

# Second-best tax policy in a growing economy with externalities\*

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## Abstract

This paper investigates the exploitation of environmental resources in a growing economy within a second-best fiscal policy framework. Agents derive utility from two types of consumption goods – one which relies on an environmental input and one which does not – as well as from leisure and from environmental amenity values. Property rights for the environmental resource are potentially incomplete. We connect second best policy to essential components of utility by considering the elasticity of substitution among each of the four utility arguments. The results illustrate potentially important relationships between environmental amenity values and leisure. When amenity values are complementary with leisure, for instance when environmental amenities are used for recreation, taxes on extractive goods generally increase over time. On the other hand, optimal taxes on extractive goods generally decrease over time when leisure and environmental amenity values are substitutes. Under some parameterizations, complex dynamics leading to nonmonotonic time paths for the state variables can emerge.

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# 1 Introduction

It has been well known since Ramsey (1927) that optimal tax rates should differ across goods depending on how each of the individual goods is valued by agents. The analysis of optimal taxation across heterogeneous goods was later extended by Sandmo (1975) and Bovenberg and van der Ploeg (1994) to consider taxes set in the presence of externalities. This analysis is the foundation for evaluating optimal policy in second-best settings with distortionary taxes and non-market goods and has been applied to issues ranging from pollution control to the management of public goods.<sup>1</sup> As interest has grown in understanding the connection between economic growth and the environment, there is an increasing need to consider the dynamic implications of growth for environmental policy with distortionary taxes.<sup>2</sup> However, analysis of second-best tax policy in dynamic settings has been limited by the competing goal of providing tractability. In the macroeconomic literature, where much of the dynamic optimal taxation work is found, a simple preference structure for agents is frequently employed.<sup>3</sup> Numerical techniques pioneered in the macroeconomic literature have been recently developed that allow a rich cross-section of taxes to be considered in dynamic analysis, but these techniques, to our knowledge, have not been applied to policy settings with externalities. The aim of this paper is to use numerical techniques to examine the connection between economic growth and optimal tax policy in settings where utility is derived from multiple sources with different degrees of substitutability and where at least one of these sources is incompletely internalized

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<sup>1</sup>For surveys of this literature, see Poterba (1993), Auerbach and Hines (2001), and Bovenberg and Goulder (2002).

<sup>2</sup>Examples of dynamic models that consider second-best tax policy with environmental externalities include Jorgenson and Wilcoxon (1993), Goulder (1995), Bovenberg and Goulder (1996), and Bovenberg and de Mooij (1997).

<sup>3</sup>A common assumption in most macroeconomic applications of dynamic optimal taxation is that a single representative good is consumed at each date. This is the case in Lucas and Stokey (1983), Judd (1985,1999), Chamley (1986), Zhu (1992), Jones, Manuelli and Rossi (1993, 1997), Chari, Christiano and Kehoe (1994), Cassou and Lansing (1998, 2006) Aiyagari, Marcet, Sargent and Seppälä (2002) and Gorostiaga (2003, 2005). In this case, the tax rate on all consumption goods available at a particular date are implicitly equal.

in markets.

The paper contributes to the analytical and numerical literature on optimal second-best tax policy in two essential ways. First, we focus on how the optimal second-best structure evolves over time under various parameterizations of the utility structure of goods. This differs from the research on the so-called “double dividend” of environmental taxation that follows Goulder (1995) and Bovenberg and Goulder (1996) to consider how optimal taxes depart from Pigovian rates in dynamic models with revenue recycling and from papers which seek to find conditions that support non-monotonic relationships between income and environmental quality over time.<sup>4</sup> This also differs from the macroeconomic literature on second-best optimal taxation that suppresses cross-sectional issues by considering a single, representative good consumed at each date. Second, we consider a fixed endowment of an environmental resource, for instance a given quantity of forestland, and examine the effect of economic growth on the marginal valuation of the resource in alternative uses as a productive input and as a source of amenity value for consumers. This allows us to derive unique connections between economic growth and dynamic second-best tax policy that can inform environmental policy.

Our analysis is framed around four essential determinants of consumer utility, two types of consumption goods –one which relies on an environmental input and one which does not– leisure, and environmental amenity value. All goods are elastically supplied, so that taxes are always distortionary, and our focus is on how the second-best optimal tax policy evolves over time as growth alters the magnitude of the various distortions in the economy.

Our main finding is that optimal tax policy in a growing economy involves distinct dynamic patterns that depend on the degree in which the various utility arguments substitute for one another. Environmental resources contribute uniquely to the dynamic time path due to the competing uses for the environment as an input in

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<sup>4</sup>Cassou and Hamilton (2005) examine conditions under which second-best policy leads to an environmental Kuznets curve (EKC), which is similar to the present goal. However, their analysis does not consider how second-best policy is influenced by the relationship between goods.

goods production of in the generation of amenity values. Under circumstances where amenity values are not fully internalized in markets, for instance when property rights for the environmental resource are incomplete, the “corrective” component of taxes on resource-extractive goods trends over time according to an evolving margin that sets rising amenity values, as consumers acquire more effective leisure with economic growth, against a rising value of resources devoted to production, as the output potential associated with a given level of extractive inputs increases over time.

The results of our analysis extend the intuitive connections between utility valuations and policy derived by Ramsey (1927) and Sandmo (1975) to dynamic settings with externalities whose magnitudes are influenced over time with economic growth. We identify two main forces which cause the magnitude of environmental externalities, and hence the required extent of corrective taxation, to trend over time. These include the degree to which environmental amenity values are complementary with leisure and the degree to which resource-extractive goods are complementary with non-extractive goods.

Economic growth raises effective leisure. In a growing economy, the corrective role for tax policy therefore depends on whether increments in leisure raise or lower the demand for environmental amenities. When amenity values are more complementary with leisure activities, this facilitates consumer demand, while the opposite occurs, consumer demand for amenities declines over time, when amenity values substitute for leisure. Put somewhat differently, if consumers enjoy spending their leisure time on the internet, as opposed to recreating outdoors, then economic growth reduces demand for environmental resources such as forestland, for which leisure can substitute, and the optimal policy increasingly encourages the conversion of forests into goods, for instance printer paper, by reducing taxes on extractive goods.

Similarly, the degree to which resource-extractive goods are complementary with non-extractive goods impacts the optimal tax policy. When resource-extractive goods are more complementary (substitutable) with non-extractive goods the optimal tax on

resource-extractive goods decreases (increases) over time.<sup>5</sup> Economic growth raises the amount of consumption goods that can be produced from a given level of inputs, including environmental ones. To the extent that technological growth is neutral, economic growth thus expands the production possibilities frontier proportionally for both types of goods. When the consumption goods are substitutes, the growing capability of the economy to produce non-extractive goods reduces the social value of environmental resources in production, and this facilitates an increasing role for corrective taxes over time. For instance, if electronic files substitute well for paper ones, then economic growth should lead to higher taxes on paper products over time.

Our analysis also reveals that complex dynamics can emerge, including non-monotonic relationships in the dynamic tax profile and in the allocation of resources. These non-monotonic time paths tend to arise in situations where there is a high degree of substitutability among some of the key utility arguments. Under these circumstances, for instance when leisure substitutes for forestland amenities at the same time that electronic files substitute for paper, the productive and consumptive margins for the environment evolve in common directions with economic growth. Dynamic reversals tend to require high degrees of substitution between goods, because substitution possibilities raise the likelihood that policy can produce large swings in the desired consumption allocations as the economy grows. These large swings in desired consumption allocations, in turn, can result in non-monotonic time paths for the optimal second-best tax policy variables.

The remainder of the paper is organized as follows. Section 2 presents the competitive economy and the Ramsey planner's problem. The numerical methods used to solve the Ramsey problem are also outlined in Section 2, and, in Section 3, results of the numerical Ramsey solutions are presented and some general statements are derived about the influence of utility valuations on optimal policy.

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<sup>5</sup>This corroborates the finding of Lopez (1994), who demonstrates that optimal environmental taxes should rise over time when polluting and non-polluting inputs are substitutes in a single production process.

## 2 The Model

Our interest in second-best, dynamic tax policy is twofold. First, we address how distortionary taxes are used to optimally allocate resources across final good production activities under conditions of economic growth. Second, we consider the effect of incomplete property rights over a fixed environmental resource endowment that is to be allocated between goods-producing and amenity-producing activities in the economy. Economic growth, which changes the marginal valuation of the environmental resource in its alternative uses, alters the corrective role for taxation, so that examining the optimal second-best tax structure can inform environmental policy. To couch these issues in a reasonably transparent model, we pursue a structure in which environmental resources are converted into final goods through a process of exogenous (and neutral) technological progress.

In what follows, we first describe the growth process through which productive inputs change over time. Next, we focus on the details of the economy which is comprised of a corporate sector, a consumer sector, and a government sector. We consider each of these sectors in turn before formalizing the Ramsey problem at the end of this section.

### 2.1 Productive resources

Let  $L_t$ ,  $A_t$  and  $k_t$  denote the population, technology level and the capital to labor ratio in the economy at time  $t$ . Population growth, which does not play an important role in the analysis, is assumed to be zero, and the population level is normalized to  $L_t = 1$  for all  $t$ . Technology and the capital-labor ratio are assumed to grow exogenously according to

$$A_t = (1 + q)A_{t-1} \quad \text{for } t \geq 0, \tag{1}$$

and

$$k_t = (1 + q)k_{t-1} \quad \text{for } t \geq 0, \tag{2}$$

where  $q$ , the rate of technical change, proportionally influences productivity growth and the growth rate in the capital to labor ratio. Such would be the outcome under balanced growth.

Productive resources are allocated across two sectors, an extractive sector, which makes use of an environmental resource and a non-extractive sector that produces goods without environmental resource inputs. The total amount of the environmental resource available in the economy is fixed at a quantity of  $f$  units, and, at time  $t$ , units of the environmental resource are either devoted to extractive goods production,  $f_{1,t}$ , or left in a natural state,  $f - f_{1,t}$ . Units of the resource left in a natural state generate amenity values that contribute to consumer utility, whereas units devoted to goods-production do not.

## 2.2 The corporate sector

The corporate sector consists of two types of producers who manufacture consumer goods. We index the producers by  $j$  and distinguish the sectors by  $j = f, h$ , where  $f$  denotes the sector which makes use of environmental resources in production and  $h$  denotes the sector which does not. Let  $y_{j,t}$ ,  $k_{j,t}$  and  $l_{j,t}$  denote output, capital input and labor input into sector  $j$  at time  $t$ . In each sector, output is created through the employment of physical capital, labor and land according to

$$y_{f,t} = A_{1f} k_{f,t}^{\alpha-\eta} (A_t l_{f,t})^{(1-\alpha)} f_{1,t}^\eta \quad \text{for } t \geq 0, \quad (3)$$

and

$$y_{h,t} = A_{1h} k_{h,t}^\alpha (A_t l_{h,t})^{(1-\alpha)} \quad \text{for } t \geq 0, \quad (4)$$

where parameters are restricted according to  $0 < A_{1j}$  for  $j = f, h$ ,  $\alpha \in (0, 1)$  and  $0 < \eta < \alpha$ . For simplicity we assume the production functions (3) and (4) are associated with symmetric factor shares for labor inputs and that each experiences the same rate of technological progress as governed by  $A_t$  in equation (1). Productivity growth does not bias the economy towards either one of the two productive sectors, although sectoral productive capabilities may differ inherently according to the sectorial coefficients ( $A_{1f} \neq A_{1h}$ ).

Each agent in the economy begins at time  $t = 0$  with a homogeneous endowment of land which they keep for the rest of time and an equal amount capital  $k_0$  which evolves over time according to (2). Capital, labor and land resources are assumed to be freely mobile. At each date, productive resources are allocated across sectors towards their highest return. Implicitly, this assumes that the allocation of the environmental resource between production and its natural state is a reversible decision. In equilibrium, resource mobility has the effect of equating returns across activities, so that

$$r_t = r_{f,t} = r_{h,t} = p_{f,t}(\alpha - \eta) \frac{y_{f,t}}{k_{f,t}} = p_{h,t} \alpha \frac{y_{h,t}}{k_{h,t}} \text{ for } t \geq 0, \quad (5)$$

$$w_t = w_{j,t} = p_{j,t}(1 - \alpha) \frac{y_{j,t}}{l_{j,t}} \text{ for } j = f, h, \quad t \geq 0, \quad (6)$$

and

$$v_t = p_{f,t} \eta \frac{y_{f,t}}{f_{f,t}} \text{ for } t \geq 0, \quad (7)$$

where  $r_t$ ,  $w_t$  and  $v_t$  denote the market capital rental rate, market wage rate and market environmental resource rental rate at time  $t$  and  $p_{j,t}$  denotes the price of output in sector  $j$  at time  $t$ . We choose good  $h$  as the numeraire, so that  $p_{h,t} = 1$ .

### 2.3 The consumer sector

The private sector consists of many identical, infinitely-lived agents who provide capital, labor and land to the corporate sector in exchange for income used to purchase consumption goods. The representative agent acquires instantaneous utility at time  $t$  through the consumption of resource extractive goods,  $c_{f,t}$ , and non-extractive goods  $c_{h,t}$ . Consumers also obtain utility from the environmental resource through “visitation”. The visitation value of the environmental resource, denoted by  $a_t$  for amenity value, is an increasing function of the quantity of the environmental resource set aside from production,  $a_t = f - f_{1,t}$ .

Utility is derived from the consumption of both extractive goods and nonextractive goods, from the time devoted to leisure, and from the amount of the environmental resource devoted to amenity production. The utility function takes the CES



form,

$$\sum_{t=0}^{\infty} \beta^t \left( \varepsilon_1 c_t^\theta + (1 - \varepsilon_1) b_t^\theta \right)^{\frac{1}{\theta}}, \quad (8)$$

where  $0 < \beta < 1$  is the discount factor,

$$c_t = \left( \varepsilon_2 c_{f,t}^\psi + (1 - \varepsilon_2) c_{h,t}^\psi \right)^{\frac{1}{\psi}}, \quad (9)$$

is a composite consumption good, and

$$b_t = \left( \varepsilon_3 a_t^\sigma + (1 - \varepsilon_3) [A_t(1 - l_t)]^\sigma \right)^{\frac{1}{\sigma}}, \quad (10)$$

is an amenity-leisure composite. The parameters are restricted according to  $\theta \leq 1$ ,  $\psi \leq 1$  and  $\sigma \leq 1$ . These parameters are related to the elasticity of substitution between the consumption aggregate  $c_t$  and the amenity-leisure aggregate  $b_t$ ,  $1/(1 - \theta)$ , the elasticity of substitution between the consumption levels  $c_{h,t}$  and  $c_{f,t}$ ,  $1/(1 - \psi)$ , and the elasticity of substitution between the amenity  $a_t$  and leisure  $A_t(1 - l_t)$ ,  $1/(1 - \sigma)$ .<sup>6</sup> Positive values of  $\phi$ ,  $\psi$  and  $\sigma$  arise for substitutes and negative values arise for complementary goods.

Amenity value arising from the quantity of the environmental resource set aside from production is allowed to have common property aspects, which reflects potentially incomplete property rights. Specifically,

$$a_t = f_{2,t}^\gamma \bar{f}_{2,t}^{1-\gamma}, \quad (11)$$

where  $\bar{f}_{2,t}$  is the uninternalized value of the resource amenity, and  $0 < \gamma \leq 1$  represents the extent of property rights. When  $\gamma \rightarrow 0$ , the amount of the environmental resource an individual sets aside from production contributes negligibly to aggregate utility (a common property resource), whereas when  $\gamma = 1$  each individual is

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<sup>6</sup>Note,  $l_t \neq L_t = 1$ , since  $l_t$  is used to denote the portion of the total time endowment ( $L_t$ ) spent working and  $1 - l_t$  is used to denote the total time spent in leisure. Notice also that the value of time in leisure grows at the same rate as the value of time in production. The implicit assumption is that technological advances that improve the production of goods (e.g., “fish finders”) lead to proportional advances in home production techniques (e.g., recreational fishing). If technology does not proportionally stimulate production and leisure, the consumer’s leisure time allocation would be unstable and balanced growth would not be possible.

fully compensated for the amenity value associated with ownership of an undisturbed environmental resource. It is assumed in equilibrium that  $\bar{f}_{2,t} = f_{2,t}$ .

Consumers face a budget constraint given by

$$\sum_{j=f,h} (1 + \tau_{j,t}) p_{j,t} c_{j,t} = \sum_{j=f,h} r_{j,t} k_{j,t} + \sum_{j=f,h} w_{j,t} l_{j,t} + v_t f_{1,t}, \quad \text{for } t \geq 0, \quad (12)$$

where the right hand side indicates income earned at time  $t$  through the provision of inputs to the corporate sector and the left hand side indicates expenditures at time  $t$  on consumption goods. Consumers take the consumption tax on good  $j$  at time  $t$ ,  $\tau_{j,t}$ , as exogenously chosen by the government.

## 2.4 The government sector

The government engages in two types of activities. First, the government purchases goods from sector  $j$  at time  $t$  at a level denoted by  $g_{j,t} \geq 0$ . These purchases are assumed to be nonproductive. Second, the government chooses a tax policy which serves both to raise revenue for financing its expenditures and to correct the externality in amenity formulation that arises when  $\gamma \neq 1$ . The tax instruments available for this purpose consist of a consumption tax on each of the two goods, denoted by  $\tau_{j,t}$  for  $j = f, h$ , where a negative value for a tax may be interpreted as a subsidy. We assume the government runs a balanced budget at each date given by,

$$\sum_{j=f,h} p_{j,t} g_{j,t} = \sum_{j=f,h} \tau_{j,t} p_{j,t} c_{j,t}, \quad \text{for } t \geq 0. \quad (13)$$

It is assumed that nonproductive government spending grows in proportion to the total level of output according to  $\sum_{j=f,h} p_{j,t} g_{j,t} = \phi \sum_{j=f,h} p_{j,t} y_{j,t}$ , where  $\phi \geq 0$ . Under this restriction, spending remains a constant share of output over time in which the share,  $\phi$ , can be thought of as being decided in an exogenous political process. (It is also possible to interpret  $\phi$  as administrative loss.) If  $\phi = 0$ , then taxes serve a purely corrective role.

Finally, we assume the government contributes at least a minimum level of expenditures in each of the two sectors. This is imposed by assuming that there are

minimum values for  $\phi_{f,t}$  and  $\phi_{h,t}$  where  $\phi_{j,t} = \frac{g_{j,t}}{y_{j,t}}$  for  $j = f, h$ . We denote these minimums by  $\phi_f^M$  and  $\phi_h^M$  and formally write the constraint as

$$\phi_{j,t} \geq \phi_j^M \quad \text{for } j = f, h \quad \text{and } t \geq 0. \quad (14)$$

A natural minimum value is  $\phi_f^M = \phi_h^M = 0$ , as values less than zero would allow the government to manufacture goods without a production function.

## 2.5 Competitive equilibrium

The competitive equilibrium is defined by several types of market clearance conditions. First, input market clearance requires that capital across sectors adds up to the total capital stock,

$$k_t = \sum_{j=f,h} k_{j,t} \quad \text{for } t \geq 0, \quad (15)$$

that the total time allocation adds up to the total time available,

$$l_t = \sum_{j=f,h} l_{j,t} \quad \text{for } t \geq 0, \quad (16)$$

and that the resource allocation adds up to  $f$ ,

$$f = \sum_{i=1,2} f_{i,t} \quad \text{for } t \geq 0. \quad (17)$$

Second, goods market clearance requires that  $c_{j,t} + g_{j,t} = y_{j,t}$  for  $j = f, h$  and  $t \geq 0$ .

A competitive equilibrium is defined as follows. Given a sequence of technology and capital stocks  $\{A_t, k_t : t \geq 0\}$  and the total amount of the resource  $f$ , allocations  $\{c_{f,t}, c_{h,t}, f_{1,t}, \bar{f}_{2,t}, f_{2,t}, l_t, l_{f,t}, l_{h,t}, k_{f,t}, k_{h,t}, y_{f,t}, y_{h,t} : t \geq 0\}$ , prices  $\{p_{f,t}, p_{h,t}, r_t, w_t, v_t : t \geq 0\}$  and policies  $\{\tau_{f,t}, \tau_{h,t}, g_{f,t}, g_{h,t}, \phi_{f,t}, \phi_{h,t} : t \geq 0\}$  constitute a competitive equilibrium if the following conditions are satisfied:

- (i)  $\{c_{f,t}, c_{h,t}, l_t, l_{f,t}, l_{h,t}, k_{f,t}, k_{h,t}, f_{1,t}, f_{2,t} : t \geq 0\}$  maximizes the consumer objective function subject to the temporal sequence of budget constraints given the average level of resource amenities  $\{\bar{f}_{2,t} : t \geq 0\}$ , prices  $\{p_{f,t}, p_{h,t}, r_t, w_t, v_t : t \geq 0\}$  and taxes  $\{\tau_{f,t}, \tau_{h,t} : t \geq 0\}$ ;

- (ii) At each date  $t$ ,  $\{k_{f,t}, l_{f,t}, f_{f,t}, y_{f,t}\}$  maximizes firm profits in the resource extractive sector given prices  $\{p_{f,t}, r_t, w_t, v_t\}$  and  $\{k_{h,t}, l_{h,t}, y_{h,t}\}$  maximizes firm profits in the nonextractive sector given prices  $\{p_{h,t}, r_t, w_t\}$ ;
- (iii) At each date  $t$ , the average level of resource amenities in equilibrium is  $\bar{f}_{2,t} = f_{2,t}$  and therefore  $a_t = f_{2,t}$ ;
- (iv) The government budget constraint (13) and minimum expenditure constraint (14) hold;
- (v) At each date  $t$ , all markets clear.

## 2.6 The Ramsey Planner

Optimal policy in all cases is described by the actions of a Ramsey planner who wishes to maximize consumer utility subject to the various conditions summarized in the definition of a competitive equilibrium. Because the capital stock grows exogenously and the government runs a balanced budget, the Ramsey planning problem in itself has no intertemporal consequences. The optimal second-best tax problem therefore reduces to a sequence of temporal optimization problems. Despite this simplification, the utility function is still sufficiently complicated that closed form expressions do not arise for the demand equations, and, as a consequence, a primal approach to the Ramsey problem simplifies the numerical calculations. The primal approach requires an implementability constraint, which is provided in the following proposition.

**Proposition 1:** A competitive equilibrium allocation must satisfy

$$\sum_{j=f,h} U_{c_{j,t}}(\cdot) c_{j,t} + U_{l_t}(\cdot) \left[ \frac{\alpha}{1-\alpha} \left( \frac{l_{h,t}}{k_{h,t}} \right) k_t + l_t \right] - U_{a_t}(\cdot) \gamma f_{1,t} = 0, \quad \text{for } t \geq 0, \quad (18)$$

$$\frac{\alpha}{\alpha - \eta} \frac{k_{f,t}}{l_{f,t}} = \frac{k_{h,t}}{l_{h,t}}, \quad (19)$$

and

$$\gamma(1-\alpha) U_{a_t} f_{1,t} + \eta U_{l_t} l_{f,t} = 0, \quad (20)$$

at each date  $t$ , where  $U_{c_f,t}(\cdot)$ ,  $U_{c_h,t}(\cdot)$ ,  $U_{l_t}(\cdot)$ ,  $U_{a_t}(\cdot)$  are specified in the appendix. Given allocations which satisfy (18), (19) and (20), it is possible to construct policies and prices which, together with these allocations, generate a competitive equilibrium. In the exercises described in the next section, the numerical solutions were calculated based upon this approach.

### 3 Policy Implications

This section presents the optimal policy results. These results are found by using numerical methods to investigate the decisions made by a Ramsey planner. The results are organized into three subsections in order to provide greater clarity. The first subsection describes a baseline calibration of the model. The second subsection demonstrates that the minimum spending level is always binding in the extractive sector whenever an externality exists. This result is demonstrated by plotting the utility surface under different parameterizations of the model. The third subsection discusses the dynamic aspects of fiscal policy in a growing economy. Here results of the Ramsey planning outcome are plotted for an exogenously growing capital stock under various degrees of substitution between arguments in the utility function.

#### 3.1 Baseline Calibration

For the baseline calibration of the model, both goods-producing sectors of the economy are assumed to be equally productive,  $A_{1f} = A_{1h} = 1$ . Next, since roughly sixty percent of output in industrial nations is used to pay labor inputs, we select  $\alpha = 0.4$ , a value widely used in macroeconomic studies. The parameter  $\eta$ , the output elasticity of the environmental resource in extractive goods production, is not a widely calibrated value in other studies, and we consider a range of values in our analysis. This range is given by  $[0, .2]$ , with the upper limit chosen to equal the implied lower limit for the capital elasticity in the extractive sector (i.e.  $.2 = \alpha - \eta$ ). For the baseline calibration, a value of  $\eta = 0.1$  is employed.

One of the main objectives of this study is to investigate how optimal policy varies

with the degree to which utility arguments are substitutable or complementary. For the baseline case, all of these parameters were set to  $\psi = \theta = \sigma = -0.1$ , a value close to the natural log preference value of 0. We also explore a range of values for the degree to which agents internalize property rights for the environmental amenity. In particular, we investigate  $\gamma \in (0, 1]$  and use the midpoint value of  $\gamma = 0.5$  for the baseline case. The weights in the utility function were chosen in part to obtain a labor supply equal to one third of the time endowment, a widely used calibration point. With this objective,  $\varepsilon_2 = .5$  was selected to place equal weights on resource extractive and non-extractive goods, while  $\varepsilon_1 = .4$  and  $\varepsilon_3 = .1$  ensured that the labor supply allocation remained at one third of the time endowment.

For government consumption, we chose a government consumption rate of twenty percent of output,  $\phi = .2$ , the approximate value observed in the U.S. Values for  $\phi_f^M$  and  $\phi_h^M$  are allowed to vary between 0 and .2, and the minimum spending level in each sector is set at .05 in the baseline calibration. Finally, we set  $f = 100$ ,  $k_0 = 5$  and  $A = 1$ . This allows us to consider situations where natural resources are relatively plentiful as well as situations in which they are in less abundance relative to the capital stock. We refer to situations in which they are relatively plentiful as less developed periods and situations in which they are less abundant as developed periods.

### 3.2 Optimal Government Spending on Extractive Goods

When an externality exists in allocating the environmental resource, the minimum spending limit on extractive goods,  $\phi_f^M$ , is generally binding in the Ramsey planning outcome.<sup>7</sup> This is because, when  $\gamma < 1$ , the social valuation of the amenity is larger than the private valuation, so that agents excessively devote the resource to production in the market. To correct this distortion, the policy maker raises the tax on resource extractive goods and reduces its spending level in the sector. Since

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<sup>7</sup>The situations when  $\phi_f^M$  is not binding occur when  $\gamma$  is near 1 and  $\phi_f^M$  is significantly larger than  $\phi$ . Because this occurs when  $\phi_f^M > \phi$ , we don't consider this to be realistic enough to elaborate on here. Further intuition can be found in Cassou, Gorostiaga and Gutiérrez (2006).

total government spending across both sectors is fixed, shifting spending away from extractive goods is less distortionary than raising the tax on extractive goods, so that the boundary,  $\phi_f^M$ , is always reached.<sup>8</sup>

To illustrate this intuition, we plot a feasible set of competitive equilibria over which a Ramsey planner can select fiscal policy. Figure 1 plots utility values for the competitive equilibria that emerge for various spending levels and property rights structures for the case of equal sectoral taxation,  $\tau_{f,t} = \tau_{h,t} = 0.25$ .<sup>9</sup> In the figure, total government spending is held constant at the baseline value of  $\phi = .2$ . However, spending on extractive goods,  $\phi_f$ , ranges from 0 to .4 (and spending on nonextractive goods,  $\phi_h = \phi - \phi_f$ , ranges in an opposing fashion). All other parameters are maintained at the baseline values. With small values of  $\gamma$ , the utility surface slopes steeply upward from the right side of the picture to the left, which indicates that utility is maximized at the boundary where  $\phi_f^M = 0$  occurs. As  $\gamma$  increases the utility surface increases and flattens out. The social and private valuation of the amenity converge as  $\gamma \rightarrow 1$ , and the role for corrective policy disappears. Nonetheless, as  $\gamma$  increases, the slope of the surface maintains an upward slope from right to left for values of  $\phi_f \leq \phi$ , which indicates that utility is maximized for any  $\gamma < 1$  at the  $\phi_f^M$  boundary.

### 3.3 Dynamic optimal tax structure

In this section we carry out two types of analysis which are broken into separate subsections. The first subsection investigates incremental changes from the baseline, one parameter at a time. This analysis provides intuition into how each parameter impacts optimal tax policy. Of primary interest is how the optimal tax rates change for variation in the elasticity of substitution between leisure and resource amenities,  $\sigma$ , and for variation in the elasticity of substitution between resource-extractive goods and non-extractive goods,  $\psi$ . Second, we move multiple parameters away from the

<sup>8</sup>To see that taxes are more distortionary, note that (symmetric) taxes of  $\tau = 0.25$  are needed to provide a government spending level of  $\phi = 0.2$ .

<sup>9</sup>Figure 1 is robust to other tax structures. In particular, for any exogenous tax pair  $(\tau_{f,t}, \tau_{h,t})$ , the utility surface has the same general shape.

baseline values at once and highlight several cases that lead to non-monotonic dynamic tax patterns over time. Identifying circumstances that lead to non-monotonic time paths for the policy variables has important implications for environmental policy, because it may be the case that rising taxes on resource-extractive goods are optimal at early stages of development and falling tax rates are optimal at later stages of development.

### 3.3.1 Baseline variations

In this section we investigate how small increments for five key parameters impact tax policy in a developing economy. We explore various changes in the value of the utility elasticity parameters,  $\sigma$ ,  $\psi$  and  $\theta$ , the degree of property rights,  $\gamma$ , and the importance of the environmental resource in production of the extractive good,  $\eta$ . These results are summarized by the pattern of optimal dynamic tax rates in Figures 2-6.

To understand the policy role of the Ramsey planner, notice that the baseline value of  $\gamma = .5$  implies incomplete property rights over environmental resources devoted to amenity production. Absent policy, amenity values are undersupplied in the economy. Accordingly, the Ramsey planner desires to increase the supply of environmental resources allocated to amenity services and decrease the supply of environmental resources to the production of extractive goods. This corrective role of taxes implies that the Ramsey planner selects a higher tax on the resource extractive good than on the non-extractive good.

Next, consider the dynamic aspects of policy as the economy grows. As economic growth takes place, the capital stock and the effective labor supply increase over time, but the level of the environmental resource remains fixed. Consequently, the value of the environmental resource in extractive production is bid up over time. Balancing this effect is the increase in amenity value of the environmental resource. The optimal allocation of the environmental resource is determined by the interaction of these competing interests. Moreover, because the direct effect of these competing interests



is fully offset by factor price equalization under the optimal tax policy, the dynamic nature of the externality is influenced by the degree to which the alternative uses of the environmental resource are complementary or substitutable with other arguments in the utility function.

Figure 2 plots the optimal dynamic tax profile for different degrees of complementarity between amenity values and leisure,  $\sigma$ . Notice in Figure 2a that when amenity values are more complementary with leisure, the optimal tax on resource extractive goods increases over time, while the opposite occurs when amenity values substitute for leisure. In the case of complementary values, a rise in (effective) leisure over time facilitates consumer demand for amenities, and the amenity value of environmental resources goes up more quickly than the value of environmental resources in extractive production. For any amount of aggregate goods production, consumers desire relatively equal levels of amenities and leisure when these products are highly complementary, and this exacerbates the externality created by incomplete property rights. The planner responds by steering resources away from extractive goods production and towards the production of amenity values by raising the tax on extractive goods. The opposite occurs when leisure and amenity values are substitutes. In this case, increased leisure over time facilitates substitution possibilities between leisure and amenity values, and the production value of resources rises more quickly than resource amenity values. The Ramsey planner responds by guiding environmental resources towards production by lowering the tax on extractive goods over time. (In the case of perfect substitutes, the externality vanishes asymptotically as growth occurs.)

Figure 2b indicates an outcome for the tax on the non-extractive good that mirrors that of the extractive tax. This outcome reflects the planner's need to raise a constant share of output as tax revenue, so that taxes on other goods decline with larger taxes on extractive goods. In the remaining simulations, we consider only the dynamic pattern of taxes on extractive goods and allow the tax on non-extractive goods to remain implicit.

Figure 3 depicts the dynamic optimal tax profile for variations in the elasticity of substitution between consumption of extractive goods and non-extractive goods,  $\psi$ . The optimal tax on extractive goods generally rises over time, and the tax increases at a faster pace when the goods are more substitutable. Under circumstances in which non-extractive goods can be readily substituted for extractive goods in utility, the opportunity cost of allocating environmental resources to amenity production falls as the economy grows, compounding the rate at which extractive goods are oversupplied in the market. This means the amenity externality increases over time, and the Ramsey planner responds by raising taxes on resource extractive goods. When consumption goods are highly complementary, the opposite occurs: taxes on the extractive good fall over time. This is because the utility value of extractive goods rises faster than the amenity value as economic growth increases consumption of non-extractive goods resulting in a greater allocation to amenity and a reduction in the oversupply to the market. This means the planner has less to correct and thus accordingly reduces taxes on extractive goods over time to redirect resources into production.

Figure 4 shows the optimal tax path for incremental changes in the elasticity of substitution between the consumption good aggregate,  $c$ , and the leisure-amenity aggregate,  $b$ . These figures exhibit similar characteristics as those in Figure 3, although the tax on extractive goods is higher in all periods for greater values of the substitution parameter between goods,  $\psi$ , than they are for greater values of  $\theta$ . This is because a higher degree of substitution arising from  $\psi$  values allows the tax to be focused on the resource extractive good while a higher degree of substitution arising from  $\theta$  does not.

Figure 5 depicts the optimal tax path for variations in property rights,  $\gamma$ , and Figure 6 shows the outcome for different output elasticity values of the environmental resource input in production,  $\eta$ . The diagrams are similar in appearance, because they both reflect the extent to which resources are misallocated to production. In Figure 5, the tax on the extractive good falls as  $\gamma$  rises, reflecting the vanishing

amenity externality as property rights become more complete. Similarly, as  $\eta$  decreases in Figure 6, less environmental resources are necessary for the production of extractive goods, reducing the magnitude of the externality in amenity provision.

### 3.3.2 Substitute goods

In this section we show that optimal policy in the extractive goods sector need not follow a monotonic path over time. We demonstrate this with a simulation in which resource extractive goods and non-extractive goods are highly substitutable and vary the substitution possibilities between leisure and amenity values.

Figures 7a, 7b, and 7c illustrate the dynamic outcomes when consumption goods are highly substitutable,  $\psi = 0.7$ , for various parameterizations of  $\sigma$ . Notice in Figure 7a that a reversal in the tax on extractive goods occurs when leisure and amenities are highly substitutable,  $\sigma = .7$ , whereas a reversal in the tax on other goods occurs when  $\sigma$  reflects a high degree of complementarity in Figure 7b. Nonetheless, the amount of the environmental resource devoted to production follows a monotonic time path in all cases in Figure 7c, with an increasing amount of the resource devoted to production over time when leisure and amenities are substitutes and a decreasing amount of the resource devoted to production over time when leisure and amenities are complements.

The intuition for these outcomes are as follows. In early periods of a growing economy resources are relatively plentiful (i.e.,  $f = 100$  and  $k_0 = 5$  in the initial period), and, because extractive goods and non-extractive goods are close substitutes, it is optimal to keep taxes on extractive goods low to take advantage of the relative abundance of environmental resources in this initial “development stage”. To do this, the planner sets taxes on extractive goods initially “low” and then increase them over time as environmental resources become less abundant. If leisure and amenity values are complements, this also facilitates increasing taxes on extractive goods over time, and the effects are reinforcing. However, if leisure and amenity values are highly substitutable, the planner faces a trade-off between using environmental resources

to substitute for scarce capital in production and using environmental resources to substitute for scarce leisure in consumption. In early periods, the tax on extractive goods increases over time to reduce the flow of environmental resources into production, but as effective leisure grows the magnitude of the externality decreases, and the Ramsey planner eventually wishes to reverse course and steer environmental resources back toward goods production. Notice that the inflection point in the extractive goods tax occurs when environmental resources and capital allocated to production are roughly equal (e.g.  $k$  is close to  $f_1$ ). At this point, the capital and effective labor are in greater relative abundance than the environmental resource and utility begins to rise more quickly when the environmental resource is devoted to amenity production.

Next consider the case where there is a reversal in the tax rate on other goods (i.e. the cases where  $\sigma$  is small). Figure 7b shows that the initial development stage characterized by abundant environmental resources ( $f_1 > k$ ) is marked by a falling tax on the non-extractive good over time, followed by an eventual reversal around the point where environmental resources and capital are roughly equally abundant. This is because, when amenity values and leisure are highly complementary, the planner wishes to maintain relatively balanced levels of amenities and leisure in the economy. In the early stages of development when effective leisure is small, the demand for amenities is also small, whereas, as the capital stock and effective labor supply grow, environmental resources can be readily replaced in production by capital and labor and demand for amenities increases with the commensurate rise in effective leisure. To guide this movement of environmental resources from the production of goods to the production of amenities, the planner raises taxes on resource-extractive goods (see Figure 7a), and this allows taxes on non-extractive goods to be temporarily reduced. As environmental resources become relatively less abundant, however, the planner is ultimately forced to increase taxes on non-extractive goods as the share of government activities financed from taxes on extractive goods becomes smaller.

## 4 Conclusion

This paper investigated the dynamic policy decisions of a Ramsey planner who designs policy to allocate an environmental resource between alternative uses in the production of goods and the production of amenity values. The optimal policy is corrective in the sense that property rights over amenity values are incomplete, and the magnitude of the externality component to fiscal policy evolves over time according to the elasticity of substitution in utility among the various utility arguments.

The analysis reveals some qualified statements about the optimal dynamic tax structure. First, the substitution elasticities between environmental resource amenity values and leisure and between extractive goods and non-extractive goods have the most pronounced impact on the dynamic policy profile. The reason for this is that effective leisure grows over time in the economy, which alters the magnitude of the externality involved with amenity production. Under circumstances where amenity value is complementary with leisure, the growth in leisure increases demand for environmental resource amenities, and the optimal policy adjusts by increasing taxes on extractive goods over time to steer resources towards the production of amenity values. Conversely, when amenity values and leisure are substitutes, a growth in effective leisure over time causes consumers to substitute away from amenity values in utility, and the optimal tax profile involves asymptotically declining taxes on extractive goods over time. Under circumstances where extractive goods and non-extractive goods are substitutes, taxes on extractive goods increase over time with the growth of capital allocated to non-extractive production techniques, as consumers are better able to replace extractive goods consumption with the consumption of non-extractive goods over time.

The overall pattern of development of the economy depends in a complex way on the interaction between these elasticity values. For instance, for an environmental resource-abundant economy in an early stage of development, the dynamic pattern of taxes may be non-monotonic. It was shown that when a high degree of substitution

possibilities exist between amenity value and leisure and between resource-extractive and non-extractive consumption goods, taxes on extractive goods may rise over time for a period to capitalize on the potential for extractive goods to substitute for non-extractive goods, but then subsequently fall as the growth in leisure facilitates substitution away from amenity values in utility. Complexity of the dynamic optimal tax problem highlights the need to carefully understand the relationship between environmental values and other goods in consumer utility functions. Understanding this relationship is particularly important for environmental resources that have dual uses. Moreover, these results show that environmental policy need not be generic, but instead should reflect the individual aspect of amenities provided by a particular environmental resource. For instance, forest resources may be characterized by more complete property rights than fishery resources, but also provide a greater degree of complementarity with leisure. Optimal dynamic policies on forest and fishery resources should reflect these differences.

The paper also suggests the need for further study on the nature of technical change. Technical change may be important not just to expand the capability of non-resource-extractive production techniques to substitute for resource-extractive techniques in the economy, but also to facilitate complementarities between resource amenity values and leisure. Television and computers may increase the degree of substitution between leisure and environmental amenities associated with outdoor recreation, so that optimal policy may encourage extractive uses of environmental resources over time. In spatial markets, automobiles and airplanes allow consumers to recreate in more distant locations, which may change the relative degree of complementarity between leisure and nearby environmental resources, for instance open space within an urban region, and between leisure and more distant environmental resources. In growing economies for which effective leisure rises over time, understanding the complex implications of economic growth on the intrinsic value of resource amenities, particularly as this pertains to resources with poorly-defined property rights, is essential for the design of efficient dynamic environmental policies.

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## A Appendix - Proof of Proposition 1

The Lagrangian for the consumer's problem to be written as

$$L(\cdot) = \sum_{t=0}^{\infty} \beta^t \left\{ U(c_{f,t}, c_{h,t}, a_t, l_t) + \lambda_{1,t} \left[ r_{h,t}k_{h,t} + r_{f,t}(k_t - k_{h,t}) + \sum_{j=f,h} w_{j,t}l_{j,t} + v_t f_{1,t} - \sum_{j=f,h} (1 + \tau_{j,t})p_{j,t}c_{j,t} \right] \right\}$$

Solving for the first-order conditions yields the private market equilibrium conditions

$$\frac{\partial L(\cdot)}{\partial c_{j,t}} : U_{c_{j,t}}(\cdot) - \lambda_{1,t}(1 + \tau_{j,t})p_{j,t} = 0, \text{ for } j = f, h, \quad (21)$$

$$\frac{\partial L(\cdot)}{\partial l_{j,t}} : U_{l_t}(\cdot) + \lambda_{1,t}w_{j,t} = 0, \text{ for } j = f, h, \quad (22)$$

$$\frac{\partial L(\cdot)}{\partial f_{1,t}} : -U_{a_t}(\cdot)\gamma(f - f_{1,t})^{\gamma-1} \bar{f}_{2,t}^{1-\gamma} + \lambda_{1,t}v_t = 0, \quad (23)$$

$$\frac{\partial L(\cdot)}{\partial k_{h,t}} : r_{h,t} - r_{f,t} = 0, \quad (24)$$

$$\frac{\partial L(\cdot)}{\partial \lambda_{1,t}} : \sum_{j=f,h} r_{j,t}k_{j,t} + \sum_{j=f,h} w_{j,t}l_{j,t} + v_t f_{1,t} - \sum_{j=f,h} (1 + \tau_{j,t})p_{j,t}c_{j,t} = 0. \quad (25)$$

Substituting (21), (22), (23) and (24) into (25) and making use of (5), (6), (11), (15), (16) and (17) gives (18). Next note that (19) follows from (5) and (6). Finally, note that (20) follows from (6), (7), (11), (22) and (23).

To establish the reverse implication begin by defining  $p_{h,t} = 1$  and  $p_{f,t} = \frac{\alpha}{\alpha-\eta} \left( \frac{k_{f,t}}{k_{h,t}} \right) \left( \frac{y_{h,t}}{y_{f,t}} \right)$ . Use these prices with the allocations to define capital rental rates, wages and resource rental rates according to (5)-(7) (Note that condition (19) ensures that the rental rates are equal across industries.) Next define taxes by  $(1 + \tau_{j,t}) = \frac{-U_{c_{j,t}}w_{j,t}}{U_{l_t}p_{j,t}}$  for  $j = f, h$ . Defining  $\lambda_{1,t} = \frac{-U_{l_t}(\cdot)}{w_{j,t}}$ , thus ensuring (22). The definition of  $(1 + \tau_{j,t})$  ensures (21) holds and (20) ensures (23) holds. Also note that by construction  $r_{h,t} = r_{f,t}$  and thus (24) holds. So it only remains to demonstrate that (25) holds. But this can be shown by substituting these various conditions in to (18) and essentially working backwards to get (25).

Figure 1









