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2012

Working Paper Series: IL. 64/12

Departamento de Fundamentos del Análisis Económico I

Ekonomi Analisiaren Oinarriak I Saila



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Games with Perceptions*

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October 23, 2012

Abstract

We assume that 2×2 matrix games are publicly known and that players perceive a dichotomous characteristic on their opponents which defines two types for each player. In turn, each type has beliefs concerning her opponent's types, and payoffs are assumed to be type-independent. We analyze whether the mere possibility of different types playing different strategies generates discriminatory equilibria. Given a specific information structure we find that in equilibrium a player discriminates between her types if and only if her opponent does so. We also find that for dominant solvable 2×2 games no discriminatory equilibrium exists, while under different conditions of concordance between players' beliefs discrimination appears for coordination and for competitive games. A complete characterization of the set of Bayesian equilibria is provided. (JEL C72)

Keywords: 2×2 matrix games, incomplete information.

*This research is supported by the Spanish Ministerio de Ciencia e Innovación under project ECO2009-11213, co-funded by ERDF, and by Basque Government funding for Grupo Consolidado GIC07/146-IT-377-07. We thank Carlos Alós-Ferrer, Fabrizio Germano, Federico Grafe, Dov Samet, Federico Valenciano, José Vila, Oscar Volij for some discussion and comments, as well as Ulf-Dietrich Reips for reference help in Psychology.

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1 Introduction

As observed by Aumann [4], players' perceptions about their opponents may play a significant role in solving normal form games in the following coordination game played by Alice and Bob.

	<i>c</i>	<i>d</i>
<i>c</i>	9, 9	0, 8
<i>d</i>	8, 0	7, 7

Each pure equilibrium has something going for it; (c, c) is Pareto-dominant but (d, d) is much safer against any eventual deviation of the opponent. No equilibrium is self-enforcing even with pre-play communication and the final choice of each player may depend on whether s/he is careful and prudent and fears that the other not trust him/her or impulsive and optimistic and believes that the other is too. So Alice's question is: will Bob trust me or not?

Since our work deals with perceptions, we start by quoting what the *Encyclopaedia Britannica*¹ says about the term:

“Perception, in humans, is understood as the process whereby sensory stimulation is translated into organized experience. This process influences formation of judgement and can be considered as a subjective feature. Perception involves receiving signals from the environment, organizing these signals, and interpreting them. Perception is selective, subjective and largely automatic rather than conscious. Because the perceptual process is not itself public or directly observable (except to the perceiver himself), the validity of perceptual theories can be checked only indirectly. That is, predictions derived from theory are compared with appropriate empirical data, quite often through experimental research”.

Thus, perception captures signals which require one to somehow see others. Perceptual processes are analyzed by Weisbuch and Ambady in [31], which reviews experiences where subjects are briefly exposed to each other. They find that complete strangers often need less than 10 seconds to make non-random inferences about emotional states, personality, and physical traits. It is not surprising that these inferences about others influence decisions. Moreover, individuals' perceptions about others are generated unconsciously. Consequently we consider that in one-shot games perception of opponents are private information acquired once the game has begun, but before players make any choices. Although we immediately perceive our opponents we do not know how they perceive us and this lack of information calls for incomplete information games.

To analyze the effect of perception in games we consider 2×2 games which are assumed to be publicly known. Each player perceives a characteristic² in the opponent, and that perception

¹<http://www.britannica.com/EBchecked/topic/451015/perception>

²Here a characteristic is understood as the signal that an individual perceives in her opponent.

defines the player’s type. In the aforementioned example Alice and Bob may perceive each other as trustworthy or untrustworthy. These perceptions condition their beliefs as follows: Alice is of type B_I if she perceives Bob as trustworthy (or of type B_{II} if she perceives him as untrustworthy). Similarly, Bob may be of type A_I or A_{II} . Alice and Bob know their type (that is, how they perceive their opponent) but do not know how they themselves are perceived. Probabilities are assigned to the opponent’s types, and those probabilities are conditional on their own types (that is, on their own perceptions of the opponent). If she is of type B_I (or B_{II}) these probabilities (subjective beliefs) are p_I and $(1 - p_I)$ respectively (or p_{II} and $(1 - p_{II})$). Likewise, Bob’s beliefs are represented by q_I , $(1 - q_I)$, q_{II} and $(1 - q_{II})$.

Several studies show that exposure to facial expressions of emotion (Dimberg in [11]) or listening to a happy or sad voice (Neuman and Strack in [24]) evoke congruent effects in receivers. This is what is referred to by Hatfield in [18] as emotional contagion. Accordingly we label this kind of perceptions as *contagious perceptions*. For instance, if Alice perceives Bob as trustworthy, then she assigns a higher probability of being perceived as trustworthy by Bob than if she perceives him as untrustworthy. Hence, for Alice we have $p_I > p_{II}$. Personality traits such as intelligence, empathy and extroversion, and physical traits such as beauty, are non transferable between agents and may generate *antagonistic perceptions*. As a consequence, we can expect the reverse relationship to hold: $p_I < p_{II}$. For instance, it is shown by Walter *et al.* in [30] that attractive individuals have more rigorous requirements for an acceptable date than less attractive ones do. Consequently a player assigns a lower probability to being perceived as attractive when facing an attractive opponent than when facing an unattractive one. Whether the two players share contagious or antagonistic perceptions or not generates a specific information structure over their beliefs which is assumed to be exogenous.

Once types are defined players make choices. Strategies, which depend on types, are randomized strategy vectors. Specifically, Alice’s strategy is denoted by $\alpha = (\alpha_I, \alpha_{II})$ where α_I denotes the probability of playing her “first” action when she is of type B_I , and α_{II} denotes the probability of choosing her “first” action when she is of type B_{II} . Analogously, strategies for Bob are denoted by $\beta = (\beta_I, \beta_{II})$. Strategies may or may not be different depending on the types. As usual, each type is assumed to maximize her conditional expected payoff. Players, types, strategies and expected payoffs define *games with perceptions*, which are a specific case of Bayesian games insofar as uncertainty about opponents’ perceptions is included. Note that the existence of a common prior is not required.

We determine the Bayesian Nash equilibria for 2×2 generic games with perception.³ The

³Notice the difference between our approach, in which no communication whatsoever exists between players, and the cheap talk approach, in which players may exchange costless messages, possibly with the help of a mediator. (See, for instance, the work by Vida and Forges [14] for the two-person case.)

invariance of the payoff matrix for all combinations of types allows us to focus our analysis on the effect of perceptions on the solution of these games. Given the specific information structure assumed in this paper, the question that we pose is whether the mere possibility of discriminating generated by players' beliefs about each other gives discriminatory equilibria (*i.e.* equilibria in which players act differently depending on their perception of their opponent). We first find that the set of non-discriminatory equilibria of a game with perception replicates the set of Nash equilibria of the underlying game. That is, in the said set each equilibrium consists of each player choosing the same strategy regardless of her type. We also find that discrimination is not asymmetric between types since in equilibrium a player discriminates if and only if the other player does so. A property of the discriminatory equilibria is that each player plays always one pure strategy for at least one of her types (Proposition 1). Based on these properties we find that *dominant solvable games* do not have discriminatory equilibria. For *coordination games* however, we find discriminatory equilibria if and only if the two players have beliefs induced by perceptions of the same kind (*i.e.* both contagious, or both antagonistic), while for the discriminatory equilibria to exist in *strictly competitive games* beliefs induced by different perceptions (for instance, contagious for Alice and antagonistic for Bob) are required (Theorem 1). A complete characterization of the set of equilibria is provided in the Appendix.

Our work is related to that on (subjective) correlated equilibria. The concept of correlated equilibrium (Aumann, in [2] and [3]) generalizes Nash equilibrium by allowing players to make their choices based on private and payoff-irrelevant signals. Its underlying idea is to start with a game and to find information structures which induce equilibria in the resulting Bayesian game (a subjective correlated equilibrium is defined when common priors are not assumed). Our approach differs in that given a game we assume a specific information structure, induced by contagious and antagonistic perceptions, and derive the Bayesian Nash equilibria. Unlike correlated equilibria, we do not try to find perceptions that justify certain players' behavior but rather analyze players' behavior given different perceptions. The present paper shows how the information structure induced by our categorization of perceptions provides equilibria with relevant properties.

Regarding perceptions there is an increasing evidence of systematic heterogeneity in players behavior induced by perceptible characteristics. The importance of appearance has already been addressed in the form of a "beauty premium" (Wilson *et al.* in [32]) or as a reciprocally influencing factor in ultimatum games in lab experiments (Solnick and Schweitzer [28]). Eckell and Petrie, and Mulford *et al.*, also present in [12] and [23] also present empirical evidence in this direction. But signals embodied in traits are beyond the control of the signaling actor, and what players perceive in others is not only beauty or attractiveness but all sorts of emotions.

In fact, emotions are useful for signaling intentions and this may induce modifications in one's strategic actions. This has already been studied, for instance, in the "sweet revenge game" by Gilboa and Schmeidler in [16], by Geanakoplos *et al.* in psychological games like the "gift-giving" game in [15], and in Rabin's model in [26] where players wish to act kindly in response to kind strategies. In [25] O'Neill argues that psychological games, defined for complete information, deal with emotions that depend on learning information, such as gladness, disappointment or relief, but in his opinion games with emotions would be more accurately analyzed if modelled as games of incomplete information. In his words "In social situations we feel anger or appreciation not when we learn the outcome of a random variable, but when we learn something about the other player, that their loyalty or thoughtfulness is lower or higher than we thought. This calls for incomplete information". Although we consider perception, a broader concept than emotion, this is precisely the approach followed in this paper.

Another interesting study is the inductive game theory approach to discrimination and prejudices by Kaneko and Matsui [20]. Here players have no *a priori* knowledge of the game structure, and their behavior is based on beliefs acquired from past experience at playing the game. By contrast, in our paper the game structure is publicly known and players are strangers whose exposure to each other generates beliefs.

The paper is organized as follows. Section 2 contains the preliminaries about 2×2 games and beliefs. Sections 3 and 4 present the formal model and the results respectively. Section 5 concludes. Proofs are relegated to the Appendix.

2 Preliminaries

Our object of study is two person games in which each player has two available actions. These are formally represented by two binary *action sets* S_1 and S_2 , and two *payoff functions* u_1, u_2 from $S_1 \times S_2$ to \mathbb{R} . If, for instance, we denote players' actions by $S_1 = \{H, D\}$ and $S_2 = \{L, R\}$, game G can be represented by a matrix in the following usual way:

$$\begin{array}{c} \begin{array}{cc} & L & R \\ H & a_{11}, b_{11} & a_{12}, b_{12} \\ D & a_{21}, b_{21} & a_{22}, b_{22} \end{array} \end{array}$$

It is well known (see for instance Calvo-Armengol and Eichberger *et al.* in [10, 13] respectively) that an affine transformation of the payoffs of the games defined as:

$$u_1^*(s_1, L) = u_1(s_1, L) - u_1(D, L), \text{ and } u_1^*(s_1, R) = u_1(s_1, R) - u_1(H, R), \text{ for } s_1 \in S_1, \text{ and,}$$

$$u_2^*(H, s_2) = u_2(H, s_2) - u_2(H, R), \text{ and } u_2^*(D, s_2) = u_2(D, s_2) - u_2(D, L), \text{ for } s_2 \in S_2,$$

preserves the best reply structure of the game, in the sense that any action of any player is a best reply to any randomized strategy of her opponent *before* the transformation if and only if it is also one *after* the transformation. So, since we only focus on solution concepts induced by best reply logic (for instance, Nash and Bayesian Nash equilibria are both of this kind), *i.e.* action and randomized strategy profiles where each player's move is optimal given her opponent's, there is no loss of generality in the assumption that the 2×2 game G (and in particular its payoff functions) is represented by a matrix of the following kind:

	L	R
H	a_1, b_1	$0, 0$
D	$0, 0$	a_2, b_2

If we restrict ourselves to *generic* games,⁴ the representation above allows for a simple, standard classification of 2×2 games (again, [10, 13]) in terms of their number and nature of Nash equilibria by just attending payoff parameters a_1, a_2, b_1 and b_2 (which by genericness, are all different from 0), as represented in Table 1.

Game Class	Conditions	Nash Equilibria
• Dominant solvable games	$a_1 a_2 < 0$, or $b_1 b_2 < 0$	One NE in pure strategies
• Coordination games	$a_1, a_2, b_1, b_2 > 0$, or $a_1, a_2, b_1, b_2 < 0$	Two NE in pure strategies, and one NE in mixed strategies
• Strictly competitive games	Otherwise	One NE in mixed strategies

Table 1: A taxonomy of 2×2 games

We now turn our attention to players' *perceptions* of their opponents. We assume that each player can be perceived in two different ways by his/her opponent. Thus, player 1 may perceive her opponent as B_I or B_{II} , and player 2 may perceive his/her opponent as A_I or A_{II} . Note that perception of their opponent is *interim private information* of the players, this is: it is private information that they acquire once the game is taking place but before they make any choices.

We attach *beliefs* to perceptions. A *belief* is a player's probability on the set of possible perceptions of her opponent which is conditional of her own perception on her opponent. Formally, this can be represented by maps $p : \{B_I, B_{II}\} \rightarrow \Delta(\{A_I, A_{II}\})$ and $q : \{A_I, A_{II}\} \rightarrow \Delta(\{B_I, B_{II}\})$ for player 1 and 2 respectively. By $p(\cdot | B_k)$ we refer to the image of B_k under p (and similar for q). To abbreviate we simply denote:

$$p_I = p(A_I | B_I), \text{ and } p_{II} = p(A_{II} | B_{II}), \text{ and,}$$

$$q_I = q(B_I | A_I), \text{ and } q_{II} = q(B_{II} | A_{II}),$$

⁴One reference is von Stengel in [29]. Roughly speaking, a game is *generic*, or *non degenerate*, if it has some neighborhood whose elements have the same number of Nash Equilibria as the original game.

and summarize the perception and belief profile of players in vector $B = (\mathbf{p}, \mathbf{q})$ where $\mathbf{p} = (p_I, p_{II})$ and $\mathbf{q} = (q_I, q_{II})$.

Our analysis is focused on perceptions that generate some impact on beliefs, hence $p_I \neq p_{II}$ and $q_I \neq q_{II}$. However player 1's beliefs may be either $p_I > p_{II}$ or $p_I < p_{II}$ (and the same for player 2's beliefs). The inequalities depend on the type of "characteristic" perceived in her opponent. We consider the simplest case, in which players perceive only one *dichotomous characteristic* in their opponents.

Characteristics based on emotions generate *contagious perceptions* which induce the following relationship between beliefs: $p_I > p_{II}$. For instance, the probability that player 1 assigns to being perceived as happy is higher if she perceives player 2 as happy than if she perceives player 2 as unhappy. The same relationship can be established between player 2's beliefs: $q_I > q_{II}$. By contrast characteristics based on personality traits generate *antagonistic perceptions*, which induce the following relationship between beliefs: $p_I < p_{II}$. For instance, the probability that player 1 assigns to being perceived as attractive is lower if she perceives player 2 as attractive than if she perceives player 2 as non-attractive. Likewise, for player 2's beliefs we get: $q_I < q_{II}$. However, concordant beliefs between players may not always appear and characteristics may induce situations where players belong to different communities and perceive a contagious characteristic for in-group members, and an antagonistic one for out-group members (or *viceversa*)⁵, generating *opposite* beliefs in players, so that $p_I > p_{II}$ and $q_I < q_{II}$, or $p_I < p_{II}$ and $q_I > q_{II}$.

Summarizing, whether or not the two players in the game share contagious or antagonistic perceptions gives rise to a specific information structure which is assumed to be exogenous, and which we formalize as follows:

1. *Contagious concordant beliefs*: $p_I > p_{II}$ and $q_I > q_{II}$.
2. *Antagonistic concordant beliefs*: $p_I < p_{II}$ and $q_I < q_{II}$.
3. *Discordant beliefs*: $p_I > p_{II}$ and $q_I < q_{II}$ or $p_I < p_{II}$ and $q_I > q_{II}$.

Finally, it should be noted that given some perceptions, their being concordant or discordant does not depend on how those perceptions are tagged (by tags I and II).

3 Games with Perceptions

Formally, the introduction of perception and beliefs as defined above in games in normal form leads to a Bayesian game where uncertainty centers around *payoff-irrelevant* parameters. We

⁵It is shown Rule *et al.* in [27] how Mormon participants exhibit significantly higher categorization accuracy (between being Mormon or non Mormon) than non Mormon participants. Similarly, research by Ambady *et al.* in [1] indicates that gay and lesbian participants were more accurate than heterosexual ones at judging sexual orientation. Segregation of a community could explain the existence of discordant beliefs.

follow a slight modification of the formalization of a Harsanyi game by Maschler *et al.* in [22, chapter 9]. We have:

- The set of players of game G , $I = \{1, 2\}$.
- The *type* of each player is defined by the way in which she perceives her opponent. Thus, $T_1 = \{B_I, B_{II}\}$ and $T_2 = \{A_I, A_{II}\}$. As usual, $T = T_1 \times T_2$.
- Maps $p : \{B_I, B_{II}\} \rightarrow \Delta(\{A_I, A_{II}\})$ and $q : \{A_I, A_{II}\} \rightarrow \Delta(\{B_I, B_{II}\})$ as defined in Section 2, which in this context attach beliefs to each type of each player concerning the type of her opponent. We depart from the Harsanyi doctrine and do not require that these beliefs be derived as conditionals from a *common prior* on T .⁶ Again, we summarize this data with vector $B = (\mathbf{p}, \mathbf{q})$.
- A set of *states of nature* consisting of a single element G . Therefore, any of the four possible type vectors $t \in T$ is always attached to the same state of nature, namely G .

The game with incomplete information then proceeds as follows:

1. A type vector $t = (t_1, t_2)$ is selected.
2. Each player $i = 1, 2$ knows how she perceives her opponent (*i.e.*, her type), but does not know how her opponent's perceives her (*i.e.*, her opponent's type).
3. Players choose an action simultaneously once they know their type. We represent this by randomized strategy vectors $\alpha = (\alpha_I, \alpha_{II})$ for player 1 and $\beta = (\beta_I, \beta_{II})$ for player 2, where the first component of each vector represents the probability of the corresponding player choosing her first strategy when she is of her first type, and the second the probability of the corresponding player choosing her first strategy when she is of her second type.
4. Players are maximizers of an expected payoff function induced by their own perception and their beliefs of their opponent's perception:

$$\begin{aligned} U_{1,k}(\alpha, \beta) &= p_k u_1(\alpha_k, \beta_I) + (1 - p_k) u_1(\alpha_k, \beta_{II}), \text{ for } k \in \{I, II\}, \text{ and,} \\ U_{2,k}(\alpha, \beta) &= q_k u_2(\alpha_I, \beta_k) + (1 - q_k) u_2(\alpha_{II}, \beta_k), \text{ for } k \in \{I, II\}. \end{aligned}$$

We refer to such a game as a *game with perceptions* and denote it by a pair (G, B) . Note that the underlying game is always G regardless of the information structure of the game. There is no uncertainty regarding available actions or payoffs: the only source of uncertainty lies in player's mutual perceptions and consequent beliefs.

⁶Aumann and Gul discuss the issue in *Econometrica* in [5, 17].

In this set-up we refer to a randomized strategy vector α , as simply *strategy*. We say that α is a *discriminatory* strategy if $\alpha_I \neq \alpha_{II}$, otherwise, we say that it is a *non discriminatory* strategy. A strategy profile (α, β) is a *discriminatory* profile if both α and β are discriminatory, and a *non discriminatory* profile if both α and β are non discriminatory.

4 Equilibria and Discrimination in Games with Perceptions

We next attempt to characterize the set of Bayesian Nash equilibria⁷ (henceforth referred to simply as equilibria or equilibrium in the singular) of games with incomplete information such as those described above and, more precisely, to study whether situations in which players *discriminate*, *i.e.* play different strategies depending on their perception, can occur in equilibrium. We begin with some elementary aspects of Bayesian Nash equilibria of a game with perceptions:

Proposition 1 *Let (G, B) be a game with perceptions and (α, β) a strategy profile. Then:*

1. *If (α, β) is non discriminatory, *i.e.* $(\alpha, \beta) = ((\alpha, \alpha), (\beta, \beta))$, then (α, β) is an equilibrium if and only if (α, β) is a Nash equilibrium of G .*
2. *If (α, β) is an equilibrium, then it is either discriminatory or non discriminatory.*
3. *If (α, β) is a discriminatory equilibrium, then at least one of the components of both α and β is an action.*

Part 1 of Proposition 1 characterizes the set of non discriminatory equilibria of a game with perceptions which not surprisingly, correspond with the set of Nash equilibria of the underlying game G . Parts 2 and 3 allow us to discard candidates for inclusion in the set of equilibria, namely strategy profiles in which only one player discriminates and those consisting solely of totally mixed strategies can be excluded. As it is shown in the Appendix Part 3 of this proposition greatly simplifies the issue regarding the existence of discriminatory equilibria, which is presented as our main result:

Theorem 1 *Let (G, B) be a game with perceptions. Then:*

1. *If G is a dominant solvable game, (G, B) has no discriminatory equilibria.*

⁷It should be mentioned that, as observed by Aumann in [3], that in the particular case in which beliefs can be derived from some common prior distribution, the distributions over action profiles induced by Bayesian Nash equilibria are correlated equilibria of the underlying game G , and conversely, every correlated equilibrium of game G corresponds to an equilibrium of some game (G, B) . For the general case with no common priors, the reader is referred to Brandenburger and Dekel [7].

2. If G is a coordination game, (G, B) has discriminatory equilibria if and only if beliefs are either concordant contagious or concordant antagonistic.
3. If G is a strictly competitive game, (G, B) has discriminatory equilibria if and only if beliefs are discordant.

Now, if G is a coordination or strictly competitive game, it may happen that the set of equilibria is infinite if an element of B equals a player's strategy at the Nash equilibrium in mixed strategies of G . In that case the set of equilibria can be uniquely partitioned into maximal connected subsets that may be a vertex or a line segment. The following corollary, whose proof we omit, follows straightforwardly from the tables in the proof of Theorem 1:

Corollary 1 *Let (G, B) be a game with perceptions. The number of elements of the partition of the set of equilibria in maximal connected subsets is 0, 2, 4 or 6, with at most 2 components that are line segments.*

A complete characterization of the set of equilibria for games with perceptions is provided in the tables in the Appendix, and a close look at them reveals the relationship between beliefs and discriminatory equilibria. Consider antagonistic concordant beliefs: $(p_I - p_{II}), (q_I - q_{II}) < 0$ and a coordination game with positive payoff parameters. Discriminatory equilibria satisfy the requirement that $(\alpha_I - \alpha_{II}) < 0 \iff (\beta_I - \beta_{II}) > 0$. That is, in equilibrium players discriminate in *opposite* ways. However, for a coordination game with negative payoff parameters⁸ $(\alpha_I - \alpha_{II}) > 0 \iff (\beta_I - \beta_{II}) > 0$ is obtained, so players discriminate in the *same* way. Opposite results are derived when players have contagious concordant beliefs. In the case of discordant beliefs and strictly competitive games is not so straightforward: players discriminate in the *same* way if both $(a_1 + a_2)(p_I - p_{II}) > 0$ and $(b_1 + b_2)(q_I - q_{II}) > 0$ hold, but they discriminate in *opposite* manners if $(a_1 + a_2)(p_I - p_{II}) < 0$ and $(b_1 + b_2)(q_I - q_{II}) < 0$.

Relying on our results and following the tables in the appendix an accurate answer to the example in the introduction can be offered.

Example 1 *Let Alice and Bob each focus on whether the other looks trustworthy or not, a characteristic inducing contagious concordant beliefs. Let $((p_I, p_{II}), (q_I, q_{II})) = ((9/10, 8/10), (15/16, 12/16))$ be their beliefs. Then it is found that $((\alpha_I, \alpha_{II}), (\beta_I, \beta_{II})) = ((1, 0), (1, 0))$ is an equilibrium. Hence, both players play c if their opponent looks trustworthy to them and d otherwise. The rest of equilibria are: $((13/15, 1), (31/36, 1)), ((14/15, 0), (35/36, 0))$ and $((1, 1/2), (1, 3/8))$.*

⁸This game is also called an anti-coordination game.

Finally, since concordant beliefs are not necessarily derived from a common prior, this usual assumption is not a necessary condition for the existence of discriminatory equilibria. In particular, for coordination games the results obtained without a common prior are no less sharp than those obtained assuming a common prior (this can be easily deduced from the characterization of generic discriminatory equilibria in the Appendix). Given that a common prior is incompatible with discordant beliefs, for strictly competitive games this assumption excludes the possibility of discriminatory equilibria.

5 Concluding remarks

Situations suitable for modeling as games rarely arise in real life between agents who do not see each other. Hence we believe that the inclusion of perceptions in normal form games adds realism to the study of the strategic interaction between players. In this regard Aumann and Drèze [6, p. 72] state: "A player i 's actions in a game are determined by her beliefs about other players; these depend on the game's real-life context, not only its formal description".

Our approach distinguishes between contagious and antagonistic perceptions, which are respectively sparked by emotions and personality traits recognized in the others. We would now like to quote two works that confirm the relevance of these characteristics in decision making:

- (i) Bechara and Damasio [8, p. 352]: "There are exceptions, i.e., a few theories that addressed emotion as a factor in decision making (Janis and Mann [19] or Mann [21]), however, they address emotions that are the consequence of some decision (e.g., the disappointment or regret experienced after some risky decision that worked out badly), rather than the affective reactions arising directly from the decision itself at the time of deliberation".
- (ii) Young [33, pp. 28–29]: "... "the achievement of determinate solutions for two person, non-zero-sum games through the estimation of subjective probabilities requires the introduction of an assumption to the effect that the individual employs some specified rules of thumb in assigning probabilities to the choices of the other player. But this is not a very satisfactory position to adopt within the framework of the theory of games. Logically speaking, there is an infinite variety of rules of thumb that could be used in assigning subjective probabilities, the game theory offers no persuasive reason to select anyone of these rules over the others. This problem can be handled by introducing new assumptions (or empirical premises) about such things as the personality traits of the players".

The evidence mentioned in the introduction about the information value of perceptions induced by emotions and personality traits when a strategic interaction takes place suggests that

they may be categorized as contagious and antagonistic. This has led us to an analysis of games with perceptions which has concluded with some results about players' discriminatory behavior.

Two implications of our work are worth mentioning in particular: *(i)* substantial experimental work on game theory analyzes whether results in the lab coincide with the equilibrium outcomes predicted by theoretical analysis. Our results support the idea that if subjects interact in the lab without keeping their identities confidential the results may be affected. *(ii)* Discrimination is usually understood as an asymmetric phenomenon. However in this setting players' behavior is never asymmetric: either both players discriminate or neither does.

Our approach follows to a certain extent that of Cass and Shell in [9]: introducing an external parameter in a model that in principle does not appear as relevant becomes essential and provides non standard solutions. In our study perception, a subjective parameter which does not affect the payoff matrix, may be considered an irrelevant external parameter, but it manages to generate discriminatory equilibria.

In conclusion we present some other points regarding our analytical framework. We have restricted ourselves to 2×2 games, which are an archetype for strategic interaction, and to dichotomous characteristics that generate two types for each player. More types would require players to be sharper in their ability to process characteristics, but it is hard to believe that this could occur when subjects are only briefly in contact with each other. Moreover we conjecture that adding more types would generate a greater number of discriminatory equilibria but this would not change the essence of our results.

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A Proofs

First recall that if for player 1 we define (we denote g for player 2), for any $\beta \in [0, 1]$:

$$f(\beta) = (a_1 + a_2)\beta - a_2.$$

Then, the *best reply correspondence* for player 1 in game G is:

$$br_1(\beta) = \begin{cases} \{0\} & \text{if } f(\beta) < 0, \\ [0, 1] & \text{if } f(\beta) = 0, \\ \{1\} & \text{if } f(\beta) > 0. \end{cases}$$

Now, if we look for a similar notion for game (G, B) , again, for player 1 we define (and analogously for player 2 we define g_I and g_{II}), for $\beta \in [0, 1]^2$ and $k = I, II$:

$$f_k(\beta) = f(p_k\beta_I + (1 - p_k)\beta_{II}).$$

Then, the best reply correspondence for player 1 (the one for player 2 is similar) is:

$$BR_1(\beta) = \begin{cases} [0, 1] \times [0, 1] & \text{if } f_I(\beta) = 0, \text{ and } f_{II}(\beta) = 0, \\ \{(0, y) \mid y \in [0, 1]\} & \text{if } f_I(\beta) < 0, \text{ and } f_{II}(\beta) = 0, \\ \{(1, y) \mid y \in [0, 1]\} & \text{if } f_I(\beta) > 0, \text{ and } f_{II}(\beta) = 0, \\ \{(x, 0) \mid x \in [0, 1]\} & \text{if } f_I(\beta) = 0, \text{ and } f_{II}(\beta) < 0, \\ \{(x, 1) \mid x \in [0, 1]\} & \text{if } f_I(\beta) = 0, \text{ and } f_{II}(\beta) > 0, \\ \{(0, 1)\} & \text{if } f_I(\beta) < 0, \text{ and } f_{II}(\beta) > 0, \\ \{(1, 0)\} & \text{if } f_I(\beta) > 0, \text{ and } f_{II}(\beta) < 0, \\ \{(1, 1)\} & \text{if } f_I(\beta) > 0, \text{ and } f_{II}(\beta) > 0, \\ \{(0, 0)\} & \text{if } f_I(\beta) < 0, \text{ and } f_{II}(\beta) > 0. \end{cases}$$

We say that a strategy profile (α, β) is an *equilibrium* of (G, B) if:

$$(\alpha, \beta) \in BR_1(\beta) \times BR_2(\alpha).$$

A.1 Basic and Auxiliary Results

Proof of Proposition 1.

1. For player 1, note that since for any $\beta \in [0, 1]$:

$$f(\beta) = f_I(\beta, \beta) = f_{II}(\beta, \beta),$$

then:

$$\alpha \text{ is a best reply in } G \text{ to } \beta \iff (\alpha, \alpha) \in BR_A(\beta, \beta).$$

The part corresponding to player 2 is analogous.

2. Let $((\alpha_I, \alpha_{II}), (\beta_I, \beta_{II}))$ be an equilibrium of (G, B) . Proceed by contradiction and assume without loss of generality that player 1 discriminates and player 2 does not. Then $\beta_I = \beta_{II} = \beta$. And:

$$f_I(\beta, \beta) = f_{II}(\beta, \beta),$$

and in consequence, $\alpha_I \neq \alpha_{II}$ implies that:

$$f_I(\beta, \beta) = f_{II}(\beta, \beta) = 0,$$

and thus, $\beta \in (0, 1)$.⁹ But this is not possible, since g is bijective, and $\alpha_I \neq \alpha_{II}$ implies that $q_I \alpha_I + (1 - q_I) \alpha_{II} \neq q_{II} \alpha_I + (1 - q_{II}) \alpha_{II}$.

3. Let $((\alpha_I, \alpha_{II}), (\beta_I, \beta_{II}))$ be a discriminatory equilibrium of (G, B) . Proceed by contradiction and assume without loss of generality that $\alpha_I, \alpha_{II} \in (0, 1)$. Then, it must be true that

$$f_I(\beta_I, \beta_{II}) = f_{II}(\beta_I, \beta_{II}) = 0,$$

thus:

$$p_I \beta_I + (1 - p_I) \beta_{II} = p_{II} \beta_I + (1 - p_{II}) \beta_{II},$$

and in consequence, $\beta_I = \beta_{II}$, which is a contradiction.

■

Before moving onto the proof of Theorem 1, we need to present and prove the following technical result:

Lemma 2 *Let (G, B) be a game with perceptions. If $(a_1 + a_2)(b_1 + b_2)(p_I - p_{II})(q_I - q_{II}) \leq 0$, (G, B) has no discriminatory equilibria.*

Proof. Proceed by contradiction and assume that (G, B) has one discriminative equilibrium, namely $((\alpha_I, \alpha_{II}), (\beta_I, \beta_{II}))$. Note that, due to Proposition 1:

$$\alpha_I > \alpha_{II} \implies f_I(\beta_I, \beta_{II}) - f_{II}(\beta_I, \beta_{II}) > 0,$$

and in consequence:

$$(a_1 + a_2)(p_I - p_{II})(\beta_I - \beta_{II}) > 0.$$

Similarly:

$$\alpha_I < \alpha_{II} \implies (a_1 + a_2)(p_I - p_{II})(\beta_I - \beta_{II}) < 0,$$

$$\beta_I > \beta_{II} \implies (b_1 + b_2)(q_I - q_{II})(\alpha_I - \alpha_{II}) > 0,$$

$$\beta_I < \beta_{II} \implies (b_1 + b_2)(q_I - q_{II})(\alpha_I - \alpha_{II}) < 0.$$

⁹It might be the case, depending on the parameters of the payoff matrix, that this is already a contradiction.

Now assume for instance, that our equilibrium verifies $\alpha_I < \alpha_{II}$ and $\beta_I > \beta_{II}$. Then by the implications above we have both that $(a_1 + a_2)(p_I - p_{II}) < 0$, and $(b_1 + b_2)(q_I - q_{II}) < 0$, so:

$$(a_1 + a_2)(b_1 + b_2)(p_I - p_{II})(q_I - q_{II}) > 0,$$

which is a contradiction. It can immediately be checked that for any other combination the result remains constant. ■

A.2 Main Result

Check first that it is possible, with no loss of generality, to complete the proof for any G which is not dominant solvable, and any B , assuming that both $a_1, a_2 > 0$ and $p_I < p_{II}$ hold. This requires two remarks:

1. If $a_1, a_2 < 0$, it is possible, regardless of B , to nominally interchange player 1's actions and calculate the *diagonal* representation of the *new* game, G^* :

$$\begin{pmatrix} 0, 0 & a_2, b_2 \\ a_1, b_1 & 0, 0 \end{pmatrix} \rightarrow \begin{pmatrix} -a_1, -b_2 & 0, 0 \\ 0, 0 & -a_2, -b_1 \end{pmatrix} = \begin{pmatrix} a_1^*, b_1^* & 0, 0 \\ 0, 0 & a_2^*, b_2^* \end{pmatrix}.$$

It is immediately apparent that $a_1^*, a_2^* > 0$, and that $((\alpha_I, \alpha_{II}), (\beta_I, \beta_{II}))$ is an equilibrium of (G, B) if and only if $((1 - \alpha_I, 1 - \alpha_{II}), (\beta_I, \beta_{II}))$ is an equilibrium of (G^*, B) . So if it is possible to compute the equilibria of the latter, the equilibria of the former can easily be computed too.

2. $p_I > p_{II}$, it is possible, regardless of G , to nominally interchange player 2's types and define $A_I^* = A_{II}$ and $A_{II}^* = A_I$. Note that this change implies new beliefs $B^* = ((p_I^*, p_{II}^*), ((q_I^*, q_{II}^*))) = ((1 - p_I, 1 - p_{II}), ((q_{II}, q_I)))$, where obviously, $p_I^* < p_{II}^*$. It is immediately apparent that $((\alpha_I, \alpha_{II}), (\beta_I, \beta_{II}))$ is an equilibrium of (G, B) if and only if $((\alpha_I, \alpha_{II}), (\beta_{II}, \beta_I))$ is an equilibrium of (G, B^*) , so again, if it is possible to compute the equilibria of the latter, the equilibria of the former can easily be computed too.

Proof of Theorem 1. We begin with the part regarding non existence of discriminatory equilibria:

- If G is a dominant solvable game and it is player 1 who has a dominant strategy, then $f(\alpha) > 0$ or $f(\alpha) < 0$ for any $\beta \in [0, 1]$. Thus, a best reply of player 1 can never be discriminatory. Then, by Proposition 1, there are no discriminatory equilibria.
- If G is not dominant solvable, note that in either of the remaining cases it holds that:

$$(a_1 + a_2)(b_1 + b_2)(p_I - p_{II})(q_I - q_{II}) \leq 0,$$

and apply Lemma 2.

Now move on to the part regarding existence of discriminatory equilibria. As seen at the beginning of the paragraph, it suffices to complete the proof for the case in which $a_1, a_2 > 0$, and $p_I < p_{II}$. Lemma 2 means that only the following two cases need to be considered:

- $a_1, a_2, b_1, b_2 > 0$ and $p_I < p_{II}$, $q_I < q_{II}$ and,
- $a_1, a_2 > 0, b_1, b_2 < 0$ and $p_I < p_{II}$, $q_I > q_{II}$.

Now we present the tables that include all the possible discriminatory equilibria and the corresponding conditions for their existence (any other possible strategy profile not included in the tables is discarded merely by applying 1.

1. We begin with case $a_1, a_2, b_1, b_2 > 0$ and $p_I < p_{II}$, $q_I < q_{II}$:

- **Discriminatory equilibria in pure strategies:**

Equilibria	Conditions
$\{(1, 0), (0, 1)\}$	$1 - p_{II} < a < 1 - p_I; q_I < b < q_{II}$
$\{(0, 1), (1, 0)\}$	$p_I < a < p_{II}; 1 - q_{II} < b < 1 - q_I$

- **Generic discriminatory equilibria in pure/mixed strategies:**

Equilibrium	Conditions
$\left(\left(0, \frac{b}{1-q_I} \right), \left(\frac{a}{p_{II}}, 0 \right) \right)$	$a < p_{II}; b < 1 - q_I$
$\left(\left(0, \frac{b}{1-q_{II}} \right), \left(1, \frac{a-p_{II}}{1-p_{II}} \right) \right)$	$a > p_{II}; b < 1 - q_{II}$
$\left(\left(\frac{b-(1-q_{II})}{q_{II}}, 1 \right), \left(1, \frac{a-p_I}{1-p_I} \right) \right)$	$a > p_I; b > 1 - q_{II}$
$\left(\left(\frac{b-(1-q_I)}{q_I}, 1 \right), \left(\frac{a}{p_I}, 0 \right) \right)$	$a < p_I; b > 1 - q_I$
$\left(\left(\frac{b}{q_{II}}, 0 \right), \left(0, \frac{a}{1-p_I} \right) \right)$	$a < 1 - p_I; b < q_{II}$
$\left(\left(\frac{b}{q_I}, 0 \right), \left(\frac{a-(1-p_I)}{p_I}, 1 \right) \right)$	$a > 1 - p_I; b < q_I$
$\left(\left(1, \frac{b-q_I}{1-q_I} \right), \left(\frac{a-(1-p_{II})}{p_{II}}, 1 \right) \right)$	$a > 1 - p_{II}; b > q_I$
$\left(\left(1, \frac{b-q_{II}}{1-q_{II}} \right), \left(0, \frac{a}{1-p_{II}} \right) \right)$	$a < 1 - p_{II}; b > q_{II}$

- **Non-generic discriminatory equilibria in pure/mixed strategies:**

Equilibria	Interval	Conditions
$((0, 1), (\beta, 0))$	$\left(\frac{a}{p_{II}}, \frac{a}{p_I}\right)$	$a < p_{II}; b = 1 - q_I$
$((0, \alpha), (1, 0))$	$\left(\frac{b}{1-q_I}, \frac{b}{1-q_{II}}\right)$	$a = p_{II}; b < 1 - q_I$
$((\alpha, 1), (1, 0))$	$\left(\frac{b-(1-q_I)}{q_I}, \frac{b-(1-q_{II})}{q_{II}}\right)$	$a = p_I; b > 1 - q_{II}$
$((0, 1), (1, \beta))$	$\left(\frac{a-p_{II}}{1-p_{II}}, \frac{a-p_I}{1-p_I}\right)$	$a > p_I; b = 1 - q_{II}$
$((\alpha, 0), (0, 1))$	$\left(\frac{b}{q_{II}}, \frac{b}{q_I}\right)$	$a = 1 - p_I; b < q_{II}$
$((1, 0), (0, \beta))$	$\left(\frac{a}{1-p_I}, \frac{a}{1-p_{II}}\right)$	$a < 1 - p_I; b = q_{II}$
$((1, 0), (\beta, 1))$	$\left(\frac{a-(1-p_I)}{p_I}, \frac{a-(1-p_{II})}{p_{II}}\right)$	$a > 1 - p_{II}; b = q_I$
$((1, \alpha), (0, 1))$	$\left(\frac{b-q_{II}}{1-q_{II}}, \frac{b-q_I}{1-q_I}\right)$	$a = 1 - p_{II}; b > q_I$

Where the interval represents lower and upper bounds for the mixed strategy that one of the players chooses.

It can be checked that the conditions for existence are indeed exhaustive, *i.e.* at least the conditions for existence of two different equilibria hold for any given values of a, b, p_I, p_{II}, q_I and q_{II} .

2. We go on with case $a_1, a_2 > 0, b_1, b_2 < 0$ and $p_I < p_{II}, q_I > q_{II}$:

• **Discriminatory equilibria in pure strategies:**

Equilibria	Conditions
$\{((1, 0), (0, 1))\}$	$1 - p_{II} < a < 1 - p_I; q_{II} < b < q_I$
$\{((0, 1), (1, 0))\}$	$p_I < a < p_{II}; 1 - q_I < b < 1 - q_{II}$

• **Generic discriminatory equilibria in pure/mixed strategies:**

Equilibrium	Conditions
$\left(\left(0, \frac{b}{1-q_I}\right), \left(\frac{a}{p_{II}}, 0\right)\right)$	$a < p_{II}; b < 1 - q_I$
$\left(\left(0, \frac{b}{1-q_{II}}\right), \left(1, \frac{a-p_{II}}{1-p_{II}}\right)\right)$	$a > p_{II}; b < 1 - q_{II}$
$\left(\left(\frac{b-(1-q_{II})}{q_{II}}, 1\right), \left(1, \frac{a-p_I}{1-p_I}\right)\right)$	$a > p_I; b > 1 - q_{II}$
$\left(\left(\frac{b-(1-q_I)}{q_I}, 1\right), \left(\frac{a}{p_I}, 0\right)\right)$	$a < p_I; b > 1 - q_I$
$\left(\left(\frac{b}{q_{II}}, 0\right), \left(0, \frac{a}{1-p_I}\right)\right)$	$a < 1 - p_I; b < q_{II}$
$\left(\left(\frac{b}{q_I}, 0\right), \left(\frac{a-(1-p_I)}{p_I}, 1\right)\right)$	$a > 1 - p_I; b < q_I$
$\left(\left(1, \frac{b-q_I}{1-q_I}\right), \left(\frac{b-(1-p_{II})}{p_{II}}, 1\right)\right)$	$a > 1 - p_{II}; b > q_I$
$\left(\left(1, \frac{b-q_{II}}{1-q_{II}}\right), \left(0, \frac{a}{1-p_{II}}\right)\right)$	$a < 1 - p_{II}; b > q_{II}$

• **Non-generic discriminatory equilibria in pure/mixed strategies:**

Equilibria	Interval	Conditions
$((0, 1), (\beta, 0))$	$\left(\frac{a}{p_{II}}, \frac{a}{p_I}\right)$	$a < p_{II}; b = 1 - q_I$
$((0, \alpha), (1, 0))$	$\left(\frac{b}{1-q_{II}}, \frac{b}{1-q_I}\right)$	$a = p_{II}; b < 1 - q_{II}$
$((\alpha, 1), (1, 0))$	$\left(\frac{b-(1-q_{II})}{q_{II}}, \frac{b-(1-q_I)}{q_I}\right)$	$a = p_I; b > 1 - q_I$
$((0, 1), (1, \beta))$	$\left(\frac{a-p_{II}}{1-p_{II}}, \frac{a-p_I}{1-p_I}\right)$	$a > p_I; b = 1 - q_{II}$
$((\alpha, 0), (0, 1))$	$\left(\frac{b}{q_I}, \frac{b}{q_{II}}\right)$	$a = 1 - p_I; b < q_I$
$((1, 0), (0, \beta))$	$\left(\frac{a}{1-p_I}, \frac{a}{1-p_{II}}\right)$	$a < 1 - p_I; b = q_{II}$
$((1, 0), (\beta, 1))$	$\left(\frac{a-(1-p_I)}{p_I}, \frac{a-(1-p_{II})}{p_{II}}\right)$	$a > 1 - p_{II}; b = q_I$
$((1, \alpha), (0, 1))$	$\left(\frac{b-q_I}{1-q_I}, \frac{b-q_{II}}{1-q_{II}}\right)$	$a = 1 - p_{II}; b > q_{II}$

Where again, the interval represents lower and upper bounds for the mixed strategy one of the player chooses.

It can again be checked that the conditions for existence are again exhaustive, that is: at least the conditions of existence of two different equilibria hold for any given values of a, b, p_I, p_{II}, q_I and q_{II} .

■