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The Sense of Patterns and Patterns in the Senses. An Approach to the Sensory Area of a Montessori Preschool Classroom

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Abstract This article presents and analyses a didactic proposal based on manipulative material (Knobless Cylinders) used in a Montessori classroom of 3-6-year-old pre-schoolers. Choosing this material is justified in relation to the competencies/strategies/skills used during the development of mathematical patterning. Numerous studies emphasize the importance of patterns in mathematics instruction from early childhood onward. However, there are several educational factors that have not yet been clarified, such as meaningful learning, the overuse of abstract visual patterns that are distant from the student's previous life experience, etc. This article discusses the sequence of proposed activities and certain critical issues.

Keywords early mathematic education; patterns; didactic proposal; Montessori Method; manipulative materials

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1. Introduction

This article presents an example of didactic experience related to early pattern learning in preschool students through Montessori manipulative material. The text is divided into three main parts: 1) an introduction to research on patterns in early mathematics education, followed by an overview of the Montessori method and the materials provided in the sensory area; 2) a presentation and didactic analysis of the use of a material belonging to that area; and 3) a discussion on what was previously contributed.

1.1. Learning patterns in early childhood education

Numerous studies, curricula and lesson plans focus on the importance of patterns in mathematics instruction from early education to learning algebra (NCTM, 2003; Rittle-Johnson, Fyfe, Loehr, & Miller, 2015). However, at the educational level, there is no consensus when defining the concept of "pattern" or its properties or functions (Carraher, Martinez, & Schliemann, 2008). Sarama and Clements (2009) state that the study of patterns is the search for regularities and mathematical structures, while Papic, Mulligan, & Michelmore (2011) claim that a pattern means any reproducible regularity, which can occur in three contexts: in a single object, in an ordered set of objects or between two ordered sets.

However, the importance of teaching-learning of patterns, even at preschool age, seems evident (Fyfe, McNeil, & Rittle-Johnson, 2015; Papic et al., 2011). Identifying, extending and describing predictable sequences in objects or numbers constitute basic skills, for example, knowledge of the sequence of natural numbers (Fernández Escalona, 2016) and algebraic thinking in general (Rittle-Johnson et al., 2015). Likewise, the early study of patterns prepares for future algebraic learning (Mulligan, & Mitchelmore, 2009; Papic et al., 2011) and relational reasoning (Fyfe et al., 2015). In this sense, the National Council of Teachers of Mathematics (NCTM, 2000), in the section on algebra between early childhood education and 2nd grade, argues that patterns are a child's way of recognizing, ordering and organizing their world, and at this stage, they are important in all aspects of mathematics. In addition, this same organization adds that teachers should help children become capable at working with the generalization of mathematical ideas.

Patterning instruction during the early childhood stage translates into, among other skills, the ability to recognize the unit of repeat and develop multiplicative reasoning (Papic et al., 2011), to repeat, duplicate, extend and abstract them (Rittle-Johnson et al., 2015) and to compare and order objects (NCTM, 2006).

To the lack of a solid concept of pattern must be added certain critical aspects. McGarvey (2012) warns of the lack of attention to the concept of non-pattern. In addition, this author details the scarce resources available to teachers for deciding the activities to propose for teaching children, the reliance on visual perception without taking into account the previous experience of the students, the overuse of recursive procedures versus understanding the unit of repeat with the consequent inability to generalize, etc. The difficulty in identifying the unit of repeat is also suggested by Rittle-Johnson et al. (2015). Due to these issues, this article reports on the possibility of acquiring skills necessary for subsequent pattern learning through a specific didactic experience.

1.2. Learning patterns in early childhood education

The first Montessori school, Casa dei bambini, was founded by Maria Montessori in Rome in 1907. From then until today, hundreds of schools have followed its method, more or less faithfully. The educational cycles range from early childhood education to secondary education, with early education predominating up to six years of age. It is difficult to know the number of Montessori schools that currently exist; in Europe, the largest number are in United Kingdom, approximately 700 centres (Nursery World, 2014). Worldwide, India and the US have the highest numbers, between 4000 and 8000, between public and private (Wikipedia, 2020). Notably, not all of them apply the method in the same way and with the same rigour (Chan, & Rao, 2013; Lillard, 2012). In this sense, the most prestigious and methodically faithful schools are endorsed by the Association Montessori Internationale (AMI).

The Montessori method, considered a nonconventional teaching method, is characterized by recognizing the interest and needs of students, allowing their personal choice; reinforcing self-motivated, active and autonomous, as well as collaborative, learning by mixing students of different ages; facilitating the manipulation and understanding of materials; developing self-control; respecting freedom and teaching responsibility; encouraging creativity; avoiding extrinsic motivation; not rewarding success; and encouraging trial-error techniques in tasks, among other characteristics (Lillard, 2005).

Each classroom is directed by a teacher/guide and one assistant. The peculiarities of the Montessori method make this terminological distinction necessary; the guides are limited to presenting the materials and explaining their application in the form of a task, to later observe and record how they are used, without intervening. Obviously, this procedure is not strict due to the number of situations and needs that can arise in the classroom at these ages, but it can be said that the norm in the instruction and use of materials is performed under the parameters mentioned.

The importance of using materials in the early childhood education method is noteworthy; in fact, the distribution of each classroom is planned around them, in a strictly ordered manner and in different areas or environments. These areas are classified as mathematics, sensory experience, language, practical life and cultural studies (Peng, & Md-Yunus, 2014); learning in each area is based on manipulating certain materials. The materials, made of wood and attractive colours, are designed to perform tasks in a structured and controlled manner, with precise indications for their use, following a sequence from least to greatest complexity, from concrete to abstract. Therefore, it can be understood that both the environmental order of the classroom and the structural order of the manipulative tasks aim to promote the mental and emotional stability of the students.

The use of materials in the sensory area of the classroom, in addition to meeting specific objectives, prepares students for learning mathematics. The tasks proposed in Mathematics follow a growing order as far as abstraction is concerned. The Mathematics curriculum in the early childhood stage adheres to the following sequence: numbers from 1 to 10; the decimal system; linear counting; memorization of basic arithmetic combinations; the transition to abstraction; and fractions.

The manipulation of materials in early childhood education is a common practice, as reflected in many articles. However, the data on their educational benefit are inconclusive; the suitability of their use is discussed, depending on the student's knowledge (Petersen, & McNeil, 2013), the complexity of its use, which is added to the inherent difficulty of the mathematical concept to which it is intended to represent (Brizuela, & Cayton, 2010), its approach to the concept as a symbol or as a substitute for the symbol (Uttal et al., 1997), its possible relation with abstract concepts (McNeil, & Uttal, 2009), etc.

1.3. Sensory materials

Most likely, the most well-known aspect of the Montessori method is its contribution to the value of learning through the senses through materials (Isaacs, 2015). The students who use these materials acquire a refinement of the sensory stimuli of the external environment that encourages their exploration (Montessori, 1967). The "training of the senses", in ages between 3 and 6 years, pursues a dual objective: the biological, which helps natural development, and the social, consistent with adapting the individual to their environment (Montessori, 1949). According to Montessori, this practice precedes higher intellectual faculties.

Various strategies/skills/competencies associated with the use of these sensory materials are identified: ordering, sorting, pairing, grouping and grading, as well as introducing pattern creation (Isaacs, 2015).

The materials stimulate the following senses:

- Tactile (through different textures).
- Visual (through shapes, sizes and colours).
- Thermic and baric (by materials of different temperature and weight).
- Auditory (using different tones and musical notes).
- Taste and smell (examining, for example, food and flowers).
- Stereognostic (recognizing objects by feeling their shape and other physical properties, without seeing them).

The objective of this article is to relate the use of a particular sensory material with an early approach to competencies that can influence pattern learning and the subsequent implications in mathematics education.

2. Materials and Methods

Next, the use of colour cylinders (Knobless Cylinders), a visual sensory material, is described and analysed. Notably, this material is presented to students at the age of 4.5 years after having acquired skills with other sensory materials. The presentation and exercises that will be described (in italics) have been provided by the guide of a 3-6-year-old classroom in Aiurri Montessori School (Aiurri Montessori, 2020), located in Vitoria-Gasteiz (Basque Country-Spain). This professional is certified by the AMI, in addition to having extensive teaching experience in Montessori schools.

Description of the material:

Four boxes with coloured cylinders; ten pieces of the same colour per box; two dimensions per colour (height and diameter) that vary proportionally at a ratio of 0.5 cm per spatial position. The characteristics of each colour, according to their spatial distribution, are as follows (see Figure 1):

- Reds: The diameter decreases from 5.5 to 1 cm. The height remains constant at 5.5 cm.
- Yellows: The diameter varies as in the reds, while the height decreases from 5.5 to 1 cm.
- Greens: The diameter and height increase from 1 to 5.5 cm.
- Blues: The diameter remains constant at 2 cm, while the height decreases, as in the yellows.

Presentation of the material:

1. We invite you to become familiar with the material.
2. Two boxes (yellow and green cylinders) are placed on the carpet.
3. All cylinders are mixed.
4. A colour is chosen and ordered from largest to smallest diameter (see Figure 1).
5. The same procedure is performed with the other colour.
6. Observe the spatial distribution from different visual angles.
7. A colour is chosen, and the cylinders are superposed, two by two, maintaining the previously established order (see Figure 2).
8. Step 6 is repeated.
9. It is taken down, and step seven is repeated, inverting the order of colours.
10. Step 6 is repeated, and the exploration and free use of the material is encouraged.

The first competence worked on in this presentation is sorting or grouping by colour. Subsequently, the diameters are organized in descending order for both colours (steps 4 and 5). Steps 4 and 5 offer an interesting strategy: attention is focused on the diameter dimension, while it can be perceived, implicitly, that height gets lower in one case and higher in another.

On the other hand, two cylinders are superimposed according to both colour and spatial position; this gives rise to double alternating towers of the same height in both steps (7 and 9) but with permuted colours.

Exercises:

a) Perform the same work with any two of the boxes.

The exercise allows one to see the graduation of the other two colours (red and blue); the diameter decreases for the first and invariably for the second. In addition, the exercise helps to visualize two possible cases of double alternating towers: 1) the towers are constructed with the yellow or green and red cylinders, obtaining the same distribution of heights (decreasing or increasing) because the red cylinders do not vary in height; and 2) any colour combined with blue, although the tower can be built, the bases are not the same diameter except for the 2-cm cylinders because the blue cylinders have a fixed diameter of 2 cm. The latter case invites one to reflect on the difference in diameters in some cases and the identical sizes in others, as well as the suitability of building towers with "ideal" bases (the case of the initial presentation) or unequal in terms of diameter.

b) Perform the same work with any three of the boxes.

As in exercise a), the different growth-decrease patterns obtained are worked on, and triple alternating towers are constructed (see Figure 3) with the possible handicap of the different bases mentioned above.

c) Perform the same work with four boxes.

The competences for exercises a) and b) are repeated with different height and diameter distributions.

d) Build towers using all cylinders of one colour (or alternating).

This activity allows mounting towers of the same colour, drawing attention to the diameter of the cylinders in the three cases that vary (see Figure 4). It also allows the execution of sequences of alternating towers that produce a colour pattern (for example, red-yellow-green-blue).

e) Find the five equal pairs (ignoring colour).

Pairing is explored by paying attention to the dimensions of height and diameter.

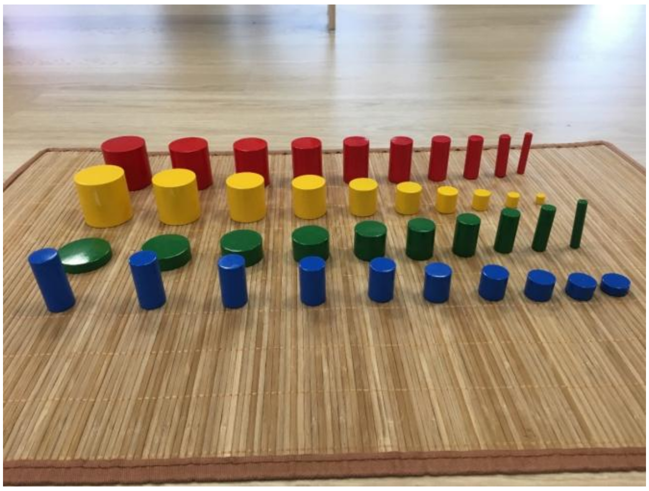

Figure 1. Knobless Cylinders.

Figure 2. Double alternating towers with equal bases.

Figure 3. Triple alternating towers with equal bases.

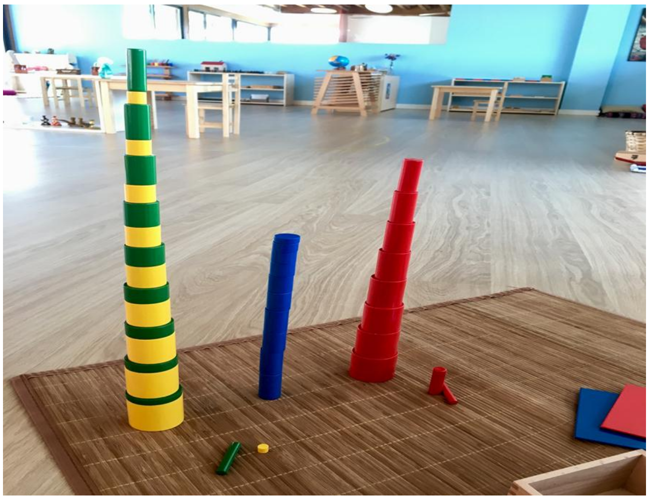

Figure 4. Towers of the same colour (with all cylinders) and alternating colour.

3. Conclusion

The educational tasks shown by the use of Knobless Cylinders promote the skills of ordering, pairing, sorting and grading. Maria Montessori (1949) indicates the importance of isolating the quality of an object while the other aspects remain fixed in order to centre and focus without distractions. This ability can be practiced with the families of red and blue cylinders thanks to the exclusive variation in diameter and height, respectively. Sensory discrimination organizes cognitive structures and is influenced by early sensory experience (Lillard, 2005). Another significant aspect mentioned in previous educational practice is the implicit combination of two qualities; although it focuses on one dimension as a variable, the other changes in a decreasing or increasing way.

Beginning in preschool, organizations such as NCTM (2003) report on activities to be carried out in relation to the study of patterns, such as selecting, sorting, recognizing, ordering and organizing. All these actions are implemented through the use of the manipulative material presented here. In this sense, another report describes the main preschool curriculum focal points, listing, comparing and ordering in the measurement of objects (for example length) and in their attributes (equal, different...) (NCTM, 2006). This document includes the identification, duplication and extension of simple numerical patterns and growing patterns (figures); the dimensional grading of the Knobless Cylinders would be in line with these skills. Finally, reference should be made to the repetition of a pattern, considered key for effectively learning these patterns (Fyfe et al., 2015; Rittle-Johnson et al., 2015). The formation of towers of the same height, with cylinders of alternating colours and with the same diameter, could enable the recognition and understanding of this concept.

Analogous to what is stated in this didactic experience, pattern learning offers basic tools for algebraic thinking beyond the strict management of regularity or repetition. Despite this finding, neither the teaching programmes nor the research on this subject clearly show prior knowledge or teaching concerning this complex learning at young ages. Inappropriate use of frequently chromatic visual perception, the lack of distinguishing between patterned and nonpatterned images, and the excessive insistence on repetition to the detriment of generalization (McGarvey, 2012), among other factors, may suggest that learning about patterns is not significant, nor does it foster relational reasoning (Collins, & Laski, 2015; Rittle-Johnson et al., 2015).

Another critical issue is the omission of the student's previous experience when recognizing patterns (McGarvey, 2012). Sensory materials transcend the visual sense, can be manipulated experimentally and avoid the abstraction of images. Maria Montessori (1967) describes these materials as "materialized abstractions" because they impersonate, in nature, objects that are mathematically but not physically present. In this same direction, some studies highlight the social and cultural component of the teaching-learning process (Vogel, 2013).

In conclusion, the material discussed serves as a starting point to acquire valid competencies/strategies/skills to understand patterning and what their learning entails later. Although Montessori schools are not accessible to just anyone and the application of its method requires professional guides prepared to exhaustively apply it in all its complexity, the sequence of activities proposed gives clues to educational practices that can contribute to mathematics instruction at an evolutionary stage as important as early childhood education.

REFERENCES

[1] Aiurri Montessori (2020). *Aiurri*. Retrieved from: http://www.aiurrimontessori.eus/.

[2] Brizuela, B. M., & Cayton, G. A. (2010). Writing-down numbers from kindergarten to second grade: The impact of using two representation systems when presenting numbers. *CULTURA Y EDUCACION*, *22*(2), 149-167. https://doi.org/10.1174/113564010791304500

[3] Carraher, D. W., Martinez, M., & Schliemann, A. D. (2008). Early algebra and mathematical generalization. *ZDM. The International Journal of Mathematics Education*, *40*(1), 3-22. https://doi.org/10.1007/s11858-007-0067-7

[4] Chan, W. L., & Rao, N. (2013). Variation in the Qualities of Instruction Methods Adopted by. *International Journal of Early Childhood, 19*(2), 21-41.

[5] Collins, M. A., & Laski, E. V. (2015). Preschoolers' strategies for solving visual pattern tasks. *Early Childhood Research Quarterly, 32*, 204-214. https://doi.org/10.1016/j.ecresq.2015.04.004

[6] Fernández Escalona, C. M., (2016) Una propuesta didáctica para trabajar la secuencia numérica en el segundo ciclo de educación infantil. *Enseñanza de las Ciencias, 34*(2), 185-204. http://dx.doi.org/10.5565/rev/ensciencias.1798

[7] Fyfe, E. R., McNeil, N. M., & Rittle‐Johnson, B. (2015). Easy as ABCABC: Abstract language facilitates performance on a concrete patterning task. *Child development, 86*(3), 927-935. https://doi.org/10.1111/cdev.12331

[8] Isaacs, B. (2015). *Bringing the Montessori approach to your early years practice*. Third edition. New York: Routledge.

[9] Lillard, A. S. (2005). *Montessori: The Science Behind the Genius*. New York: Oxford University Press.

[10] Lillard, A. S. (2012). Preschool children's development in classic Montessori, supplemented Montessori, and conventional programs. *Journal of School Psychology, 50*(3), 379-401. https://doi.org/10.1016/j.jsp.2012.01.001

[11] McGarvey, L. M. (2012). What is a pattern? Criteria used by teachers and young children. Mathematical *Thinking and Learning, 14*(4), 310-337. https://doi.org/10.1080/10986065.2012.717380

[12] McNeil, N. M., & Uttal, D. H. (2009). Rethinking the use of concrete materials in learning: Perspectives from development and education. *Child development perspectives, 3*(3), 137-139. https://doi.org/10.1111/j.1750-8606.2009.00093.x

[13] Montessori, M. (1967). *The discovery of the child*. New York: Ballantine.

[14] Montessori, M. (1949). *The Absorbent Mind*. Madras: The Theosophical Publishing House.

[15] Mulligan, J., & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal, 21*, 33-49. http://dx.doi.org/10.1007/BF03217544

[16] National Council for Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA : Author.

[17] NCTM (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*. Reston, VA: Author.

[18] Nursery World (2014). *Demand grows among parents for Montessori schools in UK*. Retrieved from: https://www.nurseryworld.co.uk/news/article/demand-grows-among-parents-for-montessori-schools-in-uk.

[19] Papic, M. M., Mulligan, J. T., & Mitchelmore, M. C. (2011). Assessing the development of preschoolers' mathematical patterning. *Journal for Research in Mathematics Education, 42*(3), 237-268.

[20] Peng, H. H., & Md-Yunus, S. A. (2014). Do Children in Montessori Schools Perform Better in the Achievement Test? A Taiwanese Perspective. *International Journal of Early Childhood, 46*(2), 299-311.

[21] Petersen, L. A., & McNeil, N. M. (2013). Effects of perceptually rich manipulatives on preschoolers' counting performance: Established knowledge counts. *Child development, 84*(3), 1020-1033. https://doi.org/10.1111/cdev.12028

[22] Rittle-Johnson, B., Fyfe, E. R., Loehr, A. M., & Miller, M. R. (2015). Beyond numeracy in preschool: Adding patterns to the equation. *Early Childhood Research Quarterly, 31*, 101-112. https://doi.org/10.1016/j.ecresq.2015.01.005

[23] Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research; learning trajectories for young children.* New York: Routledge.

[24] Uttal, D. H., Scudder, K. V., & DeLoache, J. S. (1997). Manipulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of applied developmental psychology, 18*(1), 37-54. https://doi.org/10.1016/s0193-3973(97)90013-7

[25] Vogel, R. (2013). Mathematical situations of play and exploration. *Educational Studies in Mathematics, 84*(2), 209-225. https://doi.org/10.1007/s10649-013-9504-4

Wikipedia (2020). *Montessori in the United States*. Retrived from: https://en.wikipedia.org/wiki/Montessori_in_the_United_States#Montessori_schools