Strategic Behavior and Collusion: An Application to the Spanish Electricity Market

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Abstract

The paper has two major contributions to the theory of repeated games. First, we build a supergame oligopoly model where firms compete in supply functions, we show how collusion sustainability is affected by the presence of a convex cost function, the magnitude of both the slope of demand market, and the number of rivals. Then, we compare the results with those of the traditional Cournot reversion under the same structural characteristics. We find how depending on the number of firms and the slope of the linear demand, collusion sustainability is easier under supply function than under Cournot competition. The conclusions of the models are simulated with data from the Spanish wholesale electricity market to predict lower bounds of the discount factors.

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1 Introduction

It is well-known that collusive practices are prosecuted, because of the misallocation of resources that results from the output restriction and higher prices. Therefore, Antitrust laws explicitly prohibit coordination among firms to reduce output, increase price, prevent entry, exclude actual competitors, and other practices that weakens competition.

Stigler's classic (1964) paper already remarked that, the study of collusion in a static oligopoly model was very limited. Since his pioneer work, game theory has been widely used. Friedman (1971) proved that an oligopolistic supergame is a better approach to study collusion sustainability. Firms involved in those practices may find profitable to deviate if the subsequent punishment is sufficiently small; that is, when the discount factor is close to one, then tacit collusion can support any Pareto Optimal outcome. Furthermore, the industrial organization literature aims to identify conditions that facilitate collusion, among them: product homogeneity, differences among purchasers, fewness of sellers, high barriers to entry, vertical integration, and a low elasticity of demand. Thus, ceteris paribus, we should expect collusion to occur more often when these conditions are met[1].

The supply function strategic oligopoly model has been widely developed by Klemperer and Meyer (1989). This approach relates the quantity a firm sells to the price the market will bear: such a supply function allows the firm to adapt better to changing conditions than does a traditional Bertrand competition (a commitment to a fixed price) or Cournot competition (a commitment to a fixed quantity). The Spanish Pool market resembles some of those characteristics: every day and for each hour, generators submit a supply schedule. Some applications of the supply function approach to electricity markets can be found in Green (1992, 1996), Green and Newbery (1991) and Powell (1994) for the British case and in Baldick, and Grant and Khan (2000) for the California case.

The paper has a first contribution to the theory of repeated games. We build a supergame-theoretic model to show how a convex cost function, the demand conditions, and the number of generators affect the sustainability. of tacit collusion in those markets where firms compete offering supply schedules. Various authors have remarked that one important factor that may affect the sustainability. of collusion is the elasticity of demand. Jacquemin and Slade (1989) note that "the elasticity of the individual firms demand curve is an important factor' affecting the incentive to cheat by cutting price and increasing sales". Collie (2003) develop a model using a constant elasticity of demand model to explain how elasticity affects the sustainability of collusion in a Cournot framework.

We consider supply function competition for two main reasons. First, theoretical literature on repeated games lacks this kind of competition structure to model collusion sustainability. Second, we are going to test the predictions

[1]Even the Spanish antitrust authority, Tribunal de Defensa de la Competencia, also considers these factors in the merger guidelines and dominant position abuse cases.

obtained from the theoretical model using data from the Spanish wholesale electricity market, that, as we mentioned, resembles characteristics of that type of competition.

The paper also compares the model under SFC with the traditional Cournot model in which firms compete offering a single quantity-bid stretch. Such comparison is done to explain two facts. First, these two approaches are mostly used to study electricity markets[2]. For example, Borenstein and Bushnell (1997) use a Cournot simulation model to evaluate the performance of the California market. Ramos, Ventosa and Rivier (1998) also use a production-cost model under Cournot competition, that is relevant in the new regulatory framework of the electricity market, to provide a detailed representation of the electric system. The supply fuction approach is attractive because it offers a more realistic view of electricity markets; at least offers the possibility of developing some insight into the bidding behaviour of firms. Second, if we use the slope of demand as aproximation of the elasticity of demand, we find that the conclusions about collusion sustainability significantly differ from one model to the other.

We have two major theoretical findings. First, as we expected, both strategic approaches are sensitive to the slope of demand parameter. Our model predicts that when the slope of the linear demand is low enough, collusion is easier to sustain under SFC reversion than under Cournot competition reversion.

The paper is organized as follows. Section 2 is the set up of the stage game with firms competing in supply functions. We do not consider strategic behavior on the demand side. The assumption is not so restrictive, since in the Spanish wholesale electricity market, distributors are vertically related to generators[3]. Section 3 solves the supergame and get the predictions. Section 4 makes a comparison between the incentives to deviate from collusion under SFC reversion and under Cournot competition reversion. Section 5 provides empirical evidence using data from the Spanish wholesale electricity market, covering the period May 2001 to December 2003. We report estimates of the slope of demand, the elasticity of demand around the system marginal price, and the arch-elasticity of demand, under a linear specification, then we simulate values of the discount factor for the two model specifications. Section 6 concludes and gives some policy recommendations.

2 The Stage-Game

We consider a market with n identical and independent generators. Firms face both the cost of generation and the cost of transmission. We do not consider the problem of entry of new generators. The transmission network has a fixed capacity $\overline{K}$, then each firm gets a k_i, the transmission quota, for the generation and they pay the corresponding fee for access to the grid, so that $\overline{K} = \sum_{i=1}^{n} k_i$.Therefore, suppose that each firm minimizes the variable cost, for

[2] Borenstein, Bushnell, Khan and Stoft (1995) provide a general classification of the different markets and competitive equilibria in the electricity industry.

[3] See Machado and Kühn (2003) and Gutiérrez-Hita (2004)

simplicity just the use of input x at cost per unit r,

$$\min \; rx_i$$
$$s.a: \; q_i = k_i\sqrt{x_i}$$

Then $x_i = (q_i/k_i)^2$ and $C(q_i) = (r/k_i^2)q_i^2$. Without loss of generality, we can normalize $r/k_i^2 = c/2$ (we abstract away from transmission congestions). Thus the final specification is,

$$C_i(q_i) = \frac{c}{2}q_i^2$$

The cost structure makes also explicit the fact that there are capacity constraints in electricity generation and transmission, in contrast with most of the existing models, that assume constant marginal cost, but it does not address the problem of star-up costs because the cost function would be non-convex for some generation levels, and equilibrium may not be defined. The strategy for each generator is to offer a supply function defined as follows,

$$S_i(p) = \beta_i p, \qquad i = 1, .., n$$

with slope β_i, which is indeed the strategic variable of each firm i. Observe how we impose a supply function that starts at the origin. Imposing a more general linear form like $S_i(p) = A_i + \beta_i p$ yields to the same conclusions since in equilibrium $A_i = 0$ (See Greene, 1994, for a discussion on the solution to first order linear differential equations). Therefore the total supply of electricity at the wholesale market is given by, $S(p) = \sum_{i=1}^{n} \beta_i p$.

We consider a linear demand function which is the sum of the demand bids of the former vertically related (before liberalization) distributors and independent retailers,

$$D(p) = s - \alpha p$$

The demand has a non-zero x-intercept, s, that measures the size of the market, and a constant negative slope, α[4]. Firms simultaneously choose a supply function to cover the market demand. There is a unique price at which market demand equals total supply[5]

$$\sum_{i=1}^{n} \beta_i p = s - \alpha p$$

Solving for p we can define the equilibrium wholesale price in terms of the vector β as,

$$p(\beta) = \frac{s}{\alpha + \sum_{i=1}^{n} \beta_i}, \quad \text{where } \beta = (\beta_1, ..., \beta_i, ..., \beta_n)$$

[4]It is easy to show the positive relation between elasticity of demand, ϵ, and α under a linear demand specification.

[5]Note that, given our assumptions this point is unique. Market demand is linear and downward sloping in p. Supply functions for each firm i is upward sloping in p so, $\sum_{i=1}^{n} S_i(p)$ is upward sloping as well. Then, these two functions match only once.

Therefore the profit function also depends on β. Each firm i maximizes profits choosing β_i

$$\max_{\beta_i} \pi_i(\beta) = \beta_i p(\beta) - C[\beta_i p(\beta)]$$

The first order condition $\partial \pi_i(\beta_i, \beta_{-i})/\partial \beta_i$ is,

$$\left([p(\beta)]^2 + \beta_i \frac{\partial [p(\beta)]^2}{\partial \beta_i} \right) \left(1 - \frac{\beta_i}{2} \right) - \frac{1}{2} \left(\beta_i [p(\beta)]^2 \right) = 0, \qquad i = 1, 2..., n \quad (1)$$

Solving for the system of first order conditions, we obtain the equilibrium value of the slope of the supply schedule,

$$\beta^{SF}(n, c, \alpha) = \frac{(n-2) - c\alpha + \phi}{2c(n-1)} \quad (2)$$

$$\text{where } \phi = \sqrt{(c\alpha + n)^2 - 4(n-1)}$$

We compare β^{SF} with the slope of the supply function under perfect competition (PC) where price equals marginal cost, and this is our measure of market power and the strategic interaction effect among the n generators in the market,

$$\beta^{PC} = \frac{1}{c}$$

Clearly, as we expected, the inverse of the slope of the supply function under oligopolistic competition is smaller than the inverse of the slope of the supply function under perfect competition, for every $n \geq 1$. Increasing the number of firms enhances competition among participants, therefore individual supply function falls closer to marginal cost.

Lemma 1 *In equilibrium, as β^{SF} becomes smaller, the profits of the firms are larger; that is,*

$$\frac{\partial \pi(\beta^{SF})}{\partial \beta^{SF}} < 0$$

and the value of β^{SF} depend of the number of the firms, (i) if $n \to \infty$ then $\beta^{SF} \to \beta^{PC}$, and (ii) if $n = 1$ then $\beta^{SF} \to \beta^{M}$, where β^{M} is the optimal strategy under monopoly outcome.

Proof. See appendix 2. ■

We summarize the properties of the supply function to changes in the other structural parameters (α, c); thus ceteris paribus:

Property 1 More price-response consumers, shifts up the supply function,

$$\frac{\partial \beta^{SF}}{\partial \alpha} = \frac{1}{2(n-1)} \left[\frac{n + c\alpha}{\phi} - 1 \right] > 0$$

If the consumers are more price-sensitive, given the size of the market, then non-competitive firms compete for the existing customers by bidding lower every amount of output.

Property 2 Increases in the cost of production parameters shifts up the supply function

$$\frac{\partial \beta^{SF}}{\partial c} = \frac{-2\alpha^2}{\left[(n-2)^2 + cn\alpha\right]\phi - (n-2)\phi^2} < 0$$

An increase in the cost of production, due to for example more expensive inputs or transmission access, will cause an increase in the slope of the oligopolistic supply function.

We report now the equilibrium price, quantities and profits for each firm,

$$\begin{aligned} q^{SF} &= [2(2n+c\alpha)]^{-1}[2+n+c\alpha-\phi]ns \\ p^{SF} &= ns\left[2c\alpha - n^2 + n(2-c\alpha+\phi)\right][2\alpha(2n+c\alpha)]^{-1} \\ \pi^{SF} &= cs^2[(n+c\alpha)^2 + (n+c\alpha)\,\phi - 2n]^{-1} \end{aligned}$$

We can also find the effect of changes in the structural parameters on the equilibrium profits.

Property 1 More price-response consumers decrease profits,

$$\frac{\partial \pi^{SF}}{\partial \alpha} = -\frac{(cs)^2[n+c\alpha+\phi]^2}{\phi[(n+c\alpha)\phi + (n+c\alpha)^2 - 2n]^2} < 0$$

Property 2 Increases in the cost of production decreases profits

$$\frac{\partial \pi^{SF}}{\partial c} = \frac{s^2\left[3n^3 - (c\alpha)^3 + n^2(7c\alpha-4) + n(5c^2\alpha^2+4) - (n(3n+2) - c\alpha(4n+c\alpha))\phi\right]}{4\phi[2n+c\alpha]^2} < 0$$

Recall that if consumers become more price-sensitives, then firms respond by bidding lower, but it is not compensated by the reduction in the total consumer surplus available for the firms, therefore the total effect is a reduction in the profits. Increasing cost as well as more intense competition, without changing consumer preferences, clearly shrink profits.

3 The Supergame

We seek conditions for the sustainability of the most collusive outcome in the infinitely repeated game. In the supergame a strategy must specify what action plays each firm i at each period t, so a strategy is defined by β_{it}. We assume generators play trigger strategies; all the firms colude (β^*) as long as there has been cooperation in the past (included herself), otherwise the punishment is to revert to supply function competition one period after the violation has been detected (β^{SF}),

$$\beta_{i1} = \beta_i^*$$

$$\beta_{it} = \begin{array}{ccc} \beta^* & if & \beta_{i\rho} = \beta^*, \quad for\ \rho < t \\ \beta^{SF} & if & \beta_{i\rho} \neq \beta^*, \quad for\ \rho < t \end{array} \tag{3}$$

$$i = 1, 2, ..., n; \qquad t, \rho = 2, ..., \infty$$

where β^* is the optimal collusion strategy for each firm. We suppose that all the firms reach a collusive agreement which depend of the each supply function per firm[6], $\Pi^*(\beta) = \sum_{i=1}^{n} \pi_i(\beta_i, \beta_{-i})$. Therefore, the cartel put in the market a joint supply function, which is the sum of the n (identical) individual supply functions. Therefore the problem can be written as,

$$\max_{\{\beta_i\}_{i=1}^{n}} \quad \Pi^*(\beta)$$

which yields a system of n first order conditions of the form,

$$\left[[p(\beta)]^2 + \beta_i \frac{\partial[p(\beta)]^2}{\partial\beta_i}\right]\left(1 - \frac{\beta_i}{2}\right) - \frac{\beta_i}{2}[p(\beta)]^2 + \sum_{j \neq i}^{n}\left(\beta_j - \frac{\beta_j^2}{2}\right)\frac{\partial[p(\beta)]^2}{\partial\beta_i} = 0 \tag{4}$$

$$\text{for } i = 1, 2, .., n$$

We get the optimal symmetric strategy for the cartel participants,

$$\beta^* = \frac{\alpha}{n + c\alpha}$$

We summarize the properties of the equilibrium slope of the supply function under collusion.

Property 1 Increases in the slope of the demand function shifts up the slope of the colluding supply function

$$\frac{\partial\beta^*}{\partial\alpha} = \frac{n}{(n + c\alpha)^2} > 0$$

Property 2 Increases in the cost of production parameters shifts down the supply function

$$\frac{\partial\beta^*}{\partial c} = \frac{-\alpha^2}{(n + c\alpha)^2} < 0$$

Property 3 Increasing the number of firms in generation reduces quantity bid for every price

$$\frac{\partial\beta^*}{\partial n} = \frac{-\alpha}{(n + c\alpha)^2} < 0$$

Increasing the number of firms in the market also increases the number of the participants in the cartel agreement, therefore individual supply function shift further from the marginal cost since the profit maximizing level of output does increase in the same proportion.

[6] There are no incentives to form partial cartels, that is with $k < n$ number of firms, because the potential cartels would not be internally stable

As we expected, the profit maximizing strategy restricts output, for every price, more than the oligopolistic level, therefore we get that

$$\beta^* < \beta^{SF} < \beta^{PC}$$

The equilibrium price, quantity, and profits for each firm are given by,

$$\begin{aligned} p^* &= \frac{s(c\alpha + n)}{\alpha(c\alpha + 2n)} \\ q^* &= \frac{s}{c\alpha + 2n} \\ \pi^* &= \frac{s^2}{2\alpha\left(c\alpha + 2n\right)} \end{aligned}$$

The profit function for the colluding firms behaves in the same way to changes in the structural parameters as the oligopolistic competition profit function, namely it is decreasing when the slope of the demand function increases, decresing to increases in the cost function, and decreasing to the number of colluding firms.

We look for values of the discount factor δ, such that the strategy is a Subgame Perfect Nash Equilibria (SPNE) for the repeated game. A path is sustainable by a SPNE if and only if it is sustainable by the penal code specified (i.e. the infinite reversion to the symmetric Nash equilibrium). At any stage $t > 1$, the choice of a supply schedule depends on the previous actions of the firms. We compare the discounted value of the stream of profits of collusion the profits of deviation plus the infinite sequence of discounted profits under oligopolistic supply function competition. Profits from deviation accrue when, under the trigger strategies, a firm finds, at a given stage, more profitable to violate the agreement that sticking to it, despite of the reversion to competition in subsequent periods. A firm by cheating puts in the market an individual supply function which is different form the agreed upon one in order to get extra profits. Let us choose i as the deviator, the rest of the firms believe that firm i maintains collusion and stick to it. Call β_i^D the optimal strategy obtained from $\arg\max \pi_i(\beta_i, \beta_{-i}^*)$, where β_{-i}^* is an $n-1$ vector that specifies that the rest of the firms remain under collusive supply functions. Firm i solves the following problem,

$$\max_{\beta_i} \quad \beta[p(\beta_i, \beta_{-i}^*)]^2 \left[1 - \tfrac{1}{2}\beta_i^*\right]$$

The first order condition for optimization is,

$$\left([p\left(\beta_i^D, \beta_{-i}^*\right)]^2 + \beta\frac{\partial[p(\beta_i^D, \beta_{-i}^*]^2}{\partial\beta_i^D}\right)\left(1 - \frac{\beta_i^D}{2}\right) - \frac{1}{2}\left(\beta[p(\beta_i^D, \beta_{-i}^*]^2\right) = 0 \quad (5)$$

Solving the equation we obtain the slope of the supply function for the deviator,

$$\beta^D = \frac{[2n + c\alpha - 1]\,\alpha}{c^2\alpha^2 + 2c\alpha n + n}$$

As we expect it holds that the slope of the supply function of the deviator is between the collusion and the oligopolistic competition ones, $\beta_i^* < \beta_i^D < \beta_i^{SF}$. Finally, the equilibrium price, quantities, and profits are,

$$\begin{aligned}
p^D &= [s\left(n + 2nc\alpha + c^2\alpha^2\right)][2n^2\alpha + 3nc\alpha^2 + c^2\alpha^3]^{-1} \\
q^D &= [ns\,(2n + c\alpha - 1)][2n^2 + 3nc\alpha + c^2\alpha^2]^{-1} \\
\pi^D &= \frac{s^2\,(n + c\alpha)^2}{2\alpha\,(c\alpha + 1)\,(c\alpha + 2n - 1)\,(2n + c\alpha)}
\end{aligned}$$

It holds that the highest possible profits are for the stage game when all the rivals collude and firm i deviates, that is,

$$\pi^D > \pi^* > \pi^{SF} > \pi^{PC}$$

The necessary and sufficient condition for the strategy profile to be SPNE of the infinitely repeated game is,

$$\frac{1}{1-\delta}\pi^* \geqslant \pi^D + \frac{\delta}{1-\delta}\pi^{SF} \tag{6}$$

equation [6] holds if,

$$\delta \geqslant \frac{\pi^D - \pi^*}{\pi^D - \pi^{SF}} \tag{7}$$

let us call $\underline{\delta}$ the minimum value for the discount factor such that collusion is sustainable. We characterize $\underline{\delta}$ in terms of the structural parameters in the following proposition:

Proposition 1 *In the infinitely repeated game, when firms compete in supply functions , collusion is sustainable if $\delta \geq \underline{\delta}$, where*

$$\underline{\delta} = \left[\frac{2(n-1)^2}{g(n) + c^2\alpha^2(1 + c^2\alpha^2) - (n + c\alpha)(1 + c\alpha)(c\alpha + 2n - 1)\phi}\right]$$

where $g(n)$ is a polynomial function of grade 3[7]*. $\underline{\delta}$ depends on the structural parameters in the following way:*

1. *$\underline{\delta}$ depends on the slope of the demand function,*

$$\begin{aligned}
\frac{\partial\underline{\delta}}{\partial\alpha} &> 0, \text{ if } n < 4 \\
\frac{\partial\underline{\delta}}{\partial\alpha} &< 0, \text{ if } n \geqslant 4
\end{aligned}$$

for all $c, n \in \mathbb{R}^+$

[7] The explicit form for $g(n)$ is,

$$g(n) = 2n^3(c\alpha + 1) + n^2(5c^2\alpha^2 - 3) + n(2 + 2c\alpha) + c^2\alpha^2$$

2. *$\underline{\delta}$ depends on the cost of generation in the same way of α, $\frac{\partial \delta}{\partial \alpha} > 0$, if $n < 4$ and $\frac{\partial \delta}{\partial \alpha} < 0$, if $n \geqslant 4$.*

3. *$\underline{\delta}$ depend positively on the number of firms, $\frac{\partial \delta}{\partial n} > 0$, for all $\alpha, c \in \mathbb{R}^{+}$.*

Proof. See appendix 2. ■

The first result is not apparently intuitive. If $n < 4$ the model predicts that as the willingness to pay decreases, it also decreases collusion sustainability, that is when α increases, the one-shot deviation gain $(\pi^D - \pi^*)$ increases more than $(\pi^* - \pi^{SF})$, then,

$$\frac{\partial(\pi^D - \pi^*)}{\partial \alpha} > \frac{\delta}{1-\delta} \frac{\partial(\pi^* - \pi^{SF})}{\partial \alpha} \quad \text{if} \quad n < 4$$

so $\underline{\delta}$ must increase for get the inequality to hold. The intuition for this result is the following: when there are few firms in the market an increase in α has a higher marginal impact on deviation profits than in the in the punishment, because supply function competition reversion with a low number of firms implies a lower level of oligopolistic competition. As a result collusion sustainability is more difficult and $\underline{\delta}$ increases.

The result does not hold if $n \geqslant 4$. The reason is that if the oligopoly has more firms an increase in α decreases the incentive to cheat on the agreement. Two factors leads to this result: (i) first, the possibility of coordination is lower, due to the large number of firms; (ii) second, a deviation gain when α increase is lower than the subsequent punishment; then, $(\pi^D - \pi^*)$ does not compensate the net present value of the stream of subsequent losses due to supply function reversion $(\pi^* - \pi^{SF})$; i.e. SFC is a severe punishment. Then,

$$\frac{\partial(\pi^D - \pi^*)}{\partial \alpha} < \frac{\delta}{1-\delta} \frac{\partial(\pi^* - \pi^{SF})}{\partial \alpha} \quad if \quad n \geqslant 4$$

so $\underline{\delta^*}$ must decrease to get the equality[8].

The result about c is explained in a simmilar way of α. When the number of firms and the cost are low is easy to sustain collusion because the extra profits of a deviation are not to much in comparison with the subsequent punishment, but if the number of firms rise and c is small a deviation is more profitable because now, the profits of the cartel must be divided betwen more firms, however, the subsequent punishment is not several because all of us have an small c parameter, so maintain collusion is difficult (a high $\underline{\delta}$ is expected). As the parameter c increase the punishment becomes more relevant in the former case, and the number of firms plays an small role in the latter case, so the critical value of $\underline{\delta}$ converges.

Increasing both the number of firms and/or increasing the cost of production, also raise the cost of reaching an agreement and also the coordination problem is worsened. Under supply function competition, when α is low, an increase in the

[8]It is esy to check that both, $\frac{\partial(\pi^D - \pi^*)}{\partial \alpha} > 0$ and $\frac{\partial(\pi^* - \pi^{SF})}{\partial \alpha} > 0$.

number of firms makes more difficult sustain collusion because a deviation from collusion increases the one-stage profits so as to compensate the punishment phase, for a wider range of the discount factor. But when the number of firms is big enough, increases α do not compensate the punishment in a wider range of the discount factor. As a result, for low values of α an increase in the number of firms has a great impact in δ, but the effect is alleviated when $\alpha \to \infty$. Then,

$$\lim_{\alpha\to\infty} \frac{\partial \underline{\delta}}{\partial n \partial \alpha} \to 0$$

4 Supply Function vs Cournot competition

We explore under what type of reversion, either SFC or Cournot competition, collusion is more easily sustainable. The motivation to compare both types of strategic interaction is related to the fact that a substantial amount of literature models electricity markets as a Cournot game (see [4], [12]). We are interested in the economic implications to use one or another market competition game for collusion sustainability. We showed in the previous section how a high willingness to pay together with a convex cost function facilitate tacit collusion when there is supply function competition reversion. On the contrary, we show how under Cournot reversion is more difficult to sustain collusion when the market has the same structural conditions, regardless the number of firms.

First, let us solve the model with the same market structure conditions but under Cournot competition. Now, the strategic variable for each firm is quantity, q_i, so we consider the inverse demand function

$$p(Q) = \alpha^{-1}(s - Q)$$

where $Q = q_i + \sum_{j\neq i}^{n} q_j$. We compare the results of the stage-game and the supergame under both types of strategic interaction.

4.1 The Cournot stage game

The equilibrium values of the symmetric equilibrium under Cournot competition (C) are[9],

$$\begin{aligned} q^C &= \frac{s}{c\alpha + n + 1} \\ p^C &= \frac{s(c\alpha + n)}{c\alpha^2 + \alpha(n+1)} \\ \pi^C &= \frac{s^2(2 + c\alpha)}{2\alpha(1 + n + c\alpha)^2} \end{aligned}$$

Even though firms put just a single quantity in the market, for each price and given the supply of the others, the underlying supply function under Cournot

[9] See appendix 1 for the details on the calculations

competition has slope,

$$\beta^C = \frac{\alpha}{c\alpha + n}$$

It is easy to check that $\beta^{SF} < \beta^C < \beta^{PC}$.

Under Cournot competition, firms can choose only their quantities as a result of demand's changes (measured by α) because they not have control on prices; but under Supply Function competition, firms can make a simultaneous changes on quantities and prices in order to save profits, because they have the ability of change both variables; as a result, Cournot firms decrease equilibrium quantity lesser than Supply Function firms, which offer a quantity-price pair in which: (i) quantity decrease more than under Cournot regime but; (ii) equilibrium price decrease lesser than under Cournot regime; saving more profits.[10]

Proposition 2 *In the one-shot game, an increase in the price-response of the consumers (α), makes under Cournot competition bigger losses than under Supply Function competition. Besides, quantity decrease more under Supply Function competition than when firms compete à la Cournot.*

Proof. See appendix 2. ■

The intuition that underlies the proposition is that under supply function firms are less sensitive to changes in α than under Cournot competition because they have also can choose the price and not only the quantity as strategic variable. Increasing competition through more firms in the market mitigates the result.

4.2 Collusion sustainability

We are interested in finding the minimum value of the discount factor, $\underline{\delta}$, such that collusion is sustained under trigger strategies for the infinitely repeated game. This strategy profile is standard in the literature (see, for instance Friedman' 1971). Formally, a strategy in the infinitely repeated game is,

$$q_{i1} = q_i^{cll}$$

$$q_{it} = \begin{cases} q^{cll} & if \quad q_{i\rho} = q^{cll}, \quad for\ \rho < t \\ q^C & if \quad q_{i\rho} \neq q^{cll}, \quad for\ \rho < t \end{cases}, i = 1, 2, ..., n; \qquad t, \rho = 2, ..., \infty$$

We show that $\underline{\delta}^{CTC}$ solves the equation,

$$\underline{\delta}^C = \frac{\pi^{DC} - \pi^{cll}}{\pi^{DC} - \pi^C} \tag{8}$$

where $\pi^{DC}, \pi^C, \pi^{cll}$ are profits under deviation from collusion, Cournot competition and collusion, respectively. Because of the convexity of the cost function, the critical value of the discount factor, depends on the structural parameters (α, c, n).

[10] This result is hold for changes in the c parameter.

Proposition 3 *In the infinitely repeated game with the firms competing à la Cournot, the minimum value of the discount factor $\underline{\delta}^C$ such that collusion is sustainable is,*

$$\underline{\delta}^C = \frac{(n + c\alpha + 1)^2}{n^2 + n(4c\alpha + 6) + (2c^2\alpha^2 + 4c\alpha + 1)}$$

1. *$\underline{\delta}^C$ depends negatively on the elasticity of demand,* $\frac{\partial \underline{\delta}^C}{\partial \alpha} < 0$
2. *$\underline{\delta}^C$ depends positively on the number of firms,* $\frac{\partial \underline{\delta}^C}{\partial n} > 0$
3. *$\underline{\delta}^C$ depends negatively on the cost of production* $\frac{\partial \underline{\delta}^C}{\partial c} < 0$

Let us consider first the effect of changes in the slope of the demand. As Friedman shows, when the marginal cost is constant the minimum discount factor to sustain collusion only depends on n. The convex cost function together with a low slope of demand, increases market power. Therefore, the extra profits from deviation, are larger because the higher prices of the market. Considering that from the period $\tau + 1$ ownwards the rest of firms make Cournot-Nash competition, this punishment is not strong enough to dissuade firms to cheat: firms only refused to cheat if the elasticity of demand increase sufficiently. When elasticity rise, the incentives to deviate decrease become less attractive and,

$$\frac{\partial(\pi^{DC} - \pi^{cll})}{\partial \alpha} < \frac{\delta}{1-\delta} \frac{\partial(\pi^{cll} - \pi^{C})}{\partial \alpha} \text{ for all } n$$

and the discount factor to maintain collusion must decrease. A symmilar result is showed by Collie, R. D (2003) in a model with constant elasticity of demand but without a cost specification (assume zero marginal cost). A computational advantage of the Cournot framework is that constant elasticity demand curves are straightforward to represent, as in Borestein and Bushnell (1999). The expression obtained is the same at Supply Function competition with $n \geqslant 4$. This reveal to us that collusion sustainability about this two forms of competition, à la Supply Function and à la Cournot, are affected in a different way by α and n. This merely a wide explanation.

Exploring the evolution of the discount factors $\underline{\delta}$ and $\underline{\delta}^C$, when we focus in the variations of the demand elasticity (measured in our model by α) under this two forms of competition, Supply Function and Cournot, we find that the values of δ to sustain collusion vary in a different way. When the number of firms rise values of δ present a different evolution. The theoretical model suggest that when demand elasticity and the number of firms is low, the sustainability of collusion is easier under Supply Function competition than under Cournot competition, but this result falls when the number of firms rise and α increases.

The explanation and economic intuition of this result is the following. In a Cournot framework firms have capacity to change quantities and, as a result, market price is obtained. In contrast, when firms submit supply bids, they have the ability to change price and quantities for each β and the ability to exploit market power is high. For a reduce number of firms cheating is lesser profitable under Supply Function than under Cournot competition because the high value of the subsequent present value of the future punishments doesn't compensate to cheat; as a result, firms prefer to remain under collusion. This is true for all values of the elasticity of demand (values of α) if $n < 6$. On the contrary, when the environment is Cournot competition, the ability to exploit market power is lower but the subsequent punishment per period are lower too. This reason makes that, when elasticity of demand is low, put in the market a quantity $q_i + \epsilon$ by the deviator firm generate an extra profits which compensate competition à la Cournot from the rest of the periods. The main point is the ability to exploit market power and the severe of the punishment.

When elasticity of demand rise the result is affected for the number of firms. Rising number of firms makes more easy collusion agreements under Cournot if α is sufficient high. The reason for this fact is that market power losses under Supply Function are more severe than under Cournot so, $\pi^D - \pi^{DC}$ goes down as α increase, so cheating is relatively less attractive under Cournot. Besides, the gap between $\pi^C - \pi^{SF}$ goes down more rapidly than $\pi^{DC} - \pi^D$. Then, for $n \geqslant 6$ there is a value $\widehat{\delta}$ in which collusion sustainability is equally probably to sustain under the two regimens. Finally, this $\widehat{\delta}$ value increase when the number of firms and elasticity rise. More precisely,

Proposition 4 *(i) When the number of firms is $n < 6$, for all values of α it is easier to sustain collusion under Supply Function than under Cournot Competition, so $\underline{\delta} < \underline{\delta}^C$ is hold; if the number of firms is $n \geqslant 6$, collusion sustainability is easier under Cournot than under Supply Function competition, i.e.$\underline{\delta} > \underline{\delta}^C$ (ii) besides, the value of $\widehat{\delta}$ decrease with the number of firms when α rise,*

$$\frac{\partial \widehat{\delta}}{\partial n \partial \alpha} < 0$$

Proof. See appendix 2. ■

5 Empirical evidence: The Spanish Wholesale Electricity Market

The aim of the empirical study is to check whether the Spanish wholesale electricity market is, according to our theoretical predictions, conducive to collusive practices. The competition authorities already filed a case against the three major companies, Endesa (EN), Iberdrola (IB), and Union Fenosa (UF), charging for anti-competitive practices the days 19th, 20th and 21st of November 2001. The section is structured as follows. We briefly describe the rules of the market, which is administered as a pool. We continue with a description of the data. Then we estimate the demand functions for each time period after running a test of linearity. Finally we use those estimations to simulate values of the minimum discount factor we obtained in the theoretical model as a function of the structural parameters under both types of oligopolistic reversion.

5.0.1 The Pool

The Spanish Wholesale Electricity Market is a mandatory pool (day-ahead market). It was created in 1998 (after the Law 54/1997). The pool works as follows. Before 11:00 am, qualified buyers and sellers of electricity present their offers for the following day. Each day is divided into 24 periods, one for each hour. Sellers in the pool present offers consisting of up to 25 different prices and the corresponding energy quantities, for each of the 24 periods and for each generating unit they own; the prices must be increasing. An offer that no includes any restriction is called a "simple offer". However, a seller may present an offer with restrictions, according to the rules of the system, then it is called a "complex offer". At the same time qualified buyers present offers[11]. Purchase bids state a quantity and a price of a power block and there can be as many as 25 power purchasing blocks for the same purchasing unit, with different prices for each block; the prices must be decreasing.

The system operator, OMEL, constructs, with the selling bids and the purchasing bids, an aggregate supply and an aggregate demand schedule respectively. In a session of the daily market, it combines these offers matching demand and supply for each of the 24 periods and determines the equilibrium prices for each period (it is called the system marginal price) and the amount traded (market clearing quantity). This matching is the "base daily operating schedule", (PBF). After the PBF schedule is settled, the pool administrator evaluates the technical feasibility of the assignment; if the required technical restrictions are met then the program is feasible; if not, some previously accepted offers are eliminated and others included to obtain the "provisional feasible daily schedule", (PVP). This reassignment ends at 14:00. By 16:00 the "final feasible daily schedule", (PVD) is obtained taking into account the ancillary services assignment procedure. There is also an intra-day market to make any necessary adjustments between demand and supply[12]. The result is called the "final hourly schedule", (PHF).

5.1 Descriptive Analysis of the Data

We have hourly data from May 2001 until December 2003, so there are overall 23401 hour-observations. Each hour-observation contains all the demand-bids by qualified agents and all the supply-bids by generators. The hours are classified into peak, off-peak, and valley, to distinguish high demand, intermediate demand, and low demand hours respectively, following the market operator's classification.system (OMEL).

[11] From January 1st 2003, all buyers of electricity are considered qualified buyers. Before that day qualified buyers were those with consumption greather or equal to 1 GWh per year.

[12] This intra-day market started working on April 1st, 1998. The first three monts it had 2 sessions per day; in 2002 has 6 sessions per day. The volume of energy traded is usually very low, around 2% of the total.

5.1.1 The Demand Schedule

We summarize the mean and the standard deviation of the following descriptive variables:

1. **Block-bids**: The number of price-quantity pairs submitted by a purchasing unit to the system operator.

2. **Bidders**: Number of demand-bidders for each hour-observation. They are distributors, external agents, qualified agents, retailers, and pumping stations.

3. **Price-inflexible demand segment**: That is the horizontal segment of the demand schedule, which is at a price higher or equal to 17.99 cents of euro. The price-cap is 18.03 cents of euro.

4. **Price**: This is the weighted equilibrium price for each hour-observation as reported by OMEL, after including technical restrictions.

5. **Quantity**: This is the weighted equilibrium quantity for each hour-observation as reported by OMEL.

Table 1. Demand Descriptive Statistics, by type of hours

	All	Peak	Off-Peak	Valley
Block-bids	2.69 (0.58)	2.84 (0.44)	2.74 (0.64)	2.641 (0.56)
No. Bidders	130 (8.4)	133.6 (9.8)	130.7 (8.06)	129.7 (8.2)
Price inflexible demand	18054.9 (3156.7)	21023.2 (2372.3)	20133.6 (1989.7)	16427.6 (2723.8)
Price	3.725 (1.51)	4.953 (1.745)	4.508 (1.351)	3.095 (1.19)
Quantity	20808.08 (3114.4)	24142.8 (2320.9)	22925.5 (1922.4)	19101.8 (2526.2)

It is noteworthy how the average number of block-bids is almost three (with a very low stadard deviation), regardless the type of hour, even though, by law, the maximum number of allowed bids is 25. Demanders facing uncertainty and very low real-time response consumers, bid as to get power so there are no disruptions in the service, therefore they have an incentive to bid high for most of the energy ther buy. As a result the price inflexible demand amounts for 90 percent of the energy traded.

5.1.2 The supply schedule

There are four major firms in the market that amount for the bulk of the total electricity generation

5.2 Linear demand specification

We begin the empirical analysis with a test on the assumption of linearity of the demand. We estimate a demand function of the form,

$$Q\left(P_t\right) = s + \alpha P_t^{\gamma} + \varepsilon_t \begin{cases} \text{If } \gamma > 1 \rightarrow \text{ concave} \\ \text{If } \gamma = 1 \rightarrow \text{ linear} \\ \text{If } \gamma < 1 \rightarrow \text{ convex} \end{cases}$$

where (s, α, γ) are the parameters to be estimated. Thus if γ is significantly close to one, we can proceed the analysis under a linear demand assumption. We estimate the demand functions corresponding to each hour of the entire data set using non-linear least squares (See Judge et al., 1985). Therefore, after the parameters' estimation, we have a time series for the exponent $\{\widehat{\gamma}_t\}_{t=1}^{T=23401}$. We run a test of means where the null hypothesis is the linearity of the demand function. The results reveal that on average $\overline{\gamma} = 1.015$, however it is not significantly different from 1. We also perform a test by type of hour, that is whether it is peak $(\overline{\gamma}^P)$, intermediate $(\overline{\gamma}^I)$ or low demand $(\overline{\gamma}^L)$ respectively. The test is not rejected only during low demand hours, where $\overline{\gamma}^L = 0.992$. Still this is just an average value, and this is precisely the target of our empirical study. This preliminary test should be interpreted as a starting point. It is not entirely clear that the aggregate demand schedule is approximately linear, specially during the year 2003. The variability in the estimation of the coefficients is quite large, which explains the frequency of values different from 1.

5.3 Slope of the Demand

We fit a linear demand function of the form,

$$Q\left(P_t\right) = s + \alpha P_t + \epsilon_t \text{ if } P_t \leq 18.000$$

where the expected signs of estimation are $\widehat{s} > 0$ and $\widehat{\alpha} < 0$ respectively. Recall, there is a price cap at $P_t = 18.030$, therefore not only equilibrium prices cannot be above this level, but also price-dids are not considered. First, we report the non-weigthed average results in table 2 with the standard deviations into brackets. The price is measured in cents per KWh and the quantity in MWh.

Table 2: Non-weighted LinearDemand Estimation				
Period	Type of Hour	Slope ($\widehat{\alpha}$)	Intercept ($\widehat{s}$)	R^2
2001-2003	All hours	-363.7 (30.01)	23157.3 (185.4)	0.78
	Peak	-311.14 (24.1)	26819.1 (177.5)	0.76
	Off-peak 1	-379.43 (28.77)	25754.2 (183.08)	0.75
	Off-peak 2	-361.27 (31.7)	21140.7 (187.9)	0.79
2001	All hours	-335.01 (37.4)	21982.6 (168.21)	0.81
	Peak	-300.82 (26.16)	25848 (156.61)	0.87
	Off-peak 1	-355.06 (33.87)	24579.3 (160.75)	0.82
	Off-peak 2	-329.4 (40.84)	20071.3 (173.78)	0.78
2002	All hours	-233.85 (18.7)	22755.5 (163)	0.75
	Peak	-245.54 (19.03)	26332.4 (171.16)	0.77
	Off-peak 1	-248.44 (18.52)	25203 (163)	0.72
	Off-peak 2	-223.7 (18.72)	20728.8 (161.37)	0.76
2003	All hours	-562.83 (38.5)	24749.5 (230.7)	0.79
	Peak	-424.98 (30.6)	28379.6 (204.5)	0.75
	Off-peak 1	-571.55 (37.6)	27518.2 (229.11)	0.74
	Off-peak 2	-580.71 (40.31)	22662.3 (235.8)	0.82

The estimation results reveal that, on average, the slope of the demand function is quite low, regardless whether the hour is peak, off-peak or valley. That is the demand is not price-responsive to changes in the price. In general, the goodness of fit is quite important, on average is around 80 percent. But, as demand grows it is becoming more price-responsive since more qualified agents are also becoming agents in the market without signing binding contracts with distributors.

5.4 Elasticity of Demand

We use $\widehat{\alpha}$ to compute estimations of the elasticity of demand around the system marginal price (SMP) as reported by the market operator. By definition, he elasticity of the demand, ε, at hour h around the system marginal price (P_h^*) is,

$$\varepsilon_h = \frac{\partial Q_h}{\partial p_h} \frac{P_h^*}{Q_h^*}$$

A local measure of the elasticity around the equilibrium is the arch-elasticity of demand. Let us consider P_h^*, and the market clearing quantity, $Q_h^* = D(P_h^*)$. We take the highest price before the equilibrium price, P_h^{high}, such that $D(P_h^{high}) < D(P_h^*)$, and the smallest price after the equilibrium price, P_h^{low}, such that $D(P_h^*) < D(P_h^{low})$. The (absolute value of the) arch-elasticity of demand, for hour h, is defined as,

$$\varepsilon_h = -\frac{D(p_h^{high}) - D(p_h^{low})}{p_h^{high} - p_h^{low}} \frac{p_h^{high} + p_h^{low}}{D(p_h^{high}) + D(p_h^{low})}$$

Table 3 reports the estimation results of both the elasticity of demand and the arch elasticity of demand.

Table 3: Non-weighted Elasticities of Demand around P^*			
Period	Type of Hour	Elasticity	Arch-Elasticity
$2001-2003$	All hours	0.058 (0.001)	0.36 (0.008)
	Peak	0.056 (0.001)	0.48 (0.009)
	Off-peak 1	0.067 (0.001)	0.19 (0.006)
	Off-peak 2	0.054 (0.001)	0.13 (0.004)
2001	All hours	0.056 (0.001)	0.43 (0.021)
	Peak	0.061 (0.001)	0.62 (0.034)
	Off-peak 1	0.066 (0.001)	0.16 (0.011)
	Off-peak 2	0.019 (0.001)	0.13 (0.015)
2002	All hours	0.042 (0.001)	0.37 (0.012)
	Peak	0.048 (0.001)	0.53 (0.035)
	Off-peak 1	0.049 (0.001)	0.17 (0.012)
	Off-peak 2	0.037 (0.001)	0.11 (0.001)
2003	All hours	0.082 (0.001)	0.26 (0.007)
	Peak	0.063 (0.002)	0.29 (0.023)
	Off-peak 1	0.092 (0.002)	0.24 (0.016)
	Off-peak 2	0.079 (0.001)	0.15 (0.015)

As we expected, the elasticity of demand is higher during peak hours than during the rest of the hours. Furthermore, as more qualified consumers enter the market, the demand schedule has more steps and becomes stepper. At the same time the arch elasticity of demand has reduced since there are more bolck-bids and the distance between $\left(D(p_h^{high}), p_h^{high}\right)$ and $\left(D(p_h^{low}), p_h^{low}\right)$ is smaller.

5.5 Simulation values for δ

Finally, we simulate values for $\underline{\delta}$ using the $\widehat{\alpha}$ estimations and for different number of firms, n. We would also like to have estimations for c, but it is outside the scope of the paper. We normalize $c = 1$, in the context of the model it is correct as long as we are not considering firm asymmetries due to, for example different technology mix. Therefore we get $\widehat{\underline{\delta}}(n, \widehat{\alpha})$ as the estimated values for the lower bound of the discount factor such that collusion is sustainable using the estimations of the slope obtained in the previous section. We consider different values of the number of firms under supply function competition and Cournot competition with increasing costs. The slope of the demand function has been normalized, in both cases, by the size of the market, s[13].

Table 4: Simulation values for $\widehat{\delta}(n, \widehat{\alpha})$

	Supply Competition $\widehat{\underline{\delta}}(n, \widehat{\alpha})$				Cournot competition $\widehat{\underline{\delta}}^C(n, \widehat{\alpha})$			
	All	Peak	O-P 1	O-P 2	All	Peak	O-P 1	O-P 2
$n = 2$	0.2954	0.2893	0.2941	0.2973	0.5290	0.5291	0.5291	0.5290
$n = 3$	0.4474	0.4466	0.4472	0.4476	0.5707	0.5709	0.5708	0.5707
$n = 4$	0.5619	0.5620	0.5619	0.5619	0.6089	0.6091	0.6089	0.6088
$n = 5$	0.6384	0.6388	0.6385	0.6382	0.6419	0.6421	0.6419	0.6418
$n = 10$	0.8080	0.8085	0.8081	0.8079	0.7504	0.7507	0.7505	0.7503
$n = 20$	0.9012	0.9015	0.9013	0.9011	0.8455	0.8458	0.8456	0.8454

The data consists of 21192 hour observations spanning from May 2001 until September 2003

We can conclude from the analysis of the demand of the Spanish electricity wholesale market that three structural characteristics could play an important role in the favor of collusive practices. One is the form of the competition à la Supply Function, which, despite the limitations of the model, we believe it is a better representation of the strategic interaction than Cournot or Bertrand competition. The second is about the nature of the demand curve, it is highly inelastic. The third one is the reduce number of firms.The following corollary summarize the results.

Corollary 1 *Collusive practices in electricity markets are easier to arise when the following conditions hold*

- *the form of the competition is in supply functions as compared to Cournot.*
- *the demand curve is steeper.*
- *the number of firms is small.*

5.5.1 Policy recommendations

Advocates of the Spanish Electricity Market restructuring argue that it should increase production and consumption efficiency, by promoting competition among

[13]The reason for this normailzation is that the intercept in the X-axis and Y-axis are affected by s. Divided the demand function specified we obtain $\frac{q}{s} = 1 - \frac{\alpha}{s}p$ where $\frac{q}{s} \in [0, 1]$ and the values of δ become relevant.

generators. We show how, besides the supply-side studies reviewed in the introduction, demand-side rigidities may actually be welfare-worsening, since firms find easy to sustain collusive outcomes. Therefore, price-cap regulation when the price is set in the neighborhood of the marginal cost, is more desirable. The theory of demand-side price incentives is developing mechanisms to make demand more responsive towards price variability[14]. The problem to solve is how to get a more active demand in the electricity market. Some contractual figures could be implemented. Static time-varying prices, which are preset for determined hours and days, are actually not that difficult to implement since a classification of demand hours is available, and accordingly different tariffs. Dynamic time-varying prices are similar in nature, but are allowed to vary within short notice. The main barrier is to make information available to final consumers on the benefits of the use of these type of tariffs. For that reason independent electricity retailers is a prerequisite, and break the vertically related tights among generators and distributors.

6 Conclusions

The paper has two major contributions to the literature. First, it is a contribution to the theory of repeated games, and second, another application to the empirical studies on electricity markets. First, we develope a supergame-theoretic framework for firms that compete in supply functions; we evaluate collusion sustainability under this form of competition and for a certain market structure. We have shown how tacit collusion is easier to sustain as the willingness to pay for the good, given the size of the market, increases, and firms compete in supply functions. These results are in contrast with the traditional Cournot framework: Collusion is more difficult to sustain.

Second, we simulate the theoretical model with data from the Spanish Wholesale Electricity Market. Empirical evidence shows that if the Spanish pool can be modeled as a supply function competitive market, firms could sustain tacit collusion in a wide range of the discount factor. The result has to be interpreted carefully since we just point out there are characteristics of the market that facilitate collusion, but the nodel is unable to detect actual collusion.

[14] For a survey on the literature of demand-side incentives, see Borenstein et al., UCEI-WP, October 2002.

7 Appendix 1: Further Calculations

7.1 Supply Funtion Competition

- **JOINT-PROFIT CARTEL:** The joint profit function is,

$$\Pi^*(\beta_i, \beta_{-i}) = \left[\frac{(n-1)\beta_{-i} + \beta_i}{(\alpha + (n-1)\beta_{-i} + \beta_i)^2}\right] - \\ - \frac{c(n-1)}{2}\left[\frac{\beta_j}{\alpha + (n-1)\beta_{-i} + \beta_i}\right]^2 - \frac{c}{2}\left[\frac{\beta_i}{\alpha + (n-1)\beta_{-i} + \beta_i}\right]^2$$

therefore. there are n first order conditions

$$\frac{\partial \Pi^{COL}(\beta_i, \beta_{-i})}{\partial \beta_i} = \frac{s[\alpha - \beta_i(c\alpha + 1) - (c\beta_i + 1)(n-1)\beta_{-i} + (n-1)c\beta_{-i}]}{(\alpha + \beta_{-i} + (n-1)\beta_i)^3}$$

imposing symmetry $\beta_i = \beta_{-i} = \beta^*$ and solving for β^* we obtain

$$\beta^* = \frac{\alpha}{n + c\alpha}$$

- **PROFITS FROM DEVIATION:** Given the equilibrium price and quantity for the colluding firms, there is an incentive for deviation since the marginal revenue is above the marginal cost for each firm. Thus the first order condition is,

$$\frac{\partial \pi_i(\beta_i, \beta_{-i}^*)}{\partial \beta_i} = -\frac{s^2(n + c\alpha)^2[(c^2\alpha^2 + 2nc\alpha + n)\beta_i - (c\alpha + 2n - 1)\alpha]}{[(n + c\alpha)\beta_i + (c\alpha + 2n - 1)\alpha]^3}$$

solving in β_i (assuming symmetry) we obtain the optimal supply function for a potential deviator,

$$\beta^D = \frac{[c\alpha + 2n - 1]\,\alpha}{c^2\alpha^2 + 2nc\alpha + n}$$

- **SUPPLY FUNCTION REVERSION:** If competition occurs, then first order condition for each firm is,

$$-s^2\left[\beta_i - \alpha + c\alpha\beta_{-i} + (c\beta_i - 1)(n-1)\beta_{-i}\right]\left[\alpha + \beta_i + (n-1)\beta_{-i}\right]^{-1} = 0$$

solving in β we obtain the optimal strategy β^{SF} for each firm,

$$\beta^{SF} = [2c\,(n-1)]^{-1}\left[(n-2) - c\alpha + ((c\alpha + n)^2 - 4(n-1))^{1/2}\right]$$

Replacing β^{SF} in the price expression $p(\beta^{SF})$, we obtain

$$p^{SF} = ns[2c\alpha - n^2 + n(2 - c\alpha - ((c\alpha + n)^2 - 4(n-1))^{1/2})][2\alpha(2n + c\alpha)]^{-1}$$

7.2 Cournot Competition

- **JOINT-PROFIT CARTEL:** The joint maximization problem is

$$\max_{\{q_i\}_{i=1}^{n}} \Pi(q_1, .., q_i, .., q_n) = \sum_{i=1}^{n} \pi_i(q_i, q_{-i})$$

with n first order condition for interior solutions,

$$-\alpha^{-1}q_i + s\alpha^{-1} - \alpha^{-1}[q_i - \sum_{j\neq i}^{n} q_j] - cq_i = 0, \qquad i = 1, 2, .., n$$

impose symmetry we obtain collusion quantity for each firm,

$$q^{cll} = \frac{s}{c\alpha + 2n}$$

replacing in the demand function, we obtain collusion price,

$$p^{cll} = \frac{s(c\alpha + 2n - 1)}{\alpha(2n + c\alpha)}$$

and the profits for each firm are,

$$\pi^{cll} = \frac{s^2}{4n\alpha + 2c\alpha^2}$$

- **PROFITS FROM DEVIATION:** firm i is the deviator firm. Firms labelled $-i$ remain producing collusion quantities q^*_{-i}. The maximization problem for the deviator firm is

$$\max_{q_i} \pi^{Cd}(q_i, q^*_{-i}) = [s\alpha^{-1} - \alpha^{-1}(q_i + (n-1)q^*)]q_i - \frac{c}{2}q_i^2$$

with first order condition for interior solutions,

$$-\alpha^{-1}q_i + s\alpha^{-1} - \alpha^{-1}[q_i - (n-1)q^*] - cq_i = 0$$

Solving we obtain,

$$q^{Cd} = \frac{s(n + c\alpha + 1)}{(2 + c\alpha)(2n + c\alpha)}$$

thus the expressions for the rest of the endogenous variables are,

$$p^{Cd} = \frac{s(c^2\alpha^2 - 3 + n(3 + 2c\alpha))}{\alpha(2 + c\alpha)(2n + c\alpha)}$$

$$\pi^{Cd} = \frac{s^2(1 + n + c\alpha)^2}{2\alpha(2 + c\alpha)(2n + c\alpha)^2}$$

- **COURNOT REVERSION:** The maximization problem for each firm i is

$$\max_{q_i} \pi_i^C(q_i, q_{-i}) = [s\alpha^{-1} - \alpha^{-1}(q_i - \sum_{i \neq j} q_j)]q_i - \frac{c}{2}q_i^2$$

where q_{-i} is the vector with the strategies of the rest $j \neq i$ firms. First order condition for interior solutions,

$$-\alpha^{-1}(2 + c\alpha)q_i - \alpha^{-1}[(n-1)q_j - s] = 0$$

under symmetry across firms,

$$q^C = s[c\alpha + n + 1]^{-1}$$

$$p^C = \frac{s(c\alpha + n)}{c\alpha^2 + \alpha(n+1)}$$

$$\pi^C = \frac{s^2(2 + c\alpha)}{2\alpha(1 + n + c\alpha)^2}$$

8 References

References

[1] Allaz, B., Vila, J-L. (1991).”Cournot Competition, Forward Markets and Efficiency”. Journal of Economic Theory, vol 59, 1-16.

[2] Baldick, R., Grant, R and Kahn, E. (2000).”Linear Supply Function Equilibrium: Generalizations, Application, and Limitations”. POWER, PWP-078.

[3] Bolle, F.(1992).”Supply Fuction Equilibria and the Danger of Tacit Collusion: the Case of Spot Markets for Electricity”, Energy Economics, 94-102.

[4] Borenstein, S., Bushnell, J.(1999). ”An empirical analysis of the potential for market power in California’s electricity industry”. Journal of Industrial Economics 47(3), 285-323.

[5] Borenstein, S., Bushnell, J., Khan, e., Stoft, S.(1995).” Market Power in California electricity markets”. Utilities Policy 5 (374), 219-236.

[6] Collie R. D, (2004). ”Collusion and the elasticity of demand” Economics Bulletin, Vol. 12, No. 3 pp. 1-6.

[7] Day, C., Hobbs, B., and Pang, J-S.(2002).”Oligopolistic Competition in Power Networks: A Conjectured Supply Function Approach”, POWER, PWP-090.

[8] Fabra, N. (2003).”Tacit Collusion in Repeated Auctions: Uniform versus Discriminatory”, Journal of Industrial Economics, Vol. L1, No. 3 (September), pp. 271-293, 2003.

[9] Green, R.(1999)."The Electricity Contract Market in England and Wales". The Journal of Industrial Economics, vol XLVII, 107-124.

[10] Green, R.(1996)."Increasing Competition in the British Electricity Spot Market". The journal of Industrial Economics, vol XLIV, 205-216.

[11] Green, R.(2001)."Failing Electricity Markets: Should we shoot the Pools?". Working Paper, University of Hull.

[12] Jacquemin, A and Margaret E. Slade, (1989). "Cartel, Collusion, and Horizontal mergers" in Handbook of Industrial Organization, Vol 1, North Holland, Amsterdam.

[13] Klemperer, P. and Meyer, M.(1989) "Supply Function Equilibria in Oligopoly Under Uncertainty". Econometrica. vol 57, n^o 6, 1243-1227.

[14] Ramos, A ,Ventosa, M., and Rivier, M.(1998)."Modelling Competition in Electricity Energy Markets by Equilibrium Constraints". Utilities Policy, vol 7,n^o 4, 223-242.

[15] Powell, A.(1993)."Trading Forward in an Imperfect Market: The case of Electricity in Britain". The economic Journal, vol 103, 444-453.

[16] Selten, R., "A Simple Model of Imperfect Competition When 4 Are Few and 6 Are Many", International Journal of Game Theory 2 (1973), 141-201

[17] Stoft, S, Power Market Economics: Designing Markets for Electricity", Wiley & Sons, 2000

[18] Wolak, F. A. (2003). "Measuring Unilateral Market Power in the Wholesale Electricity Markets: The CAlifornia Market, 1998-2000". AEA Papers and Proceedings, vol 93, NO. 2. May 2003, 425-430.

9 Appendix 2: Proofs

- **PROOF OF LEMMA 1**. We are going to proceed in two steps. First, we proof the chain $\beta^{PC} > \beta^{SF} > \beta^{M}$. When $1 < n < \infty$ we obtain β^{SF} as a result of the profit maximization of the firm's profits. When $n \to \infty$ we expected to obtain the result of perfect competition[15]; then,

$$\lim_{n\to\infty}\left[\frac{(n-2)-c\alpha+\phi}{2c\,(n-1)}\right]=\frac{\infty}{\infty}$$

[15] This result is hold fixing exgenously p; then,

$$\max_{\beta_i}\pi_i(\beta)=\beta_i p - C[\beta_i p]$$

with an optimal strategy $\beta^{PC}=\frac{1}{c}$.

dividing both, numerator and denominator with n we obtain

$$\lim_{n\to 1}\left[\frac{\frac{(n-2)-c\alpha}{n}+\frac{\phi}{n}}{2c\frac{(n-1)}{n}}\right]=\frac{1}{c}$$

When $n\to 1$ we expected to obtain the result of a monopoly[16]; then,

$$\lim_{n\to 1}\left[\frac{(n-2)-c\alpha+\phi}{2c\,(n-1)}\right]=\frac{\infty}{\infty}$$

numerator and denominator are derivables so applying L'Hopital's theorem we obtain an equivalent limit

$$\lim_{n\to 1}\left[(1+\frac{c\alpha+n-2}{\phi})/2c\right]=\frac{\alpha}{1+c\alpha}$$

if we solve the lateral limit $n\to 1^{+}$ with $1^{+}=1+\varepsilon$ we obtain,

$$\lim_{n\to 1^{+}}\left[\frac{(n-2)-c\alpha+\phi}{2c\,(n-1)}\right]=\frac{\alpha}{1-\varepsilon+c\alpha}$$

then, if $n>1$ we find that $\beta^{SF}>\beta^{M}$. Of course, $\frac{1}{c}>\frac{\alpha}{1-\varepsilon+c\alpha}\Rightarrow 1-\varepsilon+c\alpha>c\alpha\Rightarrow 1>\epsilon$ because $\epsilon\to 0$. As a result $\beta^{PC}>\beta^{SF}>\beta^{M}$. In the second, we want to proof that

$$\frac{\partial\pi(\beta^{SF})}{\partial\beta^{SF}}=\frac{s^{2}(\alpha-(n+c\alpha)\beta^{SF})}{(\alpha+n\beta^{SF})^{3}}<0$$

the denominator is always positive and s^2 too. Then, $(\alpha-(n+c\alpha)\beta^{SF})$ must be less than zero, or $\frac{\alpha}{n+c\alpha}<\beta^{SF}$. We know that $\beta^{SF}>\beta^{M}=\frac{\alpha}{1+c\alpha}>\frac{\alpha}{n+c\alpha}$, so $\beta^{SF}>\frac{\alpha}{n+c\alpha}$. This complete the proof.■

- **PROOF OF PROPOSITION 1**. Let us normalize $c=1$. The expression for $\underline{\delta}$ is obtained by substituting profits of collusion (π^{COL}), deviation (π^{DEV}) and supply function competition (π^{SF}). These three profit functions depend on the number of firms and the slope of demand function, then the right hand side of equation [7] can be written as

$$\underline{\delta}(\alpha,n)=\frac{\pi^{D}(\alpha,n)-\pi^{*}(\alpha,n)}{\pi^{D}(\alpha,n)-\pi^{SF}(\alpha,n)}$$

taking the partial derivate respect to α we get,

$$\frac{\partial\underline{\delta}(\alpha,n)}{\partial\alpha}=\frac{4(n-1)^{2}(-2+6n-n^{2}-2n^{3}+n^{4}+10n\alpha-8n^{2}\alpha+}{\phi(2n^{3}-3n^{2}+2n+2n\alpha+2n^{3}\alpha+\alpha^{2}+5n^{2}\alpha^{2}+}$$

[16] This result is hold by fixing $n=1$ and

$$\max_{\beta}\pi(\beta)=\beta p(\beta)-C[\beta p(\beta)]$$

with an optimal strategy $\beta^{M}=\frac{\alpha}{1+c\alpha}$

$$\frac{+6n^3\alpha+5\alpha^2-4n\alpha^2+11n^2\alpha^2+8n\alpha^3+2\alpha^4-(n+\alpha)\phi(1+n^2+4n\alpha+\alpha^2))}{+4n\alpha^3+\alpha^4+(1+\alpha)(n+\alpha)(2n+\alpha-1)\phi)^2}$$

We are interested in the sign of the effect. The denominator is always positive. In particular, we want to know when this polynomial function change the value. Then, making it equal to zero and solving in α, we get an expression that depends on n. Let us rename the expression as $\Psi(n)$,

$$\Psi(n)=\frac{\sqrt{(n-1)^3}}{\sqrt{(n-3)}\sqrt{2}}-n$$

The function $\Psi(n)$ takes positive values for $n<4$ and negative values for the rest. ■

- **PROOF OF PROPOSITION 2**. This proof has two parts. We can eliminate s^2 from the expressions of π^C , π^{SF} , q^C and q^{SF} and makes $c=1$ because the effect of this parameter in profits is the same of the α one, and cualitative result are mantained. First, we prove that an increase in the price-response of the consumers (α) makes under Cournot competition bigger losses than under Supply Function competition. Then we are interested in the value of $\frac{\partial \pi^C}{\partial \alpha}$ and $\frac{\partial \pi^{SF}}{\partial \alpha}$. The values of the partial derivates are,

$$\frac{\partial \pi^C}{\partial \alpha}=-\frac{3\alpha+\alpha^2+n+1}{\alpha^2(\alpha+n+1)^3}$$

$$\frac{\partial \pi^{SF}}{\partial \alpha}=-\frac{(\alpha+n+\phi)^2}{\phi[(n+c\alpha)^2+(n+c\alpha)\,\phi-2n]^2}$$

This two expressions are always negative. Finally, we are interested to show that $\left|\frac{\partial \pi^C}{\partial \alpha}\right|-\left|\frac{\partial \pi^{SF}}{\partial \alpha}\right|\geq 0$, that is,

$$\phi[3\alpha+\alpha^2+n+1][(n+\alpha)^2+(n+\alpha)\,\phi-2n]^2-\alpha^2(\alpha+n+1)^3(\alpha+n+\phi)^2\geq 0$$

for all values of α and n. Second, we prove that quantity decrease more under Supply Function competition than when firms compete à la Cournot. Then, we are interested in the value of $\frac{\partial q^C}{\partial \alpha}$ and $\frac{\partial q^{SF}}{\partial \alpha}$. The values of the partial derivates are,

$$\frac{\partial q^C}{\partial \alpha}=-\frac{1}{(\alpha+n+1)^2}$$

$$\frac{\partial q^{SF}}{\partial \alpha}=-\frac{n(n^2+n(4+\alpha-\phi)+2(\phi-2))}{2(\alpha+2n)^2\phi}$$

This two expressions are always negative. Finally, we are interested to show that $\left|\frac{\partial \pi^C}{\partial \alpha}\right|-\left|\frac{\partial \pi^{SF}}{\partial \alpha}\right|\geq 0$, that is,

$$2(\alpha+2n)^2\phi-n(n^2+n(4+\alpha-\phi)+2(\phi-2))(\alpha+n+1)^2\geq 0$$

for all values of α and n. This complete the proof. Note: upon request, we report an extension of the proof with calculus and grafics with the Mathematica Program. ■

- **PROOF OF PROPOSITION 3**.Let us normalize $c = 1$. From the expressions [7] and [8] we are interested to check when both expressions are equal. Taking the right hand side,

$$\pi^{Cd} + \frac{\delta}{1-\delta}\pi^{C} \stackrel{?}{=} \pi^{D} + \frac{\delta}{1-\delta}\pi^{SF}$$

It could be equal when $\delta^{Q} \to \delta^{*}$; let us call this value $\widehat{\delta}$. Then,

$$\pi^{D} - \pi^{Cd} = \frac{\widehat{\delta}}{1-\widehat{\delta}}(\pi^{C} - \pi^{SF}) \qquad (9)$$

The intuition of the equation is the following: when the gap of the deviation's profitability between Supply function and Cournot is equal to the discounted gap between Cournot and supply function reversion, collusion under Cournot and supply function are equally sustained. Solving equation [9],

$$\widehat{\delta} = \frac{(\pi^{D} - \pi^{Cd})}{(\pi^{D} - \pi^{Cd}) - (\pi^{C} - \pi^{SF})}$$

when $n < 6$ there isn't any value of $\widehat{\delta} \in (0, 1)$ which can balance the equation. For values $n \geqslant 6$ we find a positive relation between α, n, and $\widehat{\delta}$. That is,

$$\frac{\partial\widehat{\delta}}{\partial n \partial\alpha} = \frac{2(n-1)(n^{3} - 13 - 26\alpha - 18\alpha^{2} - 4\alpha^{3} - n^{2}(2\alpha + 7) - n(13 + 20\alpha + 6\alpha^{2}))}{(1 + n^{2} + 4\alpha + 2\alpha^{2} + n(6 + 4\alpha))^{3}} > 0$$

This complete the proof. ■