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Recovering autonomous work after the pandemic: analysis in Calculus for incoming Students in Technical Education degrees

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The use of active methodologies in the first years of university technical education is a difficult task to approach from two different points of view: from the student's point of view, since it implies a greater participation in the teaching-learning process, to which he/she is not used to; and from the teacher's point of view, because of the difficulty involved in the design of activities, particularly in basic subjects of the first year. This article shows an experience of autonomous work using Problem Based Learning applied to the study of real functions of a real variable, as part of the subject Calculus of several Engineering degrees. The result is positive with an improvement in autonomous work when active methodologies are used. From the analysis performed, and using a dataset spanning three academic years and including a diverse sample of 527 students, our study provides a solid basis for researchers, designers and especially mathematics educators to improve student performance by introducing active methodologies. The increased generalizability and implications for educational practice place our study as a valuable contribution to the field, making the way easier for using Problem Based Learning in early grades.

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Introduction

The establishment of the European Higher Education Area (EHEA) marked a significant shift in teaching and learning methodologies at European universities. In Spain, following the guidelines of the European Union, the Framework Document for the Integration of the Spanish University System at the Spanish Ministry of Education, Culture and Sport (2003) outlined several key principles for the new university model. Notably, it emphasized that "the organization of the teachings will revolve around the learning, using active methodologies in which the student is the fundamental element" (Spanish Ministry of Education, Culture and Sport 2003; Hernández Trasobares and Lacuesta Gilaberte 2007).

However, the European report on the implementation of the Bologna Process made in 2015 highlighted the need for continued efforts toward the convergence of the EHEA, citing insufficient progress on several points. Among these, the focus on student-centred learning and clearly defined objectives was particularly emphasized (European Commission/EACEA/Eurydice 2015). This notion has been reinforced by subsequent documents advocating for new assessment methods and the development of student-centred learning (ESU 2015; European Commission/EACEA/Eurydice 2018). The latest report from the EU, in this way, is very clear: student-centred learning remains under-developed in many parts of the continent (European Commission 2020).

To enable students to take an active role in their learning, traditional teaching methodologies, where the teacher has the important role, must be replaced with active methodologies that place the student at the heart of the educational process. One of these active methodologies is the Problem Based Learning (PBL).

In the 1960s, Postman and Weingartner, when publishing the book Teaching as a Subversive Activity, challenged the prevailing educational paradigms by proposing revolutionary ideas about education. They advocated for a student-centered approach that encourages learners to ask questions. According to the authors, "The new education has as its purpose the development of a new kind of person, one who -as a result of internalizing a different series of concepts- is an actively inquiring, flexible, creative, innovative, tolerant, liberal personality who can face uncertainty and ambiguity without disorientation, who can formulate viable new meanings to meet changes in the environment which threaten individual and mutual survival" (Postman and Weingartner 1969).

This approach was implemented at McMaster University (Canada) where professionals recognized that health issues were not always adequately addressed by healthcare providers. They sought to reform how these professionals acquired the knowledge, competencies, and skills necessary for their roles (Font Ribas 2004). The success of this methodology made it expand throughout other universities, entering Europe through Maastricht University in 1974, and later at Aalborg University in Denmark, which utilized a derivative approach known as project-based learning (Hernández Trasobares and Lacuesta Gilaberte 2007).

At this point, the question arises: apart from Medicine, could PBL be applied to other sciences? Thus, the overall question is if PBL fits mathematics in a higher education curriculum. Burton (2002) stated that:

- Learning in the mathematics classroom is social, not individual.
- Coming to know mathematics depends on active participation in the enterprises so valued in that community of mathematics practice that they are accepted within that community.

This perspective is echoed by several authors in recent years (Mustafa et al. 2019; Noble et al. 2020; Hidayat et al. 2020; Shidqiya et al. 2023), who argue that mathematical knowledge is constructed within the classroom community. A learner "knows" mathematics based on the values upheld within that community (Schettino 2016).

Aalborg University offers a mathematics program at the Faculty of Engineering and Science that is based on a PBL model. This student-driven approach focuses on exemplary problems typical of the professions students will enter after graduation (Dahl 2018).

Related literature

The implementation of medical studies at McMaster University using active learning methodologies is grounded in Barrows' perspective, the founder of PBL, who considers it "a learning method in which the problems, as starting point, are used for the acquisition of new knowledge" (Barrows and Tamblyn 1980). Through PBL, students enhance their learning opportunities when:

- Previous knowledge is activated to integrate new information.
- There are opportunities for application.
- New knowledge is learned in the context in which it will be utilized.

The initial definition of PBL has evolved as it has been adopted by universities worldwide, leading to a broader and more varied understanding of the concept. Gijselaers (1996) relates PBL to the principles of theoretical learning, emphasizing knowledge construction, meta-learning, and contextual learning (Kolmos 2004). Subsequently, Savin-Baden and Howell Major (2004) outlined various PBL models, which are detailed in Table 1.

Other authors (Naji et al. 2020; Smith et al. 2022; Ban 2023) advocate for the PBL methodology as a framework for both teaching and learning in engineering education. Holgaard and Kolmos (2021) further elevate its significance by defining it as a conceptual platform for learning, while allowing each teacher or institution to define their own version of PBL based on common principles: utilizing real-life problems, adopting a constructivist approach, integrating theory and practice, promoting self-directed leadership, and emphasizing the ongoing reflection necessary for continuous development.

The introduction of PBL models at universities such as Maastricht or Aalborg has been relatively straightforward, as these models were developed alongside the establishment of the institutions. However, the design of PBL varies from one university to another, needing to adapt to the educational policies and cultural contexts of the countries in which they operate. Additionally, academic institutions, faculty, and students typically participate in this design process, which represents a significant cultural and educational shift that few universities have the opportunity to undertake.

According to Barrow´s description (1984), three main educational objectives are addressed simultaneously:

- acquisition of knowledge that can be retrieved and applied in a professional setting,
- development of skills to extend and enhance one's own knowledge,
- cultivation of professional problem-solving abilities.

In PBL methodology, the problem serves as both a challenge to reasoning skills and a focal point for the learning process (Turcotte et al. 2022; He and Qi 2022). Learning is viewed as an active process of constructing and reconstructing knowledge, and PBL aligns well with these findings (Perrenet et al. 2000). Project-based learning is an engaging method that involves exploring real problems and requires various skills, such as planning,

Table 1 Comparison of the forms of active learning (adapted from Savin-Baden and Howell Major 2004).

Method	Organization of the knowledge	Form of knowledge	Student's role	Tutor's role
Problem-based learning	Open situations and problems	Contingent and built	Active participants and critical researchers that are authors of their own learning experiences	Cause learning opportunities
Project-based learning	Given by the tutor. Structured tasks	Performative and practical	Complete a project or member of a project team that develops a solution or strategy	Task manager and project supervisor
Problem solving-based learning	Logical solution, step by step, through the knowledge provided by the teacher	Mostly propositional, but can it be practical	Problem solver, that acquires knowledge through the resolution of a limited problem	Guide for the knowledge and the correct solution.
Active learning	Discussion directed in groups, reflection of the action.	Personal and performative	Self-advisory, achievement of objectives through reflection and action	Facilitator of reflection

implementation, and evaluation of solutions applicable in specific contexts (de la Puente Pacheco et al. 2020).

From an educational perspective, the Spanish educational system has traditionally relied on master classes as the primary method for transmitting knowledge to students. According to Li and Stylianides (2018), despite the documented benefits of Problem-Based Learning for student learning, there has been limited research on its application in mathematics education, where most teachers are accustomed to lecture methods and students are familiar with this style of teaching. The university system reflects this tradition, with a strong emphasis on traditional educational practices. The University of the Basque Country is no exception.

The design of new degrees within the European Higher Education Area (EHEA) presented an opportunity for change in teaching and learning methodologies. However, as Idoia Fernández and Itziar Alcorta noted, "The university was not ready to undertake a change of model of such magnitude, we didn't even consider this to be the way to follow" (Fernández and Alcorta 2014). Nevertheless, the university administration supports the implementation of active teaching methodologies and provides training for faculty interested in applying these methods across various teaching modalities, including master classes, classroom practices, seminars, and laboratories.

It is also important to consider the impact of these active strategies on student learning. While numerous studies have reported positive outcomes in health-related fields (such as Medicine and Nursing), evidence in other disciplines is less prevalent, particularly due to the challenges associated with designing activities for first-year university students in foundational subjects. In this context, the conclusion by LoPresto and Slater (2016) is noteworthy, when they compare active learning strategies to traditional lectures: "The results of this study add weight to the notion that most modern pedagogies are superior to traditional lecture, and that although the relative impacts of particular pedagogies are mostly indistinguishable from one another, they are all are better than traditional lecture alone".

Overall, literature indicates that active learning methods employed in both classroom and large lecture settings can enhance students' grades and lead to positive perceptions of their learning experiences (Sinnayah et al. 2019; Reyes et al. 2018).

Methodology

Participants and data. At the University of the Basque Country, within the School of Engineering in Bilbao, degrees in Industrial Technology Engineering, Environmental Engineering, and Organization Engineering are offered. In the first year, which is common to all three degrees, the subject of Calculus is taught, corresponding to 12 ECTS. This is an annual course, with the following teaching modalities: master classes accounting for 10.5 ECTS and seminars for 1.5 ECTS. The correspondence is 1 ECTS = 10 h of class time plus 15 h of student work.

Due to the high enrolment in this subject, there are four groups taught in Spanish, three in Basque, and one in English. The average number of students in each group is 60, and for the seminars, three subgroups of 20 students are formed. In our research, we have denoted these three subgroups as GA, GB and GC. The faculty teaching this subject consists of at least eight instructors, as master classes and seminars may be taught by different individuals in some groups. To ensure equitable assessment across all groups and subgroups, the institution mandates that a portion of the evaluation be common to all.

We have collected data from the last three academic years, resulting in 188 students in the GA subgroup, 172 in GB, and 167 in GC, for a total of 527 students.

Active methodologies implementation in the first-year undergraduate groups. The faculty involved in this article is engaged in research related to innovative education (Bilbao et al. 2015; Varela et al. 2017) and has undergone training in active methodologies through various courses. In their commitment to introducing a more active and participatory teaching-learning approach, they have designed a learning experience based on problem-solving (referencing the classification in Table 1) to be implemented in the Calculus course.

Given that some of the teachers who teach this subject prefer to maintain traditional lectures as their sole teaching method (and not active methodologies), the faculty participating in this article decided to apply Problem-Based Learning (PBL) specifically in the seminar sessions. This choice was made because seminar instructors have the autonomy to assess students according to their own criteria. The number of students in the seminars allows for the division into work teams of a maximum of 4 students. Therefore, the implementation of PBL has been done during 10 weeks, with sessions lasting an hour and a half (totalling 1.5 ECTS). The learning results (LR) addressed include:

- Study of real functions with one real variable: Elementary functions (LR1), Limits (LR2), Continuity (LR3), Derivability (LR4) and Differentiability (LR5),
- Riemann integral and its applications for calculating areas and volumes of revolution (LR6).

Most of these concepts were introduced to the participants during high school, prior to their university entrance. Therefore, we anticipate that they will not encounter significant difficulties when engaging with these topics without traditional lectures. This new teaching-learning methodology aims to deepen and reinforce their understanding of concepts previously learned through conventional methods.

To support this approach, six distinct documents have been prepared, one for each learning outcome, all following a consistent structure.

Firstly, some activities are presented for the student to do autonomously and individually before attending the seminar. Subsequently, the rest of the activities are developed to be carried out in the classroom, in work teams and also autonomously, allowing students to engage both with their peers and the instructor to address any questions that may arise. Each activity is timed taking into account the duration of the seminar (90 min), and adding the work to be done by the student on their own. Students are provided with bibliographic and multimedia resources to assist them in understanding and completing the activities. Additionally, the first few minutes of each session are dedicated to addressing any questions that may have come up, with responses provided by either the students or the teacher. Each document begins with a timetable and guidelines on how to approach the proposed activities.

In the next subsection, we present part of the document that has been prepared for the study of the continuity of a real function of real variable.

Design of activities for the study of continuity. In the session where we work the continuity, which is the third session of the ten into which the seminars are divided, the concept of continuity of a function is reviewed, and how it is related to the concept of limit seen in the previous session will be studied.

Next, Table 2 shows the tasks to be carried out and the way in which they should be done.

The first task is stated as follows: "We are going to design a skateboard track. In order to do so, we have to distribute in an appropriate way structures that we can slide or jump on: parabolic tubes, inclined planes…". We try to add figure (as Fig. 1) to students get into the role of the problem.

Then, students are asked to draw the profiles of some of the tracks they would like to design and to try to define functions of one variable that fit (even if only approximately) those profiles they have drawn. They are encouraged to start with elementary functions that can represent small segments of the tracks and then to connect these segments to create longer tracks.

In the second task, we pose questions related to continuity: "In the above designs, are there abrupt changes in slopes? It is obvious that, following the profiles, skaters will have to jump through the air to get from one section to another. Can you identify these jumps with a mathematical concept you are familiar with?".

This type of questions are particularly focused on mathematical graphs in the task 6.

Next, we present the eight tasks developed in our classes related to the topic of continuity of functions of one variable.

- Task 1: We are going to design a skateboard track. In order to do so, we have to distribute in an appropriate way structures that we can slide or jump on: parabolic tubes, inclined planes…
  Draw the profiles of some tracks you would like to design, and try to define functions of one variable that (roughly) match those profiles. Start using Elementary Functions to define small parts of the track. Then try to connect those pieces to form longer tracks.
- Task 2: In the above designs, are there abrupt changes in slopes? It is obvious that, following the profiles, skaters will have to jump through the air to get from one section to another. Can you identify these jumps with a mathematical concept you are familiar with?
- Task 3: Suppose the following functions have been defined to design a skateboard track:

$$y = f(x) = \begin{cases} 2 & 0 \le x \le 1 \\ 3 - x & 1 < x \le 2 \\ 1 & 2 < x \le 4 \\ 5 - x & 4 < x \le 5 \\ 0 & 5 < x \le 6 \\ 1/2 & x > 6 \end{cases}$$

$$y = g(x) = \begin{cases} 3 & 0 \le x \le 2 \\ \frac{3}{4}x^2 - 6x + 12 & 2 < x \le 4 \end{cases}$$

Table 2 Tasks and way of working for continuity.

	Plan	Way of working
Task 1	10 min	Individual, prior to the seminar
Task 2	5 min	Individual, prior to the seminar
Task 3	15 min	Individual, prior to the seminar
Task 4	15 min	Teacher, in the classroom
Task 5	10 min	Work team, in the classroom
Task 6	10 min	Work team, in the classroom
Task 7	15 min	Work team, in the classroom
Task 8	45 min	Work team, in the classroom
	2 h and 5 min.	

Fig. 1 Skateboard track example given to students.

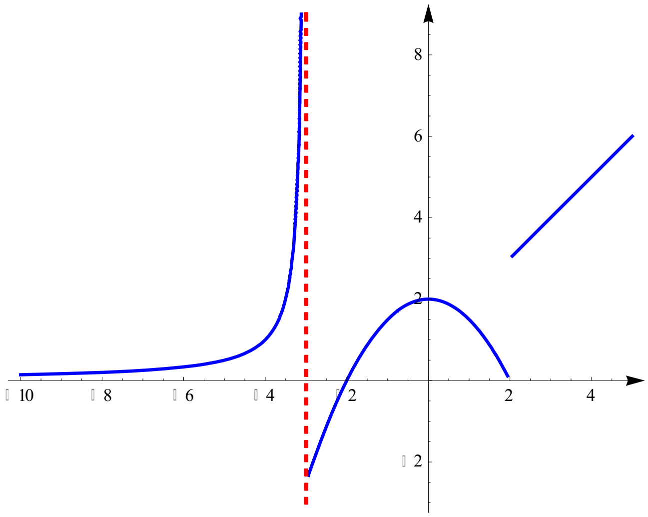

Fig. 2 Graph given to students for the Task 6a and 6b.

$$y = h(x) = \begin{cases} 2x^2 - 20x + 50 & 5 \le x \le 6 \\ 2 & 6 < x \le 7 \\ 1 - \sqrt{1-(x-8)^2} & 7 < x \le 9 \\ 1 & x > 9 \end{cases}$$

If the variable x is the horizontal position occupied by the track, and the variable y (which depends on x) is the height of the track (both in meters), plot the above functions.

It is clear that if these profiles are followed, the skaters will have to jump through the air to get from one part of the track to another. Can you match those jumps with math concepts you know?

- Task 4: The teacher will spend a few minutes to clarify the questions raised in the previous tasks. He will write the definition of continuity, as well as the types of discontinuity that can be.
- Task 5: Given the piecewise function $f(x) = \begin{cases} \frac{x^2+5x+4}{x+1} & \forall x < -1 \\ x+7 & \forall x \in [-1, 0) \\ x^2 + 2x - 1 & \forall x \in [0, 1) \\ \frac{1}{x-1} & \forall x > 1 \end{cases}$, answer the following questions:

a. Determine the points where a special continuity study should be done.
How is the function at the other points?
b. For each of these points, study the continuity, and, in case of discontinuity, determine the type of discontinuity.
c. Calculate all types of asymptotes of the function.
d. Plot, approximately, the graph of the function.

- Task 6: After seeing the below graph, answer the following questions:

a. At which points is the function continuous without the need to make a particular study at those points? Justify your answer. (Fig. 2)
b. Decide and justify at which points it is necessary to carry out a particular study of the continuity and justify your answer. (Fig. 2)
c. Looking at the following graph, determine the domain of continuity, and indicate the types of discontinuity: (Fig. 3)

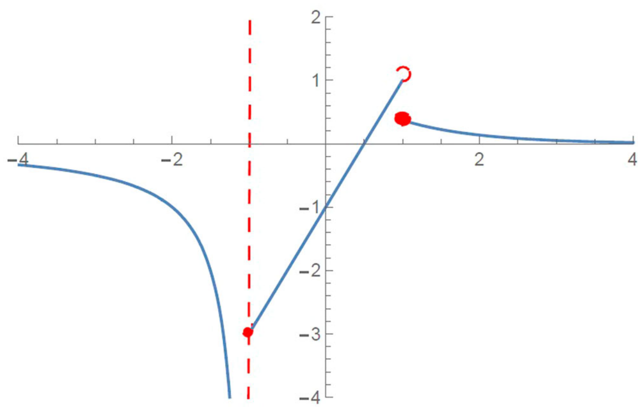

Fig. 3 Graph given to students for the Task 6c.

- Task 7: Analyse the continuity of the following functions, indicating the type of discontinuity:

a. $f(x) = \begin{cases} \frac{x \cdot Lx}{x^2-1} & \forall x \neq 1 \\ 1/2 & x = 1 \end{cases}$, at point $x = 1$
b. $f(x) = \frac{|x|}{x}$, at point $x = 0$
c. $f(x) = \frac{\sin x}{x}$, at point $x = 0$
d. $f(x) = e^{1/x}$, at point $x = 0$

- Task 8: Resolution of some exercises on continuity in the seminar book.

Results and discussion

At the end of all the sessions, the acquisition of the learning outcomes is assessed through a written test. This test is common to all the subgroups within the same group and includes problems similar to those addressed in the seminars. In addition, to gauge student satisfaction and identify potential improvements in the active learning experience, students are required to complete an anonymous satisfaction survey during the final seminar session. This survey has 18 questions designed on a Likert scale, ranging from 1 = "not at all agree" to 5 = "totally agree". It collects information regarding the students, the design of the activities, and their perceptions of how the methodology used has influenced their development of autonomous work.

The questions in the survey can be categorized as follows:

- Discriminant questions (items) or variables, considered to be those that are not related to the used methodology,
- Evaluative questions (items) or variables, which are those variables equivalent to items on a scale of attitudes or opinions towards the object to which these items can be applied.

Discriminant variables. Discriminant items tend to have a higher contrast between extreme groups, and are not easily accepted (or rejected) by all subjects. As we have previously said, discriminant variables are considered to be those that are not related to the used methodology. In our case, the discriminant variables are:

- Group and subgroup to which the students belong.
- Gender
- High School mark: the mark obtained in the subject of Mathematics in the last year of High School.
- Mark obtained in the university entrance exam.

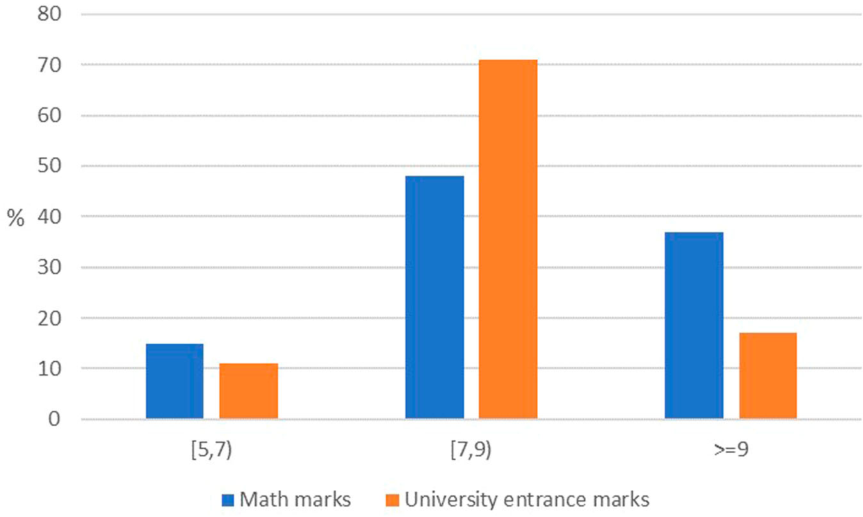

Fig. 4 Percentage of students. High School mark in Mathematics (blue) and university entrance mark (orange).

The descriptive study of the variables mentioned above shows that of the 159 students who took part in the experience, 57% were boys and 42% girls.

Figure 4 shows the mark obtained in the last year of High School in Mathematics and the university entrance mark. As can be seen, on the one hand, in general, the Mathematics mark of pre-university level is greater than 7 (almost 90%) and 37% of students have 9 or greater marks in this subject, which is logical if we consider that these students have voluntarily opted for a technical grade (Engineering). On the other hand, it is noteworthy that almost 90% of these students have an entrance exam mark of over 7. In this case (university entrance mark), "only" 17% of students have 9 or greater. We have to notice that there are subjects such as Language, History, etc. in which our students traditionally obtain a lower mark in the entrance exam.

Our research aims to explore the relationship between the introduction of active methodologies and the improvement of autonomous work and seminar grades in the Calculus course for new engineering students.

By implementing the active Problem Based Learning methodology, and designing engaging activities for students, so that they can work autonomously, we have observed a positive impact on the participating students. This approach has prompted them to reflect and engage with concepts that they might otherwise have memorized without truly understanding.

Additionally, there has been an increase in grades for the portion of the course where active methodologies were applied (that is, in the seminars) compared to previous years

Evaluative variables. All variables related to teaching and the type of applied methodology were considered to be evaluative variables. The description of these variables is as follows:

- TCA1: Work prior to the seminar
- TCA2: Improvement in the understanding of activities after the seminar.
- IAT1: Methodology helps to increase interest and motivation for the subject
- IAT2: Methodology helps to examine the work done on my own
- IAT3: The methodology helps to develop my autonomy in studying.
- AC1: The wording of the activities is clear
- AC2: The time for doing the activities is adequate
- RM1: Number of videos watched
- RM2: The explanations of the videos are easily understood
- RM3: The videos help to understand the theory.

Taking into account the context to which they refer, these variables have been grouped as follows:

- Variables related to personal work prior to the seminar: TCA1 and TCA2.
- Variables related to the promotion of active methodology in the development of autonomous work: ITA1, ITA2, and ITA3.
- Variables related to the proposed activities: AC1 and AC2.
- Variables related to the use of multimedia resources to facilitate independent work: RM1, RM2, and RM3.

Cronbach's Alpha, with a value of 0.8 for the variables mentioned above (except RM1), indicates the suitability of the questionnaire.

Autonomous work. Table 3 shows the percentages of the evaluative variables included in the study. The variable RM1 has been removed as it indicates the number of explanatory videos that have been watched by our students, and it does not make sense to evaluate it together with the other variables, so it is analysed below. We highlight the high percentage of students who choose the neutral option "neither agree nor disagree" in all of them, and especially in the variable ITA2.

Analysing the cumulative percentages in the table, we can note that almost 68% of the students have not done the previous work before attending the seminar (TCA1), but 59.7% think that the methodology helps them to increase interest and motivation in the subject (ITA1).

With regard to the variable RM1, that is, number of videos watched by our students, it can be seen in Table 4 that 12% of the students did not watch any video and only 10% watched all videos. And taking into account Table 3, the accumulated percentage of students who think that the videos are valid exceeds 50% with respect to the variable RM2, reaching 64% in the variable RM3.

Comparing the three questions related to methodology and autonomous work (IAT1, IAT2, and IAT3), it can be seen in Fig. 5 that, for a high percentage of students, the methodology helps them to develop their autonomous work (ITA3).

When analysing by means of a cross-tabulation whether or not this variable is independent of the group to which the students belong, the Chi-square statistic with a significance of 0.437 shows the independence of these variables.

Analysing the behaviour of the variables by gender, the percentage of boys who work autonomously beforehand is 32%, compared to 33% of girls who do so. However, the percentage difference increases with respect to the use of the used multimedia resources, where 37% of boys and 41% of girls have seen almost all or all of the explanatory videos prepared to understand autonomously the theoretical basis to be used in the seminar sessions. The percentage of students who pass the seminars is around 58% for both girls and boys.

Seminar mark. In order to check the influence of the autonomous work on the obtained mark in the seminar, we employed a cross-table analysis using the Chi-square statistic. The asymptotic significances obtained are shown in Table 5. Pearson's Chi-square tests the hypothesis that the two variables, autonomous work and seminar marks, are independent. The actual value of the statistic does not usually contain information of great relevance, but it is the significance value (asymptotic significance) that contains the information we are looking for. The lower the significance value, the less likely it is that the two variables are independent (that is, unrelated).

The dependence of the first two variables on the seminar mark is verified, as the significance obtained in both cases is less than 0.05.

Moreover, in order to know whether the completion of all the activities is related to the group in which it has been carried out, we make a contingency table or cross-tabulation for nominal variables. Pearson's chi-squared statistic for this crossing of variables with a value of 8.09 and 2 degrees of freedom has an asymptotic significance of 0.017 (less than 0.05), so the null hypothesis of independence of variables is rejected, and it is concluded that both variables are related. Having said that, as shown in Table 6, the values of Phi, Cramer's V and Contingency coefficient statistics are above 0.3. Therefore, the association between the variables is moderate.

In the combination of variables related to the proposed activities and multimedia material, it is worth noting the strong relation that the variable "group" has with "seminar mark". As shown in Table 7, we can say with a probability of 95% that these variables are not independent and besides, as the value of the Phi statistic is 0.695 (higher than 0.5), it is concluded that this relation is moderate.

Doing a detailed analysis of the residuals, in order to see which group contributes most to each category of the variable "seminar mark", we can confirm (Table 8) that the group of students belonging to group GA is the one with the highest weight in the category [10-8], compared to the group GC, which is the one that contributes the most to the sum of the failure categories ([5-4] + [4-2] + [2-0]). Therefore, it is concluded that the students of group GA contribute the most to the relation between both variables. This is logical since the university sorts students into subgroups (GA, GB, and GC) according to their university entrance grades, with the GA subgroup containing the students with the best grades for university entrance and the GC subgroup containing the students with the worst grades.

Table 3 Percentages of the evaluative variables included in the study.

	TCA1	TCA2	ITA1	ITA2	ITA3	AC1	AC2	RM2	RM3
1	11.3	4.4	3.2	4.4	4.4	3.7	5.0	9.4	6.4
2	20.1	17.1	10.7	17.0	15.7	14.5	14.5	11.3	8.3
3	**36.5**	**31.0**	**26.4**	**45.9**	**35.8**	**38.4**	**29.6**	**25.8**	**21.2**
4	23.9	38.6	45.9	28.3	39.0	35.2	35.2	32.7	41.0
5	8.2	8.9	13.8	4.4	5.1	8.2	15.7	20.8	23.1

Table 4 Number of videos watched by the students, RM1.

		Frequency	Percent	Valid percent	Cumulative percent
Valid	None	19	11.9	12.0	12.0
	One	33	20.8	20.9	32.9
	Two	44	27.7	27.8	60.8
	Three	46	28.9	29.1	89.9
	All	16	10.1	10.1	100.0
	Total	158	99.4	100.0	
Missing	System	1	0.6		
Total	159	100.0			

Table 5 Chi-square statistic and asymptotic significances.

	Pearson Chi-Square	Asymptotic Significance
ITA1*seminar mark	72,973	0.000
ITA2* seminar mark	49,194	0.039
ITA3* seminar mark	44,147	0.048

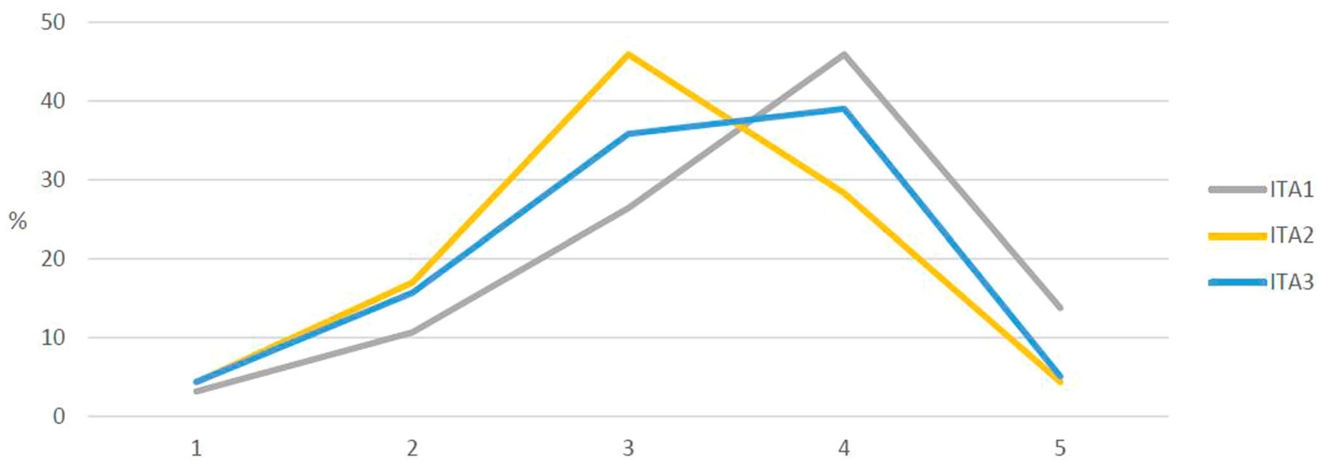

Fig. 5 Percentage of students for variables related to the promotion of active methodology in the development of autonomous work.

Table 6 Cross tabulation. Symmetric measures.

		Value	Sig. aprox.
Nominal by Nominal	Phi	0.382	0.017
	Cramer's V	0.382	0.017
	Contingency coefficient	0.371	0.017
N of valid cases		527	

Table 7 Cross tabulation. Symmetric measures.

		Value	Sig. aprox.
Nominal by Nominal	Phi	0.695	0.000

Table 8 Crossing of group with mark of seminars.

		[10-8]	[8-6]	[6-5]	[5-4]	[4-2]	[2-0]
GA	Count	17	51	67	30	18	5
	Residual	7.6	6.8	−4.4	−7.4	−1.8	−0.8
	Std. residual	2.5	1.0	−0.5	−1.2	−0.4	−0.3
	Adjusted residual	3.8	1.5	−0.6	−1.8	−0.3	−0.2
GB	Count	12	44	57	26	19	14
	Residual	3.8	6.2	−0.4	−7.6	−2.2	0.2
	Std. residual	1.3	1.0	−0.1	−1.3	−0.5	0.1
	Adjusted residual	1.4	1.4	0.0	−2.0	−0.4	0.0
GC	Count	9	32	53	42	24	7
	Residual	1.8	2.2	0.8	−1.6	−2.8	−0.4
	Std. residual	0.7	0.4	0.1	−0.2	−0.5	−0.1
	Adjusted residual	0.5	0.3	0.1	−0.2	−0.5	−0.1
Total	Count	38	127	177	98	61	26

Table 9 Pass rates.

	GA	GB	GC
Previous research	64.32	59.77	44.38
Research period	71.81	65.70	56.29

In the crossing of the variables about the methodology with the discriminant variables, only a relation has been established between the "group" variable and the "general assessment" variable, because the Phi statistic has a significance of 0.038 (less than 0.05), which means that there is dependency between the variables. Nevertheless, it is moderate since the value of Phi is of 0.37, which is between 0.3 and 0.5.

Finally, we would like to highlight the improvement in the students' seminar grades, if we compare them with those obtained in previous years. As can be seen in Table 9, the improvement in the grade involves all subgroups.

Conclusions

This study investigates the relationship between the introduction of active methodologies and the improvement of autonomous work and seminar grades in the Calculus course for new engineering students.

In the years prior to entering the university, it was observed that lectures were the predominant method for knowledge transmission, leaving students unfamiliar with active methodologies and lacking the skills necessary for independent study.

Designing activities that effectively convey the desired learning outcomes while also engaging students in autonomous work is a challenging task for educators aiming to implement active methodologies. To address these challenges, we utilized mathematical concepts that new students had encountered in their pre-university courses, making them more relatable and familiar.

The study employed "problem-solving" (Problem-Based Learning) as an active methodology, and the results indicate a positive impact on participating students. Most reported enjoying the experience, as it encouraged them to reflect and engage with concepts they had previously memorized without fully understanding.

Additionally, there was an increase in the grade for the part of the course where active methodologies were applied (that is, in the seminars) compared to previous years.

We believe that this approach, which emphasizes directed autonomous work, helps students reflect on learning outcomes from a perspective beyond mere memorization, thereby providing a more solid foundation of knowledge.

For future implementations, we emphasize the importance of preparatory work prior to seminars, as it enhances autonomous learning. Looking ahead, we suggest extending this experience by introducing new concepts within the Calculus curriculum or applying the methodology to other subjects that students are required to take.

Data availability

All data generated or analysed during this study are included in this published article. Particularly, the tasks used in our research and in our classes are included in this article.

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Author contributions
Conceptualization, JB and OG; methodology, JB, EB, OG, and CR; formal analysis, JB; investigation, JB, EB and OG; writing—original draft preparation, JB; writing—review and editing, JB, EB and CR; supervision, JB; project administration, JB.

Competing interests
The authors declare no competing interests.

Ethical approval
This study was performed in line with the principles of the Declaration of Helsinki. Approval was granted by the Ethics Committee of the University of Basque Country (Ethics approval number: M10_2020_027; Approval date: 28 Sep 2020).

Informed consent
Before taking part in the study, all participants were made aware of the details and agreed to take part. Participants in this study chose to take part of their own volition. Prior to the commencement of the survey, we informed the participants of the study's objective. The participants agreed to take part in this study of their own accord, having been given the assurance of anonymity by the researchers and that their answers were solely for educational purposes. The interviewee was aware of the confidentiality and privacy regulations prior to consenting to the interview. They agreed to take part in the study of their own accord and that by responding to the interview, they were giving their approval.

Additional information
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