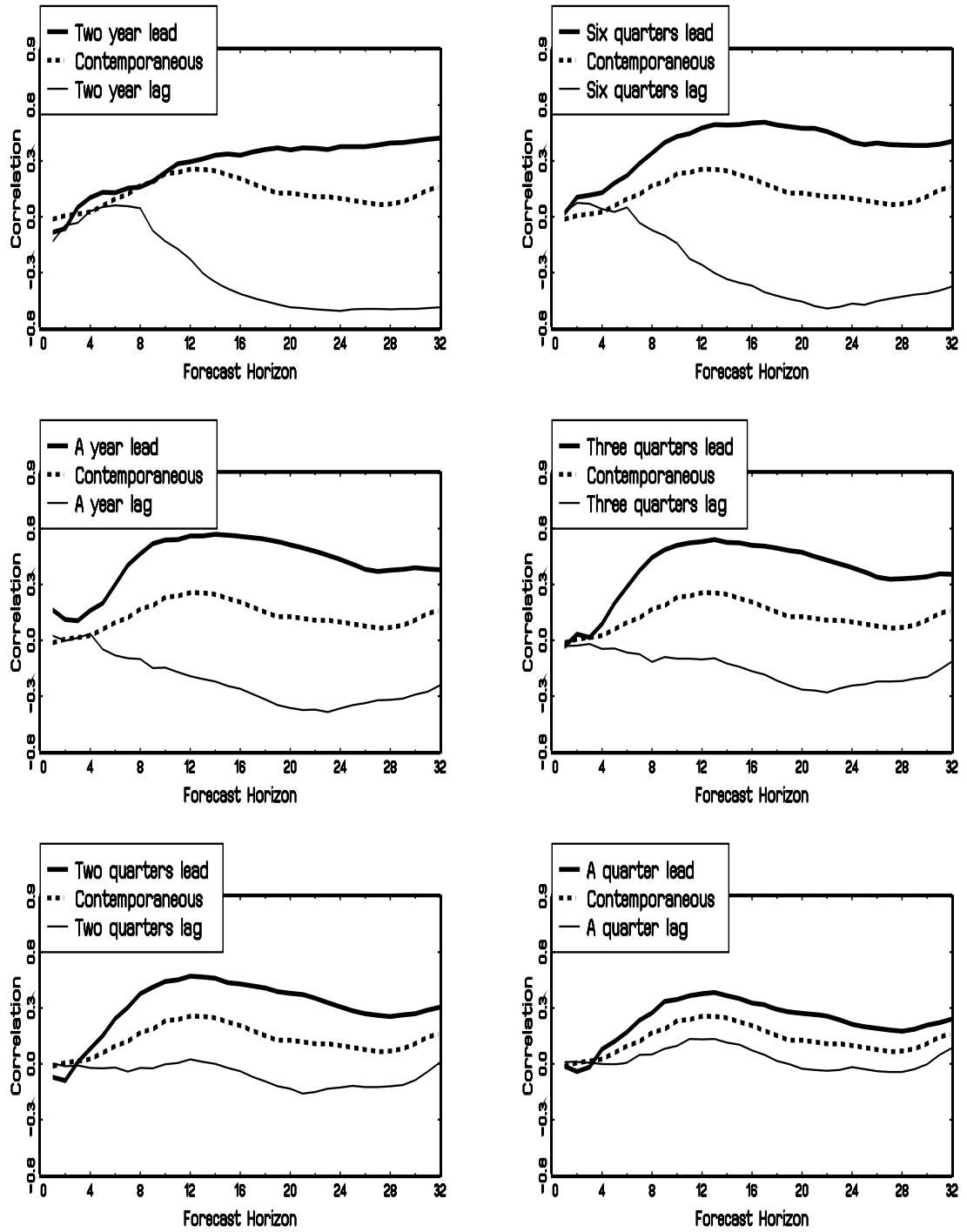
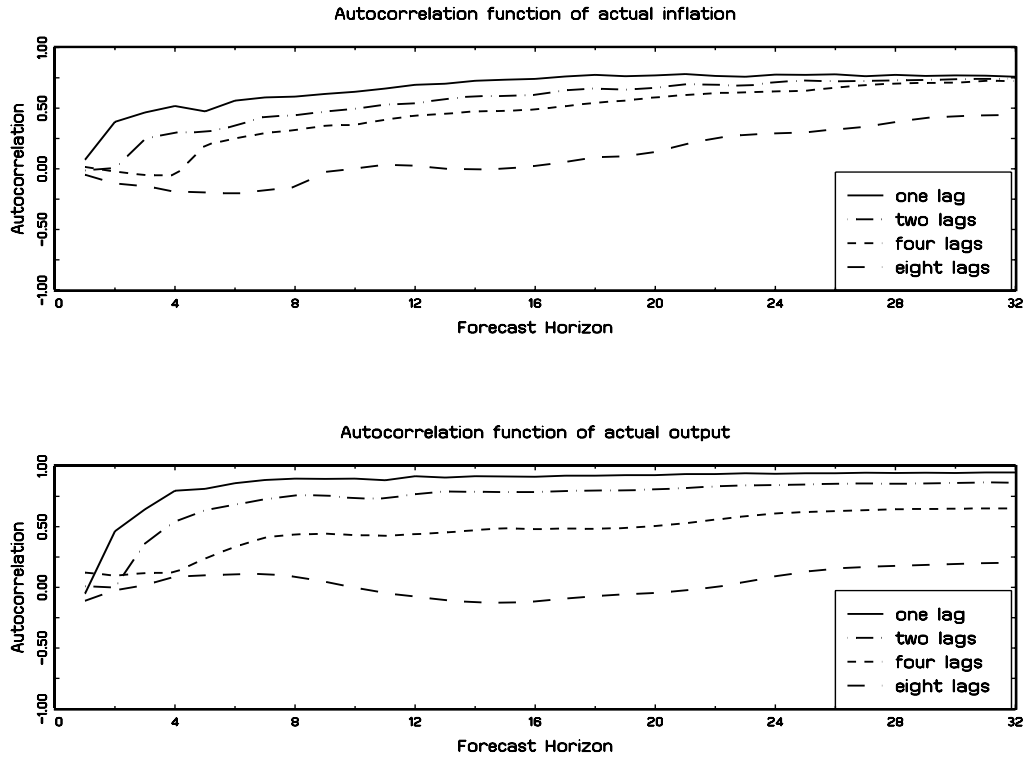Figure 2: Actual comovement between output and inflation

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Figure 3: Inflation and Output Persistence

3   A New Keynesian Model with built in persistence

This section describes a NKM with a number of different structures designed to induce persistence. Many of these structures are quite standard in the literature, so we will briefly review them. The purpose for describing this model is so that later we can explore which of these persistence structures are most effective at generating the type of comovement and persistence patterns seen in the output and inflation data.

The model is a general equilibrium model with three key equations that jointly influence the way that the economy behaves. In the IS curve, persistence is induced through a generalized habit persistence structure as well as through a shock process with persistence. In the NKPC, persistence is induced through a so called, "hy-

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brid" structure suggested by Galí and Gertler (1999) as well as through a persistent shock process. And in the monetary policy rule, persistence is introduced through a rule that incorporates several data features for guiding interest rates, including policy inertia and the standard connection to output and inflation, but also adding a connection to the growth rate of output as suggested by Smets and Wouters (2007).

3.1 An IS curve based on generalized habit persistence

The demand for goods, or IS curve, is based on an optimizing agent structure. Here we add to the standard IS derivation a generalized habit persistence formulation which induces considerable persistence in demand.

The IS curve is derived from a representative consumer optimization problem in which the consumer maximizes

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1 - \frac{1}{\tau}} \right) \left(c_t - \frac{\gamma(\alpha c_{t-1} + \alpha^2 c_{t-2} + \alpha^3 c_{t-3} + \alpha^4 c_{t-4})}{\sum_{i=1}^{4} \alpha^i} \right)^{\left(1 - \frac{1}{\tau}\right)} \right\}$$

subject to

$$c_t + s_t = y_t + R_t s_{t-1},$$

where c_t, y_t, s_t and R_t denote consumption, income, savings and gross real return at period t, respectively. The parameter τ denotes the intertemporal elasticity of substitution. The parameters γ and α control the habit persistence structure. When $\gamma = 0$, habit persistence disappears and the associated IS curve collapses into the standard IS curve described in the basic NKM. When $\gamma > 0$, habit persistence is present and the parameter α comes into play. This parameter controls the way in which habit persistence enters the model. When $\alpha = 1$, the habit consists of an equally weighted index of consumption over the last four quarters. When $\alpha < 1$, the habit overweights the most recent consumption level, while when $\alpha > 1$, the habit overweights consumption four quarters earlier.[12] As $\alpha \to 0$, the model approaches the standard one-period lag habit model.

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[12] The habit formation structure introduced here could be a potential candidate for capturing seasonal patterns in the data. Since we are dealing with seasonal adjusted data in this paper, as in the related literature, there is no need to say much about this feature here. But this structure

Using standard optimization techniques followed by standard linearization methods, the IS curve can be shown to be given by

$$
\begin{aligned}
&-\Delta\widetilde{y}_{t-3} + \left(\frac{\gamma\beta}{K} - \frac{1}{\alpha}\right)\Delta\widetilde{y}_{t-2} + \left(\frac{\gamma\beta^2}{K} + \frac{\gamma\beta}{K\alpha} - \frac{1}{\alpha^2}\right)\Delta\widetilde{y}_{t-1} \\
&+ \left(\frac{\gamma\beta^3}{K} + \frac{\gamma\beta^2}{K\alpha} + \frac{\gamma\beta}{K\alpha^2} - \frac{1}{\alpha^3}\right)\Delta\widetilde{y}_t + \left(\frac{\gamma\beta^4}{K} + \frac{\gamma\beta^3}{K\alpha} + \frac{\gamma\beta^2}{K\alpha^2} + \frac{\gamma\beta}{K\alpha^3} + \frac{K}{\gamma\alpha^4}\right)E_t[\Delta\widetilde{y}_{t+1}] \\
&+ \left(\frac{\gamma\beta^4}{K\alpha} + \frac{\gamma\beta^3}{K\alpha^2} + \frac{\gamma\beta^2}{K\alpha^3} - \frac{\beta}{\alpha^4}\right)E_t[\Delta\widetilde{y}_{t+2}] + \left(\frac{\gamma\beta^4}{K\alpha^2} + \frac{\gamma\beta^3}{K\alpha^3} - \frac{\beta^2}{\alpha^4}\right)E_t[\Delta\widetilde{y}_{t+3}] \\
&+ \left(\frac{\gamma\beta^4}{K\alpha^3} - \frac{\beta^3}{\alpha^4}\right)E_t[\Delta\widetilde{y}_{t+4}] - \left(\frac{\beta^4}{\alpha^4}\right)E_t[\Delta\widetilde{y}_{t+5}] \\
&+ \frac{K}{\gamma\alpha^4}\left(\tau(\gamma - 1)\right)\left(1 - \frac{\gamma}{K}(\beta + \beta^2 + \beta^3 + \beta^4)\right)[i_t - E_t[\pi_{t+1}]] + \frac{K}{\gamma\alpha^4}g_t = 0. \quad (2)
\end{aligned}
$$

where Δ is the first-difference operator, and $\widetilde{y}_t$, π_t and i_t denote output, inflation and the nominal interest rate deviations from their respective steady state values.[13] g_t denotes an IS shock which is assumed to follow the process

$$
g_t = \rho_g g_{t-1} + \varepsilon_{gt}, \quad (3)
$$

where ε_{gt} are innovations which are identical and independently distributed over time with variance σ_g^2.

3.2   The hybrid Phillips curve

The supply of goods in a NKM is captured by the NKPC. Most economists who work with the NKPC have a preference for the strictly forward looking version given by

$$
\pi_t = \beta E_t \pi_{t+1} + \kappa\widetilde{y}_t + z_t. \quad (4)
$$

This preference for the strictly forward looking NKPC follows because the equation can be motivated by the standard Calvo (1983) and Taylor (1980) contracting story as described in Galí and Gertler (1999). Under this formulation, the parameter

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may be useful in other contexts where unadjusted data is important. Some authors have warned about the bias introduced in empirical analysis when considering seasonal adjusted data instead of raw (unadjusted) data.

[13]These calculations can be obtained from the authors upon request. The use of the first-difference operator is just to simplify the IS curve expression a little bit, which is nevertheless quite cumbersome.

$\beta \in (0,1)$ is the firm's discount factor, κ measures the slope of the NKPC and is related to other structural parameters by

$$\kappa = \left(\frac{1}{\tau}\right)\frac{(1-\theta)(1-\beta\theta)}{\theta},$$

where θ denotes Calvo's probability, i.e. the fraction of firms that do not adjust prices optimally in a particular period. The variable z_t is an inflation-push shock and is assumed to be governed by

$$z_t = \rho_z z_{t-1} + \varepsilon_{zt}, \tag{5}$$

where ε_{zt} are independent over time, as well as from the ε_{gt} terms, and they have variance σ_z^2.

Some presentations further augment the NKPC to include a backward looking component such as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa\widetilde{y}_t + \omega\pi_{t-1} + z_t, \tag{6}$$

in Galí and Gertler (1999). This NKPC is typically refered to as hybrid NKPC. In these formulations, the additional component $\omega\pi_{t-1}$ is typically motivated by an empirical need rather than microfoundations, and because of this lack of a formal foundation, the hybrid version is considered less attractive.

Since the strickly forward looking NKPC is a special case of the hybrid curve which imposes $\omega = 0$, we will work with this more general possiblity. We wish to investigate the degree to which it is possible to match the data dynamics best. As we show below, contrary to numerous studies that have focused on single equation NKPCs, we are able to match the data dynamics well when $\omega = 0$, in our model.

3.3   A persistent policy function

To complete the model, we consider a policy rule that is borrowed from Smets and Wouters (2007). According to this rule, nominal interest rate policy responds to output, inflation and the growth rate of output according to

$$i_t = \rho i_{t-1} + (1-\rho)[\phi_1\pi_t + \phi_2\widetilde{y}_t] + \phi_3(\widetilde{y}_t - \widetilde{y}_{t-1}) + v_t, \tag{7}$$

where ϕ_1, ϕ_2 and ϕ_3 are the sensitivities of policy to the various economic variables, ρ captures policy inertia and the shock v_t is independent over time and from the ε_{gt} and ε_{zt}, and has variance σ_v^2. One attraction of this formulation for policy is that it has the popular Taylor rule as a special case. The Taylor rule arises when $\rho = 0$ and $\phi_3 = 0$.[14]

3.4 Model simulations

The model is simulated using the method suggested by Lubik and Schorfheide (2003) that builds on Sims (2001) approach. This approach is straightforward to apply and simply requires writing the system of equations (2), (6), (7), (3) and (5), along with two identities which relate current output and inflation to last periods expected output and inflation and the corresponding forecast errors, in a matrix form

$$\Gamma_0 X_t = \Gamma_1 X_{t-1} + \Psi \varepsilon_t + \Pi \eta_t, \tag{8}$$

where

$$
\begin{aligned}
X_t &= (\widetilde{y}_t, \pi_t, i_t, E_t \widetilde{y}_{t+1}, E_t \pi_{t+1}, g_t, z_t)', \\
\varepsilon_t &= (\varepsilon_{gt}, \varepsilon_{zt}, v_t)', \\
\eta_t &= (\widetilde{y}_t - E_{t-1} \widetilde{y}_t, \pi_t - E_{t-1} \pi_t)'.
\end{aligned}
$$

These equations are then programmed into computer code and simulated using routines available on the web.[15]

4 Fitting the model to the data

In Section 2, we described several data characteristics that have been important dimensions for evaluating model performance. In this section, we investigate whether

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[14] We also considered an alternative policy rule with forward looking components given by

$i_t = \rho i_{t-1} + (1-\rho)[\frac{\phi_1}{4} \sum_{j=1}^{4} E_t \pi_{t+j} + \frac{\phi_2}{4} \sum_{j=1}^{4} E_t \widetilde{y}_{t+j}] + \phi_3(\widetilde{y}_t - \widetilde{y}_{t-1}) + v_t$ in some of our preliminary exercises. This rule did not produce anything noteworthy and as a result was dropped from the final draft of the paper.

[15] The GAUSS code for computing the equilibria of LRE models was downloaded from Schorfheide's web-site.

our general NKM is able to capture those features. Our approach is to fit the model using a type of semi-formal calibration which uses a number of data moments as fitting targets.

In order to keep our exercise clear, we have organized this section into two subsections. In the first subsection, we describe our fitting approach. We then apply this approach and calibrate our model using a number of different groups of fitting targets. Because these different groups have different fitting targets, the implied parameters resulting from the fitting exercise imply different performance characteristics for the fit model.

The next subsection describes the performance results of the various fitting exercises. That subsection is broken into two smaller sections, each focusing on a different aspect of the data characteristics. The first focuses on the lead and lag patterns between output and inflation, while the next one focus on the persistence patterns of inflation and output.

4.1 The calibration approach

The paper uses a calibration approach which is similar to GMM methods matching impulse response functions as in Rotemberg and Woodford (1997) and Christiano, Eichenbaum and Evans (2005), among others. As in these papers, we split the model parameters in two groups. The first group is formed by the pre-assigned parameters β, τ and κ. We fix these parameter values because we do not want our evaluation of the NKM being 'contaminated' by some unreasonable calibrated parameter values. Accordingly, we set $\beta = 0.99$, $\tau = 0.5$ and $\kappa = 0.25$ corresponding to standard values assumed in the relevant literature for the discount factor, the consumption intertemporal elasticity and the Phillips curve slope, respectively.[16] In a similar vein, we set reasonable support intervals for the remaining parameters being calibrated.

The approach here has two key differences from the methods matching impulse response functions. First, we match a different set of moments, including lead and

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[16]Notice that these parameters are consistent with a value of the Calvo's probability equal to 0.81 which implies that firms revise roughly their optimal prices every five quarters on average.

lag correlations between output and inflation, variances of inflation and output, and the autocorrelation function of inflation. Second, because we are not interested in testing, we do not compute standard errors and other formal statistics, and instead simply report the parameter values which match the summary statistics the best.

The fitting approach works as follows. First, K summary statistics are obtained from the observed data. Then the model is simulated J times for an equal number of periods as the number of periods in the observed data.[17] For each simulation, the same summary statistics are computed. These summary statistics are then averaged over the J simulations and these averages are then compared to the summary statistics in the data to, in essence, compute a summary statistic of the statistical comparisons, which is then minimized via standard optimization methods. This objective can be written formally as

$$\text{Minimize } F = \sum_{k=1}^{K} \left(S_{k,d} - \frac{1}{J} \sum_{j=1}^{J} S_{k,j} \right)^2,$$

where $S_{k,d}$ denotes the k-th summary statistic of the observed data and $S_{k,j}$ denotes the k-th summary statistic of the j-th simulation.

For our exercise, the K summary statistics included both correlation and variance information as well as moments capturing inflation persistence. All fitting exercises were based on calculations where the number of simulations, J, was set to 50.[18] We carried out many different calibration exercises using different groups of statistics. The purpose for looking at these different groups of statistics is to evaluate the model performance when different characteristics were emphasized. The results of five of these fitting exercises are presented in Table 1.

Column 1 of Table 1 provides a list of the parameter values that were used in the calibration searches while Column 2 shows the support interval chosen for each parameter. These support intervals reflect our view of the set of reasonable parameter

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[17]To be more precise, there are 177 observations in the data. To match this length, the model is simulated for 354 periods and the first half periods are discarded to move the model away from its initial conditions.

[18]A robustness exercise was carried out by using $J = 100$, and the results were not found to be sensitive to the choice of J.

values based on the estimates provided by a vast empirical literature estimating alternative versions of the NKM model. The remaining columns provide calibrated parameter values for various calibration exercises.

The results in Columns 3, 4 and 5 provide parameter values for fitting exercises that emphasized the lead and lag patterns between output and inflation described in Section 2 as well as the variances for output and inflation, while the calibrations in Columns 6 and 7 added additional information on the inflation persistence to the set of calibration statistics. Our motivation for these exercises is described in the next subsection, but for now we will describe the exercises in a more mechanical fashion.

For the Columns 3, 4 and 5 calibrations 162 summary statistics were used. The first 160 summary statistics included the lead, lag and contemporaneous correlations that are plotted in the bottom two diagrams of Figure 2. Since the contemporaneous correlations in both diagrams are the same, in effect the calibration fits the correlations represented by only 5 lines plotted: the two lead plots, the two lag plots and the one contemporaneous correlation plots. Each of these lines has 32 correlations, so the 5 together gave us 160 summary statistics. Since this is a large number, we decided not to use the lead and lag plots from the other four diagrams in Figure 2. Next, since these 160 summary statistics are only correlations, they do not necessarily fit variances very well. So, in order to target the variances better, we added 2 standard deviation statistics. These include the standard deviation of inflation and the standard deviation of HP detrended output. Altogether, this gave us 162 summary statistics for these calibrations exercises.

The differences between the Column 3, 4 and 5 calibrations reflect an interest in investigating the range of possible model structures that can capture the lead and lag patterns between output and inflation. These three calibrations explore alternative model possibilities for persistence. The first exercise in Column 3 is an unrestricted calibration in which all parameters were free to adjust. The results show that only one of the demand shocks is important. In particular, the IS shock drove the fit, while the policy shock was essentially zero with a value of $\sigma_v = 2.4e - 05$. This

unimportant role for monetary policy shocks to reproduce the lead and lag pattern is in line with the results in Smets and Wouters (2007, p.601) based on a cross-covariance decomposition of shock contributions.

To investigate if it was possible to obtain a good fit without the IS shock persistence, a restricted calibration, summarized in Column 4, in which the IS shock persistence was restricted to be zero, $\rho_g = 0$, was undertaken. This calibration resulted in the policy shock becoming more important for the fit with a value of $\sigma_v = 0.0034$ and the IS shock declining to essentially zero with a value of $\sigma_g = 3.3e - 06$. Next Column 5 constrained both the persistence in the IS shock and the policy inertia parameter to be zero, $\rho_g = \rho = 0$. This exercise resulted in, (i) an increase of the size of demand shock innovations and, (ii) the habit persistence structure becoming important for inducing persistence in the model with $\gamma = 0.7634$ and $\alpha = 0.7906$. Since $\alpha < 1$, the habit persistence structure weights recent consumption more heavily than consumption further in the past.

The Columns 6 and 7 calibrations were undertaken in an effort to improve the inflation persistence fit beyond what was achieved in the other calibrations. Again, we will explain the details more fully below, but for now a mechanical explanation will suffice. Here, we added to the 162 summary statistics in the earlier calibrations, additional summary statistics which were obtained from the inflation autocorrelation plots in Figure 3. In particular, we added the 32 first-order autocorrelations of the inflation forecast errors and the 32 eigth-order autocorrelations of the forecast errors, bringing the total number of summary statistics up to 226. The calibration in Column 6 represented an unrestricted calibration, while the one in Column 7 constrained the persistence in the IS shock to be zero.

The bottom panel in Table 1 shows the standard deviations of actual inflation and (Hodrick-Prescott detrended) output data together with the standard deviations of simulated data obtained from the alternative parameter values. In general, we see the model underestimates output volatility whereas the opposite is true for inflation volatility.

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Table 1 - Calibration Results

		$K = 162$ Restrictions			$K = 226$ Restrictions	
	Support	None	$\rho_g = 0$	$\rho_g = \rho = 0$	None	$\rho_g = 0$
γ	$(0, 1)$	0.0000	$9.8e - 06$	0.7634	0.0022	0.0081
α	$(0, \infty)$	1.1799	0.0797	0.7906	1.5345	0.0789
ω	$(0, 1)$	$8.9e - 05$	0.0486	0.3114	0.0012	0.3311
ρ	$(0, 1)$	0.8550	0.8922	0.0000	0.8340	0.8565
ϕ_1	$(1, 2.5)$	2.4963	2.4930	2.4930	2.4963	2.4893
ϕ_2	$(0, 1)$	0.0000	0.0000	0.0000	0.0000	0.0000
ϕ_3	$(0, 1)$	$2.6e - 04$	0.0158	0.0000	0.1674	0.0156
ρ_g	$(0, 0.99)$	0.9100	0.0000	0.0000	0.9666	0.0000
ρ_z	$(0, 0.99)$	0.6596	0.6003	0.7800	0.7885	0.6072
σ_g	$(0, \infty)$	0.0018	$3.3e - 06$	0.0071	0.0013	$3.6e - 06$
σ_z	$(0, \infty)$	0.0038	0.0040	0.0020	0.0024	0.0027
σ_v	$(0, \infty)$	$2.4e - 05$	0.0034	0.0247	$2.4e - 05$	0.0055
Volatility statistics						
		$K = 162$ Restrictions			$K = 226$ Restrictions	
	Actual	None	$\rho_g = 0$	$\rho_g = \rho = 0$	None	$\rho_g = 0$
σ_y	2.1101	1.1519	1.0798	0.8585	1.0576	1.0385
σ_π	2.3604	2.7604	2.7575	2.7512	2.6723	2.7639

4.2 Simulated data performance

4.2.1 Lead and lag patterns in inflation and output

The calibration results displayed in Columns 3-5 of Table 1 were focused on achieving the lead and lag pattern between output and inflation seen in the data. These results show that there are many ways to achieve the lead and lag pattern. In fact, there are more ways to achieve a lead and lag fit than just the ones displayed, but we have only included these three to illustrate what is needed to get the pattern right. The basic requirements are: 1) the model needs to have both demand equations and supply equations with their own stochastic elements; 2) the model needs to get the relative proportions for the supply and demand shock variances just right; and 3) the model needs to get the relative persistence for the supply and demand shocks just right.

Before turning to the simulated model results and understanding the specifics of how the model works, let us first consider a few more general insights about these

requirements. The need for a general equilibrium structure stressed in this paper, with both demand and supply equations that have stochastic elements, can be recognized by noting the difficulty with which earlier papers that focused on single equation NKPC models had in achieving the lead and lag pattern. However, here, we see that such a pattern can be obtained under many different structures.

Next, comparing Column 3 and Column 4 illustrates the importance of getting the relative variances for supply and demand shocks right, but the flexibility for the origin of the actual shocks. So for instance, in Column 3, the model has a supply shock from the Phillips curve, a demand shock from the IS curve and virtually no demand shock from the policy curve, while in Column 4, the model has a supply shock from the Phillips curve, a demand shock from the policy rule and essentially no demand shock from the IS curve. Both of these models achieve the lead and lag patterns yet the structure for the demand shocks in each fit is quite different.

Next, comparing all three calibrations illustrates the importance of getting the right relative persistence for the supply and demand shocks, but the unimportance of the origin of the persistence. In Column 3, the demand persistence comes about through persistence in the IS curve shock itself, while in the Column 4 model, the persistence comes about via the persistence in the policy rule and in the Column 5 model, the persistence comes about via the habit persistence structure. All that is needed to achieve the lead and lag pattern is that there is some element in the demand structure that induces enough persistence relative to the supply persistence.

To better understand how the model works, now focus on results from simulations based on the calibration in Column 3.[19] Figure 4 plots lead and lag patterns based on simulations of the model of length 177, which is the same length as the observed data. To compute these graphs, 50 simulations were generated, then the lead and lag patterns for each simulation were computed and finally the leads and lags were

---

[19] The models in Columns 4 and 5 produced qualitatively similar simulations and diagrams for the lead and lag plots. Also, just for the record, the small value of $\omega = 2.2e - 06$ in Column 3 is not responsible for the lead and lag patterns. To confirm this, we used all the parameters in Column 3 except for setting $\omega = 0$. Simulations based on that calibration produced identical plots as in Figure 4.

averaged across the 50 simulations.

Figure 4 shows that this basic general equilibrium NKM, in spite of its simplicity, is able to reproduce some of the lead and lag patterns observed in actual data as shown in Figure 2. Although the model does fall short, in that it does not fully reproduce the size nor the hump shape seen in Figure 2, it is still a success relative to other papers which have had difficulty replicating the pattern. Focusing only on the comparison between the contemporaneous plots and the lead plots, we see that when the lead correlation is positive, output leads inflation for leads up to one year and that this lead is mostly due to medium and long term components in the simulated data. Focusing only on the comparison between the contemporaneous plots and the lag plots, we see that when the lead correlation is negative, inflation leads output at lags up to one year, and that the lead is due to medium and longer term components in the simulated data.

In order to dissect the origins for the lead and lag correlations, it is useful to consider impulse response functions. Figure 5 plots impulse response functions for the three economic variables in the model based on the calibration in Column 3 (solid line) and on the calibration in Column 6 (solid line with squares). Later we will compare the two sets of impulse responses, but for now we will focus only on the solid lines. These plots show the impulse responses for the IS curve (demand) shock and the NKPC (supply) shock. Each of these shocks is important for understanding a different part of the lead and lag pattern. In particular, the demand shocks are important for generating the lead of output over inflation, which occurs when output and inflation move together (the upper right plot of Figure 1), while the supply shocks are important for generating the lead of inflation over output, which occurs when output and inflation move in opposite directions (the lower left plot of Figure 1). Striking the right balance between the demand and supply shocks is crucial for reproducing this lead and lag pattern.
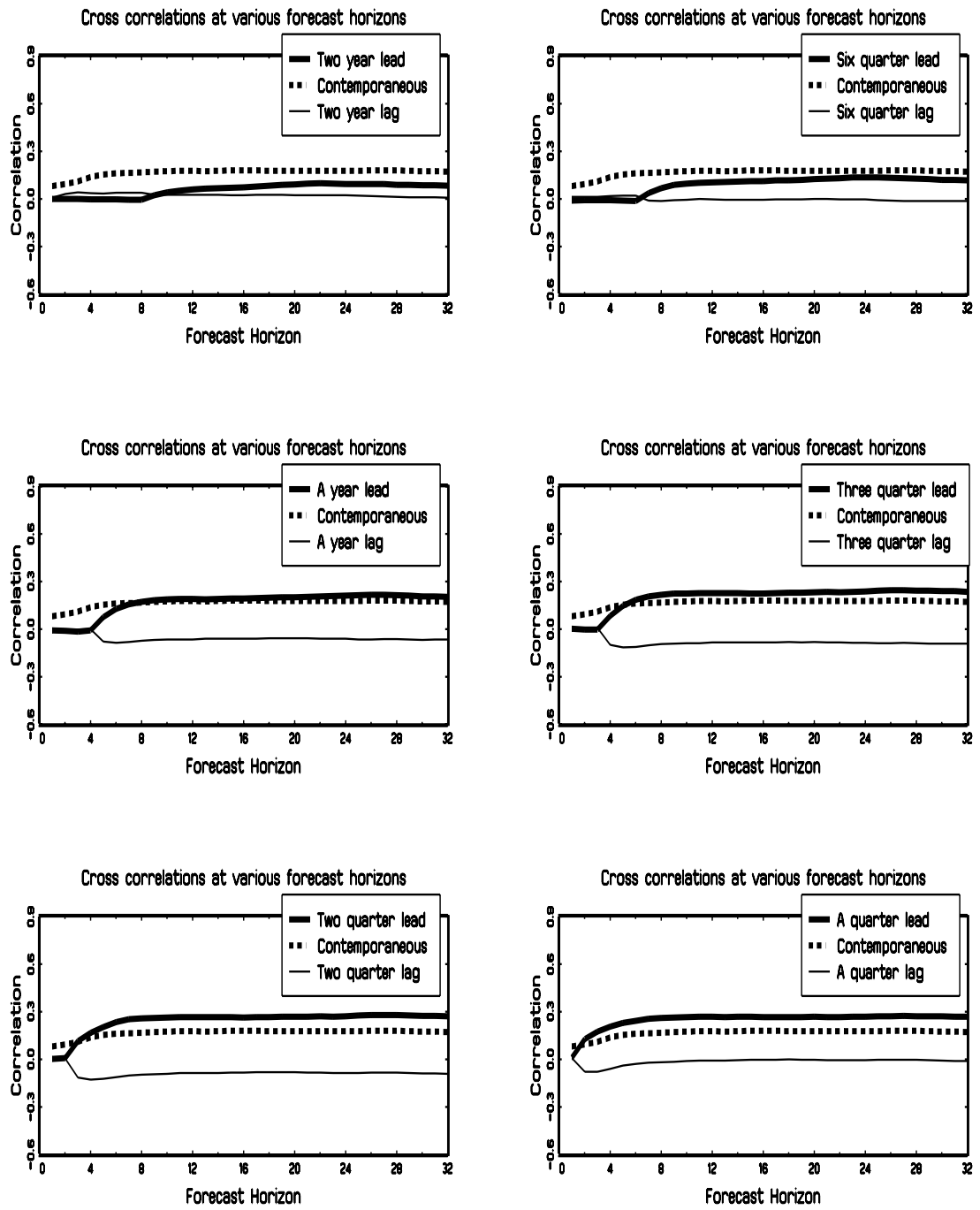
Figure 4: Simulated comovement between output and inflation

To be more clear about how the mechanics of the model work, first focus on the demand shock impulse. Note that a demand shock results in a spike up in both output and inflation. To understand the lead of output over inflation, note that the impact on output dies out more quickly than the impact on inflation. This relatively persistent inflation value means that output portents future inflation.

On the other hand, to understand the lead of inflation over output, note that a supply shock results in a spike up in inflation and a spike down in output, i.e. a negative relationship. Next note that here, the negative output impact dies out more slowly than the positive inflation impact. In this case, the relatively persistent output value means that inflation portents future output.

It is these two different response patterns which together produce the lead of output over inflation when the correlation between the two is positive, and the lead of inflation over output when the correlation between the two is negative, as seen in the off diagonal plots of Figure 1. However, to get these plots right, the response patterns need to be balanced just right. In other words, the demand and supply shocks need to be balanced just right. If either one overwhelms the other, then one of the lead relationships disappear. Furthermore, it is also important to emphasize that the degree to which there is persistence to the demand or supply shocks is also critical. This persistence impacts the persistence displayed in the impulse response functions and without that persistence, one of the lead relationships will also disappear.

Finally, it is important to note once again that there is not a unique calibration to produce these lead and lag patterns. So for instance, if one used the calibrations from Column 4 or Column 5 where the demand shock from the policy rule is important or the persistence in the model comes from the habit persistence in the consumer's problem, qualitatively similar lead and lag patterns as well as similar impulse response functions can be found. What is critical for generating the lead and lag patterns is that there is both a demand and a supply shock, that these shocks have persistent components and that there is just the right balance of the sizes of the shock variances and the persistence features built into the model.
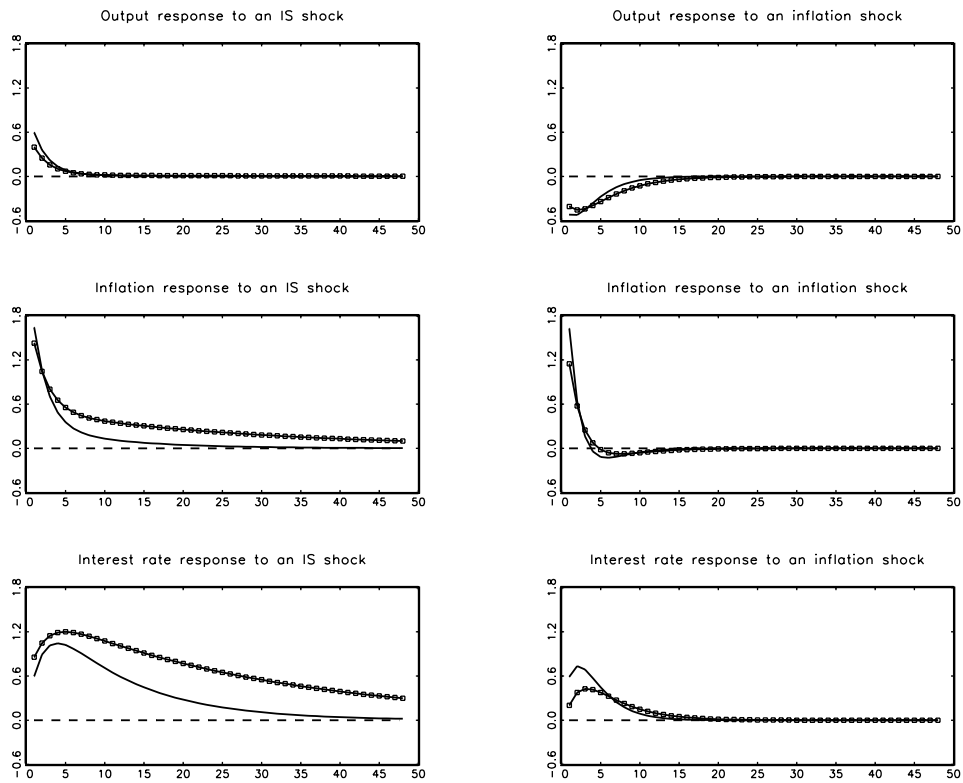
Figure 5: Impulse response functions

4.2.2   Inflation and output persistence

Two other key features of the inflation and output data are that, 1) both series are persistent and, 2) inflation is relatively more persistent than output. To investigate what modeling features were needed to match these two features, we began by investigating the model using the Column 3 calibration. These simulations produced inflation and output persistence values that are plotted in Figure 6 below. These diagrams show that this calibration achieves the joint persistent feature. However, the diagram shows that inflation is roughly equally persistent to output in contrast with the evidence on relative persistence found in actual data.
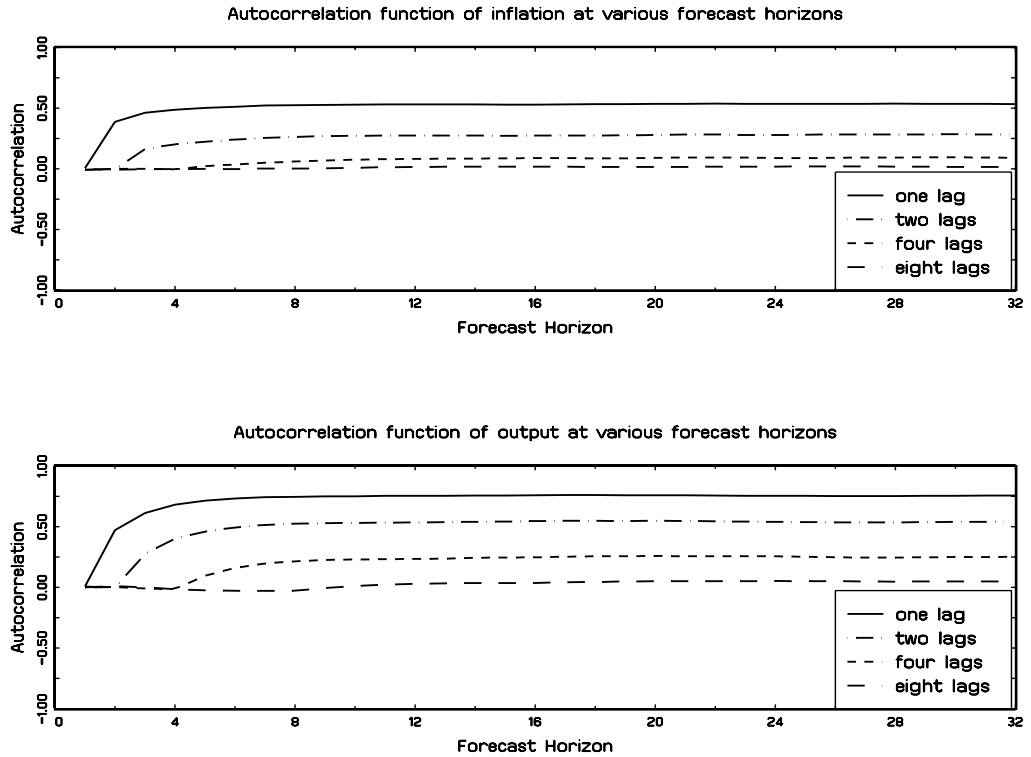
29

Figure 6: Simulated Inflation and Output Persistence

In an attempt to match the inflation persistence better, we undertook the calibration exercises summarized by the results in Columns 6 and 7 of Table 1. These exercises added to the original 162 moments additional moments summarizing the autocorrelation patterns for inflation summarized in Figure 2. Figure 7 plots the inflation and output autocorrelations for the calibration in Column 6. This calibration shows that inflation is now more persistent than in the Column 3 calibration. In addition, Figure 7 shows that inflation is now more persistent than output. As shown in Column 6, we see that the additional moments resulted in the output growth rate coefficient in the policy rule, ϕ_3, increasing. However, a positive output growth rate coefficient is not important for getting the dynamic comovement right. Indeed, by

setting $\phi_3 = 0$ and using the remaining parameter values in Column 6, there is no change in the persistence pattern displayed in Figure 7 and the lead and lag pattern shown in Figure 8 below.
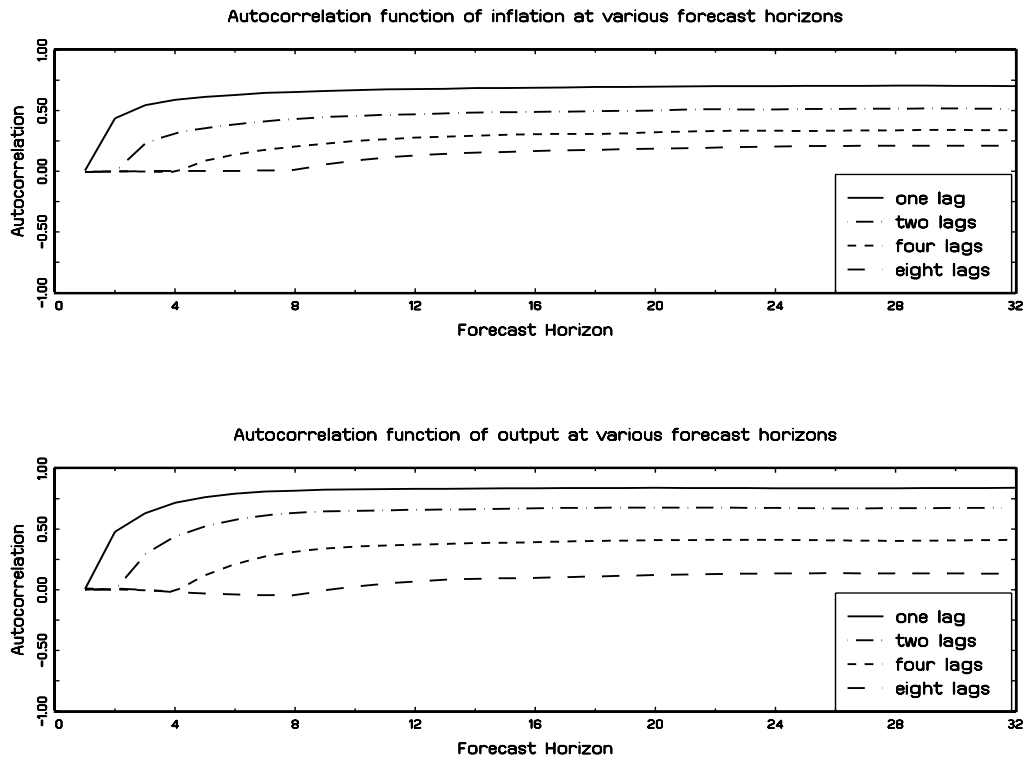

Figure 7: Simulated Inflation and Output Persistence with K=226

Another interesting result is that the calibration that included the inflation autocorrelations actually results in a slightly better lead and lag plot than the earlier calibrations. Figure 8 plots the lead and lag diagrams for the Column 6 calibration. As this diagram shows, the leads and lags now extend to two years. The reason for this better set of lead and lag plots is that this fitting algorithm induced inflation to be relatively more persistent than the previous calibrations. Using intuition built in the discussion of the impulse response functions displayed in Figure 5, and compar-

ing the two solid lines in this figure, we see this longer inflation persistence results in output portending future inflation for a greater number of years (solid line with squares) than the models without so much inflation persistence (solid line).

One last exercise, summarized in Column 7 of Table 1, is to investigate what happens if we restrict the IS curve shock persistence to be zero. Column 7 shows that this calibration results in the backward looking part of the hybrid NKPC, ω, increasing to 0.33 which is in the middle of the range of estimated values reported by Galí and Gertler (1999). Interestingly, this model falls short of reproducing the inflation persistence and the lead and lag pattern between output and inflation. This exercise shows that in a general equilibrium model with both demand and supply shocks of just the right proportions (i.e. the Column 6 parameterization), we are able to achieve the lead, lag and persistence patterns seen in the data, but if we eliminate the IS shock persistence, then the backward looking term in the hybrid NKPC and the relative importance of monetary policy shocks substantially increase However, these two features are unable to provide as quantitatively good a fit to the actual lead, lag and persistence patterns as the one provided by considering IS shock persistence.
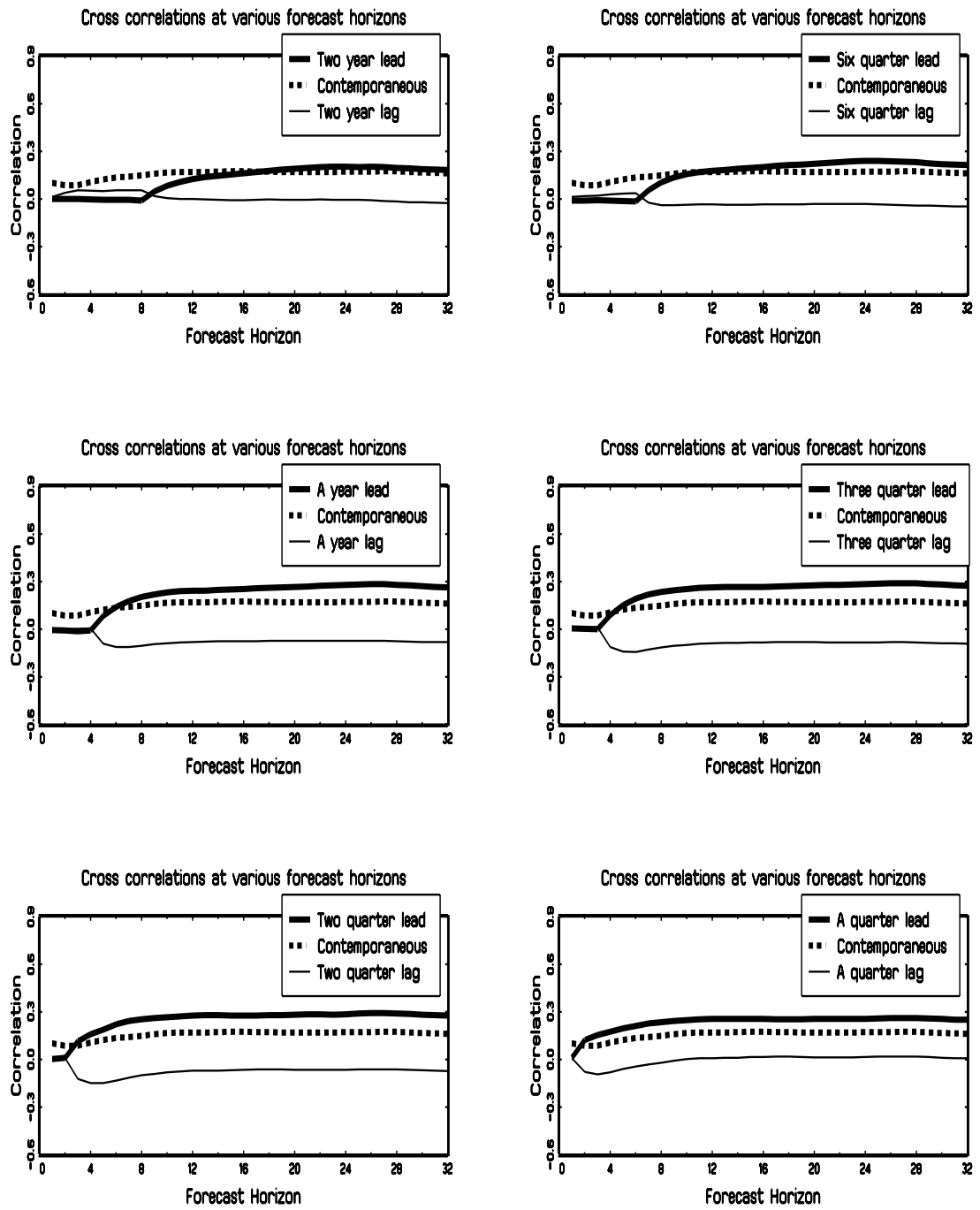
Figure 8: Simulated Comovement between Output and Inflation with K=226

5 Conclusion

This paper has contributed to our understanding of the short-run relationship between output and inflation in three important ways. First, a new statistical method that sheds light on lead and lag comovements of the data was described. This method not only identifies the lead and lag empirical regularities, but it also shows whether they are part of the short term or long term forces driving the data. Second, the paper uses these statistical techniques to describe the lead, lag and contemporaneous comovement between output and inflation, as well as inflation persistence. Here we showed that the lead and lag patterns of the data arise mostly from data components that drive the medium term forecast horizons. Third, a New Keynesian model with a rich set of modeling features is described and then studied to see which of these features are important for generating the actual patterns. It was found that demand and supply shocks are important for replicating the lead and lag patterns in the data and that IS shocks were particularly important for achieving inflation persistence while monetary policy shocks did not play an important role.

These results provide insights relative to a number of previous studies. First, NKMs that only have a NKPC, and do not have demand equations, will have a limited ability to capture the data patterns well, and this may explain their need to add backward looking components to their NKPC. Second, the model here is relatively simple compared to other general equilibrium models and shows that simply adding a structure for demand may be sufficient to explain the data patterns. In addition, the simple NKM presented here is attractive not only because of its ability to fit the data, but also because it is easy to understand the intuition behind the transmission mechanism of shocks.

Extensions of the analysis are worth considering. It is the nature of the business cycle to be asymmetric. Bringing such asymmetry into the model structure may improve the performance. As was seen in the impulse response analysis, the impact of demand and supply shocks are quite different in terms of their impact and duration

and this difference may be important in helping to understand the asymmetry of the business cycle. To pursue such analysis, further modifications to the statistical methods introduced here that treat booms and busts differently may be helpful.

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