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Abstract: A manager/mechanism designer must allocate a set of money prizes (\$1, \$2, .., \$$n$) between n agents working in a team. The agents know the state i.e. who contributed most, second most, etc. The agents' preferences over prizes are state independent. We incorporate the possibility that the manager knows the state with a tiny probability and present a simple mechanism that uniquely implement prizes that respects the true state.

Keywords: Full Implementation; Direct Mechanism; Verifiable Information; Rank-Order Tournaments. *JEL Code*: D82.

1. Introduction

Consider the following implementation problem: A manager must distribute a given set of money prizes (\$1, \$2, .., \$$n$) among an odd number of agents ($n \geq 3$) working in a team. A state of the world, s, is an ordinal ranking of the agents that mirror relative contributions i.e. who contributed most, second most, etc. (ties never occur). The agents know s and the manager knows s with some probability and she is uninformed with the residual probability (e.g. the agents are monitored with some probability). The agents do not know whether or not the manager is informed. The manager's goal is to distribute the prizes so that the agent who contributed most receives \$$n$, the agent who contributed second most receives \$$n-1$, and so on. The agents seek to maximize their reward independently of the state. Before allocating the prizes the manager conducts individual interviews where the agents announce a state. Prizes are distributed depending on the announcements and whether the manager is informed.

From a practical viewpoint, rank-order tournaments where employees are rewarded depending on their job performance relative to their peers are often observed in practice. For example, employees compete for higher shares in bonus-pool schemes (Bognanno 2001), salesmen are paid according to relative performance (Murphy and Sohi 1995), and workers compete for promotion in organizational hierarchies (Baker et al. 1994). Multi-rate assessments are one way for the management to set a standard for giving rewards and obtaining information about the employees' productivity. In such evaluations, the employees are expected to rate themselves *and* their co-workers. According to Bracken et al. (2001) over one third of U.S. companies use some type of multi-rater reporting system.

In the above context, we show that a simple outcome function exists that achieves the manager's goal (full information implementation) when the probability of her being informed is close to zero. We use the term *full* implementation in the sense that *every* equilibrium has the desired outcome. Maskin (1999) showed that any social choice rule that satisfies *monotonicity* and *no veto power* can be implemented with three or more agents. Our social choice function does not satisfy monotonicity. In fact, when the planner is uninformed with probability 1 (as in the standard setting) and the agents disregard the state the game induced by any outcome function will not vary across states and so the set of equilibria across states will not vary either.

To this end, Ben-Porath and Lipman (2012) and Kartik and Tercieux (2012) study complete-information full implementation where the agents can provide evidence. In such environments, social choice rules that are not Maskin-monotonic can be implemented. In these papers, there is complete information about the state *and* evidence. Our result crucially relies on uncertainty in this respect: If the agents know whether or not the planner has evidence that proves s our main result does not obtain.

Matsushima (2008a and 2008b), Dutta and Sen (2011), Kartik and Tercieux (2012), and Holden et al. (2012) study full implementation when the agents have a weak preference for honesty. This approach gives very permissive results for increasingly simple mechanisms. We do not assume a preference for honesty and our mechanism does not achieve the manager's goal when the agents have a small preference for truth-telling and the manager is always ignorant. These two approaches (a small preference for honesty versus a small probability that lies will get detected) are not exactly equivalent as in the current paper the cost of lying is not exogenous but can be chosen by the manager.

2. The example

A finite number of agents, n, have contributed to the completion of some task. Let n be odd and strictly greater than two. A state of the world, $s \in S$, is an ordinal ranking of the agents that mirror individual contributions (assume ties never occur). More precisely, s is a vector with n entries where the j'th entry, s_j, denote the agent with rank j. Then $S \equiv n^n$ and $|S| = n!$ We sometimes refer to the true state as s^*. The agents observe s^*. The agents' manager knows s^* with probability $\mu \in [0, 1)$ and she is uninformed with probability $1 - \mu$. This is the novel part of our model. The agents do not know whether the manager is informed or not. One can imagine that with some probability that the agents will be monitored or checked. The probability of the manager being informed is independent of s and we let

$x \in \{\emptyset, s^*\}$ denote the manager's knowledge of the state.

The problem facing the manager is to distribute a set of money prizes (\$1, \$2, .., \$$n$) to the agents so that every agent receives a prize and no agents receives the same prize. An outcome, $a \in A = S$, is a vector with n entries where the i'th entry, a_i, denote the money amount accruing to agent i. Define Δ to be the set of lotteries over outcomes. The manager's objective is given by a social choice function (SCF), which is a function $f : S \to A$. We focus on a particular SCF, $f'(s) = \{a : \forall s_i, s_{k \neq i} \text{ if } i < k \Rightarrow a_i > a_k\}$. This rule selects the "fair" outcome; if agent i contributed more than agent k then i's prize will be bigger than k's prize. Agents are expected utility maximizers and we assume that agent i's von Neumann-Morgenstern utility function takes the linear form, $u_i(a, s) = a_i$.

Consider the following direct mechanism $g = (M, \pi)$, where M is the product of individual strategy sets, $M_i = S$, and π is the outcome function, $\pi : M \times \{\emptyset, s^*\} \to \Delta$. A mechanism induces a game in each state and we let $N(g, s)$ denote the set of Nash equilibrium outcomes in the game corresponding to (g, s). We say that the mechanism (M, π) implements $f(s)$ if $\forall s \in S, f(s) = N(g, s)$. We now define the outcome function π.

Rule 1. If at least $\frac{n+1}{2}$ agents announce the same state s' and $x = s'$ or $x = \emptyset$, then the outcome is a' where $f'(s') = a'$. Suppose instead $x = s^* \neq s'$. Consider the agent, i, who announced s' and receives the lowest reward from a' among the agents announcing s' (thus i receives at most \$$\frac{n+1}{2}$ from a'). In this case the manager randomizes equally over all outcomes $a : a_i = 1$.

Rule 2. If rule 1 does not apply (i.e. no majority) and $x = \emptyset$ then the manager randomizes equally over all $a \in A$. Suppose instead $x = s^*$. Consider the agent, k, who announced $s \neq s^*$ and receives the most from a^* where $f'(s^*) = a^*$ (thus k receives at least \$$\frac{n+1}{2}$ from a^*). In this case, the manager randomizes equally over all outcomes $a : a_k = 1$.

The proposition. The mechanism (M, π) implements $f'(s)$ if and only if $\mu > 0$.

Proof. When $\mu = 0$ and given state independent preferences the set of Nash equilibrium outcomes is the same for all $s \in S$ and hence $f'(s)$ is not implemented by (M, π). Suppose $\mu > 0$. Clearly, one Nash equilibrium is when more than $\frac{n+1}{2}$ agents announce s^* and the outcome is $f'(s^*)$. We now argue that any configuration leading to an undesired outcome is unstable and thus $N(g, s) = f'(s)$.

Suppose at least $1 + \frac{n+1}{2}$ agents announce $s' \neq s^*$. Here, agent i (as in rule 1) has an incentive to deviate and announce s^*: If $x = \emptyset$ he is equal off and if $x = s^*$ he strictly benefits from deviating. Suppose exactly $\frac{n+1}{2}$ agents announce $s' \neq s^*$. Again, agent i from rule 1 has a profitable deviation. If i has a low rank according to s^* (lower half of contributors) he can announce $\bar{s} \neq s'$ which makes the claims dispersed (no majority) and let him with $\$\frac{n+1}{2}$ (in expected terms) if $x = \emptyset$ and something strictly higher than \$1 if $x = s^*$ (see rule 2). Similarly, i is made strictly better off from announcing s^* if he has a high rank according to s^* (upper half of contributors). Suppose the announcements are dispersed. In this case, agent k from rule 2 strictly benefits from claiming s^*.

If we consider mixed strategy equilibrium candidates, we reach the same conclusion. In case, with positive probability, some $s' \neq s^*$ obtains a majority of the "votes" at least one agent, e.g. agent i as in rule 1, has a profitable deviation where he puts less probability on s'. Similarly, in the situation where the agents are mixing and the claims are always dispersed then the agent who is the equivalent to agent k in rule 2 can deviate and add probability on s^*. □

Remark *(Weak preference for honesty).* Assume $\mu = 0$ and the agents report the true state if they are indifferent about truth-telling and announcing some "false" state. We shall argue that the mechanism (M, π) does not implement $f'(s)$ in such an environment. Assume $n = 3$ and consider the situation where agent 1 announce $s^* = [1, 2, 3]$ and agents 2 and 3 announce $s' = [2, 3, 1]$. This configuration constitutes a Nash equilibrium. Agent 1 cannot change the outcome and thus the truthful report is optimal. Agent 2 receives \$3 and any deviation will make him strictly worse off. If agent 3 announce s^* he will receive \$1 instead of \$2 and any "false" deviation will leave him equal off (in expected terms).

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