

# **Right and Duty: Investment Risk Under Diferent Renewable Energy Support Policies**

**Peio Alcorta<sup>1</sup>  [·](http://orcid.org/0000-0002-0625-7777) Maria Paz Espinosa1  [·](http://orcid.org/0000-0003-2189-4253) Cristina Pizarro‑Irizar1,[2](http://orcid.org/0000-0001-6204-5253)**

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# **Abstract**

Renewable energy projects are subject to risk due to the uncertainty of future electricity prices and the amount of energy produced. Public support usually consists of some form of guarantee, either on the price received per MWh supplied or the return on investment per MW installed. This support reduces the investor's risk, which is then assumed by the regulator. However, most of these policies do not only grant the investor the right to receive a guaranteed payment but also limit its potential benefts (i.e., impose an obligation). We propose a model with analytical solutions in which, considering the randomness of the market price, as well as that of the energy production, we quantify the risk removed under diferent types of regulations and assign a value to both the rights and the obligations that each policy entails. Finally, we apply the model to the case of Spain, which has undergone several changes in its green energy support system in recent years. Our results indicate that in the context of low electricity prices, the obligations imposed by most of these policies are negligible compared to the rights received. By contrast, in the context of high electricity prices and increasingly competitive renewable energy sources, the assumed obligations become more notable to the point of the support policy becoming a liability. Nevertheless, a sufficiently risk-averse investor may be incentivized by a policy with negative expected regulatory costs.

**Keywords** Energy investment · Green energy · Policy modeling · Support policies · Investment risk · Risk aversion

**JEL Classification** D04 · L51 · Q48

 $\boxtimes$  Peio Alcorta peio.alcorta@ehu.eus

<sup>&</sup>lt;sup>1</sup> Department of Economics, University of the Basque Country (UPV/EHU), Avda Lehendakari Aguirre 83, 48015 Bilbao, Spain

<sup>&</sup>lt;sup>2</sup> Basque Centre for Climate Change (BC3), UPV/EHU Science Park, Barrio Sarriena s/n, 48940 Leioa, Spain

# **1 Introduction**

Deploying some *renewable energy sources* (RES) remains more costly than other conventional sources or requires a controlled and orderly approach for technical and environmental reasons. As a result, renewable energy projects are usually subject to a regulated remuneration framework, either awarded through competitive means or established administratively.

Support schemes not only provide incentives through the level of remuneration but also remove risk from investment projects. In this paper, we argue that the risk faced by an investor in renewable energy is not only due to the price of electricity but also to the fact that production is uncertain, especially for technologies highly dependent on weather conditions, such as solar and wind. In addition, most support policies not only grant the right to charge a minimum price or receive a minimum remuneration for the energy produced but also usually limit the supplier's potential proft, i.e., impose an obligation.

Our paper aims to assess the value of risk mitigation in diferent support schemes. Due to the unique features of energy markets and the weather dependence of renewable production, analytical models for derivative pricing are not applicable without a series of strong and unrealistic assumptions. Solving this pricing problem accurately in fnancial terms would require relying on numerical methods based on time series, which must be developed and estimated for each specifc case (Caporin et al. [2012](#page-39-0)).

Instead, we propose an analytical model to study and compare the nature of the risks involved under each support scheme. This general framework can be easily applied to diferent technologies, countries, and time horizons by calibrating a few parameters in the model, and it can provide a theoretical framework for analyzing diferent regulatory schemes.

Our contribution is threefold: First, we propose a model with analytical solutions in which we jointly model market prices and generation as correlated stochastic processes and analyze the importance of both the rights and obligations that each policy entails. Second, we contribute a general framework for assessing the value of risk mitigation in RES support policies that allows us to evaluate, using analytical solutions, the risk exposure under diferent known incentive schemes compared to full merchant exposure (under no support scheme) for an investor with varying degrees of risk aversion. Third, we apply the model to the case of Spain, which has undergone numerous changes in its RES support system in recent years, implementing diferent support mechanisms. The proposed model may be helpful for policymakers as it enables them to compare and contrast diferent support policies.

The paper is structured as follows. Section [2](#page-1-0) details diferent support schemes and presents previous work on their valuation. Section [3](#page-4-0) describes the proposed methodology and the mathematical model. Section [4](#page-13-0) contains the results of the numerical application. Section [5](#page-18-0) discusses the main results and their policy implications. Section [6](#page-20-0) concludes.

### <span id="page-1-0"></span>**2 Valuation of Support Schemes: An Overview**

Whether incentives are set by the government or through an auction, they can be categorized according to the payment mechanism involved. There are two general types of "fixed" mechanisms, which we refer to as *fixed-price* (FP) and *fixed-revenue* (FR).<sup>[1](#page-1-1)</sup> Under

<span id="page-1-1"></span><sup>1</sup> We focus on price-based instruments prevalent in Europe and elsewhere, but there are also quantity-based schemes, such as green certificates (Ciarreta et al. [2014a](#page-39-1)).

an FP system, a fxed price is set for each MWh of electricity delivered. If the market price is lower than the strike (fxed) price, the producer receives a premium equal to the diference between these prices; if the market price is higher than the strike, the producer must pay the diference back to the regulator. Under this system, the producers' revenues, and thus the regulator's costs, are subject to the uncertainty of production levels. This payment mechanism corresponds to a *contract for diference* (CfD).

One of the most efective policies to promote renewable energy in the last 20 years has been the *Feed-in Tariff* (FiT) system. A FiT is a specific case of the FP contract, where the strike price is fxed in advance by the regulator, whereas, in regulations based on auctions, the fxed price is usually set during the auction itself. The benefts of FiT subsidies for renewable energy penetration have been paramount (Cardenas Rodriguez et al. [2015;](#page-39-2) Carley et al. [2017](#page-39-3)). However, their main drawback has been the runaway costs they used to entail for the regulator, especially when they were poorly designed. As a result, some countries have phased out FiTs and introduced new systems based on auctions. Indeed, by 2020, many European countries had already transitioned to auction-based support schemes. The design and characteristics of these auctions vary from year to year and country to country (Szabo et al. [2021](#page-40-0)). Most countries are still experimenting and regularly changing their auction design to improve effectiveness and efficiency (Fabra and Montero [2023](#page-39-4)).

Under an FR system, the supplier receives a fxed payment regardless of the amount of electricity produced and the price of electricity. Thus, the generator sells all of its electricity at the market price, and the regulator then pays the diference between the market revenue and the level of fxed revenue specifed in the contract. This system removes the risk of lower-than-expected revenues due to poor generation. However, if the regulator keeps all the upside, it also eliminates the possibility of higher potential revenues due to higher-than-expected generation or market prices.<sup>[2](#page-2-0)</sup> We find an example of the FR mechanism in Spain, where the Spanish government abandoned the FiT incentive mechanism for the *Rate of Return* (RoR) regulation in 2013 because the FiT system entailed excessive costs for the regulator under the incentive level offered at the time. Under this system, policymakers set an appropriate rate of return, and the renewable energy project receives the fxed revenue necessary to achieve that level of proftability.

Some variations of the FP system offer higher flexibility; in addition to guaranteeing a minimum price, they allow some potential beneft from market prices above a certain threshold. For example, in Spain, since 2021, renewable energy auctions under the *Renewable Energy Economic Regime* (REER) regulation consist of a *shared-upside* (SU) mechanism, where the investor bids in the auction for a guaranteed minimum price to be received if the market price of electricity falls below this foor. If, on the other hand, the market price exceeds the foor, the investor and the regulator share the excess remuneration according to a predefned rule that depends on the RES technology used. This mechanism is sometimes called a market-adjusted CfD. Extreme cases of this mechanism are the fxed-price, where

<span id="page-2-0"></span> $2$  It is possible to consider support schemes in which the investor has a guaranteed minimum revenue and has to split with the regulator any upside due to increased prices or production. We do not discuss this theo-retical case as it is irrelevant to our application. Jégard et al. [\(2017](#page-40-1)) offers an overview of some of the support schemes implemented in the EU.

no upside is allowed, and the *one-sided sliding premium*, where the investor has a guaranteed minimum price with unlimited upside. A more detailed explanation of the functioning of the FP and SU mechanisms is available in Alcorta et al. ([2023\)](#page-39-5).

*Real options* (RO) theory is widely used to study renewable energy investments because these projects have unique features, such as high upfront costs, long project lifetimes, and unpredictable cash fows, that make traditional investment analysis methods less efective. Although methods such as *net present value* (NPV) can address uncertainty to some extent, RO valuation directly includes the uncertainty about the future evolution of the parameters determining the value of the project. Instead of considering the likelihood of many scenarios and making individual forecasts for each outcome, a single RO model may encapsulate all possible scenarios. In addition, the RO approach may better refect how investors behave in specifc settings. For instance, Fleten et al. [\(2016](#page-40-2)) fnd that investors in hydroelectric projects in Norway behave consistently with a decision framework based on RO valuation rather than the NPV approach. Also, real options theory can provide a framework that considers investors' fexibility to adjust their investment decisions in response to changing market conditions. This approach can be applied to develop diferent models, each aiming to address diverse questions. A comprehensive review of the evolution of the RO method applied to renewable energy investment can be found in Liu et al. ([2019\)](#page-40-3).

In particular, Haar and Haar [\(2017](#page-40-4)) propose using option theory to quantify the risk the regulator faces when ofering a FiT policy to investors. By guaranteeing a fxed price to the investor, the regulator is responsible for the diference between the market price of electricity and the incentive price, creating market exposure for the regulator. This risk, ultimately transferred to consumers, could, at least in theory, be hedged by buying a series of Euro-pean put options with a selling price equal to the FiT price.<sup>[3](#page-3-0)</sup> The price of such a basket of options would represent the theoretical price of taking the risk imposed by the policy. They model the problem by considering, for each hour of a given year, the possibility of selling the electricity at the FiT-guaranteed price as a put option whose value results from using the *Black–Scholes formula* (Black and Scholes [1973](#page-39-6)). Then, the value of this hourly put multiplies the generation in that hour.

We argue that the risk involved in RES projects is not only due to the uncertainty of electricity prices but also the uncertainty of production, especially for renewables highly dependent on weather conditions. For this reason, estimating the risk removed by a given support should consider the randomness of energy production rather than treating it as an ex-ante known value. Additionally, due to how many electricity markets work, the amount of renewable energy produced is usually negatively correlated with the price of electricity due to the merit order efect (Ballester and Furió [2015](#page-39-7); Fabra and Imelda [2023](#page-39-8)). Therefore, we must consider that both stochastic processes—price and generation—can be correlated. Furthermore, there is the problem that the Black–Scholes equation is only applicable if the underlying asset follows a *geometric Brownian motion* (GBM) process, which is not realistic in the case of an hourly analysis of the problem due to all the seasonal efects present at this granularity.

Finally, in addition to granting the right to receive a guaranteed payment, most support policies impose an obligation by limiting the investor's potential return. Therefore, when analyzing these support programs, it is necessary to consider not only the selling rights granted but also the obligations imposed by the policy.

<span id="page-3-0"></span> $3$  A European put option gives the owner the right, but not the obligation, to sell the underlying asset at a predetermined price on the specifed expiration date.

## <span id="page-4-0"></span>**3 The Model**

This section presents our methodology for the valuation of a support scheme based on the value of the rights and obligations it entails (Sect. [3.1](#page-4-1)). We also calculate the risk premium associated with each incentive system for a risk-averse investor, which provides a measure of the risk under each scheme and the value for the investor of each support system (Sect. [3.2\)](#page-9-0).

#### <span id="page-4-1"></span>**3.1 Rights and Obligations Under Diferent Regulations**

In the absence of a support policy, the revenue received by an investor at the end of each period *t* is simply the product of production  $(X_t)$  and the market price  $(S_t)$  in that period. In contrast, under a support policy, the investor faces a lower risk, but the potential benefts of high market prices or high production levels are limited.

We model the value of each regulatory scheme (*V*) as the result of two opposing contributions. On the one hand, the investor may have the right to sell the electricity produced either at a minimum unit price or at a guaranteed revenue. We denote the value of these rights as *R*. On the other hand, the investor may be obliged to sell the electricity produced either at a maximum price per MWh supplied or at a capped revenue. We denote the value of these obligations as *O*. Therefore, we propose to model the value of the support policy as the diference between these two quantities:

<span id="page-4-3"></span>
$$
V = R - O.\t\t(1)
$$

Suppose that the scheme under discussion offers the investor at time  $t = 0$  the rights to trade the electricity produced at some  $t > 0$  for at least a revenue  $w_{min,t}$ , which in general depends on the total production and the price in that period ( $w_{min,t} = w_{min,t}(X_t, S_t)$ ). If the guaranteed revenue is greater than the market revenue ( $w_{min,t} > X_t S_t$ ), the investor would like to exercise the right to sell the electricity for the guaranteed revenue. Then, the payof of the policy would be the surplus over the market revenue  $(w_{min,t} - X_t S_t)$ . On the other hand, if the guaranteed revenue is less than the market revenue  $(w_{min}, \langle X, S_t)$ , then the investor would not exercise the rights, and the payoff of the policy is zero. Therefore, we defne the value of the rights provided by the regulation for time *t* as the expected value of the discounted payofs that the exercise at time *t* of these rights provides, taking into account the probability that the market will not reach the guaranteed revenue:

<span id="page-4-4"></span>
$$
R_{t} = \mathbb{E}_{0}[e^{-rt}(w_{min,t} - X_{t}S_{t})^{+}],
$$
\n(2)

where *r* is the constant discount rate and  $\mathbb{E}_0$  is the conditional expectation operator given the information at  $t = 0.4$  $t = 0.4$ 

Now suppose that accepting the regulation at time  $t = 0$  forces the investor to trade the electricity produced at a future time  $t > 0$  for a maximum revenue  $w_{max,t}$ , which again generally depends on the total production and price in that period ( $w_{max,t} = w_{max,t}(X_t, S_t)$ ). If the market revenue is higher than the maximum allowed revenue  $(X_tS_t > w_{max,t})$ , the investor is obliged to sell the electricity for the capped revenue. Thus, the investor must give up the surplus over the maximum revenue  $(X_tS_t - w_{max,t})$ . On the other hand, if the market revenue is lower than the maximum revenue  $(X_t S_t \leq w_{max_t})$ , then the investor has no obligations to fulfll, so the amount to give up is zero. Therefore, we defne the value of the

<span id="page-4-2"></span><sup>&</sup>lt;sup>4</sup> We use the standard notation  $z^+ = \max\{z, 0\}$ .

obligations imposed by the regulation for time *t* as the expected value of the discounted amount that the investor must give up under the policy in that period, taking into account the likelihood that the market will exceed the allowed cap:

$$
O_t = \mathbb{E}_0[e^{-rt}(X_t S_t - w_{max,t})^+].
$$
\n(3)

From Eq. [\(1\)](#page-4-3) it can be seen that if  $w_{min,t} = w_{max,t}$ , denoted as  $w_t$ , then subtracting Eqs. ([2](#page-4-4)) and  $(3)$  we get:

<span id="page-5-2"></span><span id="page-5-0"></span>
$$
V_t = \mathbb{E}_0[e^{-rt}(w_t - X_t S_t)].
$$
\n(4)

Due to the defnitions of *R* and *O* in Eqs. ([2](#page-4-4)) and [\(3\)](#page-5-0), respectively, the value *V* aligns with the expected regulatory cost. In our model, we adopt an approach borrowed from option theory where the rights given by the regulation can be understood as a put option and the obligations imposed as a call option. Indeed, Eq. [\(1](#page-4-3)) can be seen as an analogy of the putcall parity (Wilmott [2013\)](#page-40-5). However, electricity is a unique commodity as it cannot be stored. Therefore, demand must equal supply at all times. This feature afects the ability to hedge and proft from arbitrage, so the electricity market is incomplete. A direct implication of this incompleteness is that the Black–Scholes methodology may not be applicable, as this approach consists of a risk-neutral valuation, thus requiring a modifcation of the pricing procedure. For example, Lyle and Elliott ([2009\)](#page-40-6) and Farrell et al. [\(2017](#page-39-9)) price electricity market derivatives under the physical or real-world measure instead of the riskneutral measure. On the contrary, some authors suggest that there are arguments in favor of risk-neutral pricing as the correct approach (Aïd et al. [2009](#page-38-0); Clewlow and Strickland [2000\)](#page-39-10).

Our approach takes into account the incompleteness of the electricity market and the inability to proft from arbitrage. For a given renewable technology, we model both the *volume-weighted average price* (VWAP) of electricity and the annual electricity production during a given period (*t*), say a year, as two GBM stochastic processes  $S_t$  and  $X_t$ , respectively. The annual VWAP is the annual average price, where each hour's price is weighted by the amount of electricity produced by that technology in that hour. Since investments in renewable energy are long-term decisions, long-term price trends are particularly relevant, even when analyzing markets where electricity prices fuctuate on an hourly, daily, or monthly basis. In addition, by considering annual time steps, intra-annual seasonal efects (which are very pronounced in electricity markets) are avoided. This fact is why the assumption that prices follow a GBM process has been widely used to model long-term electricity prices (see for example Blazquez et al. [2018;](#page-39-11) Farrell et al. [2017;](#page-39-9) Zhu [2012;](#page-41-0) Alcorta et al. [2023;](#page-39-5) Tolis and Rentizelas [2011](#page-40-7); Hou el al. [2017\)](#page-40-8). We propose that the annual generation of a given renewable technology can also be approximated as a GBM as it seems reasonable to approximate it as a log-normally distributed variable.<sup>5</sup> The dynamics are described by

<span id="page-5-3"></span>
$$
\begin{cases}\ndS_t = \mu_S S_t dt + \sigma_S S_t dW_t^S \\
dX_t = \mu_X X_t dt + \sigma_X X_t dW_t^X \\
\rho dt = dW_t^S dW_t^X,\n\end{cases} \tag{5}
$$

<span id="page-5-1"></span><sup>&</sup>lt;sup>5</sup> Given that we use the VWAP instead of spot electricity prices, it is not clear that the arguments in favor of using the risk-neutral valuation method apply when weighing by energy produced. Similarly, the amount of energy produced is weather-dependent, and because many authors consider that weather derivatives should not be priced using risk-neutral probabilities (Platen and West [2004](#page-40-9)), we use real-world dynamics in our analysis.

where both Brownian processes,  $W_t^S$  and  $W_t^X$ , are correlated with correlation parameter  $\rho \in (-1, 1)$ ;  $\sigma_S$  and  $\sigma_X$  are the volatilities of each process, and  $\mu_S$  and  $\mu_X$  are the drifts characterizing the expected growth rate of  $S_t$  and  $X_t$ , respectively.<sup>6</sup>

In the following subsections, we present the closed-form solutions to Eqs.  $(2)$  $(2)$ ,  $(3)$  $(3)$  $(3)$ , and ([4\)](#page-5-2) under the most common renewable support schemes (fxed-price, fxed-revenue, and shared-upside regulations).

### **3.1.1 Fixed‑Price Regulation (FP)**

Under an FP scheme, the supplier receives a fixed price  $(K_{fp})$  per MWh generated in a given period *t* (i.e.,  $w_{min,t} = w_{max,t} = K_{fp}X_t$ ). This arrangement is usually formalized through a two-way CfD between the generator and the regulator, where  $K_{fp}$  is the strike price. The value of  $K_{fp}$  may be determined directly by the regulator or through a competitive bidding procedure. According to Eq.  $(2)$  $(2)$ , the value of the rights offered at initial time  $t = 0$  for exercise at a future time *t* > 0 is determined by  $R_{fp,t} = \mathbb{E}_0[e^{-rt}X_t(K_{fp} - S_t)^+]$ . Therefore, the total value of the rights provided by the regulation over a horizon of *T* periods is the sum of the rights for each maturity period:

<span id="page-6-3"></span><span id="page-6-1"></span>
$$
R_{fp} = \sum_{t=1}^{T} \mathbb{E}_0[e^{-rt}X_t(K_{fp} - S_t)^+] \tag{6}
$$

Under the dynamics described in [\(5](#page-5-3)), Eq. ([6](#page-6-1)) has the following analytical solution:

$$
R_{fp} = \sum_{t=1}^{T} X_0 e^{(\mu_X - r)t} \Big( K_{fp} \Phi(-d_{fp}) - S_0 e^{(\mu_S + \sigma_S \sigma_X \rho)t} \Phi(-d_{fp} - \sigma_S \sqrt{t}) \Big), \tag{7}
$$

where  $\Phi$  is the cumulative distribution function (CDF) of the standard normal distribution and

$$
d_{fp} = \frac{\log\left(\frac{S_0}{K_{fp}}\right) + \left(\mu_S + \sigma_S \sigma_X \rho - \frac{\sigma_S^2}{2}\right)t}{\sigma_S \sqrt{t}}.
$$
\n(8)

Similarly, under the FP scheme, the generator receives a maximum price  $(K_{fp})$  per MWh generated. Thus, from Eq. [\(3\)](#page-5-0), the value of the obligations imposed at the initial time for exercise in *t* > 0 is given by  $O_{fp,t} = \mathbb{E}_0[e^{-rt}X_t(S_t - K_{fp})^+]$ . Thus, the total value of the obligations imposed by the regulation over a horizon of *T* periods is the sum of the obligations for each maturity period, i.e.,

$$
O_{fp} = \sum_{t=1}^{T} \mathbb{E}_0[e^{-rt}X_t(S_t - K_{fp})^+],
$$
\n(9)

which has the following solution:

<span id="page-6-0"></span> $6$  For more details on modeling a stochastic process using a GBM, see Shreve ([2004\)](#page-40-10).

<span id="page-6-2"></span><sup>&</sup>lt;sup>7</sup> We include a mathematical appendix with the details on how the solutions to each of the expressions are obtained (see Sect. [7.2](#page-25-0) in Appendix 1).

$$
O_{fp} = \sum_{t=1}^{T} X_0 e^{(\mu_X - r)t} \Big( S_0 e^{(\mu_S + \sigma_S \sigma_X \rho)t} \Phi(d_{fp} + \sigma_S \sqrt{t}) - K_{fp} \Phi(d_{fp}) \Big). \tag{10}
$$

Finally, according to Eq. ([4\)](#page-5-2), the value at  $t = 0$  of the fixed-price scheme for  $t > 0$  is given by  $V_{f*pi*,t} = \mathbb{E}_{0}[e^{-rt}X_{t}(K_{f*pi* - S_{t})}]$ , so the total value of the regulation over a horizon of *T* periods is the sum of the values for each maturity period, i.e.,

<span id="page-7-2"></span><span id="page-7-1"></span>
$$
V_{fp} = \sum_{t=1}^{T} \mathbb{E}_0[e^{-rt}X_t(K_{fp} - S_t)],
$$
\n(11)

with the following closed-form solution:

$$
V_{fp} = \sum_{t=1}^{T} X_0 e^{(\mu_X - r)t} \Big( K_{fp} - S_0 e^{(\mu_S + \sigma_S \sigma_X \rho)t} \Big). \tag{12}
$$

#### **3.1.2 Fixed‑Revenue Regulation (FR)**

Under the FR regulation, investors receive a guaranteed revenue  $(K_f)$  for delivering all the electricity they generate in a given period *t* (i.e.,  $w_{min,t} = w_{max,t} = K_f$ ). Therefore, according to Eq. [\(2\)](#page-4-4), the value of the rights offered at initial time  $t = 0$  for exercise at  $t > 0$  is given by  $R_{fr,t} = \mathbb{E}_0[e^{-rt}(K_{fr} - X_tS_t)^+]$ . Thus, the total value of the rights provided by the regulation over a horizon of *T* periods is

<span id="page-7-0"></span>
$$
R_{fr} = \sum_{t=1}^{T} \mathbb{E}_0[e^{-rt}(K_{fr} - X_t S_t)^+] \tag{13}
$$

Under the dynamics described in  $(5)$  $(5)$ , Eq.  $(13)$  $(13)$  $(13)$  has the following analytical solution:

$$
R_{fr} = \sum_{t=1}^{T} e^{-rt} \left( K_{fr} \Phi(-d_{fr}) - X_0 S_0 e^{(\mu_S + \mu_X + \sigma_S \sigma_X \rho)t} \Phi\left( -d_{fr} - \sqrt{(\sigma_S^2 + \sigma_X^2 + 2 \sigma_S \sigma_X \rho)t} \right) \right), \tag{14}
$$

where

$$
d_{fr} = \frac{\log\left(\frac{X_0 S_0}{K_{fr}}\right) + \left(\mu_S + \mu_X + \sigma_S \sigma_X \rho - \frac{\sigma_S^2 + \sigma_X^2 + 2\sigma_S \sigma_X \rho}{2}\right)t}{\sqrt{(\sigma_S^2 + \sigma_X^2 + 2\sigma_S \sigma_X \rho)t}}.
$$
(15)

Similarly, under this policy, the generator receives a maximum revenue  $(K_f)$  for delivering all the electricity produced. Therefore, according to Eq. [\(3](#page-5-0)), the value of the imposed obligations at the initial time for exercise at *t* > 0 is given by  $O_{f r,t} = \mathbb{E}_0[e^{-rt}(X_tS_t - K_{f r})^+]$ . Therefore, the total value of the obligations imposed by the regulation over a horizon of *T* periods is

$$
O_{fr} = \sum_{t=1}^{T} \mathbb{E}_0[e^{-rt}(X_t S_t - K_{fr})^+],
$$
\n(16)

which yields

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$$
O_{fr} = \sum_{t=1}^{T} e^{-rt} \left( X_0 S_0 e^{(\mu_S + \mu_X + \sigma_S \sigma_X \rho)t} \Phi \left( d_{fr} + \sqrt{(\sigma_S^2 + \sigma_X^2 + 2 \sigma_S \sigma_X \rho)t} \right) - K_{fr} \Phi (d_{fr}) \right). \tag{17}
$$

Finally, according to Eq. ([4\)](#page-5-2), the initial value of the fixed revenue scheme for  $t > 0$  is given by  $V_{f_{r,t}} = \mathbb{E}_0[e^{-rt}(K_f - X_tS_t)]$ , so the total value of the regulation over a horizon of *T* periods is

<span id="page-8-2"></span>
$$
V_{fr} = \sum_{t=1}^{T} \mathbb{E}_{0}[e^{-rt}(K_{fr} - X_{t}S_{t})],
$$
\n(18)

which has the following closed-form solution:

$$
V_{fr} = \sum_{t=1}^{T} e^{-rt} \Big( K_{fr} - X_0 S_0 e^{(\mu_S + \mu_X + \sigma_S \sigma_X \rho)t} \Big). \tag{19}
$$

#### **3.1.3 Shared‑Upside Regulation (SU)**

In the SU scheme, the investor receives a guaranteed price  $(K_{\rm su})$  per MWh generated and also a percentage  $\alpha$  of the difference between the market price and  $K_{\alpha\mu}$  (i.e.,  $w_{min,t} = K_{su} X_t$ .<sup>[8](#page-8-0)</sup> The value of the rights offered at time  $t = 0$  for exercise in  $t > 0$  is given by  $R_{suf} = \mathbb{E}_0[e^{-rt}X_t(K_{su} - S_t)^+]$ . Therefore, the total value of the rights provided by the regulation over a horizon of *T* periods is

<span id="page-8-1"></span>
$$
R_{su} = \sum_{t=1}^{T} \mathbb{E}_0[e^{-rt}X_t(K_{su} - S_t)^+] \tag{20}
$$

which yields

$$
R_{su} = \sum_{t=1}^{T} X_0 e^{(\mu_X - r)t} \Big( K_{su} \Phi(-d_{su}) - S_0 e^{(\mu_S + \sigma_S \sigma_X \rho)t} \Phi(-d_{su} - \sigma_S \sqrt{t}) \Big), \quad (21)
$$

where

$$
d_{su} = \frac{\log\left(\frac{S_0}{K_{su}}\right) + \left(\mu_S + \sigma_S \sigma_X \rho - \frac{\sigma_S^2}{2}\right)t}{\sigma_S \sqrt{t}}.
$$
 (22)

Unlike the previous cases, the SU scheme allows the investor to beneft from higherthan-expected electricity prices. If the market price of electricity exceeds the guaranteed floor, the investor and the policymaker share the excess remuneration: if  $\alpha \in [0, 1]$ represents the predefned share of the market upside received by the investor, then  $w_{max,t} = X_t (K_{su} + \alpha (S_t - K_{su}))$ , and  $(1 - \alpha)$  represents the share of the upside retained by the regulator (i.e., the percentage of the potential upside that the investor has to give up).

<span id="page-8-0"></span><sup>&</sup>lt;sup>8</sup> Note that some common schemes, such as the fixed-price (CfD/two-sided sliding premium) or the onesided sliding premium, are special cases of this mechanism, where  $\alpha$  equals 0 and 1, respectively. However, due to the relevance of the fxed-price scheme, we have preferred to treat it separately.

Therefore, the value of the obligations imposed at the initial time for exercise in  $t > 0$  is  $(1 - \alpha)$  times the value that would apply under a fixed maximum price per MWh (if the investor had to give up all of the potential upside), i.e.,  $O_{\text{31},t} = \mathbb{E}_0[e^{-rt}(1-\alpha)X_t(S_t - K_{\text{31}})^+]$ . Thus, the total value of the obligations imposed by the regulation over a horizon of *T* periods is

$$
O_{su} = \sum_{t=1}^{T} \mathbb{E}_{0} [e^{-rt} (1 - \alpha) X_{t} (S_{t} - K_{su})^{+}], \qquad (23)
$$

which has the following solution:

$$
O_{su} = \sum_{t=1}^{T} (1 - \alpha) X_0 e^{(\mu_X - r)t} \Big( S_0 e^{(\mu_S + \sigma_S \sigma_X \rho)t} \Phi(d_{su} + \sigma_S \sqrt{t}) - K_{su} \Phi(d_{su}) \Big). \tag{24}
$$

Finally, according to Eq. [\(1](#page-4-3)), the value at  $t = 0$  of the SU regulation for  $t > 0$  is given by  $V_{\text{S}u,t} = R_{\text{S}u,t} - O_{\text{S}u,t}$ . Therefore, the total value of the regulation over a horizon of *T* periods is

<span id="page-9-1"></span>
$$
V_{su} = R_{su} - O_{su},\tag{25}
$$

where  $R_{su}$  and  $O_{su}$  are given by Eqs. ([21](#page-8-1)) and [\(24\)](#page-9-1), respectively.

#### <span id="page-9-0"></span>**3.2 Incentive Schemes and Risk Exposure**

Up to this point, we have examined the actuarial value of the regulation for the investor (i.e., the fair value relative to the unregulated benchmark) considering the discounted sum of all rights and obligations that the diferent incentive schemes entail. However, a riskaverse decision maker may value two incentive schemes with the same actuarial value differently. In this subsection, we introduce the possibility that a risk-averse investor values the stream of revenue  $w_t$  associated with a given incentive scheme according to a concave utility function. We represent the investor's preferences in each period by the isoelastic utility function:

<span id="page-9-4"></span>
$$
u(e^{-rt}w_t) = \frac{(e^{-rt}w_t)^{1-\gamma} - 1}{1 - \gamma},
$$
\n(26)

where  $e^{-rt}w_t$  represents the discounted revenues under each scheme at period *t*, and  $\gamma \in [0, \infty)$  denotes the degree of relative risk aversion.<sup>9</sup> A value of  $\gamma = 0$  corresponds to risk neutrality, while higher values of  $\gamma$  indicate increasing degrees of risk aversion (Arrow [1984\)](#page-39-12).[10](#page-9-3) Constant relative risk aversion preferences are prevalent in the fnance literature due to their analytical simplicity (Merton [1969;](#page-40-11) Samuelson [1969\)](#page-40-12).

<span id="page-9-2"></span>This function is not well-defined at  $\gamma = 1$ . In this case, we take the limit of *u* as  $\gamma$  approaches 1: the utility function tends toward logarithmic utility, expressed as  $\lim_{x \to 0} u(w_t) = \log(w_t)$ .

<span id="page-9-3"></span>For the utility function [\(26](#page-9-4)), the *Arrow-Pratt measure of relative risk aversion*  $RRA = -w \frac{u''(w)}{u'(w)} = \gamma$ . This utility function is also called the CRRA (constant relative risk aversion) utility function.

Given the stream of revenues  $w_t$ , the total expected utility over a horizon of *T* periods is as follows: $11$ 

$$
U = \frac{1}{1-\gamma} \sum_{t=1}^{T} \left\{ \mathbb{E}_0 \left[ \left( e^{-rt} w_t \right)^{1-\gamma} \right] - 1 \right\}.
$$
 (27)

The stream of revenues  $w_t$  will be different in the benchmark of an unregulated market (merchant) and for each incentive system:

<span id="page-10-1"></span>
$$
w_{m,t} = X_t S_t,\tag{28}
$$

$$
w_{fp,t} = K_{fp} X_t,\tag{29}
$$

<span id="page-10-3"></span>
$$
w_{fr,t} = K_{fr},\tag{30}
$$

$$
w_{su,t} = X_t \max\{K_{su}, K_{su} + \alpha(S_t - K_{su})\}.
$$
 (31)

Therefore, to assess the utility of each scheme according to Eq. ([27](#page-10-1)), we need the closedform expression  $\mathbb{E}_0\left[w_t^{1-\gamma}\right]$  for each case. The solutions for merchant, fixed-price, and fixed-revenue schemes are as follows (see the general solution [64](#page-25-1) in Appendix 1):

$$
\mathbb{E}_0\left[w_{m,t}^{1-\gamma}\right] = (X_0 S_0)^{1-\gamma} \exp\left[\left(\mu_X + \mu_S + (1-\gamma)\rho \sigma_S \sigma_X - \gamma \frac{\sigma_X^2 + \sigma_S^2}{2}\right) (1-\gamma)t\right], \quad (32)
$$

$$
\mathbb{E}_0\left[w_{fp,t}^{1-\gamma}\right] = (K_{fp}X_0)^{1-\gamma} \exp\left[\left(\mu_X - \gamma \frac{\sigma_X^2}{2}\right)(1-\gamma)t\right],\tag{33}
$$

<span id="page-10-5"></span><span id="page-10-4"></span>
$$
\mathbb{E}_0\left[w_{fr,t}^{1-\gamma}\right] = K_{fr}^{1-\gamma}.
$$
\n(34)

In the case of the SU regulation, it is not possible to obtain an analytical solution. This is due to the lack of a closed-form solution for the distribution of the sum of two log-normal random variables, a crucial step in solving *Usu*. Nevertheless, various analytical approaches to this problem have been proposed in the literature (Fenton  $1960$ ).<sup>[12](#page-10-2)</sup>

<span id="page-10-0"></span><sup>&</sup>lt;sup>11</sup> We assume there is no intertemporal discounting.

<span id="page-10-2"></span><sup>&</sup>lt;sup>12</sup> An alternative approach to overcome this limitation would be to rely on numerical methods. Another option would be the mean-variance approach. Markowitz [\(1959](#page-40-13)) justifes mean-variance (MV) analysis as an alternative to maximizing a given utility function using analytical or numerical methods. In this approach, the expected utility is approximated as expected returns minus the volatility of returns (multiplied by a risk-aversion parameter). Indeed, Levy and Markowitz [\(1979](#page-40-14)) found that mean-variance approximations tend to be highly accurate. However, in our particular model, this approach may not be appropriate because some schemes may have a strong asymmetry in terms of upside and downside risk, and the MV approach penalizes all risks equally.

We use the approximation method for the sum of log-normal random variables intro-duced by Lo [\(2013](#page-40-15)) to derive an analytical expression for  $\mathbb{E}_0 \left[ w_{sat}^{1-\gamma} \right]$  (for details see Sect. [7.3](#page-28-0) in the Appendix):

$$
\mathbb{E}_{0}\left[w_{\text{su},t}^{1-\gamma}\right] \approx (K_{\text{su}}X_{0})^{1-\gamma} \exp\left[\left(\mu_{X}-\gamma\frac{\sigma_{X}^{2}}{2}\right)(1-\gamma)t\right] \Phi\left(-d_{u}-(1-\gamma)\sigma_{X}\rho\sqrt{t}\right) + (\Lambda X_{0})^{(1-\gamma)} \exp\left[\left(\mu_{X}-\frac{\gamma\sigma_{Z}^{2}}{2}\right)(1-\gamma)t\right] \Phi\left(d_{u}+(1-\gamma)\sigma_{Z}\rho_{SZ}\sqrt{t}\right), \tag{35}
$$

where

<span id="page-11-1"></span>
$$
\Lambda = K_{\rm su}(1-\alpha) + \alpha S_0 e^{(\mu_S + \rho \sigma_S \sigma_X)t},\tag{36}
$$

$$
\sigma_Z = \frac{1}{\Lambda} \left[ \left( K_{su}(1-\alpha)\sigma_X \right)^2 + 2S_0 e^{(\mu_S + \rho \sigma_S \sigma_X)t} K_{su} \alpha (1-\alpha)\sigma_X(\rho \sigma_S + \sigma_X) \right. \n+ \left( \alpha S_0 e^{(\mu_S + \rho \sigma_S \sigma_X)t} \right)^2 (\sigma_S^2 + 2\rho \sigma_S \sigma_X + \sigma_X^2) \Big]^{\frac{1}{2}},
$$
\n(37)

$$
d_u = \frac{\log\left(\frac{S_0}{K_{su}}\right) + \left(\mu_S - \frac{\sigma_S^2}{2}\right)t}{\sigma_S \sqrt{t}},
$$
\n(38)

and

$$
\rho_{SZ} = \frac{1}{\sigma_Z \sigma_S t} \log \left[ 1 + \frac{K_{su}(1-\alpha)(e^{\sigma_S \sigma_X \rho t} - 1) + \alpha S_0 e^{(\mu_S + \sigma_S \sigma_X \rho)t} \left(e^{(\sigma_S \sigma_X \rho + \sigma_S^2)t} - 1\right)}{K_{su}(1-\alpha) + \alpha S_0 e^{(\mu_S + \rho \sigma_S \sigma_X)t}} \right].
$$
\n(39)

This approximated solution for  $\mathbb{E}_0\left[w_{\text{3}u,t}^{1-\gamma}\right]$  is more accurate the higher the value of  $K_{\text{3}u}$  and  $\gamma$ (see Fig. [10](#page-35-0) in Appendix [7.3](#page-28-0)).

Diferent support mechanisms imply diferent risk exposures. In our setting, investors would compare support mechanisms using a von Neumann–Morgenstern (VNM) *expected utility function* (von Neumann and Morgenstern  $1944$ ),<sup>13</sup> If two alternatives yield the same expected utility, the investor would be indiferent between them. Selling to the market and obtaining an expected utility  $U_m$  is the outside option for investors. Therefore, any

<span id="page-11-0"></span><sup>&</sup>lt;sup>13</sup> According to the *expected utility theorem* if a decision maker's preferences satisfy a set of conditions, then these preferences can be represented by a von Neumann–Morgenstern expected utility function  $U = \mathbb{E}[u] = \int_{-\infty}^{\infty} u(x)f(x)dx$ , where *x* takes the value of each possible outcome,  $u(x)$  is its valuation, and  $f(x)$ is the probability density function of *x*. Therefore, such a rational agent is an expected utility maximizer. For details, see Mas-Colell et al. [\(1995](#page-40-17)).

incentive scheme has to yield an expected utility equal to or greater than  $U<sub>m</sub>$  to be accepted by investors.

Assume the regulator tries to decide between two diferent incentive schemes, denoted *A* and *B*. The regulator will set the retributive parameters  $\Omega_A$  and  $\Omega_B$  (i.e., strike prices, duration, etc.) that yield identical expected utility to the investor than the outside option of selling to the market. That guarantees that the support policies are accepted by investors (expected utility is at least  $U_m$ ) and that the incentive schemes are not more expensive than necessary. That may result in different expected costs to the regulator  $(V_A \neq V_B)$ , which may help to decide between the incentive schemes. On the other hand, schemes with retributive parameters that imply equivalent expected costs to the regulator,  $(V_A = V_B)$ , may yield different expected utility to the investor  $(U_A \neq U_B)$ , which may affect take up.

In our framework, given the characteristics of one of the two incentive mechanisms  $(\Omega_4)$ , it is possible to determine the parameters of the other system  $(\Omega_R)$  such that either the investor's utility or the regulator's expected cost is identical under both schemes. This will be useful for the regulator to design the optimal (cost minimizing) incentive scheme that would be accepted by investors, or given the budget, to choose the scheme that increases investors' expected utility and promotes take up.

As an important part of the incentives they provide, support policies can lead to a partial or total reduction in the risk associated with production and market price uncertainties. We can measure the risk associated with support policies by their risk premium (Pratt [1964](#page-40-18)).<sup>[14](#page-12-0)</sup> In our setting, for the *T* years that the incentive scheme is in place, its *annual absolute risk premium*  $(\pi<sub>t</sub>)$  is

$$
\pi_t = \mathbb{E}_0\big[w_t\big] - \left(\mathbb{E}_0\big[w_t^{1-\gamma}\big]\right)^{\frac{1}{1-\gamma}},\tag{40}
$$

where  $\left(\mathbb{E}_{0}\left[w_{t}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}$  is the *certainty-equivalent value*. Thus, the *total absolute risk premium*  $(\pi)$  of the scheme is

<span id="page-12-1"></span>
$$
\pi = \sum_{t=1}^{T} e^{-rt} \pi_t.
$$
 (41)

The absolute risk premium is scale-dependent, i.e., the higher the expected revenues, the higher the value of  $\pi$ . A more adequate measure of the amount of risk involved in each scheme is, therefore, the *relative risk premium*  $(\zeta)$ , which we define as follows:

<span id="page-12-2"></span>
$$
\zeta = \frac{\pi}{\sum_{i=1}^{T} e^{-rt} \mathbb{E}_0\left[w_i\right]}.
$$
\n(42)

The closed-form solutions for the absolute and relative risk premium under each scheme are obtained simply by substituting into Eqs.  $(41)$  and  $(42)$ , respectively, the expressions for the expectations for each scheme in Eqs.  $(32)$  $(32)$  $(32)$ ,  $(33)$  $(33)$  $(33)$ ,  $(34)$ , and  $(35)$ .<sup>15</sup> For a given attitude toward risk  $(\gamma)$ ,  $\pi$  and  $\zeta$  are measures of how much risk there is in the income stream  $W_t$ 

<span id="page-12-0"></span><sup>&</sup>lt;sup>14</sup> The absolute risk premium  $\pi$  is the quantity that makes a rational agent indifferent between receiving a risky asset *z* and receiving the non-random amount  $\mathbb{E}[z] - \pi$  (i.e.,  $u(\mathbb{E}[z] - \pi) = \mathbb{E}[u(z)]$ ).

<span id="page-12-3"></span><sup>&</sup>lt;sup>15</sup> Note that  $\mathbb{E}_0\left[w_t\right]$  is obtained by evaluating the expressions for  $\mathbb{E}_0\left[w_t^{1-\gamma}\right]$  in  $\gamma = 0$ .

To give a measure of the value for the investor of a specifc scheme that provides a stream of income  $w_t$ , we define  $v_\tau$  as the amount of money per period that makes the investor indifferent between the support policy with a stream of income  $w_t$  and a stream of fixed income  $v_{\tau}$  (i.e.,  $\sum_{t=1}^{T} u(e^{-rt}v_{\tau}) = \sum_{t=1}^{T} u(e^{-rt}w_{t})$ ):

$$
v_{\tau} = \left(\frac{\sum_{t=1}^{T} \mathbb{E}_{0}\left[\left(e^{-rt}w_{t}\right)^{1-\gamma}\right]}{\sum_{t=1}^{T} e^{-r(1-\gamma)t}}\right)^{\frac{1}{1-\gamma}},\tag{43}
$$

and thus, the total *value to the investor* of the incentive scheme is

$$
v = \sum_{t=1}^{T} e^{-rt} v_t.
$$
 (44)

Therefore, we can express *the incentive value* of a policy for an investor with a given attitude toward risk  $\gamma$  as the difference between the value to the investor of the policy and the value to the investor of the market option (i.e.,  $v - v_m$ ).

### <span id="page-13-0"></span>**4 Application**

This section presents the results of our empirical application. Section [4.1](#page-13-1) contains the calibration and the data sources used. Sections [4.2](#page-15-0) and [4.3](#page-17-0) present the results for the case of wind power and solar PV power, respectively.

### <span id="page-13-1"></span>**4.1 Data and Calibration**

We apply the method presented in Sect. [3](#page-4-0) to the case of Spain, which has undergone signifcant changes in its green energy support policy and has implemented the three diferent incentive mechanisms discussed in the previous section. We focus on the two main renewable technologies, wind and solar photovoltaic (PV), in two diferent periods. First, in 2013, there was a transition from the Feed-in Tarif system (a fxed-price mechanism) to the Rate of Return regulation (fxed-revenue). Second, in 2021, while the previous investments were still under the Rate of Return regulation, new renewable projects were implemented under a system called the Renewable Energy Economic Regime, which consists of a shared-upside mechanism awarded through an auction.<sup>[16](#page-13-2)</sup>

Starting from each of these dates, 2013 and 2021, we consider a time horizon of 15 years  $(T = 15)$ . We choose the same duration for all systems to be able to compare them on equal terms. Similarly, as capacity is not constant over time, we examine the results for each MW of promoted capacity to avoid distortions.

Under a FiT scheme, the producer is offered a fixed price  $(K_{fp})$  for all its energy. The objective of the RoR scheme is not to guarantee a fxed price but a reasonable return on investment regardless of price and production uncertainties. The regulator sets a rate of

<span id="page-13-2"></span><sup>&</sup>lt;sup>16</sup> 2021 marks the beginning of the new REER regulation for renewable capacity deployment through competitive auctions. Moreover, it is the year of the last successful auction of renewable capacity in Spain, as in the 2022 auction, bids exceeded the reserve price set by the regulator.

return on investment  $(\theta)$  that the generator will receive annually to achieve the aforementioned reasonable revenue  $(K_f)$ . Denoting  $\kappa$  the investment in capacity:  $\kappa \theta = K_f$ . Thus, for the Rate of Return regulation to be equivalent to a fxed revenue scheme, the variable costs must be negligible, which may hold for renewable energy.

Finally, the shared upside system was established in 2021 for new renewable energy investments awarded through competitive bidding processes. In the auction, candidate projects bid for the guaranteed minimum price they are willing to accept. The regulator awards the most competitive bids, and the selected projects receive not only the guaranteed price of their bid  $(K_{\rm \tiny{cut}})$  but also a pre-determined share  $(\alpha)$  of the upside above that threshold, which is also set in advance by the regulator and depends on the technology of the project.

We calibrate the model for each technology and each year. The parameters  $K_{fp}$ ,  $K_{fr}$ ,  $S_0$ ,  $X_0$ ,  $\sigma_s$ , and  $\sigma_X$  are obtained from data provided by the Spanish regulator (Comisión Nacional de los Mercados y la Competencia, CNMC).  $K_{fp}$  is the price per MWh delivered in 2013 under the FiT regime. For the RoR system,  $K_f$  is taken as the annual revenue per supported MW that the regulator estimates suppliers should receive in 2014, immediately after the system change. The VWAP for the initial year  $(S_0)$  is the weighted average price, where for each hour of the year, the hourly price is weighted by the amount of electricity supplied by that technology in that hour.  $X_0$  is the annual electricity generation per MW of installed capacity in the initial year.

For simplicity, we assume the drift characterizing the expected growth rate of production per MW of capacity installed  $(\mu_Y)$  is zero, as the values obtained for the long-term trend are very close to this value.<sup>17</sup> For 2013, the drift characterizing the expected growth rate of annual prices  $(\mu_s)$  is obtained as the average logarithmic return of annual prices in the preceding 8-year window. Similarly, the volatilities of annual prices and generation,  $\sigma_{\rm S}$  and  $\sigma_{\rm X}$ , respectively, are obtained as the standard deviation of the logarithmic returns of their corresponding processes in that window.<sup>18</sup> For 2021, given the exceptionally high prices (see Appendix [2\)](#page-35-1), the resulting trend in previous years may not be a good indicator of expected future developments. Moreover, it was expected the increase in renewable energy would lead to lower prices in the medium to long term once the energy crisis was over. For this reason,  $\mu<sub>S</sub>$  is obtained from the electricity futures prices on the day of the 2021 renewable energy auction for the ten following years (OMIP [2021\)](#page-40-19). The regulator establishes the share  $\alpha$  depending on the renewable technology.

The correlation parameter  $\rho$  represents the correlation between the logarithmic returns of the two stochastic processes  $(X_t \text{ and } S_t)$ . Since the annual series have too few observations to estimate a correlation, we approximate  $\rho$  as the correlation between the weekly logarithmic returns of both series for the time window of the 2 years before the initial year. We obtain weekly VWAP and weekly generation per MW using hourly generation data from the Spanish TSO (Red Eléctrica de España, REE) and hourly price data from the market operator (Operador del Mercado Ibérico de Energía, OMIE).  $K_{su}$  is taken as the capacity-weighted average bid price of awarded projects in the 2021 renewable energy auction (BOE [2021b](#page-39-14)). The Weighted Average Cost of Capital (WACC) is commonly used to determine the discount rate (*r*) for energy projects (see Haar and Haar [2017](#page-40-4)). The WACC, the weighted average cost of a company's equity and debt, represents the average rate of

<span id="page-14-0"></span><sup>&</sup>lt;sup>17</sup> This is due to our application at the national level, but it may be different for other applications, such as individual projects, where changes in project productivity may occur due to technological, regulatory, or geographic reasons.

<span id="page-14-1"></span><sup>&</sup>lt;sup>18</sup> The estimation method of the GBM parameters is discussed in Wilmott [\(2013](#page-40-5)).

return for an energy project and thus is a good estimate of the opportunity cost of invest-ment (Steffen [2020](#page-40-20)).

### <span id="page-15-0"></span>**4.2 Results for Wind Power**

We first show the results of the rights  $(R)$ , obligations  $(O)$ , and regulatory value  $(V)$ obtained for each of the regulatory schemes for the case of wind power capacity. On the left of Fig. [1](#page-30-0), we present the case for 2013, when the plants enrolled in the fxed-price (FiT) system were reassigned to the fxed-revenue (RoR) scheme. We show the corresponding solutions before and after this transition. On the right side, we show the case for 2021, where the fxed-revenue and shared-upside (REER) systems coexist. In 2021, all the installed capacity previously supported under the FiT incentive scheme had been reassigned to the RoR system, while the new capacity allocated through auctions is under the REER support system. For REER, we use the incentive levels resulting from the renewable energy auction held in October 2021.<sup>19</sup>

As can be seen in Fig. [1](#page-30-0)(left), for 2013, under both the FiT and RoR regimes, the obligations imposed by the support policy are negligible compared to the rights received, suggesting that both the fxed-price and fxed-revenue mechanisms provide strong incentives for investment. Under the incentive levels set at the time for each system, both the rights received and the value of the regulation are higher for the fxed-price mechanism than for the fxed-revenue scheme.

In 2021, the scenario for renewable promotion shown in Fig. [1\(](#page-30-0)right) has drastically changed. Both under the RoR regulation and the REER, the obligations imposed by the policy have increased signifcantly. In the case of RoR, the obligations remain lower than the rights received, which have also increased, and therefore, the value of the scheme remains similar to that of 2013. In contrast, for new projects under the REER, the obligations imposed are now greater than the rights received, making the value of regulation negative. $20$ 

Figure [2](#page-31-0) shows, for 2013 at the top and for 2021 at the bottom, how these quantities (*V*, *R*, and *O*) depend on the value of the strike price (*K*) of each program. The actual strike price is represented by the vertical dotted line, while the actual values of *V*, *R*, and *O* are represented by the horizontal dashed lines. Figure [2a](#page-31-0) shows how the FiT system in 2013, with either the same rights or regulatory value as the RoR system, would imply a slightly lower FiT price level than before the system change. Conversely, a change from the RoR system to the FiT system would require an increase in the remuneration ofered under the RoR regulation.

Figure [2](#page-31-0)b shows how the RoR system in 2021, with the same rights, obligations, or regulatory value as the REER system, would require either a signifcant reduction in the guaranteed revenue per period under the RoR or a signifcant increase in the guaranteed price under the REER. The results indicate that when the incentives to the new wind power projects are awarded through a competitive process, such as the auctions under the REER system, generators receive a regulation of substantially lower value than those installed

<span id="page-15-1"></span><sup>&</sup>lt;sup>19</sup> Within each policy, O, R, and V are depicted from left to right.

<span id="page-15-2"></span><sup>&</sup>lt;sup>20</sup> This negative value raises the question of generators' participation and bidding at the renewable auc-tions. One reason for participation may be preemption (see Zhu et al. [2021](#page-41-1)). As we show in the next section, another reason may be risk aversion.

earlier and that are subject to the Rate of Return regulation. Competition between generators has substantially decreased the cost to the regulator of wind project promotion.

We proceed to analyze risk exposure under each incentive scheme from the perspective of a risk-averse investor.<sup>[21](#page-16-0)</sup> For the case of wind power, in Fig. [3,](#page-32-0) we compare the relative risk premium  $(\zeta)$  for each policy and the outside option of selling to the market  $(M)$  for different values of the investor's attitude towards risk  $(\gamma)$ . Figure [3](#page-32-0)a shows that in 2013, for any value of  $\gamma \geq 0$ , the risk associated with the merchant option is significantly higher than under the FiT system. In the case of RoR, all risk is removed because revenues are fxed and independent of generation and prices, so the risk premium is always zero. Therefore, even if the RoR regulation had a lower value in terms of rights and obligations than a FiT system in 2013 (Fig. [1\)](#page-30-0), a risk-averse investor could have preferred the RoR scheme because it entailed a lower risk.

Figure [3](#page-32-0)b shows that for 2021, the risk involved in the merchant option is again higher than in the two support policies. For the shared-upside mechanism (REER), we show both the solution obtained using the analytical approximation for the utility described in Eq. ([35](#page-11-1)) (solid line) and the real value obtained by simulating  $100,000$  trials (dashed line).<sup>22</sup> In this case, too, the REER policy ofers a signifcantly lower level of risk than the market option.

Finally, Fig. [4](#page-32-1) shows the value to a risk-averse investor of each policy concerning the benchmark of selling to the market  $(\nu - \nu_m)$ , as a function of the risk aversion parameter *𝛾*. The horizontal dashed lines represent each scheme's regulatory value *V*, and the vertical dotted line is the value of  $\gamma$  at which the order of preference changes. Note that the incentive value of a given policy to a risk-neutral investor (i.e.,  $\gamma = 0$ ) coincides with that policy's regulatory value or expected regulatory cost.

We find that depending on the investor's attitude toward risk  $\gamma$ , the preference for a given scheme may reverse. In 2013,  $M \prec RoR \prec FiT$  for  $\gamma \prec 1.837$ , whereas  $M \prec FiT \prec RoR$ for  $\gamma > 1.837$ . Thus, for an investor with a relatively low risk aversion, the FiT scheme yields higher utility than the RoR. Thus, the total incentive value  $v - v_m$  is higher for FiT than for RoR, while an investor with a higher degree of risk aversion would value the RoR more than the FiT. In 2021, REER  $\prec M \prec R$  or  $\gamma \prec 0.315$ , whereas M  $\prec REER \prec R$  or for  $\gamma > 0.315$ . This finding explains generators' participation in renewable auctions, even though the value of such regulation was negative (Fig. [1](#page-30-0)). Note that risk-averse investors may prefer the REER scheme to the outside option of selling to the market, which solves the puzzle of why investors would willingly accept a policy that yields a negative value compared to the merchant option. In the REER scheme, investors are paying an implicit risk premium.

Since the higher the risk aversion, the higher the incentive value of the diferent support policies, the regulator may exploit risk aversion to minimize the cost of providing incentives. This fact becomes relevant for the REER system in 2021, which, despite having a negative regulatory value (regulatory cost), for a sufficiently risk-averse investor, this policy may work as an incentive as it is preferred to the merchant option.

<span id="page-16-0"></span><sup>&</sup>lt;sup>21</sup> The utility function and absolute risk premia graphs are available in Appendix  $3$ .

<span id="page-16-1"></span><sup>&</sup>lt;sup>22</sup> Although the analytical approximation works best for high values of  $K_{su}$  concerning market prices, even for values of  $K_{su}$  much lower than prices, as it is the case here, the analytical approximation is quite accurate.

#### <span id="page-17-0"></span>**4.3 Results for Solar PV Power**

For the case of solar PV capacity, the graph in Fig. [5](#page-32-2)(left) shows that for 2013, the support policy entailed negligible obligations compared to the resulting benefts received from both the FiT and RoR regulations. $^{23}$  $^{23}$  $^{23}$  Therefore, both the FiT and RoR mechanisms provided strong incentives for investment. In addition, given the incentive levels set at the time for each mechanism, the fxed-price system provided both higher rights and regulatory value than the fxed-revenue system.

Figure [5](#page-32-2)(right) shows that *V* and *R* for the RoR system are higher in 2021 than in 2013. In contrast, for the REER regulation, the obligations imposed by the policy exceed the rights received, which have decreased dramatically, resulting in a negative value of the regulation. Given the incentive levels set for each regulation, the obligations imposed (rights received) are much higher (lower) for the shared-upside mechanism than for the fxed-revenue mechanism.

Figure [6](#page-33-0) shows, for 2013 (top) and 2021 (bottom), how the results depend on the value of the strike price  $(K)$  of each scheme. Figure [6](#page-33-0)a shows, in the case of solar PV capacity, how the FiT system in 2013, with either the same rights or the same regulatory value as the RoR system, would require either a lower FiT price level or a higher guaranteed revenue under RoR than before the system change. Similarly, Fig. [6](#page-33-0)b shows how the 2021 RoR system for PV projects, with either the same obligations, rights, or regulatory value as the REER system, would require in each case either a substantial reduction in the level of retribution under RoR or a substantial increase in the guaranteed price under REER. The results indicate that in 2021, new solar PV projects awarded under the REER system received signifcantly less valuable regulation than those installed earlier and subject to RoR regulation.

We now examine the level of risk associated with each system from the perspective of a risk-averse investor.<sup>24</sup> In Fig. [7,](#page-34-0) we present the relative risk premium  $(\zeta)$  for each scheme and diferent degrees of risk aversion. Figure [7](#page-34-0)b shows the relative risk premium for the systems in place in 2021. We show both the closed-form approximation (solid line) and the real value obtained by simulating 100,000 trials for REER (dashed line).<sup>[25](#page-17-3)</sup>

Again, for 2013 and 2021, the outside option of full market exposure entails the highest relative risk premium. For the RoR, as discussed above, the risk premium is always zero. We see that both FiT and REER achieved a reduction in investor risk for 2013 and 2021, respectively. It is worth noting that in this case, due to the extremely high FiT prices that were in place for solar power in 2013, the absolute risk premium associated with the market-only option is signifcantly lower than the absolute risk premium associated with the FiT scheme (see Fig. [15a](#page-38-1) in the Appendix).

Finally, Fig. [8](#page-34-1) shows the total incentive of each policy to the investor. The horizontal dashed lines represent the regulatory value *V* of each policy, and the vertical dotted line represents the value of  $\gamma$  at which the ordering of preferences changes. Depending on the investor's degree of risk aversion, the ordered preferences of the diferent incentives change. In 2013,  $M \prec RoR \prec FiT$  for  $\gamma \prec 0.54$ , whereas  $M \prec FiT \prec RoR$  for  $\gamma > 0.54$ , i.e., for an investor with an Arrow-Pratt measure of relative risk aversion lower than 0.54, the

<span id="page-17-1"></span> $23$  Within each policy, O, R, and V are ordered from left to right.

<span id="page-17-2"></span><sup>&</sup>lt;sup>24</sup> The utility function and absolute risk premia graphs are available in Appendix [3.](#page-36-0)

<span id="page-17-3"></span><sup>&</sup>lt;sup>25</sup> Again, even for very low values of  $K_{\alpha}$  (where the approximation works worst), the result obtained seems reasonably close to the real one.

FiT scheme yields higher utility than the RoR, and thus the total incentive  $v - v_m$  is higher for FiT than for RoR. In contrast, an investor with a higher degree of risk aversion would value the RoR more than the FiT. In 2021, REER  $\lt M \lt R$  or  $\gamma \lt 0.229$ , whereas  $M \prec$  REER  $\prec$  RoR for  $\gamma > 0.229$ . In both scenarios, the higher the measure of risk aversion, the higher the incentive value of the diferent support policies. As can be seen in the case of the REER system in 2021, despite having a negative regulatory value, a sufficiently risk-averse investor would prefer this policy over the market-only option.

## <span id="page-18-0"></span>**5 Discussion**

Until early 2013, Spain's main renewable energy support instrument was a mix of Feed-in Tarif and Feed-in Premium (FiP) policies. This system was very successful in achieving a signifcant deployment of renewable capacity, but regulatory costs became so high that the government had no choice but to redesign its policy. At the beginning of 2013, FiPs were abolished and all plants under this mechanism were assigned to a fxed-price FiT. By the end of 2013, all capacity supported by FiTs was reallocated to the new RoR regulation in a new attempt to reduce regulatory costs. At that point, approximately 20 GW of wind and 4.6 GW of solar capacity that had been under the FiT system began to receive an annual payment, in addition to market revenues, that was intended to provide a reasonable return on investment. This return was set at 7.4% per year (three percentage points higher than the average return on 10-year government bonds at the time).

In 2021, auctions for renewable capacity were introduced, and two diferent incentive systems coexisted. Old installations were under the RoR mechanism, while new investments were subject to the REER framework under the conditions set in the upcoming auctions.

The two frst auctions held in 2021 were quite successful. For instance, the frst auction, held in January 2021, awarded more than 3 GW of renewable capacity at average prices of around 25 €/MWh (BOE [2021a\)](#page-39-15). The second auction, held in October 2021, awarded around 2.2 GW of wind capacity and 838 MW of solar PV capacity at guaranteed prices of around 30  $\epsilon$ /MWh in a year in which the average wholesale market price was 111.97  $\epsilon$ / MWh (BOE [2021b\)](#page-39-14). In contrast, the third renewable energy auction, held in October 2022, fell short of expectations, with only 177 MW allocated out of the 520 MW put up for tender. Indeed, the thermosolar quota remained unallocated as all bids exceeded the secret reserve price set by the regulator (BOE  $2022a$ ).<sup>26</sup> Similarly, in the fourth auction held in November 2022, the regulator aimed to achieve the deployment of 3.3 GW of renewable capacity, but only 45 MW of wind power was awarded (BOE [2022b](#page-39-17)).

In Sects. [4.2](#page-15-0) and [4.3](#page-17-0), we have implemented our methodology to compute the value of a given support policy and the importance of both the rights and obligations that the policy entails. Our results indicate that for both technologies, there was a decrease in the value of regulation when switching incentive schemes. This fnding is consistent with the fact that switching incentive schemes was motivated by the attempt to decrease regulatory costs. Note that in 2013, the results obtained for the rights received and the value of regulation were much higher for solar than for wind because the FiT levels were very high in the case

<span id="page-18-1"></span><sup>&</sup>lt;sup>26</sup> Although more expensive than photovoltaic power, thermosolar technology can provide stored energy during the early hours of the night.

of solar: around 395  $\epsilon$ /MWh for solar compared to 77  $\epsilon$ /MWh for wind, at a time when average prices were often below 45 €/MWh.

We have seen how the balance between rights and obligations varies in each case, depending on the support mechanism, the year, and the technology. We defne an *incentive coefficient* that summarizes in a single indicator the relative importance of rights and obligations under a given regulation. We defne this indicator (*I*) as

$$
I = \frac{R - O}{R + O}.\tag{45}
$$

Note that the value of this parameter is  $I = 1$  when the policy grants rights and no obligations ( $O = 0$ ), whereas when the policy implies some obligations without rights ( $R = 0$ ), the value of the incentive coefficient is  $I = -1$ . If both contributions are equal  $(R = 0)$ , then  $I = 0$ . Figure [9](#page-35-2) shows the relative importance of rights and obligations in our results.

Even though 2013 was a transition year, each policy involved a subsidy and almost no burden (see Fig. [9\)](#page-35-2). In 2021, the results for solar capacity under RoR were the same as in 2013, but there is a signifcant decrease in the value of *I* for the case of wind capacity. Moreover, new projects awarded through auctions (REER) show such a reduction in the value of *I* that the obligations imposed outweighed the rights granted. An advantage of the model we have obtained with analytical solutions in terms of elementary functions is that it is straightforward to obtain closed-form expressions for sensitivities of *R*, *O*, and hence *I*, to each model parameter. It is the case that, in FiT and RoR schemes, *I* strictly increases with the fixed strike (i.e.,  $\partial I_{FT}/\partial K_{fp} > 0$  and  $\partial I_{RoR}/\partial K_{fr} > 0$ ). Similarly, in the REER scheme, *I* strictly increases with the guaranteed price and the share of the upside received by the investor (i.e.,  $\partial I_{RFER}/\partial K_{su} > 0$  and  $\partial I_{RFER}/\partial \alpha > 0$ ). Therefore, for the REER in 2021, where *I* is negative, increasing either  $K_{su}$  or  $\alpha$  would yield higher values of the incentive coefficient. Note that our discussion focuses on the value of the contract signed from the investor's perspective; a result indicating that the regulator provides a support scheme primarily with a given amount of rights but without any drawbacks does not imply that such a policy is not socially desirable due to externalities such as the social cost of carbon or the learning curve concerning the implementation of renewable technologies. By contrast, a scheme in which the downside for the investor outweighs the upside may disincentivize investment.

To understand the efect these support policies may have on renewable energy deployment and rationalize the fact that policies with negative value may still be accepted by generators, we introduced risk considerations and the perspective of investors valuing incentive schemes with a CRRA utility function.

Our results indicate there is less risk in the diferent incentive schemes than in the market-only option. However, the level of exposure varies widely across systems and technologies (more risk in solar than in wind). This is because the higher the strike price for the fxed-price or shared upside schemes, the greater the risk due to the uncertainty of the quantity produced. As expected, the only system that completely mitigates risk for the investor is the fxed-revenue (RoR).

In deciding which system is more favorable, the investor considers not only the expected earnings or the amount of risk involved; the utility function refects both considerations. According to our results, the value of the incentive of each scheme relative to the outside option depends on the measure of risk aversion. Moreover, the ranking of preferences can change as the degree of risk aversion changes. In fact, in 2013, we found that the expected regulatory costs for the FiT system were higher than for the RoR, as this transition was

made in an attempt to reduce costs. However, for sufficiently high levels of risk aversion, the value to the investor of the RoR (fxed-revenue) policy could be even higher than that of the FiT (fxed-price) policy, as the former mitigates all the risk, whereas the latter mitigates just the risk arising from exposure to market prices. Similarly, in 2021, we see that although the REER system implies expected negative regulatory costs and thus should be rejected by any risk-neutral investor, an investor with sufficient risk aversion may be incentivized by such regulation and prefer it to complete market exposure.

When comparing support policies, the usual practice is for the regulator to analyze the expected outlays under each scheme in an attempt to minimize the cost of the incentives provided to investors. Our focus is on the value to the investor of the risk removed under each policy. The idea is that, as long as the investor values certainty, some incentive schemes may reach the same objective at a lower cost for the regulator. The surplus created by an efficient distribution of risk between the regulator and the investor may render some incentive schemes more efficient than others.

# <span id="page-20-0"></span>**6 Conclusions**

The risk faced when investing in renewable energy projects highly dependent on weather conditions, such as solar and wind, is not only due to the price of electricity but also to the fact that production is uncertain. Therefore, it may not be sufficient to use option theory to study the value of a given RES support policy by considering the market price of electricity as the only source of uncertainty.

We contribute to the literature with an analytical model to estimate the value of the investment risk removed under diferent types of renewable energy policies and the importance of both the rights and the obligations that each policy entails, taking into account both the randomness of the market price and the randomness of energy production. We develop a framework for evaluating energy subsidies by modeling prices and energy production as correlated stochastic processes. We obtain analytical solutions for support policies consisting of fxed-price, fxed-revenue, or shared-upside payment mechanisms. The developed methodology enables a direct comparison of diferent incentive schemes with varying natures of risk exposure. In addition, our model can be applied to both administratively imposed and competitively auctioned subsidy schemes.

Our approach is not a valuation model for incentive schemes as if there were a fnancial market for them. Incentive schemes are not traded, so their market value cannot be properly defned. Instead, we present a model with analytical solutions that can be used as a frst approximation to analyze the level of risk and the balance of rights and duties that each type of regulation implies. In addition, our model allows us to compute the risk premium and value of a support policy to an investor with varying degrees of risk aversion. An advantage of this approach is that it can be easily applied to diferent technologies, countries, and time horizons by calibrating a few parameters.

The application of the model to the specifc case of wind and solar power in Spain shows the evolution from a situation in 2013 where the obligations imposed were negligible compared to the rights received to a scenario in 2021 where the assumed obligations for new projects awarded through auctions become more relevant, corresponding to a context of high electricity prices and increasingly competitive renewables. Changes in support policies were made to reduce the regulatory costs of these instruments. As a result,

investment incentives for wind and solar have decreased to the point of a negative value of the support policy. Our results show how investors' risk aversion can explain why investors submitted bids and projects were awarded in the 2021 auction under a regime with negative regulatory costs and thus lower expected revenues than the market.

However, the potential disincentive for investors of REER schemes should not be underestimated. As an illustration, in Spain's 2022 renewable energy auctions, policymakers failed to assess the investment environment in light of high market prices, increased costs of many renewable energy projects due to infation, supply chain bottlenecks, scarcity of critical materials, and increased transportation costs. This combination of factors resulted in only 45 MW out of the 1500 MW reserved for wind and none of the 1800 MW reserved for solar PV awarded.<sup>27</sup> In the long term, failed auctions delay the national energy transition agenda (74% of electricity generation from renewable sources by 2030 and 100% renewable generation by 2050, MITECO [2020](#page-40-21)). In the short term, lower penetration of renewables increases the wholesale market price, ultimately afecting all consumers (Ciarreta et al. [2014b;](#page-39-18) Fabra and Imelda [2023\)](#page-39-8).

When the obligations assumed are negligible compared to the rights received, investors' incentives for renewable energy projects increase, potentially leading to higher investment fows. Conversely, when the obligations exceed the rights, risk-neutral or moderately riskaverse investors may be discouraged from participating in these projects. Another implication of our fndings is that the regulator should always consider the degree of risk aversion of potential investors. As our results indicate, a sufficiently risk-averse investor may be willing to accept a contract in which the obligations outweigh the rights. This fnding has potential consequences for the type of frms awarded in renewable energy auctions. In principle, it is reasonable to think that larger frms tend to behave as risk-neutral agents, as they can hedge diferent types of risks by diversifying their portfolios. In contrast, smaller frms tend to be more risk-averse, as they may lack the hedging capabilities of larger frms. Therefore, the design of these support policies or renewable energy auctions can infuence the size of frms that receive new projects and thus impact the concentration of this market.

Our analytical model can provide valuable insights into the valuation of renewable energy support policies in diferent countries and contexts. We believe that how investors' risk aversion degree impacts the outcomes of renewable energy auctions should be further explored. The potential application of our methodology to the valuation of private contracts such as *power purchase agreements* (PPAs) deserves further research.

### **Appendix 1: Mathematical Appendix**

Let  $S_t$  and  $X_t$  be two GBM processes characterized by the following dynamics:

<span id="page-21-1"></span>
$$
\begin{cases}\ndS_t = \mu_S S_t dt + \sigma_S S_t dW_t^S \\
dX_t = \mu_X X_t dt + \sigma_X X_t dW_t^X \\
\rho dt = dW_t^S dW_t^X,\n\end{cases} \tag{46}
$$

where  $W_t^S$  and  $W_t^X$  are two correlated Brownian motions with correlation  $\rho \in (-1, 1)$  on the probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ , and let  $\mathcal{F}_t$ ,  $(0 \le t \le T)$ , be a filtration of sub- $\sigma$ -algebras of  $\mathcal F$ 

<span id="page-21-0"></span>The regulator set a secret maximum price above which bids would be rejected.

for these Brownian motions, where  $T$  is a fixed final time.<sup>28</sup> The above stochastic differential equations have the following solutions: $^{29}$  $^{29}$  $^{29}$ 

$$
S_t = S_0 e \left(\mu_S - \frac{\sigma_S^2}{2}\right) t + \sigma_S W_t^S, \tag{47}
$$

$$
X_t = X_0 e^{\left(\mu_X - \frac{\sigma_X^2}{2}\right)t + \sigma_X W_t^X},
$$
\n(48)

where  $S_0$  and  $X_0$  are the values of these processes at time  $t = 0$ . Levy's theorem allows one of the correlated Brownian motions, say  $W_t^S$ , to be expressed as a combination of mutually independent Brownian motions  $W_t^1$  and  $W_t^2$  (i.e.,  $dW_t^1 dW_t^2 = 0$ ):

$$
\begin{pmatrix} W_t^S \\ W_t^X \end{pmatrix} = \begin{pmatrix} \rho & \sqrt{1 - \rho^2} \\ 1 & 0 \end{pmatrix} \begin{pmatrix} W_t^1 \\ W_t^2 \end{pmatrix},
$$
\n(49)

where we defined  $W_t^1 = W_t^X$ , and introduced the orthogonal process  $W_t^2$ . Introducing these in  $(46)$  $(46)$  $(46)$ , we get

$$
\begin{cases}\ndS_t = \mu_S S_t dt + \sigma_S S_t \left(\rho \, dW_t^1 + \sqrt{1 - \rho^2} \, dW_t^2\right) \\
dX_t = \mu_X X_t dt + \sigma_X X_t \, dW_t^1.\n\end{cases} \tag{50}
$$

#### <span id="page-22-2"></span>**Appendix 1.1: Solution to Some Preliminary Expectations**

We are interested in obtaining the solution of the conditional expectation  $\mathbb{E}_0[X_t^m S_t^n | S_t > K]$ , where we use the shorthand notation  $\mathbb{E}_0[\cdot] \equiv \mathbb{E}[\cdot | \mathcal{F}_0]$ , being  $\mathcal{F}_0$  the sub- $\sigma$ -algebra containing the information available at time  $t = 0, K \in \mathbb{R}_+$ , and  $m, n \in \mathbb{R}$ .

The first step is to realize that  $X_t^m$  and  $S_t^n$  are in turn two correlated GBM processes  $\overline{X}_t$ and  $S_t$ , respectively:

<span id="page-22-0"></span> $28$  The notation used in the last equation in [\(46](#page-21-1)) is the most prevalent within the realm of stochastic calculus, yet it is merely a shorthand representation of  $\rho dt = d(W_t^S, W_t^X)$ , where  $\langle \cdot, \cdot \rangle$  represents the quadratic variation.

<span id="page-22-1"></span><sup>&</sup>lt;sup>29</sup> The mathematical tools used throughout this appendix can be consulted in Shreve  $(2004)$  $(2004)$ .

$$
\overline{X}_t \equiv X_t^m = \left(X_0 e \left(\mu_X - \frac{\sigma_X^2}{2}\right)t + \sigma_X W_t^X\right)^m
$$
\n
$$
= X_0^m e \left(m\mu_X - m\frac{\sigma_X^2}{2}\right)t + m\sigma_X W_t^X
$$
\n
$$
= X_0^m e \left(m\mu_X + \frac{m(m-1)}{2}\sigma_X^2 - \frac{(m\sigma_X)^2}{2}\right)t + m\sigma_X W_t^X
$$
\n
$$
= \overline{X}_0 e \left(\overline{\mu}_X - \frac{\overline{\sigma}_X^2}{2}\right)t + \overline{\sigma}_X W_t^X,
$$
\n(51)

where  $\overline{\mu}_X = m\mu_X + \frac{m(m-1)}{2}\sigma_X^2$ ,  $\overline{\sigma}_X = m\sigma_X$ , and  $\overline{X}_0 = X_0^m$ . Analogously:

$$
\overline{S}_t \equiv S_t^n = \left( S_0 e \left( \mu_S - \frac{\sigma_S^2}{2} \right) t + \sigma_S W_t^S \right)^n
$$
  
\n
$$
= S_0^n e \left( n \mu_S - n \frac{\sigma_S^2}{2} \right) t + n \sigma_S W_t^S
$$
  
\n
$$
= S_0^n e \left( n \mu_S + \frac{n(n-1)}{2} \sigma_S^2 - \frac{(n \sigma_S)^2}{2} \right) t + n \sigma_S W_t^S
$$
  
\n
$$
= \overline{S}_0 e \left( \overline{\mu}_S - \frac{\overline{\sigma}_S^2}{2} \right) t + \overline{\sigma}_S W_t^S
$$
 (52)

where  $\overline{\mu}_S = n\mu_S + \frac{n(n-1)}{2}\sigma_S^2$ ,  $\overline{\sigma}_S = n\sigma_S$ , and  $\overline{S}_0 = S_0^n$ . Thus, the differential form of these processes is described by

$$
\begin{cases}\n\,\bar{\delta}_t = \overline{\mu}_s \overline{S}_t dt + \overline{\sigma}_s \overline{S}_t \left( \rho \, dW_t^1 + \sqrt{1 - \rho^2} \, dW_t^2 \right) \\
d\overline{X}_t = \overline{\mu}_X \overline{X}_t dt + \overline{\sigma}_X \overline{X}_t \, dW_t^1.\n\end{cases} \tag{53}
$$

Therefore,  $\mathbb{E}_0 \left[ X_t^m S_t^n \mid S_t > K \right]$  can be expressed as

$$
\mathbb{E}_0\Big[\overline{X}_t\overline{S}_t \mid S_t > K\Big] = \mathbb{E}_0\Bigg[\Bigg(\overline{X}_0e^{\left(\overline{\mu}_X - \frac{\overline{\sigma}_X^2}{2}\right)t + \overline{\sigma}_X W_t^1}\Bigg)\overline{S}_t \mid S_t > K\Bigg].\tag{54}
$$

Now, according to the *Radon–Nikodym theorem*, the problem of changing the measure from  $\mathbb P$  to  $\widetilde{\mathbb P}$  reduces to finding the *Radon–Nikodym derivative*  $G = \frac{d\widetilde{\mathbb P}}{d\mathbb P}$  satisfying  $\mathbb E[G] = 1$ . Thus, for any random variable *H*:

<span id="page-23-0"></span>
$$
\widetilde{\mathbb{E}}[H] = \mathbb{E}\big[G_t H\big],\tag{55}
$$

where  $G_t = \mathbb{E}[G | \mathcal{F}_t]$  is the *Radon–Nikodym derivative process*. Hence, in [\(54\)](#page-23-0) we can defne

According to *Girsanov's theorem*, the change of measure defned in [\(56\)](#page-24-0) introduces a displacement in the Brownian motion contained in  $G_t$ . Therefore, the new Brownian motion  $\widetilde{W}_t^1$  under the new measure  $\widetilde{P}$  is described as  $\widetilde{W}_t^1 = W_t^1 - \overline{\sigma}_X t$ , and  $\widetilde{W}_t^2 = W_t^2$ , so:

<span id="page-24-0"></span>
$$
W_t^S = \rho W_t^1 + \sqrt{1 - \rho^2} W_t^2
$$
  
=  $\rho \left(\widetilde{W}_t^1 + \overline{\sigma}_X t\right) + \sqrt{1 - \rho^2} \widetilde{W}_t^2$   
=  $\widetilde{W}_t^S + \rho \overline{\sigma}_X t$ . (57)

Consequently, under the new measure  $(\widetilde{P})$ ,  $S_t$  and  $\overline{S}_t$  can be expressed as

$$
S_t^* = S_0 e^{(\mu_S - \frac{\sigma_S^2}{2})t + \sigma_S(\widetilde{W}_t^S + \rho \overline{\sigma}_X t)} = S_0 e^{((\mu_S + \rho \sigma_S \overline{\sigma}_X) - \frac{\sigma_S^2}{2})t + \sigma_S \widetilde{W}_t^S},
$$
(58)

$$
\overline{S}_t^* = \overline{S}_0 e^{(\overline{\mu}_S - \frac{\overline{\sigma}_S^2}{2})t + \overline{\sigma}_S(\widetilde{W}_t^S + \rho \overline{\sigma}_X t)} = \overline{S}_0 e^{((\overline{\mu}_S + \rho \overline{\sigma}_S \overline{\sigma}_X) - \frac{\overline{\sigma}_S^2}{2})t + \overline{\sigma}_S \widetilde{W}_t^S},
$$
(59)

where we use the superscript symbol ∗ to indicate that we are expressing them under the new measure. As the Brownian motion  $\widetilde{W}^S_t$  is normally distributed with mean zero and variance *t*, we can write  $\widetilde{W}_t^S = -\Upsilon \sqrt{t}$ , where  $\Upsilon$  is a standard normal random variable. So, ([54](#page-23-0)) reduces to

$$
\mathbb{E}_{0}\left[\overline{X}_{t}\overline{S}_{t} \mid S_{t} > K\right] = \frac{1}{\mathbb{P}(S_{t} > K)}\mathbb{E}_{0}\left[\overline{X}_{t}\overline{S}_{t}\mathbb{I}_{\{S_{t} > K\}}\right]
$$
\n
$$
= \overline{X}_{0}e^{\overline{\mu}_{X}t}\frac{1}{\mathbb{P}(S_{t} > K)}\widetilde{\mathbb{E}}_{0}\left[\overline{S}_{t}\mathbb{I}_{\{S_{t} > K\}}\right]
$$
\n
$$
= \overline{X}_{0}e^{\overline{\mu}_{X}t}\frac{1}{\mathbb{P}(S_{t} > K)}\int_{-\infty}^{\infty}\mathbb{I}_{\{S_{t}^{*} > K\}}(y)\overline{S}_{t}^{*}(y)\phi(y) dy,
$$
\n(60)

where we used the indicator function, and  $\phi(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}$  is the density of the standard normal distribution.<sup>30</sup> It is easy to verify that the conditions  $K < S_t(y)$  and  $K < S_t^*(y)$  hold if and only if  $y < d_k$ , and  $y < d_k + \overline{\sigma}_X \rho \sqrt{t}$ , respectively, where  $d_k$  is given by

<span id="page-24-2"></span>
$$
d_k = \frac{\log\left(\frac{S_0}{K}\right) + (\mu_S - \frac{\sigma_S^2}{2})t}{\sigma_S \sqrt{t}}.\tag{61}
$$

So, ([60](#page-24-2)) can be expressed as

<span id="page-24-1"></span><sup>&</sup>lt;sup>30</sup> Given a set *A*, the indicator function is defined as  $\mathbb{I}_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$ .

$$
\overline{X}_{0}e^{\overline{\mu}_{X}t} \frac{1}{\Phi(d_{k})} \int_{-\infty}^{d_{k}+\overline{\sigma}_{X}\rho\sqrt{t}} \overline{S}_{0}e^{\left(\overline{\mu}_{S} + \rho\overline{\sigma}_{S}\overline{\sigma}_{X} - \frac{\overline{\sigma}_{S}^{2}}{2}\right)t - \overline{\sigma}_{S}y\sqrt{t}} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^{2}}{2}} dy
$$
\n
$$
= \overline{X}_{0}\overline{S}_{0}e^{\left(\overline{\mu}_{X} + \overline{\mu}_{S} + \rho\overline{\sigma}_{S}\overline{\sigma}_{X}\right)t} \frac{1}{\Phi(d_{k})} \int_{-\infty}^{d_{k}+\overline{\sigma}_{X}\rho\sqrt{t}} \frac{1}{\sqrt{2\pi}} e^{-\frac{(y + \overline{\sigma}_{S}\sqrt{t})^{2}}{2}} dy
$$
\n
$$
= \overline{X}_{0}\overline{S}_{0}e^{\left(\overline{\mu}_{X} + \overline{\mu}_{S} + \rho\overline{\sigma}_{S}\overline{\sigma}_{X}\right)t} \frac{\Phi(d_{k} + \overline{\sigma}_{X}\rho\sqrt{t} + \overline{\sigma}_{S}\sqrt{t})}{\Phi(d_{k})}.
$$
\n(62)

The fnal solution can be obtained by expressing these results in terms of the original parameters:

$$
\mathbb{E}_{0}\left[X_{t}^{m}S_{t}^{n} \mid S_{t} > K\right]
$$
\n
$$
= X_{0}^{m}S_{0}^{n}e^{\left(m\mu_{X} + \frac{m(m-1)}{2}\sigma_{X}^{2} + n\mu_{S} + \frac{n(n-1)}{2}\sigma_{S}^{2} + mn\rho\sigma_{S}\sigma_{X}\right)t}\left(\frac{\Phi(d_{k} + m\sigma_{X}\rho\sqrt{t} + n\sigma_{S}\sqrt{t})}{\Phi(d_{k})}\right).
$$
\n(63)

Taking the limit  $K \downarrow 0$  in ([63](#page-25-2)), we obtain the unconditional expectation:

<span id="page-25-2"></span><span id="page-25-1"></span>
$$
\mathbb{E}_0 \left[ X_t^m S_t^n \right] = X_0^m S_0^n e^{(m\mu_X + \frac{m(m-1)}{2}\sigma_X^2 + n\mu_S + \frac{n(n-1)}{2}\sigma_S^2 + mn\rho \sigma_S \sigma_X)t}.
$$
(64)

Finally, using *Bayes' theorem* and the *law of total probability*, it is easy to obtain the following solution from Eqs. [\(63\)](#page-25-2) and ([64](#page-25-1)):

$$
\mathbb{E}_{0}\left[X_{t}^{m}S_{t}^{n} \mid S_{t} \leq K\right]
$$
\n
$$
= X_{0}^{m}S_{0}^{n}e^{(m\mu_{X} + \frac{m(m-1)}{2}\sigma_{X}^{2} + n\mu_{S} + \frac{n(n-1)}{2}\sigma_{S}^{2} + mn\rho\sigma_{S}\sigma_{X})t}\left(\frac{\Phi(-d_{k} - m\sigma_{X}\rho\sqrt{t} - n\sigma_{S}\sqrt{t})}{\Phi(-d_{k})}\right).
$$
\n(65)

### <span id="page-25-0"></span>**Appendix 1.2: Solutions for the Rights and Obligations of Each System**

### <span id="page-25-3"></span>**Appendix 1.2.1: FP Regulation**

We proceed to solve Eq.  $(6)$  $(6)$ :

$$
R_{fp} = \sum_{t=1}^{T} \mathbb{E}_{0} \left[ e^{-rt} X_{t} (K_{fp} - S_{t})^{+} \right]
$$
  
\n
$$
= \sum_{t=1}^{T} e^{-rt} \mathbb{E}_{0} \left[ X_{t} (K_{fp} - S_{t}) \mid S_{t} \le K_{fp} \right] \mathbb{P}(S_{t} \le K_{fp})
$$
  
\n
$$
= \sum_{t=1}^{T} e^{-rt} \left( K_{fp} \mathbb{E}_{0} \left[ X_{t} \mid S_{t} \le K_{fp} \right] - \mathbb{E}_{0} \left[ X_{t} S_{t} \mid S_{t} \le K_{fp} \right] \right) \mathbb{P}(S_{t} \le K_{fp}).
$$
  
\n(66)

Using the solution derived in Sect. [7.1](#page-22-2), we obtain

$$
R_{fp} = \sum_{t=1}^{T} X_0 e^{(\mu_X - r)t} \Big( K_{fp} \Phi(-d_{fp}) - S_0 e^{(\mu_S + \sigma_S \sigma_X \rho)t} \Phi(-d_{fp} - \sigma_S \sqrt{t}) \Big), \qquad (67)
$$

where

<span id="page-26-0"></span>
$$
d_{fp} = \frac{\log\left(\frac{S_0}{K_{fp}}\right) + \left(\mu_S + \sigma_S \sigma_X \rho - \frac{\sigma_S^2}{2}\right)t}{\sigma_S \sqrt{t}}.
$$
\n(68)

Analogously:

$$
O_{fp} = \sum_{t=1}^{T} \mathbb{E}_{0} \left[ e^{-rt} X_{t} (S_{t} - K_{fp})^{+} \right]
$$
  
\n
$$
= \sum_{t=1}^{T} e^{-rt} \mathbb{E}_{0} \left[ X_{t} (S_{t} - K_{fp}) \mid S_{t} > K_{fp} \right] \mathbb{P}(S_{t} > K_{fp})
$$
  
\n
$$
= \sum_{t=1}^{T} e^{-rt} \left( \mathbb{E}_{0} \left[ X_{t} S_{t} \mid S_{t} > K_{fp} \right] - K_{fp} \mathbb{E}_{0} \left[ X_{t} \mid S_{t} > K_{fp} \right] \right) \mathbb{P}(S_{t} > K_{fp})
$$
  
\n
$$
= \sum_{t=1}^{T} X_{0} e^{(\mu_{X} - r)t} \left( S_{0} e^{(\mu_{S} + \sigma_{S} \sigma_{X} \rho)t} \Phi(d_{fp} + \sigma_{S} \sqrt{t}) - K_{fp} \Phi(d_{fp}) \right).
$$
  
\n(69)

Finally, from ([67](#page-26-0)) and ([69](#page-26-1)), it is easy to verify that  $V_{fp}$  is as in Eq. [\(12\)](#page-7-1).

### <span id="page-26-3"></span>**Appendix 1.2.2: FR Regulation**

We proceed to solve the problem in Eq.  $(13)$  $(13)$ , i.e.,

<span id="page-26-2"></span><span id="page-26-1"></span>
$$
R_{fr} = \sum_{t=1}^{T} \mathbb{E}_0 \left[ e^{-rt} (K_{fr} - X_t S_t)^+ \right]. \tag{70}
$$

First, we define the new process  $Y_t = X_t S_t$ . According to *Itô*'s *lemma*, this new process is itself a GBM:

$$
dY_t = d(X_t S_t) = S_t dX_t + X_t dS_t + dX_t dS_t
$$
  
=  $S_t(\mu_X X_t dt + \sigma_X X_t dW_t^X) + X_t(\mu_S S_t dt + \sigma_S S_t dW_t^S)$   
+  $(\mu_X X_t dt + \sigma_X X_t dW_t^X)(\mu_S S_t dt + \sigma_S S_t dW_t^S).$  (71)

Using the multiplication rules of *Itô*<sup>*'*</sup>*s calculus* (i.e., *dt dt* = 0, *dt*  $dW_t^S = 0$ , *dt*  $dW_t^X = 0$ ,  $dW_t^S dW_t^S = dt$ ,  $dW_t^X dW_t^X = dt$ , and  $dW_t^X dW_t^S = \rho dt$ ) yields the following expression:

$$
dY_t = (\mu_X + \mu_S + \rho \sigma_S \sigma_X) Y_t dt + Y_t (\sigma_S dW_t^S + \sigma_X dW_t^X). \tag{72}
$$

The linear combination of the correlated Brownian motions  $dW_t^S$  and  $dW_t^X$  can be expressed in terms of a new Brownian motion  $dW_t^Y$  such that  $\sigma_Y dW_t^Y = \sigma_S dW_t^S + \sigma_X dW_t^X$ . To find the value of  $\sigma_Y$ , we can square this expression and use Itô's multiplication rules again:

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$$
(\sigma_Y dW_t^Y)(\sigma_Y dW_t^Y) = (\sigma_S dW_t^S + \sigma_X dW_t^X)(\sigma_S dW_t^S + \sigma_X dW_t^X)
$$
\n(73)

$$
\sigma_Y^2 dt = (\sigma_S^2 + \sigma_X^2 + 2\sigma_S \sigma_X \rho) dt
$$
\n(74)

$$
\sigma_Y = \sqrt{\sigma_S^2 + \sigma_X^2 + 2\sigma_S \sigma_X \rho}.\tag{75}
$$

Therefore:

$$
dY_t = \mu_Y Y_t dt + \sigma_Y Y_t dW_t^Y, \qquad (76)
$$

where the drift and volatility of the new GBM are given by

$$
\mu_Y = \mu_S + \mu_X + \sigma_S \sigma_X \rho, \tag{77}
$$

$$
\sigma_Y = \sqrt{\sigma_S^2 + \sigma_X^2 + 2\sigma_S \sigma_X \rho}.
$$
\n(78)

Thus, Eq. ([70](#page-26-2)) can be expressed as

$$
R_{fr} = \sum_{t=1}^{T} \mathbb{E}_{0} [e^{-rt} (K_{fr} - Y_{t})^{+}]
$$
  
= 
$$
\sum_{t=1}^{T} e^{-rt} \mathbb{E}_{0} [(K_{fr} - Y_{t}) | Y_{t} \le K_{fr}] \mathbb{P}(Y_{t} \le K_{fr})
$$
  
= 
$$
\sum_{t=1}^{T} e^{-rt} (K_{fr} - \mathbb{E}_{0} [Y_{t} | Y_{t} \le K_{fr}) ) \mathbb{P}(Y_{t} \le K_{fr}).
$$
 (79)

Using the solution derived in Sect. [7.1](#page-22-2) (where  $X_t \longrightarrow Y_t$ ,  $m = 1$ ,  $n = 0$ ), we get the following result:

$$
R_{fr} = \sum_{t=1}^{T} e^{-rt} \Big( K_{fr} \Phi(-d_{fr}) - Y_0 e^{\mu_Y t} \Phi \Big( -d_{fr} - \sigma_Y \sqrt{t} \Big) \Big), \tag{80}
$$

where

<span id="page-27-0"></span>
$$
d_{fr} = \frac{\log\left(\frac{Y_0}{K_{fr}}\right) + \left(\mu_Y - \frac{\sigma_Y^2}{2}\right)t}{\sigma_Y \sqrt{t}}.
$$
\n(81)

Analogously:

<span id="page-28-1"></span>
$$
O_{fr} = \sum_{t=1}^{T} \mathbb{E}_{0} [e^{-rt}(Y_{t} - K_{fr})^{+}]
$$
  
\n
$$
= \sum_{t=1}^{T} e^{-rt} \mathbb{E}_{0} [(Y_{t} - K_{fr}) | Y_{t} > K_{fr}] \mathbb{P}(Y_{t} > K_{fr})
$$
  
\n
$$
= \sum_{t=1}^{T} e^{-rt} (\mathbb{E}_{0} [Y_{t} | Y_{t} > K_{fr}] - K_{fr}) \mathbb{P}(Y_{t} > K_{fr})
$$
  
\n
$$
= \sum_{t=1}^{T} e^{-rt} (Y_{0} e^{\mu_{Y} t} \Phi (d_{fr} + \sigma_{Y} \sqrt{t}) - K_{fr} \Phi (d_{fr}) ).
$$
  
\n(82)

Finally, from ([80](#page-27-0)) and ([82](#page-28-1)), it is easy to verify that  $V_f$  is as in Eq. ([19](#page-8-2)).

### **Appendix 1.2.3: SU Regulation**

Note that the solution to Eq. [\(21\)](#page-8-1) is the same as the solution to Eq. [\(7\)](#page-6-3) developed in Sect. [7.2.1](#page-25-3), where  $K_{su}$  is substituted for  $K_{fp}$ . Similarly, the solution to Eq. [\(24](#page-9-1)) is equal to the solution to Eq. ([10\)](#page-7-2), except for the multiplying factor (1 –  $\alpha$ ) and the substitution of  $K_{su}$  for  $K_{fp}$ .

### <span id="page-28-0"></span>**Appendix 1.3: Approximation for the Investor's Expected Revenue Under the SU Scheme**

The revenues under the SU scheme are  $w_t = \max\{KX_t, Z_t\}$ , where  $Z_t = K(1 - \alpha)X_t + \alpha S_t X_t$ . So:

$$
\mathbb{E}\left[w_t^{1-\gamma}\right] = \mathbb{E}\left[w_t^{1-\gamma} \mid S_t \le K\right] \mathbb{P}(S_t \le K) + \mathbb{E}\left[w_t^{1-\gamma} \mid S_t > K\right] \mathbb{P}(S_t > K) \n= K^{1-\gamma} \mathbb{E}\left[X_t^{1-\gamma} \mid S_t \le K\right] \Phi(-d_k) + \mathbb{E}\left[Z_t^{1-\gamma} \mid S_t > K\right] \Phi(d_k) \n= (K X_0)^{1-\gamma} \exp\left[\left(\mu_X - \gamma \frac{\sigma_X^2}{2}\right)(1-\gamma)t\right] \Phi\left(-d_k - (1-\gamma)\sigma_X \rho \sqrt{t}\right) \n+ \mathbb{E}\left[\left(\mathbf{X} + \mathbf{Y}\right)^{1-\gamma} \mid S_t > K\right] \Phi(d_k),
$$
\n(83)

where  $Y_t = X_t S_t$ ,  $\mathbf{X} \sim \text{Lognormal}(v_X, \zeta_X^2)$ ,  $\mathbf{Y} \sim \text{Lognormal}(v_Y, \zeta_Y^2)$ , and

$$
v_X = \log\left(K(1-\alpha)X_0\right) + \left(\mu_X - \frac{\sigma_X^2}{2}\right)t,\tag{84}
$$

$$
v_Y = \log(\alpha Y_0) + \left(\mu_Y - \frac{\sigma_Y^2}{2}\right)t,\tag{85}
$$

$$
\zeta_X = \sigma_X \sqrt{t},\tag{86}
$$

$$
\zeta_Y = \sigma_Y \sqrt{t}.\tag{87}
$$

Note that we use the same notation for the parameters of  $Y_t$  as in Sect. [7.2.2](#page-26-3). There is no known analytical solution for the summation of log-normally distributed variables  $(X + Y)$ . Lo ([2013\)](#page-40-15) presents an analytical approach in which the summation of these correlated variables can be approximated by a new log-normally distributed variable **Z**:

$$
\mathbf{X} + \mathbf{Y} \approx \mathbf{Z} \sim \text{Lognormal}(v_Z, \zeta_Z^2),\tag{88}
$$

where

$$
v_Z = \log \left( e^{(v_X + \frac{c_Y^2}{2})} + e^{(v_Y + \frac{c_Y^2}{2})} \right) - \frac{c_Z^2}{2},\tag{89}
$$

$$
\varsigma_{Z} = \sqrt{\varsigma_{X}^{2} e^{\left(v_{X} + \frac{\varsigma_{X}^{2}}{2}\right)} + \varsigma_{Y}^{2} e^{\left(v_{Y} + \frac{\varsigma_{Y}^{2}}{2}\right)} + 2\rho_{XY}\varsigma_{X}\varsigma_{Y}e^{\left(v_{X} + \frac{\varsigma_{X}^{2}}{2}\right)} e^{\left(v_{Y} + \frac{\varsigma_{Y}^{2}}{2}\right)}},
$$
(90)

and  $\rho_{XY}$  satisfies the following condition:

$$
\rho_{XY}dt = dW_t^X dW_t^Y = dW_t^X \frac{1}{\sigma_Y} (\sigma_S dW_t^S + \sigma_X dW_t^X) = \frac{\sigma_S \rho + \sigma_X}{\sigma_Y} dt, \tag{91}
$$

i.e.,

$$
\rho_{XY} = \frac{\sigma_S \rho + \sigma_X}{\sigma_Y}.
$$
\n(92)

Because **Z** is log-normally distributed, we can use the formulas derived in Sect. [7.1](#page-22-2) to compute its partial moments. In particular, we use the solution [\(63\)](#page-25-2):

$$
\mathbb{E}_0\big[\mathbf{Z}^{1-\gamma} \mid S_t > K\big] = \exp\left[\left(\mu_Z - \frac{\gamma \sigma_Z^2}{2}\right)(1-\gamma)t\right] \frac{\Phi(d_k + (1-\gamma)\sigma_Z \rho_{SZ} \sqrt{t})}{\Phi(d_k)},\tag{93}
$$

where  $\mu_Z = \frac{v_Z}{t} + \frac{v_Z}{t}$  $\frac{\varsigma_Z^2}{2}$ ,  $\sigma_Z = \frac{\varsigma_Z}{\sqrt{t}}$ , and we define  $\rho_{SZ}$  such that  $Cov(S_t, Z_t) = \mathbb{E}[S_t] \mathbb{E}[Z_t] (e^{\rho_{SZ}\sigma_S\sigma_Z t} - 1), \text{ i.e.,}$ 

$$
\rho_{SZ} = \frac{1}{\sigma_Z \sigma_S t} \log \left[ 1 + \frac{K(1-\alpha)(e^{\sigma_S \sigma_X \rho t} - 1) + \alpha S_0 e^{(\mu_S + \sigma_S \sigma_X \rho)t} \left(e^{(\sigma_S \sigma_X \rho + \sigma_S^2)t} - 1\right)}{K(1-\alpha) + \alpha S_0 e^{(\mu_S + \rho \sigma_S \sigma_X)t}} \right].
$$
\n(94)

Figure [10](#page-35-0) shows, for the two cases studied with the parameter values given in Table [1,](#page-30-1) how the relative approximation error (i.e.,  $\varepsilon = 100\%$ )  $\frac{\text{approximated value} - \text{true value}}{\text{true value}}$ ) tends to decrease the larger *K* or  $\gamma$ . The true values are obtained by simulation (100,000 trials). true value

Parameter	Wind		Solar		Sources
	2013	2021	2013	2021	
$K_{fp}$ ( $\varepsilon$ /MWh)	77.3		395.3		CNMC (2022)*
$K_{fr}(\text{\textup{E}/MW})$	175,849	181,906	573,110	516,270	
$S_0$ ( $\epsilon$ /MWh)	38.3	104.1	45.6	101.5	
$X_0$ (MWh/MW)	2377	2134	1783	1413	
$\sigma_X$	0.07	0.05	0.33	0.17	
$\sigma_{\rm S}$	0.32	0.51	0.29	0.49	
$\mu_X$	$\Omega$	$\Omega$	$\Omega$	$\mathbf{0}$	
$\mu_S$	$-0.05$	$-0.11$	$-0.03$	$-0.12$	OMIP (2021), CNMC (2022)*
$\rho$	$-0.47$	$-0.56$	$-0.05$	$-0.22$	OMIE (2023), REE (2023)*
$K_{su}(\varepsilon/\text{MWh})$		30.2		31.6	BOE $(2021b)*$
$\alpha$		0.25		0.25	
r	0.10	0.04	0.10	0.05	Roth et al. (2021), IRENA (2023b)
$T_f$ (years)	15	15	15	15	

<span id="page-30-1"></span>**Table 1** Calibrated parameters and sources

#### \*Own calculations

 $K_{fp}$ , strike price in FP;  $K_{fr}$ , strike revenue in FR;  $S_0$ , initial price;  $X_0$ , initial production;  $\sigma_X$ , production volatility;  $\sigma_S$ , price volatility;  $\mu_X$ , production drift,  $\mu_S$ , price drift;  $\rho$ , correlation;  $K_{su}$ , minimum price in SU;  $\alpha$ , investor's upside share in SU;  $r$ , discount rate;  $T_f$ , time horizon



<span id="page-30-0"></span>**Fig. 1** Rights and obligations for wind power





<span id="page-31-0"></span>**Fig. 2** Rights and obligations as a function of *K* for wind power



<span id="page-32-0"></span>



<span id="page-32-1"></span>**Fig.** 4 Total incentive  $(v - v_m)$  for wind power



<span id="page-32-2"></span>**Fig. 5** Rights and obligations for solar PV power







<span id="page-33-0"></span>**Fig. 6** Rights and obligations as a function of *K* for solar PV power



<span id="page-34-0"></span>**Fig. 7** Relative risk premium for solar PV power



<span id="page-34-1"></span>**Fig. 8** Total incentive ( $v - v_m$ ) for solar PV power



<span id="page-35-2"></span>



<span id="page-35-0"></span>**Fig. 10** Relative error for different values of  $\gamma$  and  $K$ 

# <span id="page-35-1"></span>**Appendix 2: Evolution of Market Prices**

As shown in Fig. [11](#page-36-1), for both solar and wind, the VWAP for electricity held steady until 2020, showing year-to-year fuctuations but no clear long-term trend. However, in 2021 and 2022, these prices spiked signifcantly.



<span id="page-36-1"></span>**Fig. 11** VWAP by technology (own elaboration with data from CNMC [2022\)](#page-39-19)

# <span id="page-36-0"></span>**Appendix 3: Graphs of the Utilities and Absolute Risk Premia**

See Figs. [12](#page-36-2), [13,](#page-37-0) [14,](#page-37-1) and [15](#page-38-1).



<span id="page-36-2"></span>**Fig. 12** Utilities for wind power

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<span id="page-37-0"></span>**Fig. 13** Absolute risk premium for wind power



<span id="page-37-1"></span>**Fig. 14** Utilities for solar PV power



<span id="page-38-1"></span>**Fig. 15** Absolute risk premium for solar PV power

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**Data Availability** All data used in this paper are publicly available.

### **Declarations**

**Confict of interest** The authors have no confict of interest to declare that are relevant to the content of this article.

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