Observational constraints of $f(Q)$ gravity

Ruth Lazkoz,[1, a] Francisco S. N. Lobo,[2, b] María Ortiz-Baños,[1, c] and Vincenzo Salzano[3, d]

[1]*Department of Theoretical Physics, University of the Basque Country UPV/EHU, P.O. Box 644, 48080 Bilbao, Spain*
[2]*Instituto de Astrofísica e Ciências do EspaÇo, Facultade de Ciências da Universidade de Lisboa,*
Edifício C8, Campo Grande, P-1749-016, Lisbon, Portugal
[3]*Institute of Physics, Faculty of Mathematics and Physics,*
University of Szczecin, Wielkopolska 15, 70-451 Szczecin, Poland
(Dated: November 15, 2019)

In this work, we consider an extension of symmetric teleparallel gravity, namely, $f(Q)$ gravity, where the fundamental block to describe spacetime is the nonmetricity, Q. Within this formulation of gravitation, we perform an observational analysis of several modified $f(Q)$ models using the redshift approach, where the $f(Q)$ Lagrangian is reformulated as an explicit function of the redshift, $f(z)$. Various different polynomial parameterizations of $f(z)$ are proposed, including new terms which would allow for deviations from the ΛCDM model. Given a variety of observational probes, such as the expansion rate data from early-type galaxies, Type Ia Supernovae, Quasars, Gamma Ray Bursts, Baryon Acoustic Oscillations data and Cosmic Microwave Background distance priors, we have checked the validity of these models at the background level in order to verify if this new formalism provides us with plausible alternative models to explain the late time acceleration of the universe. Indeed, this novel approach provides a different perspective on the formulation of observationally reliable alternative models of gravity.

I. INTRODUCTION

General Relativity is traditionally described in terms of the Levi-Civita connection, which conforms the basis of Riemannian geometry. This choice relies on the assumption of a torsion and nonmetricity free geometry. Within this framework, the Ricci curvature scalar R acts as the building block of the spacetime. Although this is usually done for historical reasons it is important to keep in mind that the connection has a more general expression [1, 2] and General Relativity can be described in terms of different geometries from the Riemannian one. One of the alternatives is what is called teleparallel gravity [3], where the gravitational force is driven by the torsion, T. Although this formalism was formally proposed in [3], Einstein himself already used such a geometry in one of his unified field theory attempts [4]. Another possible alternative is symmetric teleparallel gravity [5], where one considers a vanishing curvature and torsion, and it is the nonmetricity, Q, that mediates the gravitational interaction. Some other interesting cases can be thought as, for example, a geometry where torsion carries part of the gravitational force and nonmetricity carries the rest. Along with the increasing interest on extended theories of gravity, all these alternative geometries have been explored due to the fact that the intrinsic implications and features of the gravitational theories could be different to the ones corresponding to Riemannian geometry [6–12].

Indeed, by making assumptions on the affine connection, one is essentially specifying a metric-affine geometry

[13]. Recall that the metric tensor $g_{\mu\nu}$ can be considered as a generalization of the gravitational potential, and it is essentially used to define notions such as angles, distances and volumes, while the affine connection $\Gamma^\mu{}_{\alpha\beta}$ defines parallel transport and covariant derivatives. In this context, as mentioned above, a basic result in differential geometry states that the general affine connection may be decomposed into the following three independent components [1, 2]:

$$\Gamma^\lambda{}_{\mu\nu} = \{{}^\lambda{}_{\mu\nu}\} + K^\lambda{}_{\mu\nu} + L^\lambda{}_{\mu\nu}, \qquad (1.1)$$

where $\{{}^\lambda{}_{\mu\nu}\} \equiv \frac{1}{2}g^{\lambda\beta}\left(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}\right)$ is the Levi-Civita connection of the metric $g_{\mu\nu}$; the term $K^\lambda{}_{\mu\nu} \equiv \frac{1}{2}T^\lambda{}_{\mu\nu} + T_{(\mu}{}^\lambda{}_{\nu)}$ is the contortion, with the torsion tensor defined as $T^\lambda{}_{\mu\nu} \equiv 2\Gamma^\lambda{}_{[\mu\nu]}$; and finally the disformation $L^\lambda{}_{\mu\nu}$ is given by

$$L^\lambda{}_{\mu\nu} \equiv \frac{1}{2}g^{\lambda\beta}\left(-Q_{\mu\beta\nu} - Q_{\nu\beta\mu} + Q_{\beta\mu\nu}\right), \qquad (1.2)$$

which is defined in terms of the nonmetricity tensor, $Q_{\alpha\mu\nu} \equiv \nabla_\alpha g_{\mu\nu}$.

In this work we will focus on a torsion and curvature free geometry, which is exclusively defined by the nonmetricity $Q_{\alpha\mu\nu}$. As this is a novel approach, no cosmological tests have been carried out so far, and its exploration will hopefully offer some insight on the late accelerated expansion of the universe. Within the scenario of modified gravity alternatives, we will take as an initial point the idea of generalizing Q-gravity in an analogous manner to what has been done with $f(R)$ theories. We will start from the $f(Q)$-type of theories presented in [11] and rewrite some proposals in the redshift approach [14] in order to explore the cosmological background evolution within this kind of geometry.

More specifically, in [14], the $f(R)$ Lagrangian was reformulated as an explicit function of the redshift, i.e.,

[a] ruth.lazkoz@ehu.es
[b] fslobo@fc.ul.pt
[c] maria.ortiz@ehu.eus
[d] vincenzo.salzano@usz.edu.pl

as $f(z)$. Thus, various different polynomial parameterizations $f(z)$ were proposed, including new terms which would allow for deviations from the ΛCDM model, and were thoroughly confronted with observational data. In fact, this novel approach provides a different perspective for the development of new and observationally reliable models of gravity.

The work is organized as follows: We set the stage in Sec. II, by briefly outlining the general formalism of $f(Q)$ gravity. In Sec. III, we reformulate the $f(R)$ Lagrangian as an explicit function of the redshift, and provide reasonable $f(z)$ models from the outset, which are to be tested at a later stage, by numerically solving the gravitational field equations, in order to test their validity and study their deviation with respect to the ΛCDM scenario. In Sec. IV we describe the observational data used in our analysis, namely, the expansion rate data from early-type galaxies, Type Ia Supernovae, Quasars, Gamma Ray Bursts, Baryon Acoustic Oscillations data and Cosmic Microwave Background distance priors. In Sec. V, we discuss our results and finally conclude in Sec. VI.

II. SETTING THE STAGE: $f(Q)$ GRAVITY

Consider the proposal of $f(Q)$ gravity given by the following action [11]:

$$S = \int \left[\frac{1}{2} f(Q) + \mathcal{L}_m \right] \sqrt{-g} \ d^4 x, \qquad (2.1)$$

where $f(Q)$ is an arbitrary function of the nonmetricity Q, g is the determinant of the metric $g_{\mu\nu}$ and $\mathcal{L}_m$ is the matter lagrangian density.

The nonmetricity tensor is defined as

$$Q_{\alpha\mu\nu} = \nabla_\alpha g_{\mu\nu} \,, \qquad (2.2)$$

and its two traces as follows:

$$Q_\alpha = Q_\alpha{}^\mu{}_\mu \,, \quad \tilde{Q}_\alpha = Q^\mu{}_{\alpha\mu} \,. \qquad (2.3)$$

It is also useful to introduce the superpotential

$$4P^\alpha{}_{\mu\nu} = -Q^\alpha{}_{\mu\nu} + 2Q_{(\mu}{}^\alpha{}_{\nu)} - Q^\alpha g_{\mu\nu}$$
$$- \tilde{Q}^\alpha g_{\mu\nu} - \delta^\alpha_{(\mu} Q_{\nu)} \,. \qquad (2.4)$$

One can readily check that $Q = -Q_{\alpha\mu\nu} P^{\alpha\mu\nu}$ (with our sign conventions that are the same as in Ref. [11]).

The energy-momentum tensor is given by

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta \sqrt{-g} \, \mathcal{L}_m}{\delta g^{\mu\nu}} \,, \qquad (2.5)$$

and for notational simplicity, we introduce the following definition

$$f_Q = f'(Q) \,. \qquad (2.6)$$

Varying the action (2.1) with respect to the metric, one obtains the gravitational field equation given by

$$\frac{2}{\sqrt{-g}} \nabla_\alpha \left(\sqrt{-g} f_Q P^\alpha{}_{\mu\nu} \right) + \frac{1}{2} g_{\mu\nu} f$$
$$+ f_Q \left(P_{\mu\alpha\beta} Q_\nu{}^{\alpha\beta} - 2Q_{\alpha\beta\mu} P^{\alpha\beta}{}_\nu \right) = -T_{\mu\nu} \,, \quad (2.7)$$

and varying (2.1) with respect to the connection, one obtains

$$\nabla_\mu \nabla_\nu \left(\sqrt{-g} f_Q P^{\mu\nu}{}_\alpha \right) = 0 \,. \qquad (2.8)$$

With the formalism of $f(Q)$ gravity specified, we will next consider cosmological applications, by reformulating the $f(Q)$ Lagrangian as an explicit function of the redshift, namely, as $f(z)$. Furthermore, by providing reasonable $f(z)$ models from the outset, i.e. at the level of the action, so that at a later stage one can numerically solve the dynamics of the universe, to test their validity and study their deviation with respect to the ΛCDM scenario.

III. THE $f(z)$ APPROACH

A. Cosmology

Considering a FLRW universe represented by the following isotropic, homogeneous and spatially flat line element

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j \,, \qquad (3.1)$$

and taking into account the energy-momentum tensor of a perfect fluid, given by $T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$, where ρ and p are the thermodynamic energy density and isotropic pressure, we obtain the Friedmann and Raychadhuri equations, given by [15]

$$3H^2 = \frac{1}{2f_Q} \left(-\rho + \frac{f}{2} \right) \,, \qquad (3.2)$$

$$\dot{H} + 3H^2 + \frac{\dot{f}_Q}{f_Q} H = \frac{1}{2f_Q} \left(p + \frac{f}{2} \right) \,, \qquad (3.3)$$

respectively, where the overdot is defined as $\cdot \equiv d/dt$. In addition to this, consider the energy conservation equation for standard energy-matter perfect fluids:

$$\dot{\rho}_i + 3H \left(\rho_i + 3p_i \right) = 0, \qquad (3.4)$$

where the suffix i stands for matter, radiation or any other fluid in the stress-energy tensor.

As our goal is to propose $f(z)$ models and find the evolution of the background we will manipulate Eq. (3.2) and write it in terms of the redshift. We perform thus a change of variable taking into account that in a FLRW geometry, $Q = 6H^2$ holds and then $f_Q = \frac{f_z}{6H_z^2}$ getting

$$H^2 = \frac{(H^2)_z}{f_z} \left(-\rho + \frac{f}{2} \right) \,. \qquad (3.5)$$

where the subindex z denotes the derivative with respect to the redshift.

In this work we will follow an analogous approach to the one outlined in [14]: we will reformulate an $f(Q)$ model as an explicit function of redshift, $f(z)$, solve Eq. (3.5) numerically, and then we will apply the obtained $H(z)$ to observational data.

B. Specific $f(z)$ proposals

In order to select some $f(z)$ models that could be interesting to study we first take a look at an $f(Q)$ model that mimics a ΛCDM background expansion, which is $f_\Lambda(Q) = -Q$. This can be clearly seen replacing this expression in Eq. (3.5). Assuming a universe filled with matter, radiation and a cosmological constant, the Friedmann equation reads

$$H^2(z) = \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + (1 - \Omega_m - \Omega_r), \quad (3.6)$$

where $\Omega_i = 8\pi\rho_i/(3H_0^2)$ is the dimensionless density parameter, and $i = m, r$ refers to matter and radiation, respectively.

Then, $f_\Lambda(Q)$ in terms of z gives

$$f_\Lambda(z) = -6(1 - \Omega_m - \Omega_r) - 6\Omega_m(1+z)^3 - 6\Omega_r(1+z)^4. \quad (3.7)$$

Here we can notice that a ΛCDM background in the redshift formalism would be described by a constant term, a third order term and a fourth order one. Now, we would like to choose $f(z)$ models which are somehow generalizations of Eq. (3.7). To do that we have found useful to make use of the tendencies we observe in f_Λ. Even if ΛCDM is not the definitive model to explain the late-time expansion of the universe, it does describe this late phase quite satisfactorily. Then, it is reasonable to propose models which satisfy the trends it shows at low and high redshift, that is,

$$\lim_{z \to 0} f(Q(z)) \propto \text{const}, \quad (3.8)$$

$$\lim_{z \to \infty} f(Q(z)) \propto (1+z)^4. \quad (3.9)$$

In fact, due to the relation between Q and H this will be satisfied at least for all the dark energy models studied in [14].

C. Models considered

As in [14], we will propose simple but well-motivated polynomical generalizations of the most fundamental model $f_\Lambda(z)$. As the high and low redshift limits lie within a constant term and an order four polynomial, we have constructed our proposals by adding some terms consisting on powers of $(1+z)$ which are within these two limits. Moreover, we have selected small powers as this

allows us to introduce small modifications that may induce changes in the background evolution but preserving the desired behaviour.

In the following analysis, we consider the following models:

1. $f_0 + f_3(1+z)^3 + f_4(1+z)^4,$

2. $f_{12}(1+z)^{1/2} + f_3(1+z)^3 + f_4(1+z)^4,$

3. $f_{12}(1+z)^{1/2} + f_1(1+z) + f_2(1+z)^2 + f_3(1+z)^3 + f_4(1+z)^4,$

4. $f_{14}(1+z)^{1/4} + f_3(1+z)^3 + f_4(1+z)^4,$

5. $f_{14} + f_1(1+z) + f_2(1+z)^2 + f_3(1+z)^3 + f_4(1+z)^4,$

6. $f_{16}(1+z)^{1/6} + f_3(1+z)^3 + f_4(1+z)^4,$

7. $f_{16}(1+z)^{1/6} + f_1(1+z) + f_2(1+z)^2 + f_3(1+z)^3 + f_4(1+z)^4,$

where the factors f_i are free parameters corresponding to each model.

IV. OBSERVATIONAL DATA

In this section, we outline the cosmological data used in this work. We use various current observational data to constrain the $f(z)$ models described in the previous section, and consider the data which is related to the expansion history of the universe, i.e., the ones describing the distance-redshift relations. More specifically, we use the expansion rate data from early-type galaxies, Type Ia Supernovae, Quasars, Gamma Ray Bursts, Baryon Acoustic Oscillations data and Cosmic Microwave Background distance priors.

A. Hubble data

Early-type galaxies (ETGs) displaying a passive evolution provide Hubble parameter measurements through estimations of their differential evolution. Compilations of such observations can be regarded as cosmic chronometers, and we use a sample covering the redshift range $0 < z < 1.97$, which received a recent update in [16]. For these measurement one can construct a χ_H^2 estimator as follows:

$$\chi_H^2 = \sum_{i=1}^{24} \frac{(H(z_i, \boldsymbol{\theta}) - H_{obs}(z_i))^2}{\sigma_H^2(z_i)}, \quad (4.1)$$

where $\sigma_H(z_i)$ stand for the observational errors on the measured values $H_{obs}(z_i)$, and $\boldsymbol{\theta}$ is the vector of the cosmological background parameters.

B. Pantheon Supernovae data

One of the latest Type Ia Supernovae (SNeIa) data compilation is the Pantheon compilation [17]. We choose this set of 1048 SNe, which covers the redshift range $0.01 < z < 2.26$, and use it in the usual manner to define

$$\chi^2_{SN} = \Delta\boldsymbol{\mathcal{F}}^{SN} \cdot \mathbf{C}^{-1}_{SN} \cdot \Delta\boldsymbol{\mathcal{F}}^{SN} \,. \qquad (4.2)$$

Here $\Delta\boldsymbol{\mathcal{F}} = \mathcal{F}_{\text{theo}} - \mathcal{F}_{\text{obs}}$ represents the difference between the theoretical and the observed value of the observable quantity for each SNeIa, which is the distance modulus. The term $\mathbf{C}_{SN}$ gives the total covariance matrix. Once a specific cosmological model has been chosen, the predicted distance modulus of the SNe, μ, is defined as

$$\mu(z,\boldsymbol{\theta}) = 5\log_{10}[d_L(z,\boldsymbol{\theta})] + \mu_0 \,, \qquad (4.3)$$

where D_L is the dimensionless luminosity distance given by

$$d_L(z,\theta_c) = (1+z)\int_0^z \frac{dz'}{E(z')} \,, \qquad (4.4)$$

with $E(z) = H(z)/H_0$. Notwithstandingly, our χ^2_{SN} above would contain the nuisance parameter μ_0, which in turn is a function of the Hubble constant, the speed of light c and the SNeIa absolute magnitude. This inconvenient degeneracy intrinsic to the definition of the parameters can be dealt with if we marginalize analytically over μ_0. We refer the reader to [18] for details on this procedure.

The χ^2 estimator thus obained reads

$$\chi^2_{SN} = a + \log\left(\frac{d}{2\pi}\right) - \frac{b^2}{d}, \qquad (4.5)$$

where $a \equiv (\Delta\boldsymbol{\mathcal{F}}_{SN})^T \cdot \mathbf{C}^{-1}_{SN} \cdot \Delta\boldsymbol{\mathcal{F}}_{SN}$, $b \equiv \left(\Delta\boldsymbol{\mathcal{F}}^{SN}\right)^T \cdot \mathbf{C}^{-1}_{SN} \cdot \mathbf{1}$ and $d \equiv \mathbf{1} \cdot \mathbf{C}^{-1}_{SN} \cdot \mathbf{1}$, with $\mathbf{1}$ being the identity matrix.

C. Quasars

The distance modulus μ of some 808 quasars in the redshift range $0.06 < z < 6.28$ can be constructed from their UV and X-ray fluxes. (Following Eq. (5) in [19]).

In order to test our model we are interested in the distance modulus μ, so we compute it using Eq. (5) given in [19], i.e.,

$$\mu = \frac{5}{2(\gamma-1)}[\log(F_X) - \gamma\log(F_{UV}) - \beta'] \,, \qquad (4.6)$$

where $\gamma = 0.6$ is the average value of the free parameter which relates both fluxes and β' is a free scaling factor. As before we compute the theoretical distance modulus

using Eq. (4.3) and marginalize over the additive constant terms so that the final χ^2 is given by Eq. (4.5) where now we have: $a \equiv \left(\Delta\boldsymbol{\mathcal{F}}^Q\right)^T \cdot \mathbf{C}^{-1}_Q \cdot \Delta\boldsymbol{\mathcal{F}}^Q$, $b \equiv \left(\Delta\boldsymbol{\mathcal{F}}^Q\right)^T \cdot \mathbf{C}^{-1}_Q \cdot \mathbf{1}$ and $d \equiv \mathbf{1} \cdot \mathbf{C}^{-1}_Q \cdot \mathbf{1}$.

D. Gamma Ray Bursts

We work with the Mayflower sample, which consist on 79 Gamma Ray Bursts (GRBs) and cover the redshift range $1.44 < z < 8.1$ [20]. We use Eq. (4.3) to compute the theoretical distance modulus and then marginalize over the constant additive term. This way, the final χ^2 estimator is given by Eq. (4.5) with $a \equiv \left(\Delta\boldsymbol{\mathcal{F}}^G\right)^T \cdot \mathbf{C}^{-1}_G \cdot \Delta\boldsymbol{\mathcal{F}}^G$, $b \equiv \left(\Delta\boldsymbol{\mathcal{F}}^G\right)^T \cdot \mathbf{C}^{-1}_G \cdot \mathbf{1}$ and $d \equiv \mathbf{1} \cdot \mathbf{C}^{-1}_G \cdot \mathbf{1}$.

E. Baryon Acoustic Oscillations

In this section, we consider the χ^2_{BAO} estimator for Baryon Acoustic Oscillations (BAO), which is defined in the following manner

$$\chi^2_{BAO} = \Delta\boldsymbol{\mathcal{F}}^{BAO} \cdot \mathbf{C}^{-1}_{BAO} \cdot \Delta\boldsymbol{\mathcal{F}}^{BAO} \,, \qquad (4.7)$$

where the quantity $\mathcal{F}^{BAO}$ depends on the survey which is considered. We use data from the WiggleZ Dark Energy Survey, which are evaluated at redshifts 0.44, 0.6 and 0.73, and are shown in Table 1 of [21]. The quantities we consider are the acoustic parameter

$$A(z) = 100\sqrt{\Omega_m h^2}\frac{D_V(z)}{c\,z} \,, \qquad (4.8)$$

and the Alcock-Paczynski distortion parameter, given by the following relation

$$F(z) = (1+z)\frac{D_A(z)\,H(z)}{c} \,, \qquad (4.9)$$

where D_A is the angular diameter distance

$$D_A(z) = \frac{c}{H_0}\frac{1}{1+z}\int_0^z \frac{\mathrm{d}z'}{E(z')} \,, \qquad (4.10)$$

and D_V is the geometric mean of the physical angular diameter distance, D_A, and of the Hubble function, $H(z)$, and is written as

$$D_V(z) = \left[(1+z)^2 D_A^2(z)\frac{c\,z}{H(z,\boldsymbol{\theta})}\right]^{1/3} \,. \qquad (4.11)$$

Moreover, we have considered the data from the SDSS-III Baryon Oscillation Spectroscopic Survey (BOSS) DR12, described in [22] and given by

$$D_M(z)\frac{r_s^{fid}(z_d)}{r_s(z_d)}, \qquad H(z)\frac{r_s(z_d)}{r_s^{fid}(z_d)} \,. \qquad (4.12)$$

Here, the factor $r_s(z_d)$ denotes the sound horizon evaluated at the dragging redshift z_d, the quantity $r_s^{fid}(z_d)$ is the same sound horizon but calculated for a given fiducial cosmological model used, and is equal to 147.78 Mpc [22].

We consider the approximation [23]

$$z_d = \frac{1291(\Omega_m\,h^2)^{0.251}}{1 + 0.659(\Omega_m\,h^2)^{0.828}} \left[1 + b_1(\Omega_b\,h^2)^{b2}\right] , \quad (4.13)$$

for the redshift of the drag epoch, where the factors b_1 and b_2 are given by

$$b_1 = 0.313(\Omega_m\,h^2)^{-0.419}\left[1 + 0.607(\Omega_m\,h^2)^{0.6748}\right] ,$$
$$b_2 = 0.238(\Omega_m\,h^2)^{0.223} ,$$

respectively. In addition to this, the sound horizon is defined as:

$$r_s(z) = \int_z^\infty \frac{c_s(z')}{H(z')}\mathrm{d}z' , \quad (4.14)$$

where the sound speed is given by

$$c_s(z) = \frac{c}{\sqrt{3(1 + \overline{R}_b\,(1+z)^{-1})}} , \quad (4.15)$$

and $\overline{R}_b$ is defined as

$$\overline{R}_b = 31500\Omega_b\,h^2\,(T_{CMB}/2.7)^{-4} , \quad (4.16)$$

with $T_{CMB} = 2.726$ K. We have taken into account the point $D_V(z = 1.52) = 3843 \pm 147\frac{r_s(zd)}{r_s^{fid}(z_d)}$ Mpc [24] from the extended Baryon Oscillation Spectroscopic Survey (eBOSS).

We have also added data points from Quasar-Lyman α Forest from SDSS-III BOSS DR11 [25]:

$$\frac{D_A(z = 2.36)}{r_s(z_d)} = 10.8 \pm 0.4 , \quad (4.17)$$

$$\frac{c}{H(z = 2.36)r_s(z_d)} = 9.0 \pm 0.3 , \quad (4.18)$$

in our analysis.

F. Cosmic Microwave Background data

The χ^2_{CMB} estimator for the Cosmic Microwave Background (CMB) is defined as

$$\chi^2_{CMB} = \Delta\boldsymbol{\mathcal{F}}^{CMB} \cdot \mathbf{C}^{-1}_{CMB} \cdot \Delta\boldsymbol{\mathcal{F}}^{CMB} , \quad (4.19)$$

where $\mathcal{F}^{CMB}$ is a vector of quantities used in [26], and we have considered the *Planck* 2015 data release in order to give the shift parameters defined in [27]. These parameters are related to the positions of the CMB acoustic peaks that depend on the geometry of the specific model

considered and, as such, can be used to discriminate between dark energy models of the different nature. Their definitions are:

$$R(\boldsymbol{\theta}) \equiv \sqrt{\Omega_m H_0^2}\frac{r(z_*,\boldsymbol{\theta})}{c},$$
$$l_a(\boldsymbol{\theta}) \equiv \pi\frac{r(z_*,\boldsymbol{\theta})}{r_s(z_*,\boldsymbol{\theta})} . \quad (4.20)$$

As in the previous data sets, r_s is the comoving sound horizon, evaluated at the photon-decoupling redshift z_*, given by the fitting formula [28]:

$$z_* = 1048\left[1 + 0.00124(\Omega_b h^2)^{-0.738}\right] \times$$
$$\times \left(1 + g_1(\Omega_m h^2)^{g_2}\right) , \quad (4.21)$$

where the factors g_1 and g_2 are given by

$$g_1 = \frac{0.0783(\Omega_b h^2)^{-0.238}}{1 + 39.5(\Omega_b h^2)^{-0.763}} ,$$

$$g_2 = \frac{0.560}{1 + 21.1(\Omega_b h^2)^{1.81}} ,$$

respectively, and r is the comoving distance defined as

$$r(z,\boldsymbol{\theta}) = \frac{c}{H_0}\int_0^z \frac{\mathrm{d}z'}{E(z',\boldsymbol{\theta})}\mathrm{d}z' . \quad (4.22)$$

G. Monte Carlo Markov Chain (MCMC)

We have implemented an MCMC code [29, 30] so that we can test the predictions of our theory with the available data. The goal of this technique is to minimize the total χ^2 defined as

$$\chi^2 = \chi^2_H + \chi^2_{SN} + \chi^2_Q + \chi^2_G + \chi^2_{BAO} + \chi^2_{CMB} . \quad (4.23)$$

Also, in order to set up the degree of reliability of the models, we use the Bayesian Evidence, $\mathcal{E}$, which is in general recognized as the most reliable statistical comparison tool, despite some issues related to the choice of the priors [31], which we overcome by choosing wide-enough priors ranges. We compute it using the algorithm described in [32]. After calculating the Bayesian Evidence, one can obtain the Bayes Factor, which is defined as the ratio of evidences of two models, M_i and M_j, $\mathcal{B}^i_j = \mathcal{E}_i/\mathcal{E}_j$. If $\mathcal{B}^i_j > 1$, model M_i is preferred over M_j, given the data. We have used the ΛCDM model, as the reference model M_j.

Even if the Bayes Factor is $\mathcal{B}^i_j > 1$, one cannot state how much better model M_i is with respect to model M_j. For this purpose, we choose the so-called Jeffreys' Scale [33]. Generally, Jeffreys' Scale affirms that: if $\ln\mathcal{B}^i_j < 1$, the evidence in favor of model M_i is not significant; if $1 < \ln\mathcal{B}^i_j < 2.5$, the evidence is substantial; if $2.5 < \ln\mathcal{B}^i_j < 5$, it is strong; and if $\ln\mathcal{B}^i_j > 5$, it is decisive. Values of $\ln\mathcal{B}^i_j$ which are negative can be seen as evidence against model M_i (or in favor of model M_j). In [31], it is demonstrated that Jeffreys' scale is not a fully-reliable tool to compare

models, but at the same time the statistical validity of the Bayes factor as an efficient model-comparison tool is not doubted: a Bayes factor $\mathcal{B}_j^i > 1$ firmly states that the model i is more likely than model j. In the following, we present our results in both contexts so that the reader can make his/her interpretation.

V. RESULTS OF THE OBSERVATIONAL TESTS

After implementing the MCMC code, we have obtained the values for the background parameters that appear in Table I. There are some general results that can be summarized before analyzing in detail model by model. It can be noticed that the parameters Ω_m and Ω_b do not present a relevant variation from one model to another, being in perfect agreement with *Planck*'s predictions [34] in every case. Some more interesting feature can be observed in the value of the Hubble constant, defined as $H_0 = 100 \ h$. With respect to the existing tension between the value obtained by local measurements [35] and the one predicted by *Planck*, our result for H_0 are perfectly consistent with the *Planck* value.

A. Model 1: ΛCDM

The first model we have considered consists on a constant term plus a third and a fourth order term. These are exactly the terms which correspond to a $f(z)$ theory mimicking a ΛCDM background in the Q-formalism, i.e., Eq.(3.7). According to this equation, one may expect that the values of f_0, f_3 and f_4 are somehow related with Ω_m and Ω_r (an analog computation was done in [14]). In fact, we could identity the quantities

$$f_0 = -6(1 - \Omega_m - \Omega_r), \qquad (5.1)$$
$$f_3 = -6\Omega_m, \qquad (5.2)$$
$$f_4 = -6\Omega_r. \qquad (5.3)$$

If we substitute here the values of Ω_m, Ω_r for ΛCDM according to Table I we obtain the following numerical values for Eqs. (5.1)-(5.3):

$$f_0 = -4.1, \qquad f_3 = -1.88, \qquad f_4 = -0.00053. \quad (5.4)$$

Comparing with the values of f_0, f_3, f_4 in the table for this model, we see that for the parameters f_3 anf f_4 there is an agreement at 1σ level. Nevertheless, we can also see that the parameter f_0 agrees in the order of magnitude but presents a discrepancy with respect to the one in the table.

B. Models 2-3

These two models do not include a constant term which could play the role of an effective cosmological constant.

Instead, they are characterized by including a $1/2$ power in the polynomial; and model 3 also presents an order one and an order two term, corresponding to the parameteres f_1 and f_2. It must be noticed that model 2 does not appear in the table as it does not reach a χ^2 which stabilizes and, moreover, it is higher than all other cases, which means it is clearly discarded by data. For model 3 it is not possible to make a concrete interpretation of the values of f_1 and f_2, because the corresponding histograms are highly irregular and far from being gaussian, so that the error bars are just an approximate and indicative estimation of their parameter space width. However, there is one interesting manner we can extract information from these two quantities: when looking at the parameters f_3 and f_4, we see their values are not very far (and statistically consistent) with those from model 1. Thus, we can infer that the role played by f_1 and f_2 is marginal. Moreover, the Bayes Factor of model 3 is the highest in the sample; which means that it is the most disfavoured by data.

C. Models 4-7

As well as models 2 and 3, models 4-7 do not include a constant term, but they perform much better. Model 4 consists on a $1/4$ polynomial term plus a third and a fourth order term. Model 5 includes the same free parameters plus the f_1 and f_2 terms. The value of f_{14} does not vary significantly between these two models, presenting an agreement at 1σ level. On the one hand, it is interesting to notice that the parameter f_1 is consistent with zero at approximately 2σ, while f_2 contains zero at 1σ. Thinking about results for model 3, these results could confirm that the presence of any effect of such order on cosmological scales can be quite confidently discarded. Finally, for f_3 and f_4, the obtained values are quite in concordance with those of model 1.

Model 6 and 7 have been constructed in a similar way. Model 6 is described by a $1/6$ polynomial term plus the usual third and fourth order ones, while model 7 contains, in addition, f_1 and f_2 parameters. Again, the quantity corresponding to the smallest power, f_{16} in this case, does not vary a lot between model 6 and 7. Moreover, in this case we have found more significant results for f_1 and f_2, as they are both consistent with zero at 1σ level. Again, f_3 and f_4 seem not to depend much on the model, having similar values in every model. Thus, we can infer from these results that cosmological data shows a statistical preference for small-order terms, i.e. for a lower-order dependence of the additional terms with redshift. In other words, even in the $f(Q)$ approach, data seem to confirm that any alternative solution must behave closely as a cosmological constant, at least for what concerns the cosmological background.

With respect to the reliability of the models in comparison with the ΛCDM model, the differences in the Bayes factor and Jeffreys' scale are so small that we cannot ex-

TABLE I. Results

model	Ω_m	Ω_b	h	f_0	f_{16}	f_{14}	f_{12}	f_1	f_2	f_3	f_4	$\mathcal{B}_\Lambda^i$	$\ln \mathcal{B}_\Lambda^i$
1	$0.313^{+0.007}_{-0.007}$	$0.046^{0.002}_{0.002}$	$0.69^{+0.02}_{-0.02}$	$-9.6^{+0.7}_{-0.8}$	-	-	-	-	-	$-1.95^{+0.04}_{-0.04}$	$-0.00046^{+0.00004}_{-0.00004}$	1	0
3	$0.317^{+0.006}_{-0.007}$	$0.047^{+0.003}_{-0.002}$	$0.69^{+0.02}_{-0.02}$	-	-	-	$-22.1^{+1.7}_{-2.2}$	$8.2^{+2.2}_{-3.3}$	$-0.5^{+0.3}_{-0.5}$	$-2.06^{+0.09}_{-0.10}$	$-0.00039^{+0.00004}_{-0.00005}$	0.394	-0.931
4	$0.317^{+0.007}_{-0.007}$	$0.047^{+0.002}_{-0.002}$	$0.69^{+0.02}_{-0.02}$	-	-	$-14.3^{+1.3}_{-1.4}$	-	-	-	$-2.06^{+0.05}_{-0.04}$	$-0.00040^{+0.00004}_{-0.00004}$	0.712	-0.339
5	$0.315^{+0.007}_{-0.006}$	$0.047^{+0.003}_{-0.003}$	$0.69^{+0.02}_{-0.02}$	-	-	$-13.0^{+1.6}_{-2.1}$	-	$2.2^{+0.9}_{-1.0}$	$-0.28^{0.46}_{-0.27}$	$-1.99^{+0.06}_{-0.05}$	$-0.00045^{+0.00005}_{-0.00004}$	0.791	-0.234
6	$0.314^{+0.009}_{-0.008}$	$0.046^{+0.003}_{-0.003}$	$0.69^{+0.02}_{-0.02}$	-	$-12.5^{+1.2}_{-1.4}$	-	-	-	-	$-2.06^{+0.05}_{-0.04}$	$-0.00042^{+0.00004}_{-0.00004}$	0.917	-0.087
7	$0.315^{+0.008}_{-0.007}$	$0.047^{+0.003}_{-0.002}$	$0.69^{+0.02}_{-0.02}$	-	$-12.2^{+1.2}_{-1.0}$	-	-	$0.05^{+0.14}_{-0.07}$	$0.004^{+0.037}_{-0.053}$	$-2.01^{+0.04}_{-0.04}$	$-0.00042^{+0.00003}_{-0.00004}$	1.011	0.011

tract much information from them. Although one cannot set decisive conclusions about the preference of one model with respect to the standard ΛCDM, this new formalism seems to be a viable alternative within the study of the late-time expansion of the universe.

VI. CONCLUSIONS

Within the context of modified gravity we have chosen a novel geometrical scenario based on the nonmetricity, Q, to obtain observational constraints for the background quantities of the late-time universe. We have selected the $f(Q)$ type of theories and followed the $f(z)$ approach to give some phenomenologically motivated models to test against observational data. We have used a wide variety of observational data sets to check the validity of our proposals. As an interesting result we have seen that order one and order two terms, i.e., f_1 and f_2, are, in general, compatible with zero. Moreover, the values we obtain for the rest of parameters are very similar for a model with and without these two parameters, not showing a significant dependence on their presence. This result coincides with the one obtained in the paper whose approach we are following.

We have also found that some of the modified parameters that we introduce, f_3 and f_4 more specifically, could be related in some way with the background parameters Ω_m and Ω_r. Finally, an interesting result we want to remark is that the value obtained for the Hubble parameter, H_0, lies close to the *Planck* estimation.

From the statistical point of view we cannot state a clear preference with respect to any of the considered models. In general, the values of the Bayes factor and their interpretation in terms of the Jeffreys' scale show a marginal and weak evidence in favor of the scenarios which slightly depart from the standard ΛCDM model. Nevertheless, one should not forget that that $f(R)/f(Q)$ scenarios have an inherent multidimensionality as regards the space of parameters, superior to that of standard dark energy settings. In spite of that, we have not fine-tuned our extended models with any a priori constraint, but we have left the corresponding parameters totally free to vary. Thus, the post-analysis consistency of the values of the background parameters for such models is a further asset (which somehow escapes the pure statistical evaluation) and can be considered as a hint pointing to continue the study of alternative theories of gravity within this kind of approach. This work can be understood as a first step in the observational study of a new class of modified gravity theories which lay in a geometry described by the nonmetricity. By going deeper in the cosmological and observational studies of these yet unexplored theories we may find interesting information which may contribute to our understanding of the evolution of the late-time universe.

[1] F. W. Hehl, J. D. McCrea, E. W. Mielke, and Y. Ne'eman, Phys. Rept. **258**, 1 (1995), arXiv:gr-qc/9402012 [gr-qc].

[2] T. Ortin, *Gravity and Strings* (2015).

[3] R. Aldrovandi and J. Pereira, *Teleparallel gravity. An introduction* (2014).

[4] A. Einstein, Sitzungsber. Preuss. Akad. Wiss. **217** (1928).

[5] J. M. Nester and H.-J. Yo, Chin. J. Phys. **37**, 113 (1999), arXiv:gr-qc/9809049 [gr-qc].

[6] R. Ferraro and F. Fiorini, Phys. Rev. **D78**, 124019 (2008), arXiv:0812.1981 [gr-qc].

[7] E. V. Linder, Phys. Rev. **D81**, 127301 (2010), [Erratum: Phys. Rev.D82,109902(2010)], arXiv:1005.3039 [astro-ph.CO].

[8] Y.-F. Cai, S. Capozziello, M. De Laurentis, and E. N. Saridakis, Rept. Prog. Phys. **79**, 106901 (2016), arXiv:1511.07586 [gr-qc].

[9] C.-Q. Geng, C.-C. Lee, E. N. Saridakis, and Y.-P. Wu, Phys. Lett. **B704**, 384 (2011), arXiv:1109.1092 [hep-th].

[10] L. Jarv and A. Toporensky, Phys. Rev. **D93**, 024051 (2016), arXiv:1511.03933 [gr-qc].

[11] J. B. Jiménez, L. Heisenberg, and T. Koivisto, Phys. Rev. **D98**, 044048 (2018), arXiv:1710.03116 [gr-qc].

[12] A. Conroy and T. Koivisto, Eur. Phys. J. **C78**, 923 (2018), arXiv:1710.05708 [gr-qc].

[13] L. Järv, M. Rünkla, M. Saal, and O. Vilson, Phys. Rev. **D97**, 124025 (2018), arXiv:1802.00492 [gr-qc].

[14] R. Lazkoz, M. Ortiz-Baños, and V. Salzano, Eur. Phys. J. **C78**, 213 (2018), arXiv:1803.05638 [astro-ph.CO].

[15] T. Harko, T. S. Koivisto, F. S. N. Lobo, G. J. Olmo, and D. Rubiera-Garcia, Phys. Rev. **D98**, 084043 (2018), arXiv:1806.10437 [gr-qc].

[16] M. Moresco, Mon. Not. Roy. Astron. Soc. **450**, L16 (2015), arXiv:1503.01116 [astro-ph.CO].

[17] D. M. Scolnic *et al.*, Astrophys. J. **859**, 101 (2018), arXiv:1710.00845 [astro-ph.CO].

[18] A. Conley, R. S. Ellis, and J. D. Neill, ApJS **192 1** (2011).

[19] G. Risaliti and E. Lusso, Astrophys. J. **815**, 33 (2015), arXiv:1505.07118 [astro-ph.CO].

[20] J. Liu and H. Wei, Gen. Rel. Grav. **47**, 141 (2015), arXiv:1410.3960 [astro-ph.CO].

[21] C. Blake *et al.*, Mon. Not. Roy. Astron. Soc. **425**, 405 (2012), arXiv:1204.3674 [astro-ph.CO].

[22] S. Alam *et al.* (BOSS), Mon. Not. Roy. Astron. Soc. **470**, 2617 (2017), arXiv:1607.03155 [astro-ph.CO].

[23] D. J. Eisenstein and W. Hu, Astrophys. J. **496**, 605 (1998).

[24] M. Ata *et al.*, Mon. Not. Roy. Astron. Soc. **473**, 4773 (2018), arXiv:1705.06373 [astro-ph.CO].

[25] A. Font-Ribera *et al.* (BOSS), JCAP **1405**, 027 (2014), arXiv:1311.1767 [astro-ph.CO].

[26] Y. Wang and M. Dai, Phys. Rev. D **94**, 083521 (2016).

[27] Y. Wang and P. Mukherjee, Phys. Rev. **D76**, 103533 (2007), arXiv:astro-ph/0703780 [astro-ph].

[28] W. Hu and N. Sugiyama, Astrophys. J. **471**, 542 (1996), arXiv:astro-ph/9510117 [astro-ph].

[29] R. Lazkoz, V. Salzano, and I. Sendra, Phys. Lett. **B694**, 198 (2011), arXiv:1003.6084 [astro-ph.CO].

[30] S. Capozziello, R. Lazkoz, and V. Salzano, Phys. Rev. **D84**, 124061 (2011), arXiv:1104.3096 [astro-ph.CO].

[31] S. Nesseris and J. Garcia-Bellido, JCAP **1308**, 036 (2013).

[32] P. Mukherjee, D. Parkinson, and A. R. Liddle, Astrophys. J. **638**, L51 (2006).

[33] H. Jeffreys, *The theory of probability*, 3rd ed., Oxford Classic Texts in the Physical Sciences, Vol. 4 (Oxford University Press, The address, 1998).

[34] N. Aghanim *et al.* (Planck), (2018), arXiv:1807.06209 [astro-ph.CO].

[35] A. G. Riess, S. Casertano, W. Yuan, L. M. Macri, and D. Scolnic, Astrophys. J. **876**, 85 (2019), arXiv:1903.07603 [astro-ph.CO].